## Particle-number projected Bogoliubov coupled cluster formalism

From weakly to strongly correlated systems


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Recent advances on proton-neutron pairing and quartet correlations in nuclei ESNT workshop, Sept 2-6 2019, Saclay, France


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## Ab initio nuclear chart

O Approximate methods for closed-shells

- Since 2000's
- MBPT, SCGF, CC, IMSRG
- Polynomial scaling
© Approximate methods for open-shells
- Since 2010's
- (P)BMBPT, GGF, (P)BCC, MR-IMSRG, MCPT
- Polynomial scaling

© "Exact" methods
- Since 1980's
- Monte Carlo, CI, ...
- Factorial/exponential scaling

N Bold = symmetry breaking (\&restoration) single-reference methods

## Single-reference expansion many-body methods

## Nuclear Hamiltonian

$$
H=T+V^{2 \mathrm{~N}}+W^{3 \mathrm{~N}}
$$

Symmetry group
U(1) dealt with today $[H, S]=0 \quad$ where $\quad S \equiv A, J^{2}, J_{z} \ldots$

## Mean-field reference state

| $H=H_{0}+H_{1}$ such that | $\left[H_{0}, S\right]=0$ |
| :---: | :---: |
| $H_{0}\left\|\Phi_{0}^{\mathrm{S}}\right\rangle=\mathcal{E}_{0}^{\mathrm{S}}\left\|\Phi_{0}^{\mathrm{S}}\right\rangle$ | Exactly solvable |

Closed-shell


Non-degenerate Good starting point

Open-shell


NoDedfegrenatate

IRRpoppestextitiggpiotit

A-body eigenvalue problem

$$
H\left|\Psi_{0}^{\mathrm{S}}\right\rangle=E_{0}^{\mathrm{S}}\left|\Psi_{0}^{\mathrm{S}}\right\rangle \quad \mathrm{N}^{\mathrm{A}} \text { cost where } \mathrm{N}=\operatorname{dim} \mathcal{F}_{1}
$$

Many-body expansion

$$
H=H_{0}+H_{1}
$$

$$
\left|\Psi_{0}^{S}\right\rangle=U^{S}(\infty)\left|\Phi_{0}^{S}\right\rangle
$$

Wave operator Reference state

- Accounts for « weak/dynamical » correlations
- Expand as a series (MBPT, CC...) + truncate $=\mathrm{N}^{\mathrm{p}}$ cost
$\left[H_{0}^{\prime}, S\right] \neq 0$
$\left[H_{1}^{\prime}, S\right] \neq 0$


Symmetry breaking
$H=H_{0}^{\prime}+H_{1}^{\prime}$

$$
\left|\Psi_{0}^{\mathrm{S}}\right\rangle=\underline{U(\infty)\left|\Phi_{0}\right\rangle} \begin{aligned}
& \text { More general } \\
& \text { reference state }
\end{aligned}
$$

-Accounts for "strong/non-dynamical" correlations

- Expand (BMBPT, BCC...) + truncate $=\mathrm{N}^{\mathrm{p}}$ cost

1) Truncated series breaks symmetry
2) Exact symmetry must eventually be restored

## Single-reference expansion many-body methods and symmetries

## Nuclear Many-Body Methods



## Single-reference expansion many-body methods and symmetries

Nuclear Many-Body Methods


## Single-reference expansion many-body methods and symmetries

Nuclear Many-Body Methods


Today: BCC and Projected BCC formalism applied to the pairing (Richardson) Hamiltonian

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## Operators

## Nuclear Hamiltonian

$H \equiv \frac{1}{(1!)^{2}} \sum_{p q} t_{p q} c_{p}^{\dagger} c_{q}$
$+\frac{1}{(2!)^{2}} \sum_{\text {pqrs }} \bar{v}_{\text {pqrs }} c_{p}^{\dagger} c_{q}^{\dagger} c_{s} c_{r}$
$\left.+\frac{1}{(3!)^{2}} \sum_{p q r s t u} \bar{w}_{p q r s t u} c_{p}^{\dagger} c_{q}^{\dagger} c_{r}^{\dagger} c_{u} c_{t} c_{s}\right]$

Grand potential
When working in Fock space
$\Omega \equiv H-\underline{\lambda} A$
Chemical potential
$\downarrow$

