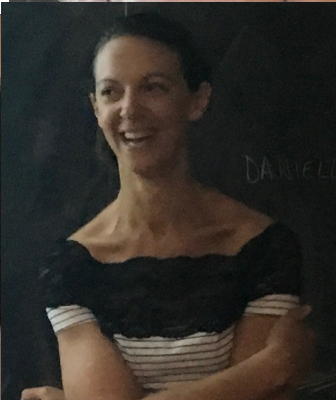
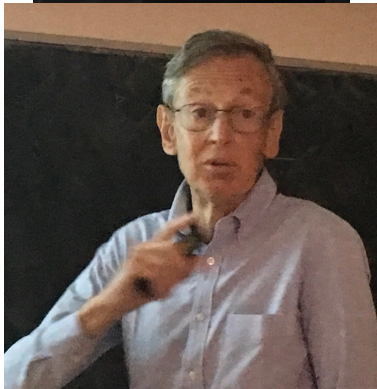
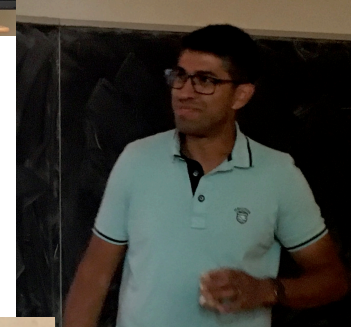
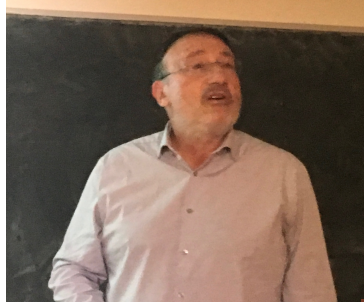
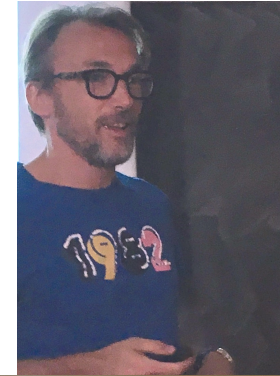
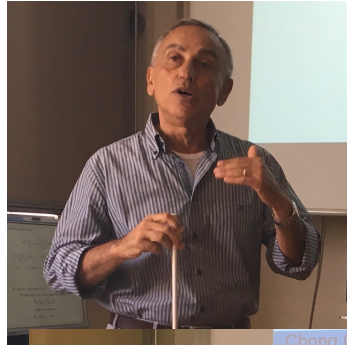
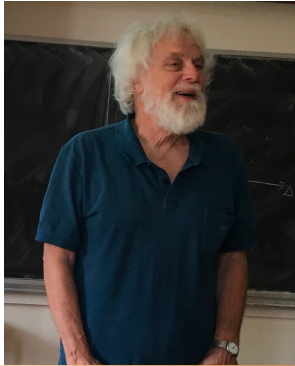
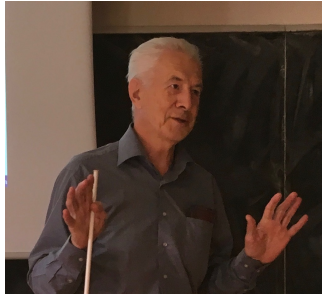


*Recent advances on proton-neutron pairing and
quartet correlations in nuclei, Session II*

Summary Talk

My goal:

My goal:



ESNT Workshop on Recent Advances on Proton-Neutron pairing... Sept 2-6, 2019

My goal:



ESNT Workshop on Recent Advances on Proton-Neutron pairing... Sept 2-6, 2019

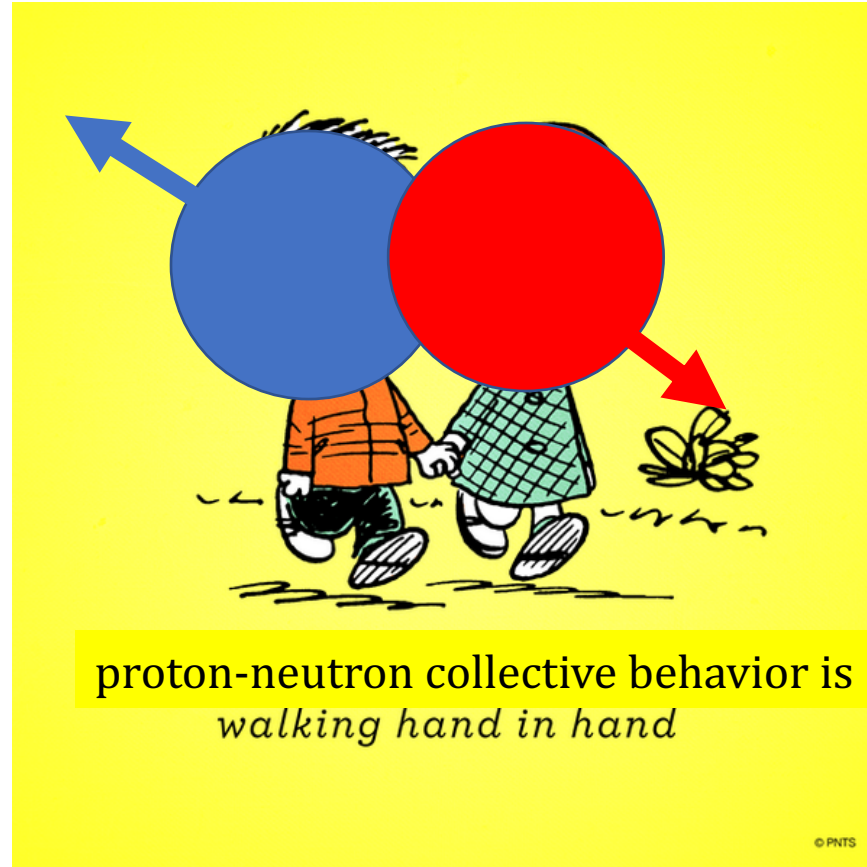
Workshop themes

Simple pictures of cooperative behavior



Workshop themes

Simple pictures of cooperative behavior

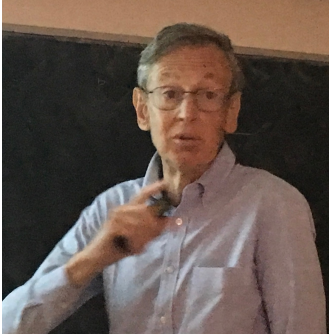
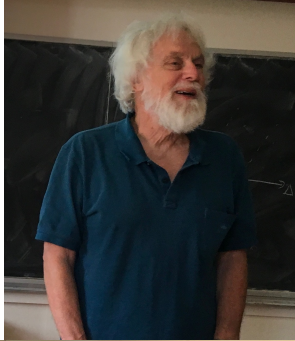
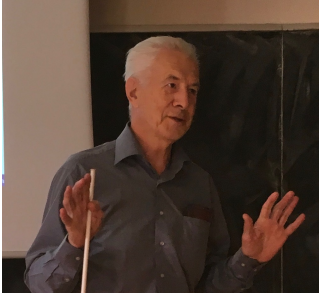


Workshop themes



Keep in mind
this is a 'mean-field' summary,
that is,
I won't get all the high-resolution details,
but instead focus on, well,
the average picture....

