IS and IV pairing correlations and its Phase transition

”Recent advances on proton-neutron pairing
and quartet correlation in nuclei”

Hiroyuki Sagawa RIKEN/University of Aizu

1. Introduction
2. Role of Isoscalar Pairing on Spin-isospin response
3. Spin-Isospin response of N=Z+2 nuclei amd Wigner SU(4) symmetry
4. Competition between IS and IV pairing correlations in deformed HFB calculations
5. Summary and future perspectives.

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RIKEN NiSHiNA CENTER

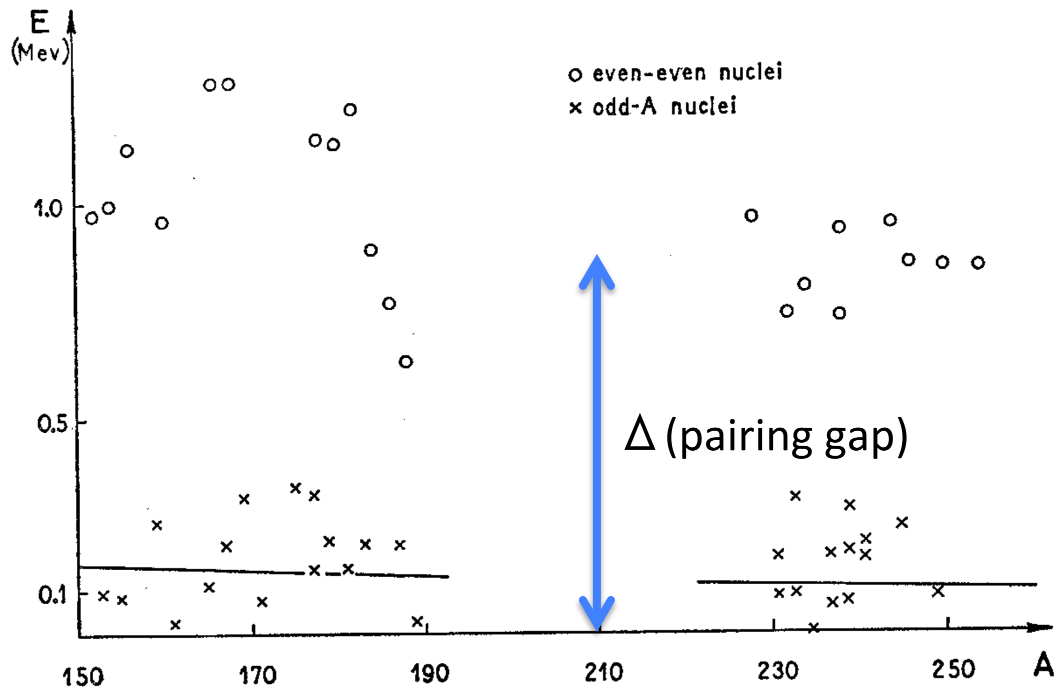
PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

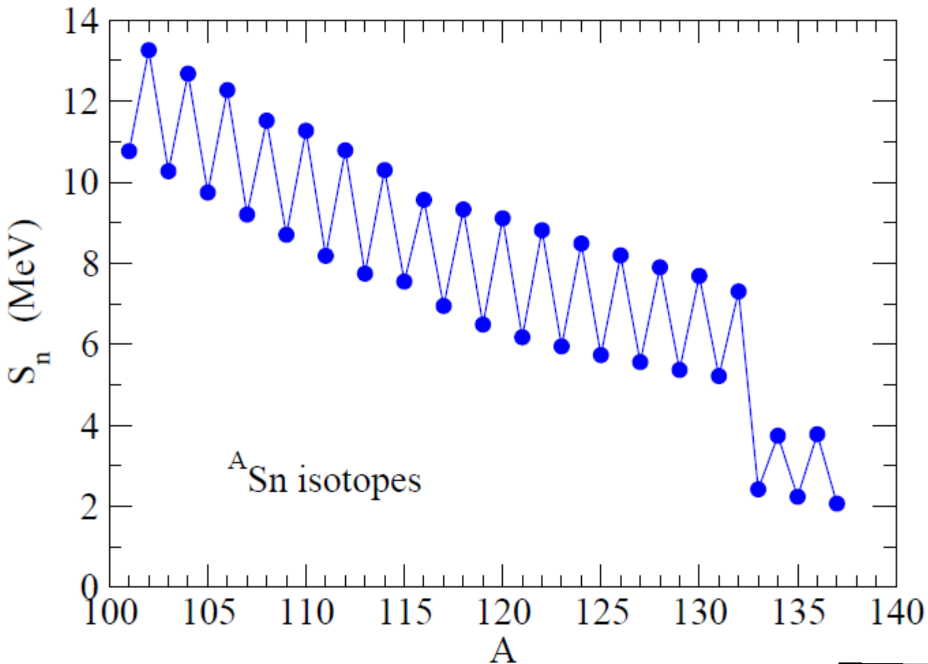
(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.



BCS (Bardeen-Cooper-Schrieffer:1957)
Theory of Pairing correlations in metalic superconductor

➢binding energy

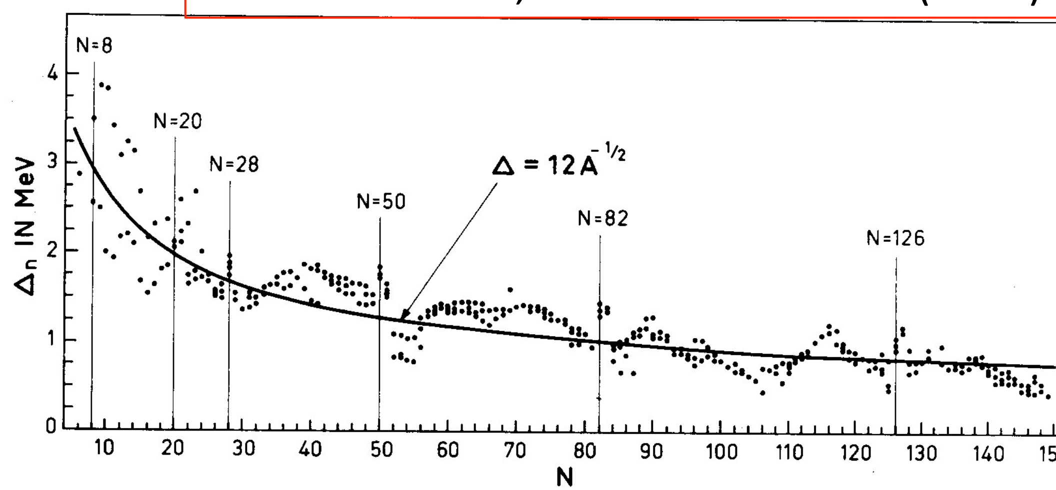


pairing gap indicator

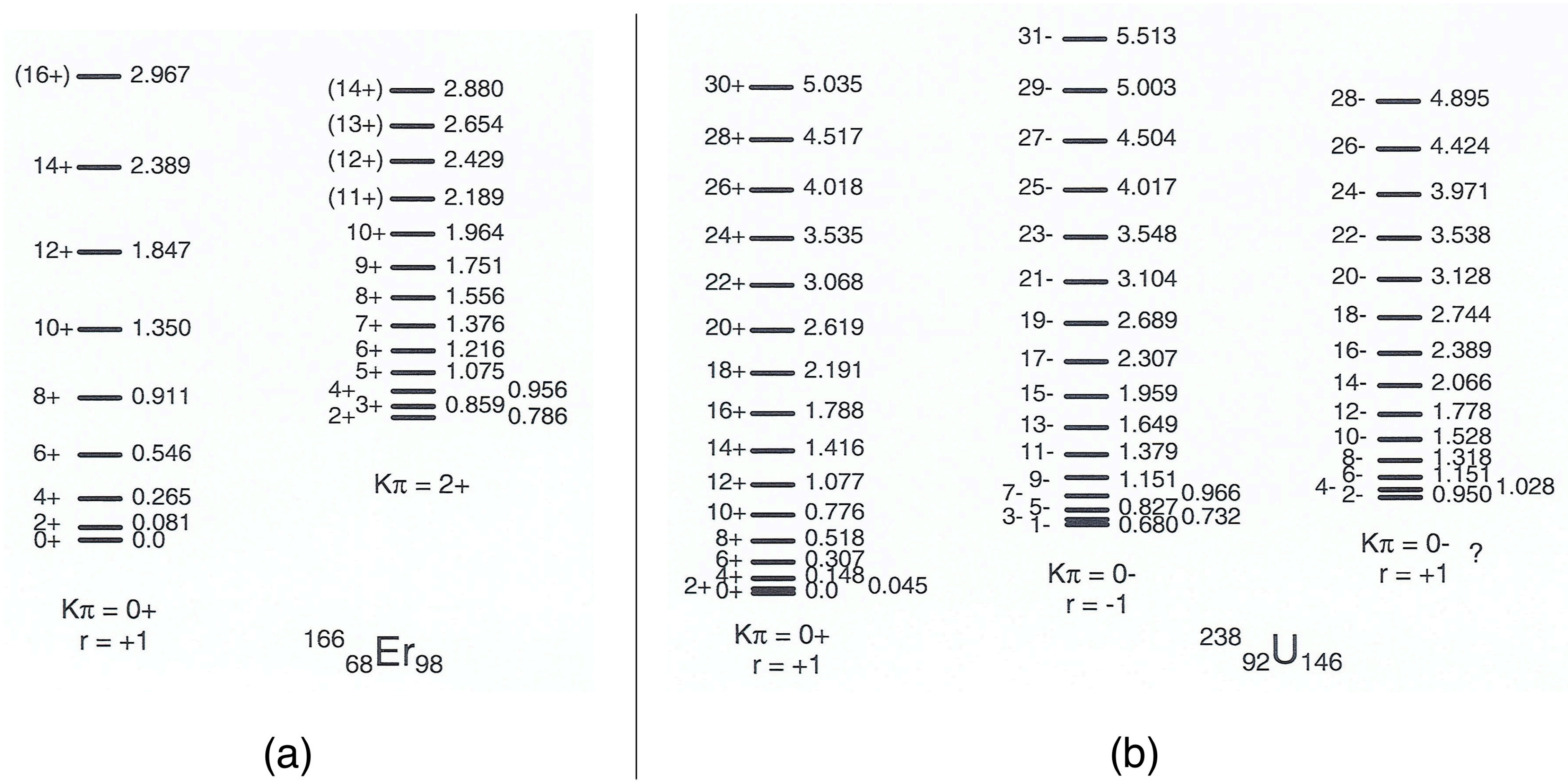
$$\Delta(N) = \frac{(-)^N}{2}(B(N-1) - 2B(N) + B(N+1)) = \frac{(-)^N}{2}(S_n(N-1) - S_n(N))$$

$$S_n(N) = B(N) - B(N-1)$$

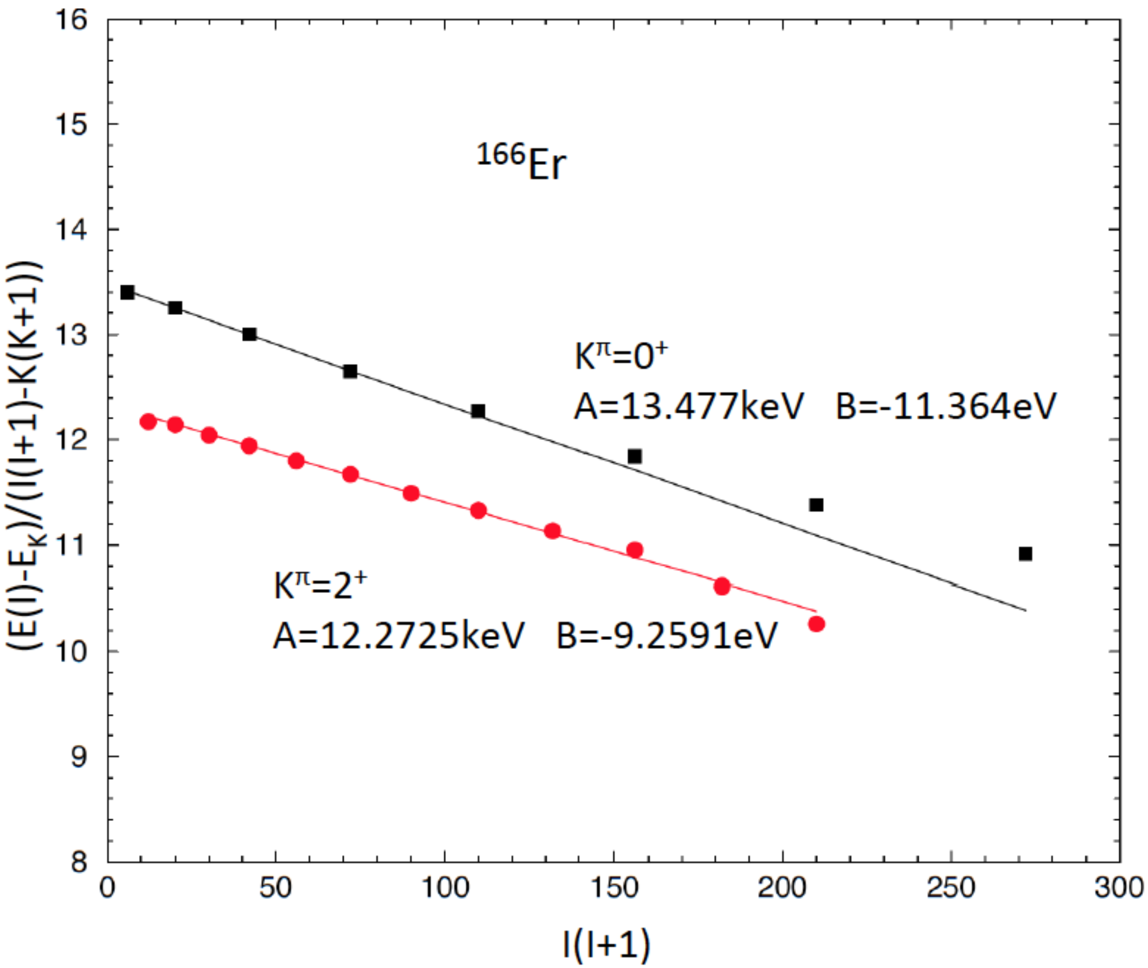
Bohr-Mottelson, Nuclear Structure I (1969)



Deformed Nuclei: Rotational Bands



Rotational bands

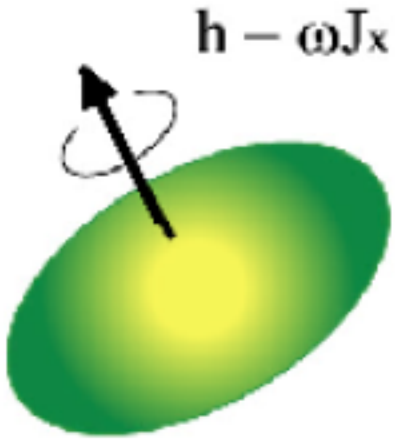


$$E(I) = \frac{I(I+1)}{2\Im}$$

$$E(I) = AI(I+1) + BI^2(I+1)^2$$

Moment of Inertia

Bohr-Mottelson, Nuclear Structure II (1975)



