



DE LA RECHERCHE À L'INDUSTRIE

## Clustering & Quantum Phase Transitions

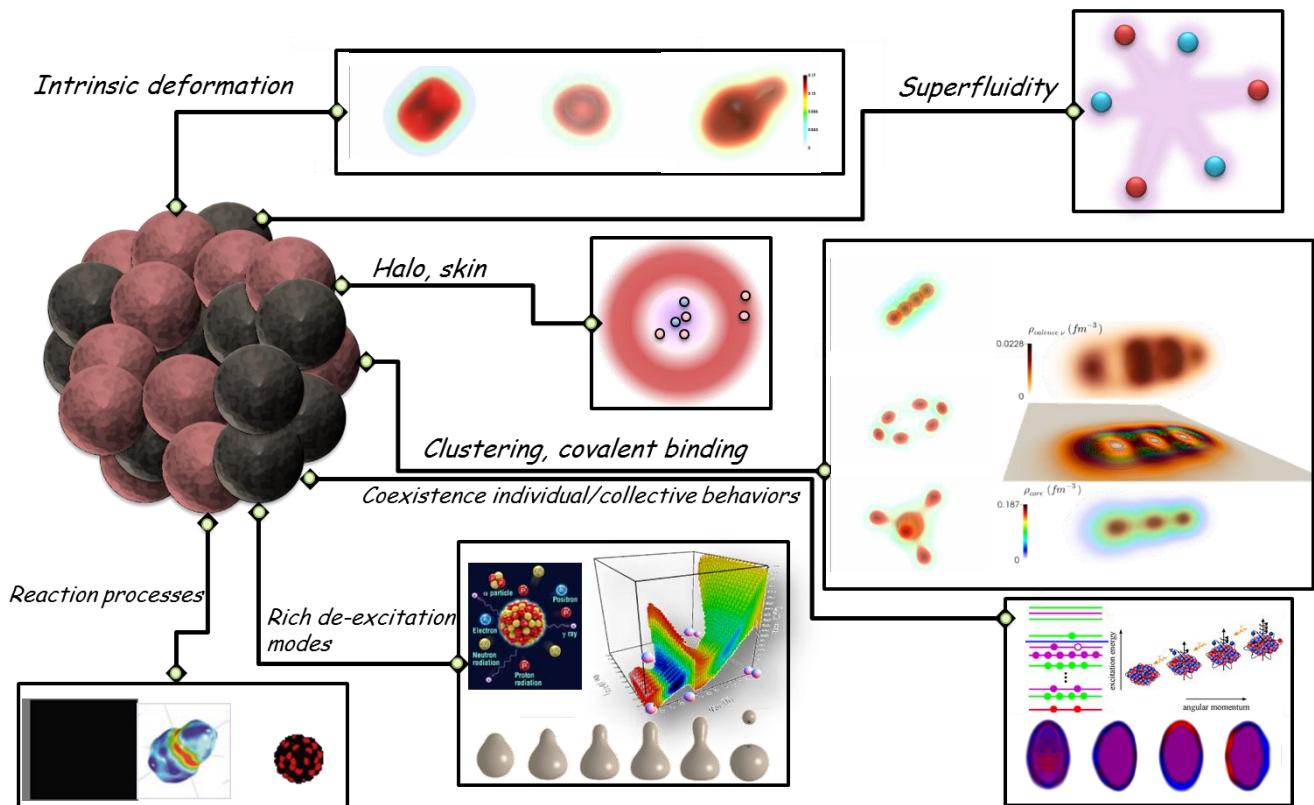
Jean-Paul EBRAN (CEA,DAM,DIF)



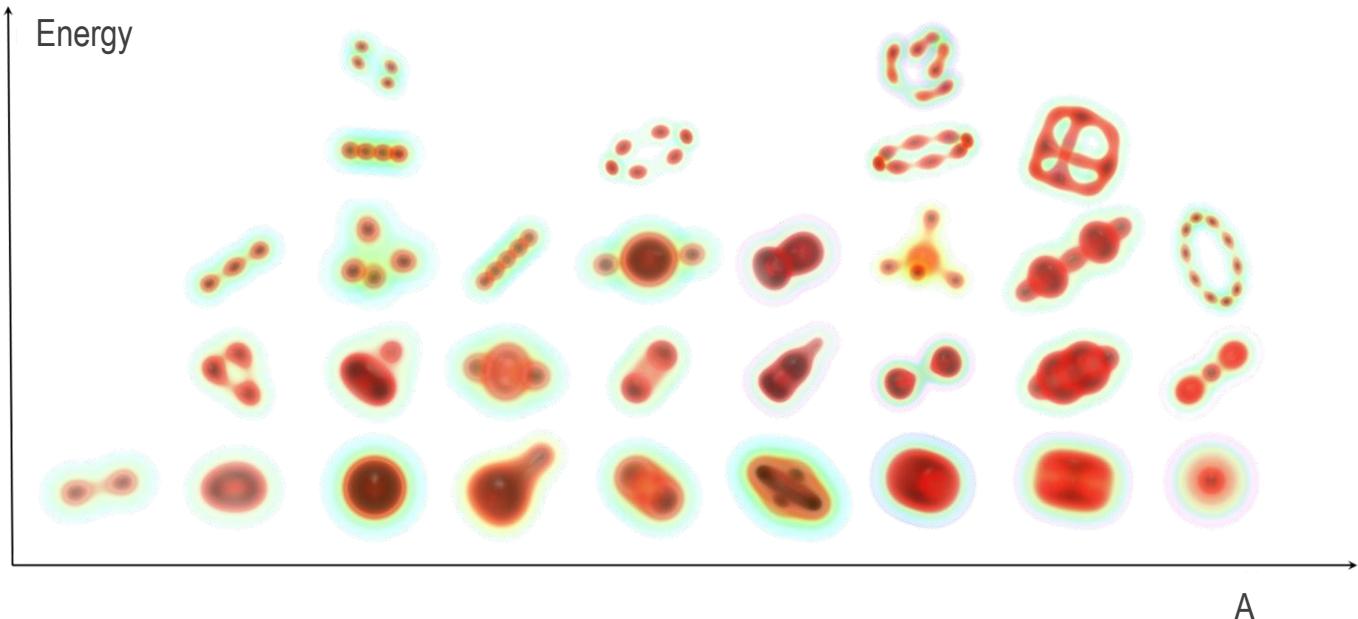
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## Clustering & Quantum Phase Transitions & Quartetting

Jean-Paul EBRAN (CEA,DAM,DIF)

**- HOW DO NUCLEONS SELF-ORGANIZE IN NUCLEI -**

- ★ Clustering = nucleons clumping together into sub-groups within the nucleus

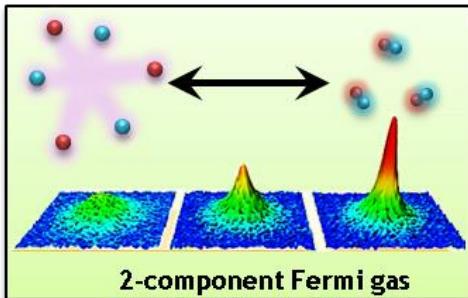


- ★ What triggers the occurrence of clusters?

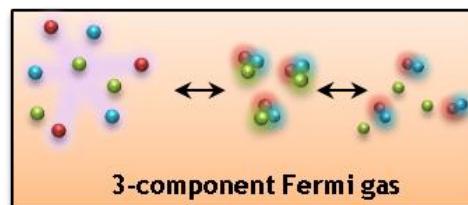
- ★ Nucleus = 4-component Fermi system



— *proton*  
— *neutron*



What about 4-component Fermi systems ?



- ★ Nucleus = 4-component Fermi system



— *proton*  
— *neutron*

- ★ Start with some Hamiltonian  $H = H_0 + \mathcal{V}_{\text{res}}$  with

$$H_0 = \int d^3r \sum_{\alpha} \varepsilon_{\alpha} \psi_{\alpha}^{\dagger}(\mathbf{r}) \psi_{\alpha}(\mathbf{r})$$

$$\mathcal{V}_{\text{res}} \sim V_{\text{pair}} = - \int d^3r \left[ g^{\text{T}=1} \sum_{\nu=\pm 1,0} P_{\nu}^{\dagger}(\mathbf{r}) P_{\nu}(\mathbf{r}) + g^{\text{T}=0} \sum_{\mu=\pm 1,0} Q_{\mu}^{\dagger}(\mathbf{r}) Q_{\mu}(\mathbf{r}) \right]$$

- ★ Correlated pair operators

$$P_{\nu}^{\dagger}(\mathbf{r}) \equiv \sqrt{\frac{1}{2}} \sum_l \sqrt{2l+1} \left\{ \psi_{l,s=\frac{1}{2},t=\frac{1}{2}}^{\dagger}(\mathbf{r}) \psi_{l,s=\frac{1}{2},t=\frac{1}{2}}^{\dagger}(\mathbf{r}) \right\}_{M_L=0,M_S=0,M_T=-\nu}^{(L=0,S=0,T=1)}$$

$$Q_{\mu}^{\dagger}(\mathbf{r}) \equiv \sqrt{\frac{1}{2}} \sum_l \sqrt{2l+1} \left\{ \psi_{l,\frac{1}{2},\frac{1}{2}}^{\dagger}(\mathbf{r}) \psi_{l,\frac{1}{2},\frac{1}{2}}^{\dagger}(\mathbf{r}) \right\}_{M_L=0,M_S=\mu,M_T=0}^{(L=0,S=1,T=0)}$$

- ★ One-to-one correspondence with spin-3/2 fermion system with Hamiltonian

$$H = \int d^3r \left\{ \sum_{\alpha} \varepsilon_{\alpha} \varphi_{\alpha}^{\dagger}(\mathbf{r}) \varphi_{\alpha}(\mathbf{r}) - g_0 S_{0,0}^{\dagger}(\mathbf{r}) S_{0,0}(\mathbf{r}) - \sum_{m=\pm 2, \pm 1, 0} g_{2,m} D_{2,m}^{\dagger}(\mathbf{r}) D_{2,m}(\mathbf{r}) \right\}$$

$$S_{0,0}^{\dagger} = \sum_{\alpha\beta} \langle \frac{3}{2} \frac{3}{2}; 00 | \frac{3}{2} \frac{3}{2} \alpha\beta \rangle \varphi_{\alpha}^{\dagger} \varphi_{\beta}^{\dagger}$$

Singlet (S=0) pairing operator

$$D_{2,m}^{\dagger} = \sum_{\alpha\beta} \langle \frac{3}{2} \frac{3}{2}; 2m | \frac{3}{2} \frac{3}{2} \alpha\beta \rangle \varphi_{\alpha}^{\dagger} \varphi_{\beta}^{\dagger}$$

Quintet (S=2) pairing operator

with  $S_{0,0}^{\dagger} = P_0^{\dagger}$ ,  $D_{2,0}^{\dagger} = Q_0^{\dagger}$ ,  $D_{2,\pm 1}^{\dagger} = P_{\pm 1}^{\dagger}$  and  $D_{2,\pm 2}^{\dagger} = Q_{\pm 1}^{\dagger}$

★ Sp(4) ~ SO(5) symmetry without fine tuning the coupling constants

★ Generators of  $\mathfrak{so}(5)$        $\Gamma^{ab} \equiv -\frac{i}{2} [\Gamma^a, \Gamma^b]$     ( $1 \leq a, b \leq 5$ )

$$\Gamma^1 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}, \quad \Gamma^{2,3,4} = \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

★ Bilinears classified according to their behavior under SO(5) (scalar, vector, tensor,... )

*Particle-hole channel*

$$\begin{aligned} n(\mathbf{r}) &= \sum_{\alpha} \varphi_{\alpha}^{\dagger}(\mathbf{r}) \varphi_{\alpha}(\mathbf{r}), \\ n_a(\mathbf{r}) &= \frac{1}{2} \sum_{\alpha\beta} \varphi_{\alpha}^{\dagger}(\mathbf{r}) \Gamma_{\alpha\beta}^a \varphi_{\beta}(\mathbf{r}), \\ L_{ab}(\mathbf{r}) &= -\frac{1}{2} \sum_{\alpha\beta} \varphi_{\alpha}^{\dagger}(\mathbf{r}) \Gamma_{\alpha\beta}^{ab} \varphi_{\beta}(\mathbf{r}). \end{aligned}$$

*Particle-particle channel*

$$\begin{aligned} \eta^{\dagger}(\mathbf{r}) &= \frac{1}{2} \sum_{\alpha\beta} \varphi_{\alpha}^{\dagger}(\mathbf{r}) C_{\alpha\beta} \varphi_{\beta}^{\dagger}(\mathbf{r}), \\ \xi_a^{\dagger}(\mathbf{r}) &= -\frac{i}{2} \sum_{\alpha\beta} \varphi_{\alpha}^{\dagger}(\mathbf{r}) (\Gamma^a C)_{\alpha\beta} \varphi_{\beta}^{\dagger}(\mathbf{r}), \\ \dot{C} &= \Gamma^1 \Gamma^3 \\ S_{0,0}^{\dagger} &= -\frac{\eta^{\dagger}}{\sqrt{2}}, \quad D_{2,0}^{\dagger} = -i \frac{\xi_4^{\dagger}}{\sqrt{2}}, \quad D_{2,\pm 1}^{\dagger} = -\frac{\xi_3^{\dagger} \mp i \xi_2^{\dagger}}{\sqrt{2}}, \quad D_{2,\pm 2}^{\dagger} = \frac{\mp \xi_1^{\dagger} + i \xi_5^{\dagger}}{\sqrt{2}} \end{aligned}$$

- ★ If  $g_0 = g_2 = g$ , singlet and quintet pairing states are degenerate and can be recasted into a sextet pairing state  $\Rightarrow$  SU(4) symmetry
- ★ Competing superfluid orders :

◎ Sp(4)-singlet BCS pairing phase  $\eta^\dagger(\mathbf{r})$

◎ SU(4) molecular superfluid phase formed from bound states of 4 fermions ( $\alpha$ )

$$A^\dagger(\mathbf{r}) \equiv \varphi_{\frac{3}{2}}^\dagger(\mathbf{r}) \varphi_{\frac{1}{2}}^\dagger(\mathbf{r}) \varphi_{-\frac{1}{2}}^\dagger(\mathbf{r}) \varphi_{-\frac{3}{2}}^\dagger(\mathbf{r})$$

- ★ Competition manifested by a  $\mathbb{Z}_2$  discrete symmetry (coset between the center of SU(4) and the center of Sp(4))

$$\begin{aligned} \eta^\dagger &\mapsto \mathcal{U}_n \eta^\dagger \mathcal{U}_n^{-1} = -\eta^\dagger, \\ A^\dagger &\mapsto \mathcal{U}_n A^\dagger \mathcal{U}_n^{-1} = A^\dagger. \end{aligned} \quad \mathcal{U}_n = e^{in_4 \pi}$$

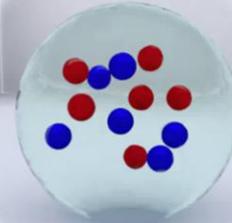
- ★  $\mathbb{Z}_2$  needs to be spontaneously broken to stabilize the BCS quasi-long range order.
- ★  $\mathbb{Z}_2$  remaining unbroken  $\Rightarrow$  strong quantum fluctuations in the spin channel suppressing Cooper pairing (2 fermions can't form a  $\mathbb{Z}_2$  singlet)  $\Rightarrow$  leading superfluid instability = quartetting

- ★ Nucleus = 4-component Fermi system



— proton  
— neutron

Competition between pairing & quartetting orders



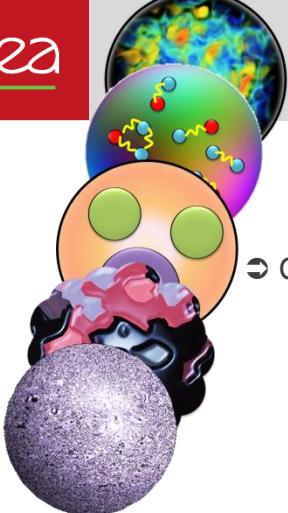
Courtesy of R.D. Lasseri

⌚ Interplay isoscalar/isovector pairing ?

⌚ Size of the pairs ?

⌚ Links  $\alpha$ -quartet &  $\alpha$ -particle ?

