ESNT workshop Recent advances on proton-neutron pairing and quartet correlations in nuclei, session II
2-6 September 2019

KTH ROYAL INSTITUTE OF TECHNOLOGY

Interplay of pairing and quadrupole interactions in N=Z nuclei

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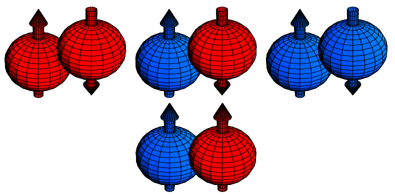
General properties of the effective interaction

- Isovector (T=1): J=0,2,..,2J-1, J=0 term attractive *(pairing),* others close to zero
- Isoscalar (T=0): J=1,3,..,2j, strongly attractive (mean field)
  - *The J=1 and 2j terms are the most attractive ones.*



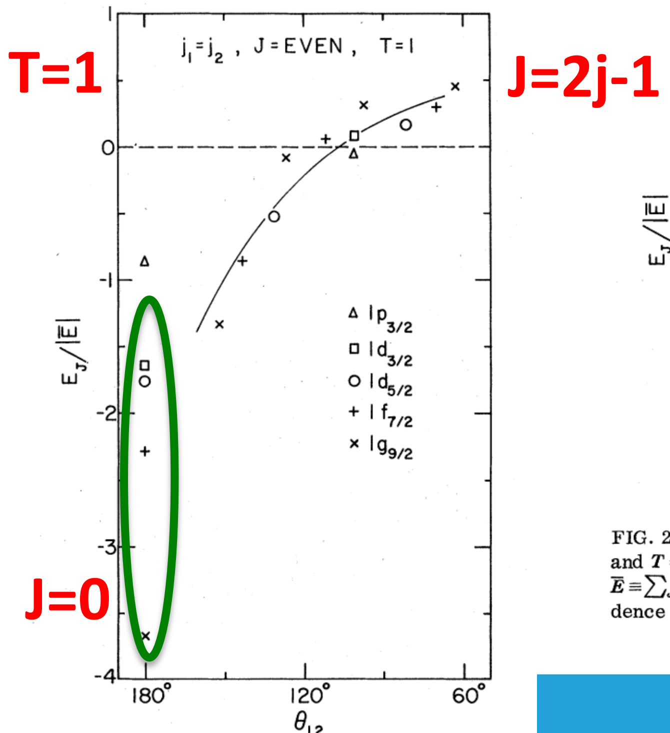


FIG. 3. Comparison of data from various multiplets with $j_1=j_2$ and $T=1$. The values of the matrix elements are divided by $\bar{E}\equiv\sum_J[J]E_J/\sum_J[J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

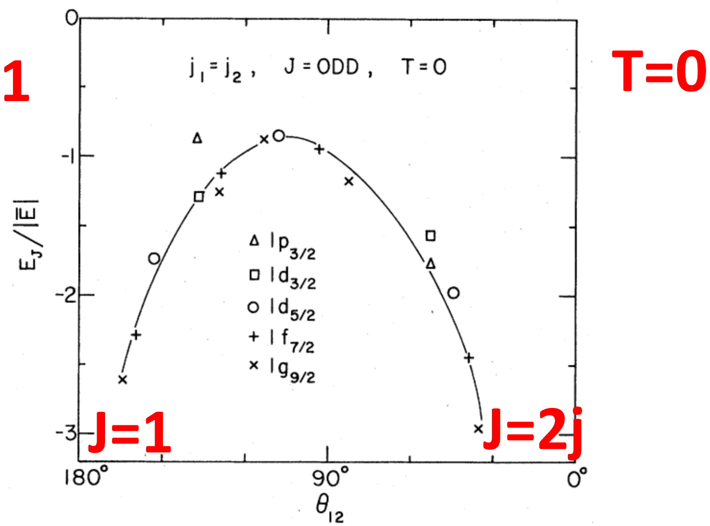


FIG. 2. Comparison of data from various multiplets with $j_1=j_2$ and $T=0$. The values of the matrix elements are divided by $\bar{E}\equiv\sum_J[J]E_J/\sum_J[J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

$$\cos q_{12} = \frac{J(J+1)}{2j(j+1)} - 1$$

J.P. Schiffer and W.W. True, Rev.Mod.Phys. 48,191 (1976)

The coupling of few neutrons and protons

In full configuration interaction 'shell model'

$$\left| \mathrm{Y}_p \, \ddot{\mathrm{A}} \, \mathrm{Y}_n \right\rangle$$

Or we can do like these

$$|J_1 \otimes J_2 \ldots\rangle_I$$

$$|\,[[J_1 \otimes J_2]_{I_{12}} \otimes J_3]_{I_{123}} \ldots\rangle_I$$

$$|[J_1 \otimes J_2]_{I_{12}} \otimes [J_3 \otimes J_4]_{I_{34}} \ldots\rangle_I$$

Generate (all) components in uncoupled M or coupled schemes and diagonalize (exactly) the Hamiltonian

The challenge is to understand the complicated full wave function: How to filter out the relevant components

Monopole Hamiltonian

Determines average energy of eigenstates in a given configuration.

- Important for binding energies, shell gaps

$$H_m = \sum_a e_a n_a + \sum_{a \le b} \frac{1}{1+\delta_{ab}} \left[\frac{3V_{ab}^1 + V_{ab}^0}{4} n_a (n_a - \delta_{ab}) + (V_{ab}^1 - V_{ab}^0)(T_a \cdot T_b - \frac{3}{4} n_a \delta_{ab}) \right]$$

n_a, T_a … number, isospin operators of orbit *a*

Monopole centroids

- Angular-momentum averaged effects of two-body interaction
- **The monopole interaction itself does not induce mixing between different configurations.**
- **Strong mixture of the wave function is mainly induced by the residual J=0 *pairing* and *QQ* np interaction**

$$V_{ab}^T = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

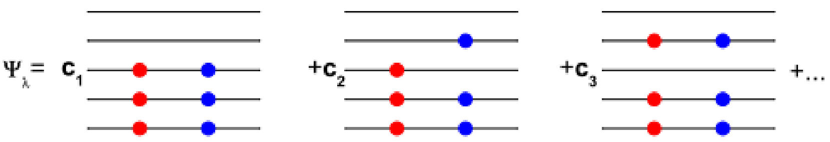
'Monopole' truncation

$$H = H_m + H_M$$

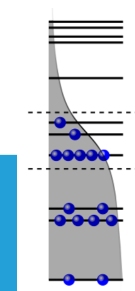
$$E^{\mathrm{SM}} = \langle \Psi_I | H | \Psi_I \rangle$$

$$\boxed{= \sum_{\alpha} \varepsilon_\alpha < \hat{N}_\alpha > + \sum_{\alpha \le \beta} V_{m;\alpha\beta} \left\langle \frac{\hat{N}_\alpha (\hat{N}_\beta - \delta_{\alpha\beta})}{1 + \delta_{\alpha\beta}} \right\rangle}$$

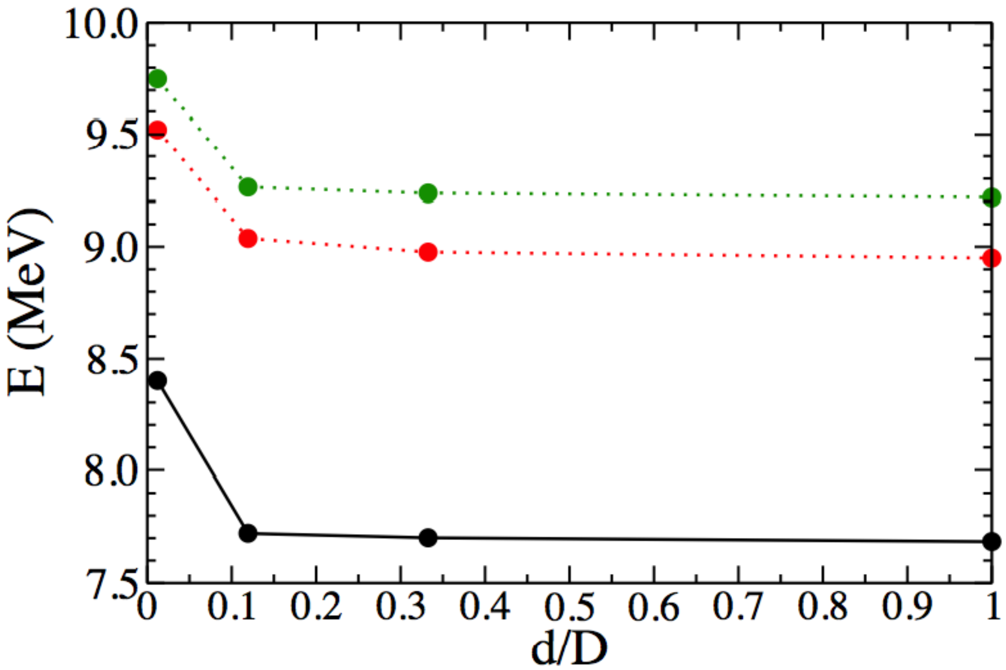
$$+ \langle \Psi_I | H_M | \Psi_I \rangle,$$



- Similar to 'npnh' and Nmax if no monopole considered.
- But monopole interaction can change significantly the (effective) mean field and invalidate npnh.
- Easy to implement and keeps the simplicity of the M-scheme algorithm
- Possibility to include certain intruder configurations



Convergence for ^{194}Pb



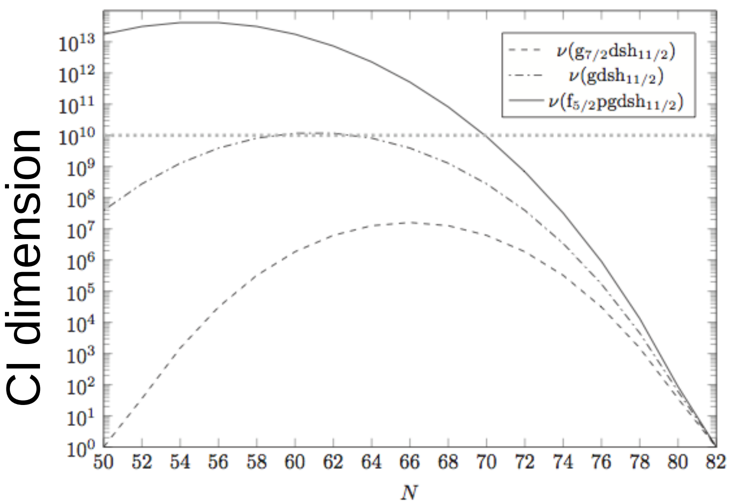
$$
\begin{aligned}
E^{\mathrm{SM}} &= \langle \Psi_I | H | \Psi_I \rangle \\
&= \sum_{\alpha} \varepsilon_\alpha < \hat{N}_\alpha > + \sum_{\alpha \le \beta} V_{m;\alpha\beta} \left\langle \frac{\hat{N}_\alpha(\hat{N}_\beta - \delta_{\alpha\beta})}{1+\delta_{\alpha\beta}} \right\rangle \\
&\quad + \langle \Psi_I | H_M | \Psi_I \rangle, \qquad (4)
\end{aligned}
$$

CQ, LY Jia, GJ Fu, PRC 2016

Seniority coupling as a result of strong J=0 pairing

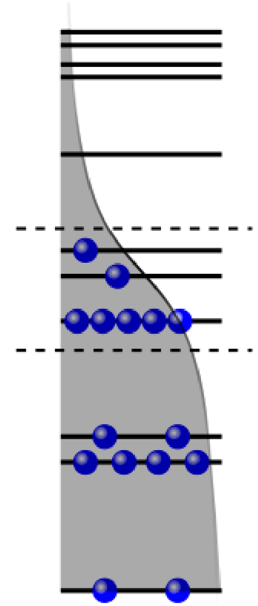
$$|\text{g.s.}\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^{n/2}|\Phi_0\rangle$$
$$|\nu = 2; JM\rangle = (P_j^+)^{(n-2)/2}A^+(j^2JM)|\Phi_0\rangle$$

Exact Diagonalization of the pairing in v=0 subspace



10^2 in seniority space

Easier to include many shells

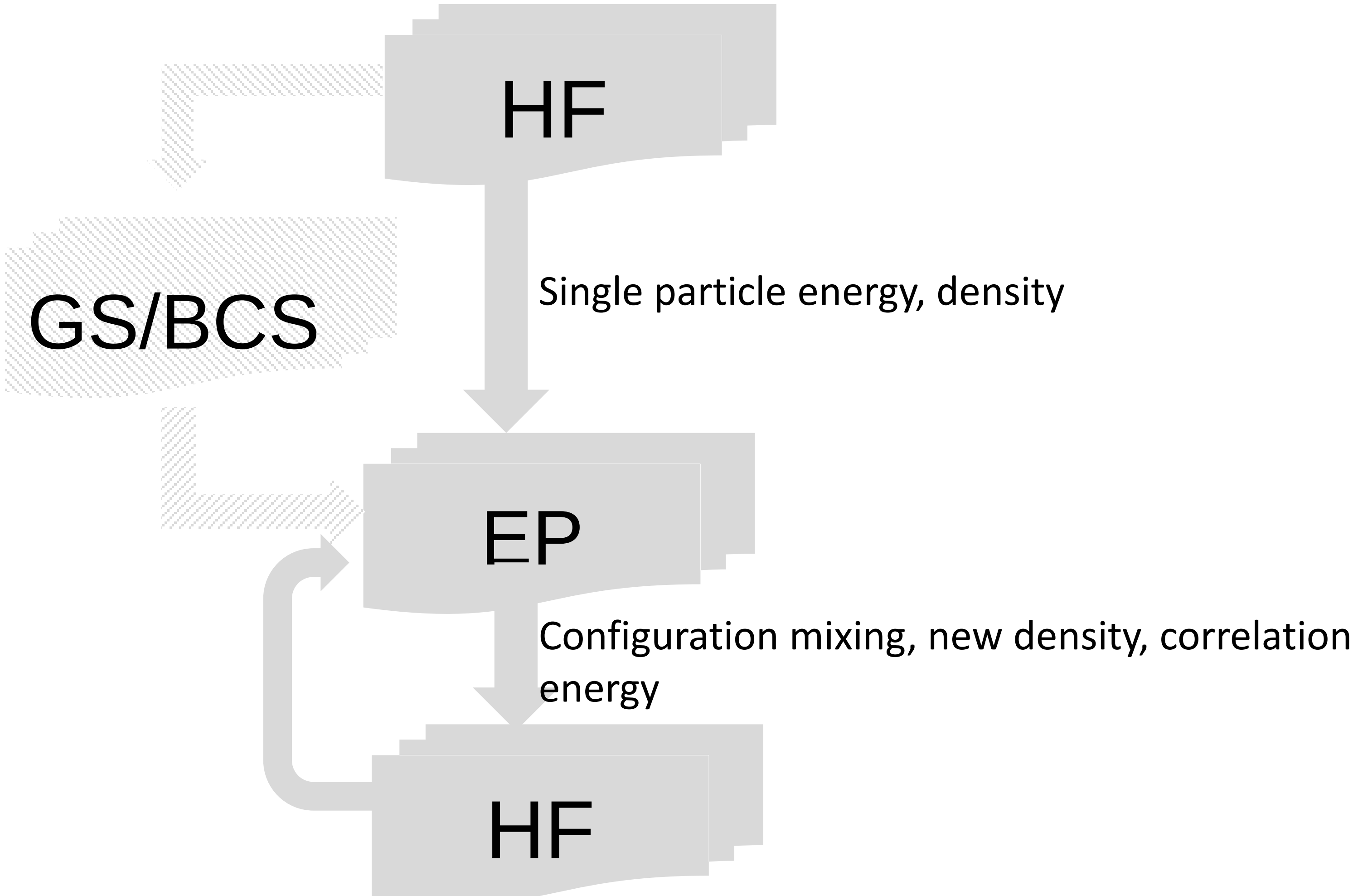


One can readily solve a half-filled system with upto 36-38 doubly-degenerate orbitals and 18-19 pairs (Dim: $9*10^9$-$3.5*10^{10}$, shell-model dimension: $4*10^{20}$-$7*10^{21}$).