Rotation of deformed nucleus

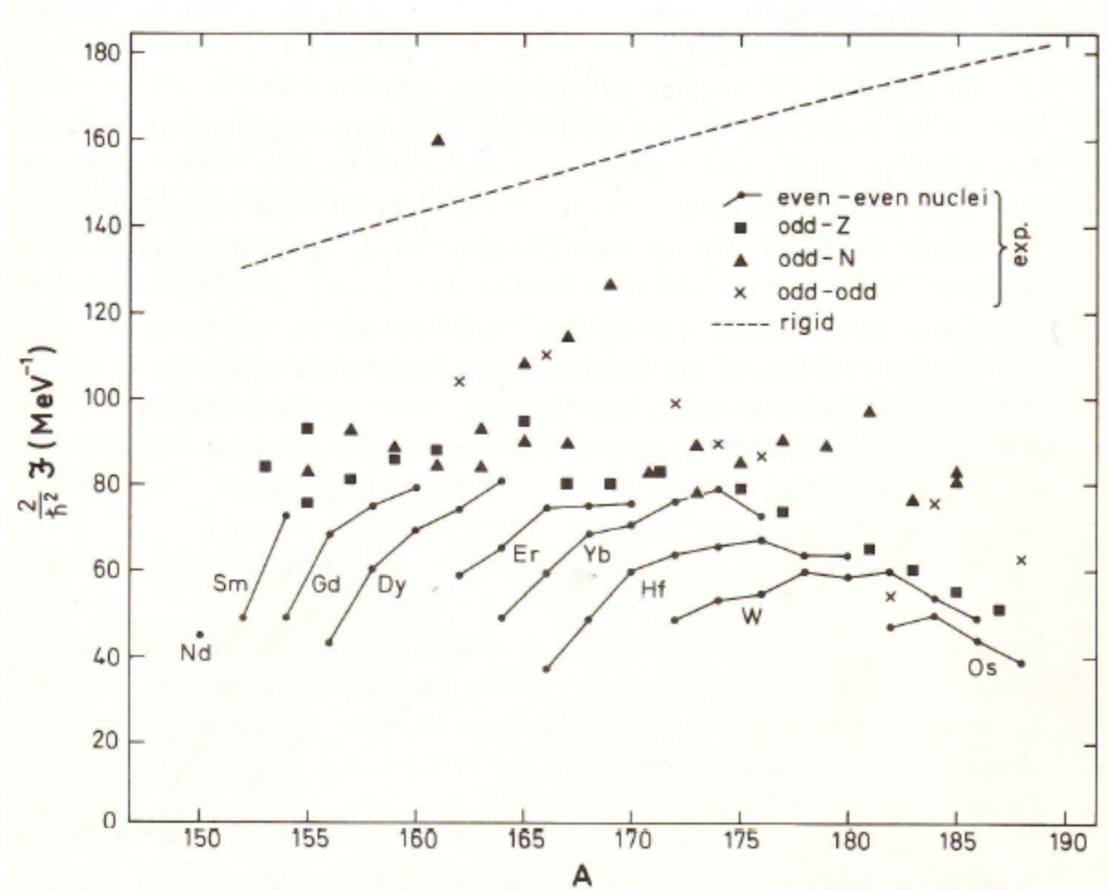


Figure 4-12 Systematics of moments of inertia for nuclei with $150 \leqslant A \leqslant 188$. The moments of inertia are obtained from the empirical energy levels in *Table of Isotopes* by Lederer *et al.*, 1967.

rigid rotor vs. superfluid rotor

$$I_{Inglis} = 2\sum_{i>j} \frac{\langle i | J_x | j \rangle^2}{\varepsilon_i - \varepsilon_j} \approx I_{rigid}$$

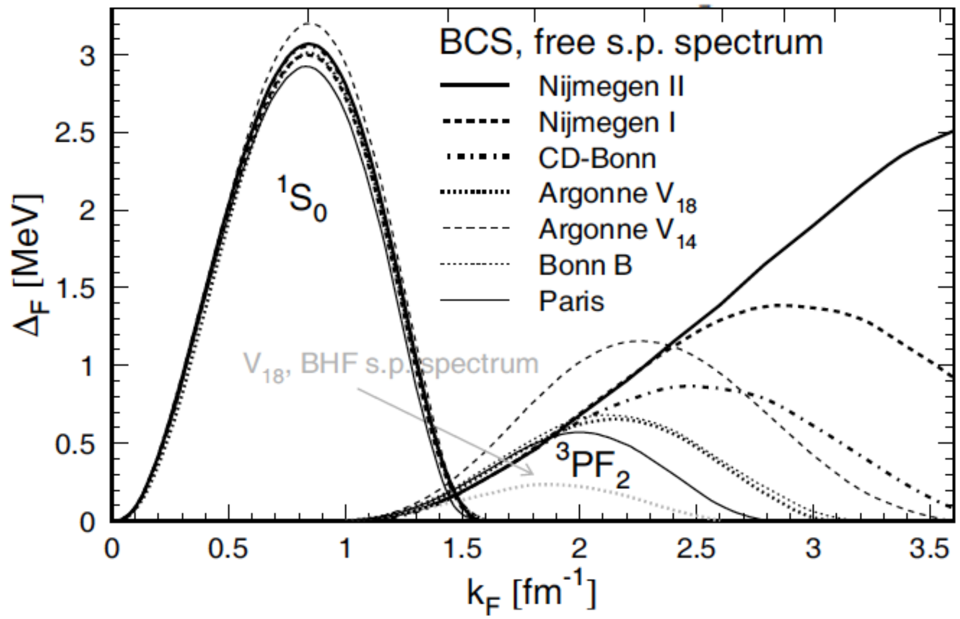
$$I_{pairing} = 2\sum_{i,j} \frac{\langle i | J_x | j \rangle^2 (u_i v_j - v_i u_j)^2}{E_i + E_j}$$

S=0 Isovector pairing interactions

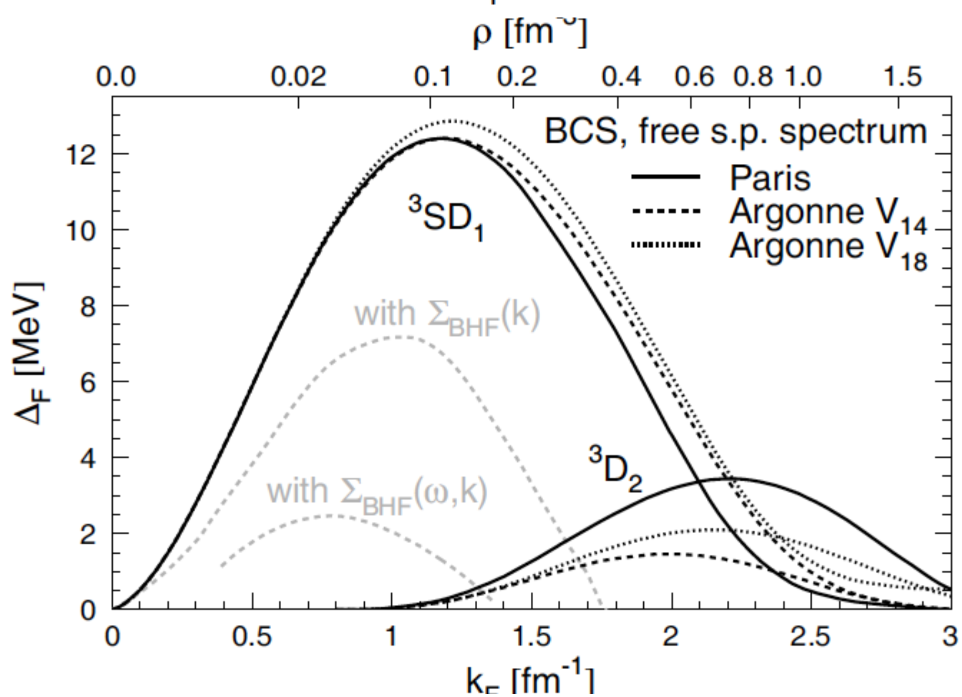
Isospin T=1 pairing (n-n, p-p, n-p pairing correlations) ➔spin singlet superfluid

- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei)
- moment of inertia of rotational band
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)

Isospin $T = 1$ $^1S_0$ and $^3PF_2$ gaps in neutron matter evaluated in BCS approximation



Int. Journ. of Mod. Phys. E14, 513 (2005)
U. Lombardo et al..



Isospin $T = 0$ $^3SD_1$ and $^3D_2$ gaps in symmetric nuclear matter

T=1 S=0 pairing and T=0 S=1 pairing interactions

T=1 pairing (n-n, p-p pairing correlations) → spin singlet superfluid

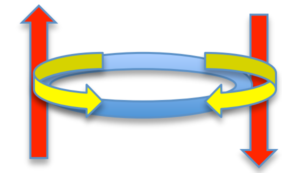
- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei
- moment of inertia
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)

Strong T=0 pairing (p-n pairing with S=1) → spin triplet superfluid ?

- deuteron (T=0,S=1) is bound, but not di-neutron (T=1,S=0)
- N=Z Wigner energy (still controversial)
- → Energy spectra in nuclei with N=Z (T=0 and J=1)
- n-p pair transfer reaction
- → low-energy super-allowed Gamow-Teller transition in N=Z and N=Z+2 between SU(4) supermultiples
- IS and IV magnetic dipole transitions in sd-shell nuclei

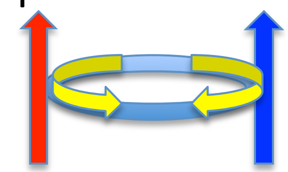
Two particle systems

T=1, S=0 pair



$$\left|(L=S=0)J=0,T=1\right\rangle \Rightarrow \left|(j=j')J=0,T=1\right\rangle$$

T=0, S=1 pair



$$\left|(L=0,S=1)J=1,T=0\right\rangle \Rightarrow$$

$$a\left|(l=l'\,j=j')J=1,T=0\right\rangle + b\left|((l=l')j,j'=j\pm1)J=1,T=0\right\rangle$$

If there is strong spin-orbit splitting, it is difficult to make (T=0,S=1)pair.

T=0 J= 1^+ state could be M1 or Gamow-Teller states in nuclei with N~Z
➔ strong M1 or GT states in N~Z nuclei

(J=0,T=1) and (J=1,T=0) are SU(4) supermultiplet in spin-isospin space

Well-known in light p-shell nuclei (LS coupling dominance)

$\boxed{\text{The spin-singlet } T{=}1 \text{ pairing}}$ $V^{(T=1)}(\mathbf{r},\mathbf{r}') = -G^{(T=1)} \sum_{i,j} P^{(1,0)\dagger}_{i,i}(\mathbf{r},\mathbf{r}') P^{(1,0)}_{j,j}(\mathbf{r},\mathbf{r}')$

$$\begin{aligned}\langle (j_i j_i) T = 1, J = 0 | V^{(T=1)} | (j_j j_j) T = 1, J = 0 \rangle \\ = -\sqrt{(j_i + 1/2)(j_j + 1/2)}\, G^{(T=1)} I^2_{ij} \end{aligned} \quad (5)$$

where I_{ij} is the overlap integral given by,

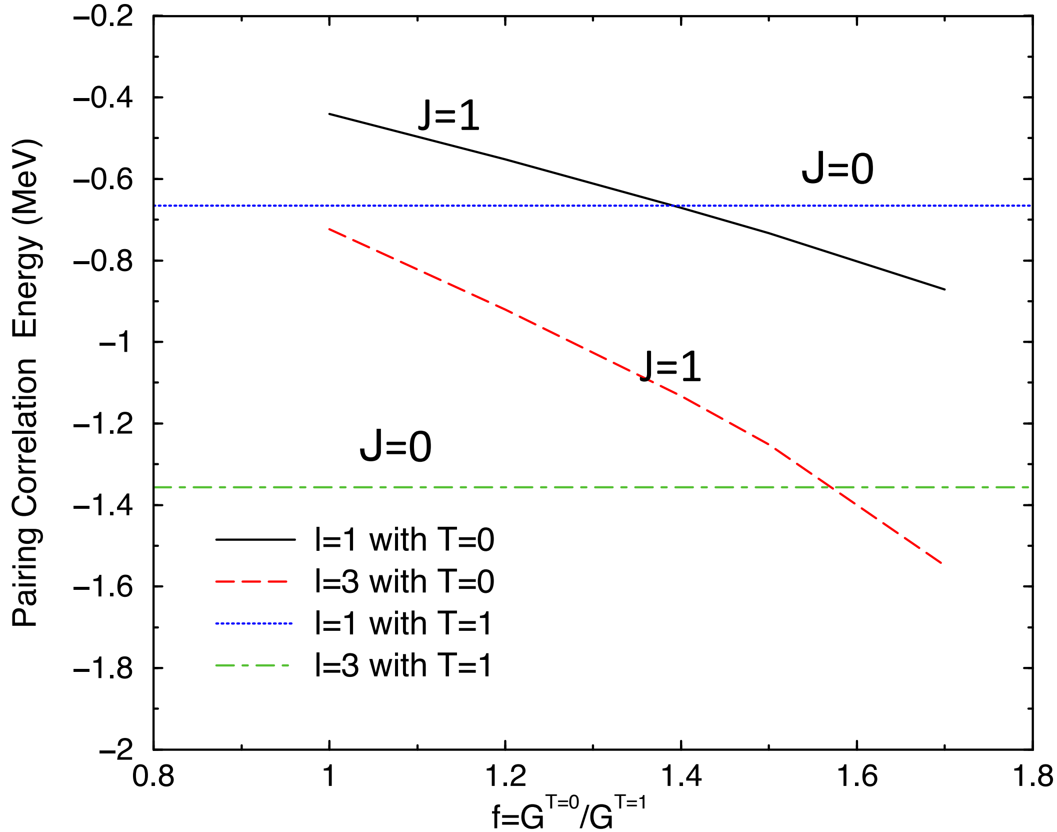
$$I_{ij} = \int \psi_i(\mathbf{r})^* \psi_j(\mathbf{r}) d\mathbf{r} \quad (6)$$

$\boxed{(T{=}0,\ S{=}1) \text{ pairing}}$ $V^{(T=0)}(\mathbf{r},\mathbf{r}') = -fG^{(T=1)} \sum_{i \geqq i', j \geqq j'} P^{(0,1)\dagger}_{i,i'}(\mathbf{r},\mathbf{r}') P^{(0,1)}_{j,j'}(\mathbf{r},\mathbf{r}')$

$$\begin{aligned}
&\langle (j_1 j_2) T = 0, J = 1 | V^{(T=0)} | (j'_1 j'_2) T = 0, J = 1 \rangle = \\
&- \left\langle \left[\left(l_1 \frac{1}{2} \right)^{j_1} \left(l_2 \frac{1}{2} \right)^{j_2} \right]^{J=1} \middle| \left[(l_1 l_2)^{L=0} \left(\frac{1}{2} \frac{1}{2} \right)^{S=1} \right]^{J=1} \right\rangle \\
&\times \left\langle \left[\left(l'_1 \frac{1}{2} \right)^{j'_1} \left(l'_2 \frac{1}{2} \right)^{j'_2} \right]^{J=1} \middle| \left[(l'_1 l'_2)^{L=0} \left(\frac{1}{2} \frac{1}{2} \right)^{S=1} \right]^{J=1} \right\rangle \\
&\times \frac{\sqrt{2l_1 + 1}\sqrt{2l'_1 + 1}}{\sqrt{1 + \delta_{j_1, j_2}}\sqrt{1 + \delta_{j'_1, j'_2}}} fG^{T=1} (I_{j_1 j'_1} I_{j_2 j'_2} + I_{j_1 j'_2} I_{j_1 j'_2}),
\end{aligned}$$

TABLE I: The transformation coefficient R between the jj coupling and the LS coupling for the pair wave functions, $R = \langle [(l\frac{1}{2})^{j}(l\frac{1}{2})^{j'}]^{J=1} | [(ll)^{L=0}(\frac{1}{2}\frac{1}{2})^{S=1}]^{J=1} \rangle$. Ω is defined as $\Omega \equiv 3(2l+1)^2$.

j	j'	R	$l=1$	$l=3$
$l+1/2$	$l+1/2$	$\sqrt{\frac{(2l+2)(2l+3)}{2\Omega}}$	$\frac{1}{3}\sqrt{\frac{10}{3}}$	$\frac{2\sqrt{3}}{7}$
$l+1/2$	$l-1/2$	$-\sqrt{\frac{4l(l+1)}{\Omega}}$	$-\frac{2}{3}\sqrt{\frac{2}{3}}$	$-\frac{4}{7}$
$l-1/2$	$l-1/2$	$-\sqrt{\frac{2l(2l-1)}{2\Omega}}$	$-\frac{1}{3}\sqrt{\frac{1}{3}}$	$-\frac{\sqrt{5}}{7}$
$l-1/2$	$l+1/2$	$\sqrt{\frac{4l(l+1)}{\Omega}}$	$\frac{2}{3}\sqrt{\frac{2}{3}}$	$\frac{4}{7}$



Pairing correlation energy of (J,T)=(0,1) and (1,0) states in *pf* shell

Even with large spin-orbit splitting for *f* -orbits, the spin-triplet correlations will be larger than the spin-singlet one for f>1.5

HS, Y. Tanimura and K. Hagino, PRC87, 034310 (2013)

TABLE I. Strengths of triplet and singlet interactions from shell-model fits and their ratios. See text for details.

