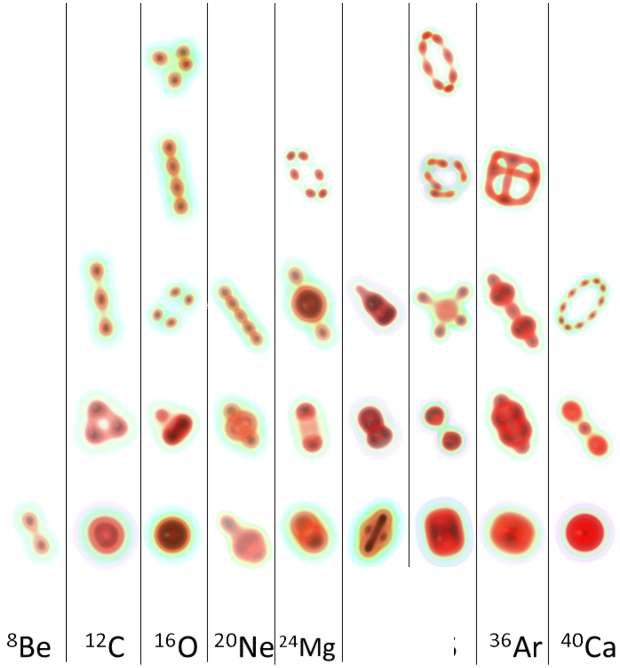
From single-particle states to α clustering



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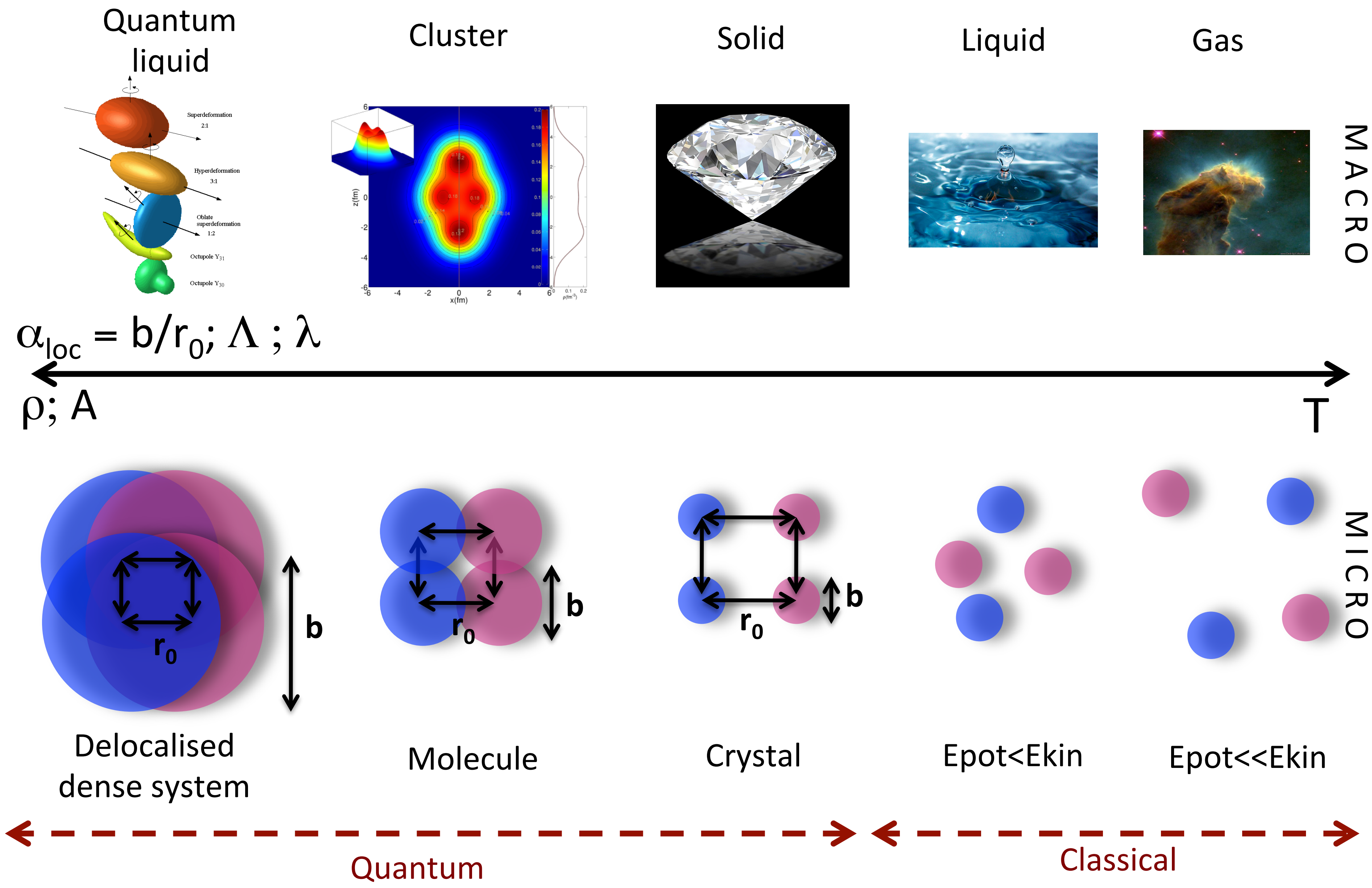
Comprendre le monde, construire l'avenir®

université PARIS-SACLAY

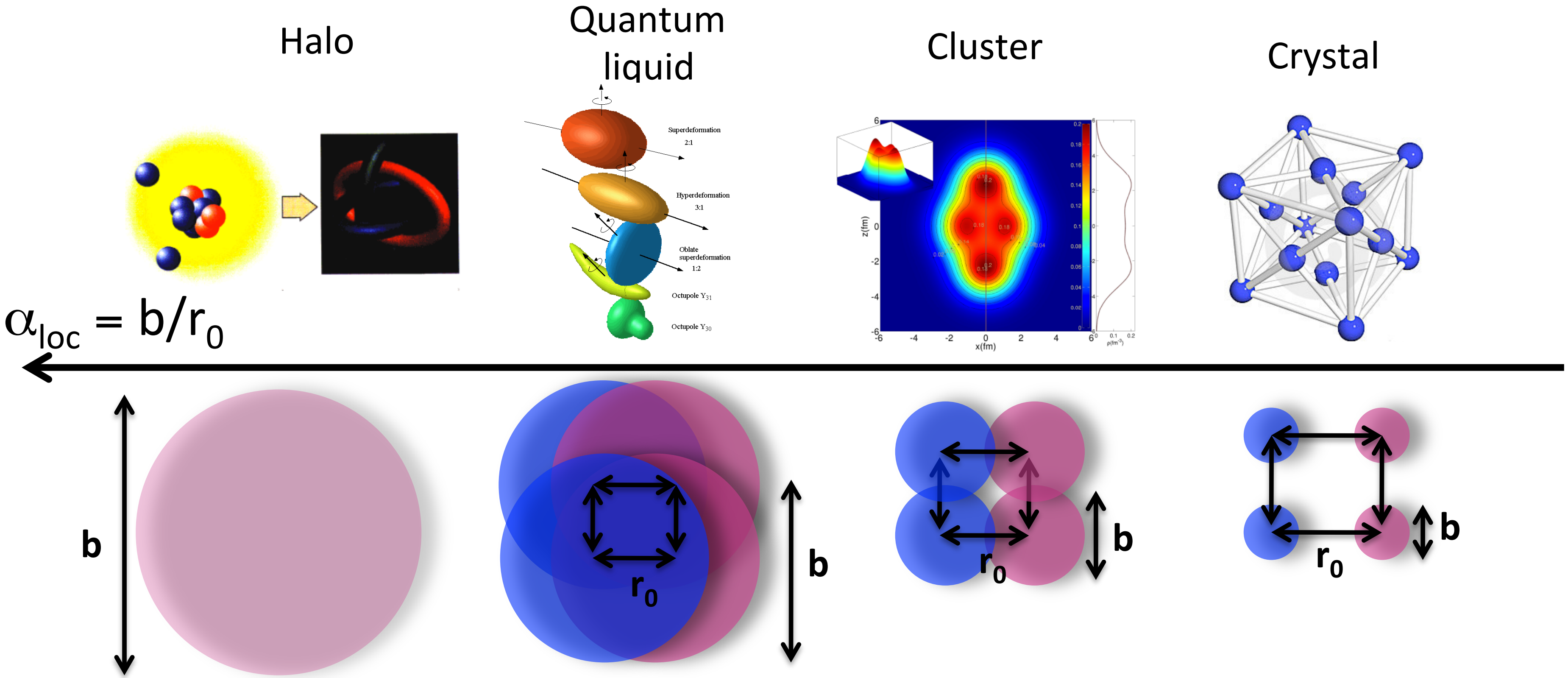
cnrs
dépasser les frontières
IN2P3
Les deux infinis

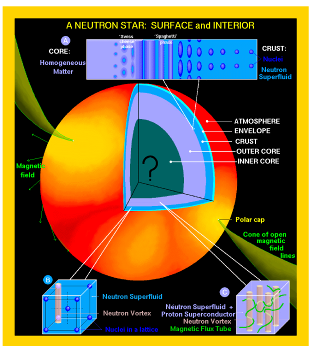
with J.-P. Ebran, R. Lasseri, P. Marevic, T. Niksic and D. Vretenar

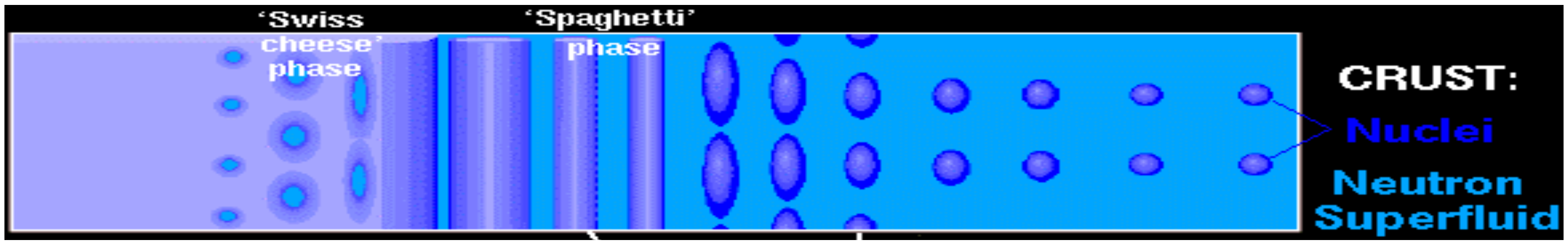
States of matter



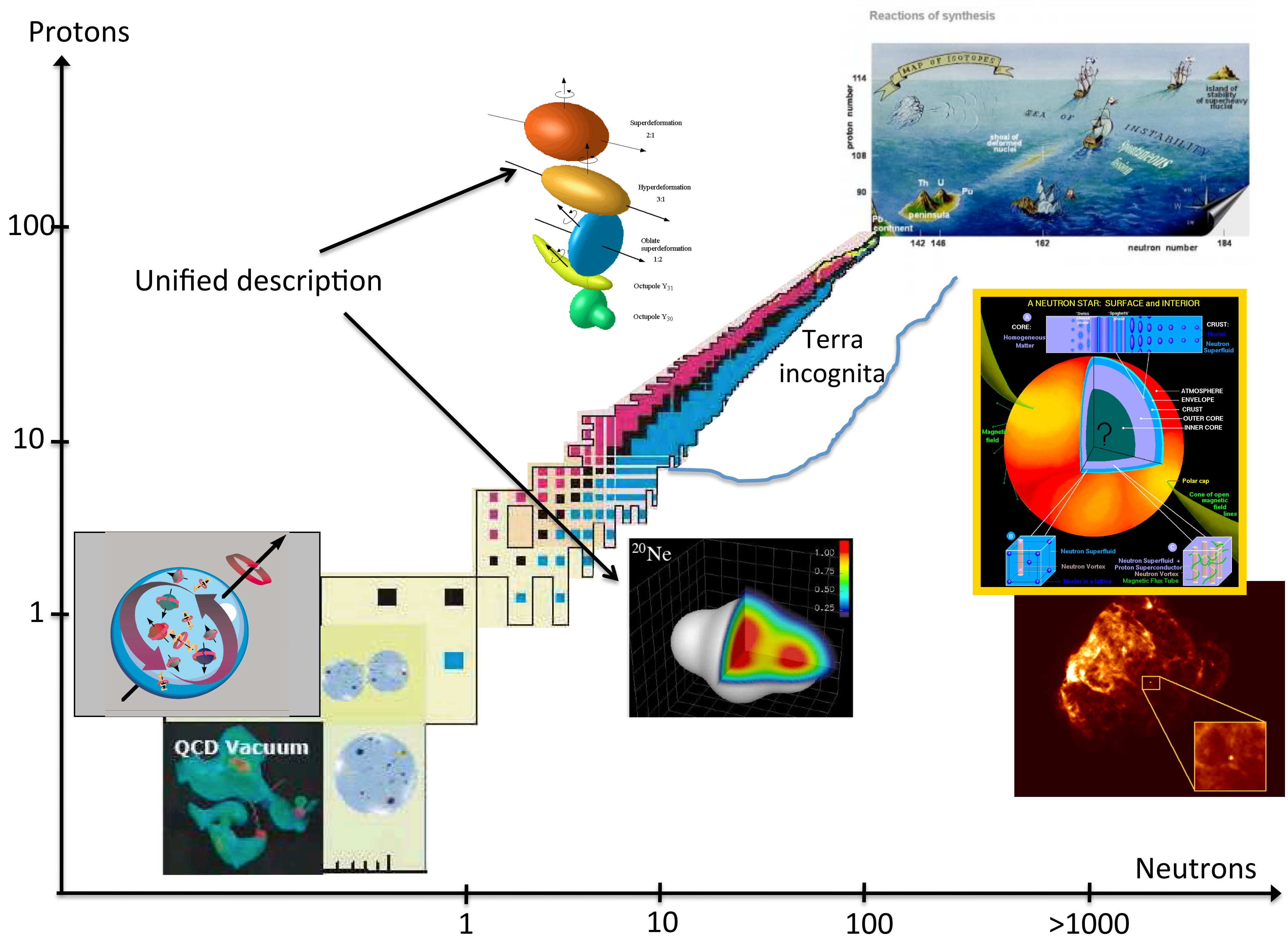
Nuclear states







J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, Nature 487(2012)341



Protons
Neutrons
100
10
1
1
10
100
>1000
Unified description
Terra incognita
Superdeformation 2:1
Hyperdeformation 3:1
Oblate superdeformation 1:2
Octupole Y_{31}
Octupole Y_{30}
Reactions of synthesis
MAP OF ISOTOPES
SEA OF INSTABILITY
island of stability of superheavy nuclei
shoal of deformed nuclei
peninsula
continent
proton number
neutron number
A NEUTRON STAR: SURFACE and INTERIOR
CORE:
Homogeneous Matter
CRUST:
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
Polar cap
Cone of open magnetic field lines
Neutron Vortex
Neutron Superfluid + Proton Superconductor
Magnetic Flux Tube
^{20}Ne
QCD Vacuum