- ❖ **Exact solution of general pairing Hamiltonian**
- ❖ **A bridge between DFT and CI->Self-consistent MF+EP**

Self-consistent HF+EP

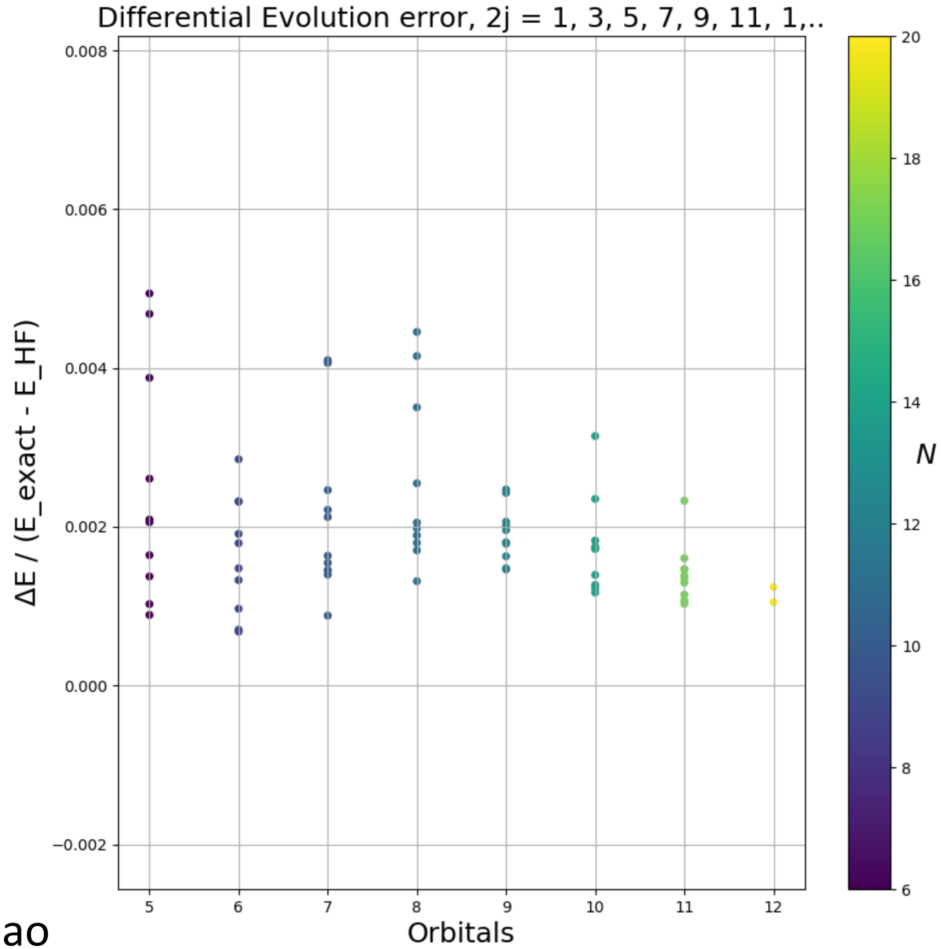
EP on top of static HF ev8 and time dependent HF Sky3d



Together with XiaoYu Liu (KTH, Lanzhou)

Approximation with generalized seniority

$$|\phi_N\rangle = \frac{1}{\sqrt{\chi_N}}(P^\dagger)^N|0\rangle,$$



Together with
D. Karlsson, L.Y. Jia, Y.X. Zhao

T(T+1) breaking terms in relation to the search for Wigner energy

For a single-j shell system

If one assumes $v = 0$ for the ground state of even-even system and $v = 1$ for that of the odd system, the expression above can be simplified as

$$\begin{aligned} E(n) &= \frac{n(n-1)}{4}G - \left[\frac{n}{2}\right](j+1)G, \qquad (9) \\ &= \left[\frac{n}{2}\right]\left(\left[\frac{n}{2}\right]-1\right)G + \delta_{v,1}\left[\frac{n}{2}\right]G + \left[\frac{n}{2}\right]E_2 \end{aligned}$$

where $[n/2]$ denotes the largest integer not exceeding n/2 and corresponds to the total number of $v = 0$ pairs. The

For a system involving equally-spaced doubly- degenerate orbital

$$\begin{aligned} E(n) \simeq & \left[\frac{n}{2}\right]\left(\left[\frac{n}{2}\right]-1\right)\mathcal{G} + \delta_{v,1}(\varepsilon_b + \delta) \\ & + \left[\frac{n}{2}\right]E_2, \end{aligned}$$

S.A. Changizi, C. Qi, R. Wyss, Nucl. Phys. A (2015)

Seniority for degenerate systems with isospin

$$
\begin{aligned}
E &= \varepsilon n + \frac{2a-G}{4} n(n-1) \\
&\boxed{+\frac{b-2G}{2}\left[\mathcal{T}(\mathcal{T}+1)\right.} - \left.\frac{3n}{4}\right] \\
&\boxed{+(j+1)G(n-v)} + G\left[\frac{v^2}{4} - v + s(s+1)\right],
\end{aligned}
$$

Vpn

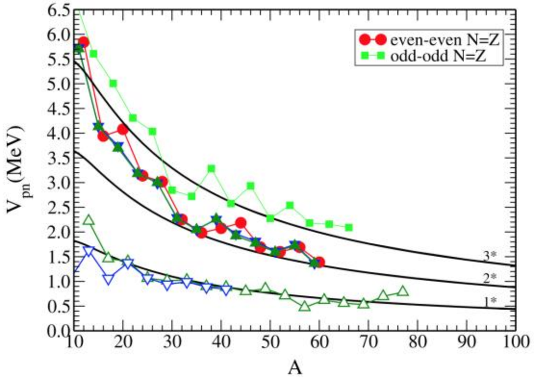


Fig. 4. (Color online.) Experimental V_{pn} values of even–even $N = Z$ nuclei (filled circles) and the adjacent odd–odd (squares) and odd-A nuclei (triangles). The filled and open triangles correspond to systems with one nucleon subtracted from and added to the even–even nuclei, respectively. The solid line labeled 1* describes the average behavior of V_{pn} in even–even $N \neq Z$ nuclei from Fig. 1. 2* and 3* denotes its twice and three time values.

For even–even nuclei with $n_\pi \neq n_\nu$,

$$V_{pn} = -\frac{4V_{m;T=1} + 2(V_{m;T=0} - V_{m;T=1})}{4} = \frac{b}{4} - a.$$

in the case of $n_\pi = n_\nu$ (i.e., $N = Z$),

$$V_{pn} = -\frac{4V_{m;T=1} + 3(V_{m;T=0} - V_{m;T=1})}{4} - \frac{G}{2}$$

$$= \frac{b}{2} - a - \frac{G}{2}.$$

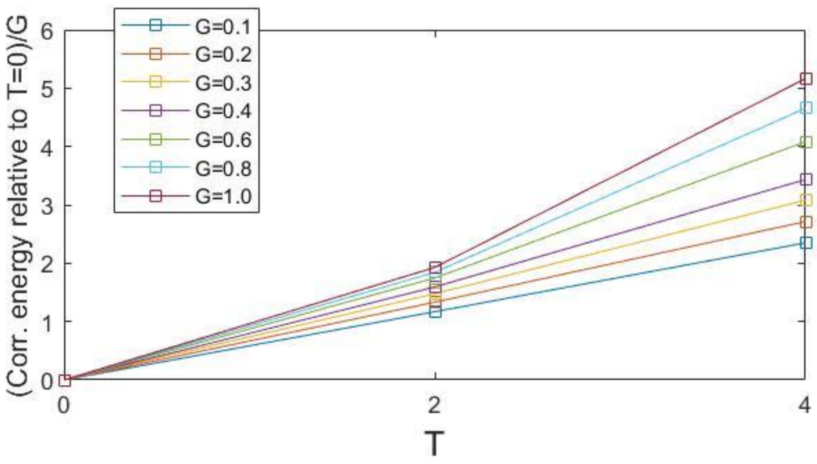
odd–odd $N = Z$

$$
\begin{aligned}
V_{pn}(Z-1, Z-1) &= B(Z-1, Z-1) + B(Z-2, Z-2) \\
&\quad - B(Z-1, Z-2) - B(Z-2, Z-1) \\
&= \frac{3b}{4} - a.
\end{aligned}
$$

CQ, Physics Letters B 717, 436 (2012)

Exact T=1 pairing in the seniority-zero symmetric subspace

Equally spaced doubly degenerate system
With constant T=1 pairing
6 n/p levels, 4 np pairs



$$A^\dagger_\mu = \sum_{i=1}^{p} A^\dagger_\mu(j_i) =$$

$$\sum_{i=1}^{p}\sum_{m_i>0}(-)^{j_i-m_i} a^\dagger_{j_i,m_i,\,\mu/2} a^\dagger_{j_i,-m_i,\,\mu/2}$$

for $\mu = 1$ or -1,

$$A^\dagger_0 = \sum_{i=1}^{p} A^\dagger_0(j_i) =$$

$$\sqrt{\frac{1}{2}}\sum_{i=1}^{p}\sum_{m_i>0}(-)^{j_i-m_i}(a^\dagger_{j_i,m_i,\,1/2} a^\dagger_{j_i,-m_i,\,-1/2} +$$

$$a^\dagger_{j_i,m_i,\,-1/2} a^\dagger_{j_i,-m_i,\,1/2})$$

Exact isovector pairing in a shell-model framework: Role of proton-neutron correlations in isobaric analog states

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[1]*Department of Physics, Rollins College, Winter Park, FL 32789, USA*
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[3]*Department of Physics, Liaoning Normal University, Dalian 116029, People's Republic of China*
(Dated: April 29, 2019)

We utilize a nuclear shell model Hamiltonian with only two adjustable parameters to generate, for the first time, exact solutions for pairing correlations for light to medium-mass nuclei, including the challenging proton-neutron pairs, while also identifying the primary physics involved. In addition to single-particle energy and Coulomb potential terms, the shell model Hamiltonian consists of an isovector $T = 1$ pairing interaction and an average proton-neutron isoscalar $T = 0$ interaction, where the $T = 0$ term describes the average interaction between non-paired protons and neutrons. This Hamiltonian is exactly solvable, where, utilizing 3 to 7 single-particle energy levels, we reproduce experimental data for 0^+ state energies for isotopes with mass $A = 10$ through $A = 62$ exceptionally well including isotopes from He to Ge. Additionally, we isolate effects due to like-particle and proton-neutron pairing, provide estimates for the total and proton-neutron pairing gaps, and reproduce N (neutron) $= Z$ (proton) irregularity. These results provide a further understanding for the key role of proton-neutron pairing correlations in nuclei, which is especially important for waiting-point nuclei on the rp-path of nucleosynthesis.

$$\hat{H} = \sum_j \varepsilon_j N_j - G \sum_{jj'\mu} A^\dagger_{j,\mu} A_{j',\mu} + \alpha\left(\hat{T}^2 - \frac{N}{2}\left(\frac{N}{2}+1\right)\right) + V_{\mathrm{Coul}}, \quad (1)$$

https://arxiv.org/pdf/1904.11555.pdf

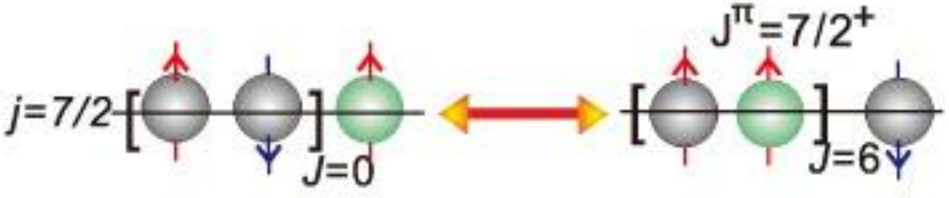
See also
I. Bentley, S. Frauendorf, Phys. Rev. C 88, 014322 (2013)

The coupling of few nucleons

The v=0 state is uniquely defined, but …

$$|\mathrm{g.s.}\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^{n/2}|\Phi_0\rangle$$
$$|\nu = 2; JM\rangle = (P_j^+)^{(n-2)/2}A^+(j^2JM)|\Phi_0\rangle$$

Three identical particles



Eigen states of QQ for a single j system

$$|J_1J_2J\rangle = -\sum_{J_1'J_2'} \hat{J}_1\hat{J}_2\hat{J}_1'\hat{J}_2' X(jjJ_1; jjJ_2; J_1'J_2'J)|J_1'J_2'J\rangle ,$$