Source	v_s (MeV fm^3)	v_t (MeV fm^3)	Ratio
sd shell [8]	280	465	1.65
fp shell [9]	291	475	1.63

G.F. Bertsch and Y. Luo, PRC81, 064320 (2010)

Theoretical models for collective excitations

Hartree-Fock (HF)+random phase approximations(RPA)
HF Bogolyuvov (HFB)+ quasi-particle RPA(QRPA)
Time-dependent DFT (Finite amplitude method)
RMF+relativistic RPA
Generator Coordinate model (GCM)
Interactive Shell model
Cluster models (AMD)
Ab initio approach (Coupled cluster, self- consistent Green's function)

Spin-Isospin Excitations

HF+RPA (RPA Green's function method)
Three-body model
Interactive Shell model

RMF+RRPA

Theoretical models

1. BCS model (1957)
2. Hartree-Fock Bogoliubov model
3. Three-body model

Bardeen Cooper Schrieffer

1972 Novel Prize

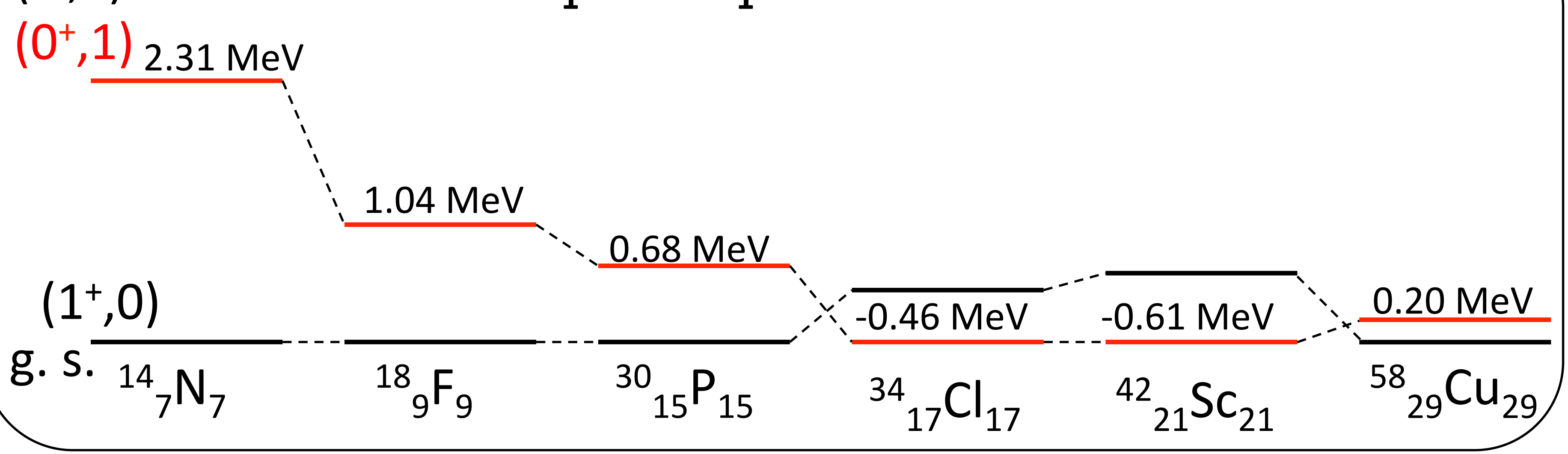
N=Z odd-odd nuclei with 3-body model

Y. Tanimura, HS, K. Hagino, PTEP 053D02 (2014)

- n-p pairing interactions
  - ✓ T=0, 1 two channels
  - ✓ T=0, S=1 is attractive stronger than T=1, S=0 pair
    cf. deuteron, matrix elements in shell models
  - ✓ In finite nuclei N>Z , the strong spin-orbit coupling
    may quench or even kill T=0 pairing

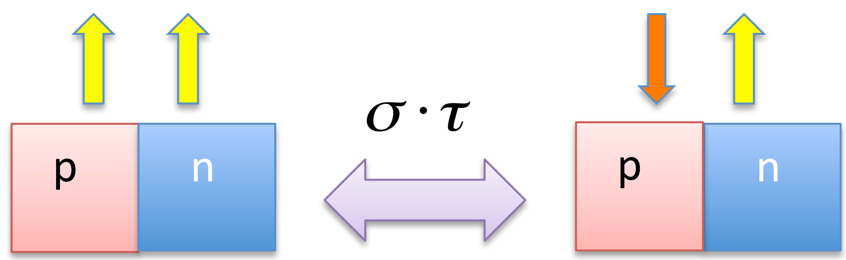
when *l* is larger , the spin-orbit is larger and T=0 pair correlations decrease

Measured 1^+_1 and 0^+_1 levels of odd-odd N=Z nuclei



Supermultiplet : Wigner SU(4) symmetry
(T=1, S=0) →(T=0, S=1) Magnetic Dipole and GT transition is allowed and enhanced .

Spacial symmetry is the same between the initial and final states



(T=0,S=1) (T=1,S=0)

$\boxed{B(GT)=6}$

Well-known in light p-shell nuclei (LS coupling dominance)

What happens in sd and pf shell nuclei with strong spin-triplet pairing interactions?

Three-body Model

Total 3-body Hamiltonian

$$H = \frac{\boldsymbol{p}_p^2}{2m} + \frac{\boldsymbol{p}_n^2}{2m} + V_{pC}(\boldsymbol{r}_p) + V_{nC}(\boldsymbol{r}_n) + V_{pn}(\boldsymbol{r}_p, \boldsymbol{r}_n) + \frac{(\boldsymbol{p}_p+\boldsymbol{p}_n)^2}{2A_C m}$$

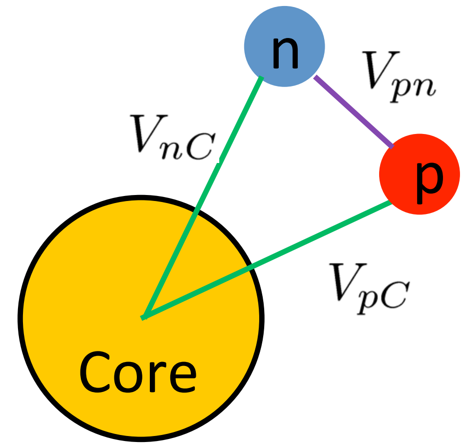
Core-N mean field

$$V_{(p/n)C}(r) = v_0 f(r) + v_{ls}\frac{1}{r}\frac{d}{dr}f(r)(\boldsymbol{l}\cdot\boldsymbol{s})(+Coulomb)$$

$$f(r) = \frac{1}{1+e^{(r-R)/a}}$$

p-n interaction

$$V_{pn} = \hat{P}_s v_s \delta(\boldsymbol{r}_p - \boldsymbol{r}_n)[1 + x_s(\frac{\rho(r)}{\rho_0})^\alpha] + \hat{P}_t v_t \delta(\boldsymbol{r}_p - \boldsymbol{r}_n)[1 + x_t(\frac{\rho(r)}{\rho_0})^\alpha]$$



Determination of parameters

v_0, v_{ls}: neutron separation energy

v_s, v_t: pn scattering length with E_{cut} (= 20 MeV)

v_s/v_t=1.7 (spin-triplet pairing is much stronger than spin-singlet)

x_S, x_t, α: 1^+, 3^+, 0^+ in ^{18}F energies are fitted

⇨

Diagonalization in a large model space

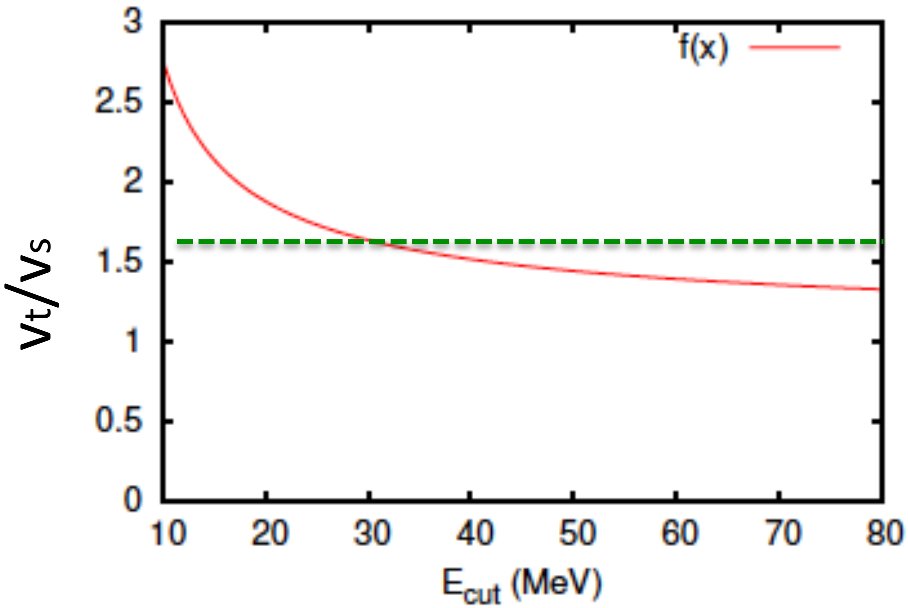
pn pairing interaction

$$V_{np}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \hat{P}_s v_s \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\left[1+x_s\left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right] + \hat{P}_t v_t \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\left[1+x_t\left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right]$$

$$\hat{P}_s = \frac{1}{4} - \frac{1}{4}\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n,\ \hat{P}_t = \frac{3}{4} + \frac{1}{4}\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n.$$

$$v_s = \frac{2\pi^2\hbar^2}{m}\frac{2a_{pn}^{(s)}}{\pi - 2a_{pn}^{(s)}k_{\rm cut}},$$

$$v_t = \frac{2\pi^2\hbar^2}{m}\frac{2a_{pn}^{(t)}}{\pi - 2a_{pn}^{(t)}k_{\rm cut}},$$



$$a_{pn}^{(s)} = -23.749 \text{ fm and } a_{pn}^{(t)} = 5.424 \text{ fm}$$

$E_{cut}=k_{cut}^2/2m$

Y. Tanimura, HS, K. Hagino, PTEP 053D02 (2014)

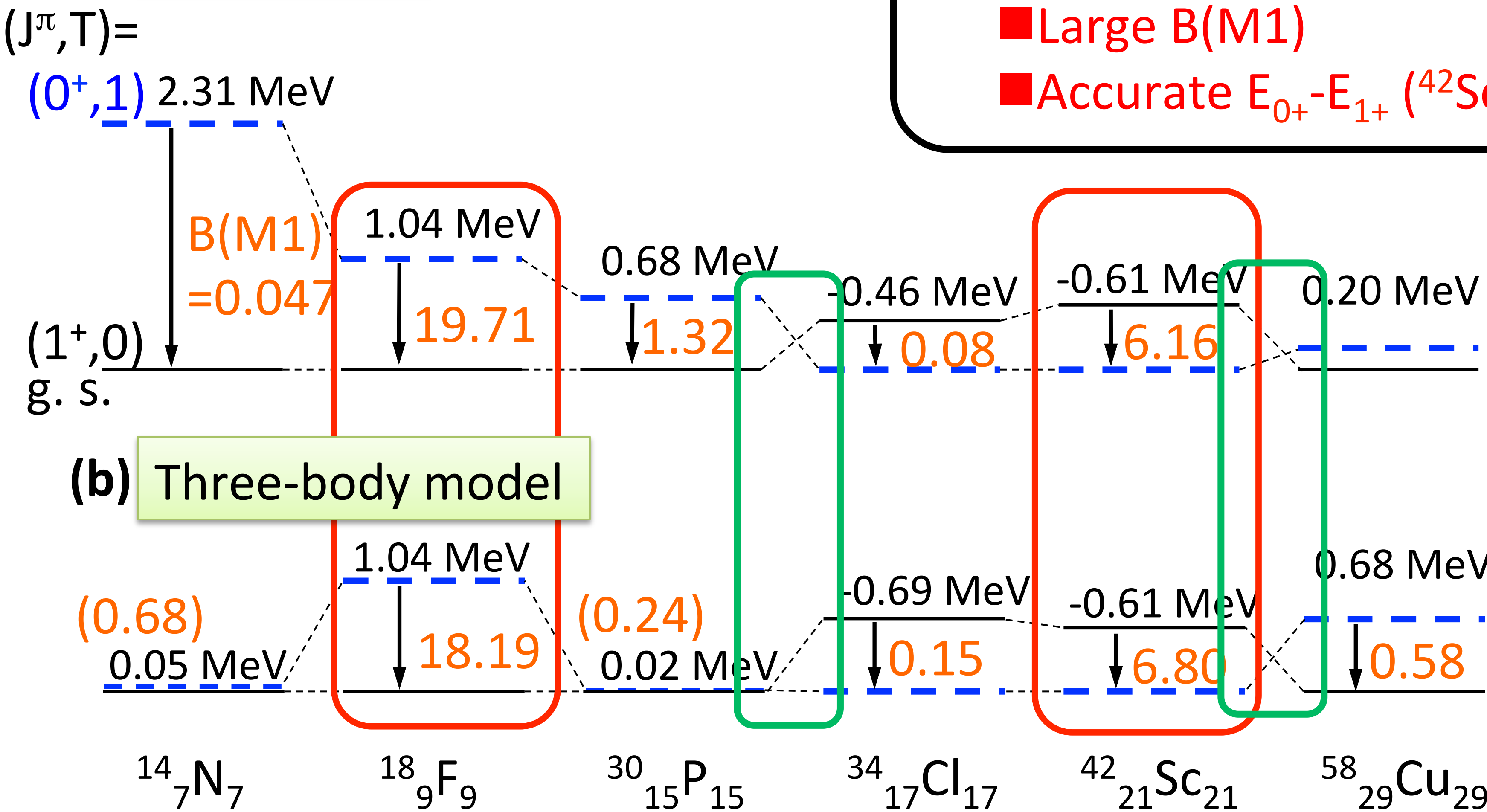
1. E_{0+}-E_{1+} and B(M1)

✓ Inversion of 1^+ and 0^+

✓ ^{18}F, ^{42}Sc

- Large B(M1)
- Accurate E_{0+}-E_{1+} (^{42}Sc)

(a) Experiment p://www.nndc.bnl.gov/)



The inversion of 1+ and 0+ shows a clear manifestation of the competition between spin-orbit and the spin-triplet pairing.

