Collective treatment of the isovector pair correlation. Boson representation.

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September 2 – 6, 2019

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The approach to consideration of the Hamiltonian with pairing forces using a technique of the finite boson representation is developed.
It is shown that a simultaneous description of the pairing vibrational state in ^{56}Ni and the pairing rotational states with T=0 in the neighboring $N = Z$ nuclei (A=60, 64) is possible if the pairing Hamiltonian takes into account only isovector monopole pairing.
The calculated energies of the pairing rotational states of $N = Z$ nuclei removed from ^{56}Ni by 12 and more nucleons exceed significantly the experimental values.

To clarify a relative role of the isoscalar pairing we consider on the same footing two types of collective states generated by pair correlation:
–pairing rotational and pairing vibrational states.
The energies of the states of the pairing rotational bands formed by the sequence of the $N=Z$ double magic nucleus $\pm n\alpha$-particles are determined by the corresponding inertia parameter. The expression for this parameter derived for the case when both isoscalar and isovector pair correlations are taken into account is

$$\Im_{pair} \sim \sum_{T,J} |\Delta_T^J|^2,$$

where $|\Delta_T^J|^2$ is a mean square value of the pairing correlation function for pairing interactions with isospin T and angular momentum J.

Moment of inertia for gauge rotations

$$z_\mu = \exp(-\imath\phi) \sum_\nu D^1_{\mu\nu} \Delta_\nu$$

$$T_{kin} = \frac{1}{2} B \sum_\mu |\dot{z}_\mu|^2$$

$$\dot{z}_\mu = \exp(-\imath\phi) \sum_\nu \dot{D}^1_{\mu\nu} \Delta_\nu$$

$$-\imath\dot{\phi} \exp(-\imath\phi) \sum_\nu D^1_{\mu\nu} \Delta_\nu + \exp(-\imath\phi) \sum_\nu D^1_{\mu\nu} \dot{\Delta}_\nu$$

$$\to -\imath\dot{\phi} \exp(-\imath\phi) \sum_\nu D^1_{\mu\nu} \Delta_\nu$$

$$T_{gauge} = \frac{1}{2} B \dot{\phi}^2 \sum_\mu |\Delta_\mu|^2$$

In contrast, a concrete pairing vibrational state is related to the concrete pairing vibrational mode: isovector with J=0 or isoscalar with different J. For instance, the well known pairing vibrational 0^+ states of ^{56}Ni with excitation energy E^*=5.004 MeV is excited in ^{58}Ni(p,t) and ^{54}Fe(^{3}He,n) reactions. It indicates that this state is generated by isovector monopole pairing.

The situation is similar to the one related to the ground state rotational band and nuclear shape vibrational states. The ground state moment of inertia contains effects of both quadrupole and octupole deformations. This is seen in behavior of the alternating pairing bands. However, quadrupole and octupole vibrational states can be distinguished by their electromagnetic transition properties.

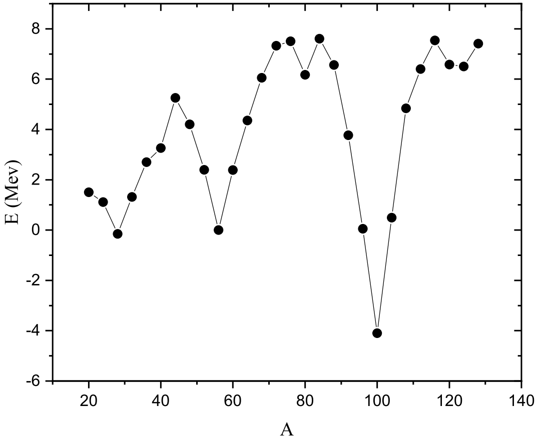
So, if it is impossible to describe both the energies of the pairing rotational and pairing vibrational excitations based on the Hamiltonian with isovector monopole pairing only, then we get an indication that other pairing modes should be taken into consideration. The opposite situation leave the problem open.