- 1 The Energy Density Functional approach
- 2 Clustering in the EDF language
- 3 EDF + QCM



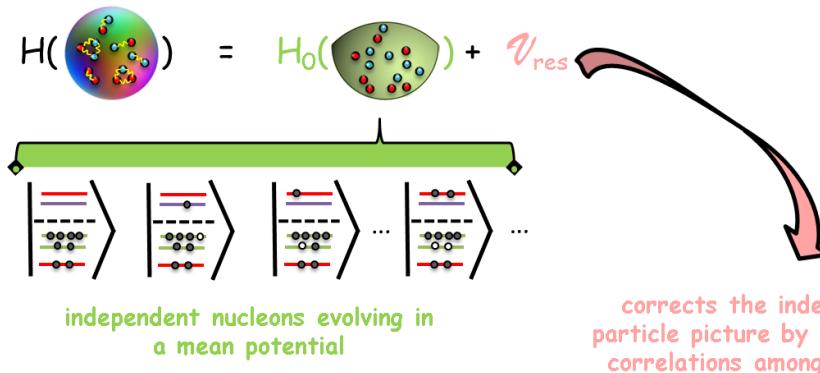
⇒ Nucleus = collection of interacting point-like nucleons

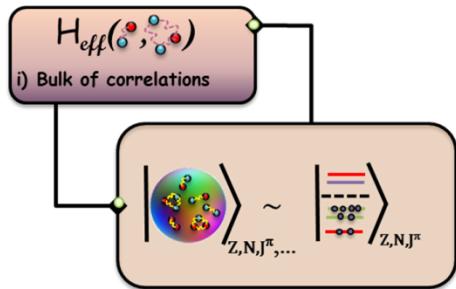
⇒ Challenge : Extract observables of interest from the nuclear Hamiltonian

$$H(\text{Nucleus}, \dots) |Z, N, J^\pi, \dots\rangle = E_{Z, N, J^\pi, \dots} |Z, N, J^\pi, \dots\rangle$$

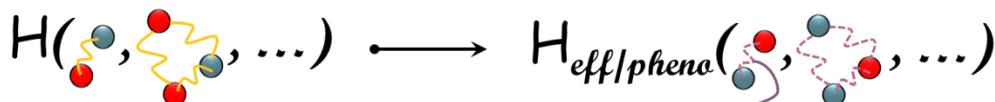
**Hard to describe**                                    **Hard to solve**

⇒ Strategy : split the total Hamiltonian into **unperturbed** and **residual** parts





i) Dominant class of correlations

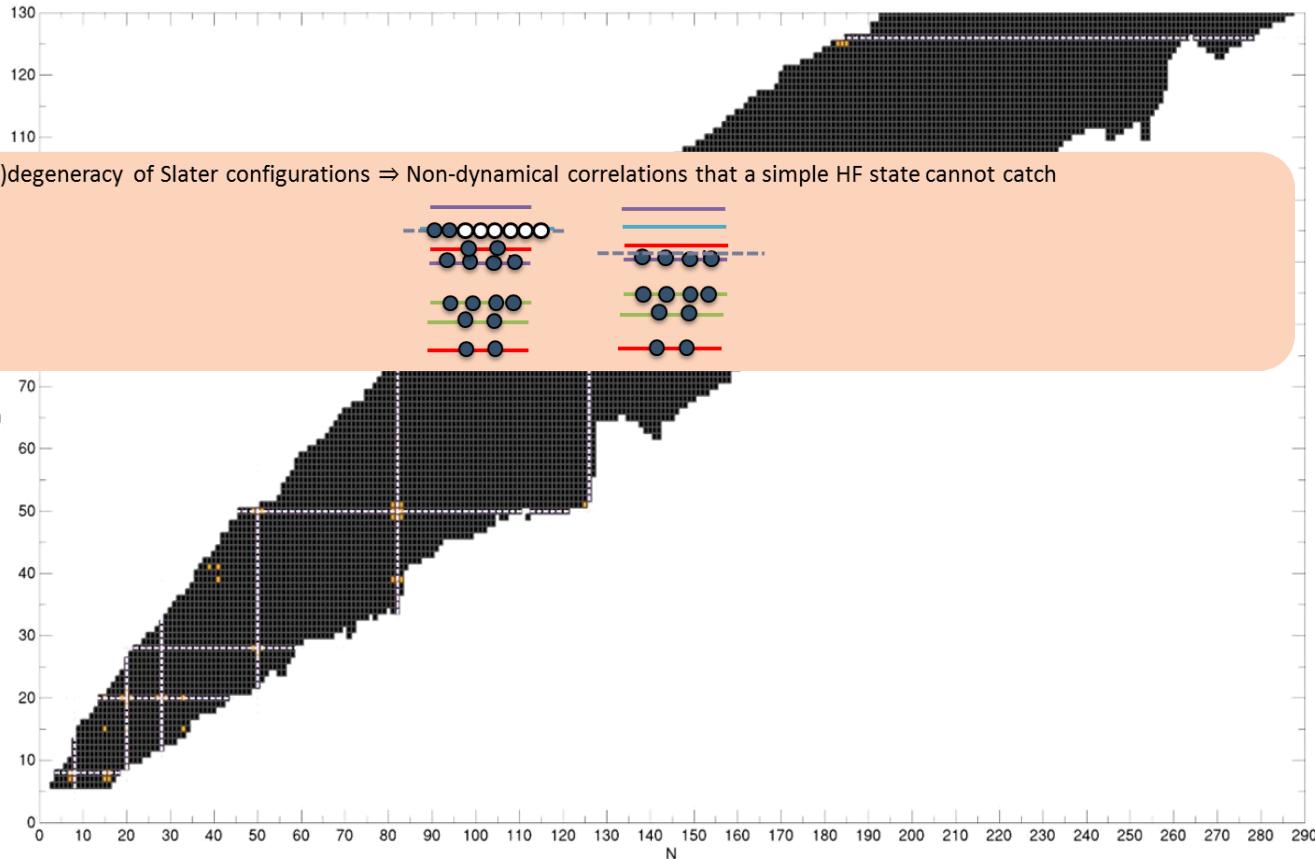


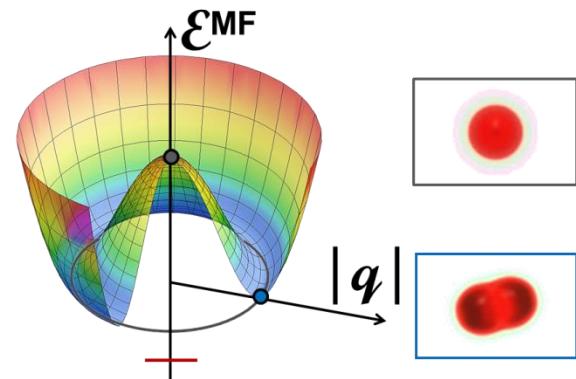
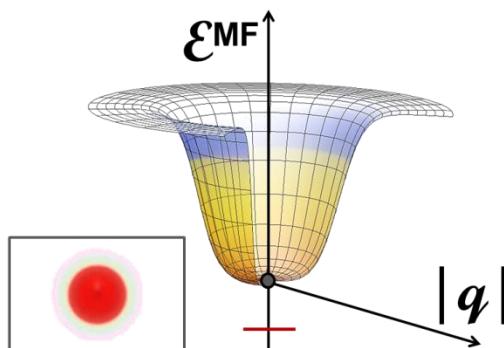
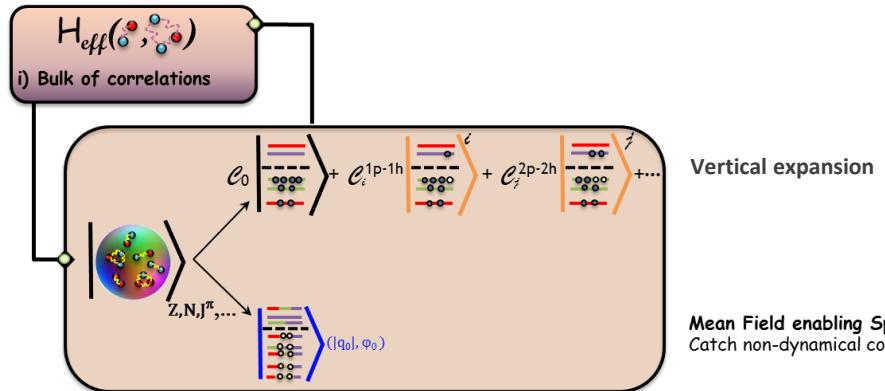
↳ Skyrme, Gogny, relativistic, ... effective in-medium interactions

Few free parameters adjusted once and for all

**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

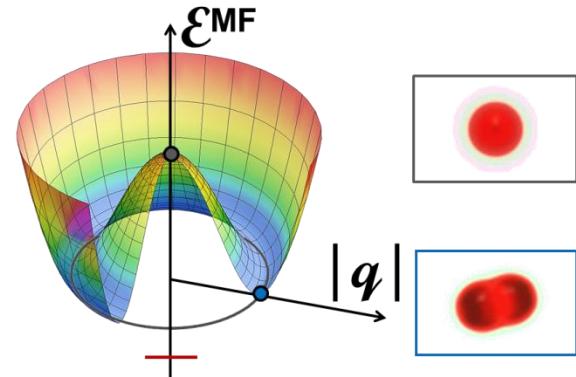
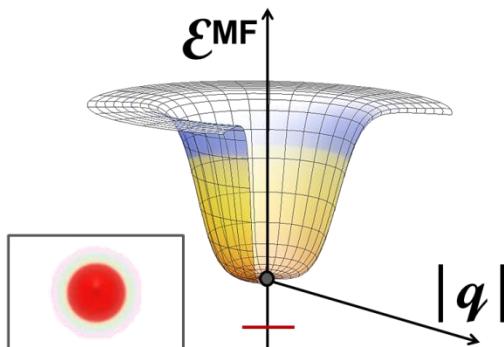
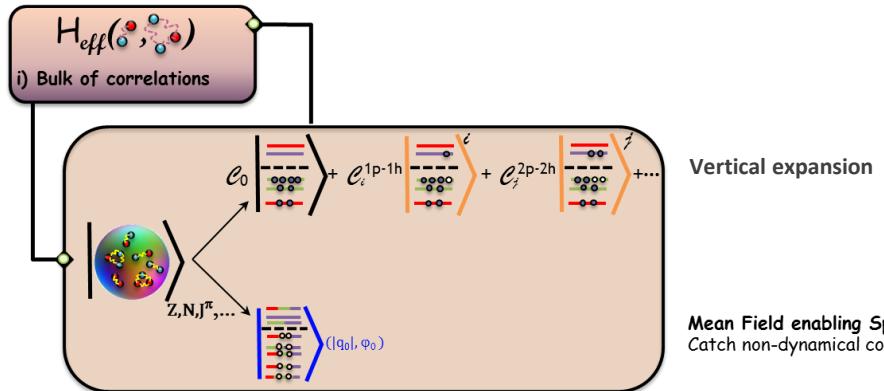
- Ground state of ~30 nuclei (spherical + non-superfluid) are described in a satisfactory way



**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

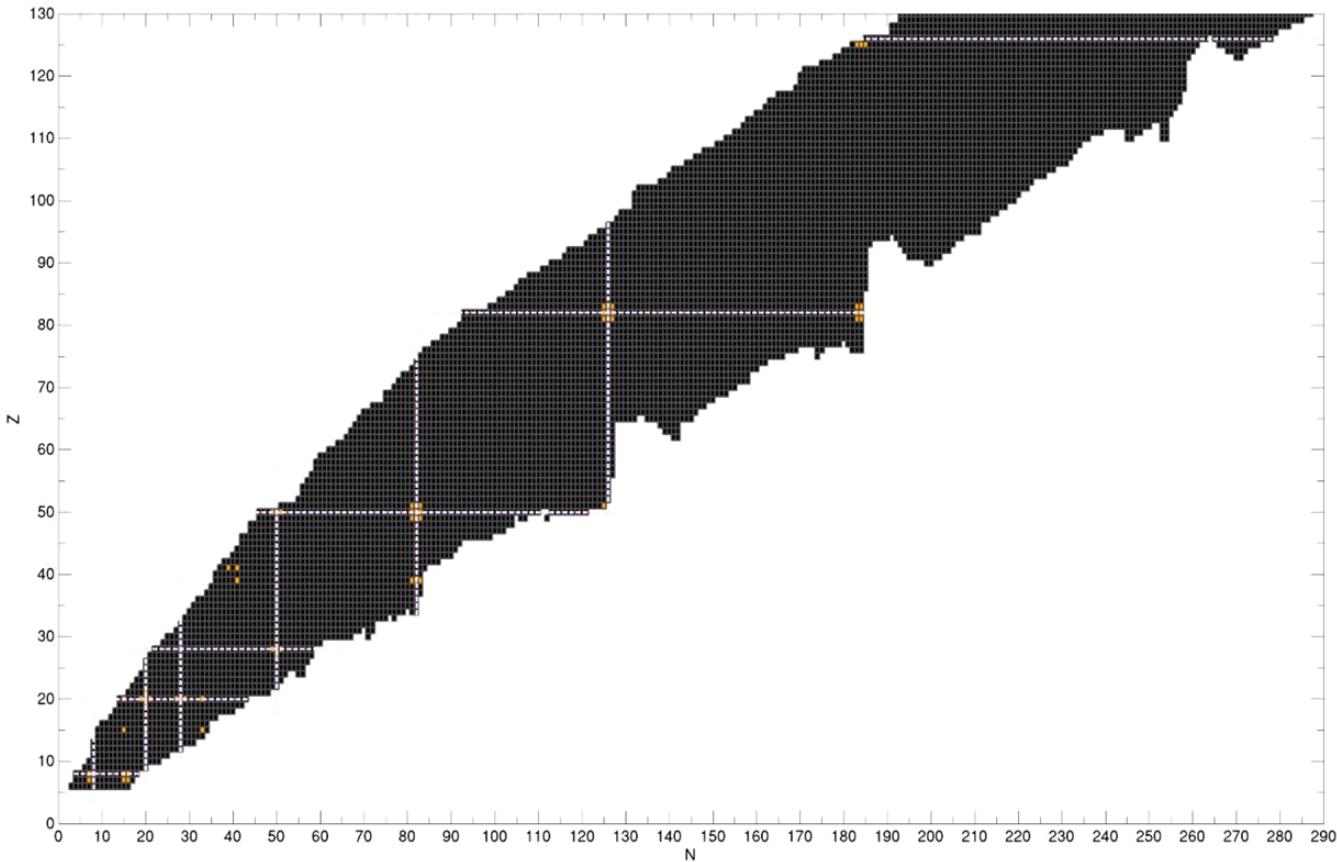
**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

- ★ Nucleus = finite system  $\Rightarrow$  No spontaneous symmetry breaking
- ★ BUT some features (rotational bands, ...) can be understood as precursors of SSBs (that would be fully-fledged SSBs in the thermodynamics limit) : call them emergent symmetry breaking
- ★ In the EDF language, such features appear as symmetry-breaking configurations in a first step (static mean-field or single-reference level) and post mean-field treatments are required to get the full answer
- ★ More than in thermodynamical functions, emergent symmetry breaking features leave their footprints in spectroscopic properties

**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

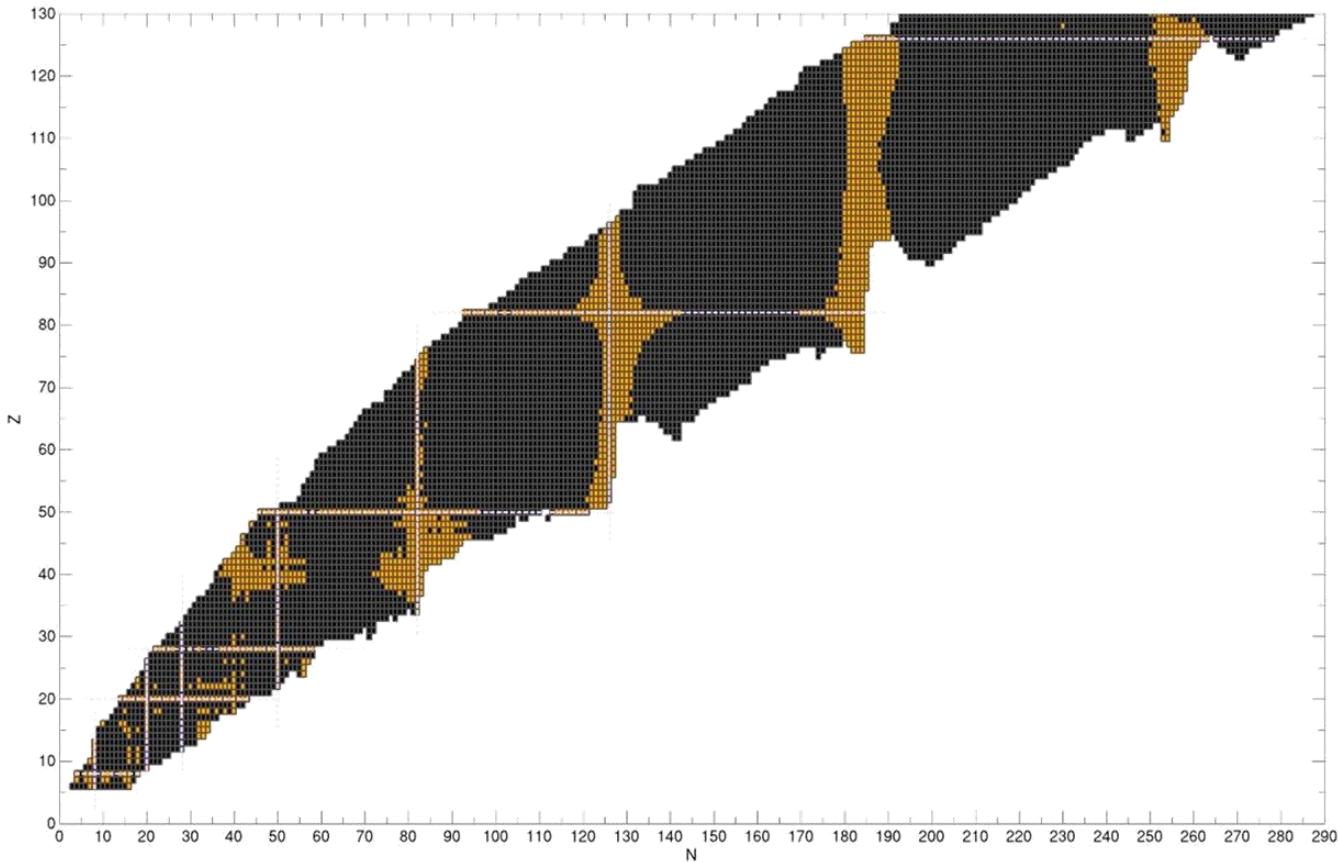
**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

- Ground state of ~30 nuclei (spherical + non-superfluid) are described in a satisfactory way



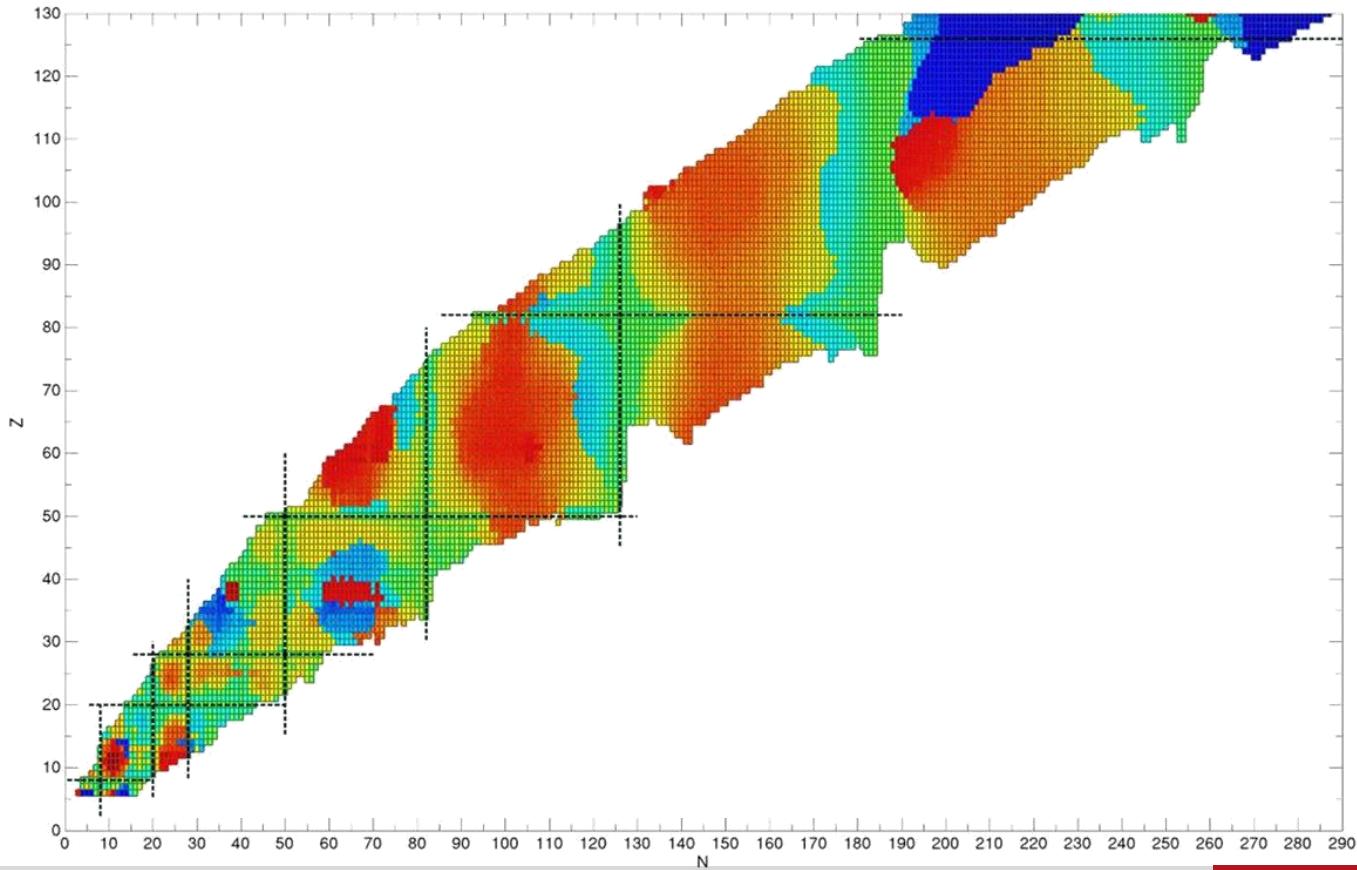
**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