Evidence for pairing/ p-n pairing /clustering



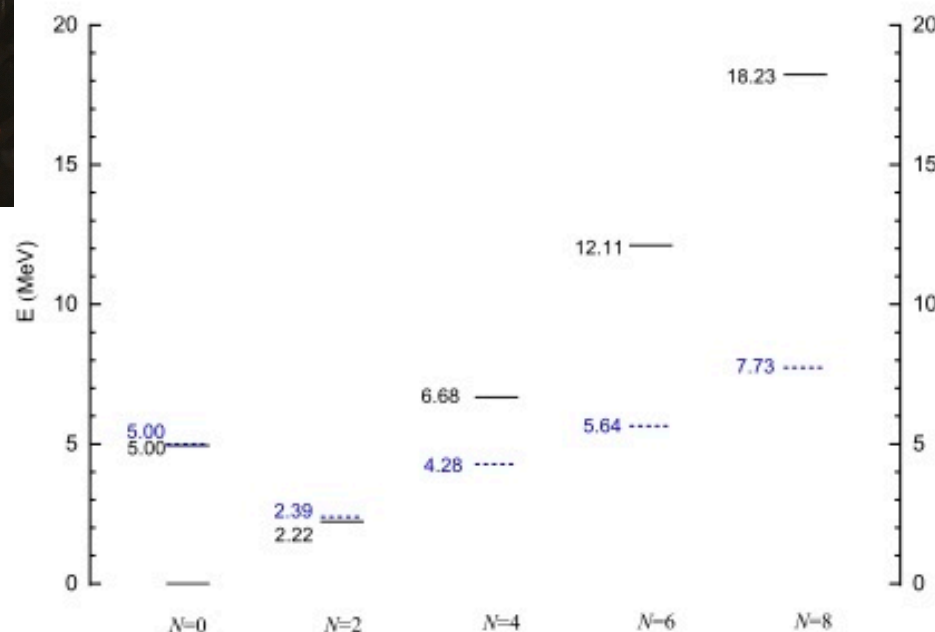
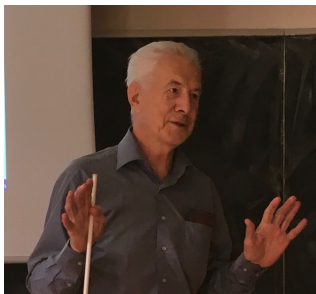
theory cluster



experimental pair
(correlation in momentum
not coordinate space...)

Evidence for pairing/ p-n pairing /clustering

Energies of the 0^+ $T=0$ states in ^{56}Ni

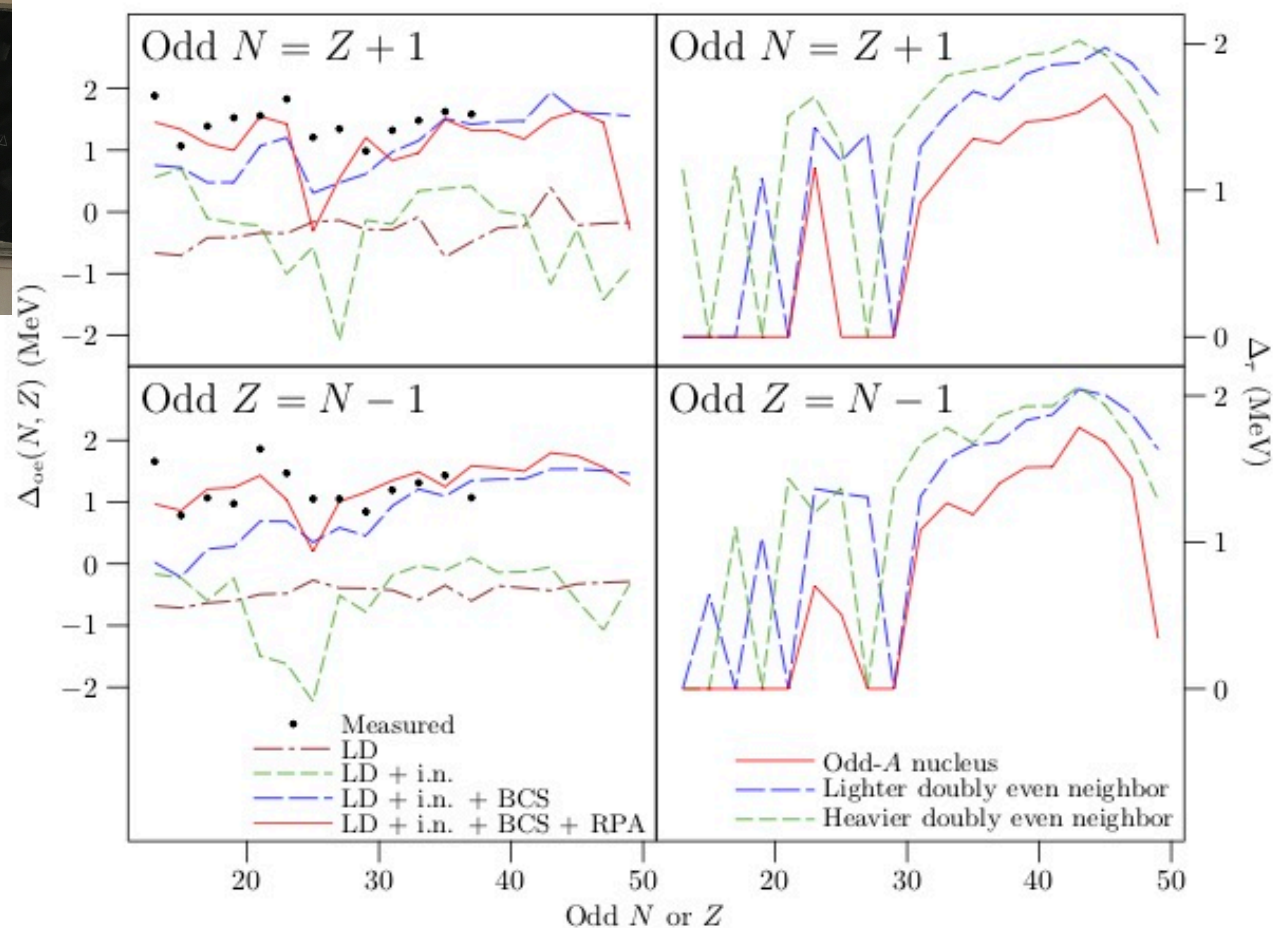
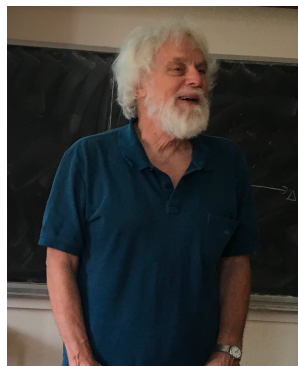