Comparison of the calculated results with the experimental data

The experimental energies are reduced to quantities which can be directly compared with the model predictions. For that, subtract those contributions which come from other sources than isovector monopole pairing.
(D.R.Bes, R.A.Broglia, O.Hansen, and O.Nathan, Phys.Rep. **34**, (1977) 1)

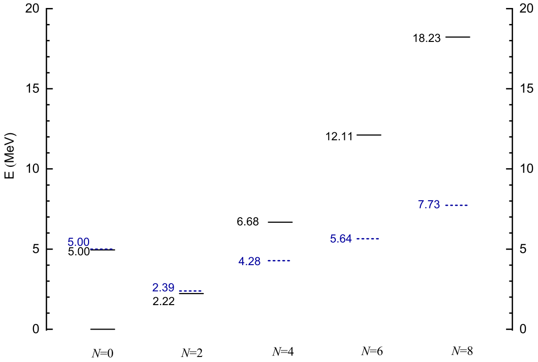
$$E(A,Z,i) = -\left(B_{exp}(A,Z,i) - B_{LD}(A,Z)\right) + \left(B_{exp}(A_0,Z_0,gs) - B_{LD}(A_0,Z_0)\right),$$

$$B_{LD}(A,Z) = 15.43A - 17A^{2/3} - 0.7Z^2\left(1 - \frac{0.76}{Z^{2/3}}\right)/A^{1/3}.$$



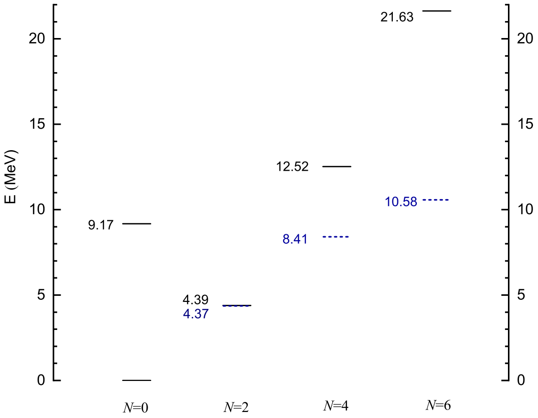
There are minima at ^{28}Si, ^{56}Ni, ^{80}Zr, ^{100}Sn. In the case of nuclei around ^{100}Sn the results of calculations are used instead of the experimental data.

Energies of the 0^+ T=0 states in ^{56}Ni



$G_1 = \frac{14.4}{A}$MeV is fixed to reproduce the energy of the pairing vibrational state in ^{56}Ni at 5 MeV. d=0.6 for ^{56}Ni

Energies of the 0^+ T=0 states in ^{100}Sn



d=0.29

- Pairing vibrational state in ^{32}S at 5.14 MeV is reproduced together with the ground state energy of ^{32}S, which is the energy of the first excited state of the pairing rotational band based on ^{28}Si.

The presented results indicate on the following problem. The calculated energies of the states of the pairing rotational band exceed significantly the corresponding experimental values when $N \geq 6$. This situation is similar to that discussed by Bohr and Mottelson in connection with the states of high angular momentum I in the long rotational bands. It was indicated that with increasing I the possible occurrence of closed shells in a selected region of I would lead to lowering of the yrast line and enhancement of stability.

Effect of the neighboring minima

In our approach the number of added or removed α-clusters plays the role of the angular momentum in gauge space. The neighboring minima of $E(A, Z, gs)$ are seen clearly. Starting from the values of N=6 and 8 determined relative to ^{56}Ni it becomes unclear whether these states should be considered as belonging to the family of pairing rotational states based on ^{56}Ni or they should be considered as pairing rotational states based on ^{28}Si or ^{80}Zr. The calculated value of the energy of the state with N=8, determined relative to ^{56}Ni, is 18.2 MeV. Let us consider this state as based on ^{80}Zr. As it is seen the ground state of ^{80}Zr is located at 6 MeV with respect to the ground state of ^{56}Ni. The state under consideration can be treated as belonging to the removal branch of the pairing rotational state with $N = 4$ determined with respect to ^{80}Zr. Its calculated excitation energy is equal to 5.2 MeV. Summing these two energies, we obtain the energy 11.2 MeV instead of 18.2 MeV, i.e. by 7 MeV closer to the experimental value, which is 7.7 MeV. This situation is a complete analog of the situation in the Shell Model or the Interacting Boson Model where the number of valence particles or bosons is related to the number of particles or holes depending on the degree of filling of the shell.

Possible influence of the noncollective modes

Let us note also, that influence of the neighboring minima on energies of pair rotational states can mean the following. The collective mode of excitation is defined for description of oscillations around a concrete minimum. Influence of the neighboring minima on the results of calculations can mean that the microscopic structure of the collective mode vary with N. It means that the microscopic structure of the collective mode has to be defined consistently for each N.