- Allowing for U(1) breaking  $\Rightarrow$  Ground state of  $\sim 300$  nuclei are described in a satisfactory way



**-SINGLE REFERENCE (AKA MEAN FIELD) IMPLEMENTATION-**

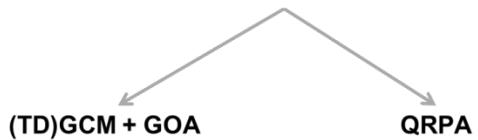
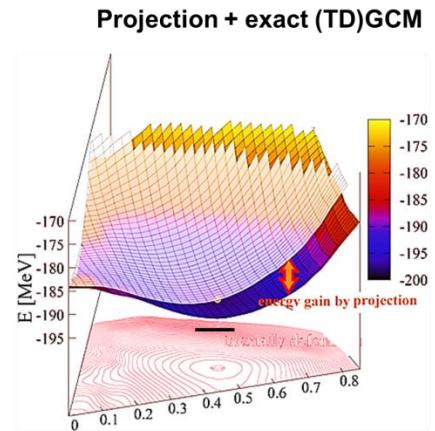
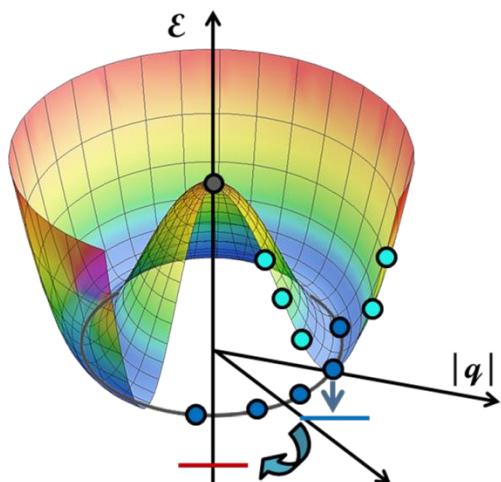
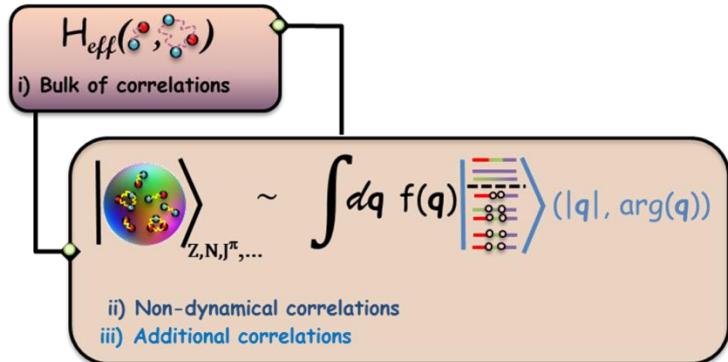
- Allowing for U(1) & SO(3) breaking  $\Rightarrow$  Ground state of all nuclei are described in a satisfactory way



**-MULTI REFERENCE (AKA BEYOND MEAN FIELD) IMPLEMENTATION-**

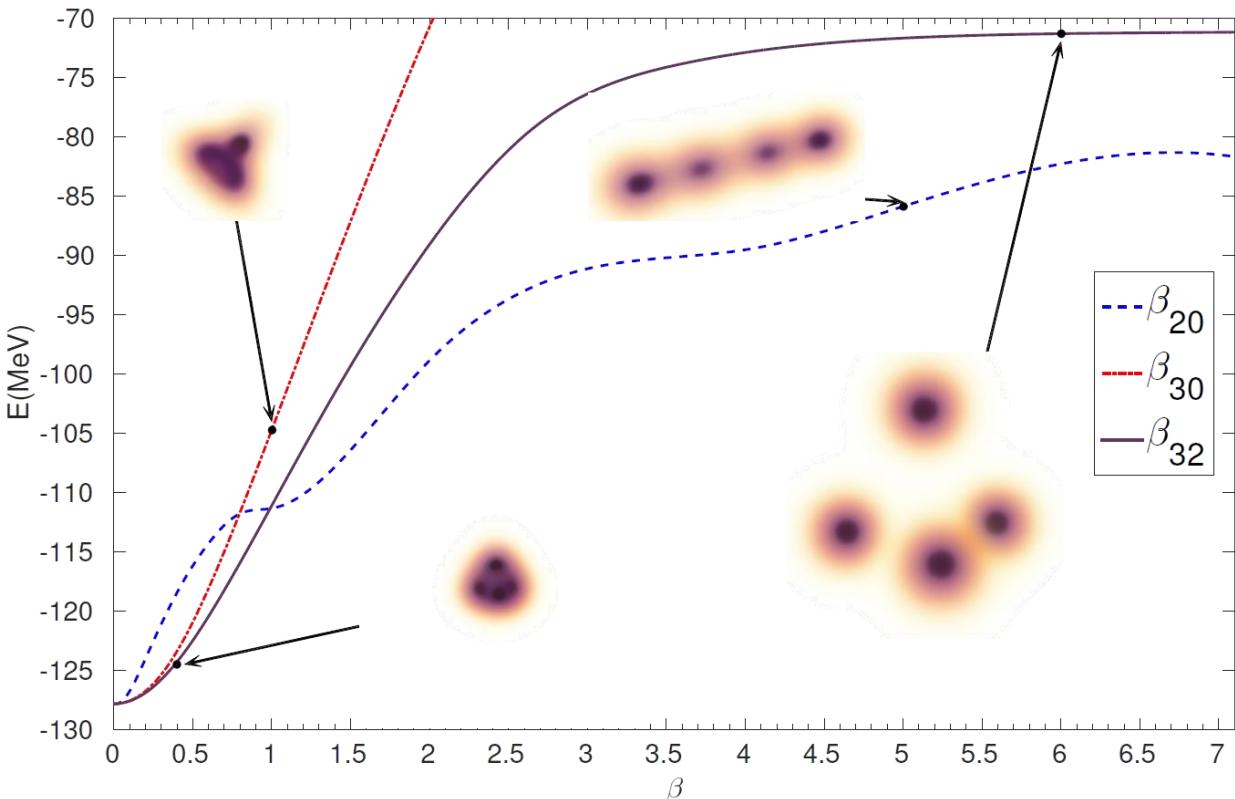
- ★ Nucleus = finite system  $\Rightarrow$  No spontaneous symmetry breaking
- ★ BUT some features (pairing, deformation, ...) can be understood as precursors of SSBs, i.e. would correspond to SSB consequences in the thermodynamics limit : call them emergent symmetry breaking
- ★ In the EDF language, such features appear as symmetry-breaking configurations in a first step (static mean-field or single-reference level) and post mean-field treatments are required to get the full answer
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## -MULTI REFERENCE (AKA BEYOND MEAN FIELD) IMPLEMENTATION-

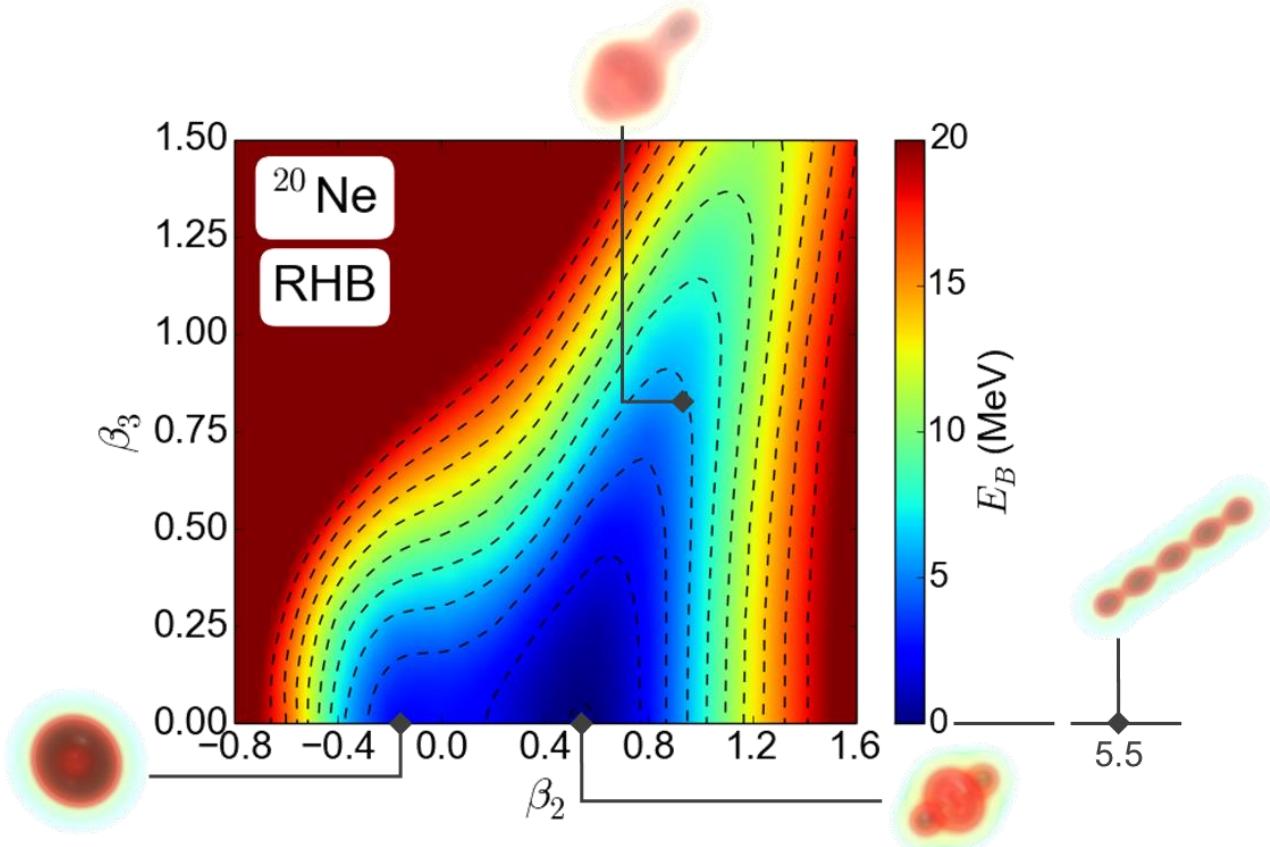


- 1 The Energy Density Functional approach
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- ★ Can the correlations causing  $\alpha$ -clustering can be recasted in those associated to a fluctuation of a collective field ?



- ★ Inhomogeneous mass distribution at the SR-EDF level



- ★ How do I know these small lumps of matter are alphas ?

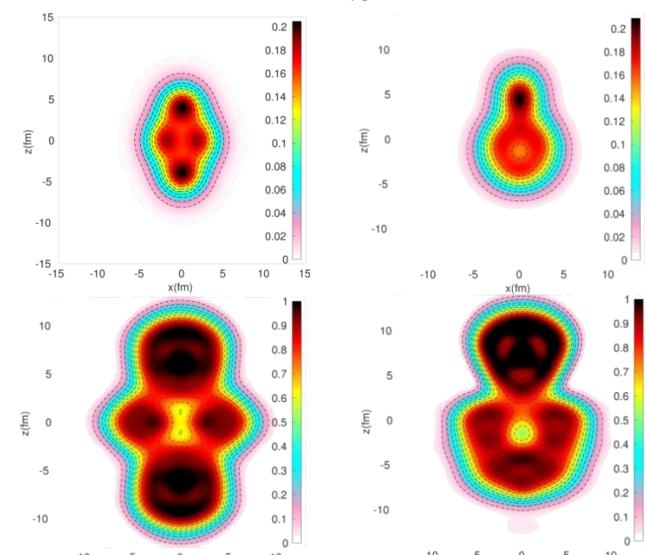
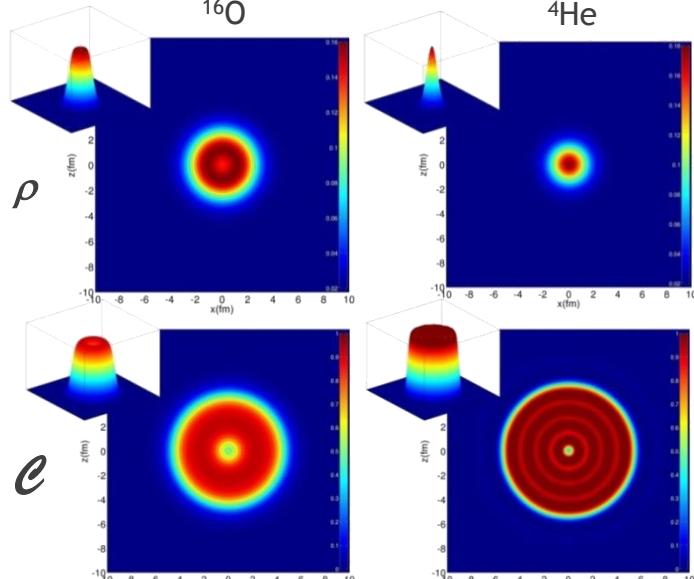
- ⦿ Integrating the density over this region gives 2 protons and 2 neutrons

$$\text{⦿ Conditional probability } R_{q\sigma}(\mathbf{r}, \mathbf{r}') = \rho_{q\sigma}(\mathbf{r}') - \frac{|\rho_{q\sigma\sigma}(\mathbf{r}, \mathbf{r}')|^2}{\rho_{q\sigma}(\mathbf{r})}$$

Reinhard et al, Phys. Rev. C 83, 034312 (2011)

$$\mathcal{C}_{q\sigma}(\mathbf{r}) = \left[ 1 + \left( \frac{\tau_{q\sigma} \rho_{q\sigma} - \frac{1}{4} [\nabla \rho_{q\sigma}]^2 - \mathbf{j}_{q\sigma}^2}{\rho_{q\sigma} \tau_{q\sigma}^{\text{TF}}} \right)^2 \right]^{-1}$$

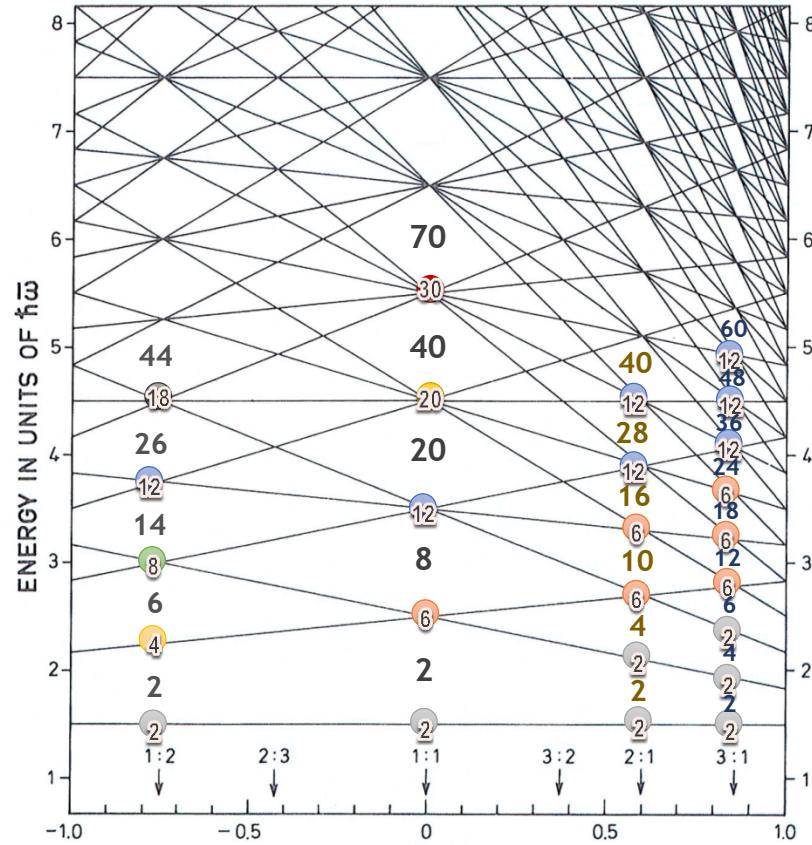
- ⦿ 0.5 : signals a nearly homogeneous Fermi gas
- ⦿ 1 : localized  $\alpha$ -like state (in  $N=Z$  systems)



## 2 ALPHA-CLUSTERING IN THE EDF LANGUAGE

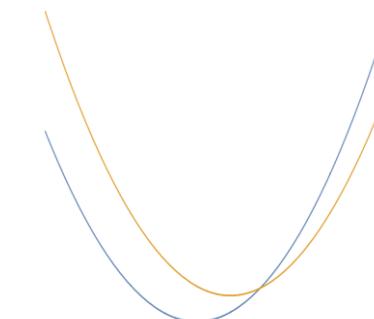
-WHAT DO WE MEAN BY "CLUSTERING" IN EDF OUTPUT ?-

- ★ Can we understand why deformation brings alpha-like structures ?



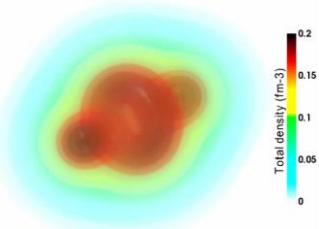
Nazarewicz et. al., AIP Conf. Proc. 259, 30 (1992)

SPHERICAL MAGIC NUMBERS	SUPERDEFORMED PROLATE MAGIC NUMBERS	SUPERDEFORMED PROLATE SPECTRUM
70	140	4
40	110	$\epsilon_F^A$
20	80	3
18	60	2
44	40	1
26	28	0
14	16	(000)
12	10	B
8	4	A
6		(001)
4		
2		
1:2		
2:3		
1:1		
3:2		
2:1		
3:1		

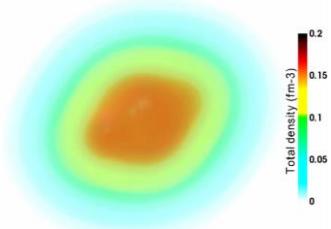


- ★ Is deformation a sufficient condition ?

$^{20}\text{Ne}$



DD-ME2



SLy4

★ When can we expect cluster structures ?