## Bogoliubov reference state and normal ordering

## Bogoliubov reference state

$$
\begin{array}{ll}
\beta_{k}=\sum_{p} U_{p k}^{*} c_{p}+V_{p k}^{*} c_{p}^{\dagger} & |\Phi\rangle \equiv C \prod_{k} \beta_{k}|0\rangle \\
\beta_{k}^{\dagger}=\sum_{p} U_{p k} c_{p}^{\dagger}+V_{p k} c_{p} & \beta_{k}|\Phi\rangle=0 \forall k
\end{array}
$$

## Breaks U(1) symmetry

$A|\Phi\rangle \neq \mathrm{A}|\Phi\rangle$

Vacuum state
Reduces to SD in $\mathscr{F}_{A}$ for closed-shell

Normal ordering via Wick's theorem in quasi-particle basis

$$
\begin{array}{rlr}
H & \equiv \sum_{n=0}^{3} \sum_{i+j=2 n} \frac{1}{i!j!} \sum_{l_{1} \ldots l_{i+j}} H_{l_{1} \ldots l_{i+j}}^{i j} \beta_{k_{1}}^{\dagger} \ldots \beta_{k_{i}}^{\dagger} \beta_{k_{i+j} \ldots \beta_{k_{i+1}}} \begin{array}{l}
\text { Hij matrix elements function of } \\
\\
\end{array} \sum_{p q} \bar{H}_{p q r s} \bar{w}_{p q r s t u} U_{p k} V_{p h}+\left[H^{20}+H^{11}+H^{02}\right]+\left[H^{40}+H^{31}+H^{22}+H^{13}+H^{04}\right]+\sum_{i+j=6} H^{i j} \\
& \equiv \sum_{n=0}^{2} H^{[2 n]}+H^{[6]} & \text { 6-qp operators }
\end{array}
$$

Six-index tensors
Too expensive to handle

NO2B approximation
$1-3 \%$ error in closed shell
[Roth et al., PRL 109 (2012) 052501]

PNO2B approximation
Particle-number conserving
[Ripoche, Tichai, Duguet, arXiv:1908.00765]

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## BCC formalism

Bogoliubov reference state

$$
|\Phi\rangle \equiv C \prod_{k} \beta_{k}|0\rangle
$$

Bogoliubov CC ansatz

Quasi-particle excitations [Signoracci et al. PRC 2015]

$$
\left\lvert\, \begin{aligned}
& \mathcal{B}^{\mu} \equiv \mathcal{B}^{k_{1} \ldots k_{2 n}}=\beta_{k_{1} \ldots}^{\dagger} \ldots \beta_{k_{2 n}}^{\dagger} \\
& \left|\Phi^{\mu}\right\rangle \equiv \mathcal{B}^{\mu}|\Phi\rangle \quad \text { Orthonormal basis of Fock space }
\end{aligned}\right.
$$

$\left|\Psi_{\mathrm{BCC}}^{* *}\right\rangle \equiv e^{U}|\Phi\rangle$ with as soon as U is truncated

$$
\begin{aligned}
& \begin{cases}U=\sum_{n=1} U_{n} & \begin{array}{l}
\text { Cluster amplitudes } \\
\text { Unknowns of the problem }
\end{array} \\
U_{n} \equiv \frac{1}{(2 n)!} \sum_{k_{1} \ldots k_{2 n}} U_{k_{1} \ldots k_{2 n}}^{2 n 0} \beta_{k_{1}}^{\dagger} \ldots \beta_{k_{2 n}}^{\dagger}\end{cases} \\
& \text { Pure excitation operators } \\
& 0=\left.\left\langle\Phi^{\mu}\right| H e^{U}|\Phi\rangle_{C}\right|^{\text {Truncate, e.g. } U=U_{1}+U_{2}} \text { (BCCSD) } \\
& \text { Solve for } n=1,2 \\
& \text { Constrained to be true in average } \\
& \text { Connected }=\text { terminating exponential } \\
& \text { Algebraic expression through Wick's theorem/diagrammatic rules }
\end{aligned}
$$

Energy and amplitude equations

Ex: for the energy

$$
E^{X}=H^{00}+\frac{1}{2} \sum_{k_{1} k_{2}} H_{k_{1} k_{2}}^{02} U_{k_{1} k_{2}}^{20}+\frac{1}{8} \sum_{k_{1} k_{2} k_{3} k_{4}} H_{k_{1} k_{2} k_{3} k_{4}}^{04} U_{k_{1} k_{2}}^{20} U_{k_{3} k_{4}}^{20}+\frac{1}{4!} \sum_{k_{1} k_{2} k_{3} k_{4}} H_{k_{1} k_{2} k_{3} k_{4}}^{04} U_{k_{1} k_{2} k_{3} k_{4}}^{40}
$$