We restrict our consideration by the Hamiltonian with a constant pairing

$$H = H_0 + H_{int},$$

$$H_0 = \sum_{j,m,\tau} (E_j - \lambda) a^+_{jm\tau} a_{jm\tau},$$

$$H_{int} = - \sum_{J,M,T,\tau} G^J_T (A^{JM}_{T\tau})^+ A^{JM}_{T\tau}.$$

$$(A^{JM}_{T\tau})^+ = \sum_j \sqrt{j+1/2} (A^{JM}_{T\tau}(j))^+,$$

$$(A^{JM}_{T\tau}(j))^+ = \frac{1}{\sqrt{2}} \sum_{m,m',t,t'} C^{JM}_{jmjm'} C^{T\tau}_{1/2t1/2t'} a^+_{jmt} a^+_{jm't'}$$

$$a^+_{jmt} = \begin{cases} c^+_{jmt}, & j \in j_+, \\ (-1)^{j-m+1/2-t} c_{j-m-t}, & j \in j_- \end{cases}$$

$$c^+_{j_+mt} c^+_{j_+m't'} \to b^+_{mt,m't'}(j_+)$$
$$- \sum_{m_1 m_2 t_1 t_2} b^+_{mt,m_1t_1}(j_+) b^+_{m't',m_2t_2}(j_+) b_{m_1t_1,m_2t_2}(j_+),$$

$$c_{j_+m't'} c_{j_+mt} \to b_{mt,m't'}(j_+),$$

$$c^+_{j_-mt} c^+_{j_-m't'} \to b^+_{mt,m't'}(j_-),$$

$$c_{j_-m't'} c_{j_-mt} \to b_{mt,m't'}(j_-)$$
$$- \sum_{m_1 m_2 t_1 t_2} b^+_{m_1t_1,m_2t_2}(j_-) b_{m't',m_2t_2}(j_-) b_{mt,m_1t_1}(j_-).$$

$$c^+_{j_\pm mt} c_{j_\pm mt} \to 2 \sum_{m_1 t_1} b^+_{mt,m_1t_1}(j_\pm) b_{mt,m_1t_1}(j_\pm),$$

In practice, this does not lead to principle complexities, since the eigenvalues of the Hamiltonian are real if the Hamiltonian is diagonalized on the physical boson subspace and other approximations are not made. A transition to the boson representation keeping hermiticity relations in the usual boson metric can be realized using some nonunitary transformation $\hat{U}$ which keeps, however, boson commutation relations. The new boson representation looks like

$$(\hat{U})^{-1}\hat{O}_D\hat{U}, \tag{1}$$

where the operator $\hat{U}$ is related to the metric operator $\hat{F}$

$$\hat{U} = (\hat{F})^{1/2}. \tag{2}$$

Here the operator $\hat{F}$ determines the new boson metric

$$\langle\Phi_\beta, \Phi_\alpha\rangle = \left(\Phi_\beta, \hat{F}\Phi_\alpha\right) \tag{3}$$

so that the hermiticity relation

$$\langle\Phi_\beta, (\hat{O})^+_D\Phi_\alpha\rangle = \langle\hat{O}_D\Phi_\beta, \Phi_\alpha\rangle \tag{4}$$

is kept. Above $\hat{O}_D$ is a Dyson type boson image of bifermion operator.

$$H = \sum_{j_+} 2(E_{j_+} - \lambda) \sum_{JMT\tau} b^{+JM}_{T\tau}(j_+) b^{JM}_{T\tau}(j_+)$$
$$+ \sum_{j_-} 2(\lambda - E_{j_-}) \sum_{JMT\tau} b^{+JM}_{T\tau}(j_-) b^{JM}_{T\tau}(j_-)$$
$$- \sum_{JMT\tau} G^J_T \left(\sum_{j_+} \sqrt{j_+ + 1/2} b^{+JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2} \tilde{b}^{JM}_{T\tau}(j_-) \right)$$
$$\times \left(\sum_{j'_+} \sqrt{j'_+ + 1/2} b^{JM}_{T\tau}(j'_+) + \sum_{j'_-} \sqrt{j'_- + 1/2} \tilde{b}^{+JM}_{T\tau}(j'_-) \right)$$
$$+2 \sum_{JMT\tau} G^J_T \left(F^{JM}_{T\tau}(+) + F^{JM}_{T\tau}(-) \right)$$
$$\times \left(\sum_{j_+} \sqrt{j_+ + 1/2} b^{JM}_{T\tau}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2} \tilde{b}^{+JM}_{T\tau}(j_-) \right)$$