- ⇒ For a quantum-mechanical system, strength of correlations measured by the non-ideality parameter :

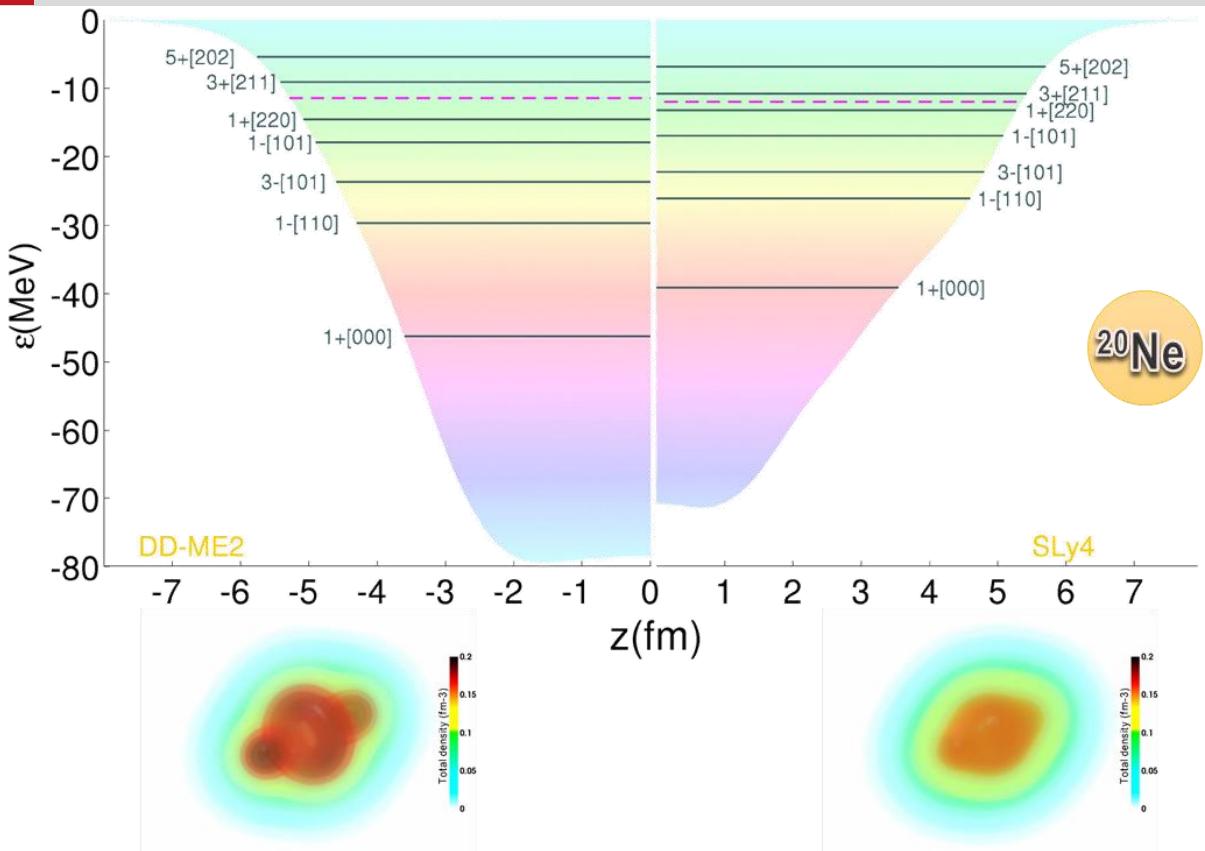
$$r_s = \frac{\langle V \rangle}{\langle K \rangle} \sim \frac{\bar{r}}{a_B} = \mathcal{N}' \frac{\sqrt{mU}}{A^{\frac{1}{3}}} \rho^{-\frac{1}{3}}$$

⇒ Clustering favored i) for low  $A$ ; ii) for deeper  $U$ ; iii) at subsaturation density

- ⇒ Formation/dissolution of bound states : Mott parameter

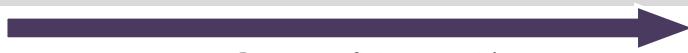
$$\chi_\alpha = \frac{R_\alpha}{\bar{r}} \Rightarrow \frac{\rho_{\text{Mott}}^\alpha}{\rho_{\text{sat}}} \sim 0.2$$

Ebran, Khan, Niksic & Vretenar Nature 487, 341-344 (2012)  
Ebran, Khan, Niksic & Vretenar PRC 87, 044307 (2013)

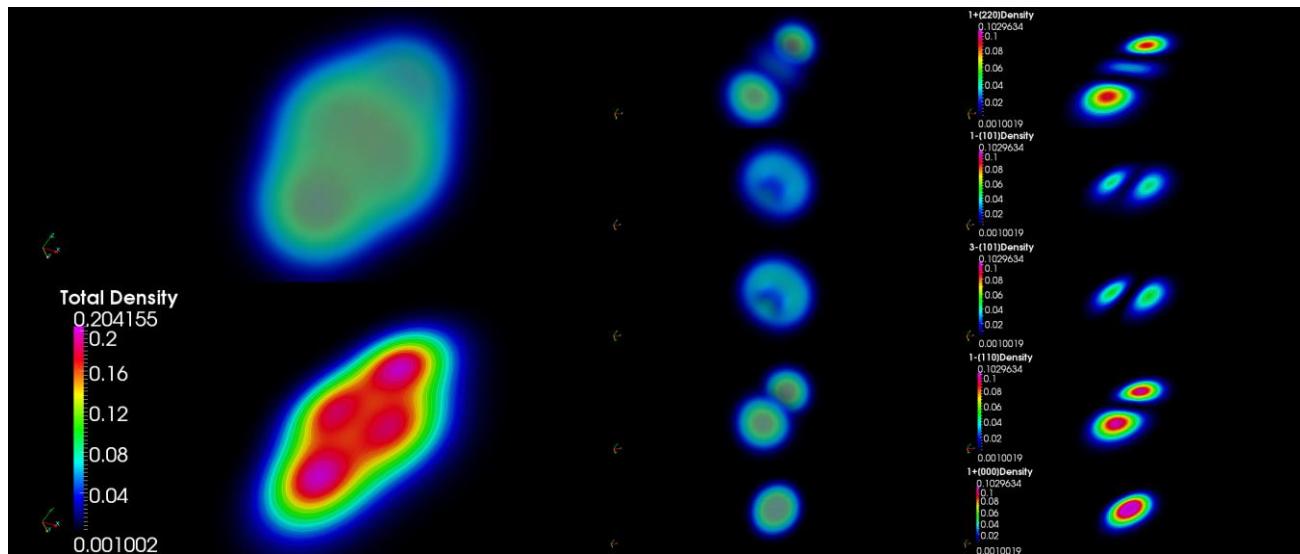
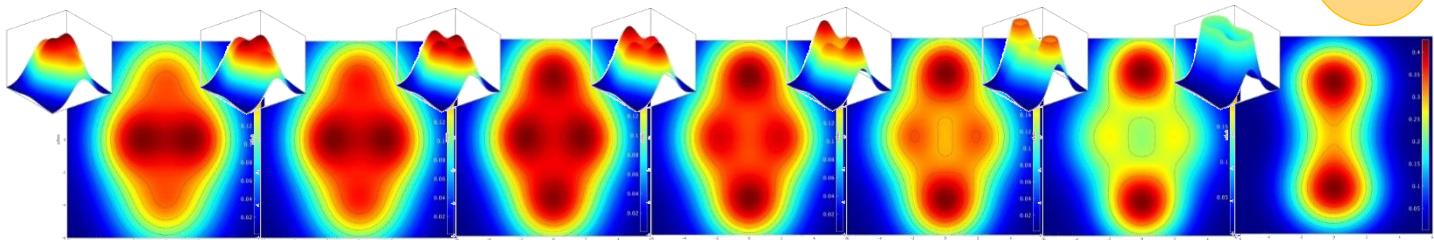


Ebran, Khan, Niksic & Vretenar Nature 487, 341-344 (2012)

Ebran, Khan, Niksic & Vretenar PRC 87, 044307 (2013)



Deeper confining potential

**20Ne**

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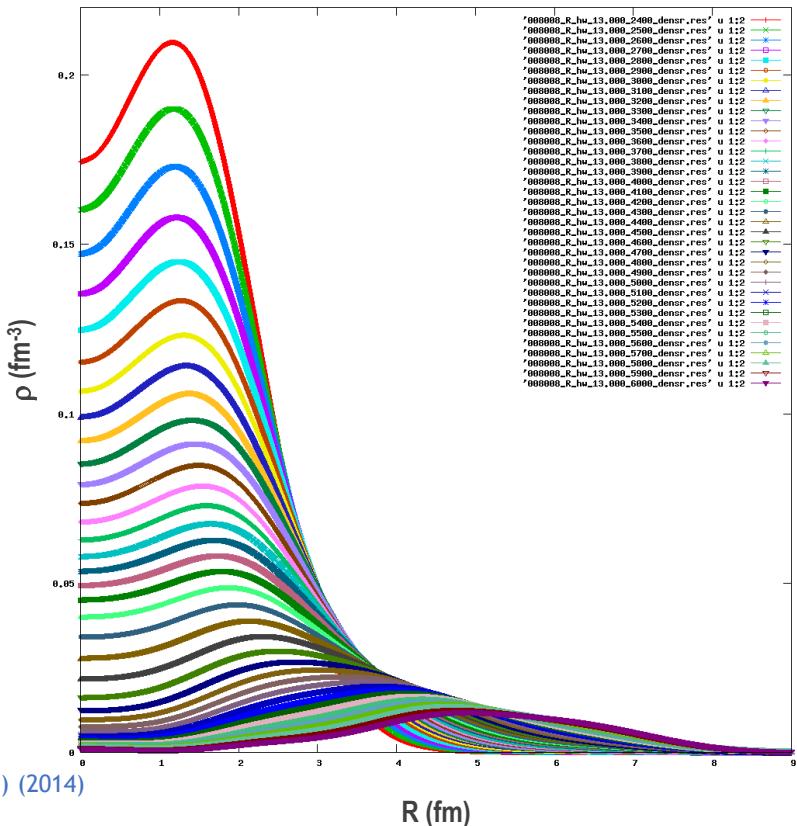
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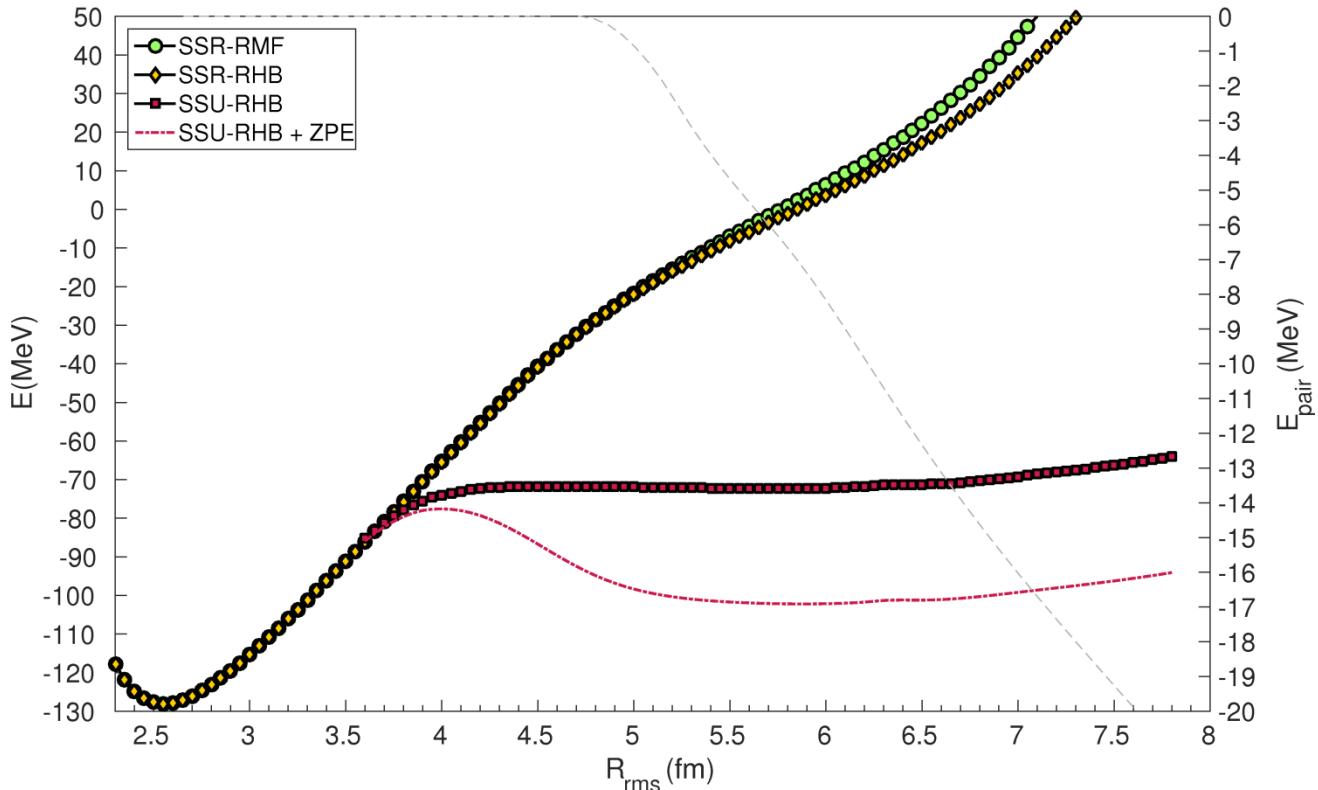
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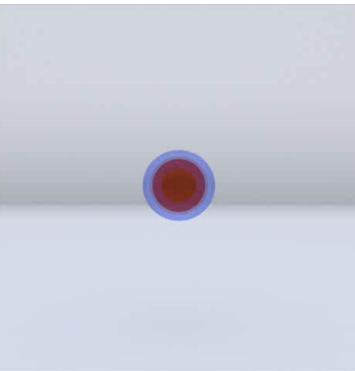
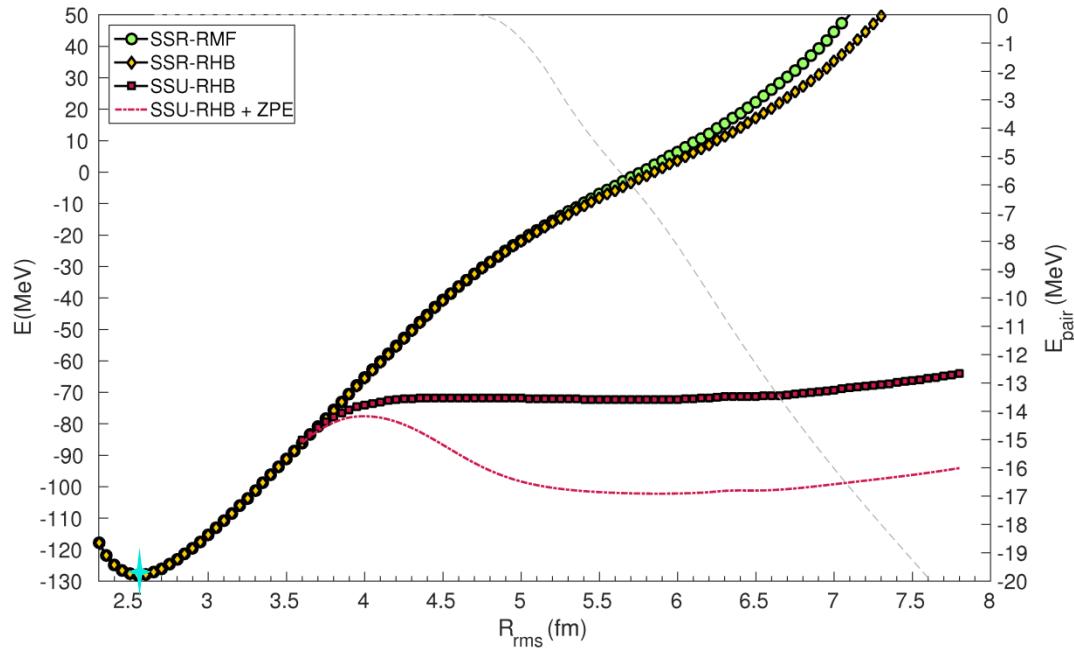
- ★ Isotropic inflation of  $^{16}\text{O}$  : constrain the r.m.s. radius while imposing a zero quadrupole moment



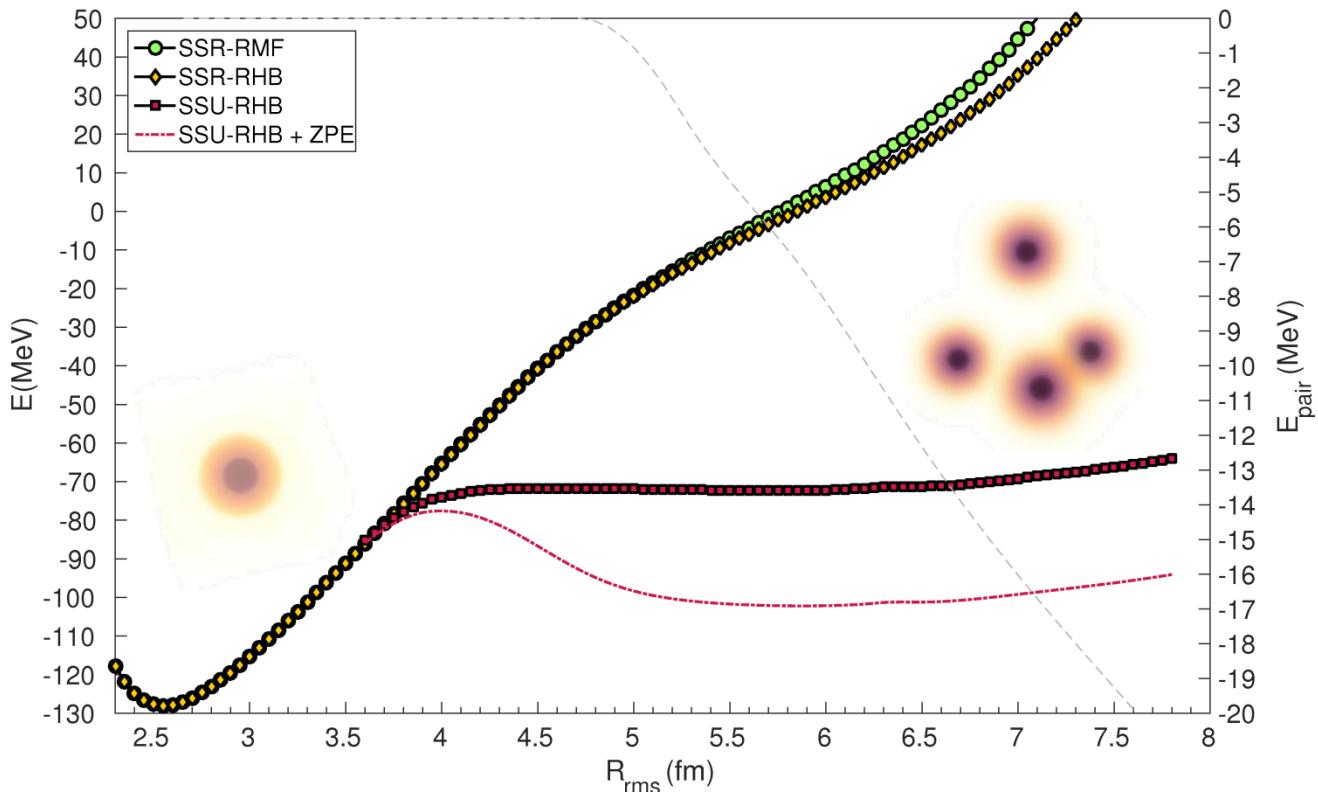
Ebran, Khan, Niksic & Vretenar PRC 89, 031303(R) (2014)  
Lasserri, Ebran, Girod, Khan & Schuck in prep