Pairing “rotations” and
“vibrations” in spectra

$G_1 = \frac{14.4}{A} \text{ MeV}$ is fixed to reproduce the energy of the pairing vibrational state in ^{56}Ni at 5 MeV. $d=0.6$ for ^{56}Ni

Evidence for pairing/ p-n pairing /clustering

Results for $Z = N - 1$



pair vibrations and
odd-even mass
differences

Evidence for pairing/ p-n pairing /clustering



EFFECT OF QUADRUPLE CORRELATIONS IN LIGHT NUCLEI

V G SOLOVIEV

Joint Institute of Nuclear Research, Dubna, USSR

Received 25 December 1959

“quadruple”= two interacting pn pairs

Fingerprints of alpha-like (quadruple) correlations

- 1) Extracting a pn pair from a even-even $N=Z$ nucleus costs more energy than adding to it a pn pair



- 2) Extracting one neutron from a even-even $N=Z$ nucleus costs more energy than from neighbouring nuclei

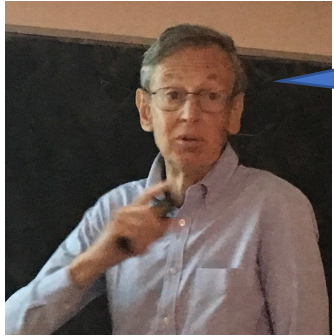
$$B(^{24}\text{Mg}) - B(^{23}\text{Mg}) = 16.6 \text{ MeV}$$

$$B(^{25}\text{Mg}) - B(^{24}\text{Mg}) = 7.3 \text{ MeV}$$

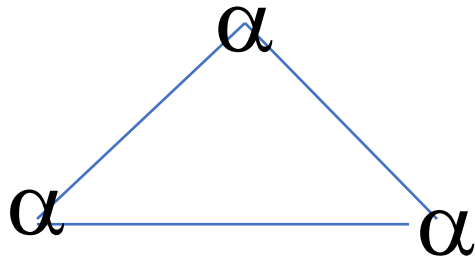
$$B(^{26}\text{Mg}) - B(^{25}\text{Mg}) = 11.3 \text{ MeV}$$

to brake a quadruple (quartet) in pairs takes about 4-5 MeV

Evidence for pairing/ p-n pairing /clustering

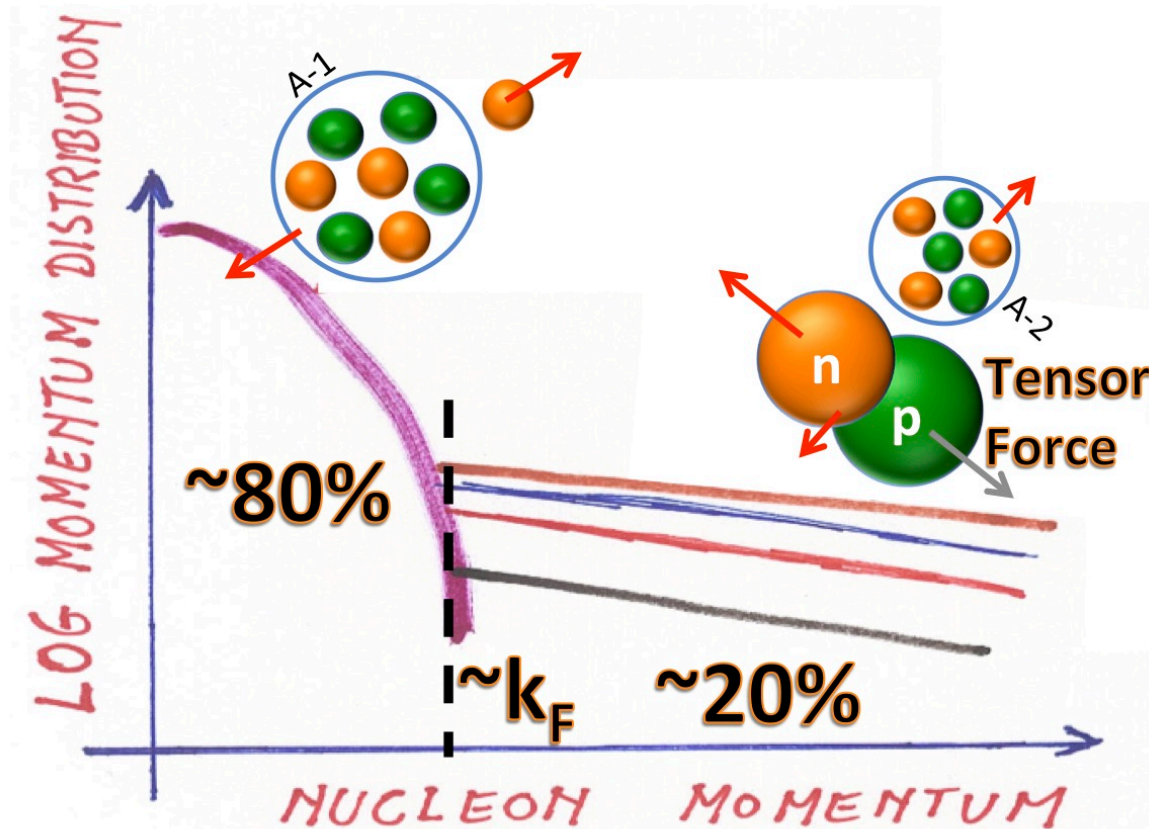


Band structure ($0^+ 2^+, 3^-$) and transition densities in inelastic α - ^{12}C shows evidence of D_{3h} discrete symmetry



Evidence for pairing/ p-n pairing /clustering

A high-momentum tail is attributed to SRCs between a pair of strongly interacting nucleons; a value of about 20% SRC contribution was indirectly inferred.