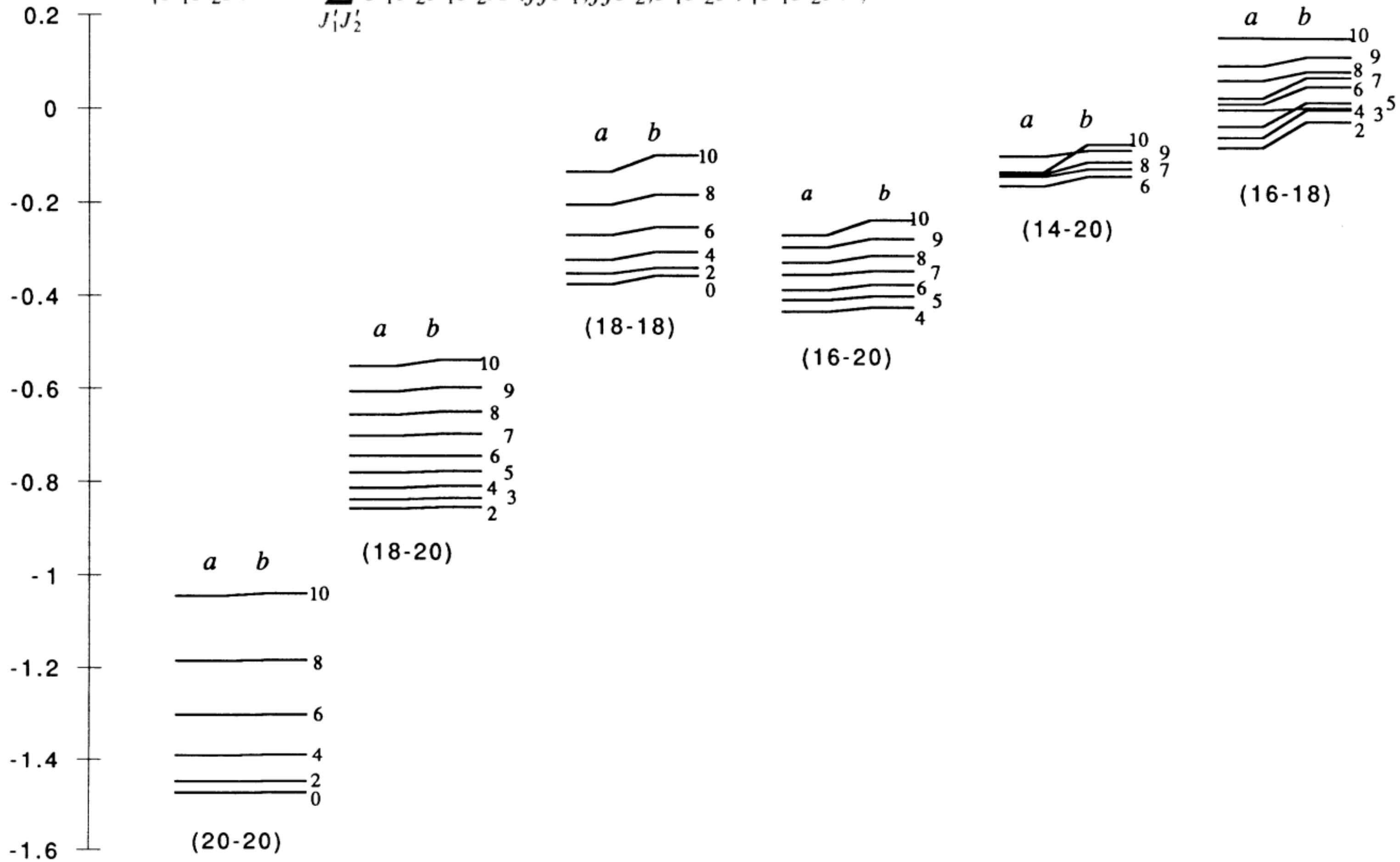
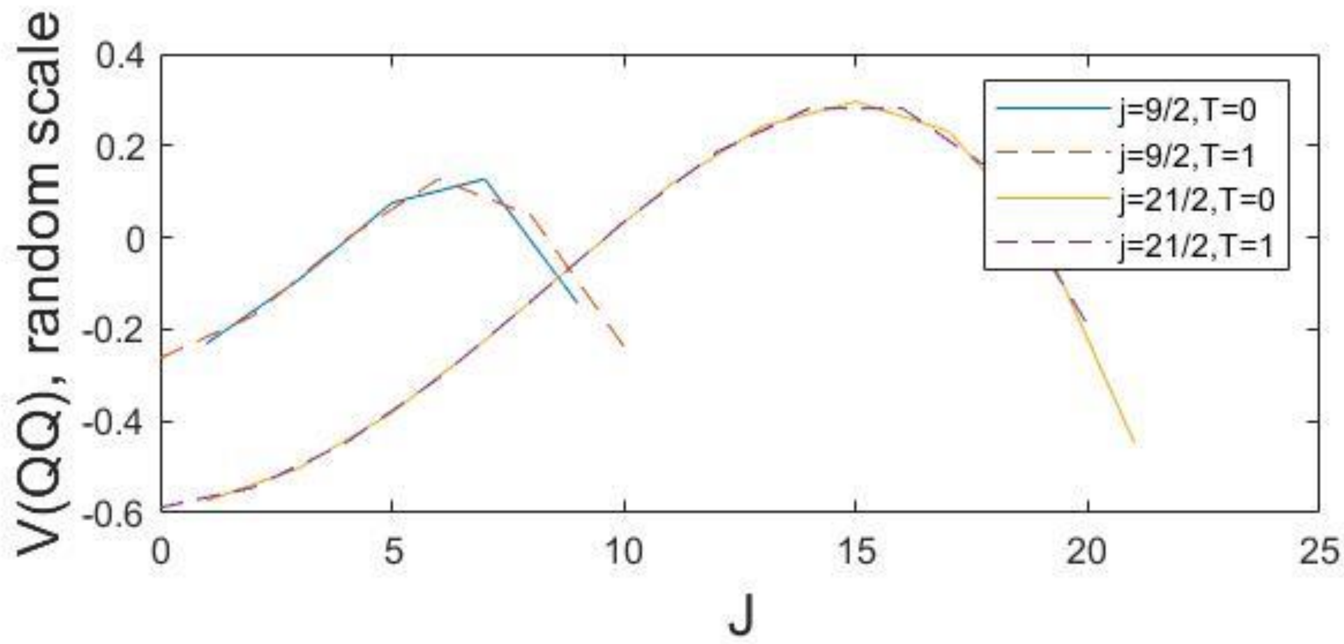


FIG. 1. The spectrum of four particles in a single-j shell ($j=\frac{21}{2}$, $H=-Q\cdot Q$, energies are in arbitrary units). Part a, the shell-model calculation; b, the GPFM calculation.

Hsi-Tseng Chen, Da Hsuan Feng, and Cheng-Li Wu Phys. Rev. Lett. 69, 418 (1992)

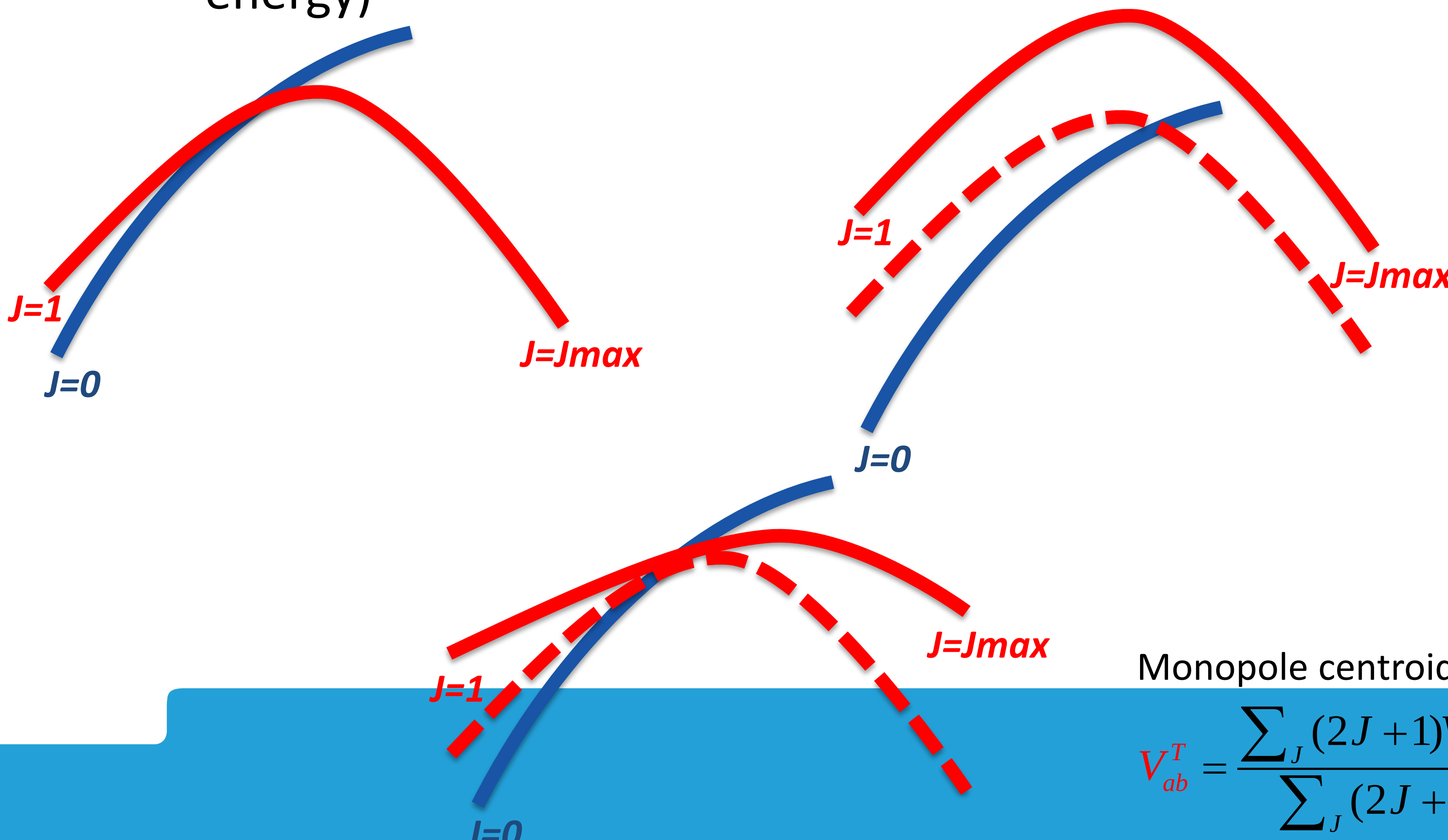
Single-j T=1/0 QQ interaction



The monopole average of QQ interaction is zero

$$V_{ab}^{T} = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

- What matter for the wave functions are the relative values between different two-body matrix elements within the same isospin (the multipole channel)
- The monopole interactions determine the relative positions of states with different total isospin (and the symmetry energy)



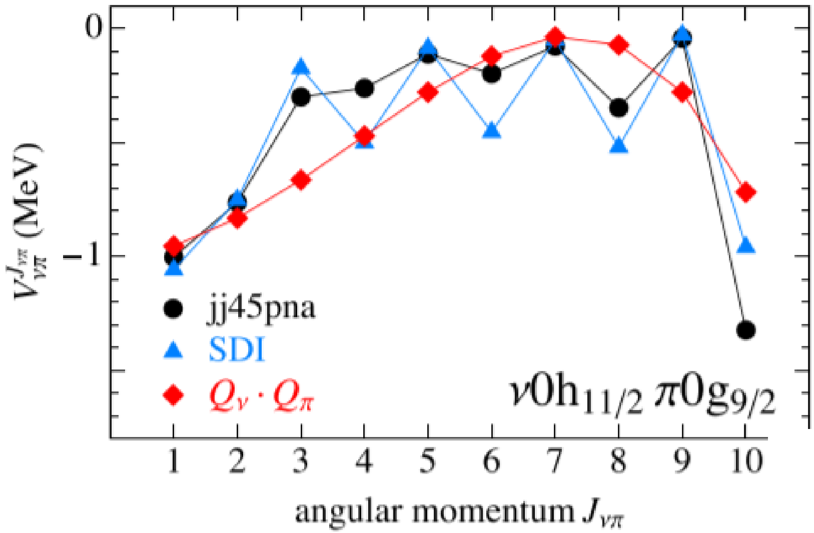
Monopole centroids

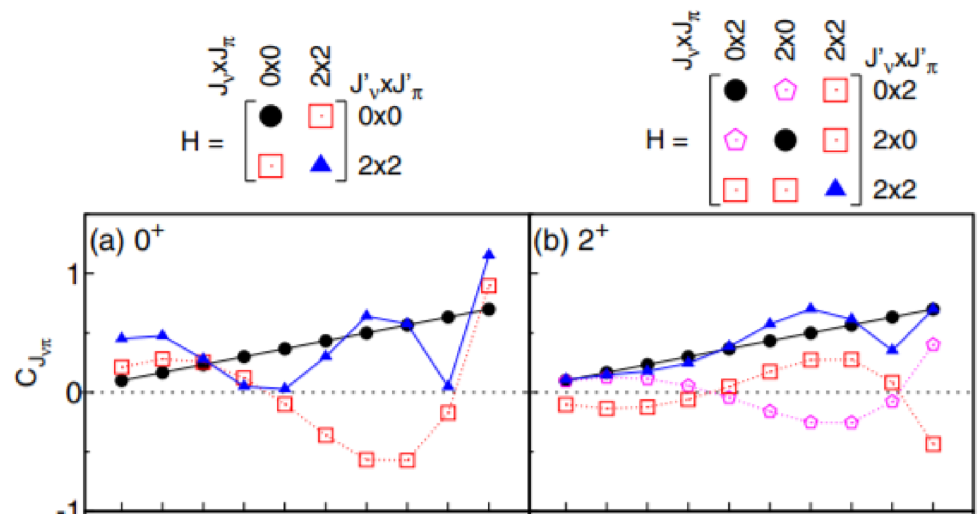
$$V_{ab}^{T} = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

PHYSICAL REVIEW C 97, 041302(R) (2018)

Generic features of the neutron-proton interaction

Y. H. Kim,* M. Rejmund, P. Van Isacker, and A. Lemasson





SU(3) model (1958) : exact solution to a QQ model

$$-\hat{Q}\cdot\hat{Q} = -2\hat{C}_2[\mathrm{SU}(3)] + 3\hat{C}_2[\mathrm{SO}(3)].$$

$$\hat{C}_2[\mathrm{SO}_\pm(3)] = \frac{N(N+1)^2(N+2)}{3}\hat{\mathcal{G}}^{(1)}\cdot\hat{\mathcal{G}}^{(1)},$$

$$\hat{\mathcal{G}}^{(1)}_\mu = \hat{G}^{(01)}_{0\mu} \pm \hat{G}^{(10)}_{\mu 0}.$$

$$\hat{C}_2[\mathrm{SU}(3)] = 3\hat{C}_2[\mathrm{U}(3)] - \hat{n}^2 = \frac{3}{2}\hat{L}\cdot\hat{L} + \frac{1}{2}\hat{Q}\cdot\hat{Q},$$

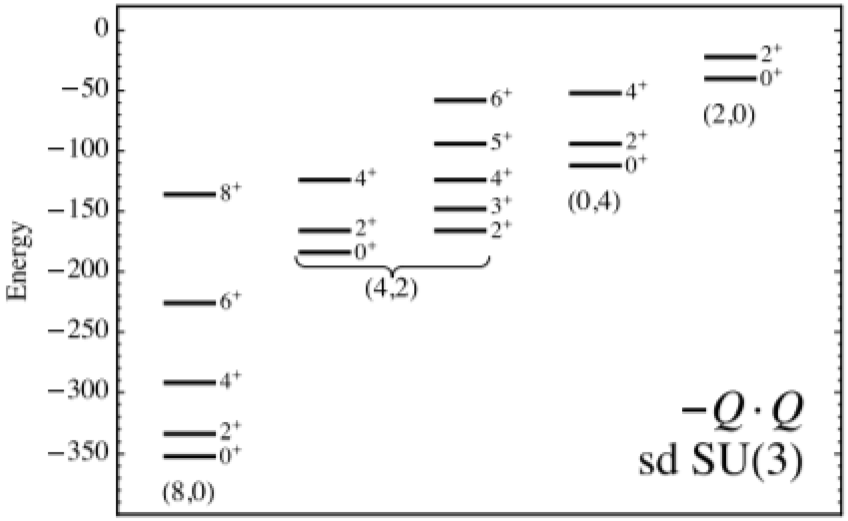


Fig. 3: The eigenspectrum of the operator $-\hat{Q}\cdot\hat{Q}$ for two neutrons and two protons in the *sd* shell. Only levels in the favoured supermultiplet $(0,0,0)$ are shown. Levels are labelled by the orbital angular momentum L and parity $\pi = +$, and by the SU(3) quantum numbers (λ,μ). All levels have $S = 0$ and therefore the total angular momentum J equals the orbital angular momentum L.