$$F^{JM}_{T\tau}(+) = \sum \sim \left((b^{+J_1}_{T_1}(j_+) b^{+J_2}_{T_2}(j_+))^{J'}_{T'} \ \tilde{b}^{J_3}_{T_3}(j_+) \right)^{JM}_{T\tau},$$

$$F^{JM}_{T\tau}(-) = \sum \sim \left(b^{+J_3}_{T_3}(j_-) \ (\tilde{b}^{J_1}_{T_1}(j_-) \tilde{b}^{J_2}_{T_2}(j_-))^{J'}_{T'} \ \right)^{JM}_{T\tau}$$

Introduction of the collective mode

$$z_{1\tau}^{+} = \sum_{j_+} w_{j_+} b_{1\tau}^{+00}(j_+) + \sum_{j_-} w_{j_-} \tilde{b}_{1\tau}^{00}(j_-),$$

$$p_{1\tau}^{+} = -i \left(\sum_{j_+} v_{j_+} b_{1\tau}^{00}(j_+) - \sum_{j_-} v_{j_-} \tilde{b}_{1\tau}^{+00}(j_-) \right).$$

Harmonic part of the collective Hamiltonian

$$H_2 = \sum_\tau \left(\omega_+ b_{1\tau}^{+00}(+) b_{1\tau}^{00}(+) + \omega_- b_{1\tau}^{+00}(-) b_{1\tau}^{00}(-)\right)$$

$$H_2 = \frac{1}{2B}\sum_\tau p_{1\tau}^+ p_{1\tau} - \frac{1}{2} C \sum_\tau z_{1\tau}^+ z_{1\tau}.$$

Calculation of the parameters of the collective Hamiltonian

$$\frac{1}{G} = \sum_{j_+} (j_+ + 1/2) \frac{2(E_{j_+} - \lambda)}{[2(E_{j_+} - \lambda)]^2 + C/4B} + \sum_{j_-} (j_- + 1/2) \frac{2(\lambda - E_{j_-})}{[2(\lambda - E_{j_-})]^2 + C/4B},$$

$$0 = \sum_{j_+} (j_+ + 1/2) \frac{1}{[2(E_{j_+} - \lambda)]^2 + C/4B} - \sum_{j_-} (j_- + 1/2) \frac{1}{[2(\lambda - E_{j_-})]^2 + C/4B}.$$

Amplitudes determining microscopic structure of the collective mode

$$w_{j_+} = G\sqrt{j_+ + 1/2}\ \frac{W2(E_{j_+} - \lambda) + V/2B}{[2(E_{j_+} - \lambda)]^2 + \gamma},$$

$$w_{j_-} = G\sqrt{j_- + 1/2}\ \frac{-W2(\lambda - E_{j_-}) + V/2B}{[2(\lambda - E_{j_-})]^2 + \gamma},$$

$$v_{j_+} = G\sqrt{j_+ + 1/2}\ \frac{-WC/2 + V2(E_{j_+} - \lambda)}{[2(E_{j_+} - \lambda)]^2 + \gamma},$$

$$v_{j_-} = G\sqrt{j_- + 1/2}\ \frac{-WC/2 + V2(\lambda - E_{j_-})}{[2(\lambda - E_{j_-})]^2 + \gamma}$$

where

$$V = \frac{1}{G}\sqrt{\frac{B}{S_+^2 + S_-^2\gamma}\left(\sqrt{S_+^2 + S_-^2\gamma} + S_+\right)},$$

$$W = \frac{1}{G}\sqrt{\frac{1}{C}\frac{1}{S_+^2 + S_-^2\gamma}\left(\sqrt{S_+^2 + S_-^2\gamma} - S_+\right)}$$

$$S_+ = \sum_{j_+} (j_+ + 1/2) \frac{2(E_{j_+} - \lambda)}{([2(E_{j_+} - \lambda)]^2 + \gamma)^2} + \sum_{j_-} (j_- + 1/2) \frac{2(\lambda - E_{j_-})}{([2(\lambda - E_{j_-})]^2 + \gamma)^2},$$

$$S_- = \sum_{j_+} (j_+ + 1/2) \frac{1}{([2(E_{j_+} - \lambda)]^2 + \gamma)^2} - \sum_{j_-} (j_- + 1/2) \frac{1}{([2(\lambda - E_{j_-})]^2 + \gamma)^2}.$$

$$W = \frac{1}{G} \sqrt{\frac{1}{C} \frac{1}{S_+^2 + S_-^2 \gamma} \left(\sqrt{S_+^2 + S_-^2 \gamma} - S_+ \right)},$$