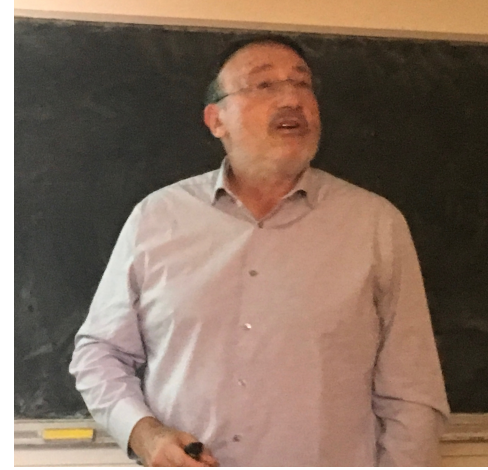


(weakly coupled experimental talk)

Duer, Nature (2018); Cohen, PRL (2018); Hen, RMP (2017); Hen, Science (2014); Hen, PLB (2013) Korover, PRL (2014); Fomin, PRL (2012); Subedi, Science (2008); Piasetzky, PRL (2007); Egiyan, PRL (2006)

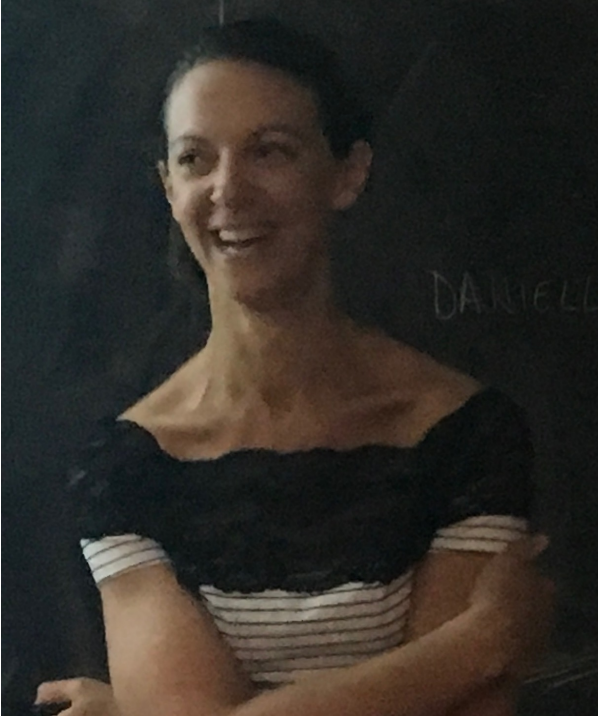
Evidence for pairing/ p-n pairing /clustering

+ bonus round on ^{40}Mg ...



(weakly coupled
experimental talk)

Evidence for pairing/ p-n pairing /clustering



Experimental searches for strong p-n pair correlations in (p,³He) transfer reactions (T=0 pairing seems weaker in *pf* shell than *sd*)

and proposed tests for α -quartets via (6Li,d) or (7Li,t) transfer reactions

What drives pairing and clustering?

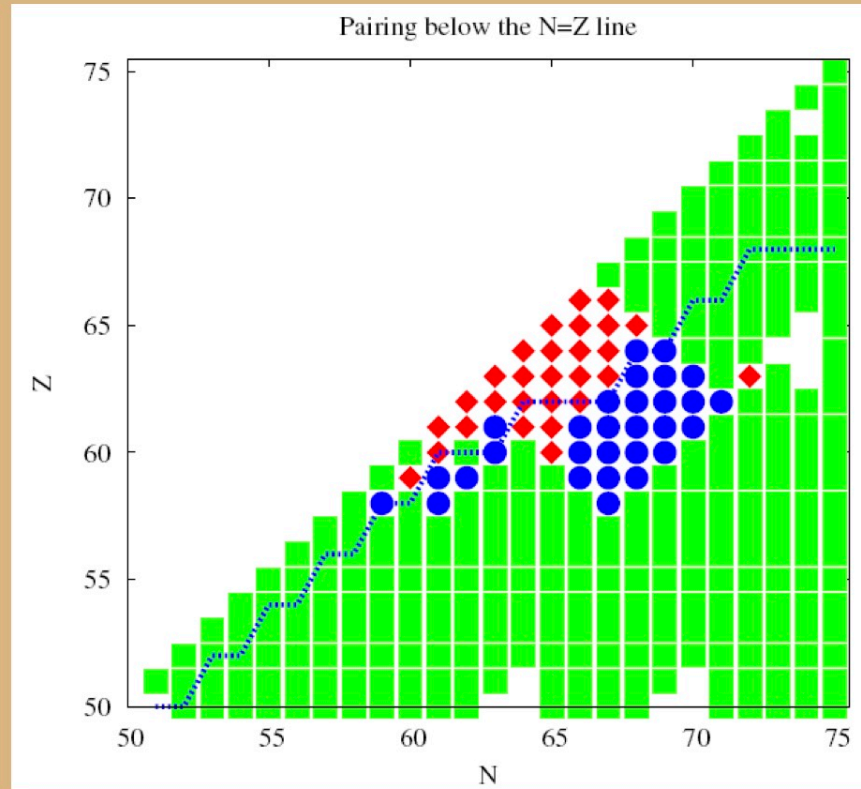


(a diffuse cluster
with long-range
correlations..?)

What drives pairing and clustering?



Pairing in heavy nuclei ($A \sim 130$)



Correlation energies

- Blue line: proton drip
- Green: spin-singlet
- Red: spin-triplet
- Blue: mixed-spin
- Spin-triplet pairing persists off $N=Z$ line
- Mixed-spin pairing appears to be energetically stable (note: no deformation)

A. Gezerlis, G. F. Bertsch, and Y. L. Luo, Phys. Rev. Lett. **106**, 252502 (2011)

What drives pairing and clustering?



Clustering occurs in states near continuum: residue of (g.s.) $A-\alpha$ scattering resonances

What drives pairing and clustering?