J. P. Elliott, Proc. R. Soc. London, Ser. A 245, 128 (1958); 245, 562 (1958). J. P. Elliott and M. Harvey, Proc. R. Soc. London, Ser. A 272, 557 (1963).

P. van Isacker, S. Pittel. Symmetries and deformations in the spherical shell model. Physica Scripta, 2016, 91 (2), 023009

PHYSICAL REVIEW VOLUME 161, NUMBER 4 20 SEPTEMBER 1967

Stretch Scheme, a Shell-Model Description of Deformed Nuclei

MICHAEL DANOS AND VINCENT GILLET
Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvett, Seine et Oise, France
and
u of Standards, Washington, D. C.

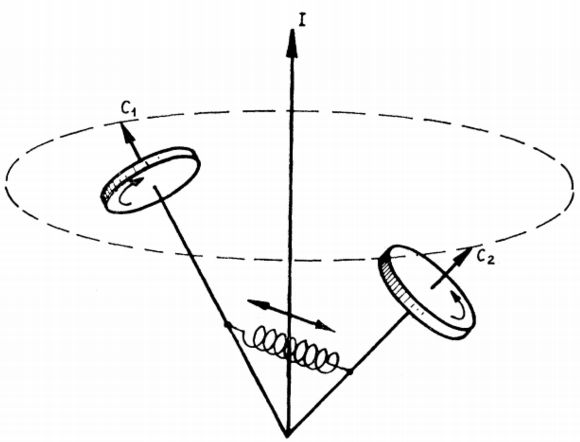
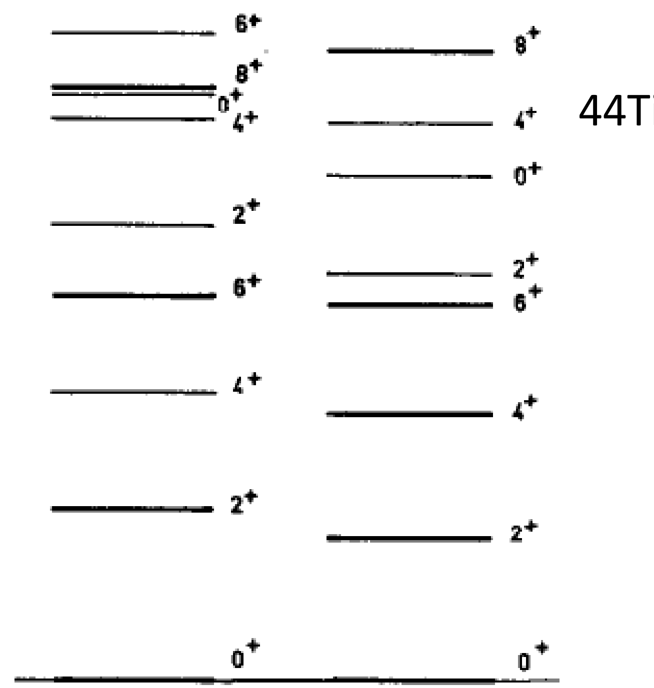


FIG. 3. A classical model for the nuclear rotations. The two



aligned np pair Full shell-model

✧Aligned np pair to explain the rotational-like spectra in ^{20}Ne and ^{44}Ti

A. Jaffrin, Nucl. Phys. A **196**, 577 (1972).

P+QQ

$$H = \alpha H_1 + (1 - \alpha) H_2 \,,$$

$$H_1 = -\chi Q \cdot Q \,, \qquad H_2 = -G S_+ S_- \,,$$

No analytic solution

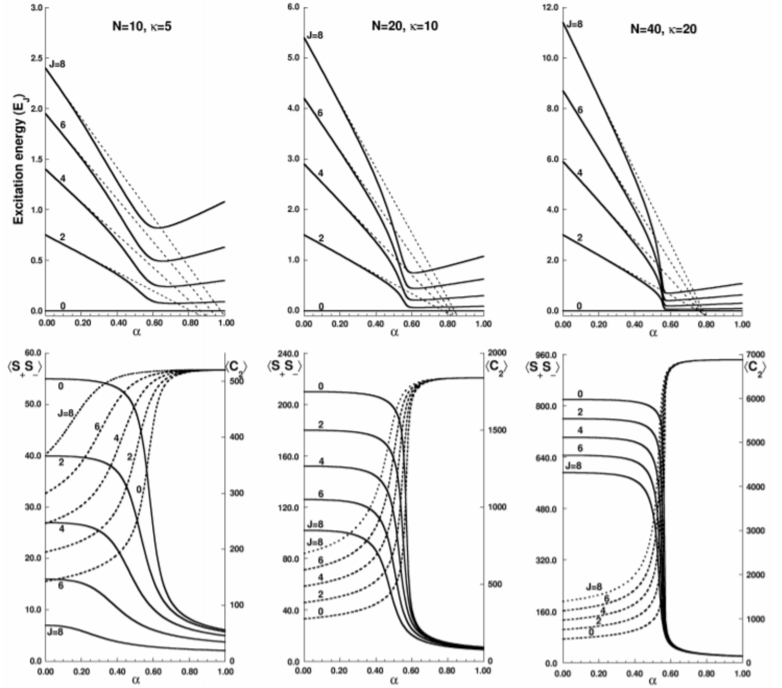


FIG. 1. The upper figures show the excitation energies for the Hamiltonian of Eq. (1) for different values of κ and $N = 2\kappa$. The lower figures show the expectations $\langle S_+ S_- \rangle$ and $\langle C_2 \rangle$ as functions of α for the corresponding κ values.

C_2 is the SU(3) Casimir operator

D. J. Rowe, C. Bahri, and W. Wijesundera, Phys. Rev. Lett. 80, 4394 (1998). C. Bahri, D. J. Rowe, and W. Wijesundera, Phys. Rev. C 58, 1539 (1998).

P+QQ for single-j shell

$$\hat{H}' = -c(xS_+S_- + (1-x)\xi\widetilde{Q}\cdot\widetilde{Q}).$$

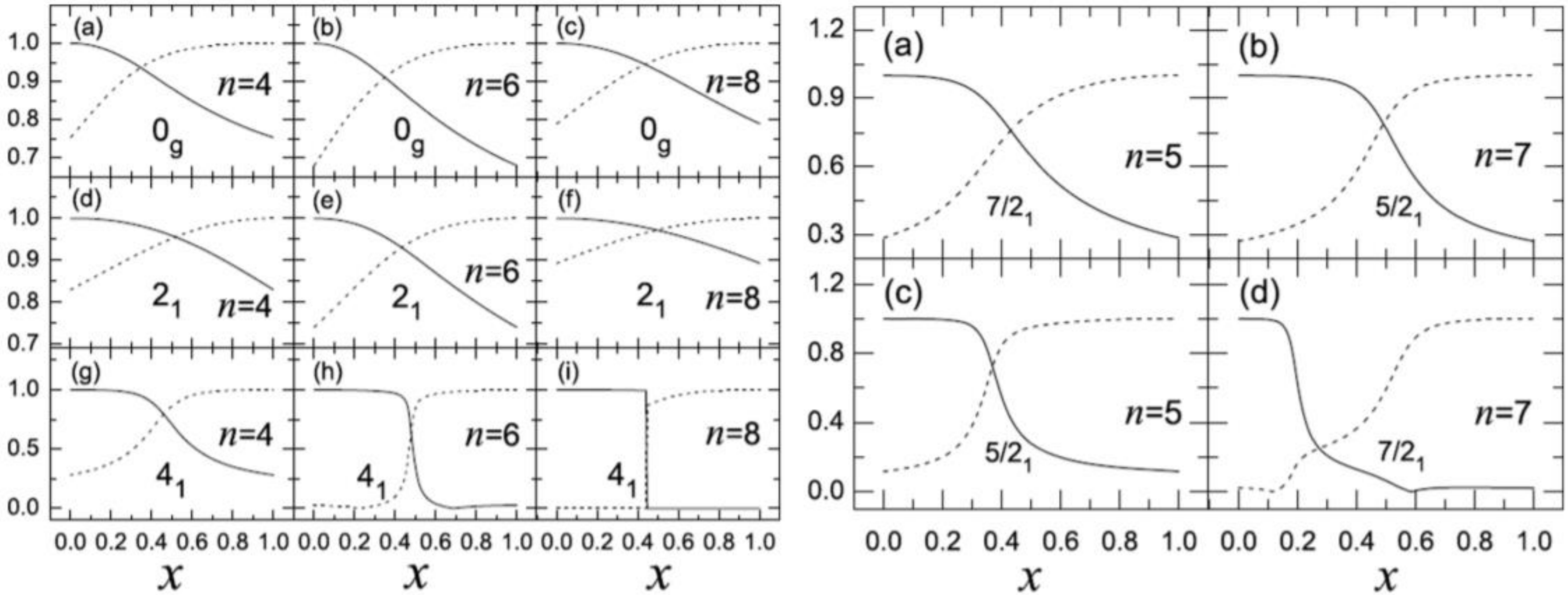


FIG. 4. The overlaps $|\langle nJ_\zeta; x = x_0|nJ_\zeta; x\rangle|$ with $x_0 = 0$ and $x_0 = 1$ for several J_1 values for n = 4, ...,8 in the $j = 15/2$ shell, where the solid line is the overlap $|\langle nJ_\zeta; x = 0|nJ_\zeta; x\rangle|$, and the dotted line is the overlap $|\langle nJ_\zeta; x = 1|nJ_\zeta; x\rangle|$.

Bo Li, Feng Pan, and J. P. Draayer
Phys. Rev. C **93**, 044312 (2016)

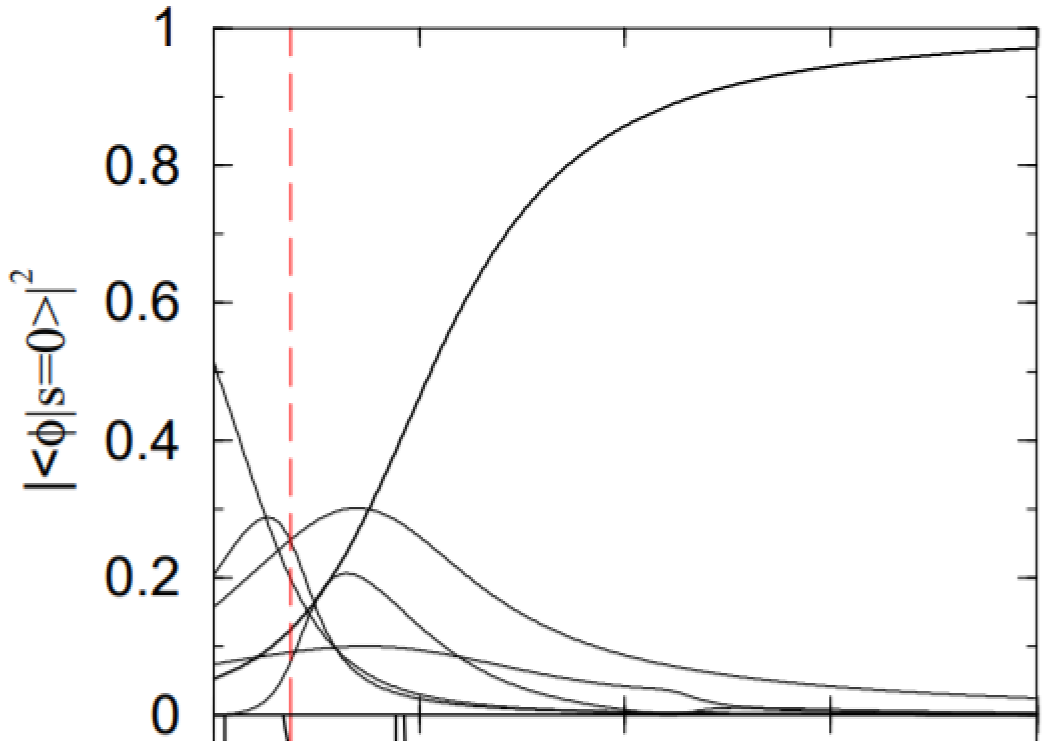
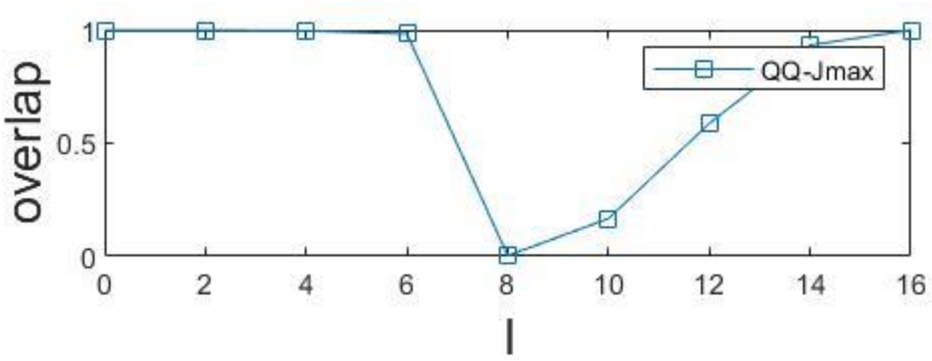


FIG. 5. Properties of the system of six particles on $j = 15/2$ orbital with the P+Q interaction are studied as a function of the parameter G; the quadrupole strength is set at $\chi_2 = 1$. The upper plot shows the overlap of all six $J = 0$ eigenstates in this system with the $s = 0$ pairing state,

https://arxiv.org/abs/nucl-th/0110067
A. Volya, Phys.Rev.C65:044311,2002

2n-2p in a single j system

LARGE overlap between the np aligned pair wave functions and the eigen state of the single-j QQ interaction



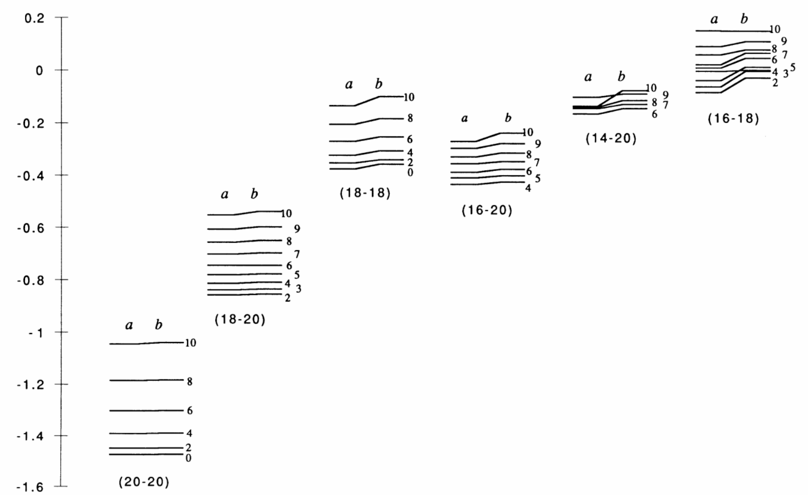


FIG. 1. The spectrum of four particles in a single-j shell ($j=\frac{21}{2}$, $H=-Q\cdot Q$, energies are in arbitrary units). Part a, the shell-model calculation; b, the GPFM calculation.

