Particle-number projected Bogoliubov coupled cluster formalism

From weakly to strongly correlated systems



Recent advances on proton-neutron pairing and quartet correlations in nuclei ESNT workshop, Sept 2-6 2019, Saclay, France



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• Breaking and restoring symmetries in quantum many-body theory

- Prolegomena
- Bogoliubov coupled cluster formalism
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- Application to Richardson/BCS pairing Hamiltonian
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Ab initio nuclear chart

- Approximate methods for closed-shells
 - \circ Since 2000's
 - \circ MBPT, SCGF, CC, IMSRG
 - Polynomial scaling

- Approximate methods for open-shells
 - \circ Since 2010's
 - (P)BMBPT, GGF, (P)BCC, MR-IMSRG, MCPT
 - Polynomial scaling



Single-reference expansion many-body methods

Nuclear Hamiltonian

 $H = T + V^{2N} + W^{3N}$

Symmetry group U(1) dealt with today [H, S] = 0 where $S \equiv A, J^2, J_z \dots$

Mean-field reference state

 $[H_0, S] = 0$ $H = H_0 + H_1$ such that $[H_1, S] = 0$ $H_0 |\Phi_0^S\rangle = \mathcal{E}_0^S |\Phi_0^S\rangle$ Exactly solvable. **Open-shell Closed-shell** Non-degenerate No**Dedregnemate**te In proper statitig opinint Good starting point

A-body eigenvalue problem

 $H|\Psi_0^{\rm S}\rangle = E_0^{\rm S}|\Psi_0^{\rm S}\rangle \quad \ {\rm N^{\rm A}\ cost\ where\ N} = \dim\ {\cal H}_1$

Many-body expansion



Wave operator Reference state

Accounts for « weak/dynamical » correlations
 Expand as a series (MBPT, CC...) + truncate = N^p cost

 $[H'_{0}, S] \neq 0$ $[H'_{1}, S] \neq 0$ $H = H'_{0} + H'_{1}$ $|\Psi^{S}_{0}\rangle = U(\infty)|\Phi_{0}\rangle$ More general reference state Accounts for "strong/non-dynamical" correlations $Expand (BMBPT, BCC...) + truncate = N^{p} cost$

- 1) Truncated series breaks symmetry
- 2) Exact symmetry must eventually be restored

Single-reference expansion many-body methods and symmetries

Nuclear Many-Body Methods



Single-reference expansion many-body methods and symmetries

Nuclear Many-Body Methods



Single-reference expansion many-body methods and symmetries

Nuclear Many-Body Methods



Today: BCC and Projected BCC formalism applied to the pairing (Richardson) Hamiltonian



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Operators

Nuclear Hamiltonian

Particle number

$$H \equiv \frac{1}{(1!)^2} \sum_{pq} t_{pq} c_p^{\dagger} c_q$$

+ $\frac{1}{(2!)^2} \sum_{pqrs} \overline{v}_{pqrs} c_p^{\dagger} c_q^{\dagger} c_s c_r$
+ $\frac{1}{(3!)^2} \sum_{pqrstu} \overline{w}_{pqrstu} c_p^{\dagger} c_q^{\dagger} c_r^{\dagger} c_u c_t c_s$

$$A \equiv \sum_{p} c_{p}^{\dagger} c_{p}$$

Genuine 3N interaction / six-legs vertex



Controls the average particle number in the system

k-body force ↓ Mode-2k tensor ↓ Basis representation dim N ↓ Storage cost N^{2k}

Bogoliubov reference state and normal ordering

Bogoliubov reference state

Breaks U(1) symmetry

$$\beta_{k} = \sum_{p} U_{pk}^{*} c_{p} + V_{pk}^{*} c_{p}^{\dagger} \qquad |\Phi\rangle \equiv C \prod_{k} \beta_{k} |0\rangle \qquad A |\Phi\rangle \neq A |\Phi\rangle$$
$$\beta_{k}^{\dagger} = \sum_{p} U_{pk} c_{p}^{\dagger} + V_{pk} c_{p} \qquad \beta_{k} |\Phi\rangle = 0 \quad \forall k \qquad \text{Vacuum state} \\ \text{Reduces to SD in } \mathcal{H}_{A} \text{ for closed-shell}$$