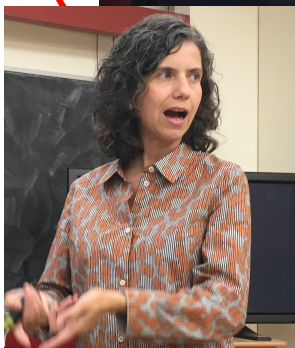
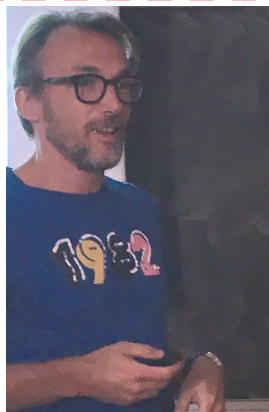


Deformation and low density
drives scattering

(not orthogonal to Volya)

How can we better model nuclei?

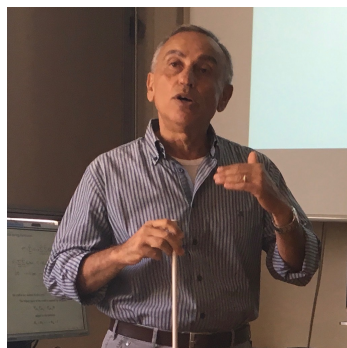
“Mean-field” + projection



(coupled via
tensor force)



What the shell model can teach us



Condensing the shell model to pairs & quartets

How can we better model nuclei?

“Mean-field” + projection



BCS in a box: symmetry restoration

$$|\psi_N\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi i} e^{-iM\phi} \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + e^{i\phi} v_{\mathbf{k}} \hat{p}_{\mathbf{k}}^\dagger \right) |0\rangle$$

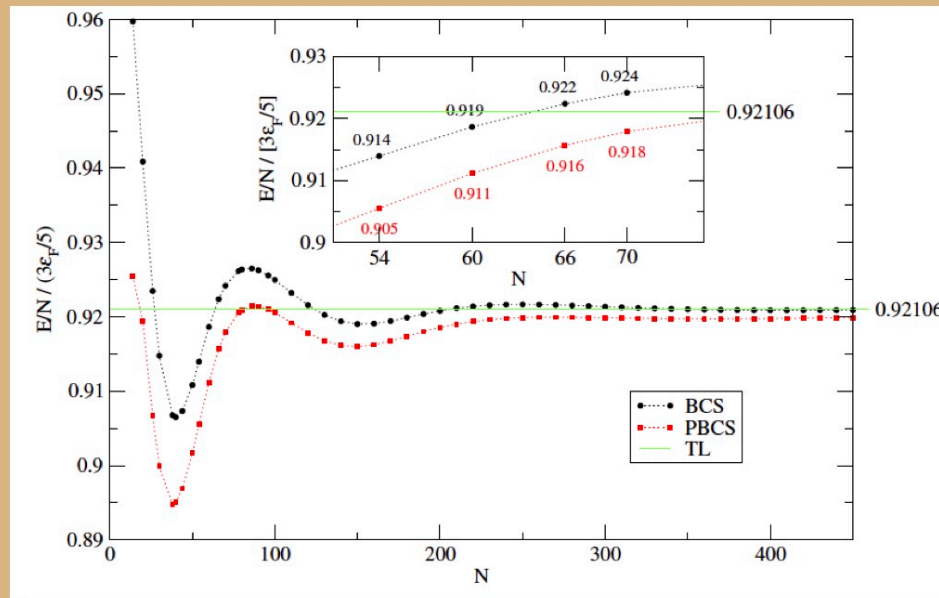
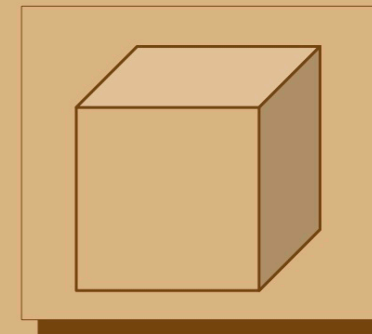
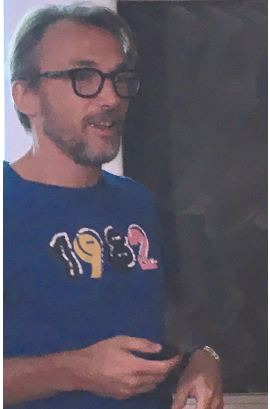


Figure by George Palkanoglou



How can we better model nuclei?

“Mean-field” + projection



Extend the reach of successful theories (coupled-cluster, MBPT) by symmetry breaking \rightarrow restoration

So far only $U(1)$ (number)
but $SU(2)$ (deformation) promised!

How can we better model nuclei?

“Mean-field” + projection



When driven to large deformation,
we can (sometimes) see clusters
arise in projected mean-field
calculations

How can we better model nuclei?

“Mean-field” + projection



Use $SO(8)$ model to
test VBP vs VAP

Pairing coexistence seen by the VAP approach!

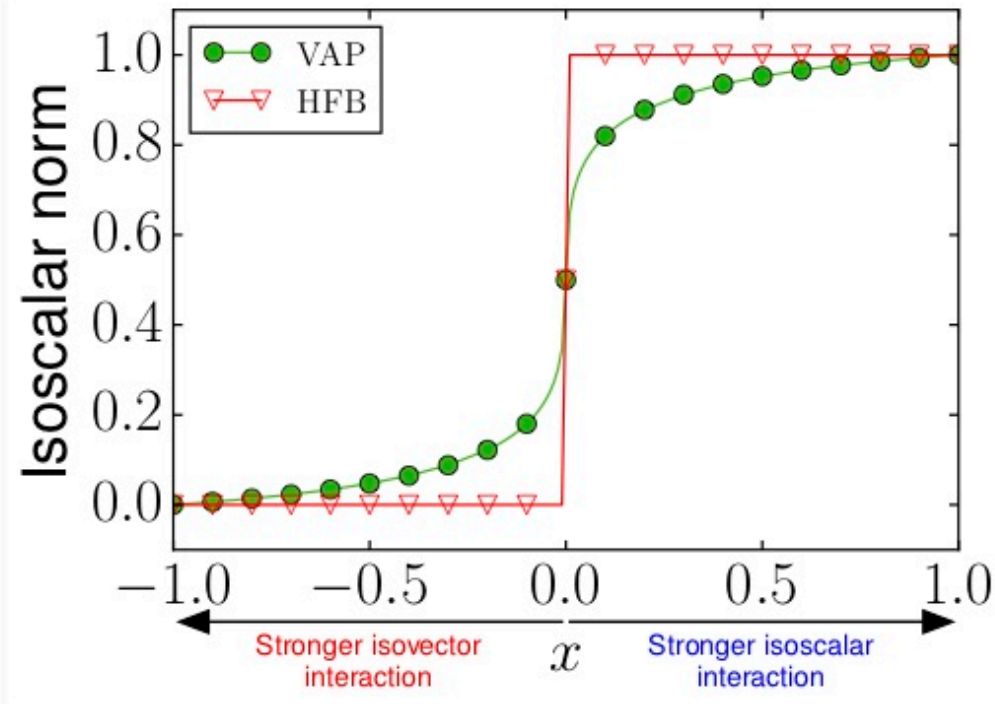


Figure 5: Norm of isoscalar pairs (contribution to the total wavefunction of the nucleus) as a function of the tuning parameter x obtained from VAP and PAV (HFB) methods.