Two complementary approaches

1) **Microscopic :EDF** (relativistic) + deformation

↕

2) **Harmonic Oscillator** : analytic, identification of key quantities

EDF method & clusters

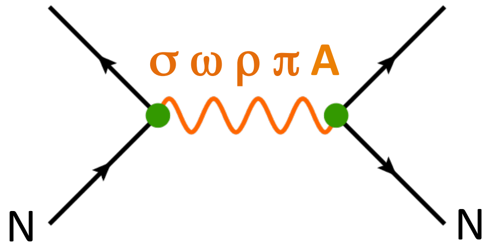
- EDF: many-body system mapped into the **one-body density** and its powers, gradient

$$\rho_0(\mathbf{r}) = \rho_0(\mathbf{r},\mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma\tau)$$
$$\rho_1(\mathbf{r}) = \rho_1(\mathbf{r},\mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma\tau)\,\tau$$
$$\mathbf{s}_0(\mathbf{r}) = \mathbf{s}_0(\mathbf{r},\mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma'\tau)\,\boldsymbol{\sigma}_{\sigma'\sigma}$$
$$\mathbf{s}_1(\mathbf{r}) = \mathbf{s}_1(\mathbf{r},\mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma'\tau)\,\boldsymbol{\sigma}_{\sigma'\sigma}\,\tau$$

$$\mathbf{j}_T(\mathbf{r}) = \tfrac{i}{2}(\nabla' - \nabla)\,\rho_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$
$$\mathcal{J}_T(\mathbf{r}) = \tfrac{i}{2}(\nabla' - \nabla)\otimes \mathbf{s}_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$
$$\tau_T(\mathbf{r}) = \nabla\cdot\nabla'\,\rho_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$
$$\mathbf{T}_T(\mathbf{r}) = \nabla\cdot\nabla'\,\mathbf{s}_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$

- **Most general** antisymmetrised product of nucleonic wavefunctions
- **Not any a priori assumption** on the nucleons' wave function
- **Correlations** beyond the mean-field effectively included by the EDF
- Investigate nuclear structure on the **whole nuclear chart**
- **Relativistic**: the depth of the central potential is **consistently predicted**

Relativistic EDF in nuclei



$$\mathcal{L}_{int} = -g_\sigma(\rho_v)\bar{\psi}\sigma\psi - g_\omega(\rho_v)\bar{\psi}\gamma_\mu\omega^\mu\psi - g_\rho(\rho_v)\bar{\psi}\gamma_\mu\vec{\rho}^\mu.\vec{\tau}\psi - \frac{f_\pi(\rho_v)}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\vec{\pi}.\vec{\tau}\psi - e\bar{\psi}\gamma_\mu A^\mu\psi$$

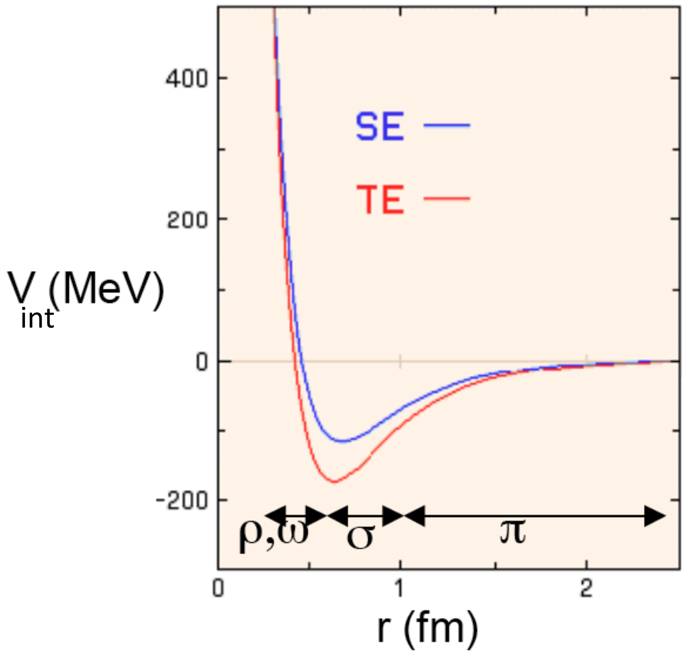
EDF [ρ ; σ,ω,ρ,π,A]

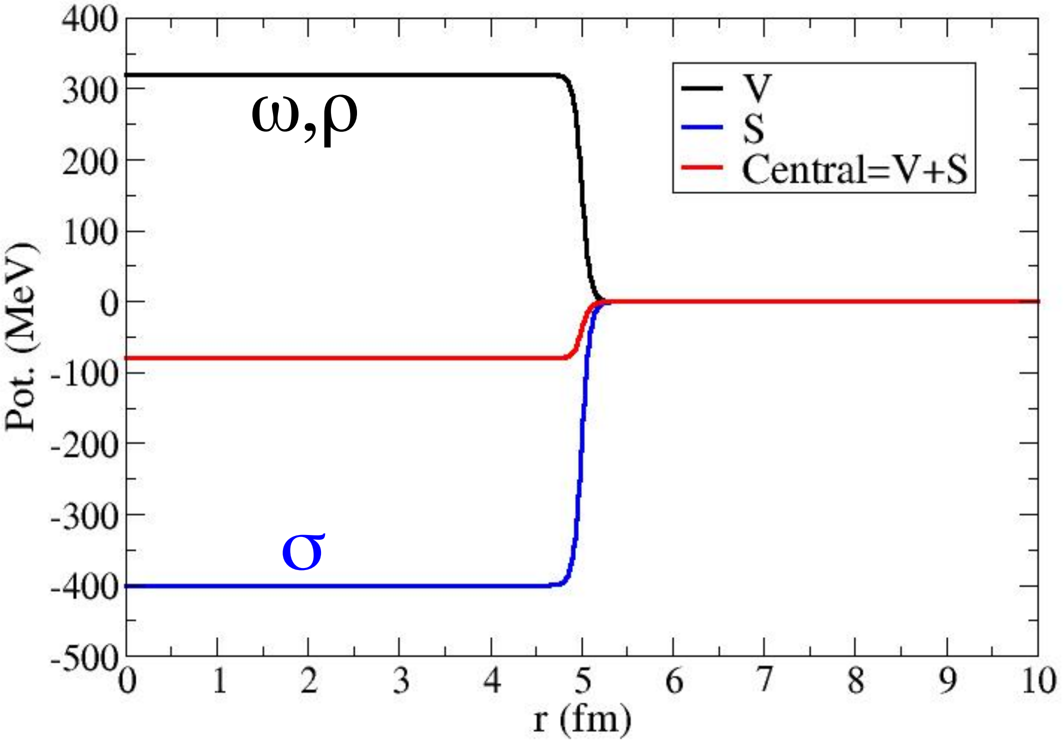
$$\left\{ \boldsymbol{p}\frac{1}{2\tilde{M}(r)}\boldsymbol{p} + W(r) + V_{ls}(r)\boldsymbol{l}.\boldsymbol{s} \right\} \varphi_i = \varepsilon_i^{NR}\varphi_i$$

$$W(r) = [V+S](r)$$

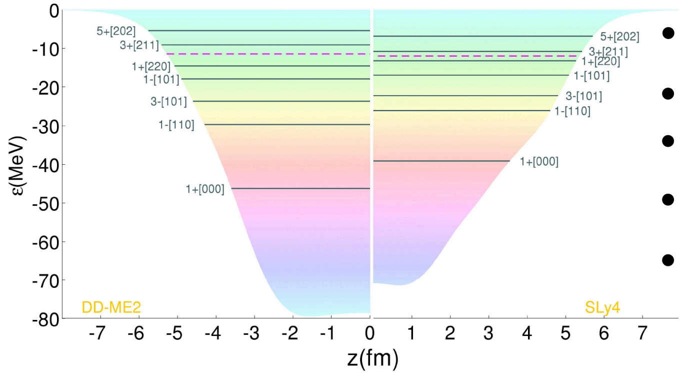
$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)}\frac{1}{r}\frac{d}{dr}(V-S)$$

V and S potentials

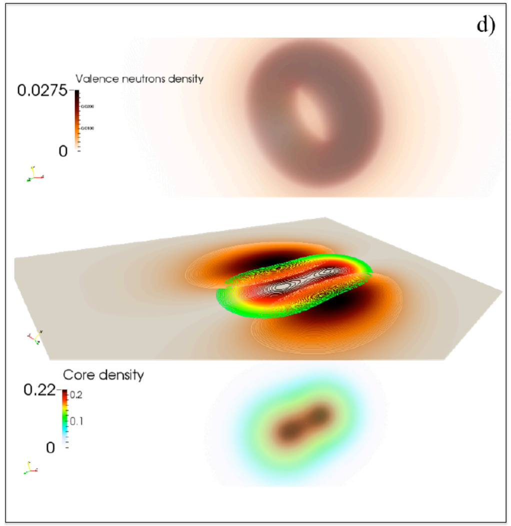




Origins of nuclear clustering



- Depth of the confining potential
- Heavy vs. Light nuclei
- Deformation / excitation energy
- Density
- Neutron excess

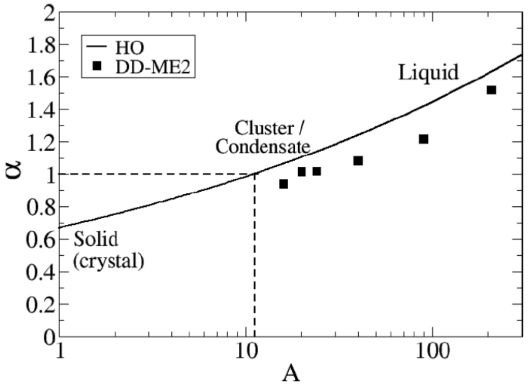


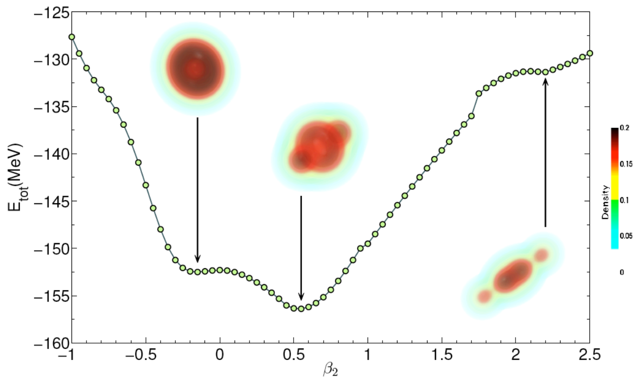
J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, Nature 487(2012)341

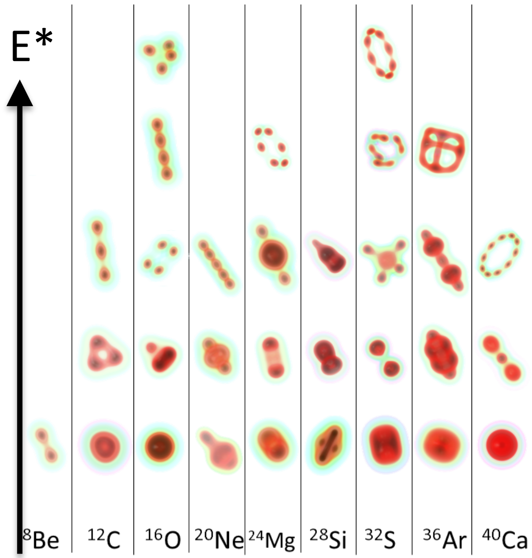
PRC87(2013)044307

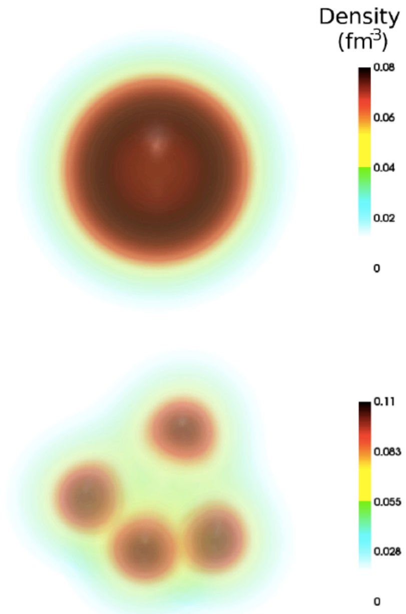
PRC90(2014)054329

PRC89(2014)031303(R)

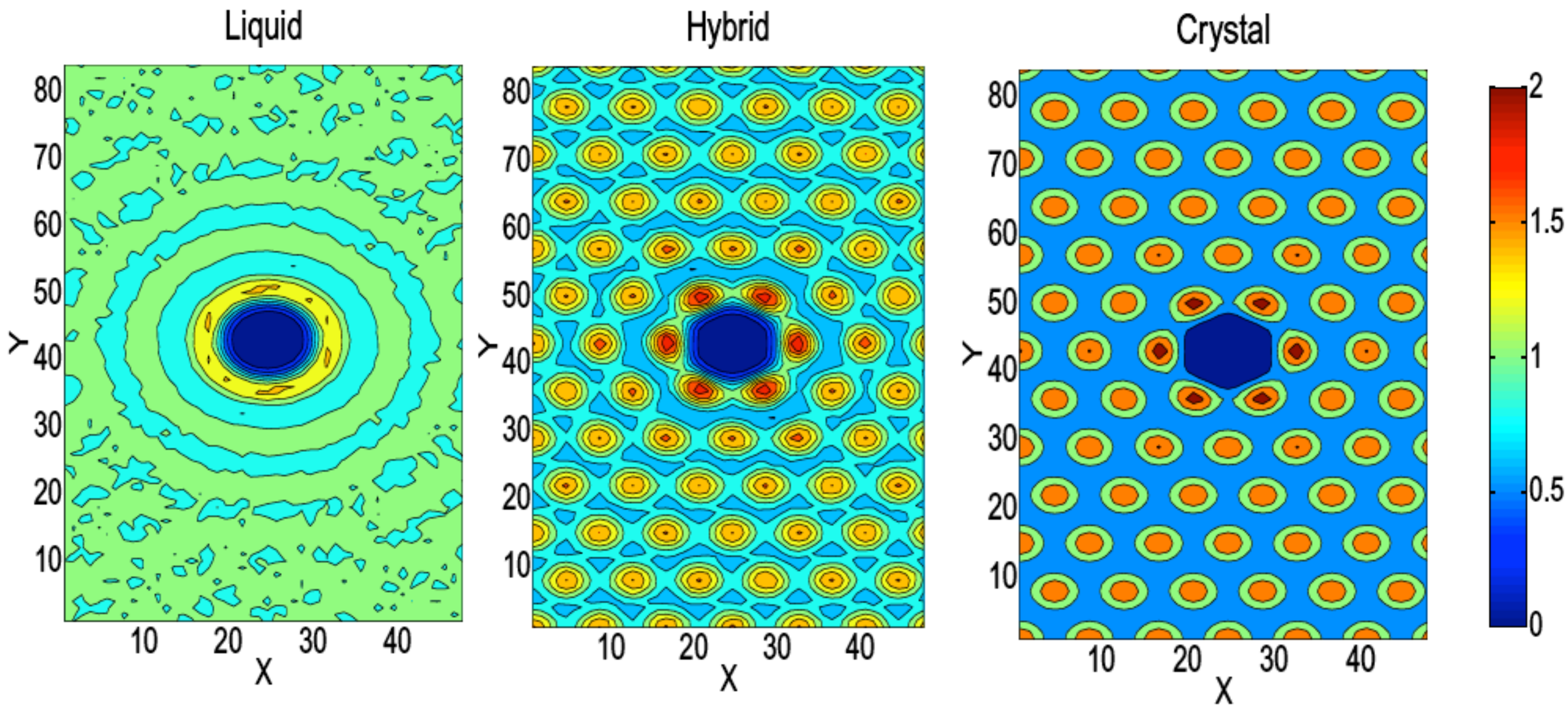








Analogies

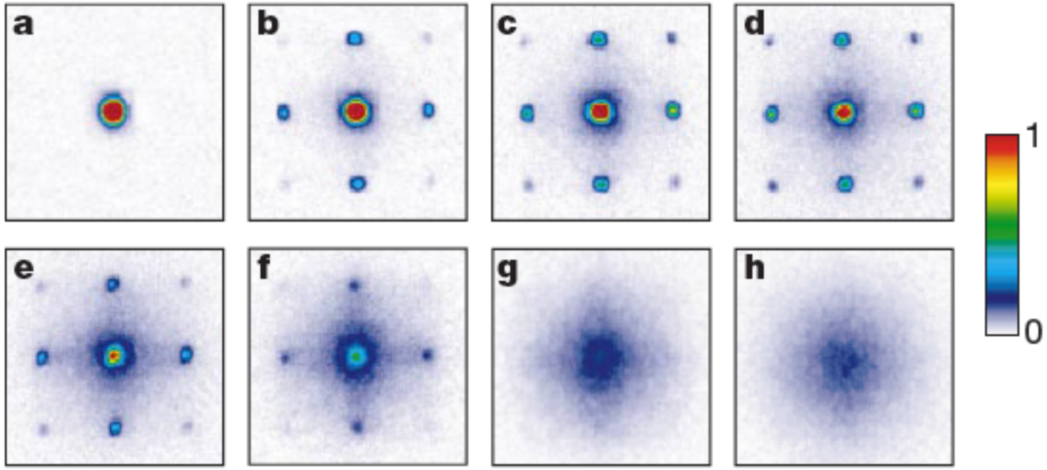


2D electronic system

H. Flakshahi and X. Waintal Phys. Rev. Lett 94 (2005) 046801

The depth of the potential

- **Ultracold atoms** : optical trap of variable depth V_0
  M. Greiner at al., Nature 415 (2002) 39