Normal ordering via Wick's theorem in quasi-particle basis

$$H \equiv \sum_{n=0}^{3} \sum_{i+j=2n} \frac{1}{i!j!} \sum_{l_1...l_{i+j}} H_{l_1...l_{i+j}}^{ij} \beta_{k_1}^{\dagger} \dots \beta_{k_i}^{\dagger} \beta_{k_{i+j}} \dots \beta_{k_{i+1}}$$

$$H^{ij} \text{ matrix elements function of}$$

$$t_{pq} \ \overline{v}_{pqrs} \ \overline{w}_{pqrstu} \ U_{pk} \ V_{pk}$$

$$\equiv H^{00} + [H^{20} + H^{11} + H^{02}] + [H^{40} + H^{31} + H^{22} + H^{13} + H^{04}] + \sum_{i+j=6} H^{ij}$$

$$\equiv \sum_{n=0}^{2} H^{[2n]} + H^{[6]} \quad 6\text{-qp operators}$$
Similarly for A and Ω
Six-index tensors
Too expensive to handle
NO2B approximation
1-3% error in closed shell
[Roth *et al.*, PRL 109 (2012) 052501]

H^{ij} matrix elements function of
$$t_{pq} \ \overline{v}_{pqrs} \ \overline{w}_{pqrstu} \ U_{pk} \ V_{pk}$$



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BCC formalism



Bogoliubov many-body perturbation formalism

Perturbative reduction of BCC

[Duguet, Signoracci JPG 2016]

- -> Code for automated generation&evaluation of many-body diagrams to arbitrary order [Arthuis et al. CPC 2018]
- → Convergence properties at high orders and resummation methods [Demol et al. to be published 2019]

OBMBPT(2) ab initio calculations of mid-mass semi-magic nuclei [Tichai et al. PLB 2018]



- → 2-3% agreement of all methods with exact results (IT-NCSM)
- Different truncation schemes yield consistent description of open-shell nuclei
- BMBPT optimal to systematically test next generation of Chiral EFT nuclear Hamiltonians

• Future implementation of BCCSD(T) for accurate ab initio calculations of open-shell nuclei



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U(1) breaking and projection



Particle-number projected BCC formalism

Projected BCC ansatz

[Duguet, Signoracci JPG 2016] [Qiu *et al.* PRC 2019]

Similarity transformed operator

 $O_Z(\varphi) \equiv e^{Z(\varphi)} O e^{-Z(\varphi)}$

$$|\Psi_{PBCC}^{A}\rangle \equiv P^{A}|\Psi_{BCC}^{A}\rangle$$
 Alw

Always true!

Projected BCC energy and particle number

$$H|\Psi^{A}\rangle = E^{A}|\Psi^{A}\rangle \stackrel{\langle\Phi|}{\Longrightarrow} = A^{A}|\Psi^{A}\rangle \stackrel{\langle\Phi|}{\to} = A^{A}|\Psi^$$

 $\begin{pmatrix} \beta(\varphi) \\ \beta^{\dagger}(\varphi) \end{pmatrix} \equiv e^{Z(\varphi)} \begin{pmatrix} \beta \\ \beta^{\dagger} \end{pmatrix} e^{-Z(\varphi)}$ $= \begin{pmatrix} 1 & 0 \\ Z^{20}(\varphi) & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \beta^{\dagger} \end{pmatrix}$

Non-unitary Bogoliubov transformation

Normal-ordered operator with ME $O_{k_1...k_{i+j}}^{ij}(\varphi)$

↑E[ρ; |q|]

 $|\Phi^{A}\rangle$

Not a pure excitation operator... with $\begin{aligned}
\mathcal{N}(\varphi) &\equiv \langle \Phi(\varphi) | e^{U} | \Phi \rangle &= \langle \Phi(\varphi) | \Phi \rangle \langle \Phi | e^{U_{Z}(\varphi)} | \Phi \rangle \\
\mathcal{H}(\varphi) &\equiv \langle \Phi(\varphi) | H e^{U} | \Phi \rangle &= \langle \Phi(\varphi) | \Phi \rangle \langle \Phi | H_{Z}(\varphi) e^{U_{Z}(\varphi)} | \Phi \rangle \\
\mathcal{H}(\varphi) &\equiv \langle \Phi(\varphi) | A e^{U} | \Phi \rangle &= \langle \Phi(\varphi) | \Phi \rangle \langle \Phi | A_{Z}(\varphi) e^{U_{Z}(\varphi)} | \Phi \rangle
\end{aligned}$

Off-diagonal norm, Hamiltonian and particle-number kernels

Particle-number projected BCC formalism



Particle-number projected BCC formalism



Integrate coupled ODEs and insert in PBCC energy

[Qiu et al. PRC 2019]

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BCS pairing Hamiltonian

Hamiltonian

$$H = \sum_{p} \left(\epsilon_{p} - \lambda \right) N_{p} - G \sum_{pq} P_{p}^{\dagger} P_{q}$$