## Bogoliubov many-body perturbation formalism

O Perturbative reduction of BCC
[Duguet, Signoracci JPG 2016]
$\rightarrow$ Code for automated generation\&evaluation of many-body diagrams to arbitrary order [Arthuis et al. CPC 2018]
$\rightarrow$ Convergence properties at high orders and resummation methods [Demol et al. to be published 2019]
O BMBPT(2) ab initio calculations of mid-mass semi-magic nuclei [Tichai et al. PLB 2018]



Calculation details
Chiral NN+3N Hamiltonian
SRG $\alpha=0.08 \mathrm{fm}^{4}$
13 major shells (1820 s.p. states)
Canonical HFB reference

## Runtime

NCSM: 20.000 hours MCPT: 2.000 hours IMSRG(2): 1.500 hours SCGF(2): 400 hours BMBPT(2): <1min!
$\rightarrow 2-3 \%$ agreement of all methods with exact results (IT-NCSM)
$\rightarrow$ Different truncation schemes yield consistent description of open-shell nuclei
$\rightarrow$ BMBPT optimal to systematically test next generation of Chiral EFT nuclear Hamiltonians
© Future implementation of $\operatorname{BCCSD}(\mathrm{T})$ for accurate ab initio calculations of open-shell nuclei

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## $\mathrm{U}(1)$ breaking and projection

Particle-number conserving states, i.e. states belonging to $\mathscr{H}_{A}$ Exact eigenstates of $\mathrm{H}:\left|\Psi_{\mu}^{\mathrm{A}}\right\rangle \quad$ Slater determinants: $\quad\left|\Phi^{\mathrm{A}}\right\rangle=\prod_{i=1}^{\mathrm{A}} a_{i}^{\dagger}|0\rangle$ Particle-number breaking states


General states on Fock space: $|\Phi\rangle$ $A|\Phi\rangle \neq \mathrm{A}|\Phi\rangle \quad|\Phi(\varphi)\rangle \equiv R(\varphi)|\Phi\rangle \neq e^{i \mathrm{~A} \varphi}|\Phi\rangle$

Particle-number projection operator

$$
P^{\mathrm{A}} \equiv \frac{1}{2 \pi} \int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} R(\varphi)
$$

Particle number projection
$|\Phi\rangle \equiv \sum_{\mathrm{A}^{\prime} \in \mathbb{N}} c_{\mathrm{A}^{\prime}}\left|\Theta^{\mathrm{A}^{\prime}}\right\rangle \quad P^{\mathrm{A}}|\Phi\rangle \equiv \sum_{\mathrm{A}^{\prime} \in \mathbb{N}} \frac{c_{\mathrm{A}^{\prime}}}{2 \pi}\left|\Theta^{\mathrm{A}^{\prime}}\right\rangle \int_{0}^{2 \pi} d \varphi e^{-i\left(\mathrm{~A}-\mathrm{A}^{\prime}\right) \varphi}=c_{\mathrm{A}} \underbrace{\left|\Theta^{\mathrm{A}}\right\rangle}$

## Particle-number projected BCC formalism

## Projected BCC ansatz

$\left|\Psi \frac{\mathrm{PBCC}}{(\mathrm{PBCC}}\right\rangle \equiv P^{\mathrm{A}}\left|\Psi_{\mathrm{BCC}}^{(\mathcal{X})}\right\rangle \quad$ Always true!

Projected BCC energy and particle number

$$
\left.\begin{array}{l}
H\left|\Psi^{\mathrm{A}}\right\rangle=E^{\mathrm{A}}\left|\Psi^{\mathrm{A}}\right\rangle \\
A\left|\Psi^{\mathrm{A}}\right\rangle=A^{\mathrm{A}}\left|\Psi^{\mathrm{A}}\right\rangle
\end{array}\right\rangle \begin{aligned}
& E^{\mathrm{A}}=\frac{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{H}(\varphi)}{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{N}(\varphi)} \\
& A^{\mathrm{A}}=\frac{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{A}(\varphi)}{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{N}(\varphi)}
\end{aligned}
$$

[Duguet, Signoracci JPG 2016] [Qiu et al. PRC 2019]