★ Isotropic inflation of  $^{16}\text{O}$ 

Lasserri, Ebran, Girod, Khan &amp; Schuck in prep

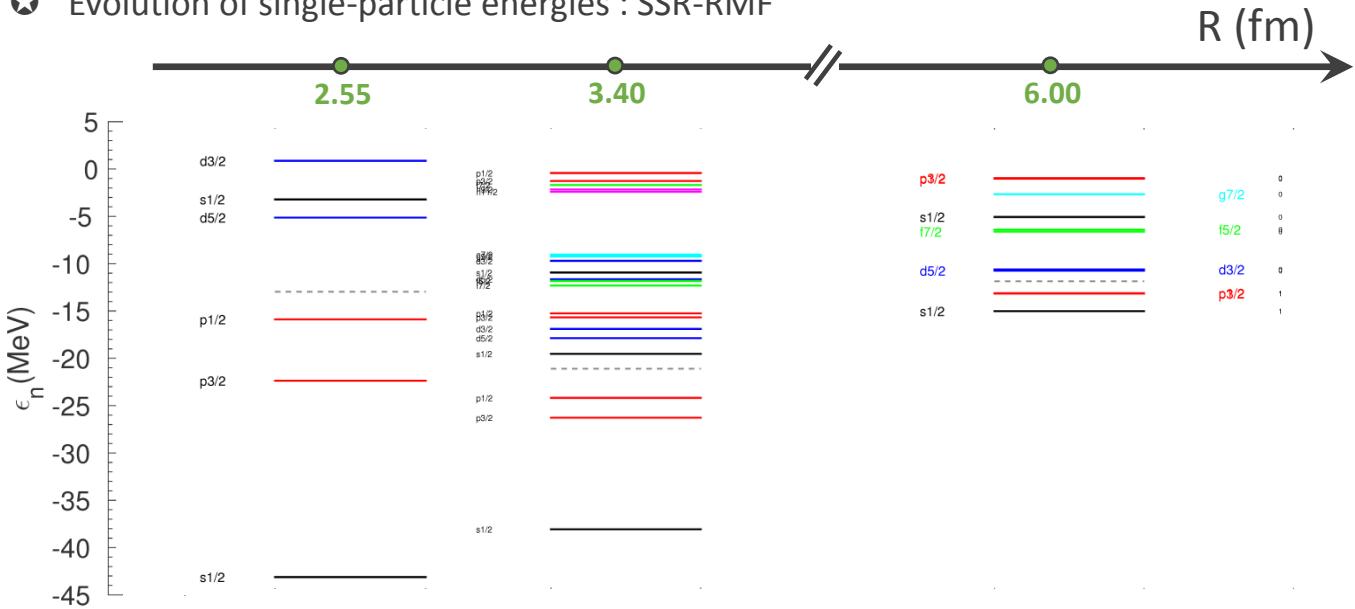
★ Isotropic inflation of  $^{16}\text{O}$ 

- ★ Mott like QPT at  $R_c = 3.7$  fm i.e.  $\rho/\rho_{sat} \sim 0.32$

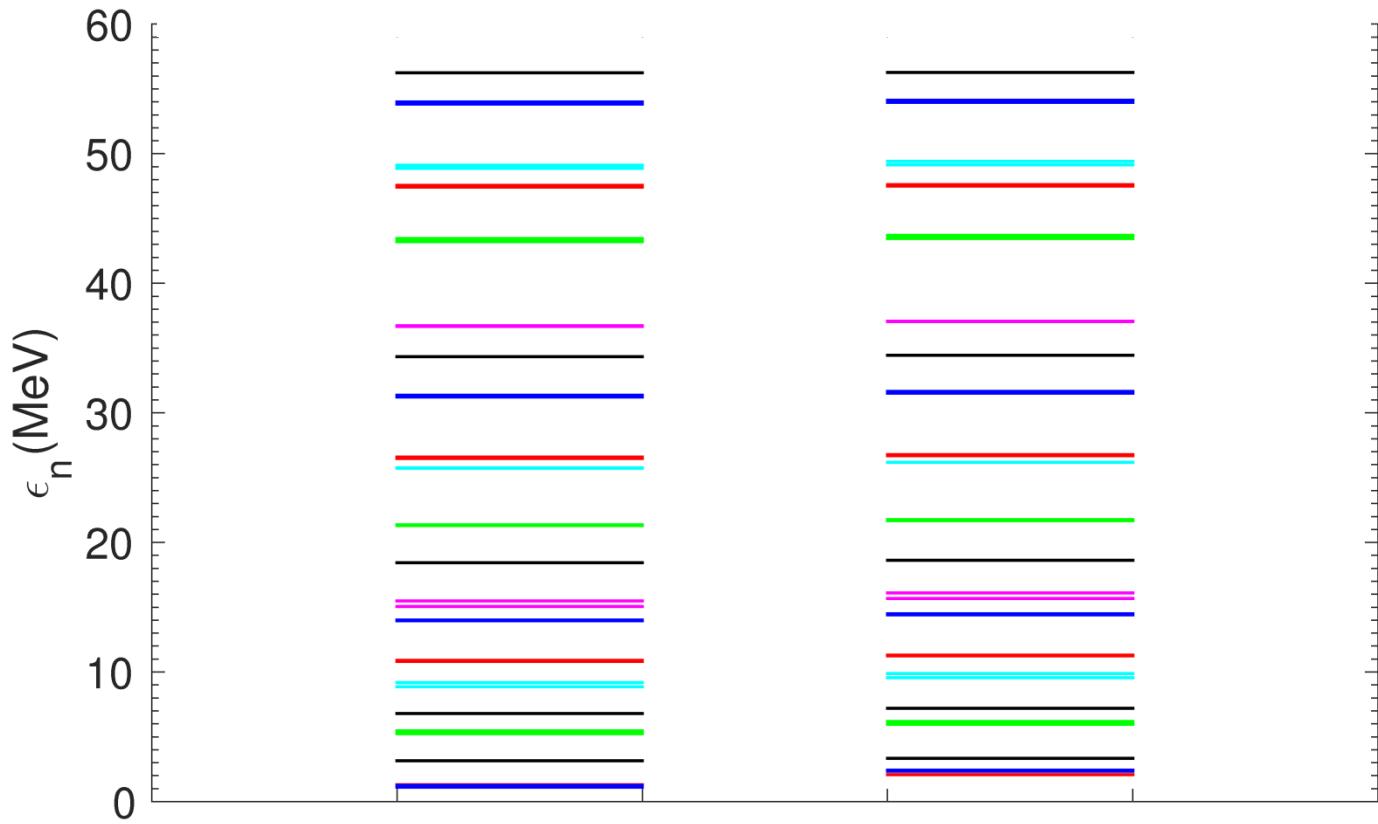


Lasserri, Ebran, Girod, Khan & Schuck in prep

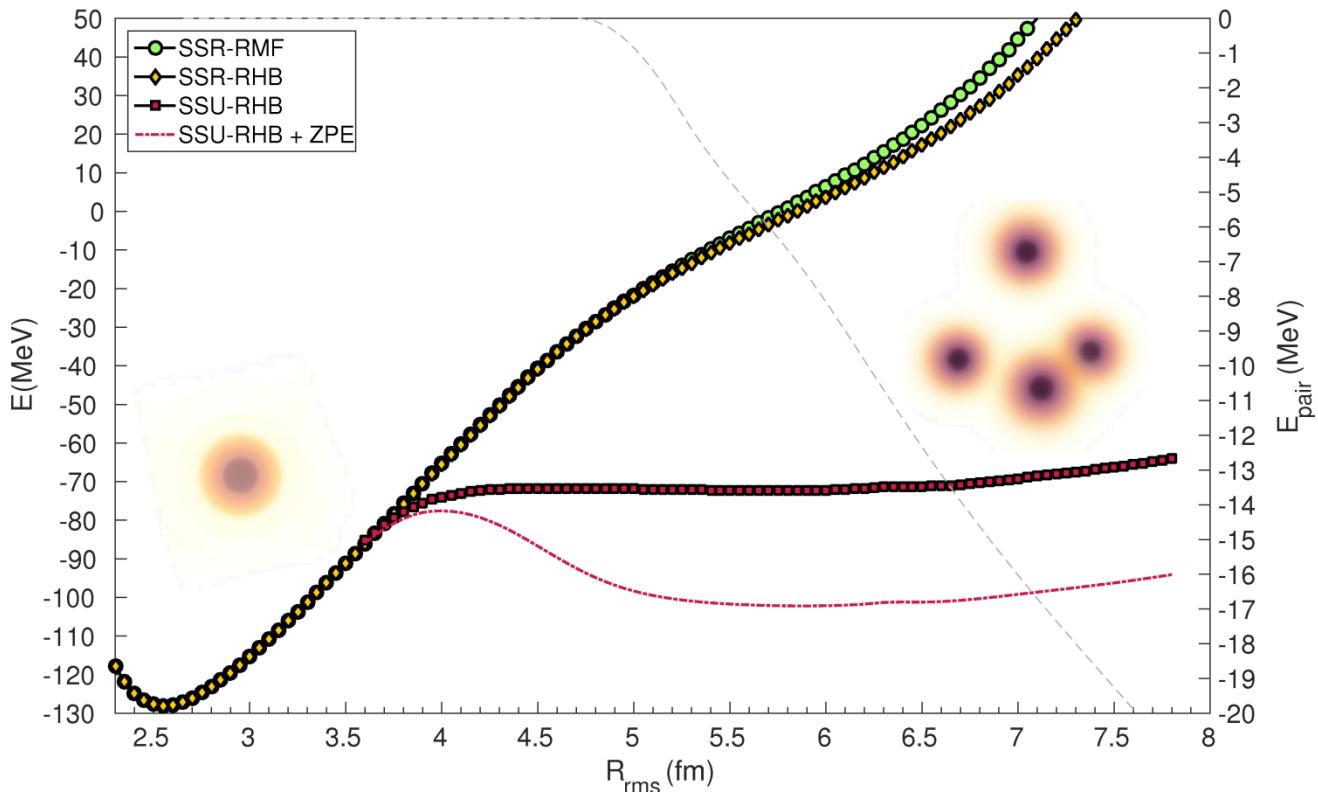
## ★ Evolution of single-particle energies : SSR-RMF



- ★ 1<sup>st</sup> way for lifting the near degeneracy : pairing correlations

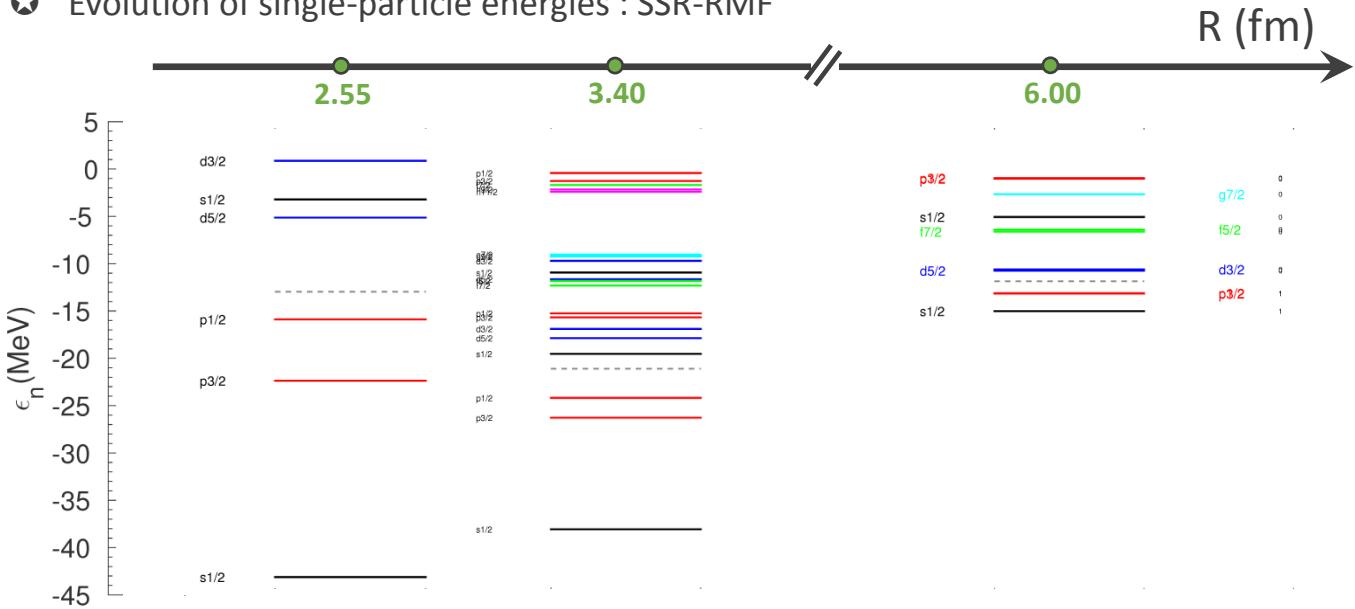


- ★ Mott like QPT at  $R_c = 3.7$  fm i.e.  $\rho/\rho_{sat} \sim 0.32$

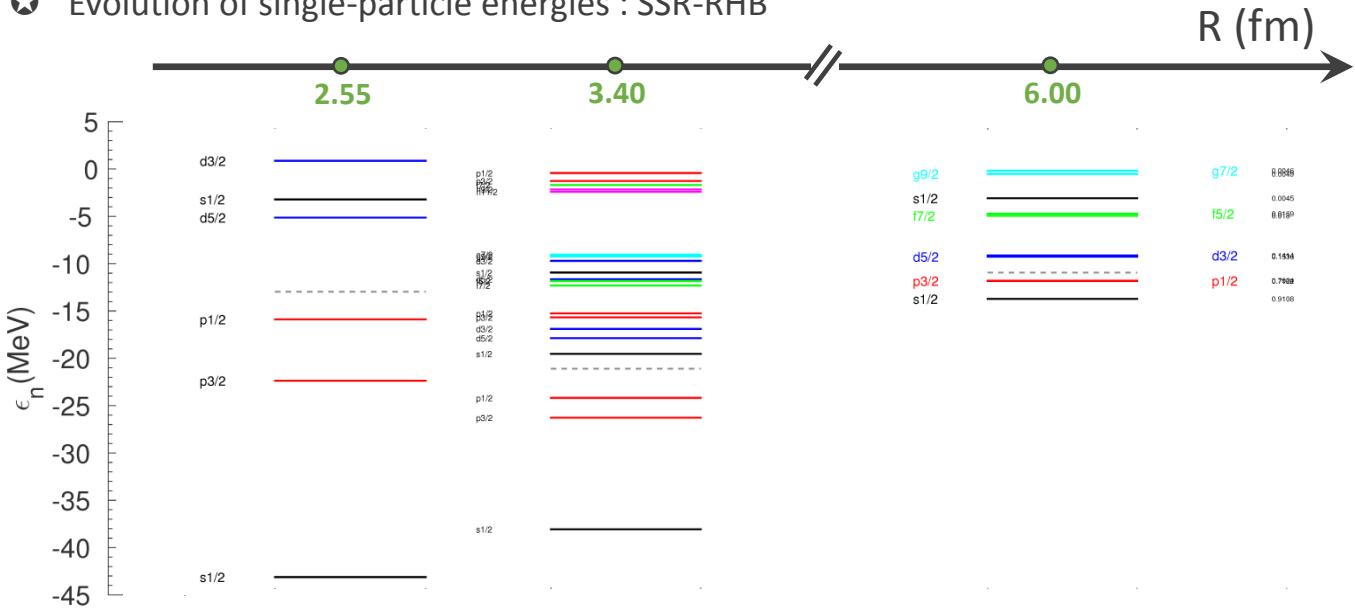


Lasserri, Ebran, Girod, Khan & Schuck in prep

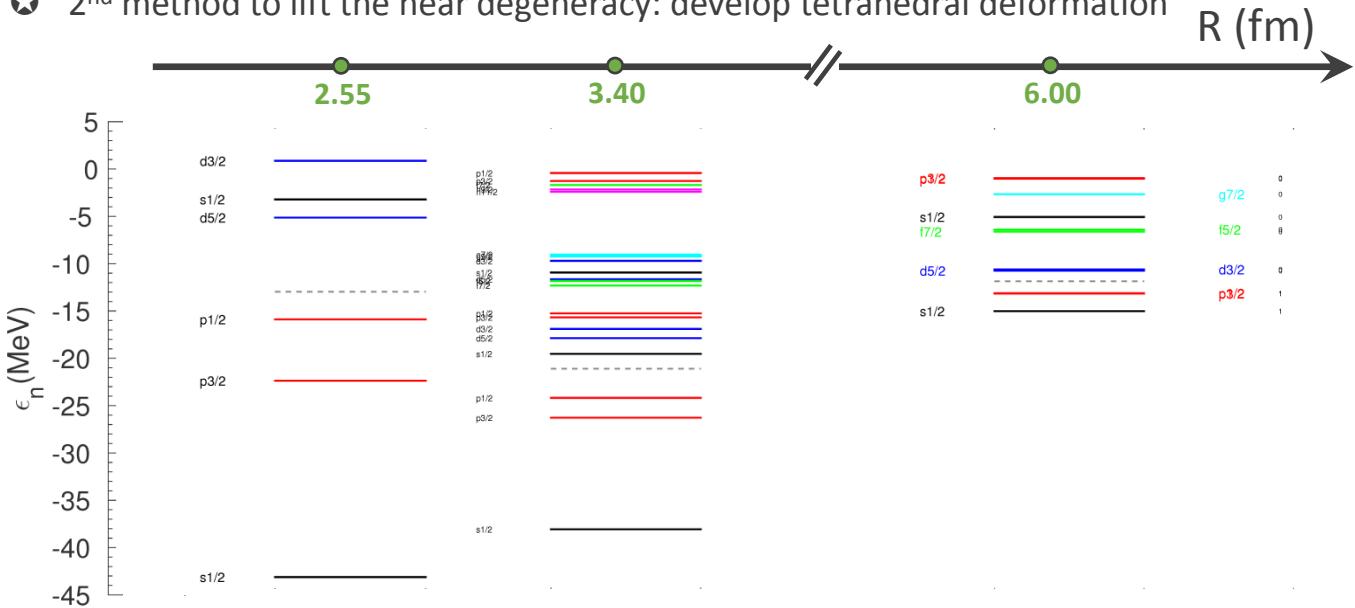
## ★ Evolution of single-particle energies : SSR-RMF