Results

Large B(M1) in ^{18}F and ^{42}Sc

^{18}F :

$$\left\{\begin{array}{l} 1^+ \rightarrow P(S=1) = 90.1\%,\ (1d)^2 \\ 0^+ \rightarrow P(S=0) = 82.2\%,\ (1d)^2 \end{array}\right.$$

1$^+$ and 0$^+$ can be considered
as the states in the same SU(4) multiplets
(LST) = (0,1,0), (0,0,1)
The same as 42Sc in 1f-orbits

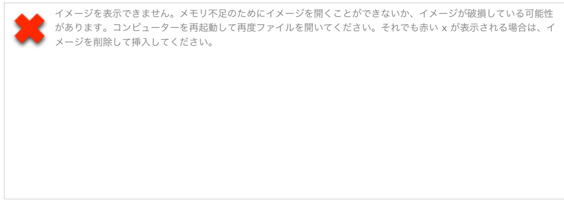
(J^π,T) =

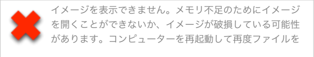
^{18}O	^{18}F	^{18}Ne
⋮	⋮	⋮
$(0^+,1)$	$(0^+,1)$	$(0^+,1)$
	$(1^+,0)$	

SU(4) multiplet

$$O(\mathrm{M1}) \propto \sum_i [g_s(i)\boldsymbol{s}(i) + g_\ell(i)\boldsymbol{\ell}(i)]$$

$$= \underbrace{g_s^{IV}}_{\text{Large}} \underbrace{\boxed{\sum_i \tau_3(i)\boldsymbol{s}(i)}}_{\text{SU(4)generator}} + \underbrace{g_s^{IS}}_{\text{(small)}} \sum_i \boldsymbol{s}(i) + \sum_i g_\ell(i)\boldsymbol{\ell}(i)$$





results

^{18}F and ^{42}Sc: large B(M1)

Separate Contribution to <f||O(M1)||i> (μ_N)

	^{14}N	^{18}F	^{30}P	^{34}Cl	^{42}Sc	^{58}Cu
Valence orbital	p1/2	d5/2	s1/2	d3/2	f7/2	p3/2
orbital	1.09	**1.28**	0.21	2.28	**2.91**	0.09
$g_s^{IV}\sum_i \tau_3(i)\boldsymbol{s}(i)$	-2.78	**7.44**	-1.21	-3.65	**6.34**	**1.47**
$g_s^{IS}\sum_i \boldsymbol{s}(i)$	5×10^{-5}	3×10^{-3}	3×10^{-5}	-1×10^{-4}	2×10^{-3}	-2×10^{-3}
B(M1)↓ (μ_N^2) Exp.	0.047	19.71	1.32	0.08	6.16	---
Calc.	0.68	**18.19**	0.24	0.15	**6.80**	0.58

✓ $(j=l-1/2)^2$ spin and orbital are cancelled
(Lisetskiy et al., PRC60, 064310 ('99))

→ ^{14}N, ^{34}Cl B(M1) **small**

✓ $(j=l+1/2)^2$ spin and orbital coherent
(Lisetskiy et al., PRC60, 064310 ('99))

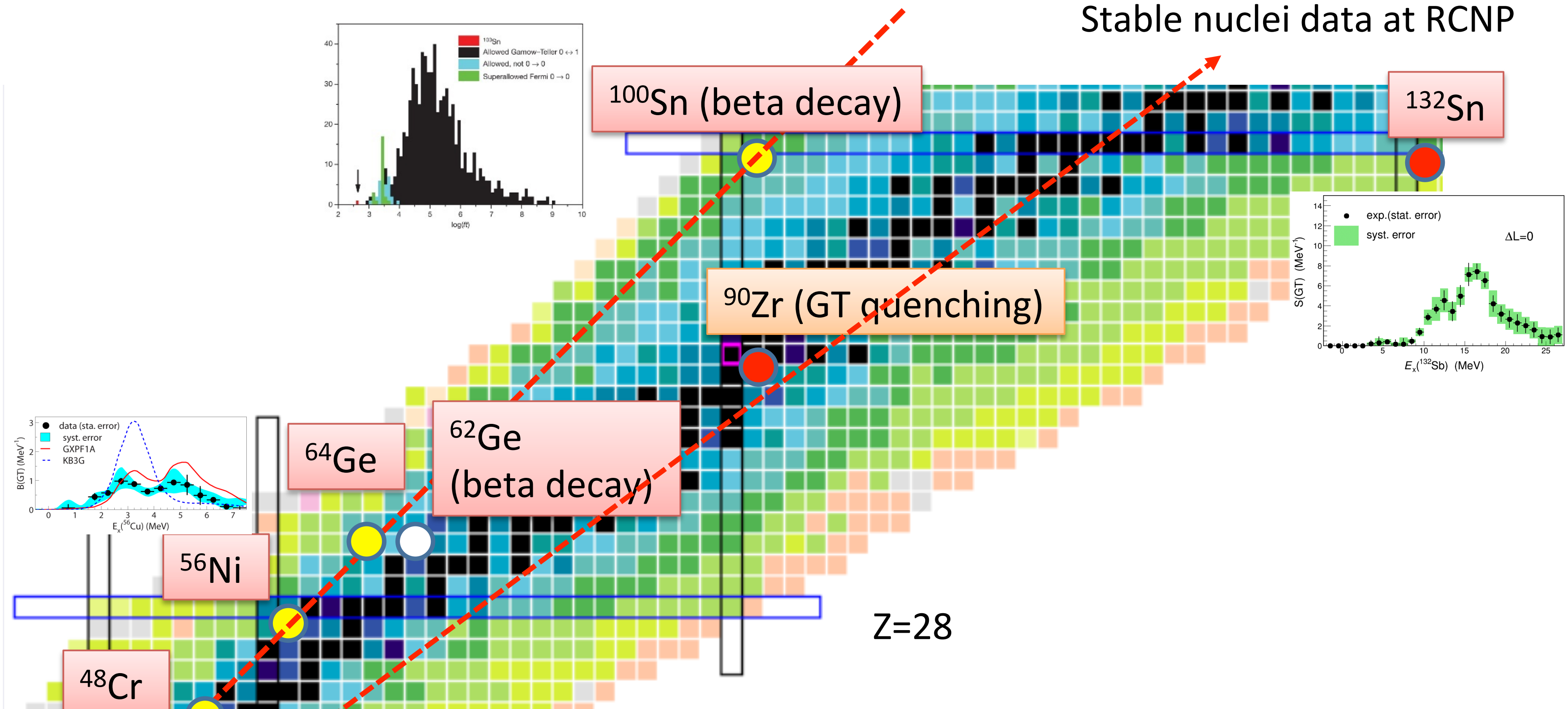
✓ **good SU(4) symmetry**

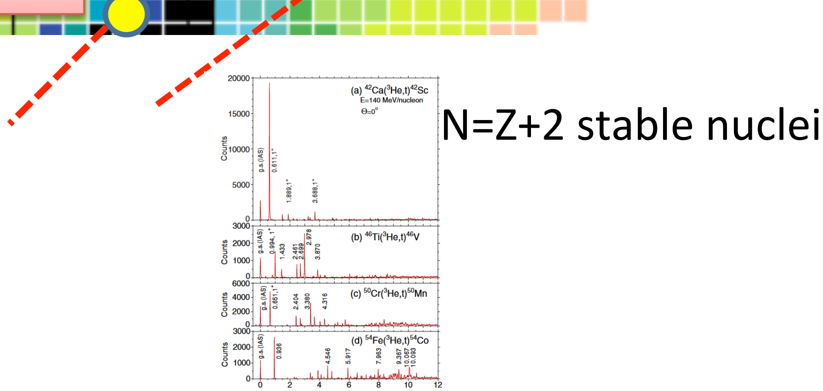
→ ^{18}F, ^{42}Sc B(M1)**large**

✓ even j=l+1/2 not good **SU(4) symmetry**

→ ^{58}Cuは(M1)**small**

Recent Progresses in GT study (courtesy of M. Sasano)





Study of Gamow-Teller transitions from ^{48}Cr and ^{64}G via the (p,n) reaction at 190 MeV
M. Sasano and Spin-Isospin Lab

Approved by RIKEN PAC DEC.2016

Gamow-Teller Transitions in nuclei with N=Z+2
C.L. Bai, HS, G. Colo, Y. Fujita et al., PRC90, 054335 (2014)

HFB+QRPA with T=1 and T=0 pairing
T=1 pairing in HFB
T=0 pairing in QRPA

$$\hat{O}(GT) = \sigma\tau_{\pm}$$

σ, τ and $\sigma\tau$ are generators of SU(4)

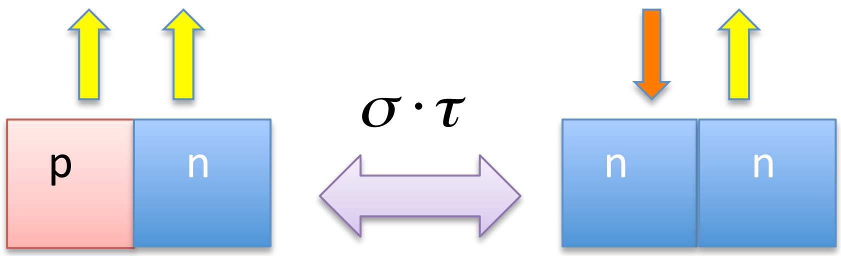
Supermultiplet : Wigner SU(4) symmetry
(E. Wigner 1937, F. Hund 1937)
(T=1, S=0) →(T=0, S=1) GT transition is allowed and enhanced .

$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0\frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_o}\right)\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (1)$$

$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = fV_0\frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_o}\right)\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (2)$$

Supermultiplet : Wigner SU(4) symmetry
(T=1, S=0) →(T=0, S=1) GT transition is allowed and enhanced .

Spacial symmetry is the same between the initial and final states



(T=0,S=1) (T=1,S=0)

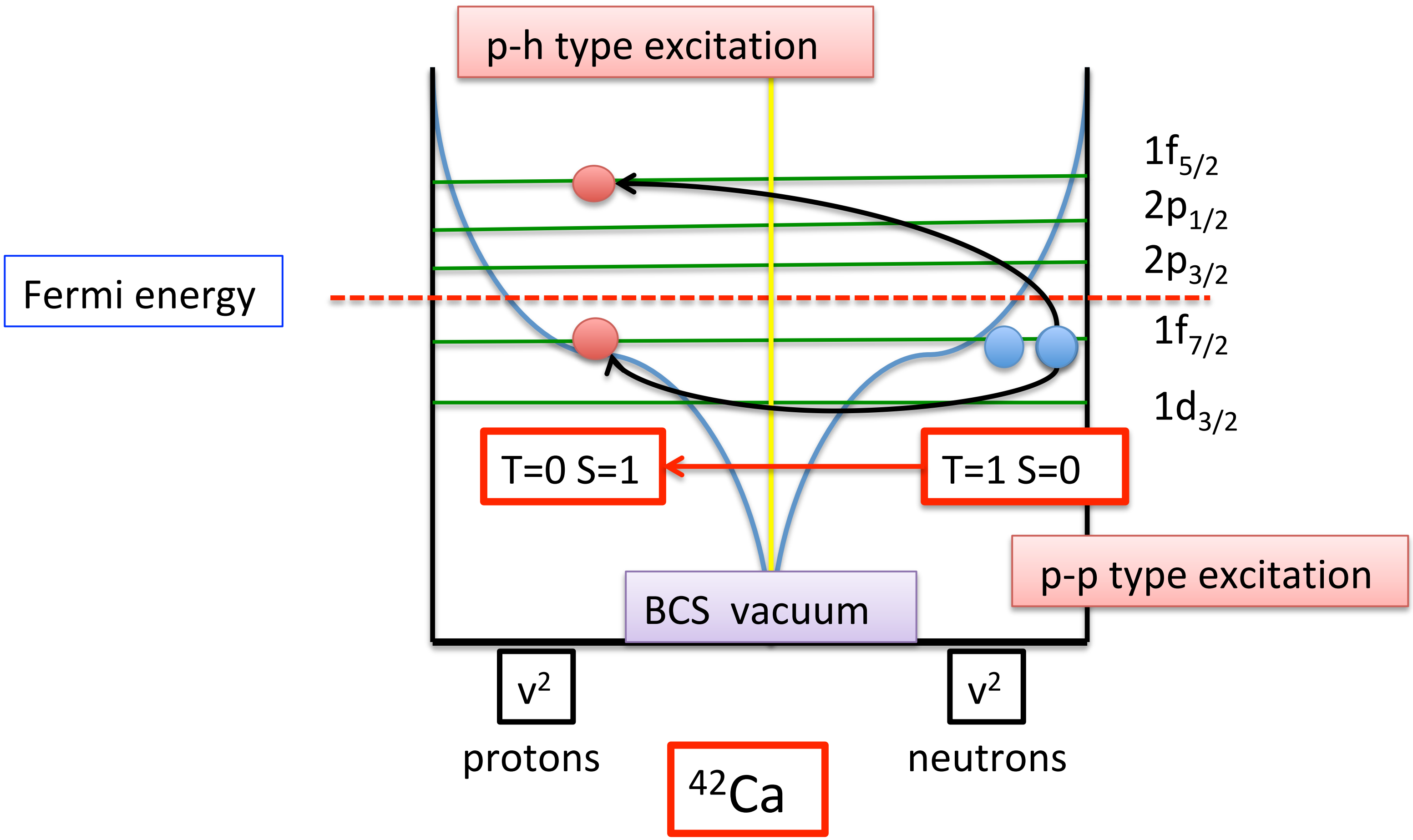
$\boxed{\text{B(GT)=6}}$

Well-known in light p-shell nuclei (LS coupling dominance)

What happens in pf shell nuclei with strong spin-triplet pairing interactions?

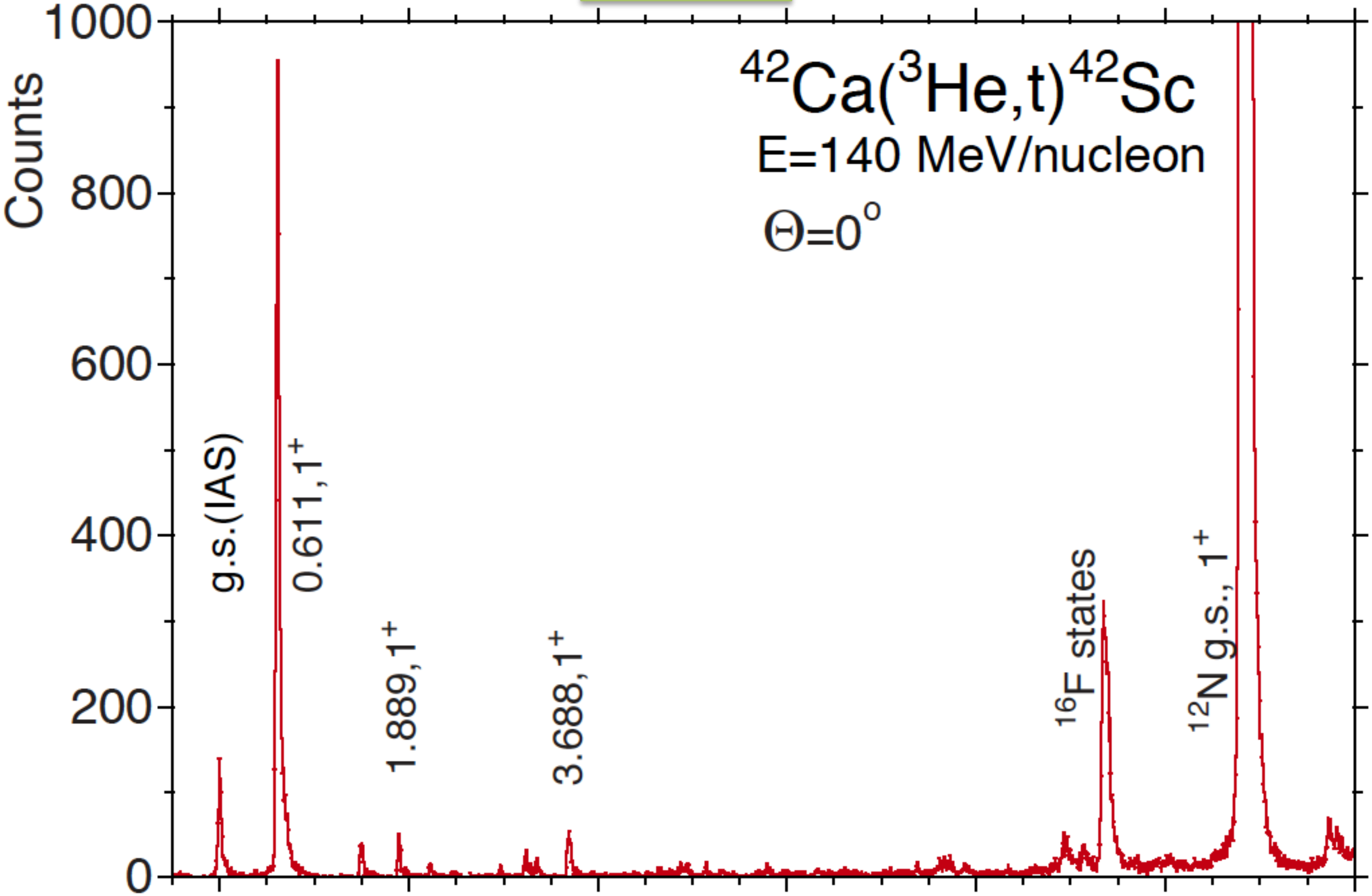
Gamow-Teller transitions in N=Z+2 *pf* nuclei