- **Nuclei** : depth of the potential **consistently** determined (relativisitic)

$$\left\{p\frac{1}{2\tilde{M}(r)}p + W(r) + V_{ls}(r)\boldsymbol{l}.\boldsymbol{s}\right\}\varphi_i = \varepsilon_i\varphi_i$$

S≈ - 400 MeV
V≈ 320 MeV ⟶ $V_0 \approx$ 80 MeV

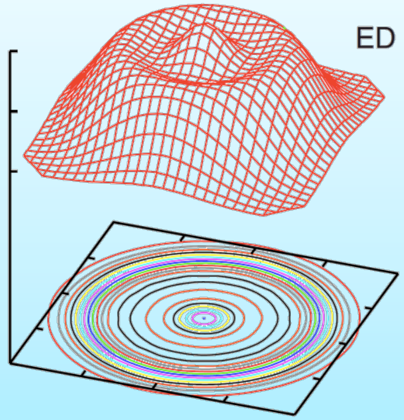
$$W(r) = [V+S](r)$$

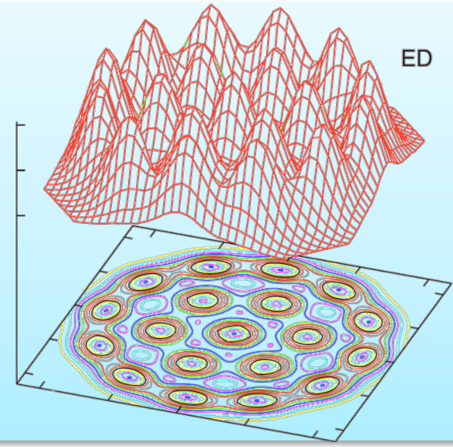
$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)}\frac{1}{r}\frac{d}{dr}(V-S)$$

Interacting many-body systems

Deeper confining potential

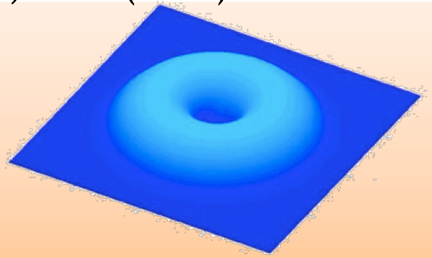
➲ Electrons in quantum dots

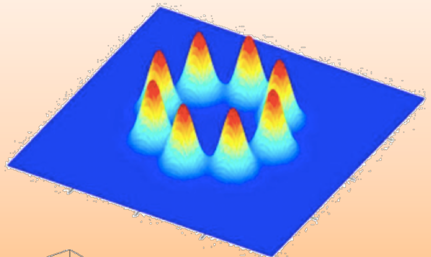




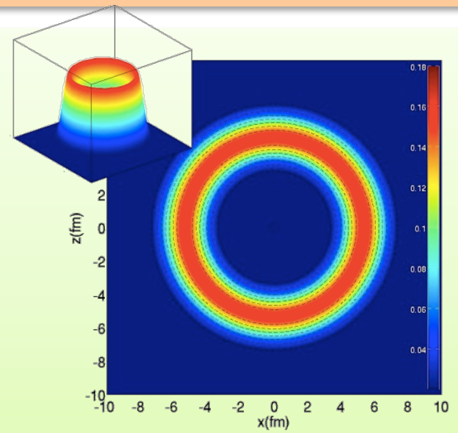
Yannouleas and Landman, Rep.Prog.Phys. 70, 2067 (2007)

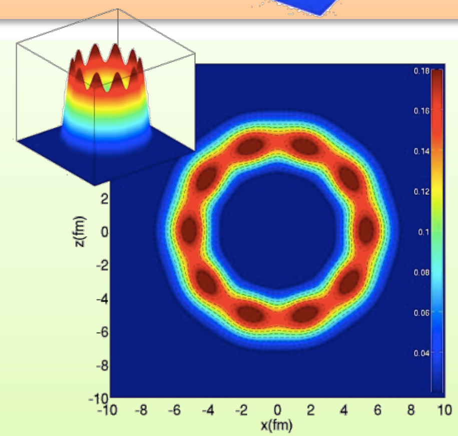
➲ Neutral bosons in rotating trap



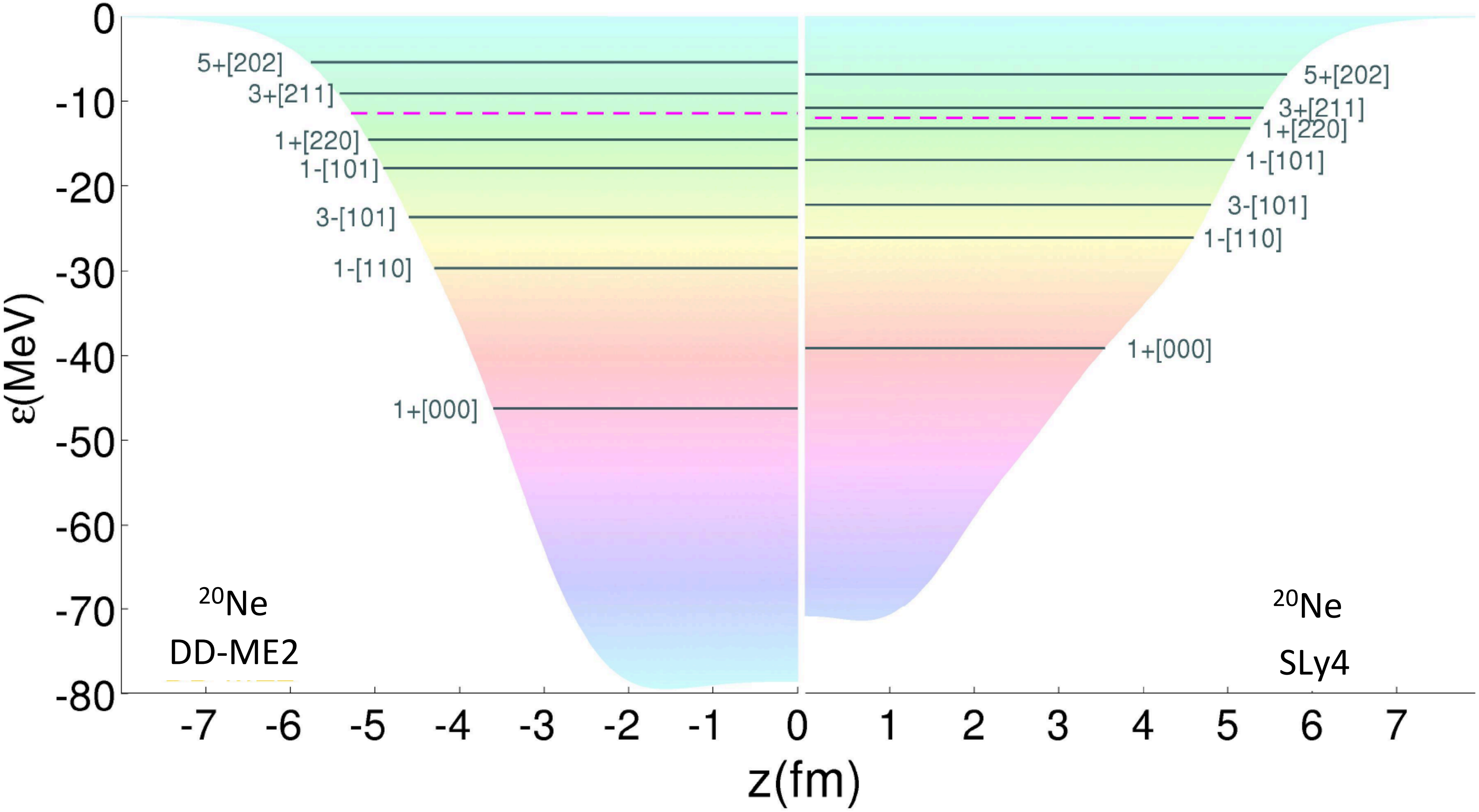


➲ Nucleons in ^{40}Ca

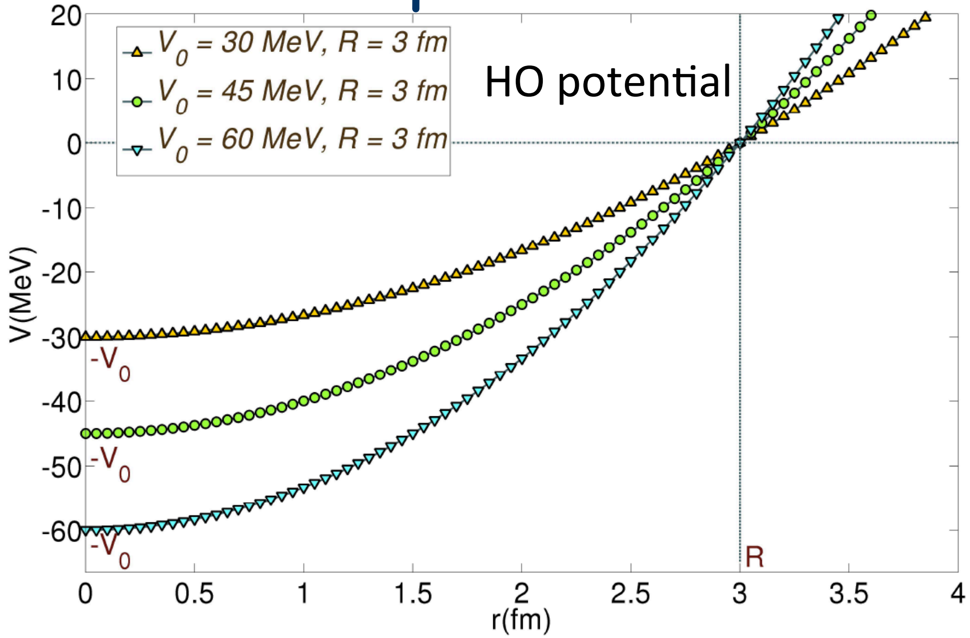




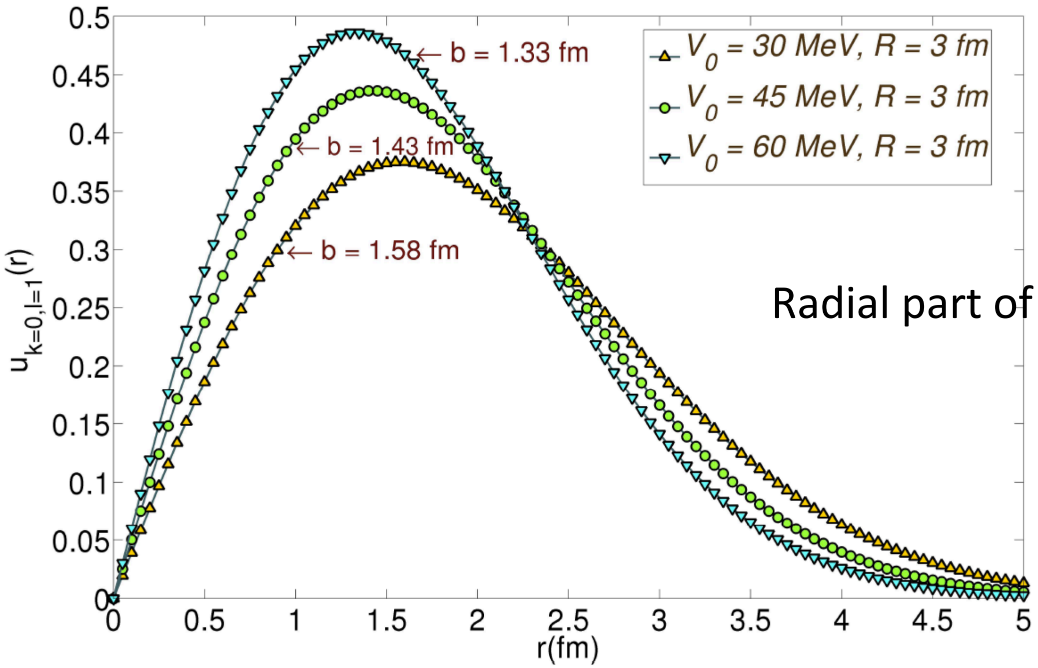
A way to vary the depth of the potential



Interpretation



HO potential



Radial part of the first p-state

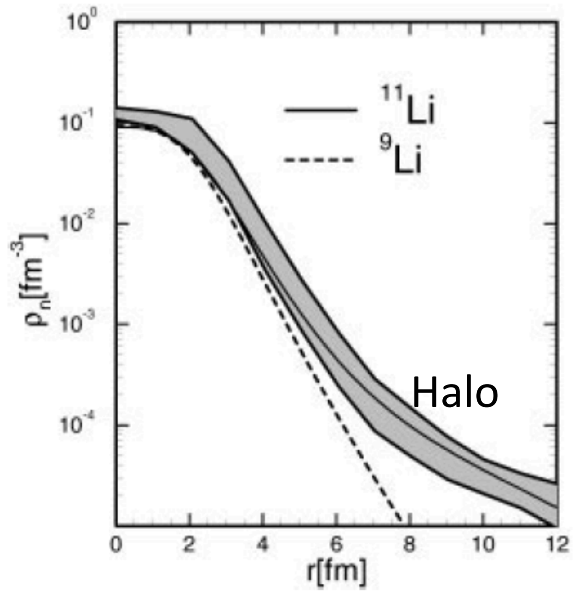
Haloes and clusters

- Halo: **binding energy** impacts spatial behavior **outside** the potential

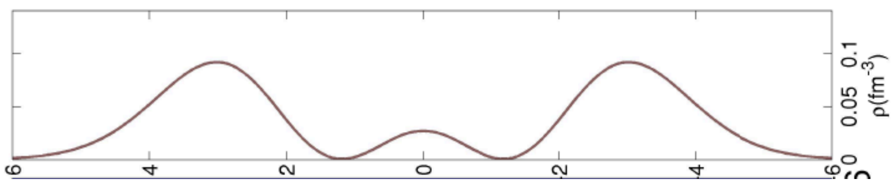
Hansen and Jonson, Eur. Phys. Lett. 4(1987)409