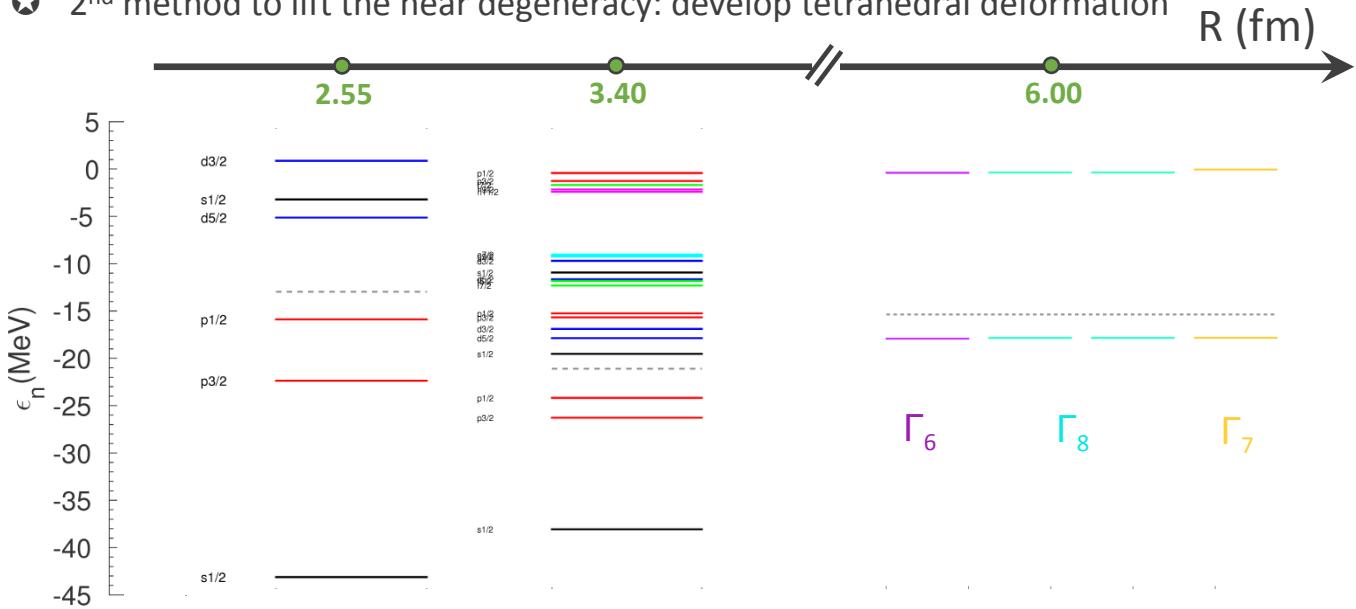
## ★ Evolution of single-particle energies : SSR-RHB



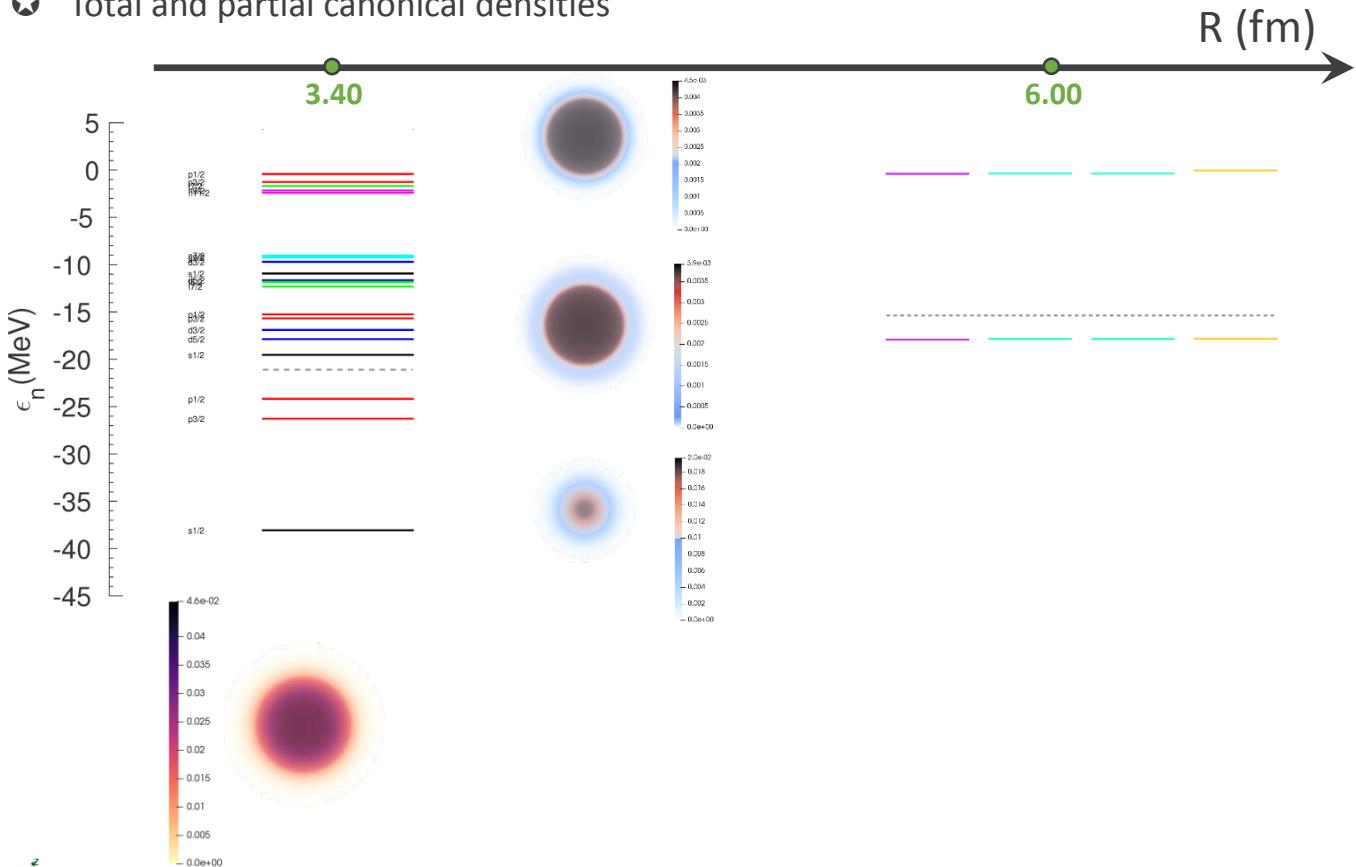
- ★ 2<sup>nd</sup> method to lift the near degeneracy: develop tetrahedral deformation



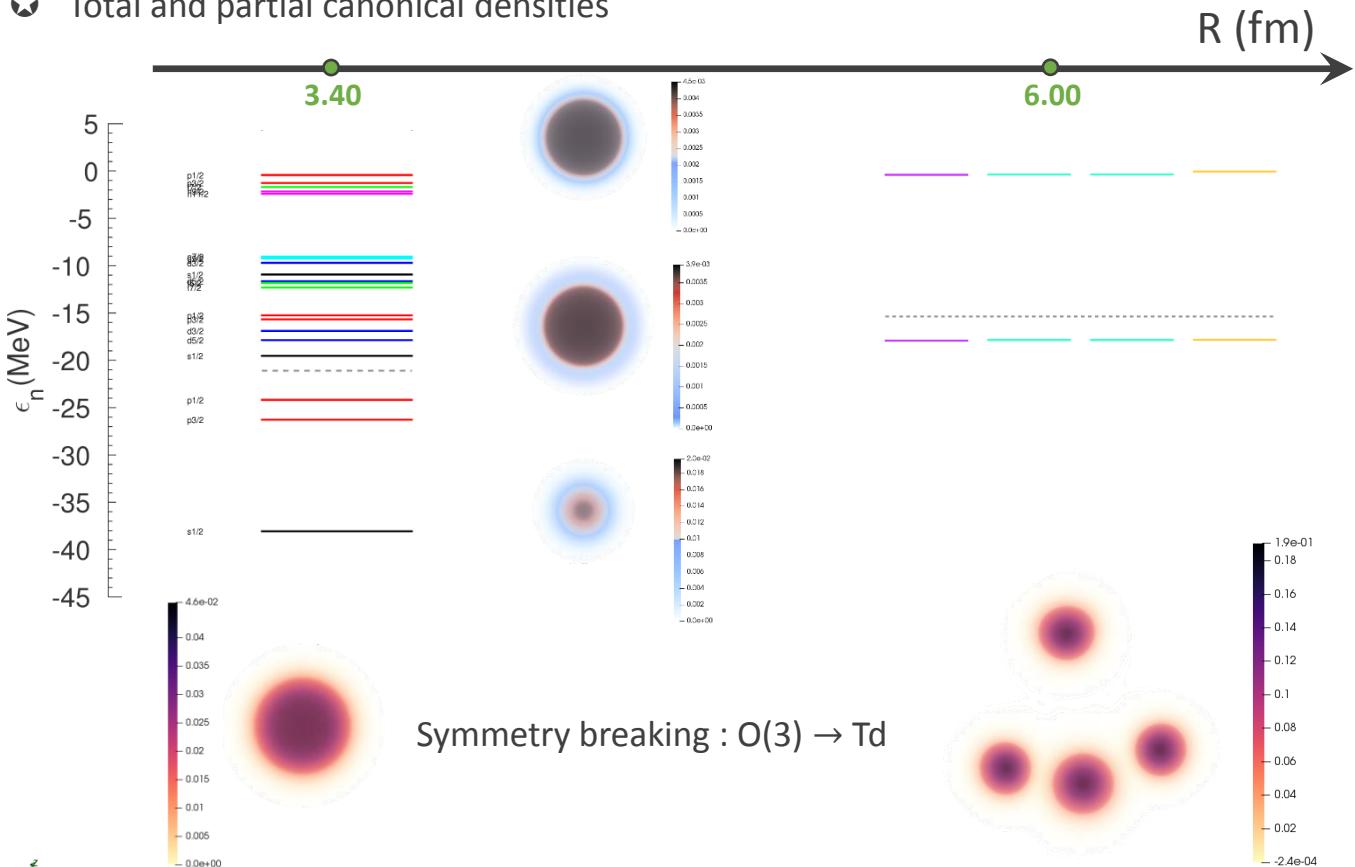
- ★ 2<sup>nd</sup> method to lift the near degeneracy: develop tetrahedral deformation



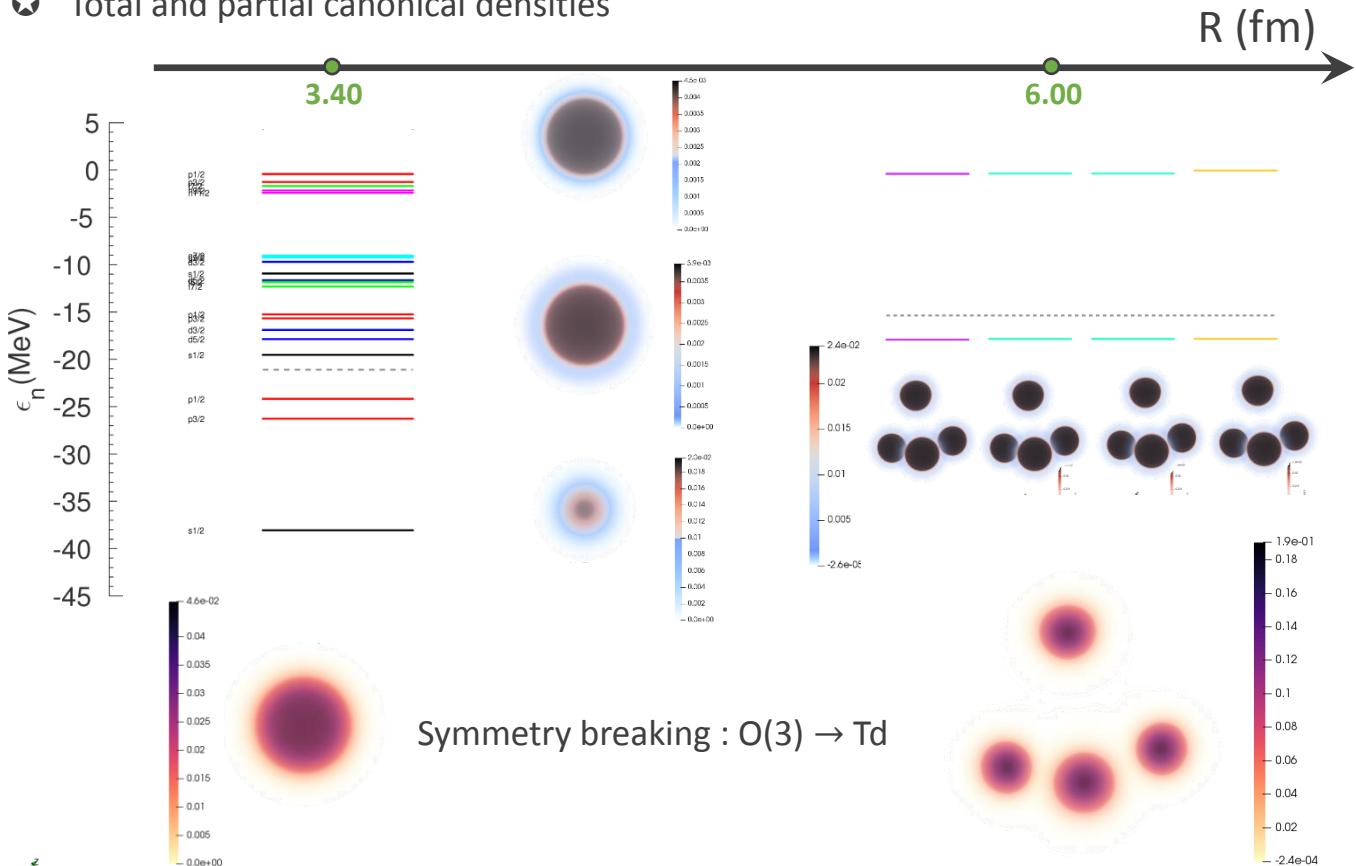
## ★ Total and partial canonical densities



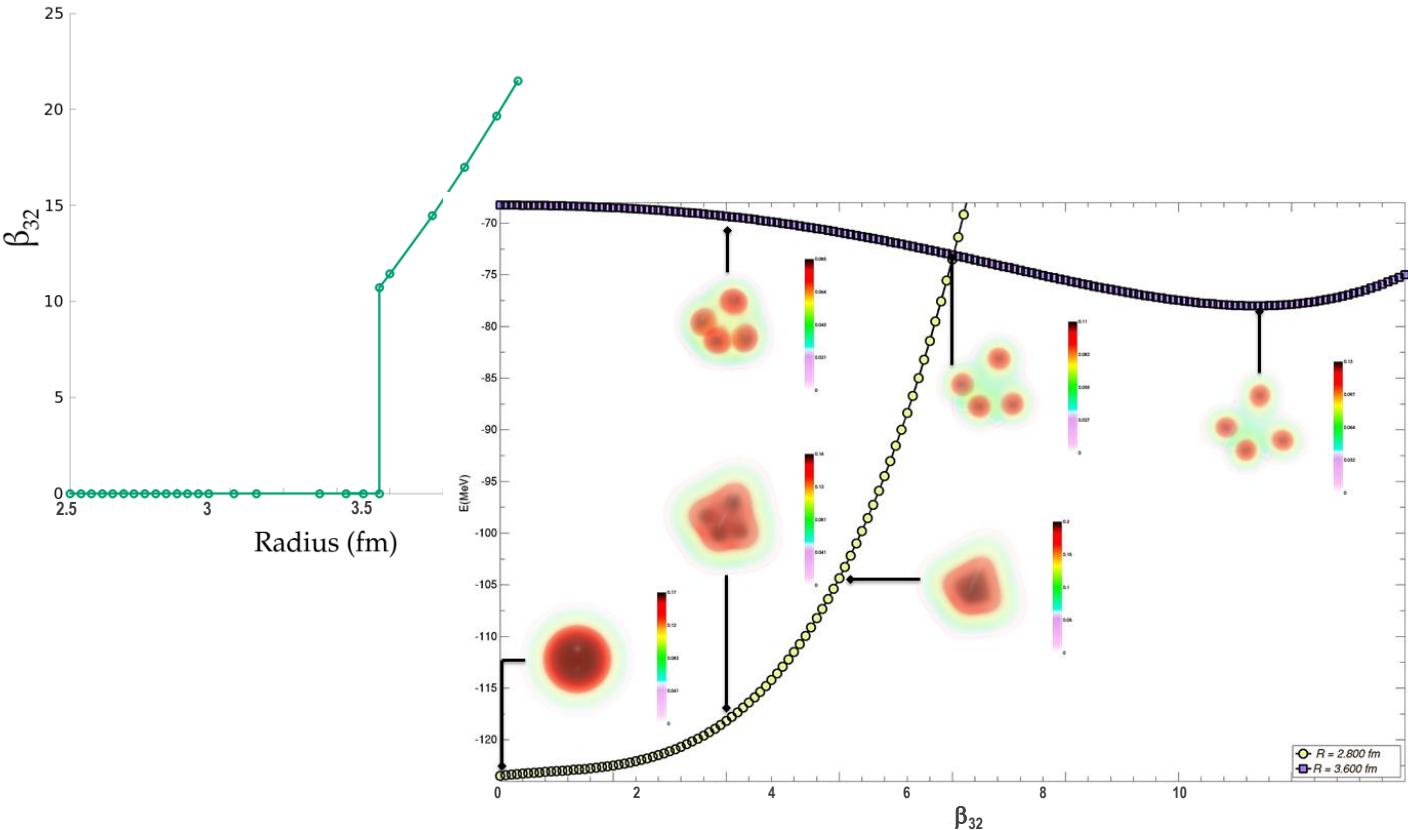
## ★ Total and partial canonical densities