How can we better model nuclei?

“Mean-field” + projection



Spin & isospin projection
seem most important!

Importance of the separate symmetry restorations

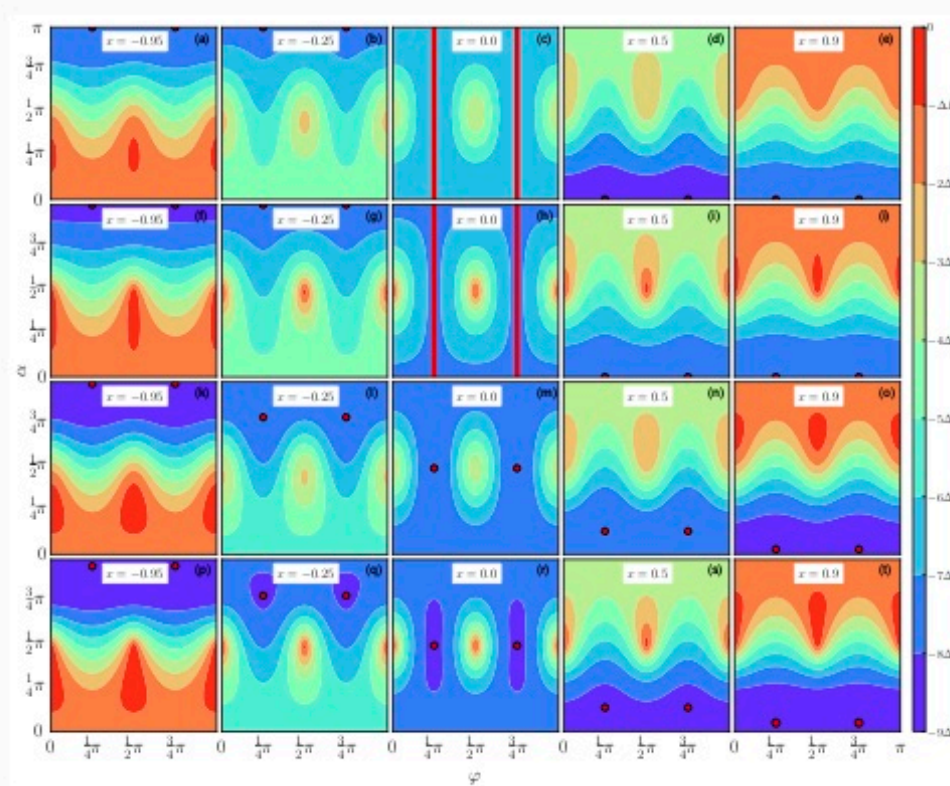
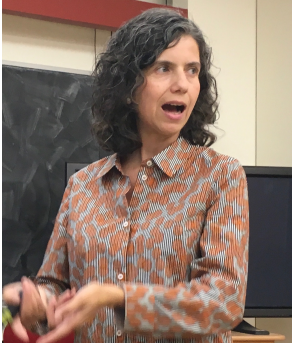


Figure 6: HFB (first row), particle-number restored (second row), spin plus isospin restored (third row) and particle number, spin and isospin restored (fourth row) energy surfaces.

How can we better model nuclei?

“Mean-field” + projection



Importance of
tensor force

WHY INCLUDING A TENSOR TERM IN THE NUCLEAR EFFECTIVE INTERACTION?	THE METHOD: HF+BCS+QRPA	LOW-LYING EXCITATIONS IN MIRROR NUCLEI	CONCLUSIONS
●●○○○○	○○○○○○○○○○○○	○○○○○○○○○○	○○

INTRODUCTION

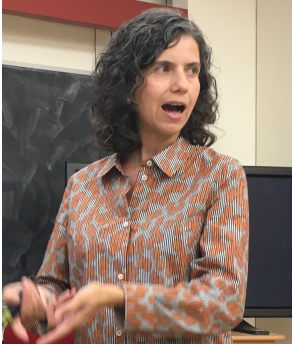
- ▶ Otsuka *et al.* showed that shell evolution cannot be studied without tensor force
Phys. Rev. Lett. **95**, 232502 (2005)
- ▶ They proposed a new parametrization for the Gogny force including a tensor-isospin term → GT2
Phys. Rev. Lett. **97**, 162501 (2006)

μ (fm)	W (MeV)	B (MeV)	H (MeV)	M (MeV)
0.7	2311	-3480	2962	-2800
1.2	-339	388	-370	260

$W_0=160 \text{ MeV fm}^5$	$x_0 = 1$	$\alpha = 1/3$
----------------------------	-----------	----------------

How can we better model nuclei?

“Mean-field” + projection



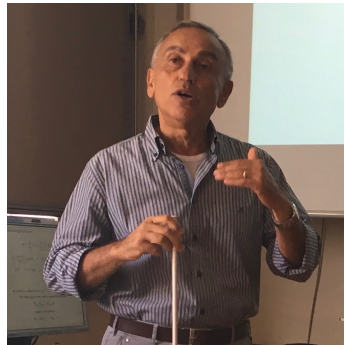
Preliminary results
in mirror nuclei

WHY INCLUDING A TENSOR TERM IN THE NUCLEAR EFFECTIVE INTERACTION? THE METHOD: HF+BCS+QRPA LOW-LYING EXCITATIONS IN MIRROR NUCLEI CONCLUSIONS						
○○○○○○ ○○○○○○○○○○ ○○●○○○○○○ ○○						
2 ⁺ AND 4 ⁺ EXCITATIONS FOR MIRROR NUCLEI: O ISOTOPES						
Nucleus	excited state	D1S	D1ST2a	D1M	D1MTd	exp
¹⁸ O	2 ⁺	2.409	2.615	2.324	2.691	1.982
	4 ⁺	3.452	3.677	3.196	3.575	3.554
¹⁸ Ne	2 ⁺	1.787	1.973	1.697	2.027	1.887
	4 ⁺	2.703	2.912	2.467	2.811	3.338
²⁰ O	2 ⁺	2.342	2.509	2.233	2.554	1.67
	4 ⁺	3.493	3.690	3.191	3.535	3.57
²⁰ Mg	2 ⁺	1.633	1.809	1.528	1.832	1.65
	4 ⁺	2.627	2.825	2.363	2.685	3.70
²² O	2 ⁺	2.839	2.936	2.447	2.716	3.199
	4 ⁺	4.067	4.430	3.423	3.876	
²² Si	2 ⁺	2.212	2.325	1.920	2.121	
	4 ⁺	3.477	3.722	2.936	3.329	

23/31

How can we better model nuclei?