- Cluster: **depth of the potential** impacts spatial behavior **inside** the potential

Ebran, Khan, Niksic, Vretenar, Nature 487 (2012) 341

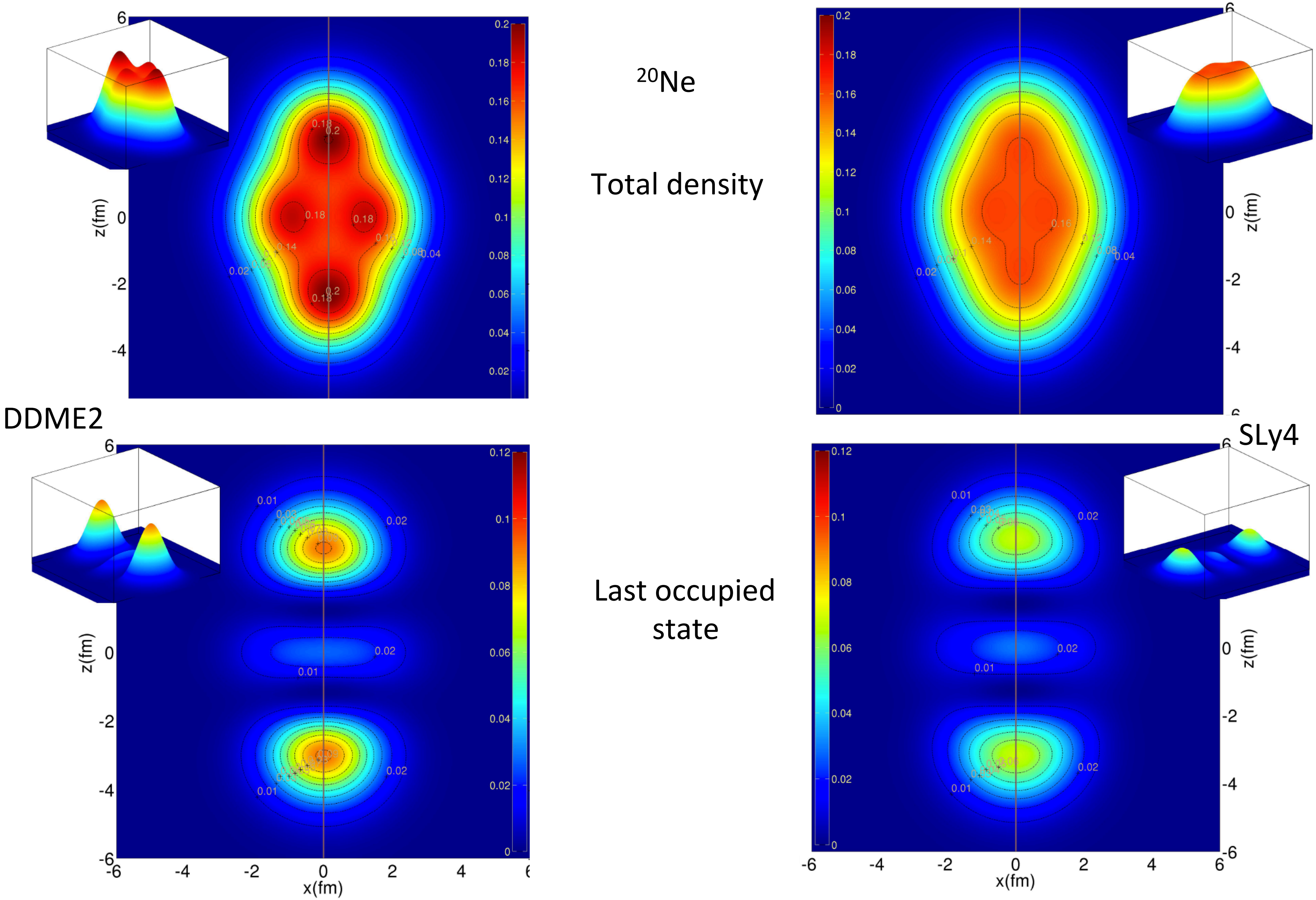


Meng, Ring, PRL77(1996)3963

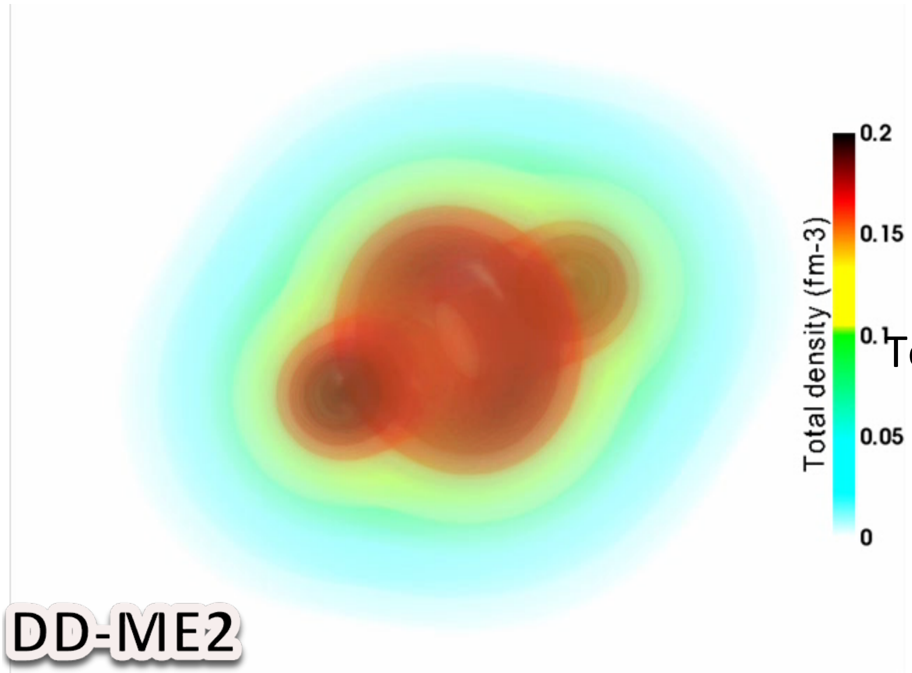


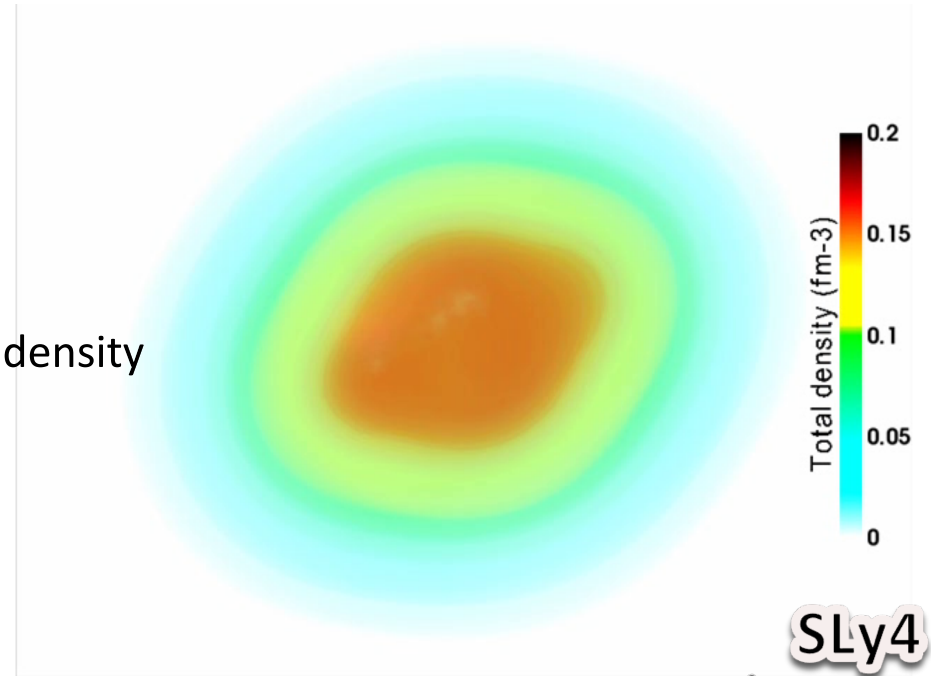
Cluster

Effect on clusterisation

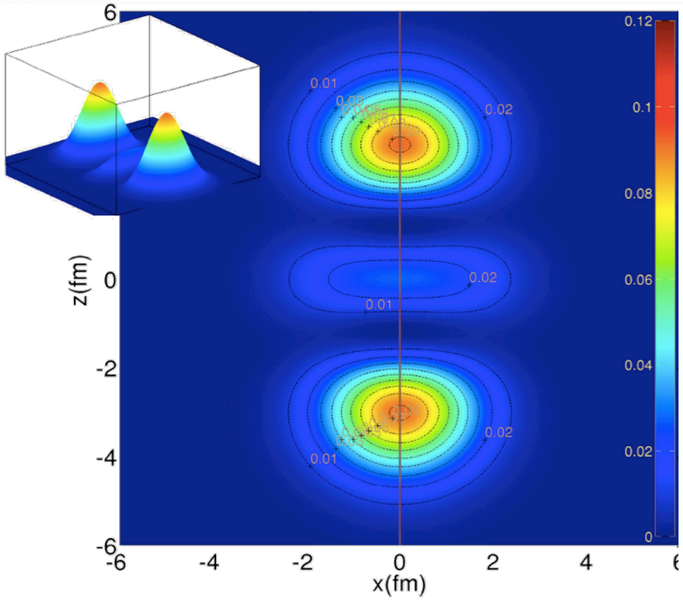


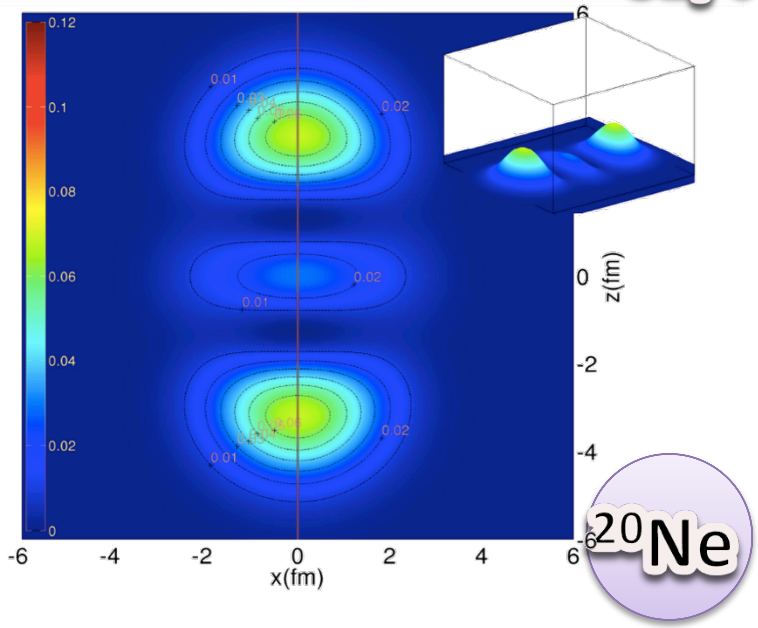
Role of the confining potential: localisation