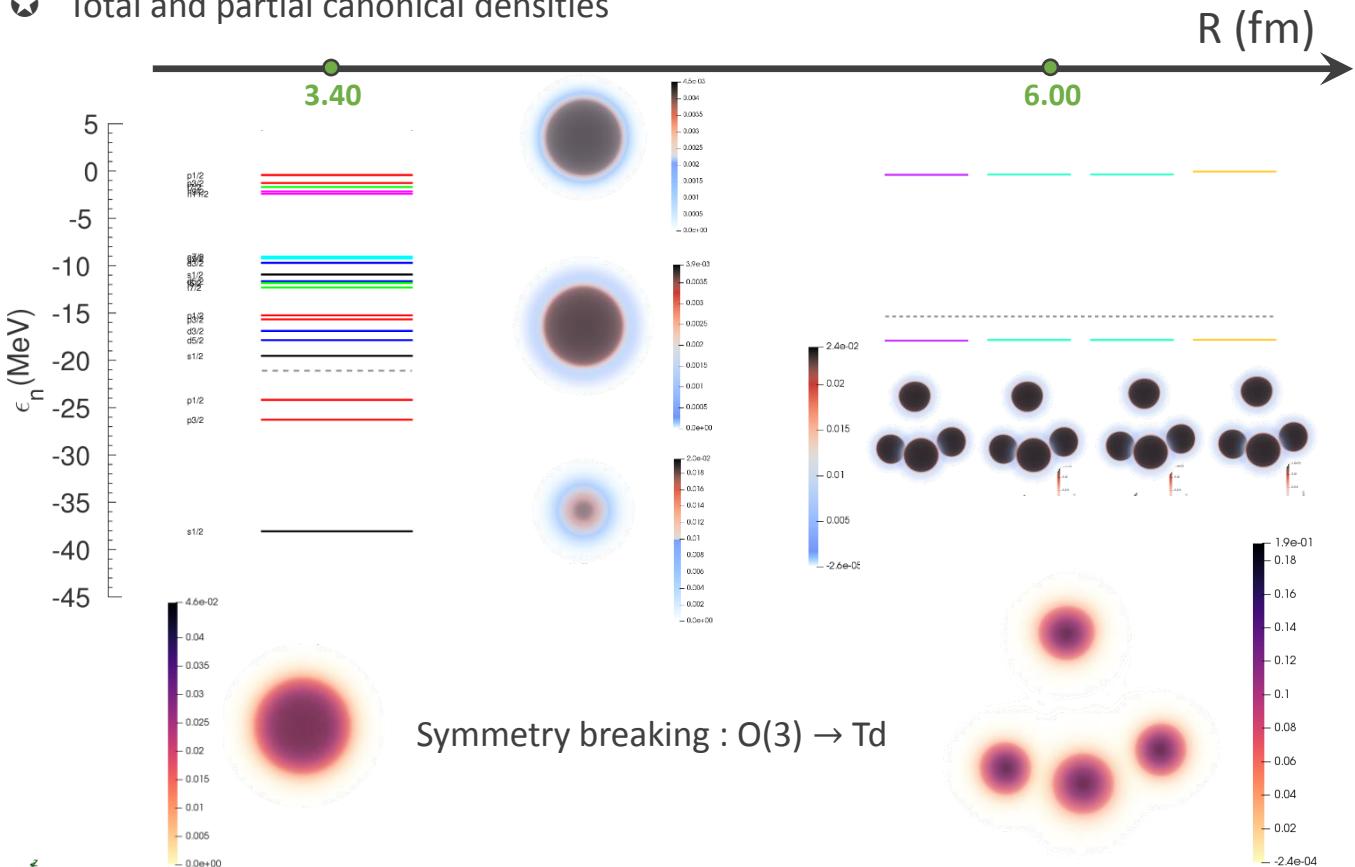
## ★ Total and partial canonical densities



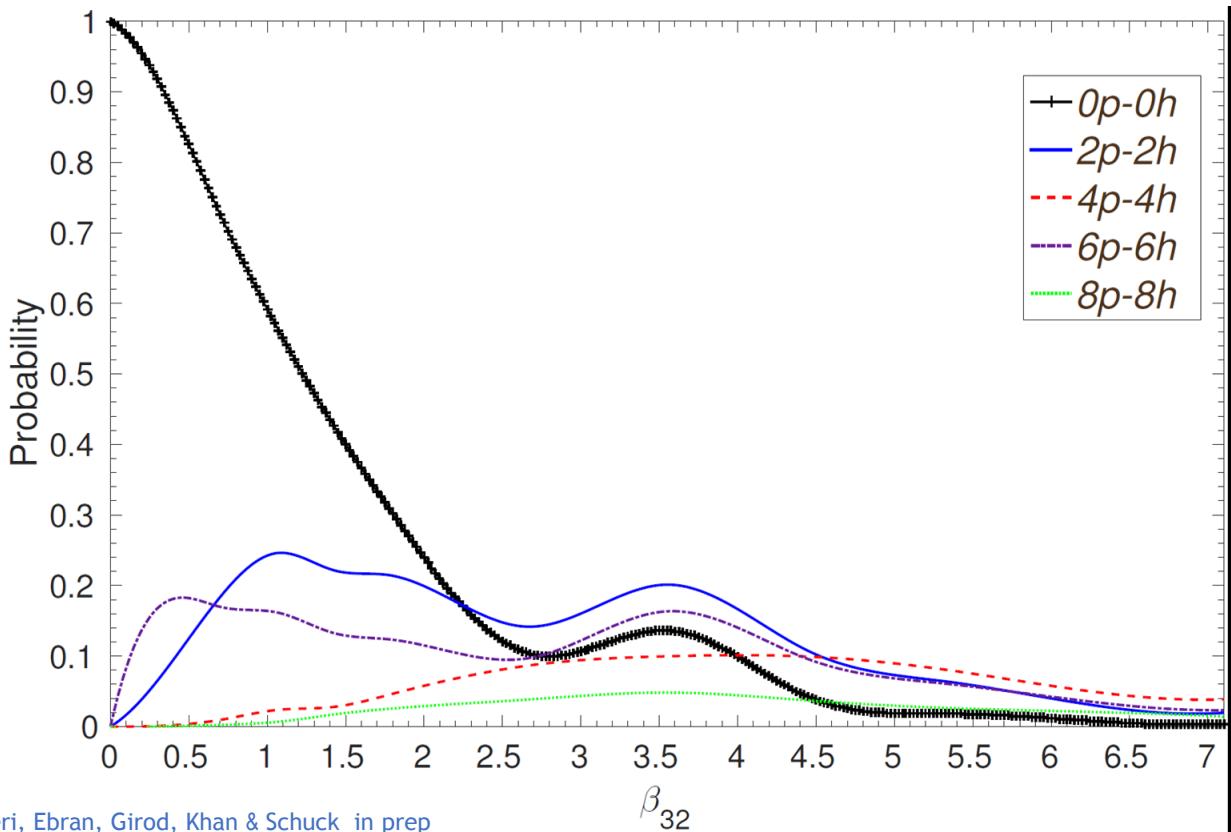
## ★ Pauli blocking



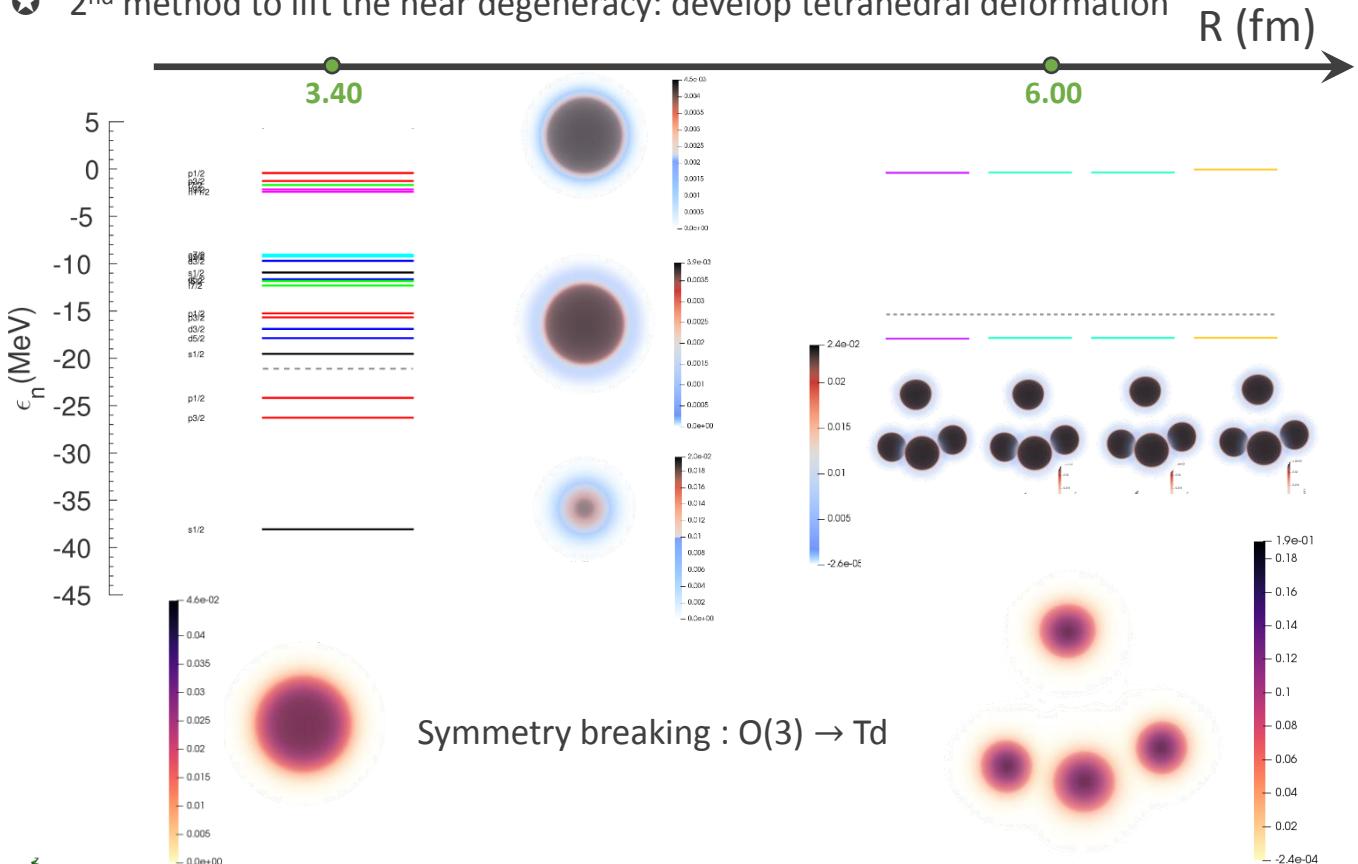
## ★ Total and partial canonical densities



- ★ Link the tetrahedral wf to the spherical ones



- ★ 2<sup>nd</sup> method to lift the near degeneracy: develop tetrahedral deformation



- ★ Link the tetrahedral wf to the spherical ones

Onishi & Sheline Nucl Phys A 165 180 (1971)

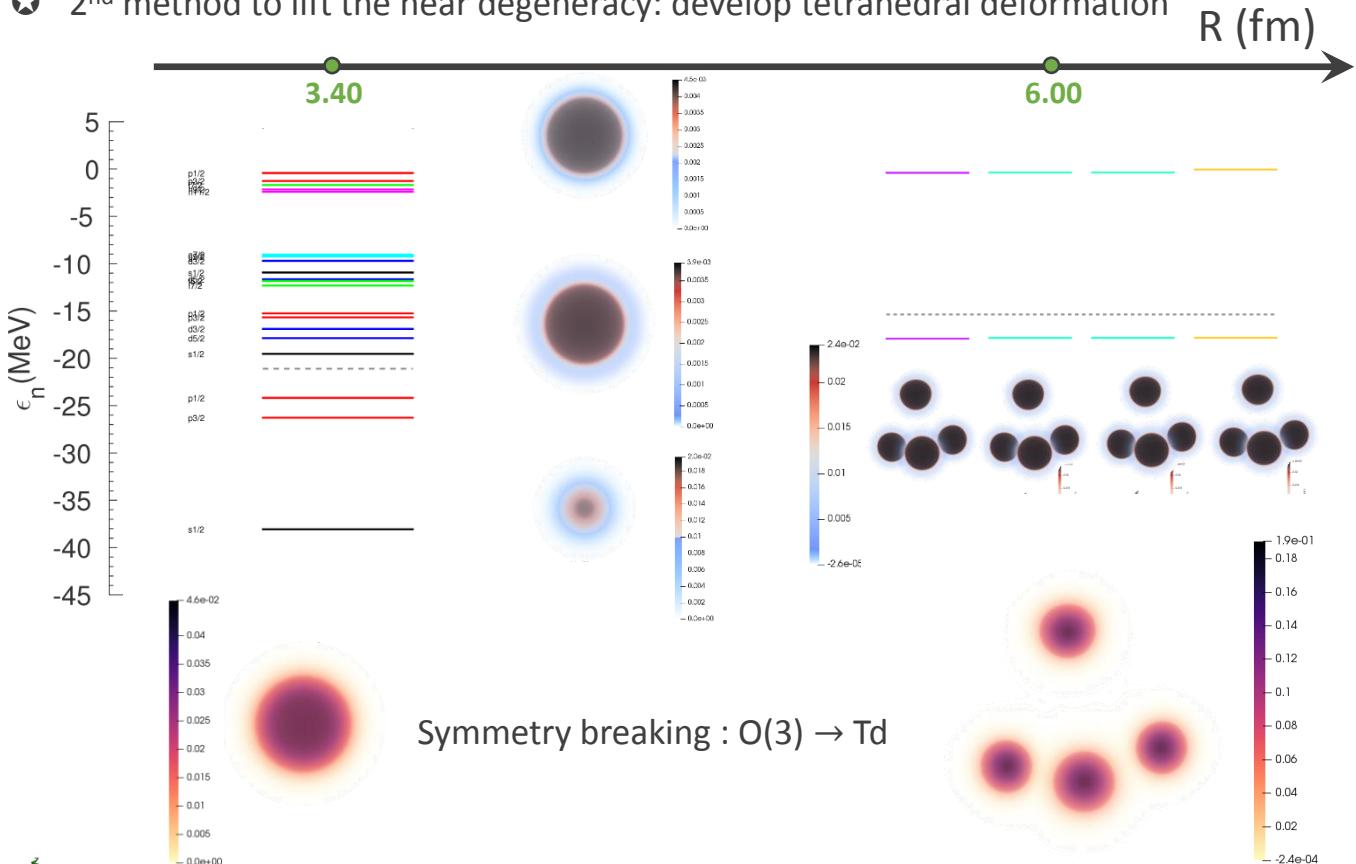
Character table for Td group

	$E$	$\bar{E}$	$8C_3$	$8\bar{C}_3$	$3C_2$	$6S_4$	$6\bar{S}_4$	$6\sigma_d$	$6\bar{\sigma}_d$
					$3\bar{C}_2$				
$\Gamma_1$	1	1	1	1	1	1	1	1	1
$\Gamma_2$	1	1	1	1	1	-1	-1	-1	-1
$\Gamma_3$	2	2	-1	-1	2	0	0	0	0
$\Gamma_4$	3	3	0	0	-1	1	1	-1	
$\Gamma_5$	3	3	0	0	-1	-1	-1	1	
$\Gamma_6$	2	-2	1	-1	0	$\sqrt{2}$	$-\sqrt{2}$	0	
$\Gamma_7$	2	-2	1	-1	0	$-\sqrt{2}$	$\sqrt{2}$	0	
$\Gamma_8$	4	-4	-1	1	0	0	0	0	

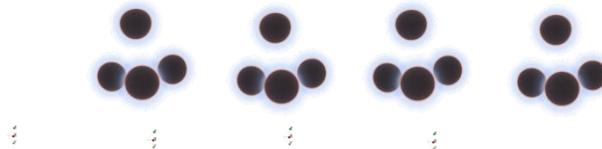
Full rotation group compatibility table for the group Td

$D_0^+$	$\Gamma_1$	$D_0^-$	$\Gamma_2$
$D_1^+$	$\Gamma_4$	$D_1^-$	$\Gamma_5$
$D_2^+$	$\Gamma_3 + \Gamma_5$	$D_2^-$	$\Gamma_3 + \Gamma_4$
$D_3^+$	$\Gamma_2 + \Gamma_4 + \Gamma_5$	$D_3^-$	$\Gamma_1 + \Gamma_4 + \Gamma_5$
$D_4^+$	$\Gamma_1 + \Gamma_3 + \Gamma_4 + \Gamma_5$	$D_4^-$	$\Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5$
$D_{\frac{1}{2}}^+$	$\Gamma_6$	$D_{\frac{1}{2}}^-$	$\Gamma_7$
$D_{\frac{3}{2}}^+$	$\Gamma_8$	$D_{\frac{3}{2}}^-$	$\Gamma_8$
$D_{\frac{5}{2}}^+$	$\Gamma_7 + \Gamma_8$	$D_{\frac{5}{2}}^-$	$\Gamma_6 + \Gamma_8$
$D_{\frac{7}{2}}^+$	$\Gamma_6 + \Gamma_7 + \Gamma_8$	$D_{\frac{7}{2}}^-$	$\Gamma_6 + \Gamma_7 + \Gamma_8$
$D_{\frac{9}{2}}^+$	$\Gamma_6 + 2\Gamma_8$	$D_{\frac{9}{2}}^-$	$\Gamma_7 + 2\Gamma_8$

- ★ 2<sup>nd</sup> method to lift the near degeneracy: develop tetrahedral deformation