$$\hat{O}(GT) = \sigma\tau_{\pm}$$



A pair of SU(4) supermultiplet

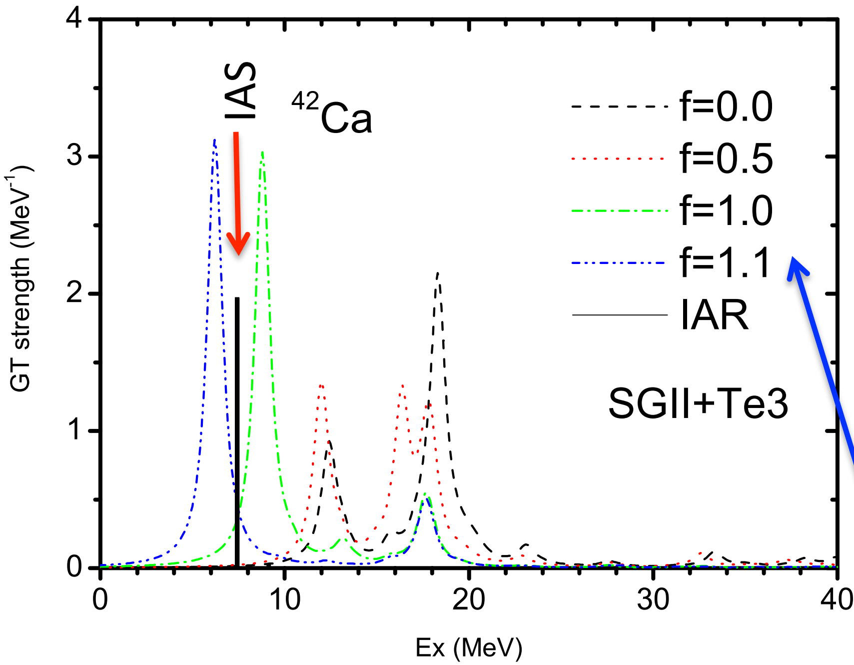
N=Z+2

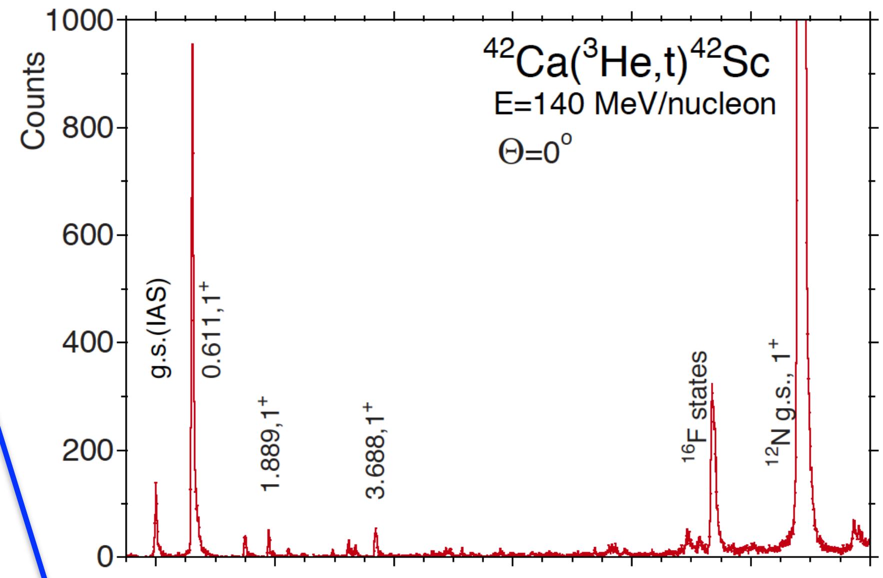


Y. Fujita et al., PRL112, 112502 (2014)

p-h spin-isospin interaction is strongly repulsive → higher energy collective Gamow-Teller states.

Spin-triplet T=0 pairing is attractive and shift down GT strength near IAS
→ SU(4) symmetry restoration

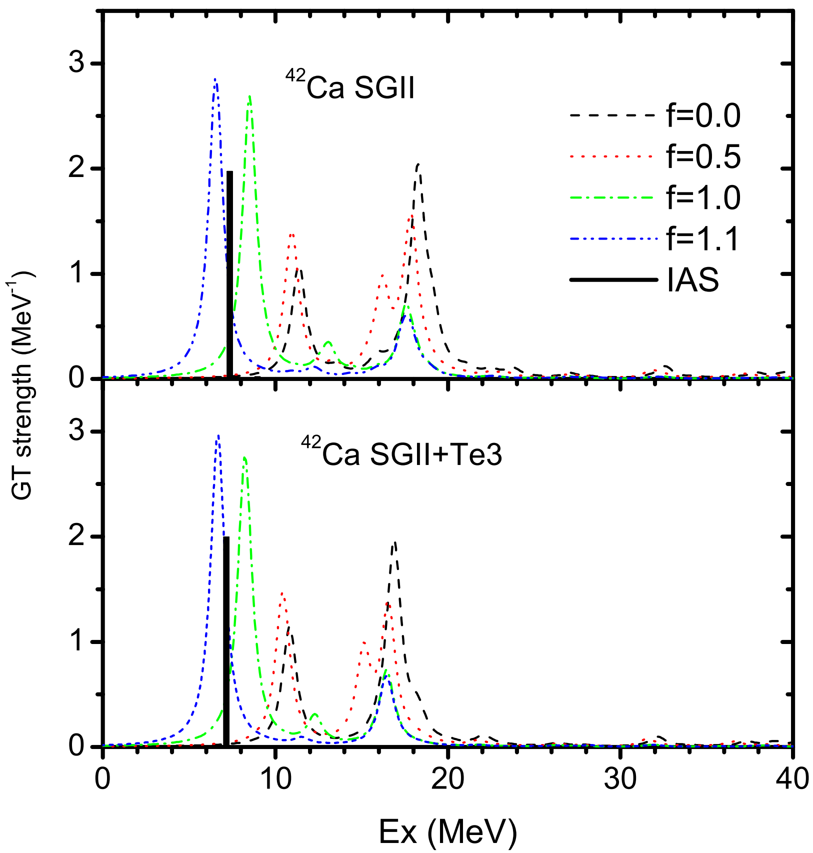


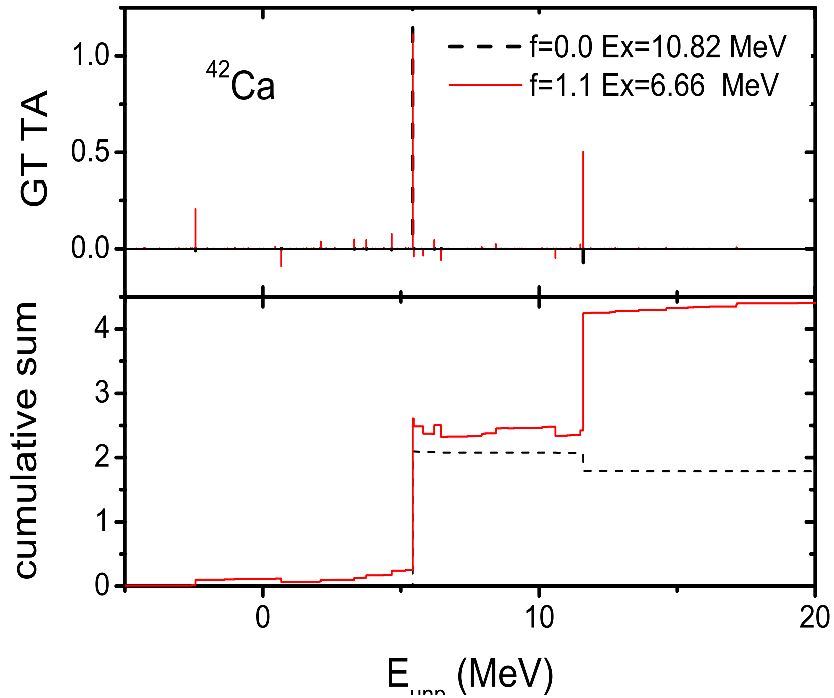


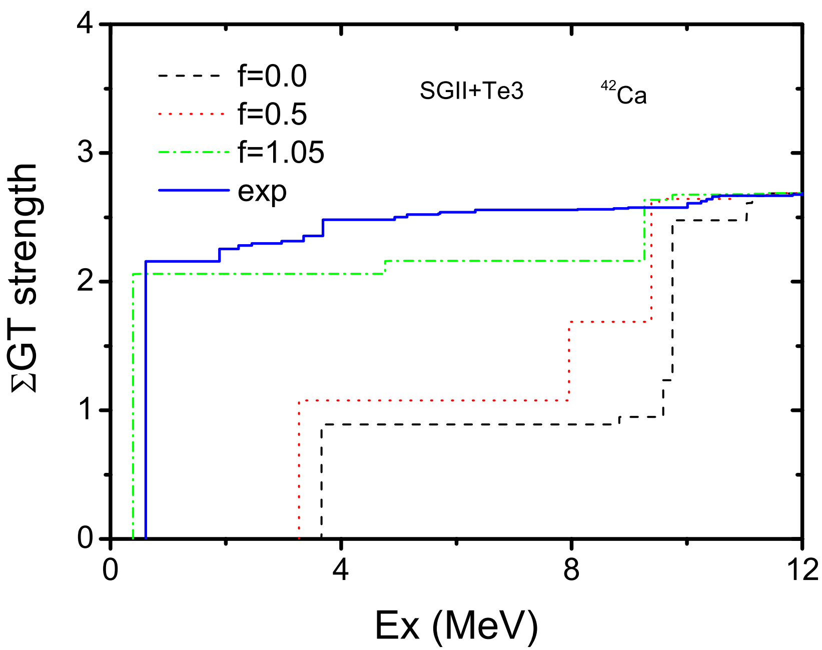
f =IS/IV pairing

HFB+QRPA with T=1 and T=0 pairing

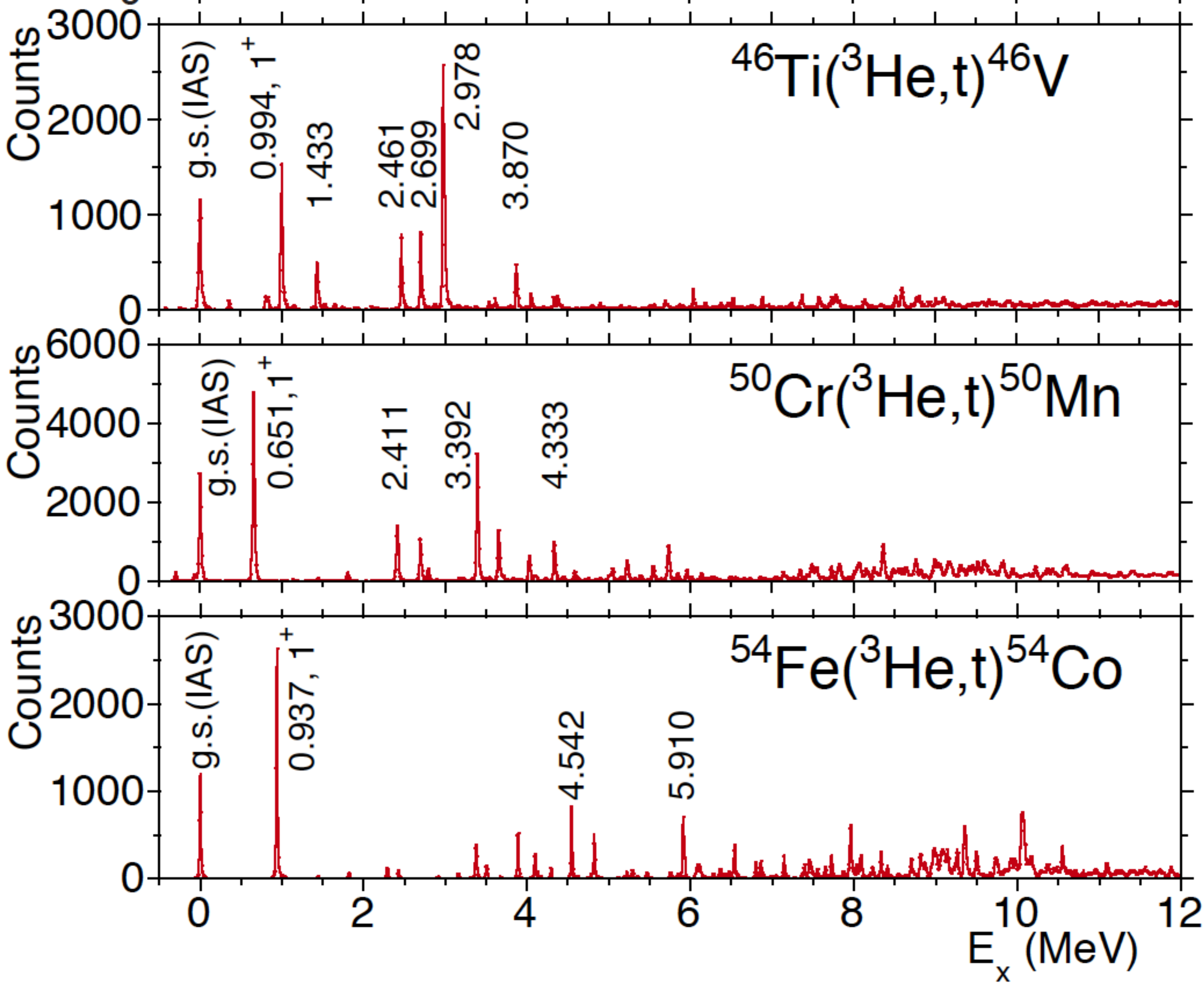
T=0 pairing in QRPA is changed as a parameter f.