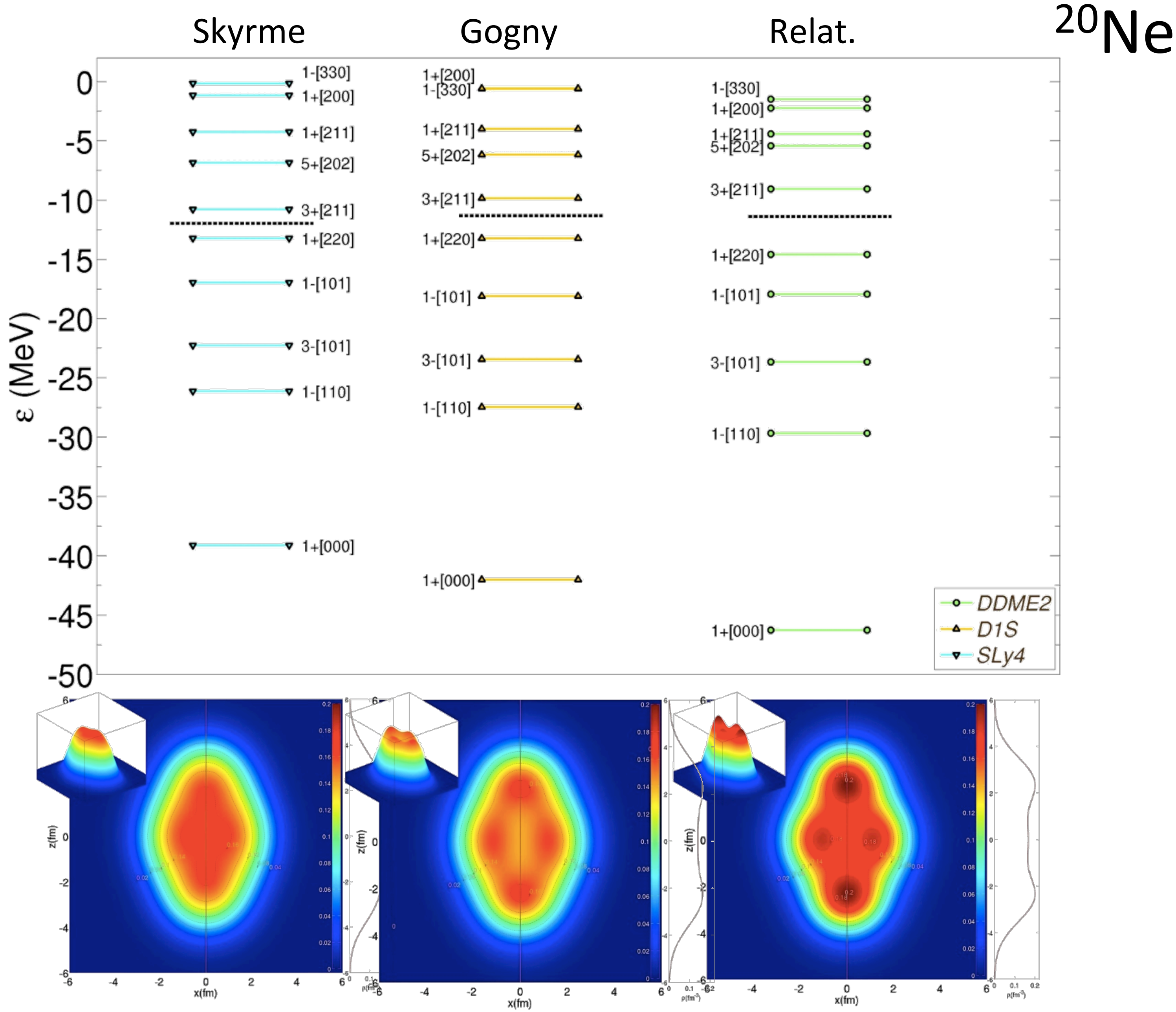
Total density



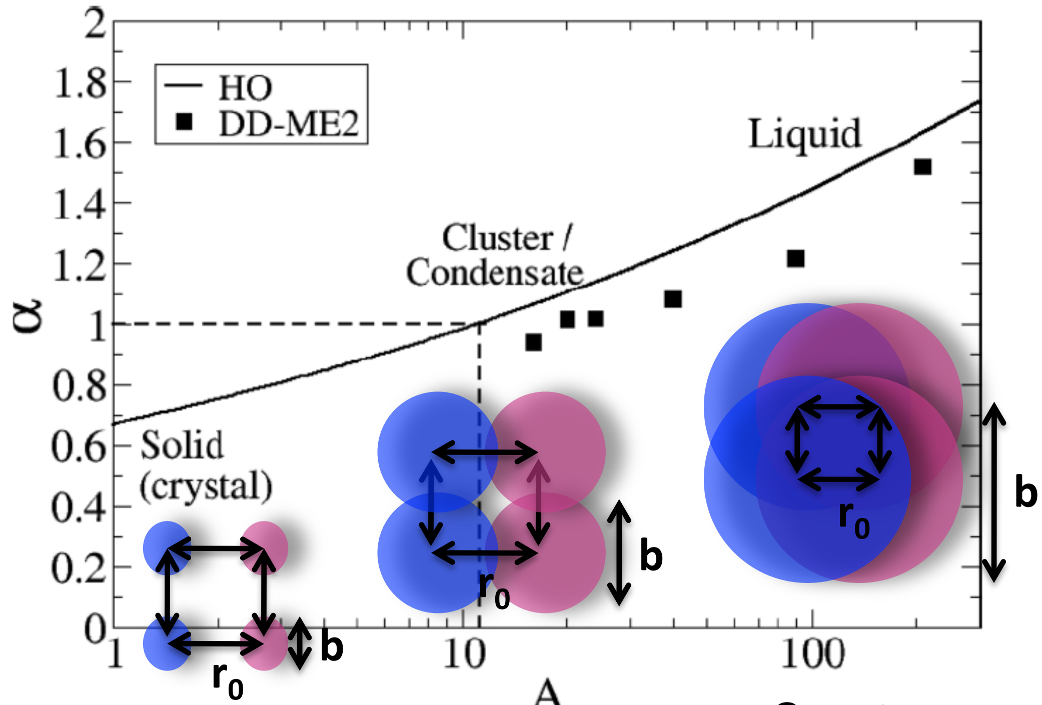


Last occupied state

Deeper potential leads to localisation



Localisation



Crystal
α_{loc} <1

Cluster
α_{loc} ~ 1

Quantum liquid
α_{loc} > 1

J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, PRC 87(2013)044307

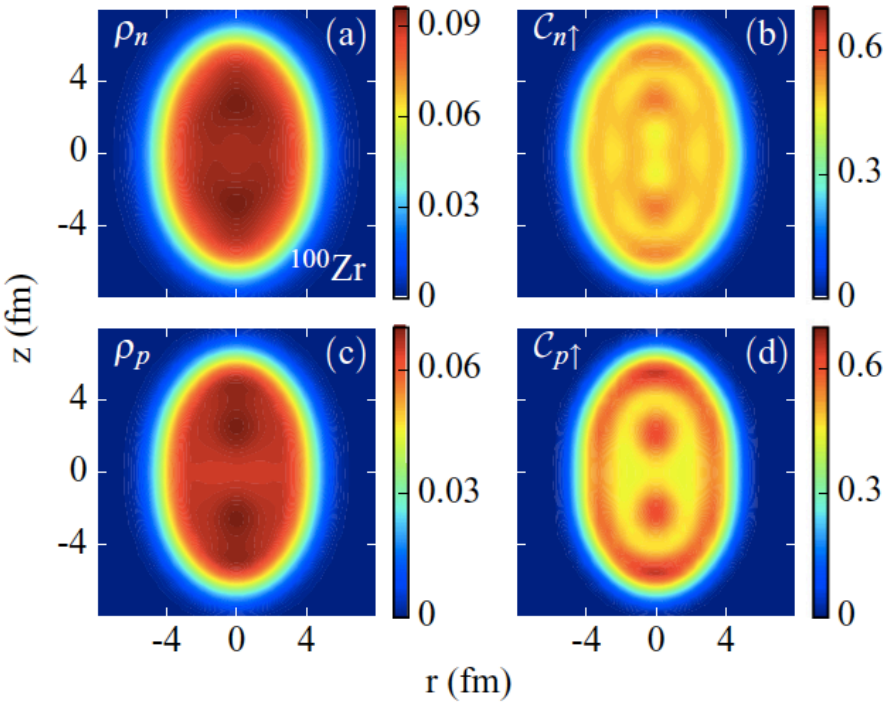
$$\alpha_{\rm loc} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0} = \frac{\sqrt{\hbar}A^{1/6}}{(2mV_0r_0^2)^{1/4}}$$

Single particle state dependence of the localisation ?

Localisation function

$$\mathcal{C}_{q\sigma}(\boldsymbol{r}) = \left[1 + \left(\frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}|\nabla\rho_{q\sigma}|^2 - \boldsymbol{j}_{q\sigma}^2}{\rho_{q\sigma}\tau_{q\sigma}^{\mathrm{TF}}}\right)^2\right]^{-1}$$

P.-G. Reinhard, J. A. Maruhn, A. S. Umar, and V. E. Oberacker, Phys. Rev. C 83, 034312 (2011).



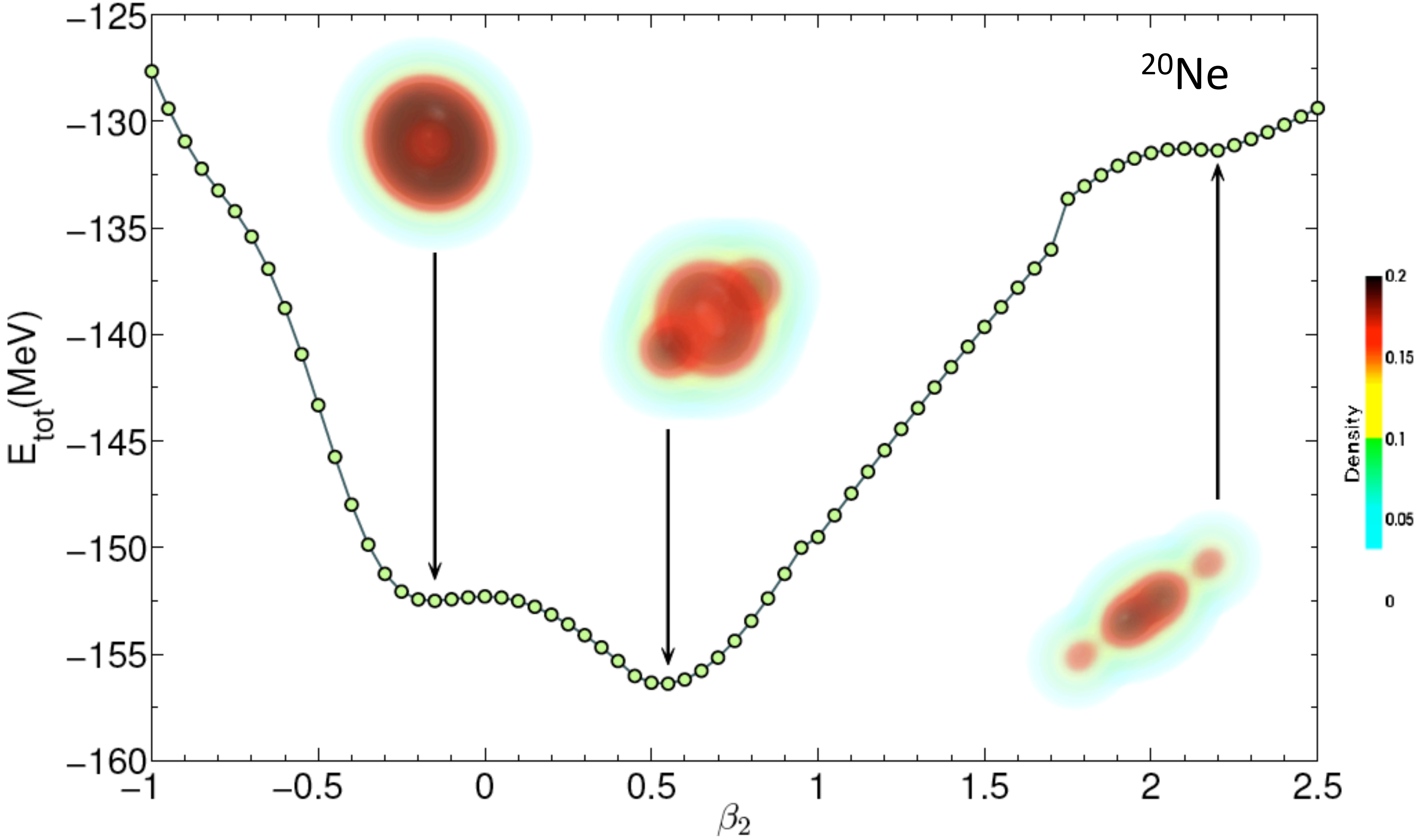
^{100}Zr

C.L. Zhang, B. Schuetrumpf, W. Nazarewicz, Phys. Rev. C 94, 064323 (2016)

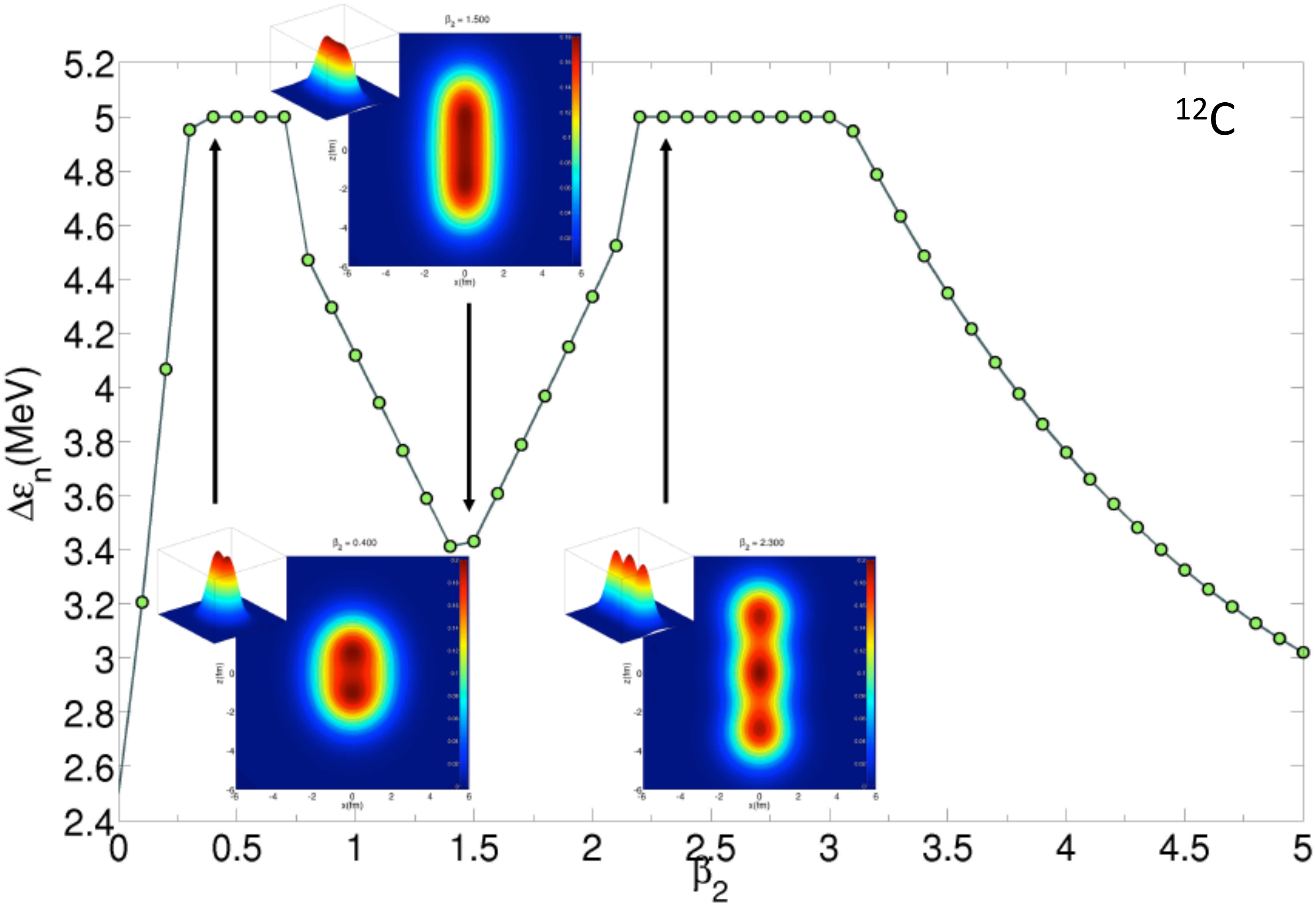
UNEDF1+application to fission and ^{294}Og
PRL 120(2018)053001