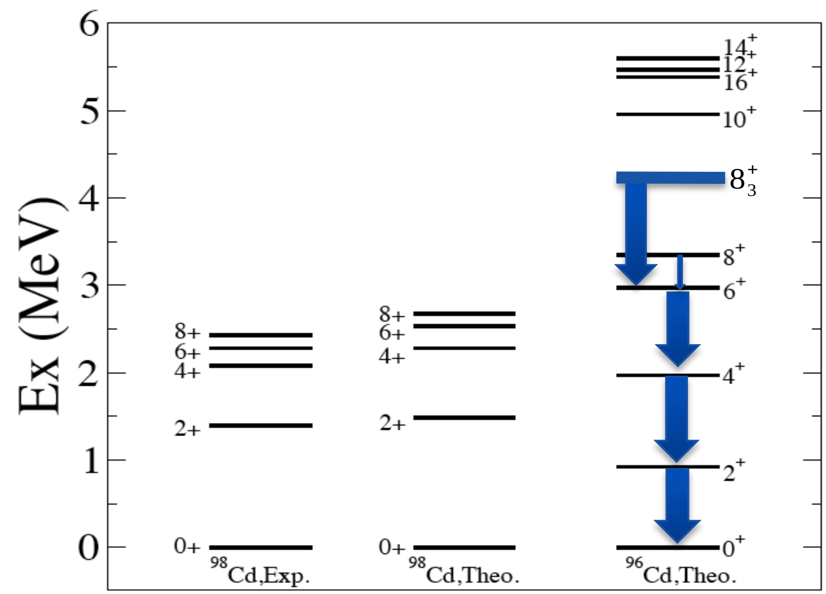
^{96}Cd (2n-2p)

Usually the wave function can be expanded as

$$|\Psi_I\rangle = \sum_{J_p, J_n} X_I(J_pJ_n)|j_\pi^2(J_p)j_\nu^2(J_n); I\rangle,$$

The thus obtained wave function is a mixture of many component as a result of the np interaction

$$\begin{aligned} |\Psi_0(gs)\rangle &= 0.76|[\pi^2(0)\nu^2(0)]_I\rangle + 0.57|[\pi^2(2)\nu^2(2)]_I\rangle \\ &+ 0.24|[\pi^2(4)\nu^2(4)]_I\rangle + 0.13|[\pi^2(6)\nu^2(6)]_I\rangle \\ &+ 0.14|[\pi^2(8)\nu^2(8)]_I\rangle. \end{aligned}$$



A striking feature is that if we project it on to np coupled terms, the wave function can be represented by a single term $(\nu\pi)_9 \otimes (\nu\pi)_9$

$$\langle [j_pj_n(J_1)j_pj_n(J_2)]_J | [j_p^2(J_p)j_n^2(J_n)]_J\rangle = -2\hat{J}_1\hat{J}_2\hat{J}_p\hat{J}_n \left\{ \begin{array}{ccc} j & j & J_p \\ j & j & J_n \\ J_1 & J_2 & J \end{array} \right\}$$

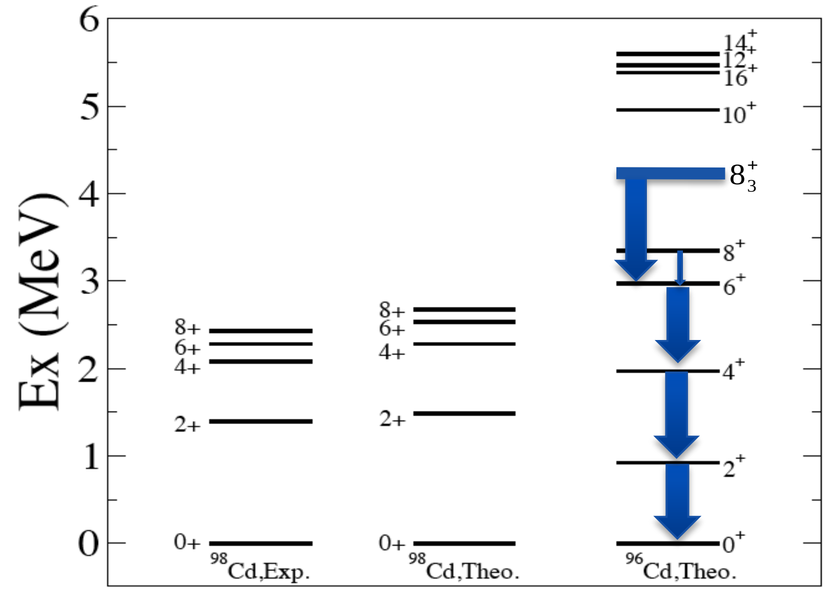
CQ et al, PRC 84, 021301(R) (2011).

^{96}Cd (2n-2p)

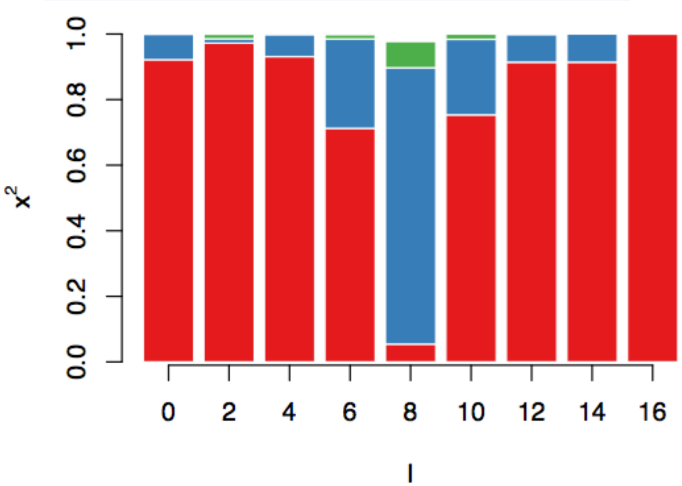
Usually the wave function can be expanded as

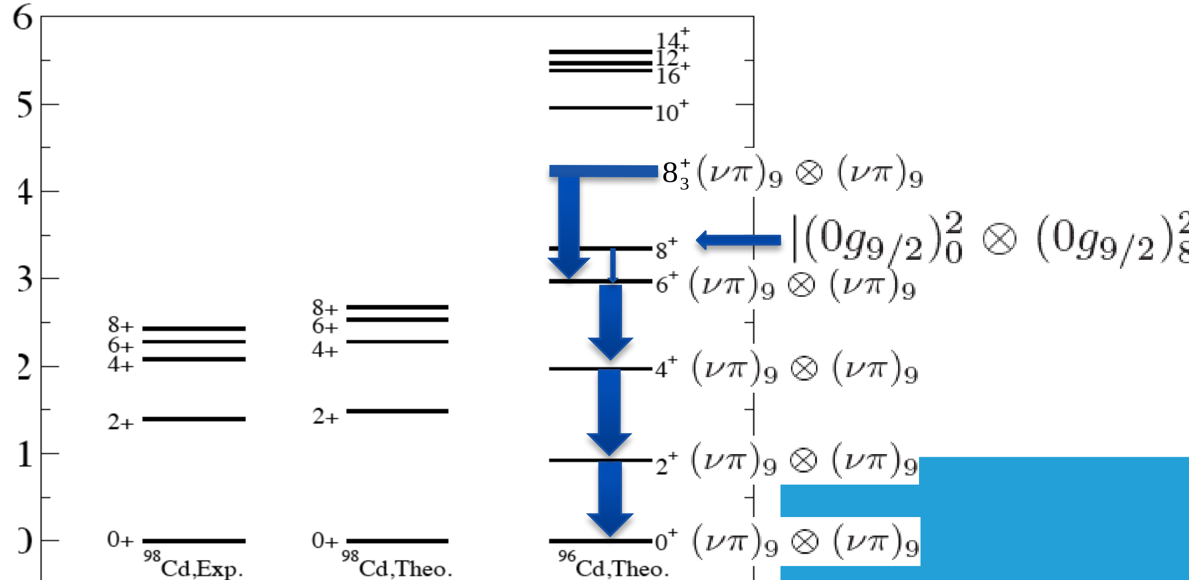
$$|\Psi_I\rangle = \sum_{J_p, J_n} X_I(J_p J_n) |j_\pi^2(J_p) j_\nu^2(J_n); I\rangle,$$

The thus obtained wave function is a mixture of many component as a result of the np interaction



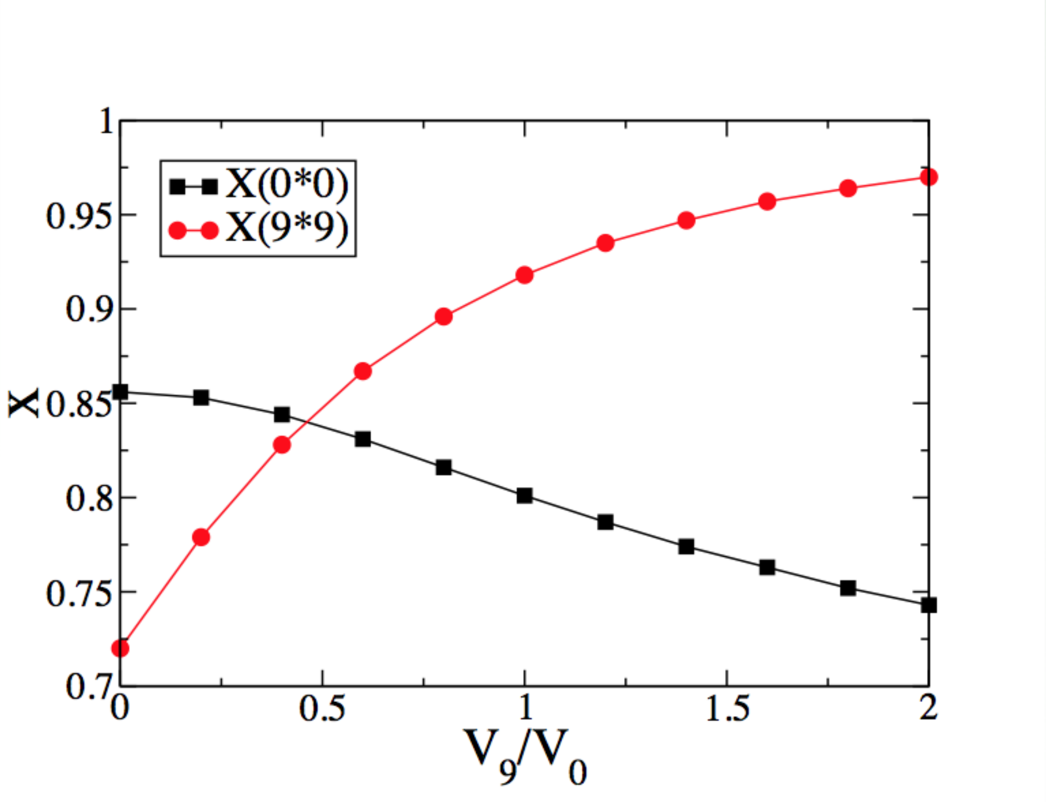
$$|\Psi_0(gs)\rangle = 0.76|[\pi^2(0)\nu^2(0)]_I\rangle + 0.57|[\pi^2(2)\nu^2(2)]_I\rangle$$
$$+ 0.24|[\pi^2(4)\nu^2($$
$$+ 0.14|[\pi^2(8)\nu^2($$





Wave function of ^{96}Cd calculated with a Hamiltonian containing $J = 0$ and 9 terms only.

- The J=9 term V_9 generates a states with pure aligned np coupling $|j_9^2 \otimes j_9^2\rangle$
- The inclusion of normal pairing is crucially important for reproducing the group state spin
- The J=9 term does not necessary to be stronger than the $J = 0$ term. It should be relatively stronger than other $T = 0$ terms. [For a simple single-j system, the relative position of $T = 0$ and 1 monopole terms does not play any effect on the wave functions.]



Quartet-like coupling as a result of T=1/0 pair coupling

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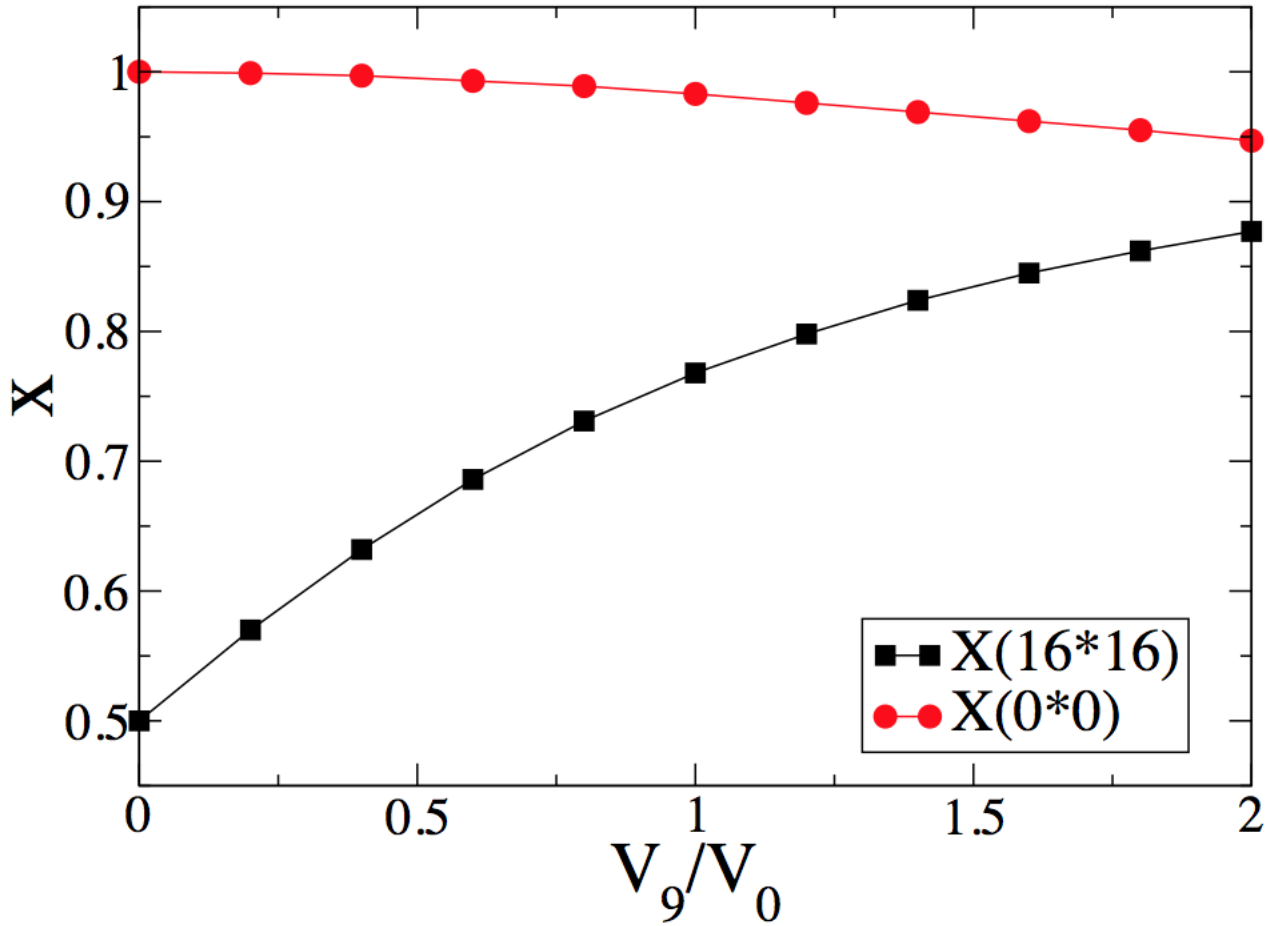