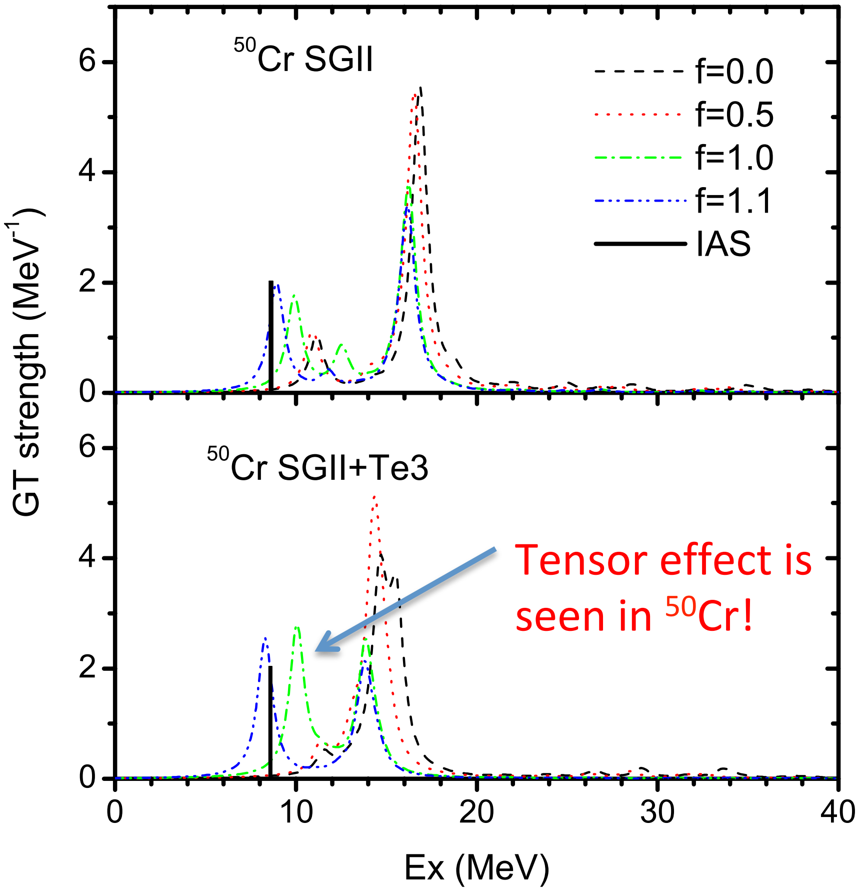


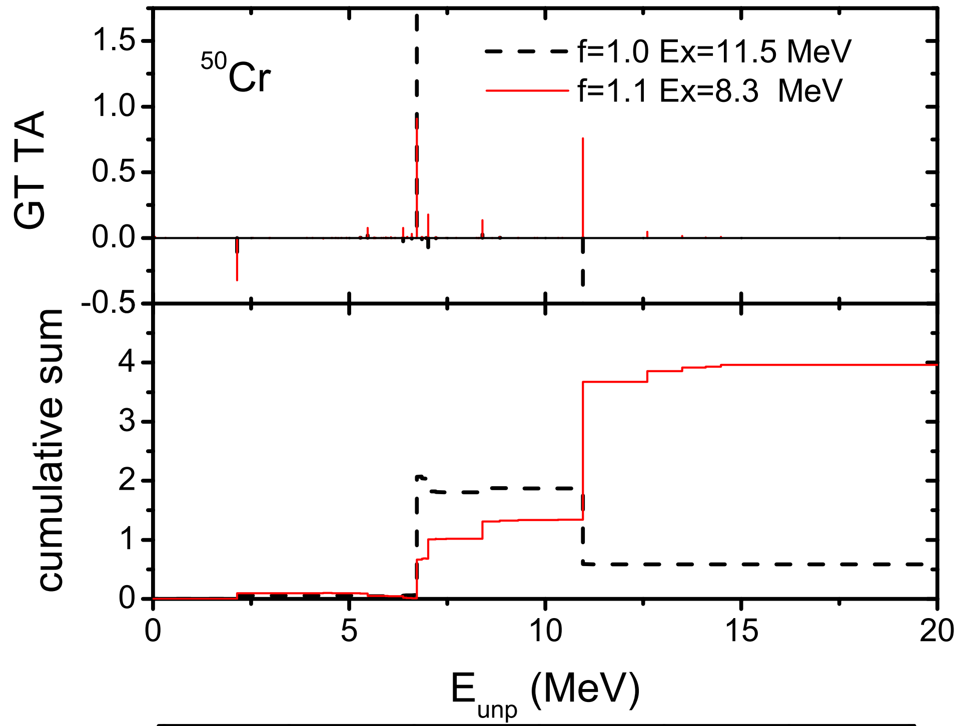


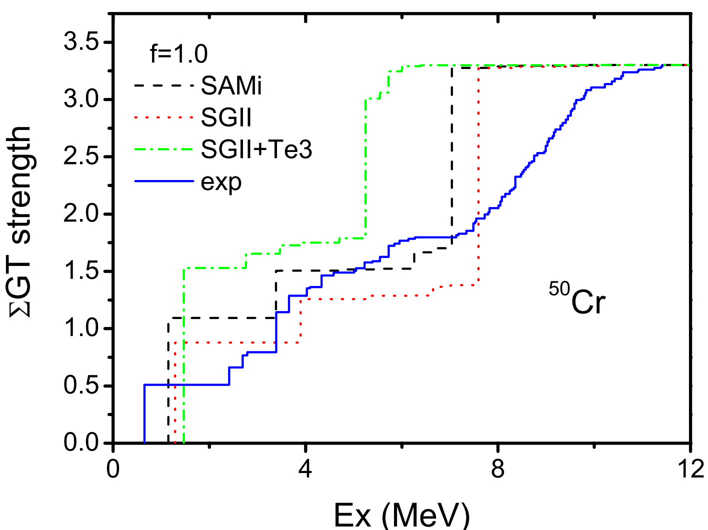
Effect of tensor correlations is small in ^{42}Ca.

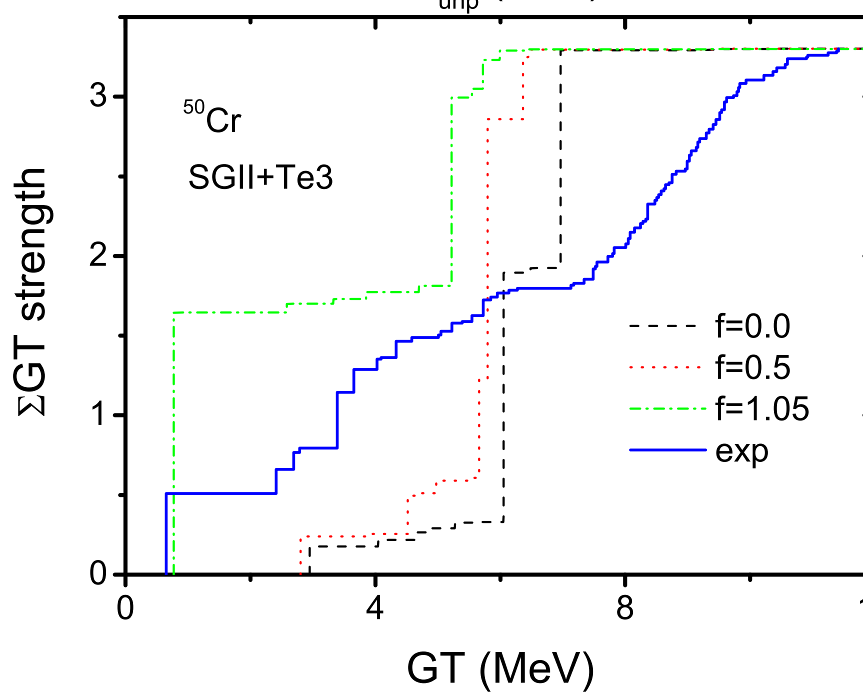


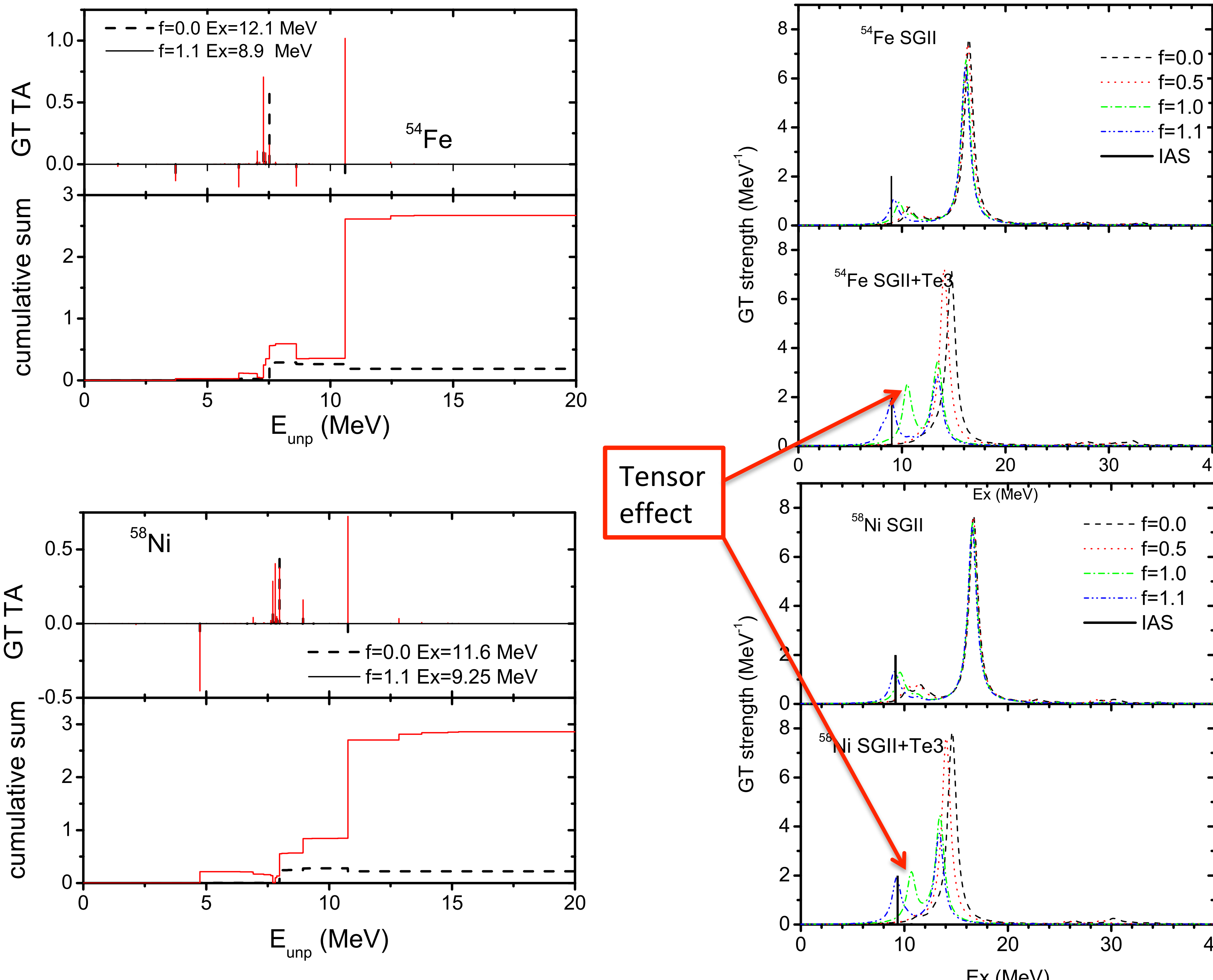
Counts
^{46}Ti(^{3}He,t)^{46}V
g.s.(IAS)
0.994, 1^{+}
1.433
2.461
2.699
2.978
3.870
^{50}Cr(^{3}He,t)^{50}Mn
g.s.(IAS)
0.651,1^{+}
2.411
3.392
4.333
^{54}Fe(^{3}He,t)^{54}Co
g.s.(IAS)
0.937, 1^{+}
4.542
5.910
E_x (MeV)


^{50}Cr SGII
^{50}Cr SGII+Te3
f=0.0
f=0.5
f=1.0
f=1.1
IAS
Tensor effect is seen in ^{50}Cr!
GT strength (MeV^{-1})
Ex (MeV)


^{50}Cr
f=1.0 Ex=11.5 MeV
f=1.1 Ex=8.3 MeV
GT TA
cumulative sum
E_{unp} (MeV)


f=1.0
SAMi
SGII
SGII+Te3
exp
^{50}Cr
ΣGT strength
Ex (MeV)

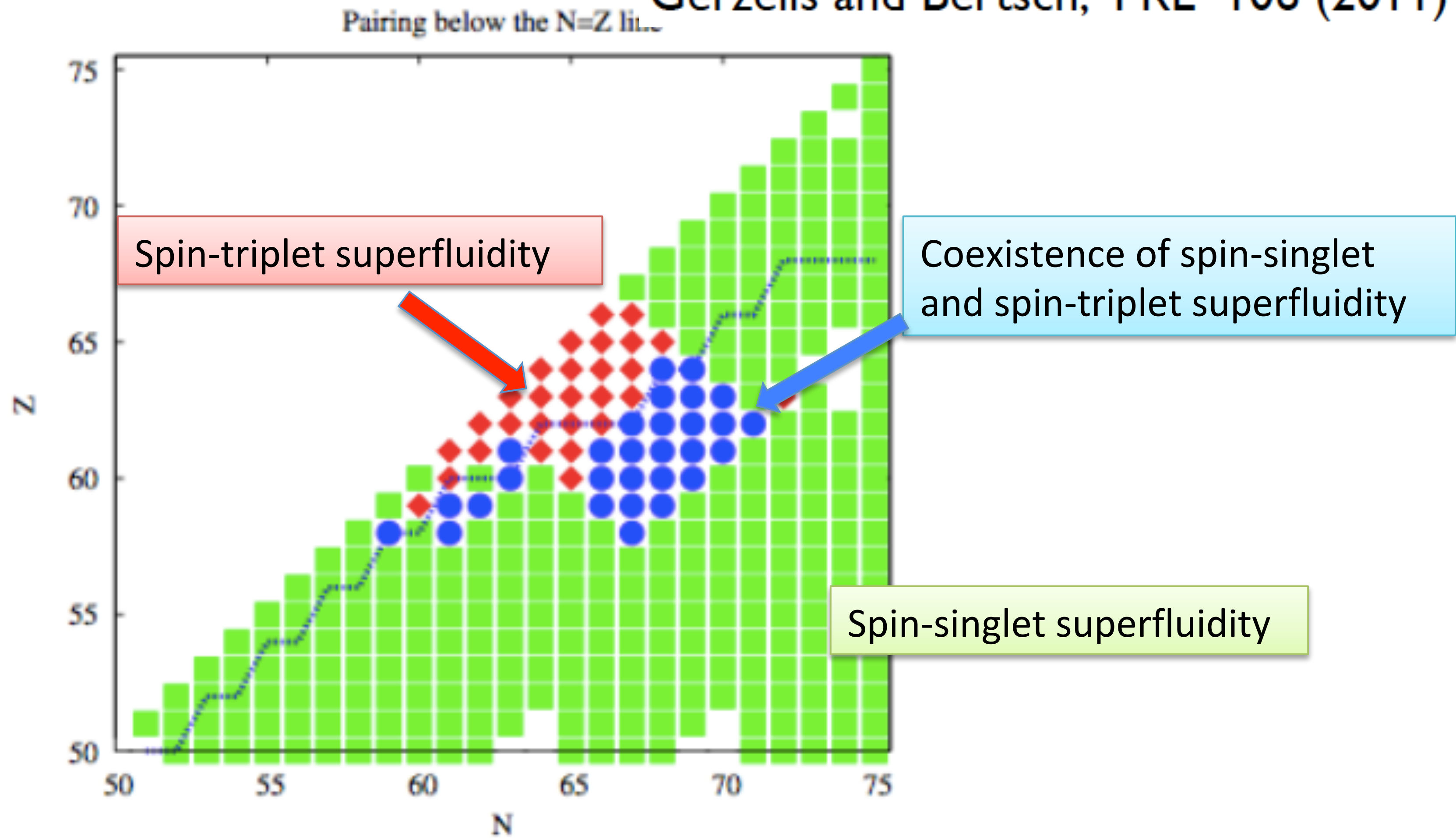

^{50}Cr
SGII+Te3
f=0.0
f=0.5
f=1.05
exp
ΣGT strength
GT (MeV)



f=0.0 Ex=12.1 MeV
f=1.1 Ex=8.9 MeV
GT TA
54Fe
cumulative sum
E unp (MeV)
58Ni
f=0.0 Ex=11.6 MeV
f=1.1 Ex=9.25 MeV
54Fe SGII
54Fe SGII+Te3
58Ni SGII
58Ni SGII+Te3
f=0.0
f=0.5
f=1.0
f=1.1
IAS
GT strength (MeV-1)
Ex (MeV)
Tensor effect

Neutron-proton pair condensates

Gerzelis and Bertsch, PRL 106 (2011)



Source	v_s (MeV fm^3)	v_t (MeV fm^3)	Ratio
sd shell [8]	280	465	1.65
fp shell [9]	291	475	1.63

G.F. Bertsch and Y. Luo, PRC81, 064320 (2010)

Deformed HFB calculations with a realistic intraction in N=Z nuclei : a competition between T=0 and T=1 pairing interactions

Eunja Ha *, Myung-Ki Cheoun †

Department of Physics and Origin of Matter and Evolution of Galaxy (OMEG) Institute, Soongsil University, Seoul 156-743, Korea

H. Sagawa ‡

RIKEN, Nishina Center for Accelerator-Based Science,

Eunja Ha, Myung-Ki Cheoun, H. Sagawa, Phys. Rev. C, **97** 024320 (2018).

Eunja Ha, Myung-Ki Cheoun, H. Sagawa, Phys. Rev. C, **97** 064322 (2018).