Convenient for Skyrme

Effect of deformation & excitation

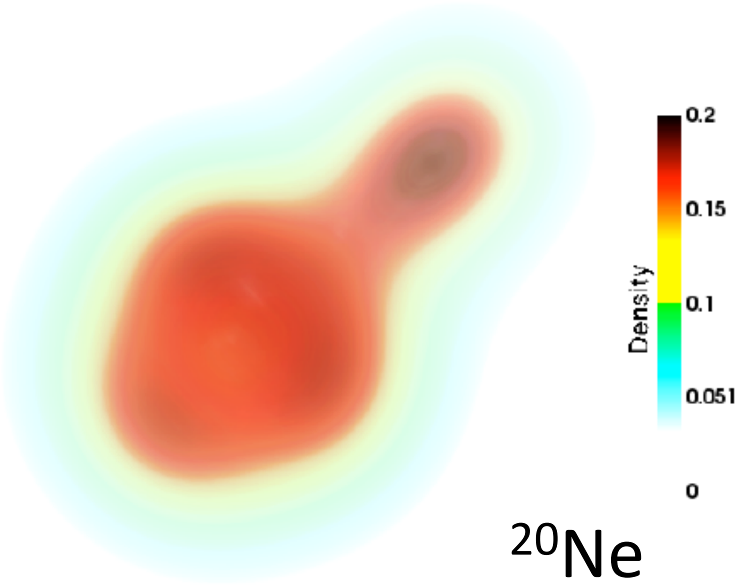


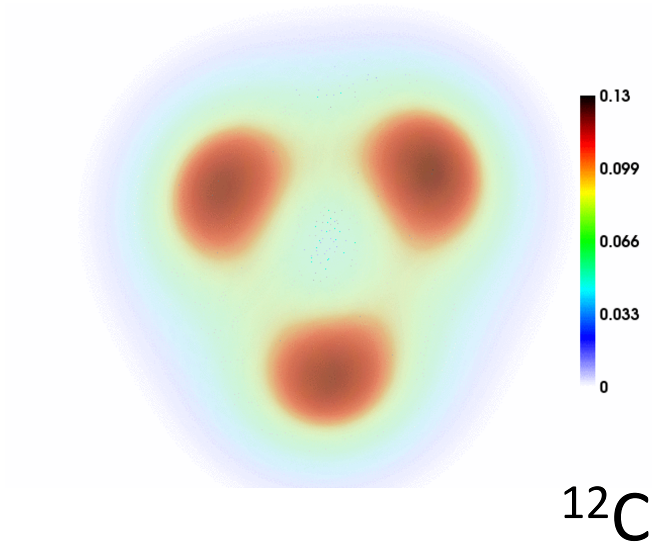
Effect of the deg. raising



J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, PRC 90(2014)054329

Quadrupole + octupole deformations

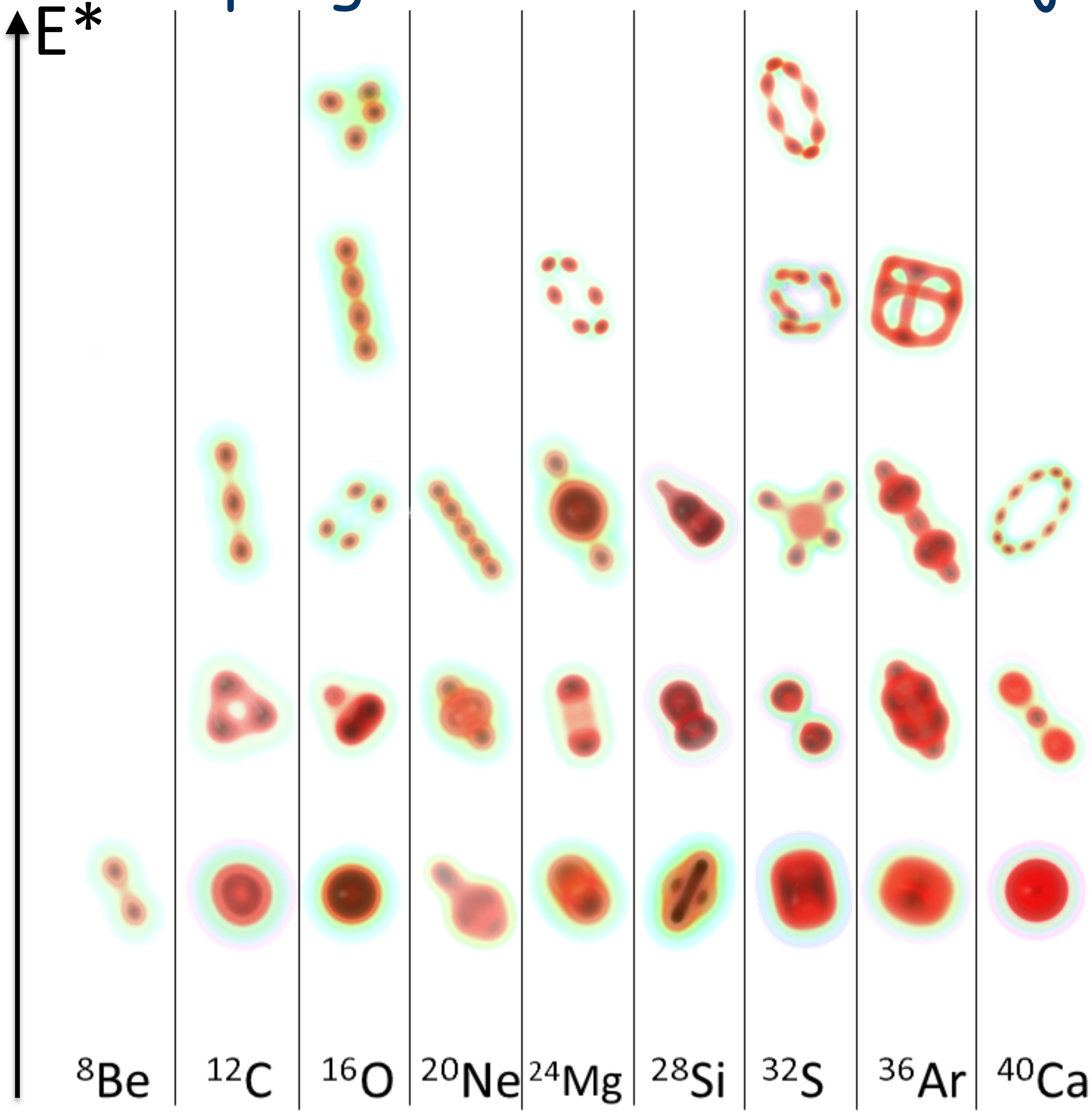




Constrained RHB (DDME2)
β_2, β_3, parity proj.

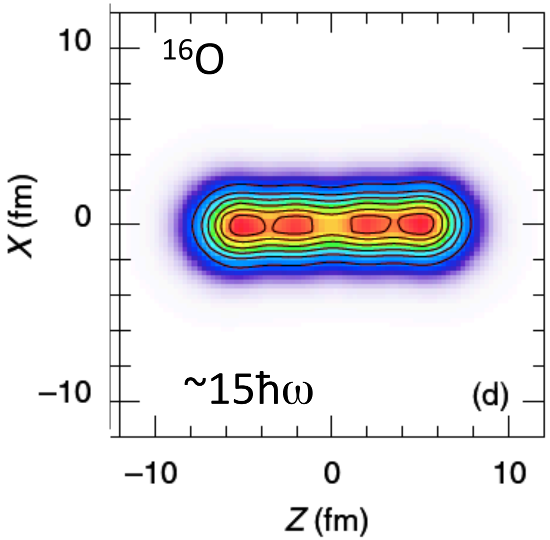
J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, PRC 87(2013)044307