Since the coefficient S_- is much smaller than S_+, we assume that $S_- = 0$. Then $W = 0$, and

$$w_{j_\pm} = \frac{1}{V}\tilde{w}_{j_\pm}, \quad \tilde{w}_{j_\pm} = \frac{1}{G_1^0 S_+} \; \frac{\sqrt{j_\pm + 1/2}}{[2(E_{j_\pm} - \lambda)]^2 + \gamma}, \tag{5}$$

$$v_{j_\pm} = V\tilde{v}_{j_\pm}, \quad \tilde{v}_{j_\pm} = G_1^0 \; \frac{\sqrt{j_\pm + 1/2}\cdot 2|E_{j_\pm} - \lambda|}{[2(E_{j_\pm} - \lambda)]^2 + \gamma}. \tag{6}$$

In this approximation

$$\sum_{j_+} \sqrt{j_+ + 1/2} b_{1\tau}^{00}(j_+) + \sum_{j_-} \sqrt{j_- + 1/2}\tilde{b}_{1\tau}^{+00}(j_-) = V z_{1\tau}. \tag{7}$$

By analogy with the assumption $S_- = 0$ we neglect below the following coefficients, that are also given by the differences of the sums over j_+ and j_-:

$$\sum_{j_+} \frac{\tilde{v}_{j_+} \tilde{w}_{j_+}^2}{\sqrt{j_+ + 1/2}} - \sum_{j_-} \frac{\tilde{v}_{j_-} \tilde{w}_{j_-}^2}{\sqrt{j_- + 1/2}} \approx 0,$$

$$\sum_{j_+} \frac{\tilde{v}_{j_+}^2 \tilde{w}_{j_+}}{\sqrt{j_+ + 1/2}} - \sum_{j_-} \frac{\tilde{v}_{j_-}^2 \tilde{w}_{j_-}}{\sqrt{j_- + 1/2}} \approx 0,$$

$$\sum_{j_+} \frac{\tilde{w}_{j_+}^3}{\sqrt{j_+ + 1/2}} - \sum_{j_-} \frac{\tilde{w}_{j_-}^3}{\sqrt{j_- + 1/2}} \approx 0.$$

Taking these approximation we assume, in fact, that the spectra of the collective states generated by the particle addition mode coincide with those generated by the particle removal mode.

Reduction to the two-level model

Approximation S-=0 and the other similar relations mean that the sums over j+ single particle states and those taken over j- states are approximately equal to each other. In all cases considered below the set of j- (j+) single particle states consists of one single particle level, whereas j+ (j-) set contains several particle levels. It means that with the approximation made above the problem is simplified effectively to a two-level model where both levels have the same degeneracy.

$$z^{+}_{1\mu} = \Delta \exp(-\imath\phi) \left(D^{1}_{\mu 0}(\psi_1,\psi_2,\psi_3)\cos\theta \right.$$
$$\left. + \frac{1}{\sqrt{2}} (D^{1}_{\mu 1}(\psi_1,\psi_2,\psi_3) + D^{1}_{\mu -1}(\psi_1,\psi_2,\psi_3))\sin\theta \right).$$
$$\hat{N} \equiv \frac{1}{2}(\hat{A} - A_0) = \imath \frac{\partial}{\partial\phi},$$

$$V = -G_1^2 \frac{C}{4B} S_{+} \Delta^2$$
$$+2G_1 \left(\sum_{j_+} \frac{\tilde{v}^3_{j_+}}{\sqrt{j_+ + 1/2}} + \sum_{j_-} \frac{\tilde{v}^3_{j_-}}{\sqrt{j_- + 1/2}} \right) \frac{1}{2} \left(1 - \frac{1}{2}\cos^2 2\theta \right) \Delta^4$$

The collective wave function is determined in the domain $0 \le \theta \le 2\pi$ once it is known in $0 \le \theta \le \frac{\pi}{4}$. All calculations involving θ can therefore be restricted to this smaller interval. In the interval $0 \le \theta \le \frac{\pi}{4}$ potential has a minimum at θ=0.