## ★ LCAO-MO picture



$$\psi_i = \sum_{j=1}^4 f_j^i \phi_j$$

- ★ Find the unknown  $f_j$  from eigenvalue problem involving the norm and energy kernels between the atomic orbitals.
- ★ Hückel approximation :  $\mathcal{N}_{ij} = 0 \forall i, j$

$$\mathcal{H}_{ij} = 0 \text{ except for adjacent } i,j \quad \epsilon \equiv \mathcal{H}_{ii} \quad -\mu \equiv \mathcal{H}_{ij}$$

$$\begin{pmatrix} \epsilon & -\mu & -\mu & -\mu \\ -\mu & \epsilon & -\mu & -\mu \\ -\mu & -\mu & \epsilon & -\mu \\ -\mu & -\mu & -\mu & \epsilon \end{pmatrix} \begin{pmatrix} f_1^i \\ f_2^i \\ f_3^i \\ f_4^i \end{pmatrix} = E_i \begin{pmatrix} f_1^i \\ f_2^i \\ f_3^i \\ f_4^i \end{pmatrix}$$

$$\psi_1 = \frac{1}{2} (\phi_1 + \phi_2 + \phi_3 + \phi_4) \quad E_1 = \epsilon - 3\mu.$$

$$\psi_2 = \frac{1}{\sqrt{2}} (-\phi_1 + \phi_2) \quad E_2 = \epsilon + \mu$$

$$\psi_3 = \frac{1}{\sqrt{2}} (-\phi_1 + \phi_3) \quad E_3 = E_2$$

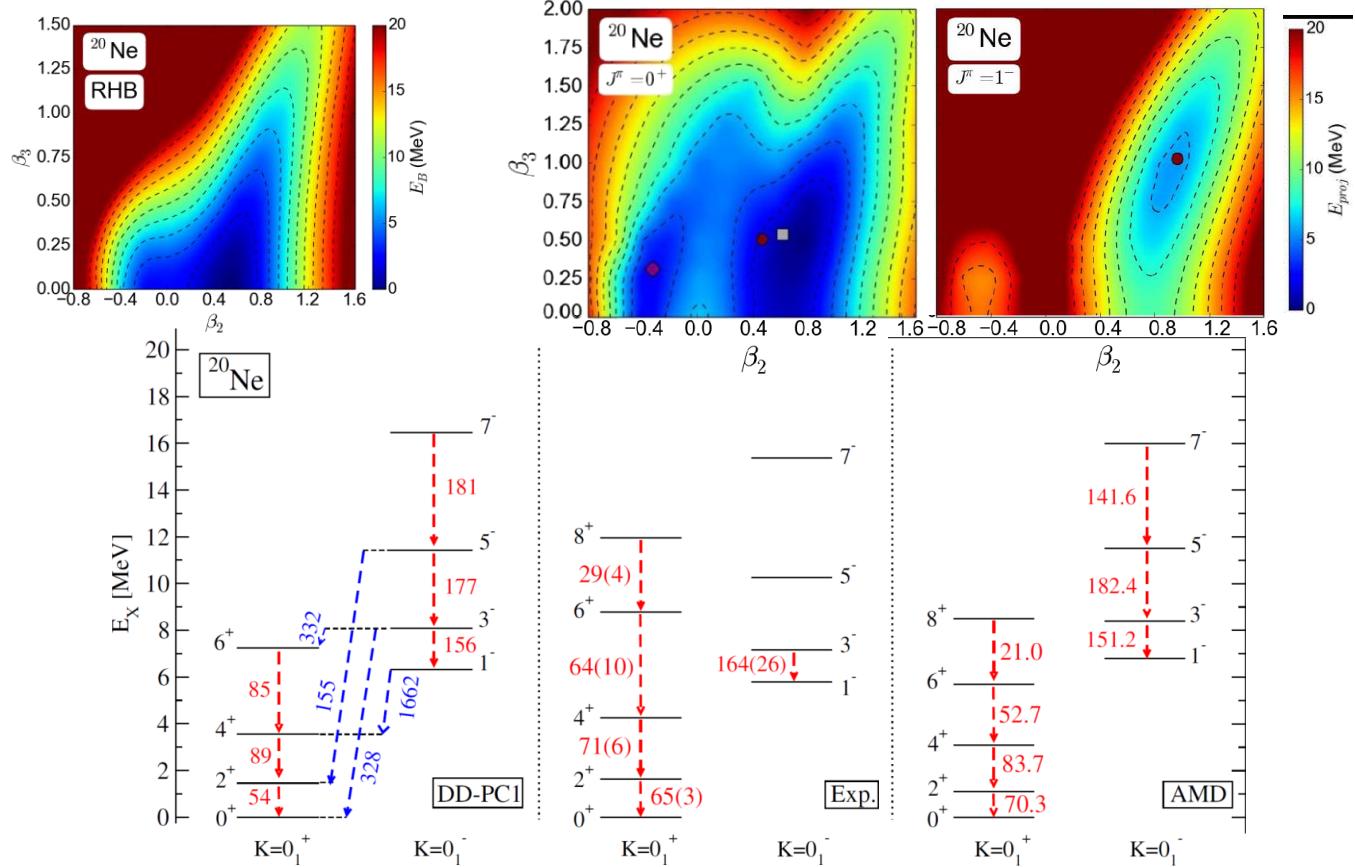
$$\psi_4 = \frac{1}{\sqrt{2}} (-\phi_1 + \phi_4) \quad E_4 = E_3 = E_2$$



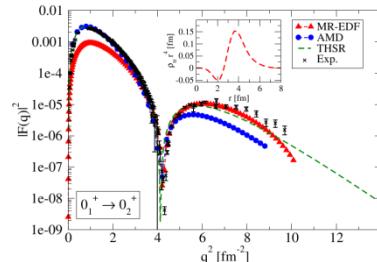
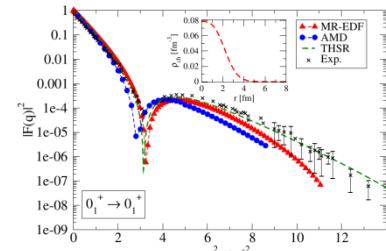
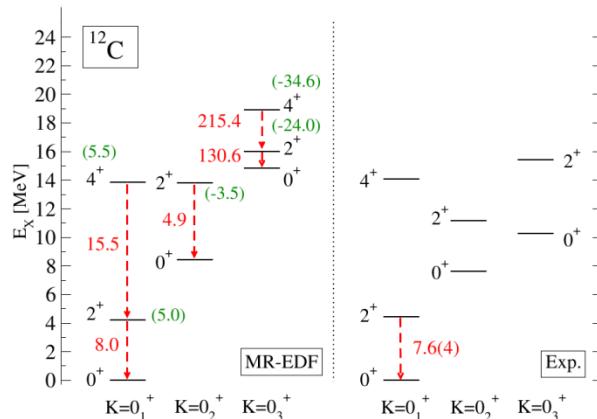
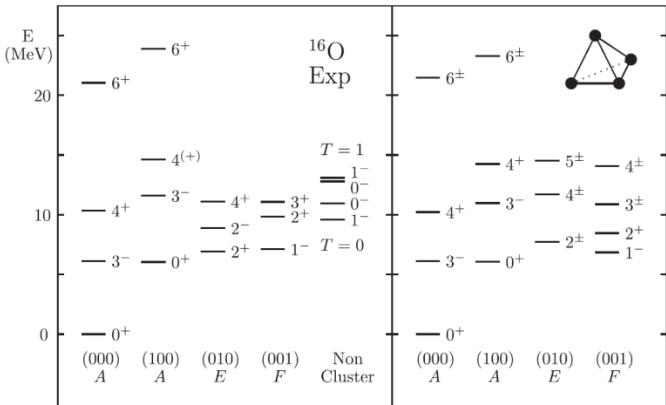
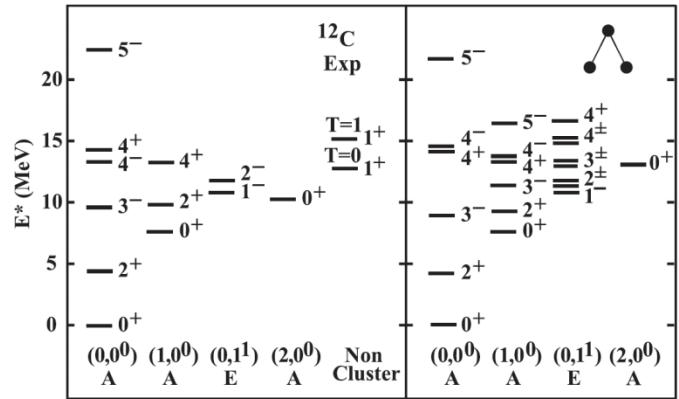
$$\psi'_2 = \frac{1}{2} (\phi_1 - \phi_2 - \phi_3 + \phi_4),$$

$$\psi'_3 = \frac{1}{2} (\phi_1 + \phi_2 - \phi_3 - \phi_4),$$

$$\psi'_4 = \frac{1}{2} (-\phi_1 + \phi_2 - \phi_3 + \phi_4).$$



Bijker (2016)



Marević, Ebran, Khan, Nikšić, and Vretenar PRC 99 034317 (2019)

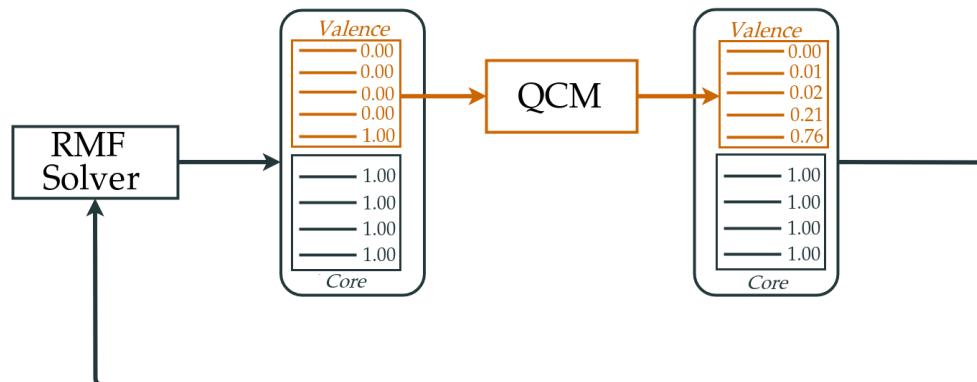
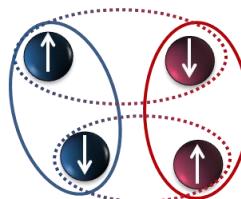
- 1 The Energy Density Functional approach
- 2 Clustering in the EDF language
- 3 EDF + QCM

## ★ RMF + QCM

$$|\Psi\rangle = (\mathcal{Q}^\dagger)^{n_q} |0\rangle$$

$$\mathcal{Q}^\dagger = 2\Gamma_1^\dagger\Gamma_{-1}^\dagger - (\Gamma_0^\dagger)^2$$

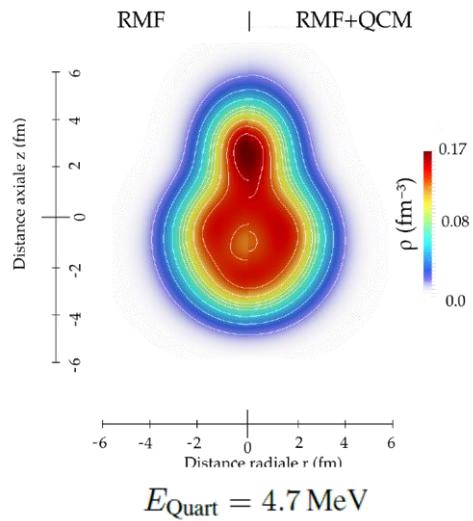
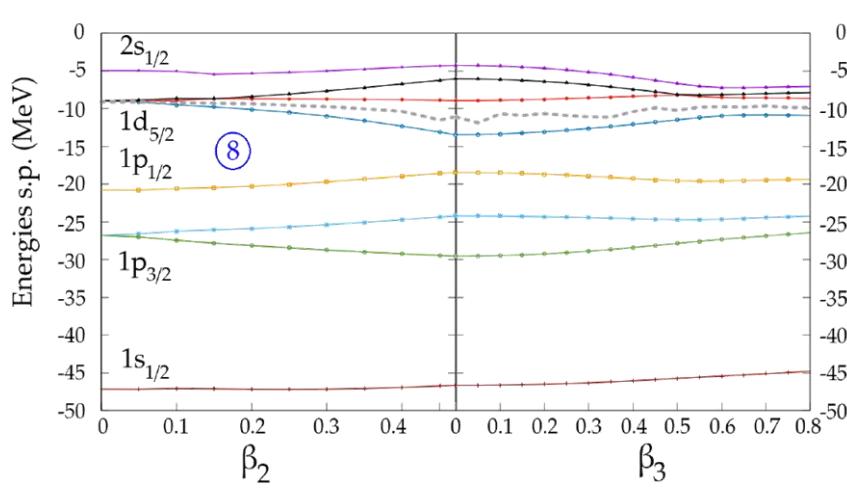
$$\Gamma_t^\dagger = \sum_k x_k P_{k,t}^\dagger$$



R.D. Lasseri, PhD Thesis (2018)

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- Example :  $^{20}\text{Ne}$  described in terms of 1 quartet on top of a Fermi vacuum associated to 16 nucleons



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## ★ Spatial distribution function

- ◎ All the information of a many-body system is contained in its Nth-order density matrix

$$D_N(1, \dots, N; 1', \dots, N') = \Psi(1, \dots, N) \Psi^*(1', \dots, N')$$

- ◎ Only keep information about p-“cluster” embedded in the medium composed by the other N-p particles :

$$\Gamma_p(1, \dots, p; 1', \dots, p') = \binom{N}{p} \int d(p+1) \dots dN \Psi(1, \dots, N) \Psi^*(1', \dots, p', p+1, \dots, N)$$

- ◎ Reduced density matrices are trace-class operators

$$\Gamma_p(1, \dots, p; 1', \dots, p') = \sum_k n_k^{(p)} \chi_k^{(p)}(1, \dots, p) \chi_k^{*(p)}(1', \dots, p')$$

Eigenfunctions provide an in-medium wave function for a subgroup built out of p particles

★ Spatial distribution function : 2-RDM

$$\begin{aligned}\Gamma(1, 2; 1' 2') &= \frac{1}{2!} \langle \Phi_B | \psi^\dagger(2') \psi^\dagger(1') \psi(1) \psi(2) | \Phi_B \rangle \\ &= \frac{1}{2} \{ \rho(1, 1') \rho(2, 2') - \rho(1, 2') \rho(2, 1') \\ &\quad + \kappa^*(1', 2') \kappa(2, 1) \}.\end{aligned}$$

★ Practical quantities

$$\Xi(1, 2; 1', 2') \equiv \chi^*(1', 2') \chi(2, 1)$$

$$\Xi^{nn}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_{1'}, \mathbf{r}_{2'}) \equiv Tr_{\text{spin}} \Xi(\mathbf{r}_1 \sigma_1 n, \mathbf{r}_2 \sigma_2 n; \mathbf{r}_{1'} \sigma_{1'} n, \mathbf{r}_{2'} \sigma_{2'} n)$$

$$\Xi^{nn}(\mathbf{R}, \mathbf{r}) \equiv \Xi^{nn}(\mathbf{R}, \mathbf{r}; \mathbf{R}, \mathbf{r}) \quad \mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}; \quad r = r_1 - r_2$$

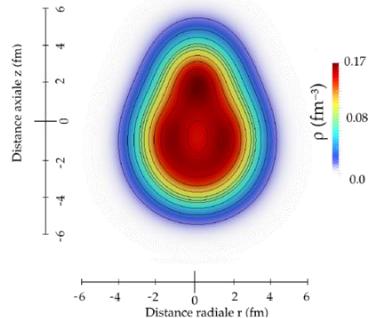
$$\Xi^{nn}(R, r) = \mathcal{N}_\Xi(4\pi R r \mid \chi(R, r) \mid)^2$$

$$\xi(R) \equiv \frac{\int dr r^2 \Xi^{nn}(R, r)}{\int dr \Xi^{nn}(R, r)}$$

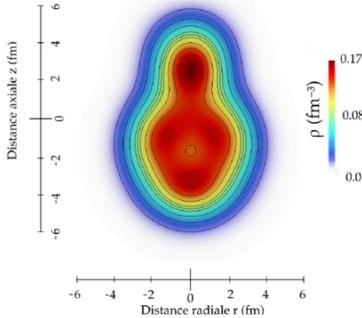
$$\xi \equiv \int dR dr r^2 \Xi^{nn}(R, r)$$

⇒  $^{20}\text{Ne}$  configuration such that  $E_{\text{corr}} = 5 \text{ MeV}$  : maximum of  $Q(r_1=r_2=r_3=r_4)$  matches the position of the  $\alpha$

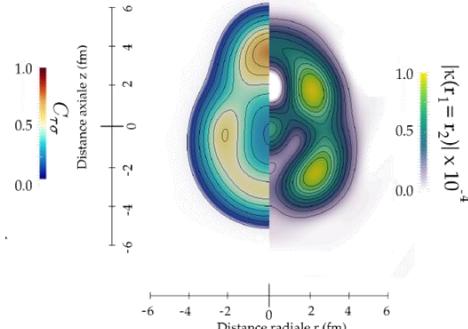
RHB



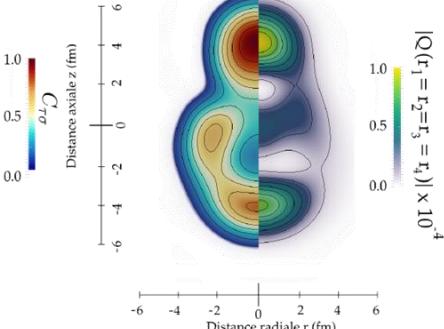
RMF+QCM



RHB



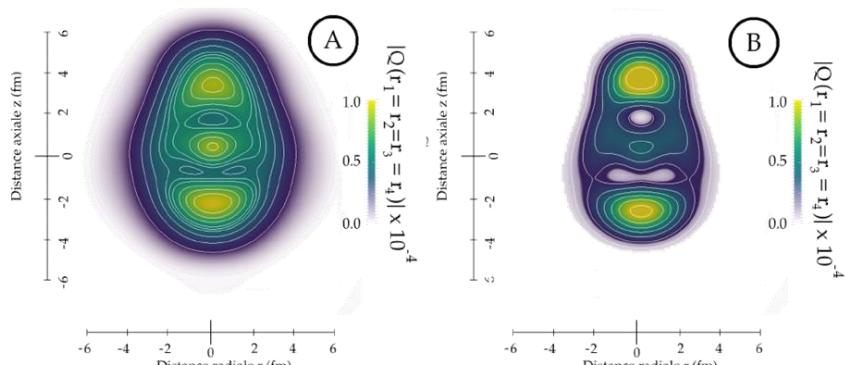
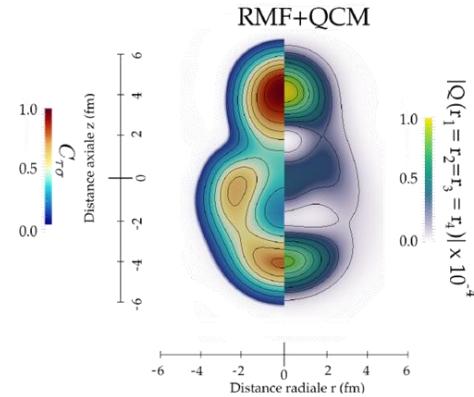
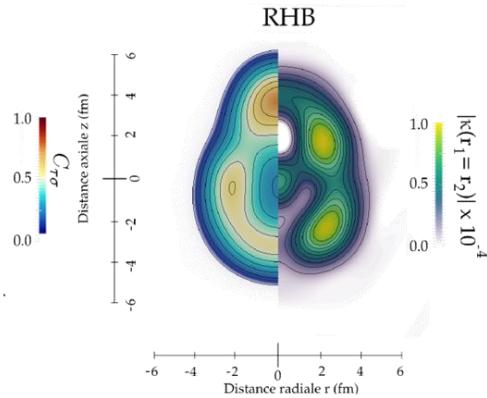
RMF+QCM



R.D. Lasseri, PhD Thesis (2018)

Lasseri, Ebran, Khan and Sandulescu in prep

⇒  $^{20}\text{Ne}$  configuration such that  $\text{Ecorr} = 5 \text{ MeV}$  : maximum of  $Q(r_1=r_2=r_3=r_4)$  matches the position of the  $\alpha$



⇒ Proton-neutron channel crucial to localize the quartet into an  $\alpha$  like structure

- ★ Hierarchy of correlations at the heart of nuclear EDFs
- ★ Clustering as emergent spatial symmetry breaking towards a discrete subgroup
- ★ Localization influenced by the depth of the confining potential, the mean density, ...
- ★ Density induced QPT of the Mott type at subsaturation density
- ★ Links alpha-clustering and quartetting, quartet coherence length, conditional probability distribution, ...