Microscopic grounds to Ikeda's conjecture

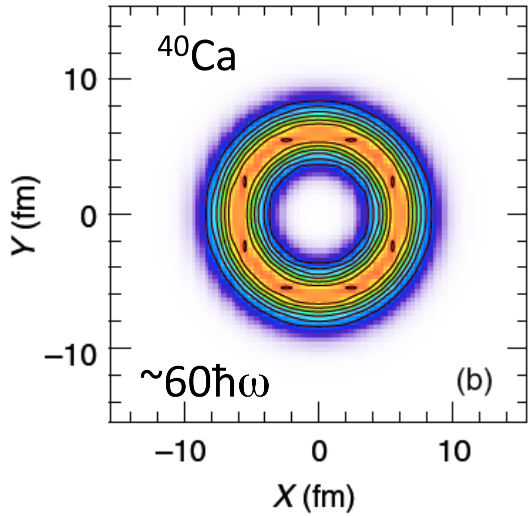


J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, PRC 90(2014)054329

Skyrme-EDF approaches to cluster states



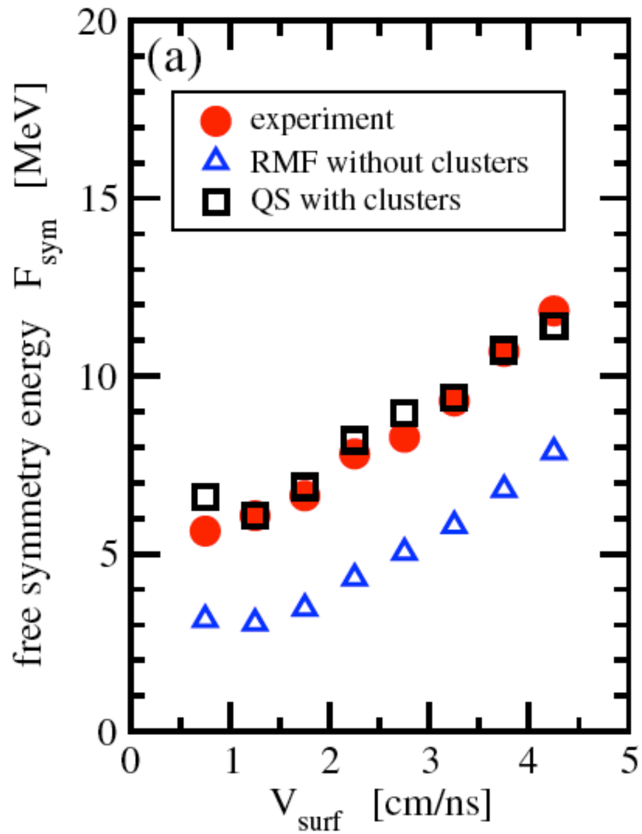
T. Ichikawa et al., PRL107(2011)112501



T. Ichikawa et al., PRL109(2012)232503

Skyrme cranked 3D HF
Stabilisation at high-spin excited states

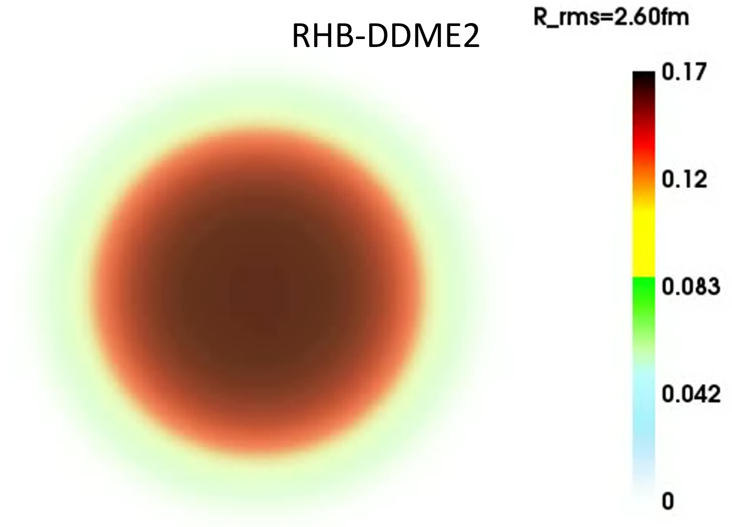
Clusters in low density nuclear matter



J.B. Natowicz et al. PRL104(2010)202501

Clusters in EoS better describe experiment
Data from heavy ion collision

Clusters in low density nuclear matter

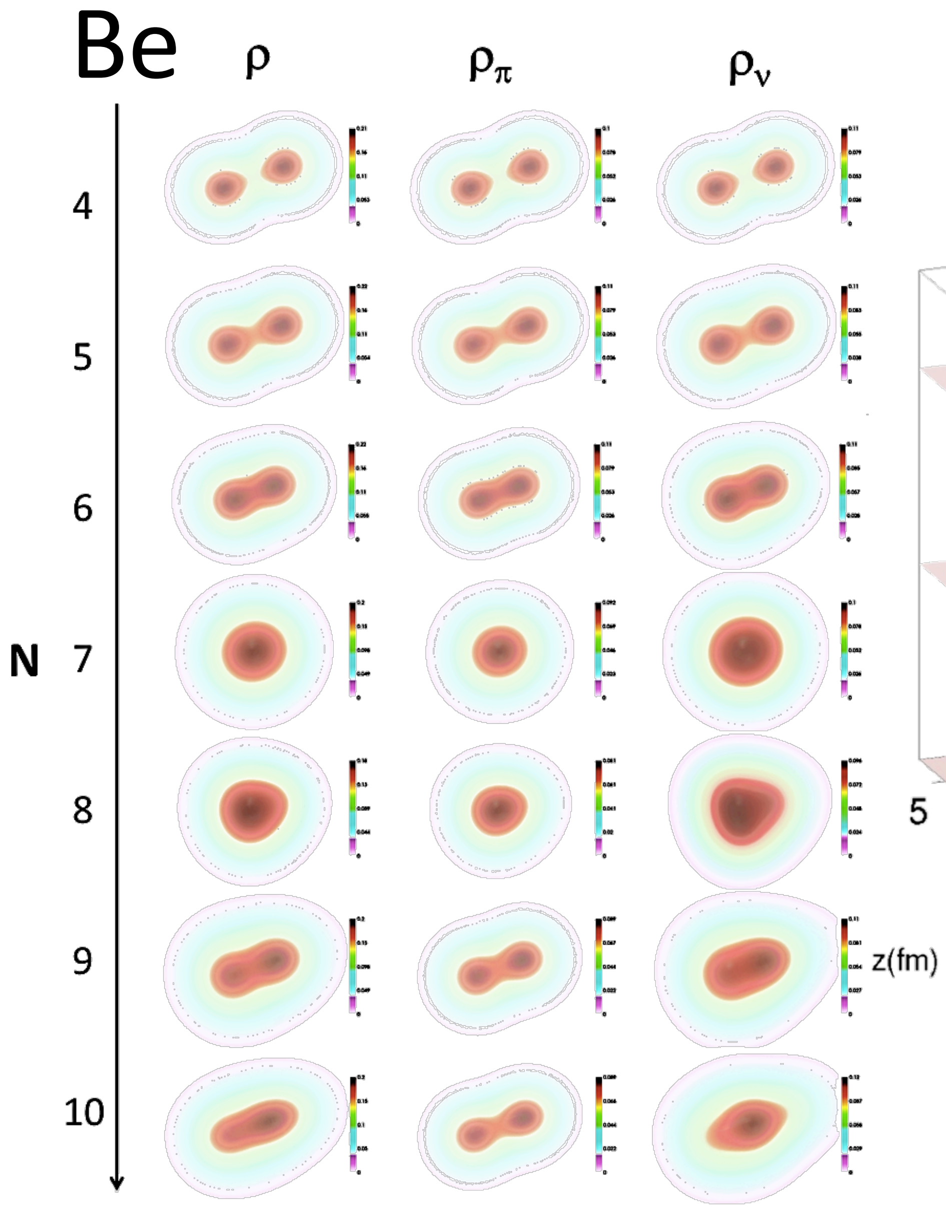


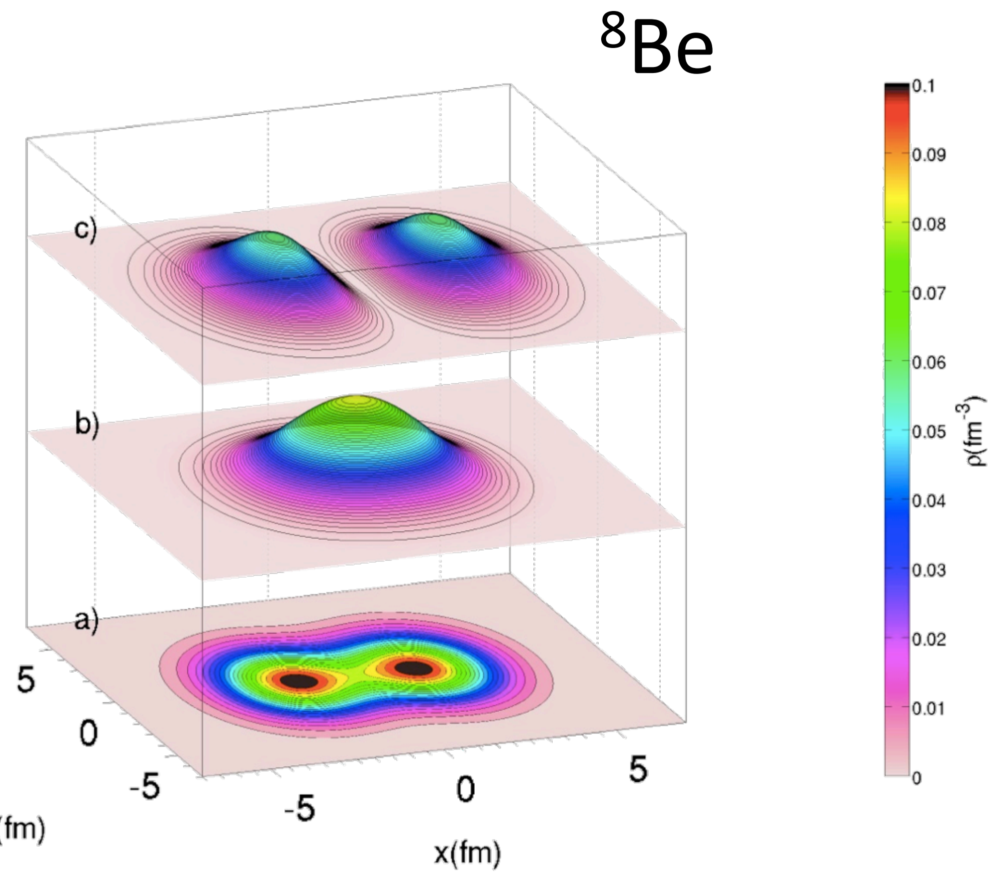
Dilution of ^{16}O

See also: P. Schuck and M. Girod PRL 111 (2013) 132503

Exp: see B. Borderie et al. PLB 755 (2016) 475

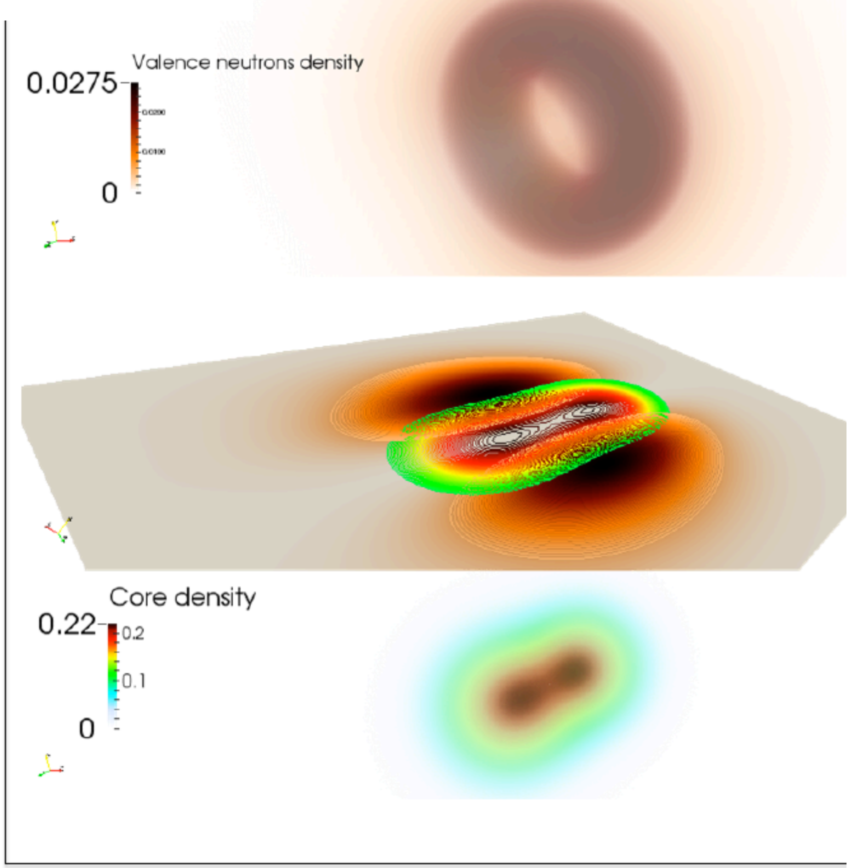
Isotopic dependence





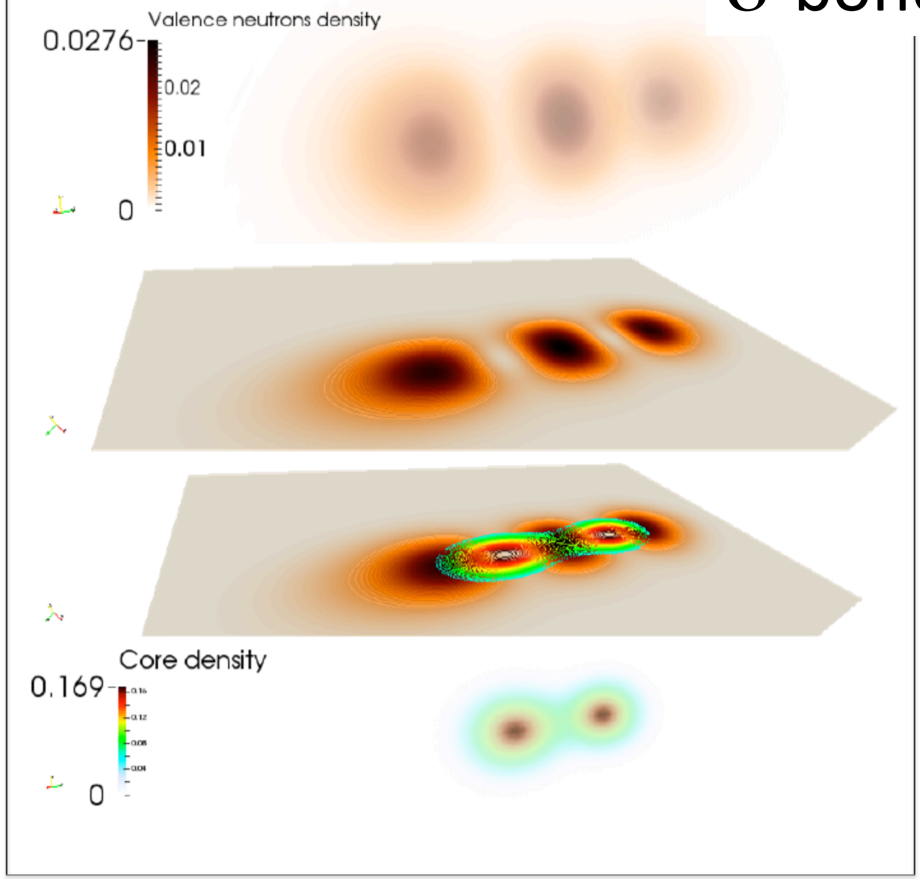
n valence molecular bond

π-bond



^{10}Be g.s.

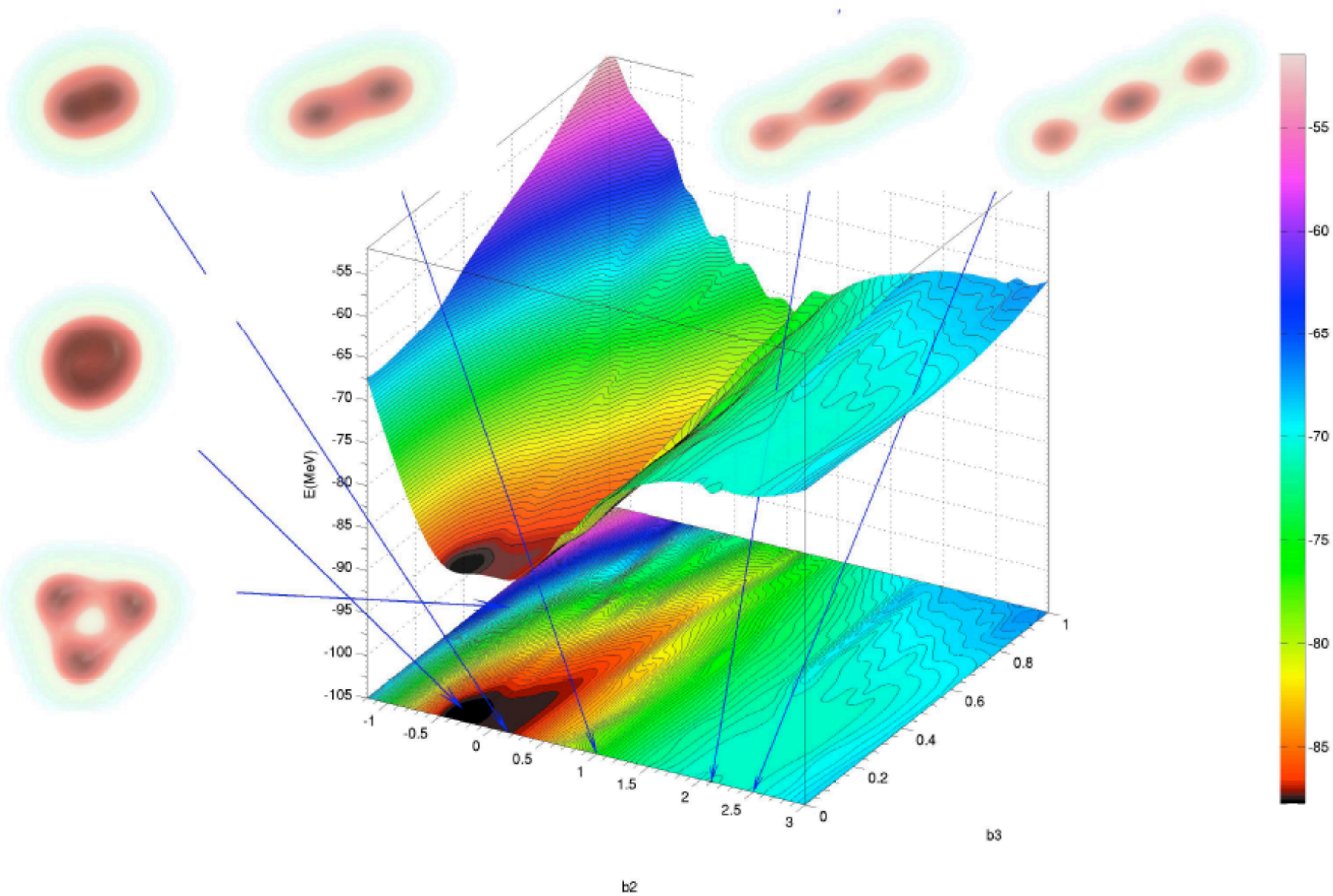
σ-bond



^{10}Be exc.

Comparison with experiment

Parity-projected quadrupole/octupole results



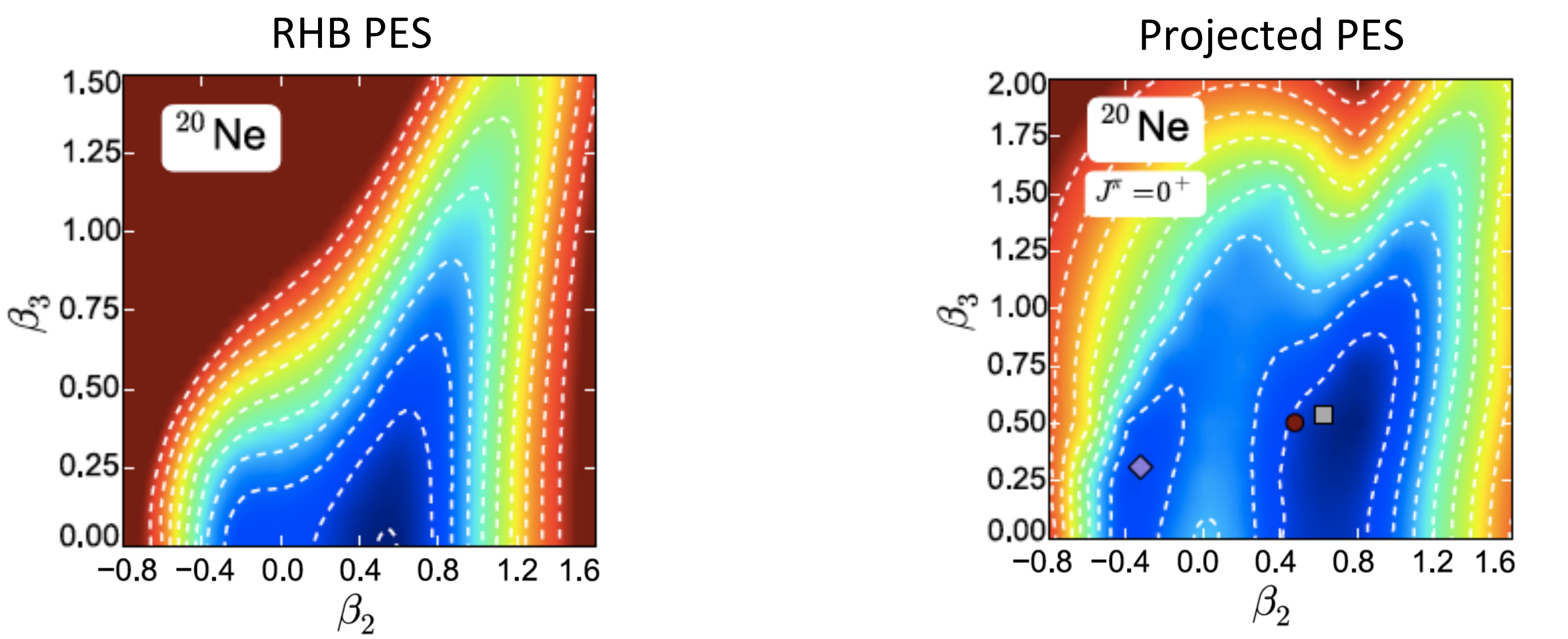
$^{12}\mathrm{C}\ (K^{\pi} = 0^{+})$ PAV

Comparison with exp. on ^{20}Ne

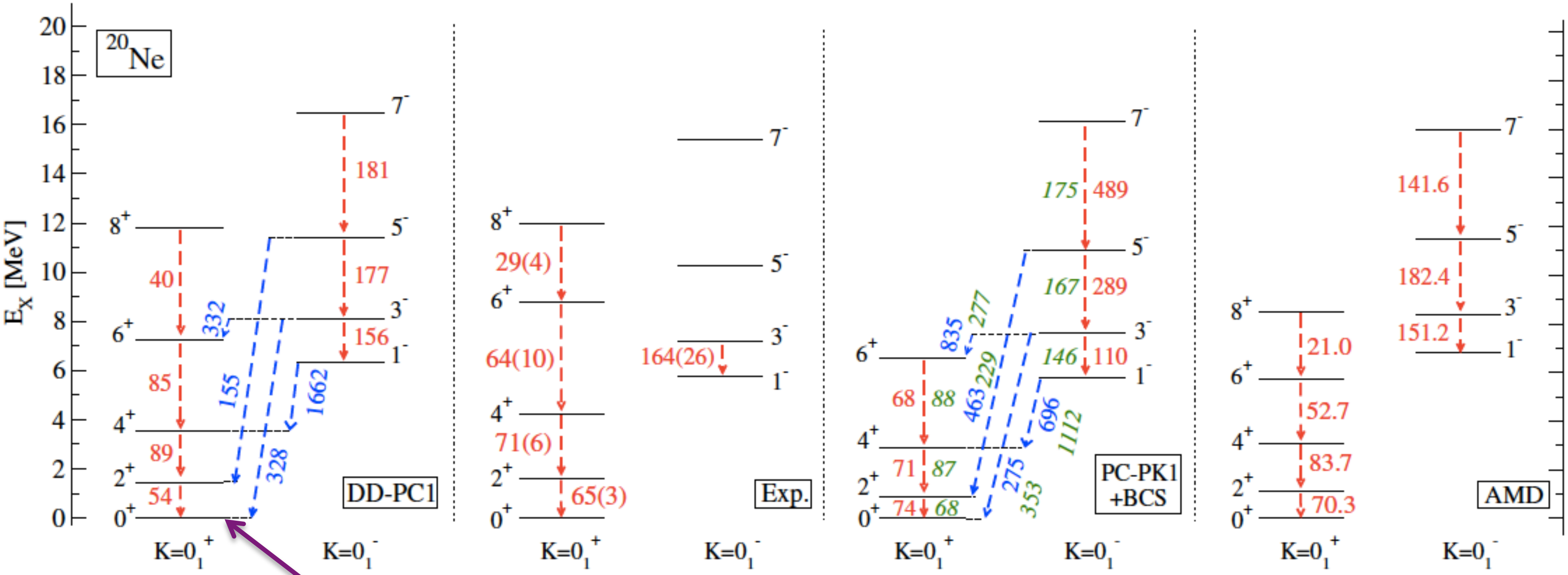
- GCM on top of axially symmetric /reflection asymmetric RHB (DD-PC1) :

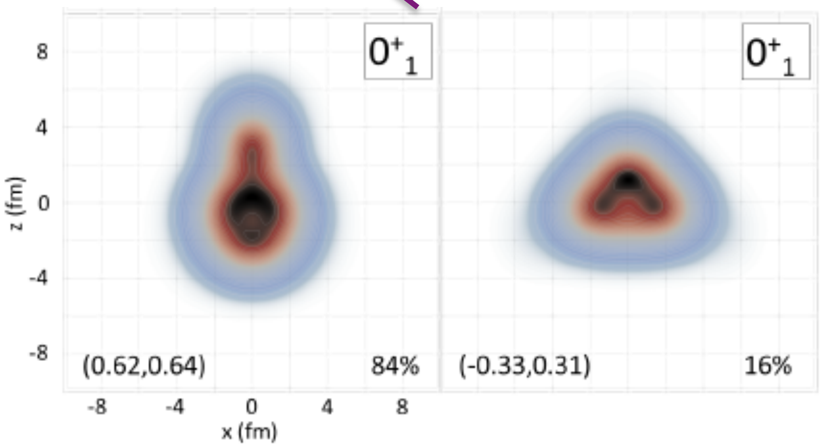
$$|JM\pi;\alpha\rangle = \sum_j \sum_K f_\alpha^{JK\pi}(q_j)\hat{P}^J_{MK}\hat{P}^\pi\,|\phi(q_j)\rangle$$