QQ leads to the stretched scheme

Nuclei around ^{100}Sn: N=Z=50 shell closures survive

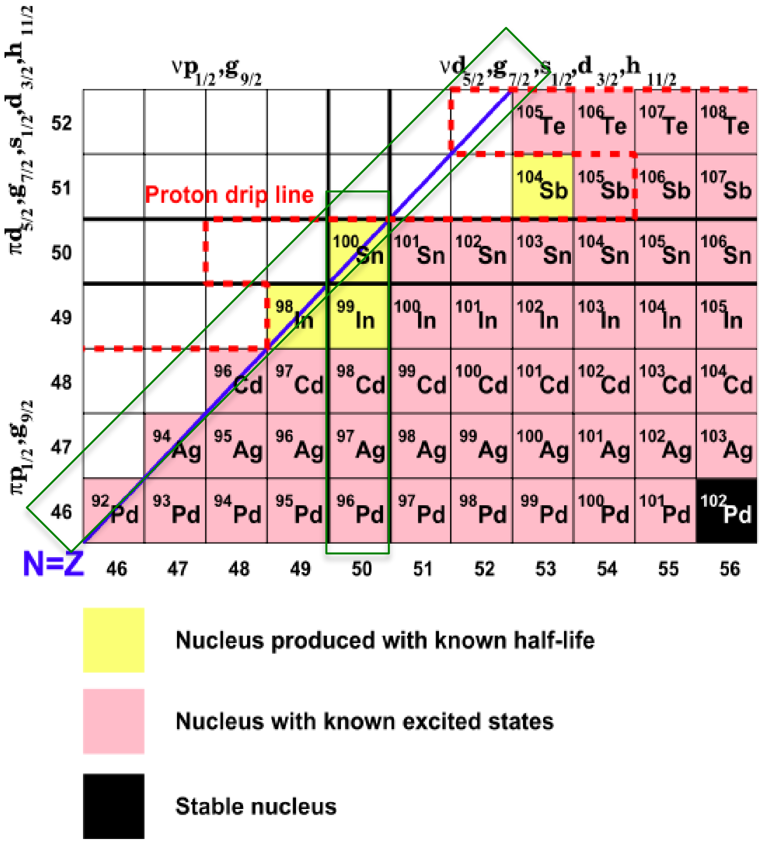


Fig. 1.1. Chart of the ^{100}Sn region showing the status of experimental observation.

T. Faestermann et al. / Prog. Part. Nucl. Phys. 69 (2013) 85–130

Superallowed Gamow–Teller decay of the doubly magic nucleus ^{100}Sn

C. B. Hinke[1], M. Böhmer[1], P. Boutachkov[2], T. Faestermann[1], H. Geissel[2], J. Gerl[2], R. Gernhäuser[1], M. Górska[2], A. Gottardo[3], H. Grawe[2], J. L. Grębosz[4], R. Krücken[1,5], N. Kurz[2], Z. Liu[6], L. Maier[1], F. Nowacki[7], S. Pietri[2], Zs. Podolyák[8], K. Sieja[7], K. Steiger[1], K. Straub[1], H. Weick[2], H.-J. Wollersheim[2], P. J. Woods[6], N. Al-Dahan[8], N. Alkhomashi[8], A. Ataç[9], A. Blazhev[10], N. F. Braun[10], I. T. Čeliković[11], T. Davinson[6], I. Dillmann[2], C. Domingo-Pardo[12], P. C. Doornenbal[13], G. de France[14], G. F. Farrelly[8], F. Farinon[2], N. Goel[2], T. C. Habermann[2], R. Hoischen[2], R. Janik[15], M. Karny[16], A. Kaşkaş[9], I. M. Kojouharov[2], Th. Kröll[17], Y. Litvinov[2], S. Myalski[4], F. Nebel[1], S. Nishimura[13], C. Nociforo[2], J. Nyberg[18], A. R. Parikh[19], A. Procházka[2], P. H. Regan[8], C. Rigollet[20], H. Schaffner[2], C. Scheidenberger[2], S. Schwertel[1], P.-A. Söderström[13], S. J. Steer[8], A. Stolz[21] & P. Strmen[15]

PRL 110, 172501 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Coulomb Excitation of ^{104}Sn and the Strength of the ^{100}Sn Shell Closure

G. Guastalla,[1] D. D. DiJulio,[2] M. Górska,[3] J. Cederkäll,[2] P. Boutachkov,[1,3] P. Golubev,[2] S. Pietri,[3] H. Grawe,[3] F. Nowacki,[4] K. Sieja,[4] A. Algora,[5,6] F. Ameil,[3] T. Arici,[7,3] A. Atac,[8] M. A. Bentley,[9] A. Blazhev,[10] D. Bloor,[9] S. Brambilla,[11] N. Braun,[10] F. Camera,[11] Zs. Dombrádi,[6] C. Domingo Pardo,[5] A. Estrade,[3] F. Farinon,[3] J. Gerl,[3] N. Goel,[3,1] J. Grębosz,[12] T. Habermann,[3,13] R. Hoischen,[2] K. Jansson,[2] J. Jolie,[10] A. Jungclaus,[14] I. Kojouharov,[3] R. Knoebel,[3] R. Kumar,[15] J. Kurcewicz,[16] N. Kurz,[3] N. Lalović,[3] E. Merchan,[1,3] K. Moschner,[10] F. Naqvi,[3,10] B. S. Nara Singh,[9] J. Nyberg,[17] C. Nociforo,[3] A. Obertelli,[18] M. Pfützner,[3,16] N. Pietralla,[1] Z. Podolyák,[19] A. Prochazka,[3] D. Ralet,[1,3] P. Reiter,[10] D. Rudolph,[2] H. Schaffner,[3] F. Schirru,[19] L. Scruton,[9] D. Sohler,[6] T. Swaleh,[2] J. Taprogge,[10,20] Zs. Vajta,[6] R. Wadsworth,[9] N. Warr,[10] H. Weick,[3] A. Wendt,[10] O. Wieland,[11] J. S. Winfield,[3] and H. J. Wollersheim[3]

PHYSICAL REVIEW C 87, 031306(R) (2013)

Transition probabilities near ^{100}Sn and the stability of the N, $Z = 50$ shell closure

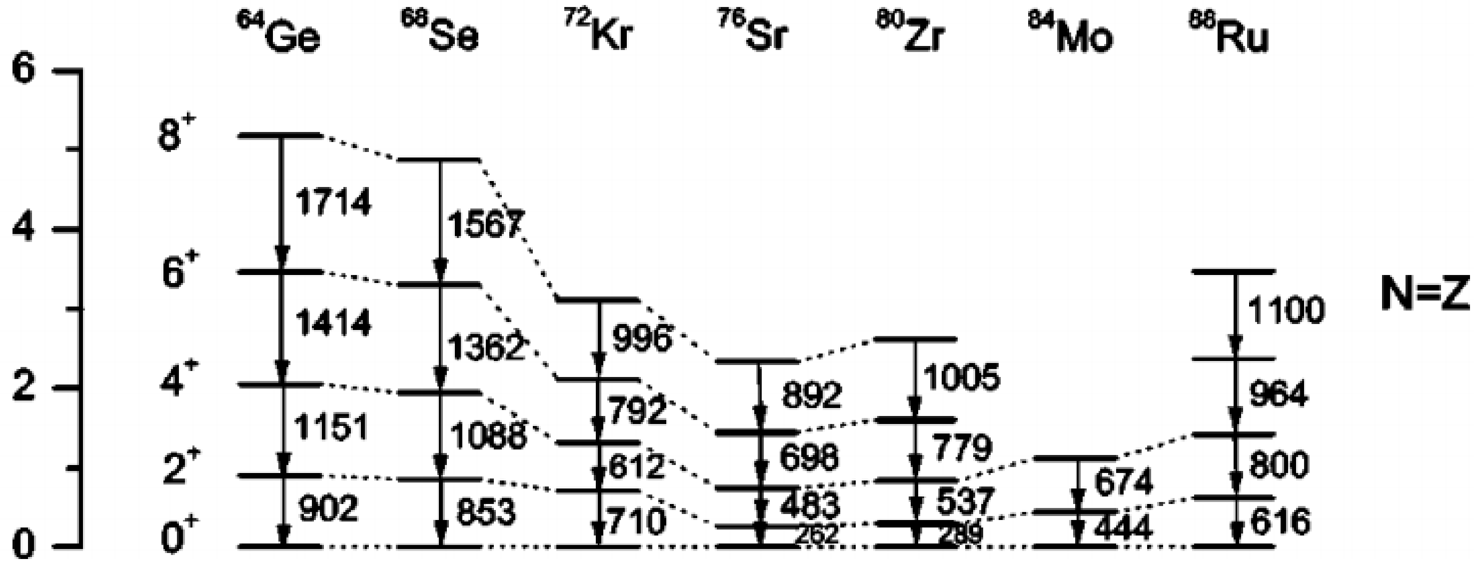
T. Bäck,[1,*] C. Qi,[1] B. Cederwall,[1] R. Liotta,[1] F. Ghazi Moradi,[1] A. Johnson,[1] R. Wyss,[1] and R. Wadsworth[2]

PHYSICAL REVIEW C 84, 041306(R) (2011)

Lifetime measurement of the first excited 2^+ state in ^{108}Te

T. Bäck,[1,*] C. Qi,[1] F. Ghazi Moradi,[1] B. Cederwall,[1] A. Johnson,[1] R. Liotta,[1] R. Wyss,[1] H. Al-Azri,[2] D. Bloor,[2] T. Brock,[2] R. Wadsworth,[2] T. Grahn,[3] P. T. Greenlees,[3] K. Hauschild,[3,†] A. Herzan,[3] U. Jacobsson,[3] P. M. Jones,[3] R. Julin,[3] S. Juutinen,[3] S. Ketelhut,[3] M. Leino,[3] A. Lopez-Martens,[3,†] P. Nieminen,[3] P. Peura,[3] P. Rahkila,[3] S. Rinta-Antila,[3] P. Ruotsalainen,[3] M. Sandzelius,[3] J. Sarén,[3] C. Scholey,[3] J. Sorri,[3] J. Uusitalo,[3] S. Go,[4] E. Ideguchi,[4] D. M. Cullen,[5] M. G. Procter,[5] T. Braunroth,[6] A. Dewald,[6] C. Fransen,[6] M. Hackstein,[6] J. Litzinger,[6] and W. Rother[6]

But many N=Z nuclei are deformed

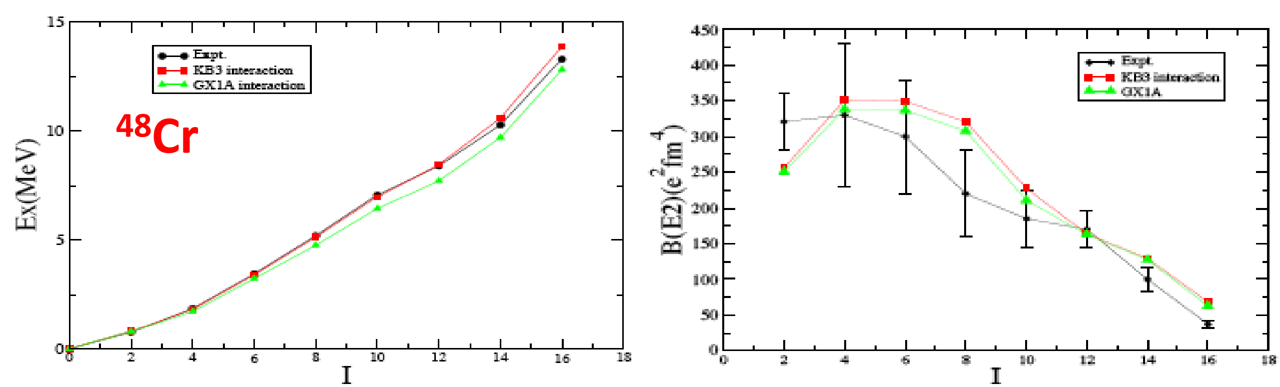


N. Mărginean et al., PRC 63, 031303(R) (2001)

◆QQ correlation induces deformation;

◆The np interaction also breaks the seniority in a major way

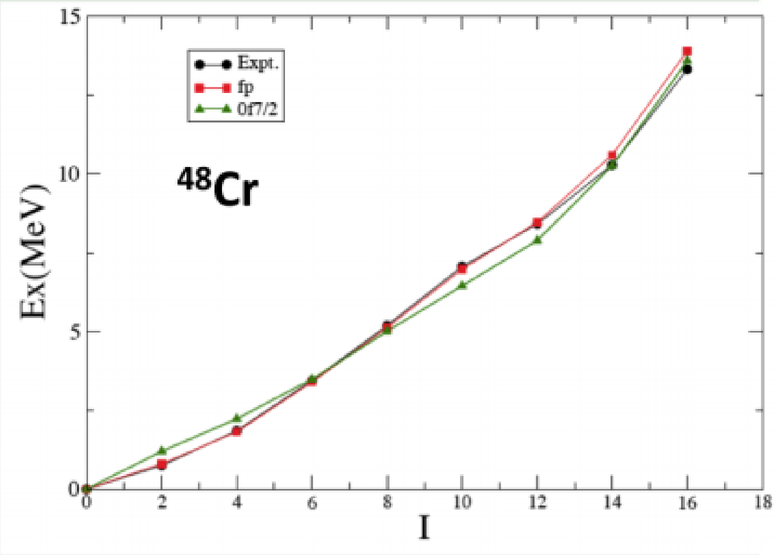
np QQ interaction between $f_{7/2}$ and $p_{3/2}$ is essential for reproducing ^{48}Cr

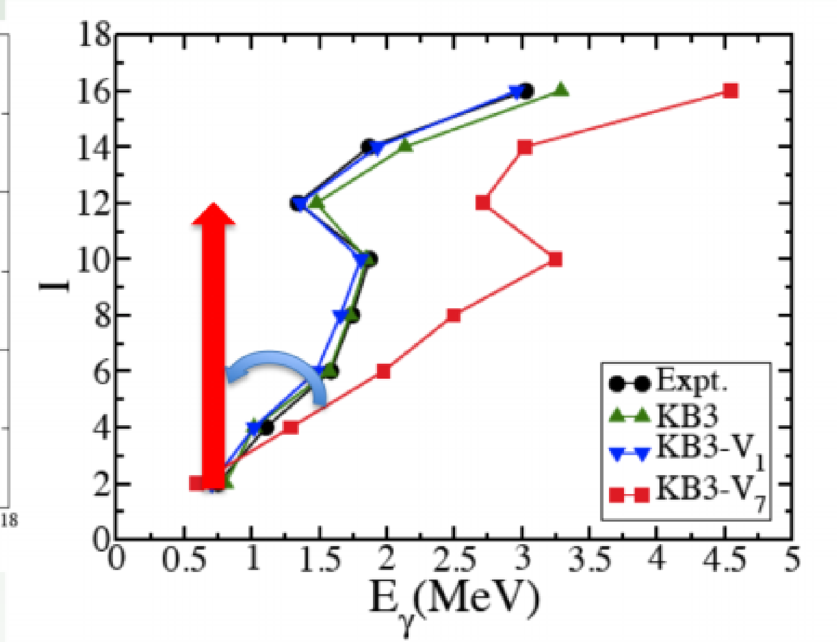


In single-j calculations, ^{44}Ti and ^{48}Cr, show vibrational like yrast specta with wave functions dominated the spin-aligned *np* coupling scheme.