Hamiltonian in the intrinsic frame

We restrict the consideration by the states with $T=0$. Potential energy has a minimum at $\theta=0$. As a consequence:

- omit the isospin operators in the Hamiltonian;
- put $\theta=0$ in the potential and the inertia coefficient.

$$T_{kin} = \frac{1}{4G_1^2 S_+}\left(-\frac{1}{\Delta}\frac{\partial}{\partial\Delta}\Delta(1+t\Delta^2)\frac{\partial}{\partial\Delta} + \frac{(1+t\Delta^2)}{\Delta^2}\hat{N}^2\right),$$

$$V = -G_1^2 S_+ \frac{C}{4B}\Delta^2 + \frac{1}{2}G_1\left(\sum_{j_+}\frac{\tilde{v}_{j_+}^3}{\sqrt{j_+ + 1/2}} + \sum_{j_-}\frac{\tilde{v}_{j_-}^3}{\sqrt{j_- + 1/2}}\right)\frac{1}{2}\Delta^4$$

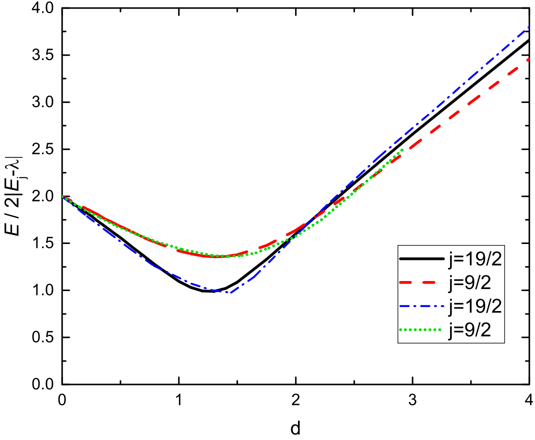
$$t = \frac{1}{2}(G_1^0)^3 S_+\left(\sum_{j_+}\frac{\tilde{v}_{j_+}\tilde{w}_{j_+}^2}{\sqrt{j_+ + 1/2}} + \sum_{j_-}\frac{\tilde{v}_{j_-}\tilde{w}_{j_-}^2}{\sqrt{j_- + 1/2}}\right).$$

$$\Delta = \frac{1}{\sqrt{t}} \sinh(x)$$

$$H = \frac{1}{8} G_1 \left(-\frac{\partial^2}{\partial x^2} + \frac{(1+\sinh^2 x) N^2}{\sinh^2 x} + \frac{(1 - 4\sinh^2 x - 4\sinh^4 x)}{4\sinh^2 x (1+\sinh^2 x)} \right.$$
$$\left. -16 \frac{(2j+1)^2}{d} \left(1 - \frac{1}{d}\right) \sinh^2 x + 16 \frac{(2j+1)^2}{d^2} \sinh^4 x \right),$$

Above $d = G_1(2j+1)/2|E_j - \lambda|$.

Comparison with the exact results for two-level model



Comparison of the results of calculations (solid and dashed lines) with the exact results (dash-dot and dot lines) for $j = 9/2$ and $j = 19/2$. T=0.

Conclusion

- The theoretical approach for a treatment of the Hamiltonian with the pairing forces using a technique of the finite boson representation of the bifermion operators is developed.
- Restricting a consideration by the isovector monopole pairing the excitation spectra of the pairing rotational and vibrational states with T=0 and $J^{\pi} = 0^{+}$ in nuclei around ^{56}Ni and ^{100}Sn are calculated. The calculations don't indicate on the necessity of introduction of the isoscalar pairing.
- It is seen from the results of calculations that the calculated energies of the states of the pairing rotational bands formed by the nuclear ground states significantly exceed the experimental values at large values of N. A possible reason of this can be an effect of the subshell closing in neighboring nuclei in which $E(A, Z, gs)$ has minima. In the case of ^{56}Ni it can be an effect of the subshell closing in ^{28}Si and ^{80}Zr. It is also possible that a microscopic structure of the collective mode depends on N and should be defined consistently with variation of N.