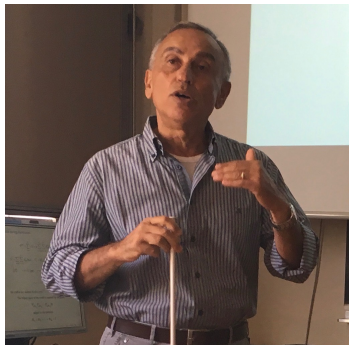
What the shell model can teach us



Condensing the shell model to pairs & quartets

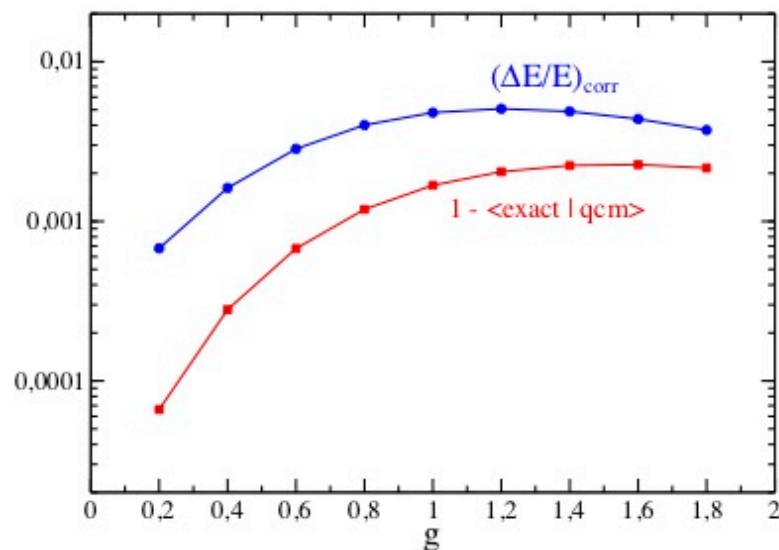
How can we better model nuclei?

Condensing the shell model to pairs & quartets



qcm vs exact results

6 protons and 6 neutrons over 6 equispaced levels



How can we better model nuclei?

Condensing the shell model to pairs & quartets



Quartet condensation versus pair condensation for isovector & isoscalar pairing

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i, j) \sum_{\tau} P_{i\tau}^+ P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i, j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}$$

$$(Q^+)^{n_q} | - > \quad (\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+)^{n_q} | - > \quad (\Gamma_{\nu\pi}^+)^{2n_q} | - > \quad = (\Delta_0^+)^{2n_q} | 0 \rangle$$

	QCM	PBC1	PBCS _{0_{iv}}	PBCS _{0_{is}}
²⁰ Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
²⁴ Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
²⁸ Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
⁴⁴ Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
⁴⁸ Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
⁵² Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
¹⁰⁴ Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
¹⁰⁸ Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
¹¹² Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- quartet condensation wins over Cooper pair condensates
- T=1 and T=0 pairing correlations **always** coexist in quartets

M. Sambataro and N.S., Phys. Rev C93, 054320 (2016)

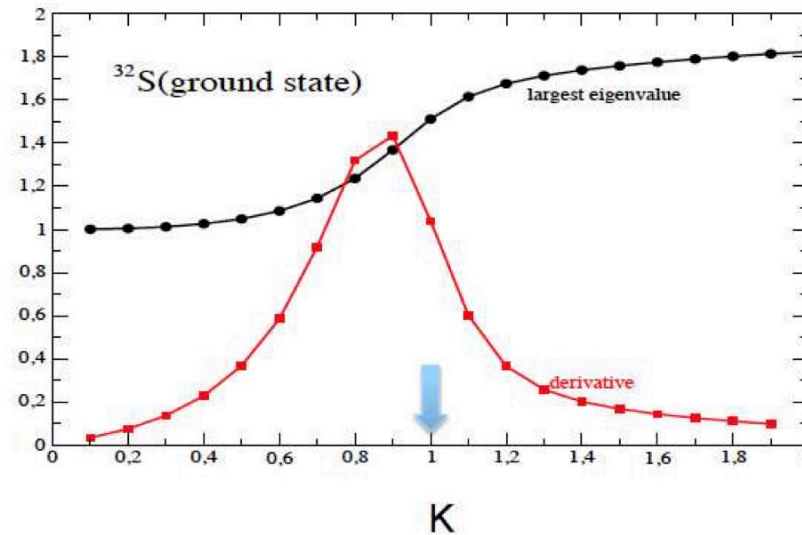
How can we better model nuclei?

Condensing the shell model to pairs & quartets



Evolution of the largest eigenvalue of 4-body density matrix: ^{32}S

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + k \sum_{ii', jj', J, T} V_{JT}(ii'; jj') [A_{ii'JT}^+ A_{jj'JT}]^{J=0, T=0}$$



Indication of a fast transition towards a quartet condensate !

I approve!



How can we better model nuclei?

Condensing the shell model to pairs & quartets



Need to include interplay with deformation...
start with simple pairing in deformed (Nilsson)
single-particle basis

How can we better model nuclei?



What the shell model can teach us

How can we better model nuclei?

What the shell model can teach us



Simple shell-model calculations, even in single- j shell, illustrate competition between deformation and pairing



Giving alpha clusters a little "boost" describes cluster states very well



Entropy may make supercomputers unnecessary

Last but far from least: Can cluster emissions tell us something?



Despite early hope for heavy cluster emissions as a signature of superheavy nuclei,

even for superheavies, α -decay always dominates

A cluster unto himself....