+preprint (2018)

Deformed HFB with a realistic interaction (CD Bonn)

T=1 channel nn,pp,np
T=0 channel np

Nuclear Hamiltonian

$$H = H_0 + H_{\rm int} \;,$$

$$H_0 = \sum_{\rho_\alpha \alpha\alpha'} \epsilon_{\rho_\alpha \alpha\alpha'} c^\dagger_{\rho_\alpha \alpha\alpha'} c_{\rho_\alpha \alpha\alpha'} \;,$$

$$H_{\rm int} = \sum_{\rho_\alpha \rho_\beta \rho_\gamma \rho_\delta, \alpha\beta\gamma\delta.\; \alpha'\beta'\gamma'\delta'} V_{\rho_\alpha \alpha\alpha' \rho_\beta \beta\beta' \rho_\gamma \gamma\gamma' \rho_\delta \delta\delta'} c^\dagger_{\rho_\alpha \alpha\alpha'} c^\dagger_{\rho_\beta \beta\beta'} c_{\rho_\delta \delta\delta'} c_{\rho_\gamma \gamma\gamma'},$$

HFB transformation

$$a^\dagger_{\rho_\alpha \alpha\alpha''} = \sum_{\rho_\beta \beta\beta'} (u_{\alpha\alpha''\beta\beta'} c^\dagger_{\rho_\beta \beta\beta'} + v_{\alpha\alpha''\beta\beta'} c_{\rho_\beta \bar\beta\beta'}),$$

$$a_{\rho_\alpha \bar\alpha\alpha''} = \sum_{\rho_\beta \beta\beta'} (u_{\bar\alpha\alpha''\bar\beta\beta'} c_{\rho_\beta \bar\beta\beta'} - v_{\bar\alpha\alpha''\bar\beta\beta'} c^\dagger_{\rho_\beta \beta\beta'}). \qquad (6)$$

$\alpha,\beta,\gamma,\delta$: real (bare) s.p. states with Ω

α',β': isospin quantum number (bare) particle (p and n)

α'',β'': isospin of quisi-particle (1 and 2)

ρ_α: sign of Ω, $\pm\Omega$ (angular momentum projection on the symmetry axis)

Deformed BCS transformation

$$a^{\dagger}_{\rho_\alpha \alpha\alpha''} = \sum_{\rho_\beta \beta\beta'} (u_{\alpha\alpha''\beta\beta'} c^{\dagger}_{\rho_\beta \beta\beta'} + v_{\alpha\alpha''\beta\beta'} c_{\rho_\beta \bar{\beta}\beta'}),$$

$$a_{\rho_\alpha \bar{\alpha}\alpha''} = \sum_{\rho_\beta \beta\beta'} (u_{\bar{\alpha}\alpha''\bar{\beta}\beta'} c_{\rho_\beta \bar{\beta}\beta'} - v_{\bar{\alpha}\alpha''\bar{\beta}\beta'} c^{\dagger}_{\rho_\beta \beta\beta'}). \qquad (6)$$

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_{\bar{1}} \\ a_{\bar{2}} \end{pmatrix}_\alpha = \begin{pmatrix} u_{1p} & u_{1n} & v_{1p} & v_{1n} \\ u_{2p} & u_{2n} & v_{2p} & v_{2n} \\ -v_{1p} & -v_{1n} & u_{1p} & u_{1n} \\ -v_{2p} & -v_{2n} & u_{2p} & u_{2n} \end{pmatrix}_\alpha \begin{pmatrix} c_p^\dagger \\ c_n^\dagger \\ c_{\bar{p}} \\ c_{\bar{n}} \end{pmatrix}_\alpha ,$$

where the u and v coefficients are calculated by the following DBCS equation

$$\begin{pmatrix} \epsilon_p - \lambda_p & 0 & \Delta_{p\bar{p}} & \Delta_{p\bar{n}} \\ 0 & \epsilon_n - \lambda_n & \Delta_{n\bar{p}} & \Delta_{n\bar{n}} \\ \Delta_{p\bar{p}} & \Delta_{p\bar{n}} & -\epsilon_p + \lambda_p & 0 \\ \Delta_{n\bar{p}} & \Delta_{n\bar{n}} & 0 & -\epsilon_n + \lambda_n \end{pmatrix}_\alpha \begin{pmatrix} u_{\alpha''p} \\ u_{\alpha''n} \\ v_{\alpha''p} \\ v_{\alpha''n} \end{pmatrix}_\alpha = E_{\alpha\alpha''} \begin{pmatrix} u_{\alpha''p} \\ u_{\alpha''n} \\ v_{\alpha''p} \\ v_{\alpha''n} \end{pmatrix}_\alpha .$$

Pairing Gaps

$$\Delta_{nn}, \Delta_{pp} : \text{ real}$$

$$\Delta_{np} : \text{ complex}$$

$$\Delta_{p\bar{p}_\alpha} = \Delta_{\alpha p \bar{\alpha} p} = -\sum_{J,c,d} g_{\rm pp} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\delta} c} G(aacd, J, T=1)(u^*_{1p_c} v_{1p_d} + u^*_{2p_c} v_{2p_d}) \ ,$$

$$\Delta_{p\bar{n}_\alpha} = \Delta_{\alpha p \bar{\alpha} n} = \ - \sum_{J,c,d} g_{\rm np} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\delta} c} [G(aacd, J, T=1) Re(u^*_{1n_c} v_{1p_d} + u^*_{2n_c} v_{2p_d})$$
$$+ \ iG(aacd, J, T=0) Im(u^*_{1n_c} v_{1p_d} + u^*_{2n_c} v_{2p_d})] \ ,$$

We do not include

Δ_{np} and $\Delta_{\overline{np}}$ explicitly, but include implicitly multiplying a factor 2 on the T=0 pairing matrix

4 point formulas for empirical gaps

$$\Delta_p^{\rm emp} = \tfrac{1}{8}[M(Z+2,N) - 4M(Z+1,N) + 6M(Z,N) - 4M(Z-1,N) + M(Z-2,N)], \qquad (14)$$

$$\Delta_n^{\rm emp} = \tfrac{1}{8}[M(Z,N+2) - 4M(Z,N+1) + 6M(Z,N) - 4M(Z,N-1) + M(Z,N-2)]. \qquad (15)$$

$$M(Z,N)_{\mathrm{odd-odd}} = M(Z,N)_{\mathrm{even-even}} + \Delta_p^{\mathrm{emp}} + \Delta_n^{\mathrm{emp}} - \delta_{np}^{\mathrm{emp}}. \tag{16}$$

Then the np pairing gap is deduced as follows:

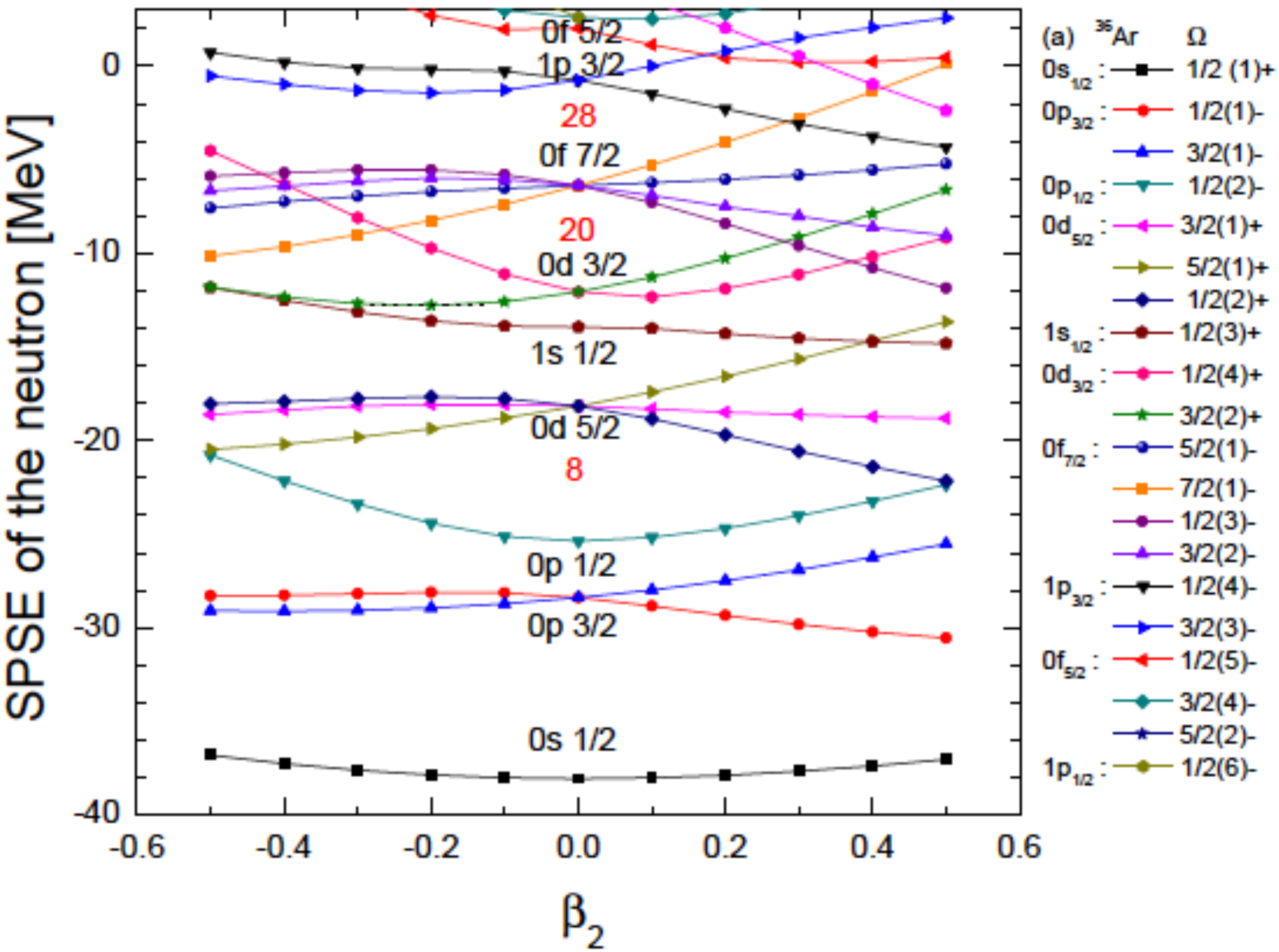
$$\begin{aligned}\delta_{np}^{\mathrm{emp}} = \pm\tfrac{1}{4}\{2[&M(Z,N+1)\\ &+M(Z,N-1)+M(Z-1,N)+M(Z+1,N)]\\ &-[M(Z+1,N+1)+M(Z-1,N+1)\\ &+M(Z-1,N-1)+M(Z+1,N-1)]\\ &-4M(Z,N)\},\end{aligned} \tag{17}$$

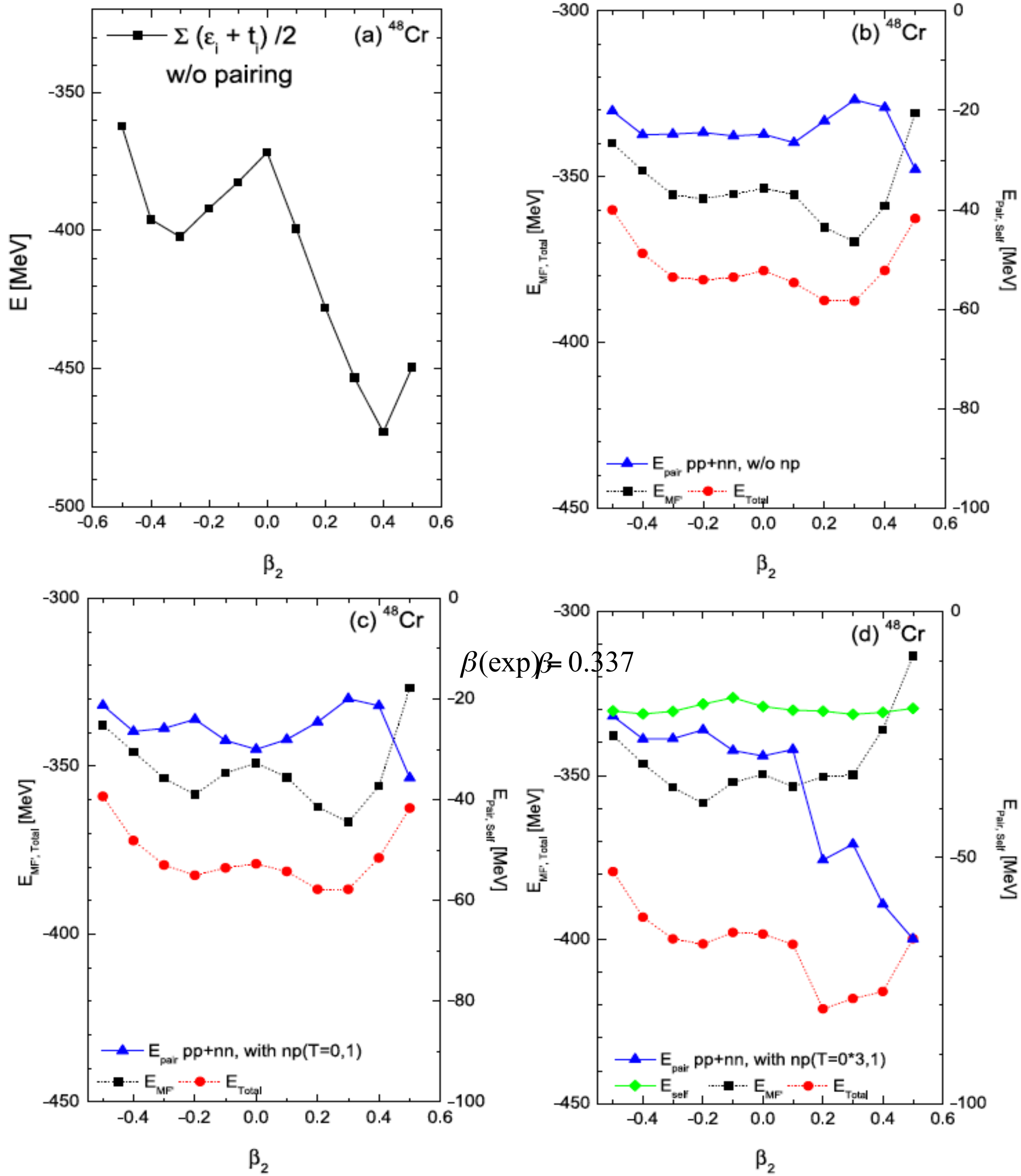
$$\delta_{np}^{th.} = -\left[\left(H_{gs}^{12} + E_1 + E_2\right) - \left(H_{gs}^{np} + E_p + E_n\right)\right]. \tag{18}$$

Here $H_{gs}^{12}(H_{gs}^{np})$ is the total deformed BCS ground-state energy with (without) np pairing and $E_1 + E_2(E_p + E_n)$ is a sum of the lowest two quasiparticle energies with (without) the np pairing potential Δ_{np} in Eq. (9). All of the pairing gaps

Nucleus	$\lvert\beta_2^{E2}\rvert$ [29]	β_2^{RMF} [30]	β_2^{FRDM} [31]	β_2^{Ours}	Δ_p^{emp}	Δ_n^{emp}	$\delta_{np}^{\mathrm{emp}}$
^{24}Mg	0.605	0.416	0.	0.300	3.123	3.193	1.844
^{36}Ar	0.256	-0.207	-0.255	-0.200	2.265	2.311	1.373
^{48}Cr	0.337	0.225	0.226	0.200	2.128	2.138	1.442
^{64}Ge	–	0.217	0.207	0.100	1.807	2.141	1.435
^{108}Xe	–	–	0.162	0.100	1.467	1.496	0.605
^{128}Gd	–	0.350	0.341	0.100	1.415	1.393	0.592