Similarity transformed operator

$$
O_{Z}(\varphi) \equiv e^{Z(\varphi)} O e^{-Z(\varphi)}
$$



$$
\begin{aligned}
\binom{\beta(\varphi)}{\beta^{\dagger}(\varphi)} & \equiv e^{Z(\varphi)}\binom{\beta}{\beta^{\dagger}} e^{-Z(\varphi)} \\
& =\left(\begin{array}{cc}
1 & 0 \\
Z^{20}(\varphi) & 1
\end{array}\right)\binom{\beta}{\beta^{\dagger}}
\end{aligned}
$$

Non-unitary Bogoliubov transformation

Normal-ordered operator with ME $O_{k_{1} \ldots k_{i+j}}^{i j}(\varphi)$

Not a pure excitation operator...
with $\left\lvert\, \begin{aligned} & \mathcal{N}(\varphi) \equiv\langle\Phi(\varphi)| e^{U}|\Phi\rangle=\langle\Phi(\varphi) \mid \Phi\rangle\langle\Phi| e^{U_{Z}(\varphi)}|\Phi\rangle \\ & \mathcal{H}(\varphi) \equiv\langle\Phi(\varphi)| H e^{U}|\Phi\rangle=\langle\Phi(\varphi) \mid \Phi\rangle\langle\Phi| H_{Z}(\varphi) e^{U_{Z}(\varphi)}|\Phi\rangle \\ & \mathcal{A}(\varphi) \equiv\langle\Phi(\varphi)| A e^{U}|\Phi\rangle=\langle\Phi(\varphi) \mid \Phi\rangle\langle\Phi| A_{Z}(\varphi) e^{U_{Z}(\varphi)}|\Phi\rangle\end{aligned}\right.$
Off-diagonal norm, Hamiltonian and particle-number kernels


## Particle-number projected BCC formalism

## Disentangled cluster operators

$\square$ Disantengling the algebra to extract pure excitation terms
$e^{U_{Z}(\varphi)}|\Phi\rangle \equiv e^{\frac{\sqrt{W(\varphi)}}{\square}}$

1) Pure excitation operator BUT contains a constant term
2) Allows algebraic expressions of kernels following standard steps
(3) Explicit relation between $\mathrm{W}(\varphi)$ and $\mathrm{U}_{\mathrm{z}}(\varphi)$ too complicated (need other approach)
$W(\varphi)=\sum_{n=0} W_{n}(\varphi) \equiv \underline{W_{0}(\varphi)}+\frac{\mathcal{T}(\varphi)}{\text { Constant }} \quad$ with $\quad W_{n}(\varphi) \equiv \frac{1}{2 n!} \sum_{k_{1} \ldots k_{2 n}} W_{k_{1} \ldots k_{2 n}}^{2 n 0}(\varphi) \beta_{k_{1}}^{\dagger} \ldots \beta_{k_{2 n}}^{\dagger}$

## Connected kernels and PBCC energy

$$
\begin{aligned}
\mathcal{N}(\varphi) & \equiv \frac{e^{W_{0}(\varphi)}}{\mathcal{H}(\varphi)}\langle\Phi(\varphi) \mid \Phi\rangle \\
h(\varphi) & \equiv \frac{\mathcal{N}(\varphi)}{\mathcal{N}\left(\varphi \mid H_{Z}(\varphi) e^{\mathcal{T}}(\varphi\right.} \\
E^{\mathrm{A}} & =\frac{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} h(\varphi) \mathcal{N}(\varphi)}{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{N}(\varphi)}
\end{aligned}
$$

$$
\begin{aligned}
& \text { But how to determine } W(\varphi) ? \\
& \text { Correlated norm kernel determined by } W_{0}(\varphi)
\end{aligned}
$$

$$
h(\varphi) \equiv \frac{\mathcal{H}(\varphi)}{}=\langle\Phi| H_{Z}(\varphi) e^{\mathcal{T}(\varphi)}|\Phi\rangle_{C}=\begin{gathered}
\text { Connected partefenergy kernel determined by } \mathcal{J}(\varphi)
\end{gathered}
$$

Consistent dynamical and static correlations [Duguet, Signoracci JPG 2016] [Qiu et al. PRC 2019]

1) Reduction to BCC

## Particle-number projected BCC formalism

## Gauge-rotated cluster amplitudes $\mathrm{W}_{\mathrm{k}}(\varphi)$



Coupled ordinary differential equations

Initial conditions
Connected off-diagonal kernel of $A^{02}$
$\frac{d}{d \varphi} W_{0}(\varphi)=\frac{i}{2} \sum_{k_{1} k_{2}} A_{k_{1} k_{2}}^{02}(\varphi) W_{k_{1} k_{2}}^{20}(\varphi)$
$\frac{d}{d \varphi} W_{k_{1} k_{2}}^{20}(\varphi){ }_{j}^{i} \sum_{k_{3} k_{4}}^{02}(\varphi)\left[\frac{1}{2} W_{k_{3} k_{4} k_{1} k_{2}}^{40}(\varphi)\right.$
$\mathcal{N}(\varphi)=e^{W_{0}(\varphi)}\langle\Phi(\varphi) \mid \Phi\rangle=e^{i \int_{0}^{\varphi} \underline{a(\phi)} d \phi}$
$\left.-W_{k_{1} k_{3}}^{20}(\varphi) W_{k_{2} k_{4}}^{20}(\varphi)\right]$
Correlated norm kernel from connected off-diagonal kernel of $\stackrel{k_{A}}{k_{3}}=$ generator of ${ }^{k}(1)$
$\frac{d}{d \varphi} W_{k_{1} k_{2} k_{3} k_{4}}^{40}(\varphi) \sum_{k_{5} k_{6}} A_{k_{5} k_{6}}^{02}(\varphi)\left[\frac{1}{2} W_{k_{5} k_{6} k_{1} k_{2} k_{3} k_{4}}^{60}(\varphi)\right.$
All ranks $A^{\mathrm{A}}=\frac{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} a(\varphi) \mathcal{N}(\varphi)}{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{N}(\varphi)}=\begin{gathered}+W_{k_{1} k_{5}}^{20}(\varphi) W_{k_{6} k_{2} k_{3} k_{4}}^{40}(\varphi) \\ +W_{k_{2} k_{5}}^{20}(\varphi) W_{k_{1} k_{6} k_{3} k_{4}}^{0}(\varphi)\end{gathered}$

$$
\begin{align*}
& W_{0}(0)=0 \\
& W_{k}(0)=U_{k}
\end{align*}
$$

Even when U truncated


Approximation on $\mathrm{P}^{\mathrm{A}}$

 Second truncation on $\mathbf{W}_{\mathbf{k}}(\varphi)$

$$
+W_{k_{4} k_{5}}^{20}(\varphi) W_{k_{1} k_{2} k_{3} k_{6}}^{40}
$$

Integrate coupled ODEs and insert in PBCC energy

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## BCS pairing Hamiltonian

## Hamiltonian

$H=\sum_{p}\left(\epsilon_{p}-\lambda\right) N_{p}-G \sum_{p q} P_{p}^{\dagger} P_{q}$
Doubly-degenerate picket fence model

$$
\epsilon_{p}=\epsilon_{\bar{p}}=p \Delta \epsilon
$$

## Exact ground-state energy

Richardson solution
[R.W. Richardson, PL 1963; PR 1966]

What about BCC? [Henderson et al. PRC 2014] BCCSD: $\mathrm{U} \approx \mathrm{U}_{1}+\mathrm{U}_{2}$

What about PBCC? [Qiu et al. PRC 2019]
$\operatorname{PBCCSD}(2): \mathbf{U} \approx \mathrm{U}_{1}+\mathrm{U}_{2}$

## Operators

$$
\begin{aligned}
& N_{p}=c_{p}^{\dagger} c_{p}+c_{\bar{p}}^{\dagger} c_{\bar{p}} \\
& P_{p}^{\dagger}=c_{p}^{\dagger} c_{\bar{p}}^{\dagger}
\end{aligned}
$$

## SU(2) algebra

$$
\begin{aligned}
& {\left[P_{p}, P_{q}^{\dagger}\right]=\delta_{p q}\left(1-N_{p}\right)} \\
& {\left[N_{p}, P_{q}^{\dagger}\right]=2 \delta_{p q} P_{q}^{\dagger}}
\end{aligned}
$$

Typical approximate methods
> BCS and projected BCS (before variation)
> Coupled cluster theory with doubles
> Self-consistent RPA


Other recent accurate methods
[Degroote et al. PRB 2016; Ripoche et al. PRC 2017]

## Connected hamiltonian kernel



PBCC energy and kernels

$$
E^{\mathrm{A}}=\frac{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} h(\varphi) \mathcal{N}(\varphi)}{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{N}(\varphi)} \quad \text { with }
$$

Exact (or symmetry-conserving) limit

$$
\begin{aligned}
\mathcal{N}(\varphi) & =e^{i \mathrm{~A} \varphi} \\
\frac{d}{d \varphi} h(\varphi) & =0
\end{aligned}
$$



$$
\begin{aligned}
& \mathcal{N}(\varphi) \equiv e^{W_{0}(\varphi)}\langle\Phi(\varphi) \mid \Phi\rangle \\
& h(\varphi) \equiv \frac{\mathcal{H}(\varphi)}{\mathcal{N}(\varphi)}=\langle\Phi| H_{Z}(\varphi) e^{\mathcal{T}(\varphi)}|\Phi\rangle_{C}
\end{aligned}
$$

○ PBCC $\leftrightarrow$ PBCS-CC(S)D here

- $\mathrm{h}(\varphi)$ real with typical bell-shape curve
- PBCC brings $h(\varphi)$ closer to constant
- Not constant $\mathrm{h}(\varphi)$ induces non-trivial projection


## Norm kernel



## PBCC energy and kernels

$$
E^{\mathrm{A}}=\frac{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} h(\varphi) \mathcal{N}(\varphi)}{\int_{0}^{2 \pi} d \varphi e^{-i \mathrm{~A} \varphi} \mathcal{N}(\varphi)} \quad \text { with }
$$

Exact (or symmetry-conserving) limit

$$
\begin{aligned}
\mathcal{N}(\varphi) & =e^{i \mathrm{~A} \varphi} \\
\frac{d}{d \varphi} h(\varphi) & =0
\end{aligned}
$$



$$
\begin{aligned}
& \mathcal{N}(\varphi) \equiv e^{W_{0}(\varphi)}\langle\Phi(\varphi) \mid \Phi\rangle \\
& h(\varphi) \equiv \frac{\mathcal{H}(\varphi)}{\mathcal{N}(\varphi)}=\langle\Phi| H_{Z}(\varphi) e^{\mathcal{T}(\varphi)}|\Phi\rangle_{C}
\end{aligned}
$$

O PBCC $\leftrightarrow$ PBCS-CC(S)D here

- $|\mathcal{N}(\varphi)|$ displays bell-shape curve and phase $\propto A$
- PBCC brings $\mathcal{N}(\varphi)$ closer to single IRREP e ${ }^{i A \varphi}$
- $\mathcal{N}(0)=1 \leftrightarrow$ Intermediate normalization


## Results - 1

Fraction of correlation energy


Dependence on system size


Absolute energy error

© BCC $\leftrightarrow$ BCS-CC(S)D here

- Extends quality of CC through phase transition
- Better than PBCS except for for $G \gg G_{c}$
- Poor in small systems

○ PBCC $\leftrightarrow$ PBCS-CC(S)D here

- Perfect from weak to strong coupling
- Perfect from small to large systems
- Dominates all other methods


## Results - 2

Filling fraction

3) For all system sizes
4) At low polynomial cost

Particle number dispersion

it.) and non-dynamical (def.+proj.) correlations
g regimes

- Poor near « closed shell»
- Reduce $\sigma_{A}$ by factor of 2 compared to BCS

○ PBCC $\leftrightarrow$ PBCS-CC(S)D here

- Perfect «throughout the shell »
- One-body properties are perfect
- $\sigma_{\mathrm{A}}=0\left(\mathrm{~W}_{3}(\varphi)\right.$ to be added for very high precision $)$


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## Conclusions

© Particle-number projected Bogoliubov coupled cluster and many-body perturbation theories

- Extends single-reference CC/MBPT methods to open-shell nuclei via symmetry breaking\&restoration
- First consistent formulation of symmetry restoration techniques beyond the mean-field
- Results obtained for $\mathbf{U}(1)$ on the solvable Richardson Hamiltonian hold great promises
© Future
- Ab initio PBCC and PBMBPT calculations of singly open-shell nuclei
[Tichai, Ripoche, Duguet, in progress]
- Apply to $\operatorname{SU}(2)$ (already formulated) for ab initio calculations of doubly open-shell nuclei
[Tichai, Hagen, Duguet, in progress]
- Combine with IT/TF techniques to go to heavier nuclei
[Tichai, Ripoche, Duguet, EPJA 2019]
- Extend BCC and PBCC to excited states
[Demol, Tichai, Duguet, planned]


## Current collaborators on ab initio many-body calculations


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