What lessons do we learn?
Final thoughts



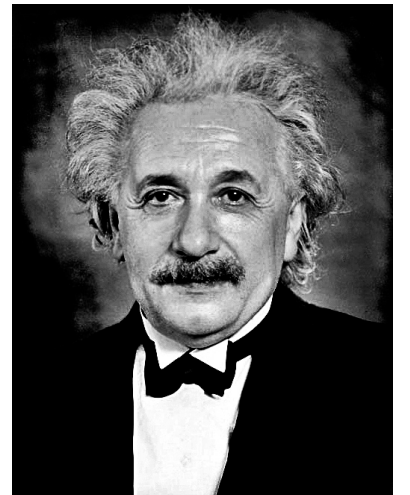
What lessons do we learn?
Final thoughts

“How do you keep track
of all those electrons?”



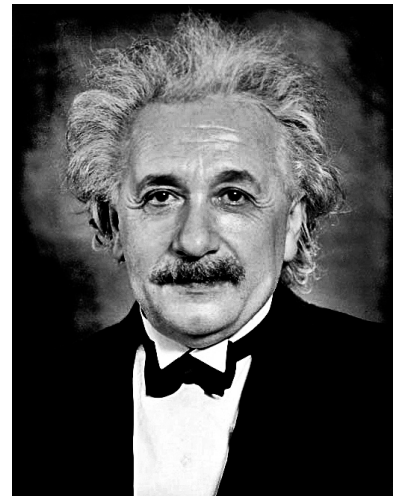
What lessons do we learn?
Final thoughts

“Everything should be made as simple as possible, but no simpler.”



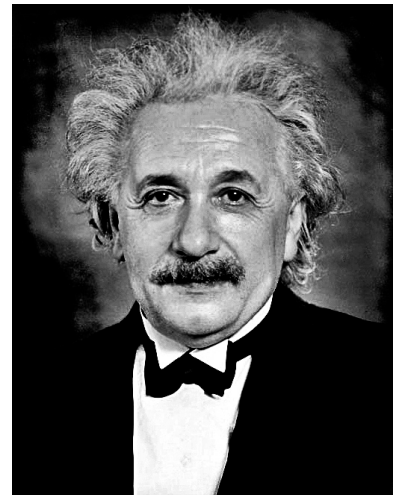
What lessons do we learn?
Final thoughts

But when is something "too simple"?



What lessons do we learn?
Final thoughts

It depends upon the question
you're asking!



What lessons do we learn?

Final thoughts

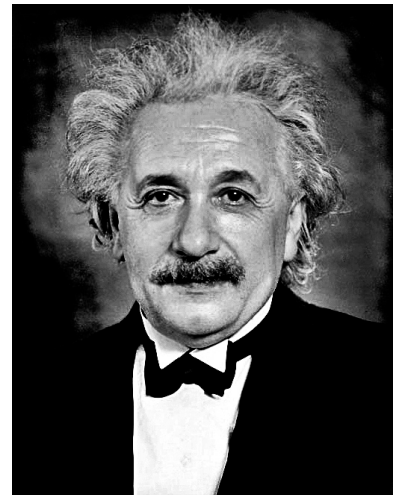
6 _____

The absolute g.s. energy of a rotational band isn't important...

4 _____

2 _____

0 _____



What lessons do we learn?

Final thoughts

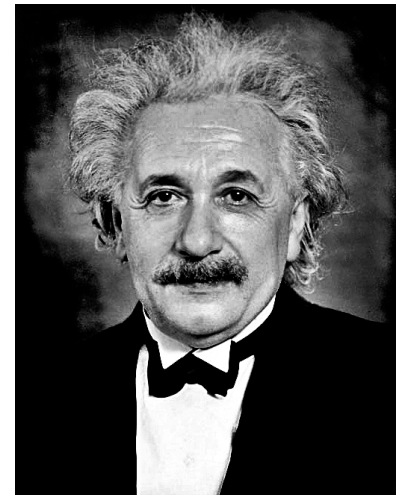
6 _____

The absolute g.s. energy of a rotational
band isn't important...
... unless you need the Q-value.

4 _____

2 _____

0 _____



What lessons do we learn?

Final thoughts

6 _____

The absolute g.s. energy of a rotational band isn't important...

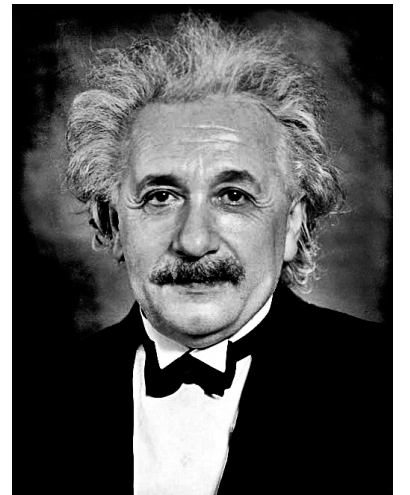
4 _____

... unless you need the Q-value.

2 _____

Only relative g.s. energies are important...

0 _____



What lessons do we learn?

Final thoughts

6 _____

The absolute g.s. energy of a rotational band isn't important...

4 _____

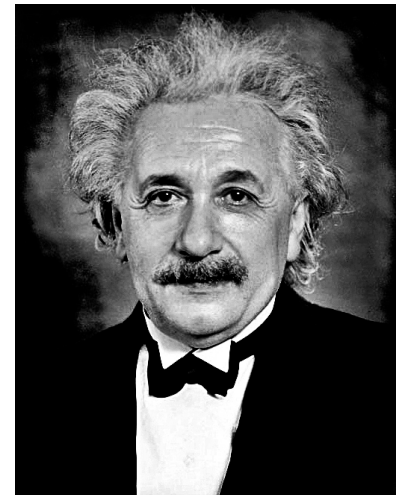
... unless you need the Q-value.

2 _____

Only relative g.s. energies are important...

0 _____

... unless you need the equation of state for neutron stars!



What lessons do we learn?

Final thoughts

6

The absolute g.s. energy of a rotational band isn't important...

4

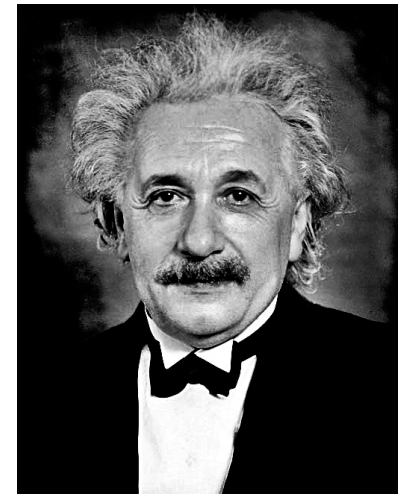
"Simple" and "important" are
relative terms!

2

...important...

0

of state for neutron star



To summarize the summary:

