Quartet structure of N=Z nuclei

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Outline

- QCM for a general Hamiltonian
- Spectra of N=Z nuclei in a quartet formalism:
  - the even-even case
  - the odd-odd case
- Bosonic approach to quartetting in even-even N=Z nuclei:
  - an application to ^{28}Si
- Conclusions

QCM for a general H

- The quartet condensate model (QCM) assumes that the ground state of an even-even N=Z nucleus has the form

$$|QCM\rangle = (Q^+)^n|0\rangle$$

where

$$Q^+ = \sum_{ii',kk',J'T'} x_{ii',kk',J'T'} [[a_i^+ a_{i'}^+]^{J'T'} [a_k^+ a_{k'}^+]^{J'T'}]^{J=0,T=0}$$

- Basic features:
  - N,T,J are exactly preserved
  - the amplitudes $x_{ii',kk',J'T'}$ are constructed variationally for each nucleus

- Previous works:
  M. Hasegawa et al., Nucl. Phys. A 592, 45 (1995)

QCM results in the sd shell

Using the USDB Hamiltonian:

	E_{corr}(SM)	E_{corr}(QCM)	$\langle SM \vert QCM \rangle$
^{20}Ne	24.77	24.77 (-)	1
^{24}Mg	55.70	53.04 (4.77%)	0.85
^{28}Si	88.75	86.52 (2.52%)	0.86
^{32}S	122.51	122.02 (0.40%)	0.98

If, in first approximation,

$$E_{QCM}(N) = NE_Q + \frac{N(N-1)}{2} V_{QQ}(N)$$

one derives (in MeV)

$$V_{QQ}(2) = -3.51, \quad V_{QQ}(3) = -4.07, \quad V_{QQ}(4) = -3.34$$

for

$$E_Q = -40.47$$

M.S. and N. Sandulescu, EPJA 53, 47 (2017)

Spectra of even-even N=Z nuclei

We want to represent an even-even N=Z nucleus in a formalism of quartets

$$Q^{+}_{\alpha,JM,TT_z} = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C^{(\alpha)}_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2} \times \left[[a^{+}_{i_1} a^{+}_{j_1}]^{J_1 T_1} [a^{+}_{i_2} a^{+}_{j_2}]^{J_2 T_2} \right]^{JT}_{MT_z}$$

- Basic questions:
  - which quartets to involve?
  - how to construct them?
  - what to do with them?

The case of ^{24}Mg

$^{24}_{12}\mathrm{Mg}_{12}$ = 4 protons + 4 neutrons outside the ^{16}O core = 2 quartets

- We want to represent its states as linear superpositions of

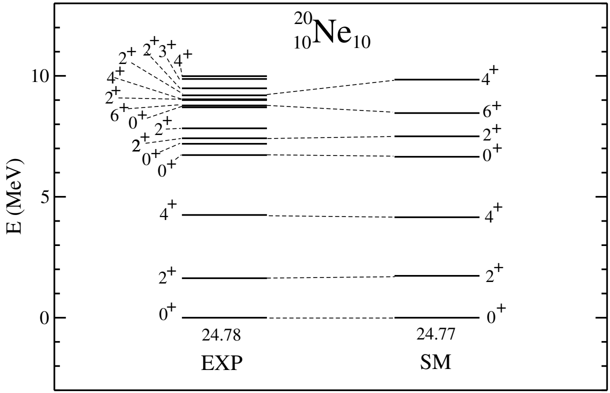
$$[Q^{+}_{\alpha J_1 T_1} Q^{+}_{\beta J_2 T_2}]^{J,T=0}_{M,T_z=0}|0\rangle$$

- We define the quartets according to the following general criterion :

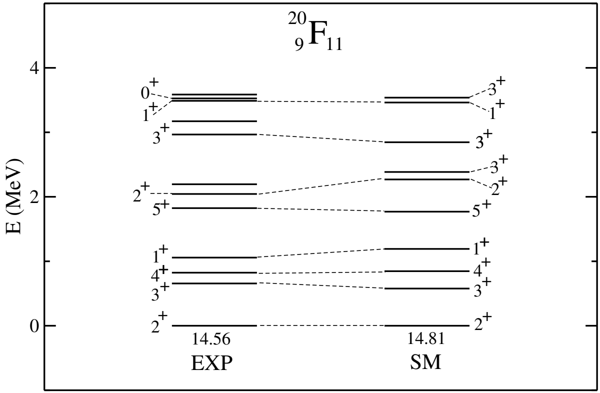
  quartets describe the low-lying states of nuclei with four active particles outside the inert core of reference

- We perform a configuration interaction calculation in the quartet space.

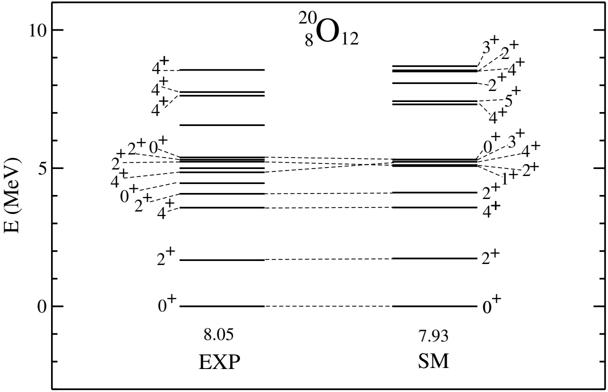
^{20}Ne: T=0 quartets



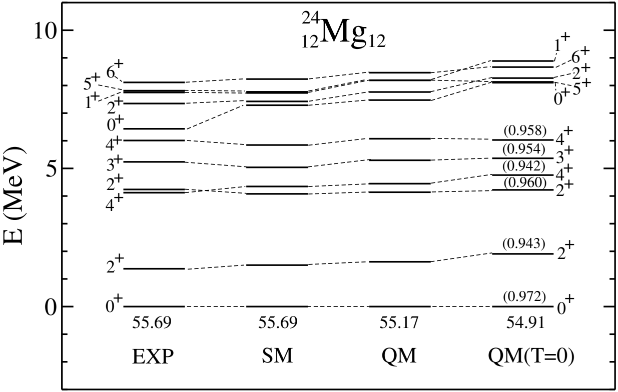
^{20}F: T=1 quartets



^{20}O: T=2 quartets

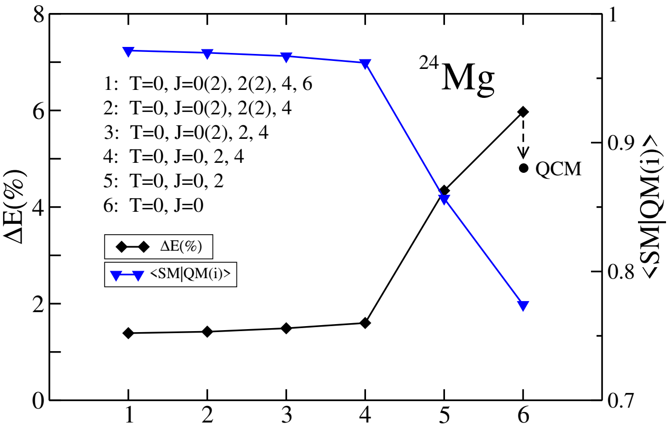


^{24}Mg: the spectrum

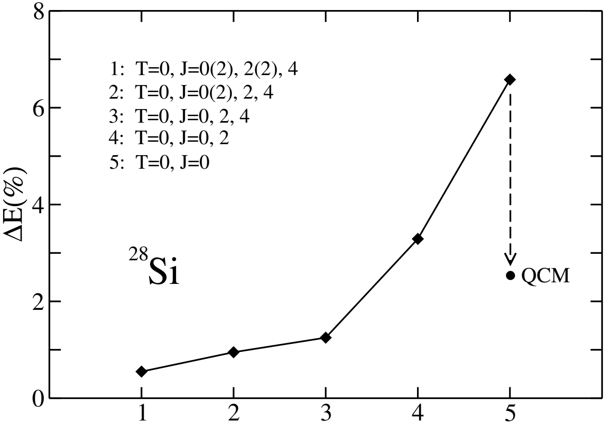


M.S. and N. Sandulescu, PRL 115, 112501 (2015)

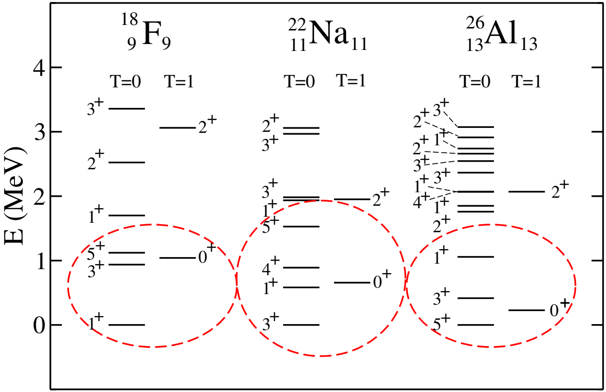
^{24}Mg: ground state correlation energy



^{28}Si: ground state correlation energy



Quartetting in N=Z odd-odd nuclei



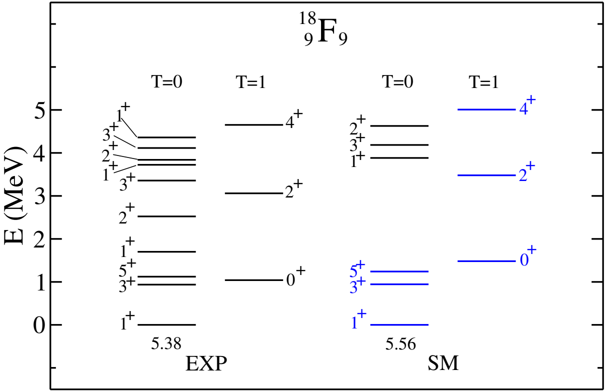
The building blocks

We describe odd-odd N=Z nuclei by resorting to two distinct families of building blocks: T=0 quartets and T=0,1 pairs.

We assume that

- T=0 quartets are those describing the lowest J=0,2,4 states of ^{20}Ne
- T=0 (T=1) pairs are those describing the lowest J=1,3,5 (J=0,2,4) states of ^{18}F

The T=0,1 pairs



The formalism

- We represent the states of ^{22}Na as linear superpositions of

$$Q^{+}_{J_1 M_1} P^{+}_{JM,T0} |0\rangle$$

  and the states of ^{26}Al as linear superpositions of

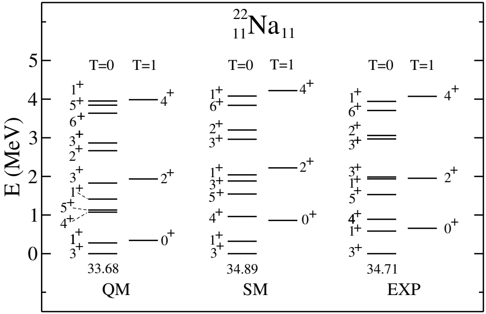
$$Q^{+}_{J_1 M_1} Q^{+}_{J_2 M_2} P^{+}_{JM,T0} |0\rangle$$

  .

- The configuration space always includes one pair plus a number of quartets varying according to the nucleus.

- The isospin T of a state coincides with that of the pair.

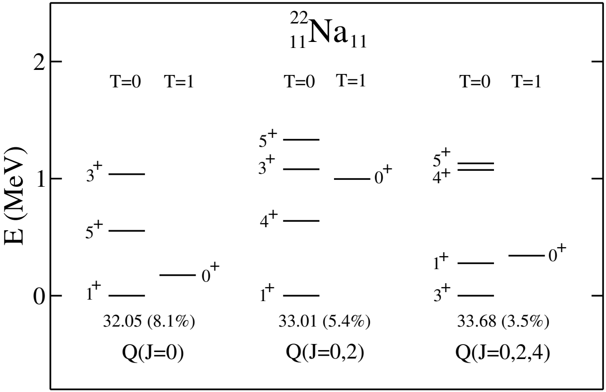
^{22}Na



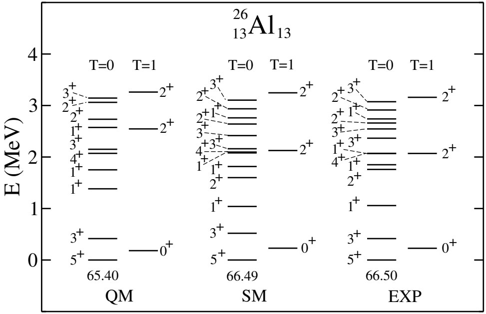
$\langle SM|QM \rangle$: 0.94(J=3), 0.93(J=1), 0.97(J=0), 0.92(J=4), 0.96(J=5)

M.S. and N. Sandulescu, PLB 763, 151 (2016)

^{22}Na for different configuration spaces

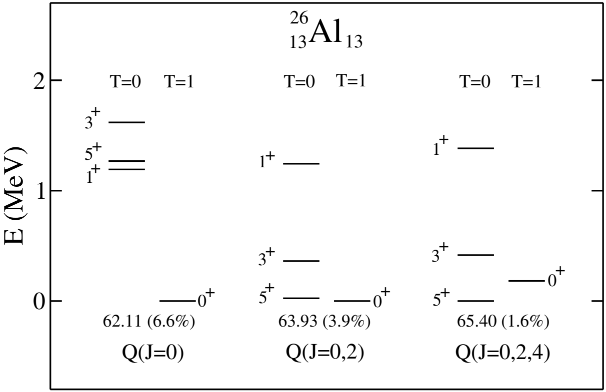


^{26}Al



$\langle SM|QM\rangle$: 0.94(J=5), 0.94(J=0), 0.95(J=3), 0.86(J=1)

^{26}Al for different configuration spaces



Bosonic approach to quartetting in even-even N=Z nuclei

- We assume two basic building-blocks:

$$Q^+_{J=0,T=0} \equiv S^+ \qquad Q^+_{J=2,T=0} \equiv D^+$$

- We replace them with two elementary bosons:

$$S^+ \qquad \Longrightarrow \qquad s^+$$

$$D^+ \qquad \Longrightarrow \qquad d^+$$

- We construct the most general one- plus two-body Hamiltonian

$$H_B = \epsilon_s \widehat{n}_s + \epsilon_d \widehat{n}_d + \sum_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2 \Lambda} V^{(\Lambda)}_{\lambda_1 \lambda_2 \lambda'_1 \lambda'_2} [[b^+_{\lambda_1} b^+_{\lambda_2}]^{(\Lambda)} [\tilde{b}_{\lambda'_1} \tilde{b}_{\lambda'_2}]^{(\Lambda)}]^{(0)}$$

- It is: $H_B \equiv H_B^{(IBM)}$ but $N_B \equiv \frac{N_B^{(IBM)}}{2}$
- Previous use of this formalism:
J. Dukelsky et al, Phys. Lett. 115B, 359 (1982)

Fermion-boson correspondence

FERMIONIC SPACE		BOSONIC SPACE
Q_i^+	$\longleftrightarrow$	b_i^+
$[Q_i, Q_j^+] = \widehat{\Delta}_{ij}$	$\longleftrightarrow$	$[b_i, b_j^+] = \delta_{ij}$
$F^{(N)} = \{Q_{k_1}^+ Q_{k_2}^+ \cdots Q_{k_N}^+ \vert 0\rangle\}_{k_i<k_j}$	$\longleftrightarrow$	$B^{(N)} = \{b_{k_1}^+ b_{k_2}^+ \cdots b_{k_N}^+ \vert 0)\}_{k_i<k_j}$
$\vert N, k\rangle$	$\longleftrightarrow$	$\vert N, k)$
$\langle N, k \vert N, k'\rangle \neq \delta_{k,k'}$	$\longleftrightarrow$	$(N, k \vert N, k') = \delta_{k,k'}$

Boson mapping of the fermion Hamiltonian

- The boson image H_B of the fermion Hamiltonian H_F is defined such that

$$(N,l|H_B|N,m) = \sum_{ij} R_{li}^{(N)} \langle N,i|H_F|N,j\rangle R_{jm}^{(N)}$$

being

$$R_{li}^{(N)} = {\sum_k}^* f_{lk}^{(N)} \frac{1}{\sqrt{\mathcal{N}_k^{(N)}}} f_{ik}^{(N)}, \qquad \sum_l \langle N,i|N,l\rangle f_{lj}^{(N)} = \mathcal{N}_j^{(N)} f_{ij}^{(N)}$$

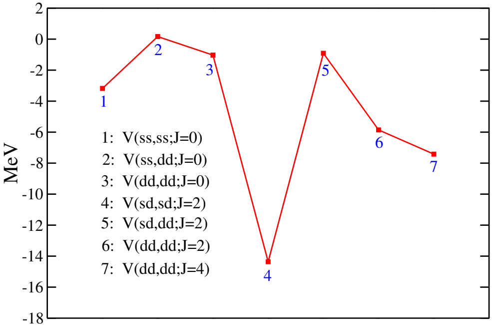
- The spectrum of H_B in $B^{(N)}$ is identical to that of H_F in $F^{(N)}$ (plus a number of zero's equal to the number of $\mathcal{N}_j^{(N)} = 0$)
- H_B is Hermitian and, in general, N-body

Parameters of H_B from the mapping procedure

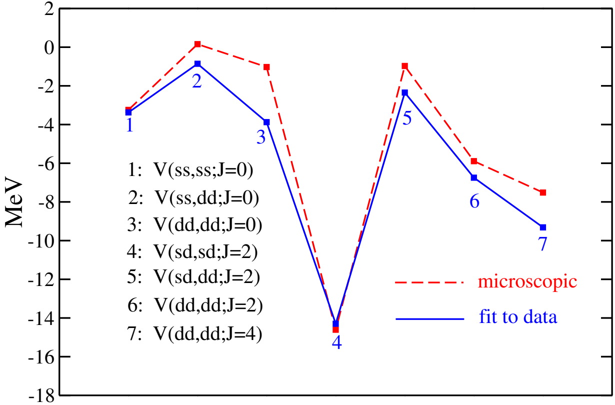
- H_B (one- plus two-body) is generated from $H_F(USDB)$
  - Single-boson energies ϵ_s and ϵ_d:

$$\begin{aligned}\epsilon_s &= \langle Q_{J=0,T=0}|H_F|Q_{J=0,T=0}\rangle \\ \epsilon_d &= \langle Q_{J=2,T=0}|H_F|Q_{J=2,T=0}\rangle\end{aligned}$$

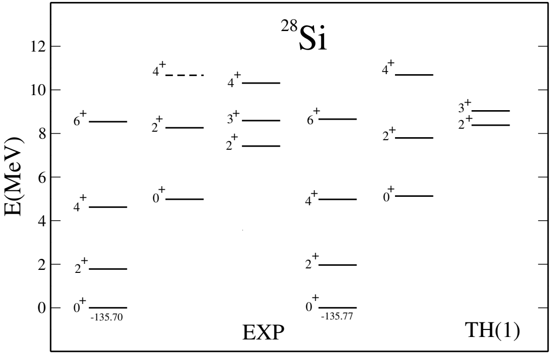
  - Two-body matrix elements:



Parameters of H_B: microscopic vs phenomenological



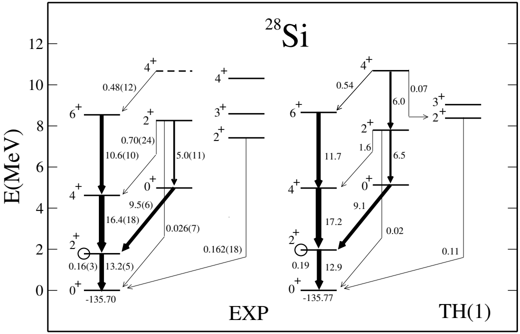
^{28}Si: spectrum



(EXP: R.K. Sheline et al., Phys. Lett. B 119, 263 (1982))

^{28}Si: spectrum and E2 transitions

$$T_\mu^{(E2)} = e_B([d_\mu^\dagger s + s^\dagger \tilde{d}_\mu]^2 + \chi[d^\dagger \tilde{d}]_\mu^2)$$



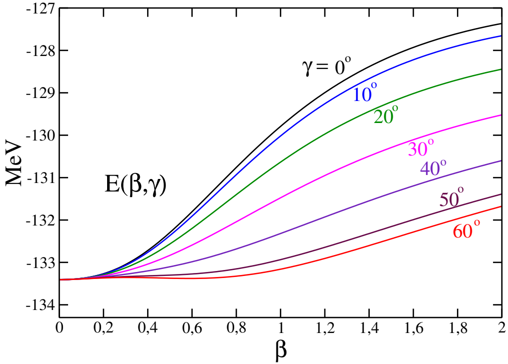
M.S. and N. Sandulescu, to appear in PLB

^{28}Si: geometric structure

$$|N;\beta,\gamma\rangle = \frac{1}{\sqrt{N!(1+\beta^2)^N}}(B^\dagger)^N|0\rangle$$

$$B^\dagger = s^\dagger + \beta[\cos\gamma\ d_0^\dagger + \frac{1}{\sqrt{2}}\sin\gamma\ (d_{+2}^\dagger + d_{-2}^\dagger)]$$

$$E(N,\beta,\gamma) = \langle N;\beta,\gamma|H_B|N;\beta,\gamma\rangle$$



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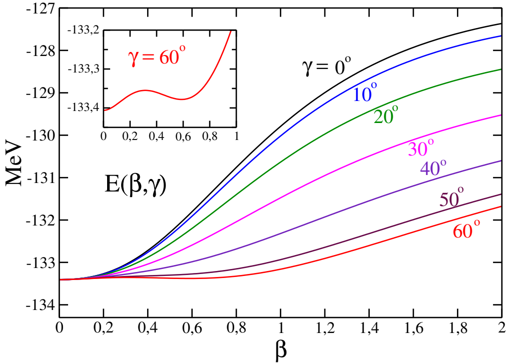
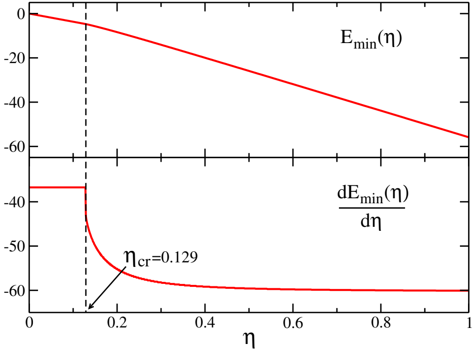


A schematic U(5)-$\overline{\mathrm{SU}(3)}$ Hamiltonian

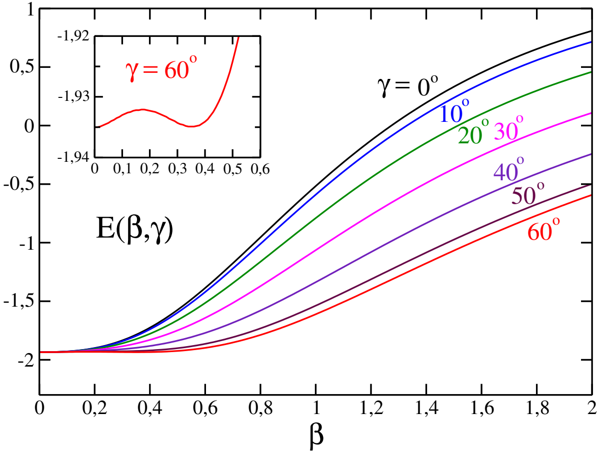
$$H_B^{(T)} = (1-\eta)\hat{n}_d - \eta(Q^\dagger \cdot Q^\dagger)$$

$$Q^\dagger = [d^\dagger s + s^\dagger \tilde{d}]^{(2)} + \chi[d^\dagger \tilde{d}]^{(2)}, \quad \chi = +\frac{\sqrt{7}}{2}$$

First-order phase transition at $\eta = \eta_{cr}$

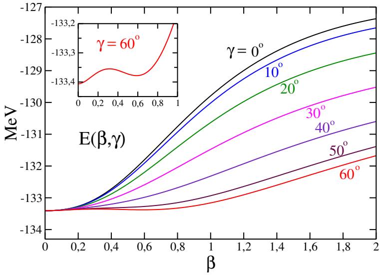


Potential energy surface at $\eta = \eta_{cr}$

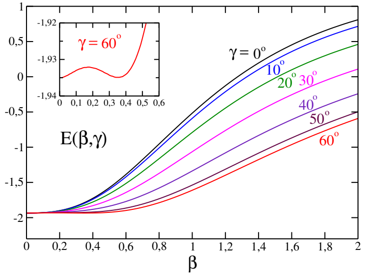


Comparison of potential energy surfaces

H_B



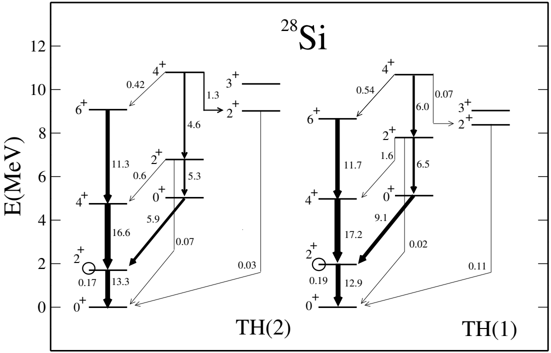
$H_B(\mathrm{U(5)}\text{-}\overline{\mathrm{SU(3)}})$ at $\eta = \eta_{cr}$



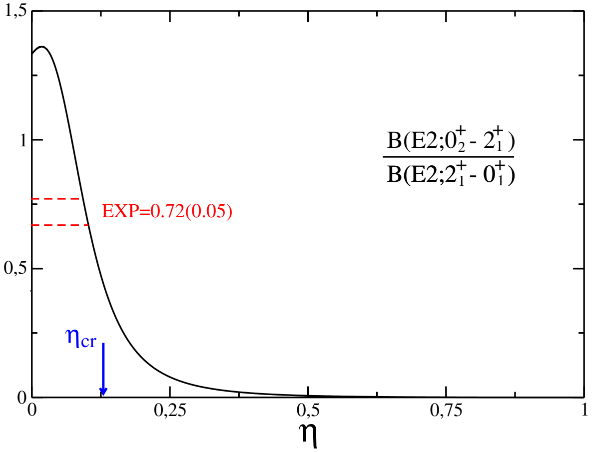
Comparison of theoretical spectra

TH(1): H_B

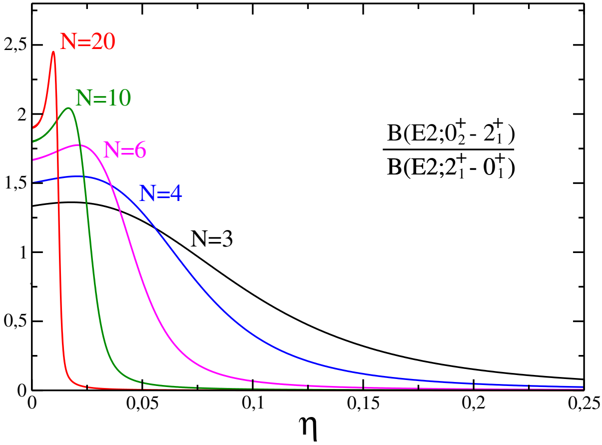
TH(2): $H_B(\text{U}(5)\text{-}\overline{\text{SU}(3)})$ at $\eta = \eta_{cr}$



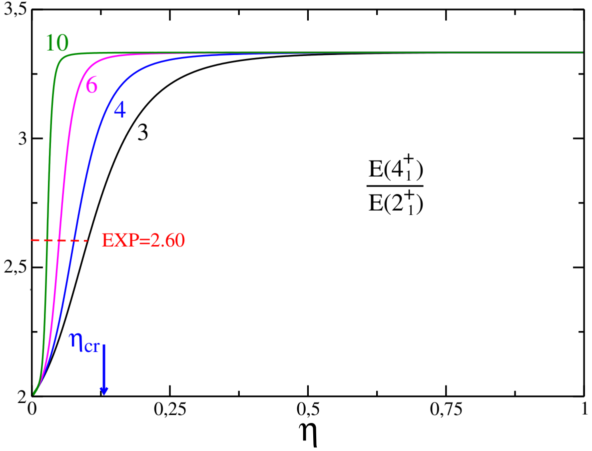
$B(E2;0_2^+ - 2_1^+)/B(E2;2_1^+ - 0_1^+)$



$B(E2;0_2^+ - 2_1^+)/B(E2;2_1^+ - 0_1^+)$



$E(4_1^+)/E(2_1^+)$



Conclusions

- N=Z nuclei have been represented in a formalism of quartets.
- T=0, J=0,2,... quartets have emerged as the basic building blocks of even-even N=Z nuclei and, jointly with an extra pair, of odd-odd N=Z nuclei.
- An IBM-like approach to even-even N=Z nuclei has been proposed, with *s* and *d* bosons representing $T = 0, J = 0$ and $T = 0, J = 2$ quartets, respectively.
- An application for ^{28}Si has been discussed.
- The analysis of the potential energy surface has placed ^{28}Si at the U(5)-$\overline{\mathrm{SU}(3)}$ phase-transitional point.