Doubly-degenerate picket fence model

 $\epsilon_p = \epsilon_{\bar{p}} = p \Delta \epsilon$

Exact ground-state energy

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Richardson solution
[R.W. Richardson, PL 1963; PR 1966]
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What about BCC? [Henderson *et al.* PRC 2014] BCCSD: $U \approx U_1 + U_2$

What about PBCC? [Qiu et al. PRC 2019]

PBCCSD(2): $\mathbf{U} \approx \mathbf{U}_1 + \mathbf{U}_2$

Operators

$$\begin{split} N_p &= c_p^{\dagger} c_p + c_{\bar{p}}^{\dagger} c_{\bar{p}} \\ P_p^{\dagger} &= c_p^{\dagger} c_{\bar{p}}^{\dagger} \end{split}$$

SU(2) algebra

$$[P_p, P_q^{\dagger}] = \delta_{pq} \left(1 - N_p \right)$$
$$[N_p, P_q^{\dagger}] = 2 \,\delta_{pq} \, P_q^{\dagger}$$

Typical approximate methods

- BCS and projected BCS (before variation)
- Coupled cluster theory with doubles
- Self-consistent RPA



Connected hamiltonian kernel



150 100 levels 100 Half filling $G/G_{c} = 1.5$ arg(*h*(*φ*)) (rad) 50 0 -50 -100PBCS-CCSD PBCS -150-1.5-1.0-0.50.0 0.5 1.0 1.5 Φ

PBCC energy and kernels

$$E^{\rm A} = \frac{\int_{0}^{2\pi} d\varphi \, e^{-i{\rm A}\varphi} \, h(\varphi) \mathcal{N}(\varphi)}{\int_{0}^{2\pi} d\varphi \, e^{-i{\rm A}\varphi} \, \mathcal{N}(\varphi)} \qquad \text{with}$$

Exact (or symmetry-conserving) limit

$$\mathcal{N}(\varphi) = e^{iA\varphi}$$
$$\frac{d}{d\varphi}h(\varphi) = 0$$

$$\mathcal{N}(\varphi) \equiv e^{W_0(\varphi)} \langle \Phi(\varphi) | \Phi \rangle$$
$$h(\varphi) \equiv \frac{\mathcal{H}(\varphi)}{\mathcal{N}(\varphi)} = \langle \Phi | H_Z(\varphi) e^{\mathcal{T}(\varphi)} | \Phi \rangle_C$$

$\textcircled{O} PBCC \leftrightarrow PBCS\text{-}CC(S)D here$

- \circ h($\phi)$ real with typical bell-shape curve
- \circ PBCC brings h(ϕ) closer to constant
- \circ Not constant h(ϕ) induces non-trivial projection

Norm kernel



Exact (or symmetry-conserving) limit

$$\mathcal{N}(\varphi) = e^{i\mathbf{A}\varphi}$$
$$\frac{d}{d\varphi}h(\varphi) = 0$$

• PBCC \leftrightarrow PBCS-CC(S)D here

 \circ $|\mathcal{N}(\phi)|$ displays bell-shape curve and phase $\propto A$

- \circ PBCC brings $\mathcal{N}(\phi)$ closer to single IRREP $e^{iA\phi}$
- $\circ \mathcal{N}(0) = 1 \leftrightarrow$ Intermediate normalization

Results - 1

[Qiu *et al.* PRC 2019]







● BCC ↔ BCS-CC(S)D here

- Extends quality of CC through phase transition
- Better than PBCS except for for G>>G_c
- \circ Poor in small systems

() PBCC \leftrightarrow PBCS-CC(S)D here

- Perfect from weak to strong coupling
- Perfect from small to large systems
- Dominates all other methods

Results - 2



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Conclusions

Output Particle-number projected Bogoliubov coupled cluster and many-body perturbation theories

Extends single-reference CC/MBPT methods to open-shell nuclei via symmetry breaking&restoration

First consistent formulation of symmetry restoration techniques beyond the mean-field

Results obtained for U(1) on the solvable Richardson Hamiltonian hold great promises

• Future

Ab initio PBCC and PBMBPT calculations of singly open-shell nuclei

[Tichai, Ripoche, Duguet, in progress]

• Apply to SU(2) (already formulated) for ab initio calculations of doubly open-shell nuclei

[Tichai, Hagen, Duguet, in progress]

• Combine with IT/TF techniques to go to heavier nuclei

[Tichai, Ripoche, Duguet, EPJA 2019]

Extend BCC and PBCC to excited states

[Demol, Tichai, Duguet, planned]

Current collaborators on ab initio many-body calculations



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