Deformed Woods-Saxon potential for s.p. energies in ^{36}Ar

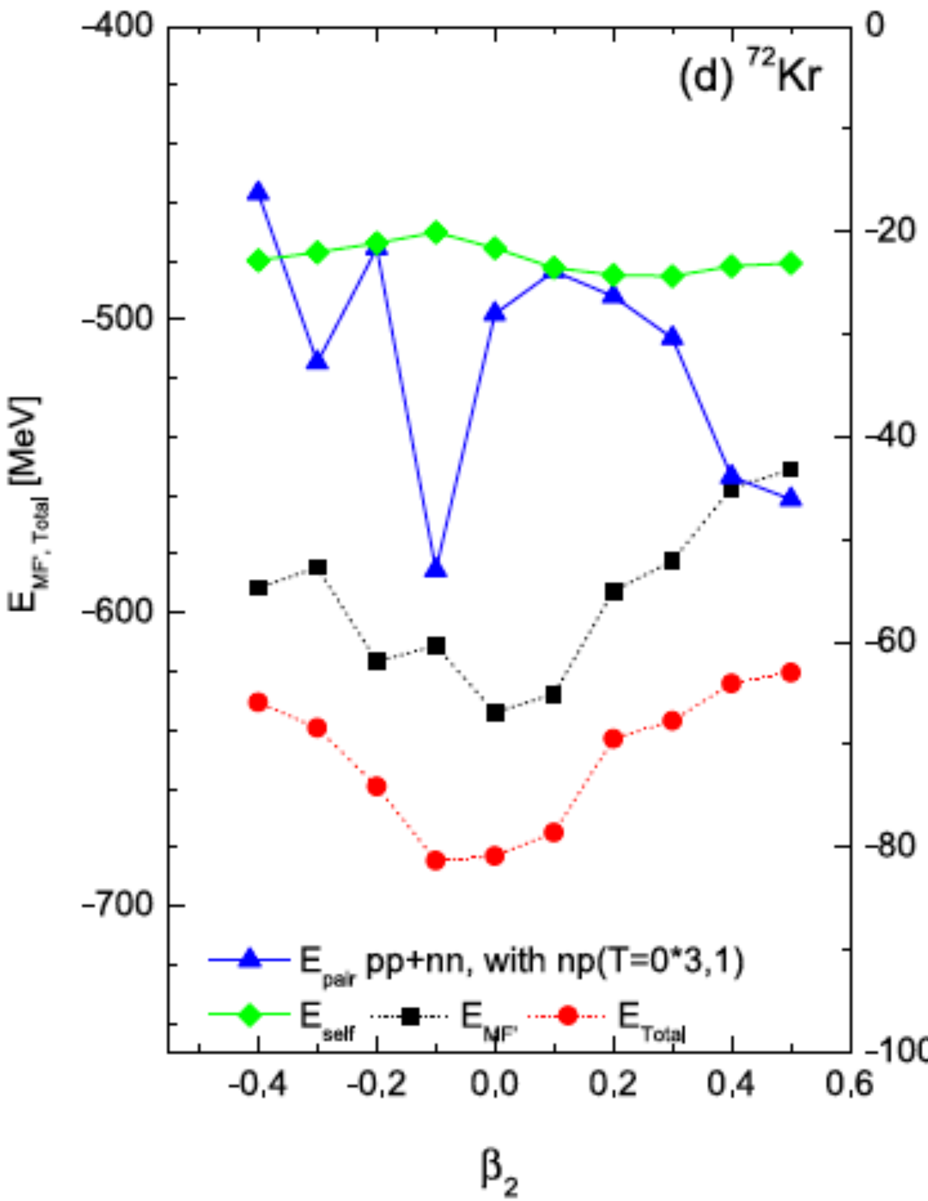




^{48}Cr

$\beta(\exp) = 0.337$

FIG. 4. Ground-state energy (GSE) for ^{44}Ti by the DBCS model based on a deformed Woods-Saxon potential [4]. Energies are estimated from the Fermi energy surface calculated by the DBCS. $E_{MF'}$ is the mean-field energy with respect to the Fermi energy, which is different from the GSE in (a) because the Fermi energy is changed by the DBCS approach owing to the pairing interactions. E_{pair} is the pairing energy indicated in the right axis label. The pairing energies are estimated by three different cases, (b) without and (c) with the np pairing and (d) with the three times enhanced $T = 0$ pairing. (d) includes the self-energy due to the pairing interactions denoted as (green) diamond.



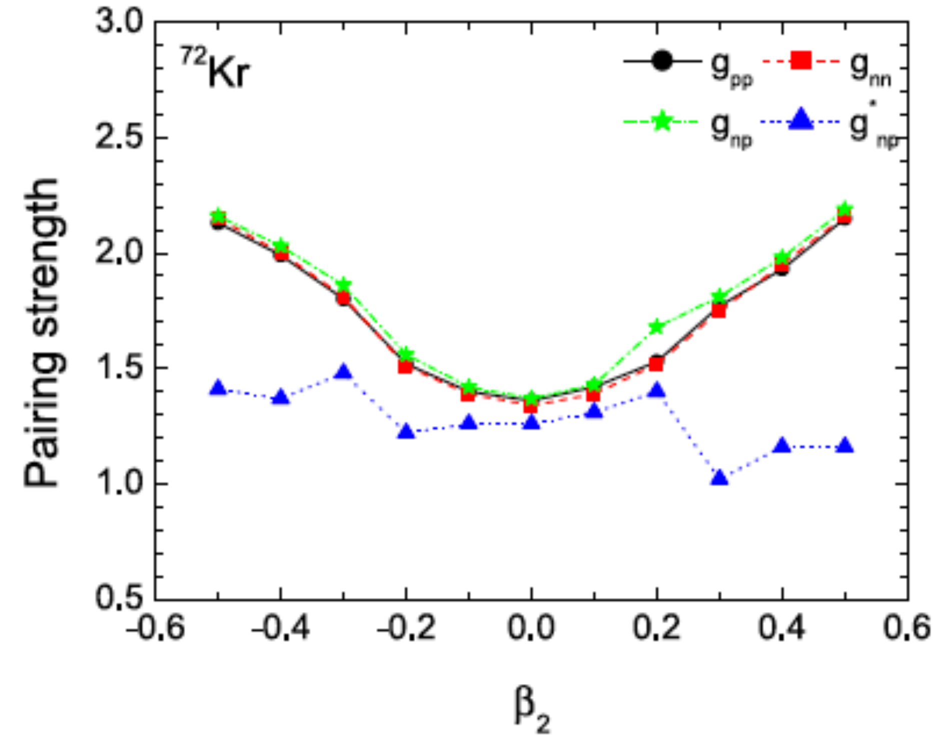
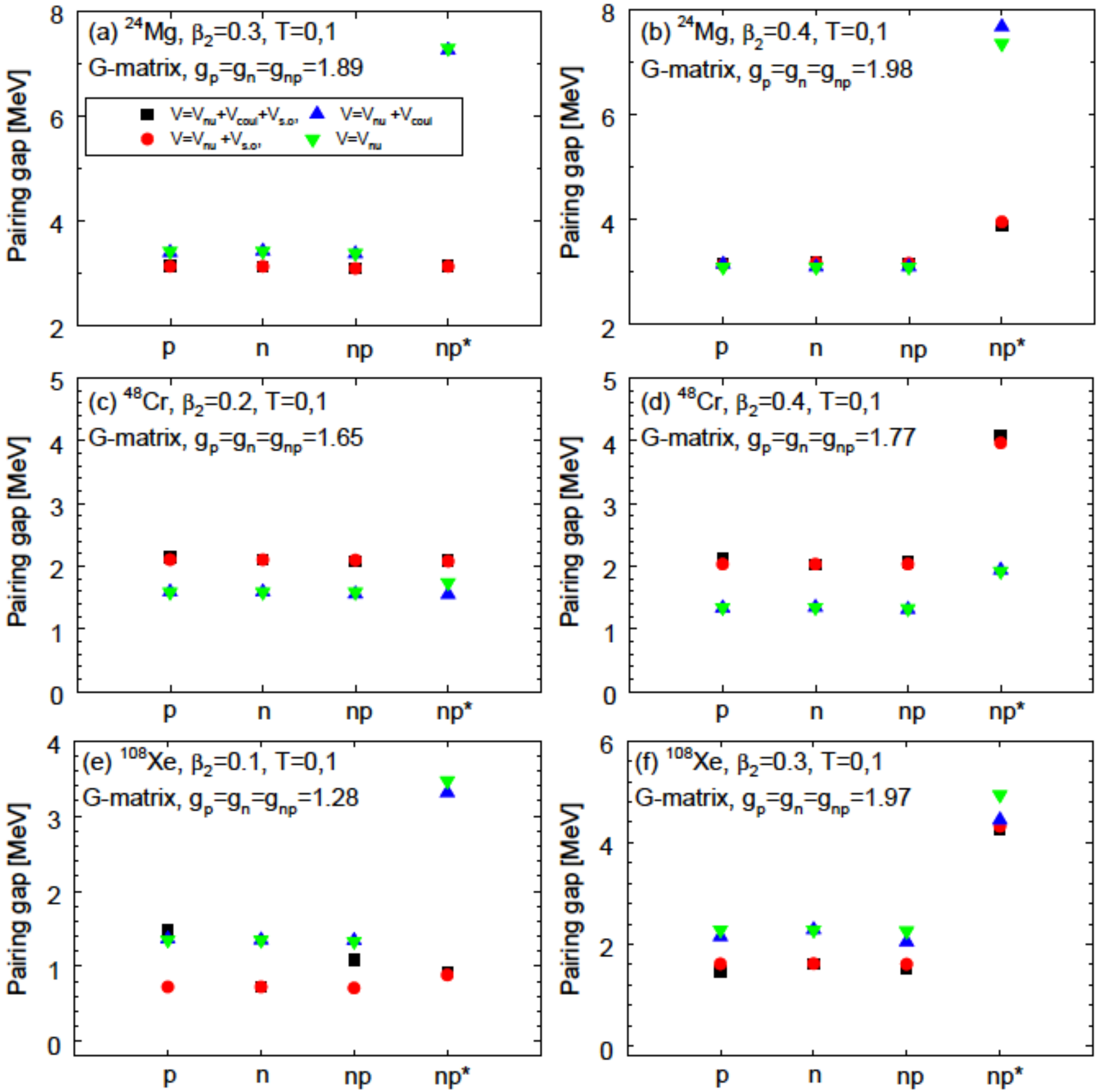
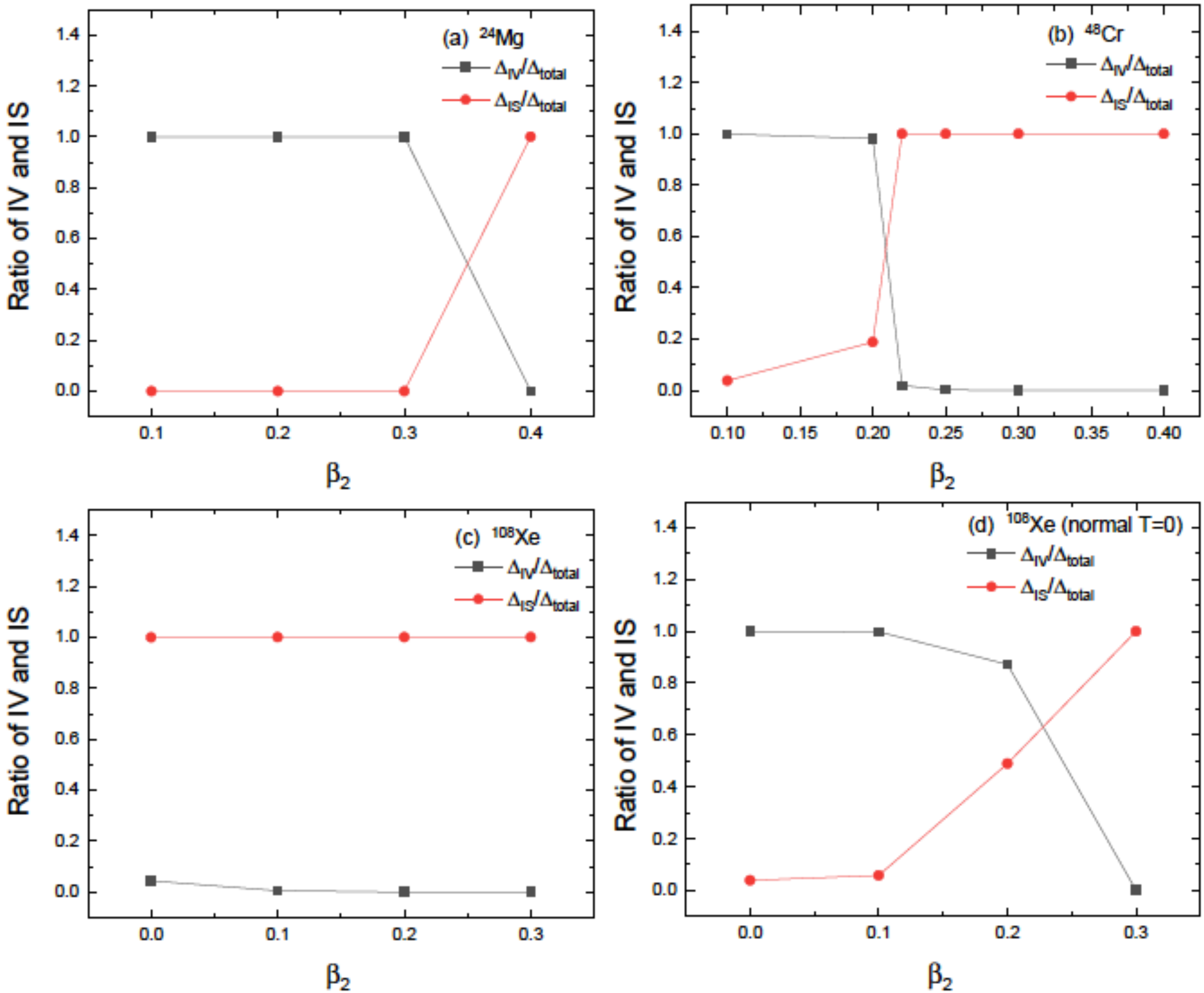


FIG. 10. Strength parameters evolution with the deformatio
Kr. Here g^*_{np} means the case by the enhanced $T = 0$ pa
orrelations.



Delta$_{np}$ includes both T=0 and 1.


(a) ^{24}Mg
$\Delta_{IV}/\Delta_{total}$
$\Delta_{IS}/\Delta_{total}$
Ratio of IV and IS
β_2
(b) ^{48}Cr
$\Delta_{IV}/\Delta_{total}$
$\Delta_{IS}/\Delta_{total}$
Ratio of IV and IS
β_2
(c) ^{108}Xe
$\Delta_{IV}/\Delta_{total}$
$\Delta_{IS}/\Delta_{total}$
Ratio of IV and IS
β_2
(d) ^{108}Xe (normal T=0)
$\Delta_{IV}/\Delta_{total}$
$\Delta_{IS}/\Delta_{total}$
Ratio of IV and IS
β_2

Summary I: N=Z GT states nucleus (3-body model)

1. Inversion of 1+ and 0+ states in the energy spectra and strong M1 transitions in odd-odd N=Z nuclei is induced by a strong T=0 pairing correlations competing with T=1 pairing and spin-orbit force.

2. Cooperative role of T=0 and T=1 pairings is studied in Gamow-Teller transitions of N=Z nuclei
3. It is pointed out that the low energy peak appear due to the strong T=0 pairing correlations in the final states.

   Supermultiplets of T=1,S=0 and T=0 and S=1 pair

Summary of N=Z+2 nuclei:RPA

1. It is pointed out the large enhancement of GT strength in the low energy peak just above IAS state in the charge exchange on N=Z+2 nuclei is induced by the strong T=0 pairing correlations in the final states.
   ➔ Restoration of supermultiplet symmetry in (T=1,J=0) and (T=0 ,J=1) states in *pf* shell nuclei
2. A cooperative effect of T=0 pairing and tensor interactions are found in nuclei at the middle of *pf* shell.

Deformed HFB

1. Deformed HFB with a realistic interaction has been performed for N=Z sd-, pf- shell sdg-shell nuclei.
2. The present HFB calculations reproduce well the deformation properties of N-Z nuclei with T=0 pairing.
3. A enhanced T=0 pairing gives the inversion of IS gap dominance of medium-heavy N=Z nuclei with a large prolate deformation.
4. Number and angular momentum projections are future pursupectives.

Collaborators

○ IS Pairing correlations (Three-body model)
Y. Tanimura (Orsay, Tohoku, University, Japan)
K. Hagino (Tohoku University, Japan)

○ Pairing and Tensor correlations on spin-isospin excitations
C.L. Bai (Sichun University 四川大学, China)
Gianluca Colo (University of Milano, Italy)

○ Deformed HFB with T=1 and T=0 pairings
Eunja Ha (Soongsil University, Seoul, Korea)
Myung-Ki Cheoun (Soongsil University, Seoul, Korea)
W. Y. So (Kangwon National University, at Dogye, Samcheok, Korea)

Review paper in Phys. Scr. 91 (2016) 083011 (23pges)
[Memorial issue for 40th anniversary of Bohr-Mottelson Rainwater Novel Prize]

「Isovector spin-singlet(T=1,S=0) and isoscalar spin-triplet (T=0,S=1) pairing interactions and spin-isospin response」
H. Sagawa, C. L. Bai and G. Colo