^{44}Ti and ^{48}Cr exhibit rotational-like ground state bands.

- For fp shell calculations, a transition from rotational-like to equidistant pattern is seen when the aligned pair getting more and more attractive





$$V_1 = \langle (0f_{7/2}^2)_{J=1} | V | (0f_{7/2}^2)_{J=1} \rangle$$
$$V_7 = \langle (0f_{7/2}^2)_{J=7} | V | (0f_{7/2}^2)_{J=7} \rangle$$

Nilsson-SU3 selfconsistency in heavy N=Z nuclei

A. P. Zuker[1], A. Poves[2], F. Nowacki[1] and S. M. Lenzi[3]

(1) Université de Strasbourg, IPHC, CNRS, UMR7178, 23 rue du Loess 67037 Strasbourg, France
(2) Departamento de Física Teórica e IFT-UAM/CSIC,
Universidad Autónoma de Madrid, 28049 Madrid, Spain and ISOLDE, CERN, CH-1211, Genève Suisse
(3) Dipartimento di Fisica e Astronomia dell'Università and INFN, Sezione di Padova, I-35131 Padova, Italy

(Dated: June 26, 2015)

It is argued that there exist natural shell model spaces optimally adapted to the operation of two variants of Elliott's SU3 symmetry that provide accurate predictions of quadrupole moments of deformed states. A selfconsistent Nilsson-like calculation describes the competition between the realistic quadrupole force and the central field, indicating a *remarkable stability of the quadrupole moments*—which remain close to their quasi and pseudo SU3 values—as the single particle splittings increase. A detailed study of the $N = Z$ even nuclei from ^{56}Ni to ^{96}Cd reveals that the region of prolate deformation is bounded by a pair of transitional nuclei ^{72}Kr and ^{84}Mo in which prolate ground state bands are predicted to dominate, though coexisting with oblate ones.

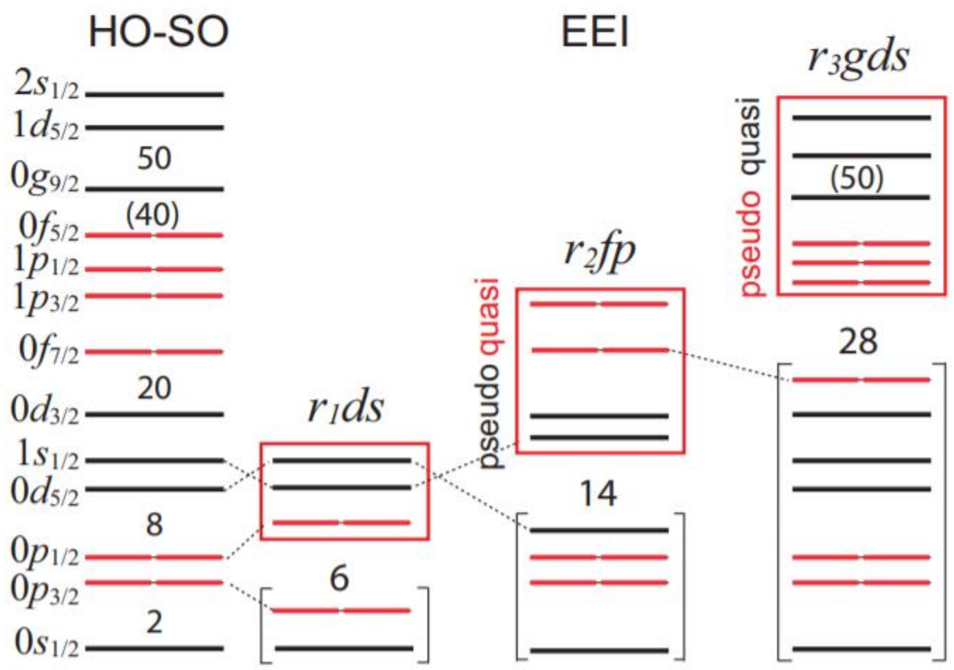


FIG. 1. (color online) Evolution of model spaces from Spin-orbit (SO) (around HO closures) to Extended Extruder-Intruder (EEI) made of Pseudo-SU3 and Quasi-SU3 subspaces

https://arxiv.org/abs/1404.0224
Phys. Rev. C 92, 024320 (2015)

- Nuclei just below ^{100}Sn are spherical
- ^{92}Pd and ^{88}Ru should be identical if g9/2 is well isolated
- Nuclei around ^{80}Zr are largely deformed and cannot be well reproduced within fpg space ($d_{5/2}$ is needed in SM but not possible for ^{80}Zr)

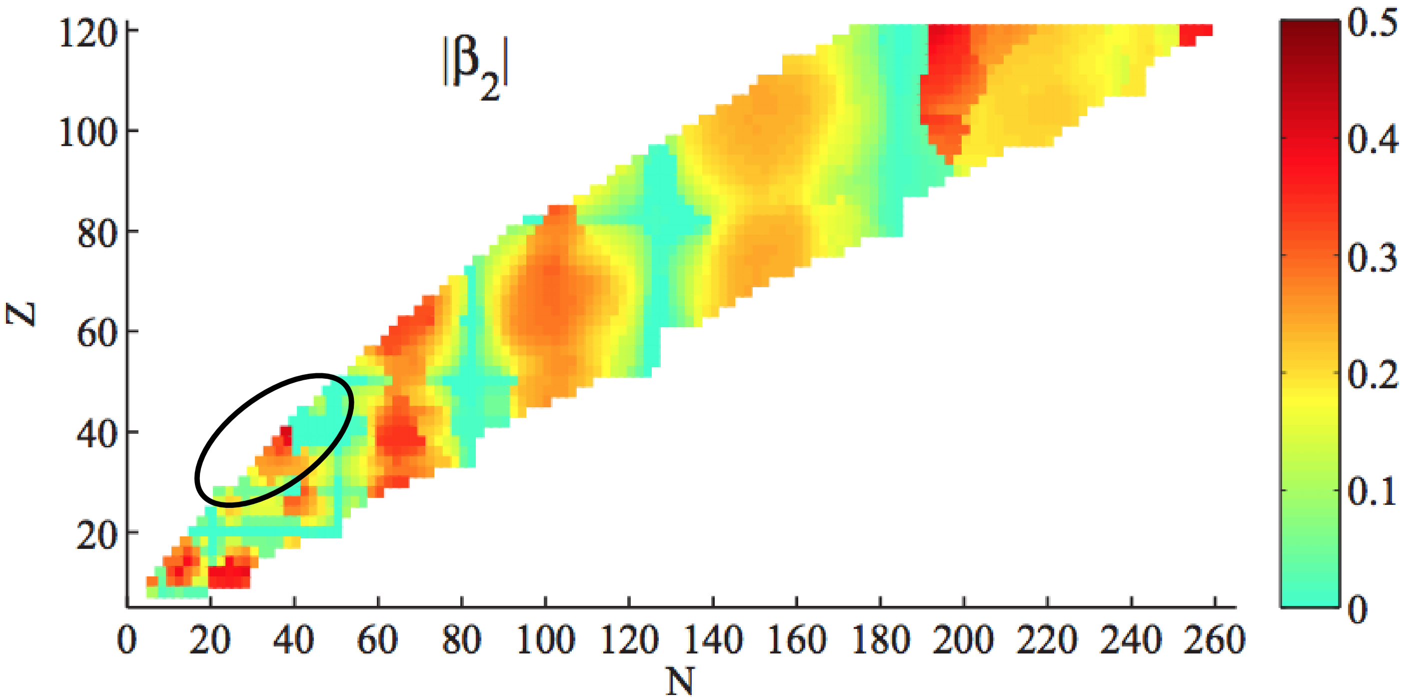


FIG. 3. (Color online) Same as Fig. 1 but for calculations with WS3 from Ref. [36].

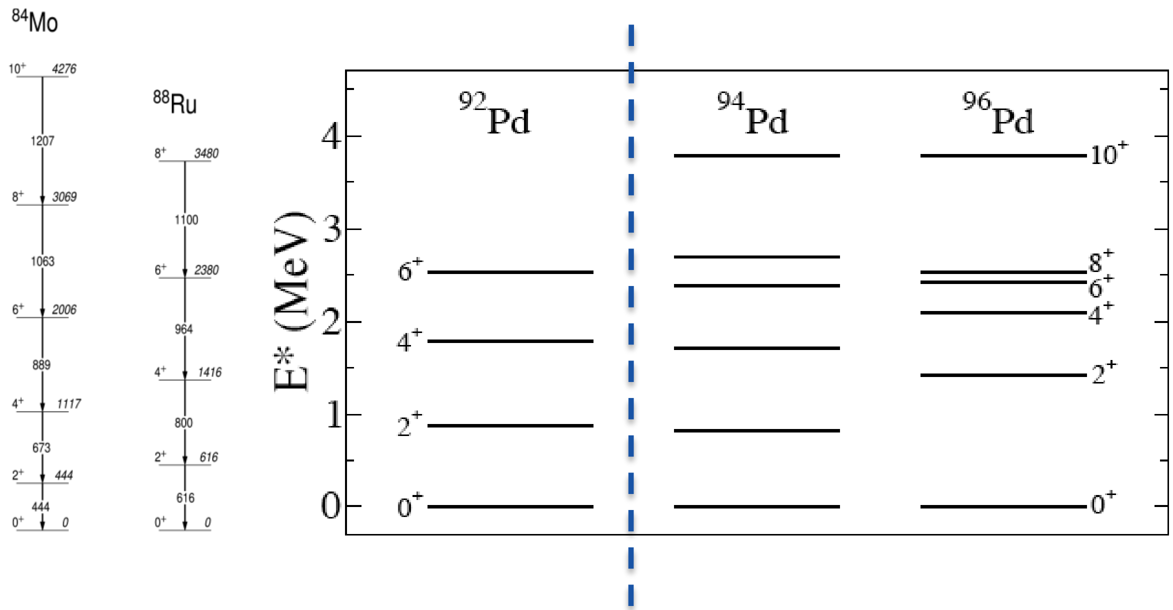
Z. Wu et al., Phys. Rev. C 92, 024306 (2015).

N=Z nuclei as a probe of np coupling scheme

Experimental status

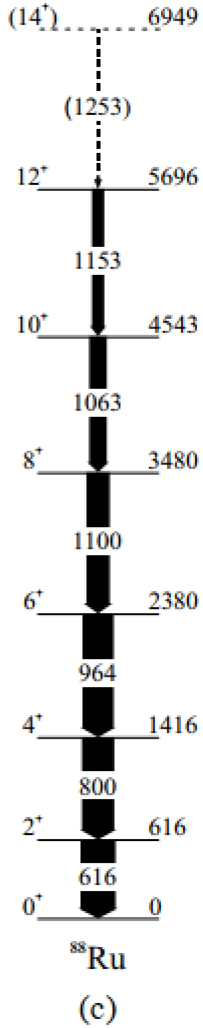
Spectra of the heaviest even-even N=Z nuclei ^{88}Ru and ^{92}Pd were reported in 2001 and 2011, respectively.

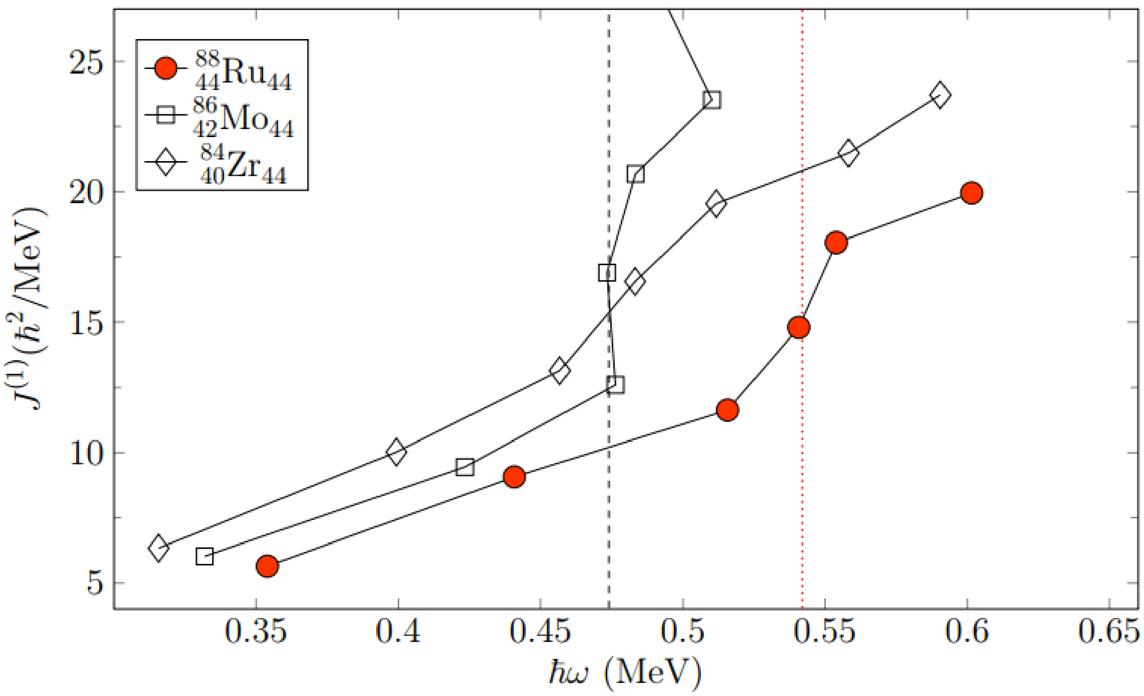
N. Mărginean et al., PRC 63, 031303(R) (2001); B. Cederwall et al., Nature 469, 68 (2011).



Rotational-like ⟺ Vibrational-like ⟺ Seniority

How to understand the difference?





From Bo Cederwall

PHYSICAL REVIEW C, VOLUME 65, 051303(R)

Delayed alignments in the $N=Z$ nuclei ^{84}Mo and ^{88}Ru

N. Mărginean,[1,2] D. Bucurescu,[2] C. Rossi Alvarez,[3] C. A. Ur,[3,2] Y. Sun,[4,5] D. Bazzacco,[3] S. Lunardi,[3] G. de Angelis,[1] M. Axiotis,[1] E. Farnea,[3] A. Gadea,[1] M. Ionescu-Bujor,[2] A. Iordăchescu,[2] W. Krolas,[6] Th. Kröll,[1,3] S. M. Lenzi,[3] T. Martinez,[1] R. Menegazzo,[3] D. R. Napoli,[1] P. Pavan,[3] Zs. Podolyak,[7] M. De Poli,[1] B. Quintana,[8] and P. Spolaore[1]

The Hamiltonian employed in the PSM calculation can be expressed as $\hat{H}=\hat{H}_\nu+\hat{H}_\pi+\hat{H}_{\nu\pi}$, where H_τ $(\tau=\nu,\pi)$ is the like-particle pairing plus quadrupole Hamiltonian, with the inclusion of quadrupole pairing,

$$\hat{H}_\tau=\hat{H}^0_\tau-\frac{\chi_{\tau\tau}}{2}\sum_\mu \hat{Q}^{\dagger\mu}_\tau\hat{Q}^\mu_\tau-G^\tau_M\hat{P}^\dagger_\tau\hat{P}_\tau-G^\tau_Q\sum_\mu \hat{P}^{\dagger\mu}_\tau\hat{P}^\mu_\tau, \qquad (2)$$

and $\hat{H}_{\nu\pi}$ is the np quadrupole-quadrupole residual interaction,

$$\hat{H}_{\nu\pi}=-\chi_{\nu\pi}\sum_\mu \hat{Q}^{\dagger\mu}_\nu\hat{Q}^\mu_\pi. \qquad (3)$$

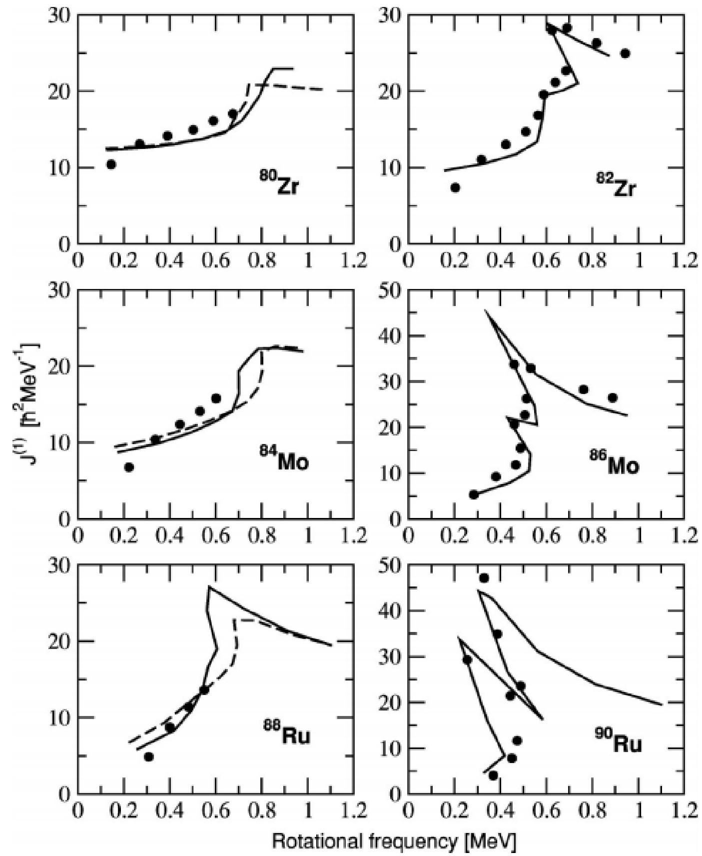


FIG. 3. Comparison of experimental data (dots) and projected shell model calculations. The experimental data are as follows: ^{84}Mo (present data), ^{86}Mo [17], ^{88}Ru [11], ^{90}Ru [20]. For continuity with the study of the $N=Z$ nuclei presented in Ref. [9], ^{80}Zr [2] and ^{82}Zr [21] are also shown. The full lines are the PSM calculations with a standard interaction, the dashed ones with an enhanced neutron-proton residual interaction (see text for details).

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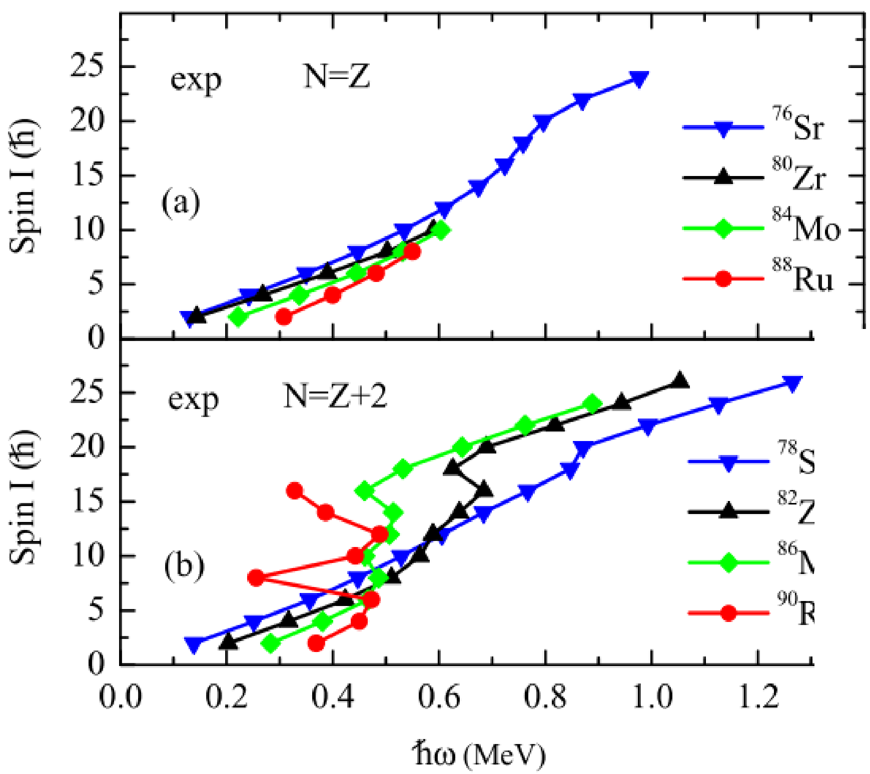
Nuclear Physics A 957 (2017) 144–153

www.elsevier.com/locate/nuclphysa

Enhancement of high-spin collectivity in $N = Z$ nuclei by the isoscalar neutron–proton pairing

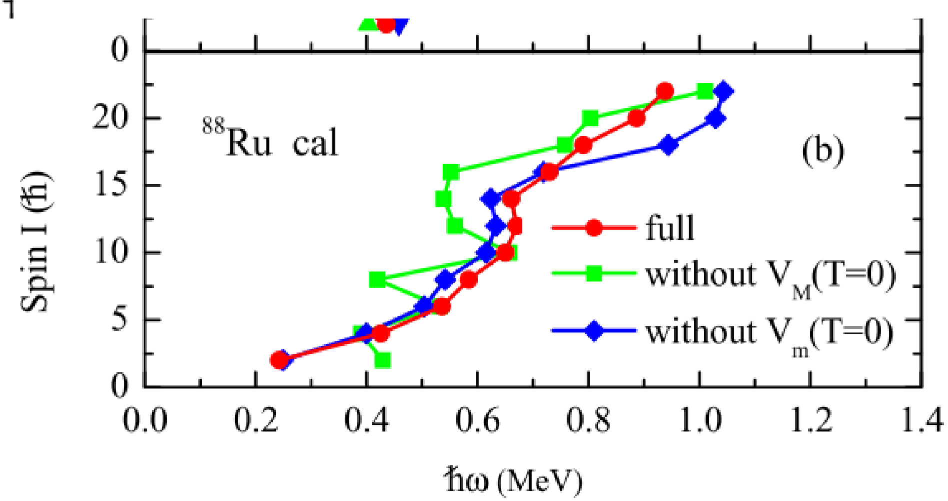
K. Kaneko [a], Y. Sun [b,c,d,*], G. de Angelis [e]



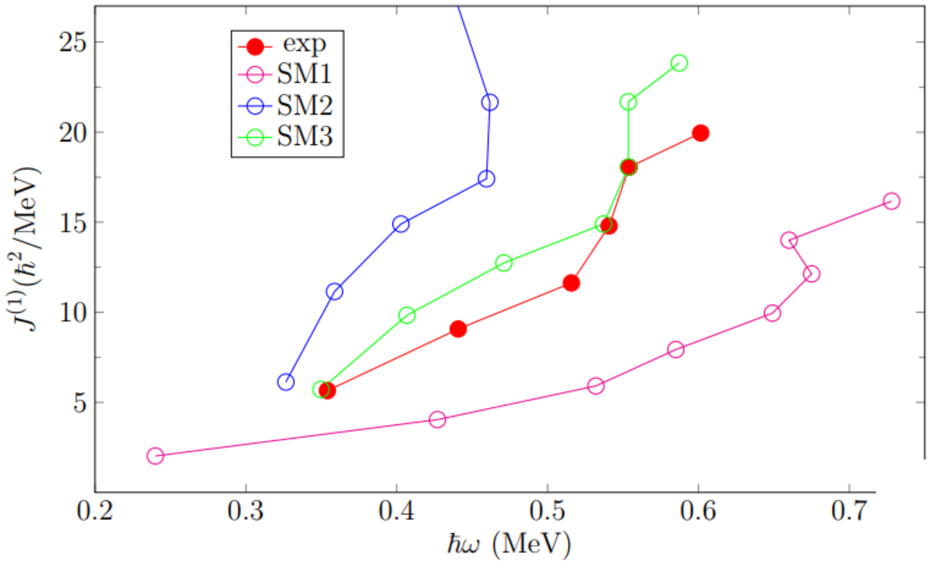
$$H = H_0 + H_P + H_M + H_m^{MU},$$

$$H_0 = \sum_\alpha \varepsilon_a c_\alpha^\dagger c_\alpha, \qquad H_P = -\sum_{J=0,2} \frac{1}{2} g_J \sum_{M\kappa} P_{JM1\kappa}^\dagger P_{JM1\kappa}$$

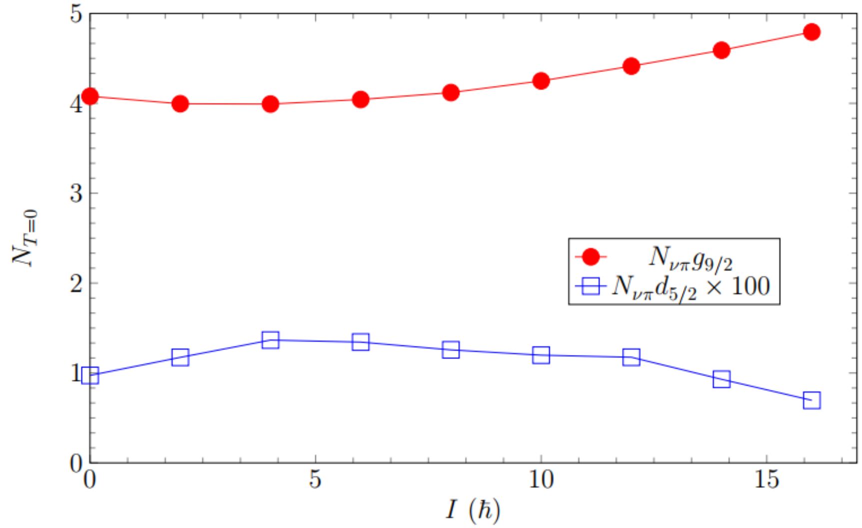
$$H_M = -\frac{1}{2}\chi_2 \sum_M : Q_{2M}^\dagger Q_{2M} : -\frac{1}{2}\chi_3 \sum_M : O_{3M}^\dagger O_{3M} :$$



Shell model calculations for 88Ru



Number of ‘pairs’



SM1: Kaneko
SM2: fpg shell model calculation with JUN45
SM3: Extension to d5/2 with a QQ

Summary

- **General feature of T=1/0 interaction**

  Strong T=1, J=0 pairing, Exact diagonalization of isovector pairing

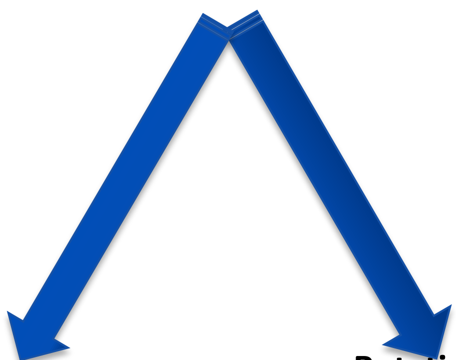
- **Competition between T=1 pairing and ‘QQ’**

  Not necessarily leading to deformation

- **Aligned np pair and QQ lead to same configuration for single-j systems**

- **Large QQ leads to strong quadrupole correlation**

  and scatters the wave function



Competition?

Seniority $\leftrightarrow$ Equidistant $\leftrightarrow$ Rotational-like

Pairing theory of the symmetry energy

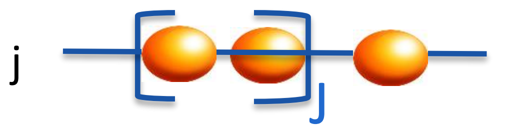
K. Neergård
*Fjordtoften 17, 4700 Næstved, Denmark**

$$E - E_{T=0} = \tfrac{1}{2}(D+\kappa)T(T+1)$$
$$-D\left(\sqrt{(aT)^2+b^2} - \sqrt{\left(\frac{T}{2}\right)^2 + \left(\frac{\Delta}{D}\right)^2} - b + \frac{\Delta}{D}\right).$$

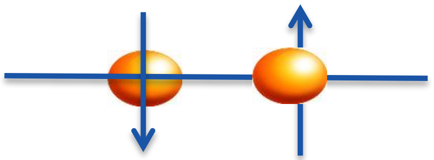
https://arxiv.org/pdf/0905.4095.pdf

Computer 'likes' uncoupled scheme

Is 'M=0' pair a relevant degree of freedom (for truncation in M-scheme)



$$|j_J^2 = 0\rangle = \sum_m f_m |jm; j-m\rangle$$



M=0

J=0,2,4…