- Angular momentum, parity and particle number projections



Comparison with the data

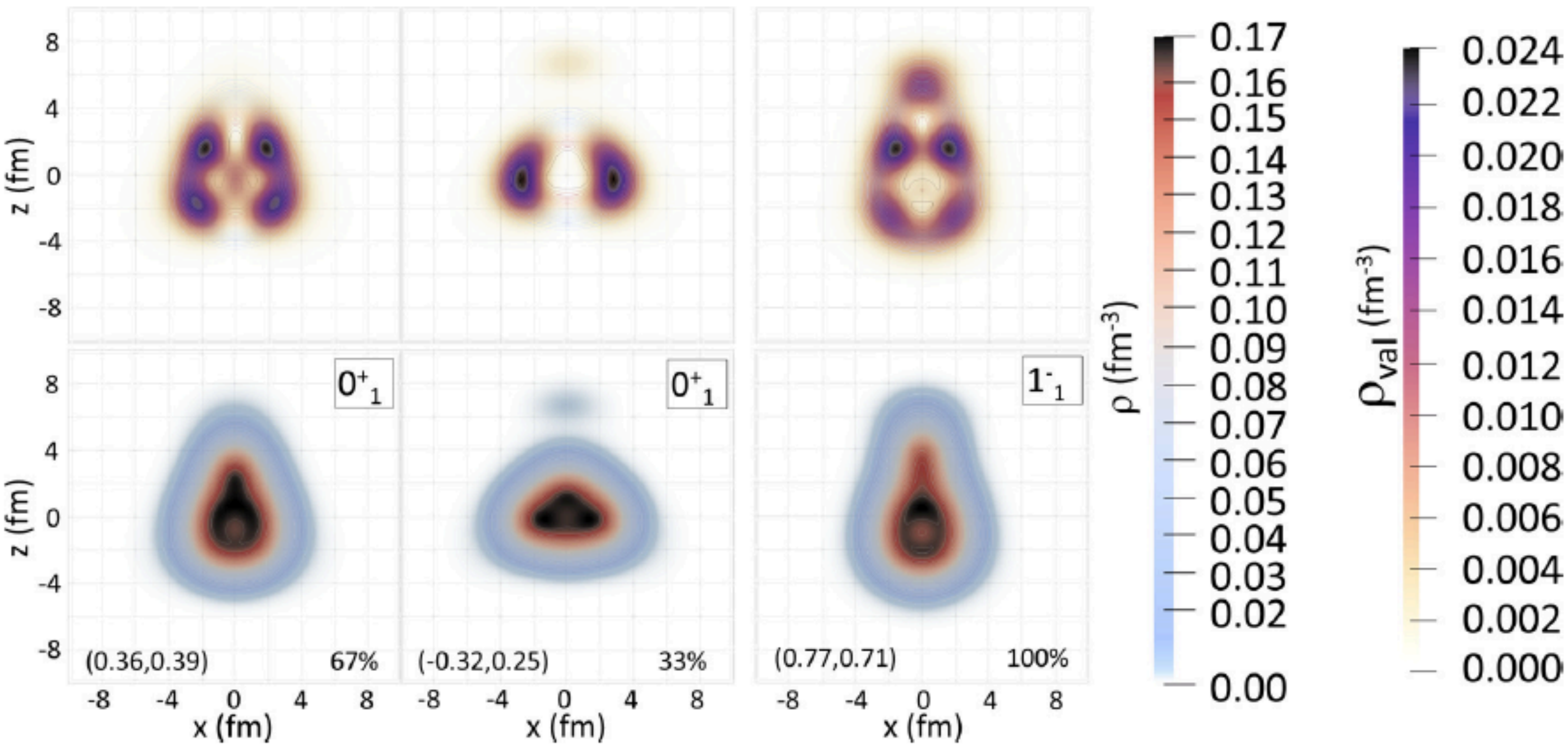




P. Marevic *, J.-P. Ebran, E. Khan, T. Niksic , and D. Vretenar, PRC 97, 024334 (2018)*

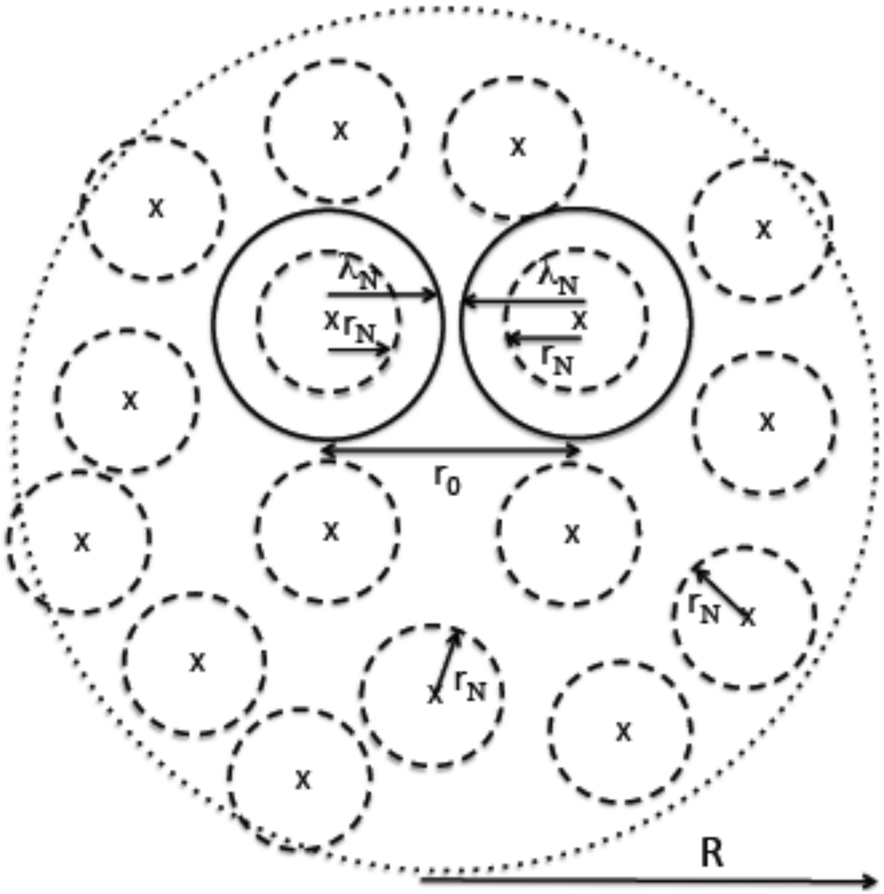
Analysis of the densities

^{24}Ne



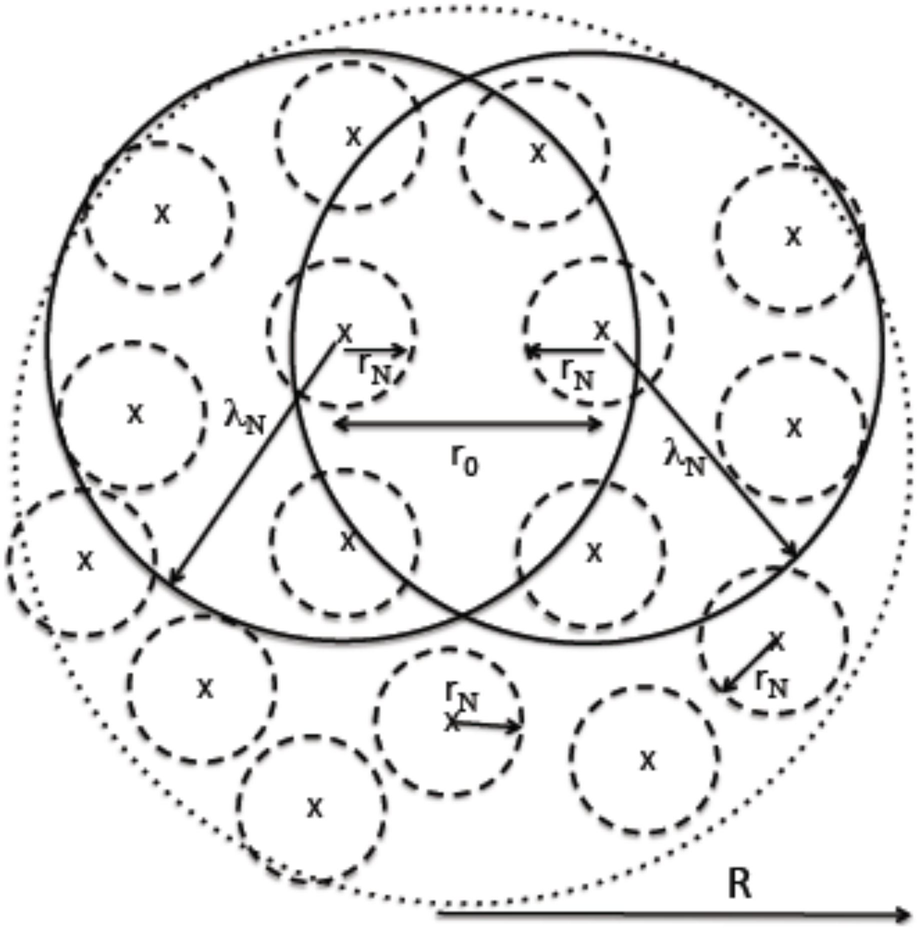
Localisation

Localisation



Localised (crystal)

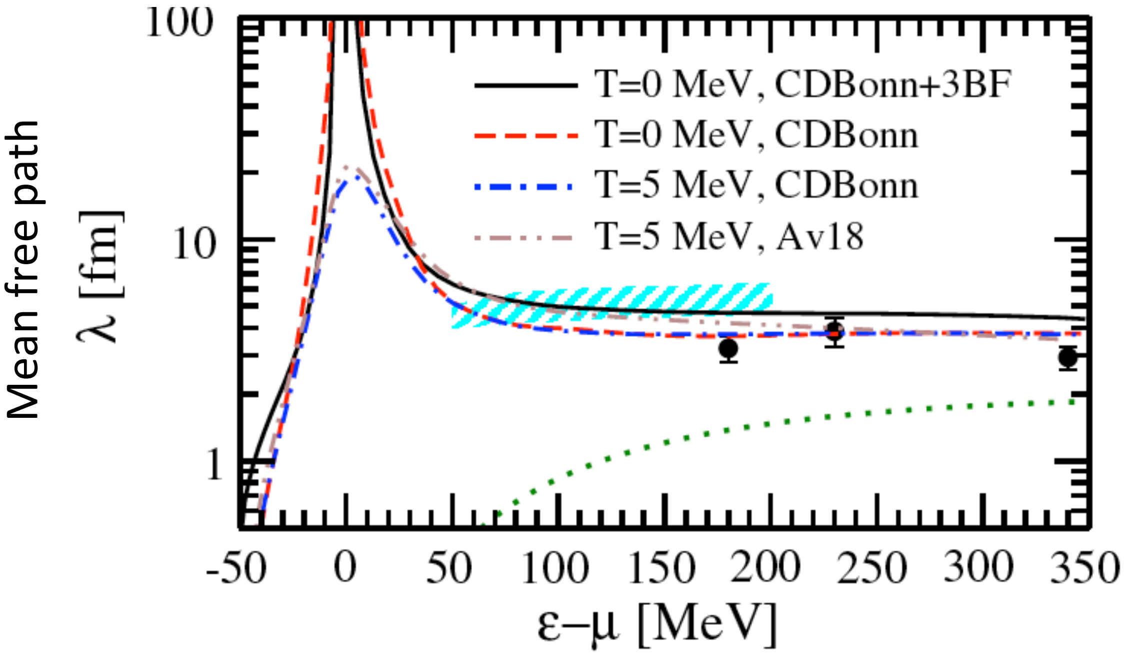
$\lambda_N < r_0$



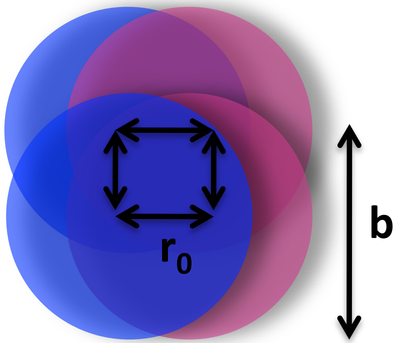
Delocalised (quantum liquid)

$\lambda_N > r_0$

Nuclei: a quantum liquid feature

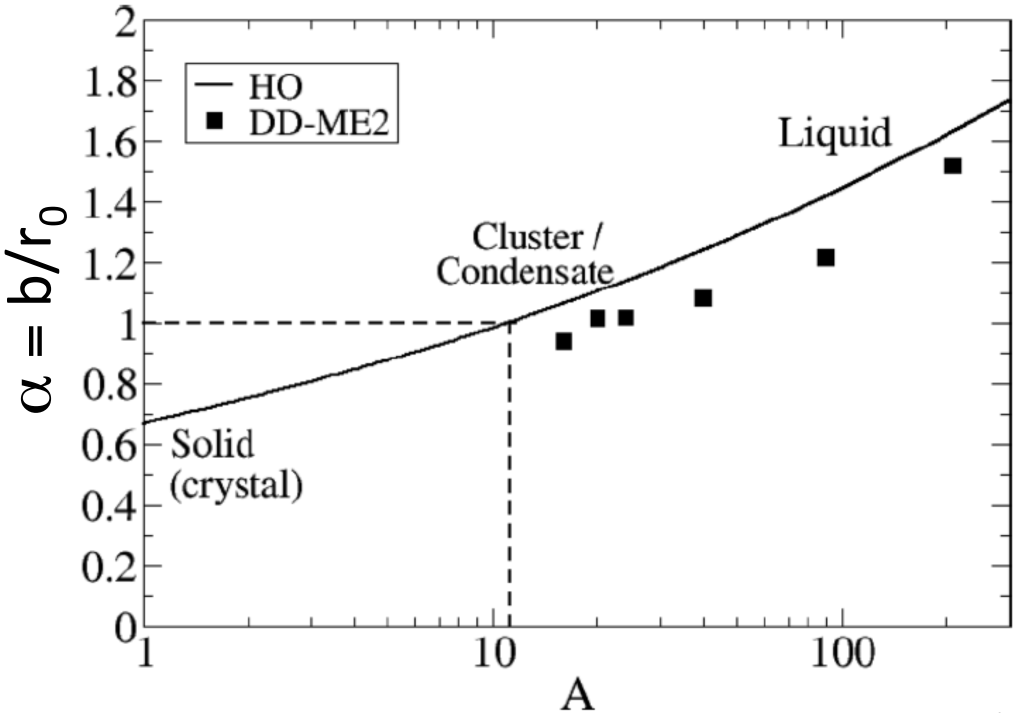


A. Rios & V. Soma PRL108(2012)012501



B. Mottelson ⇒ the concept of independent particle motion is based on the fact that the orbits of individual nucleons are delocalized and reflect the shape and radial dependence of the effective potential over the entire nucleus!

From a nuclear crystal to a nuclear liquid



J.-P. Ebran et al., PRC87(2013)044307

B. Mottelson: quantality Λ

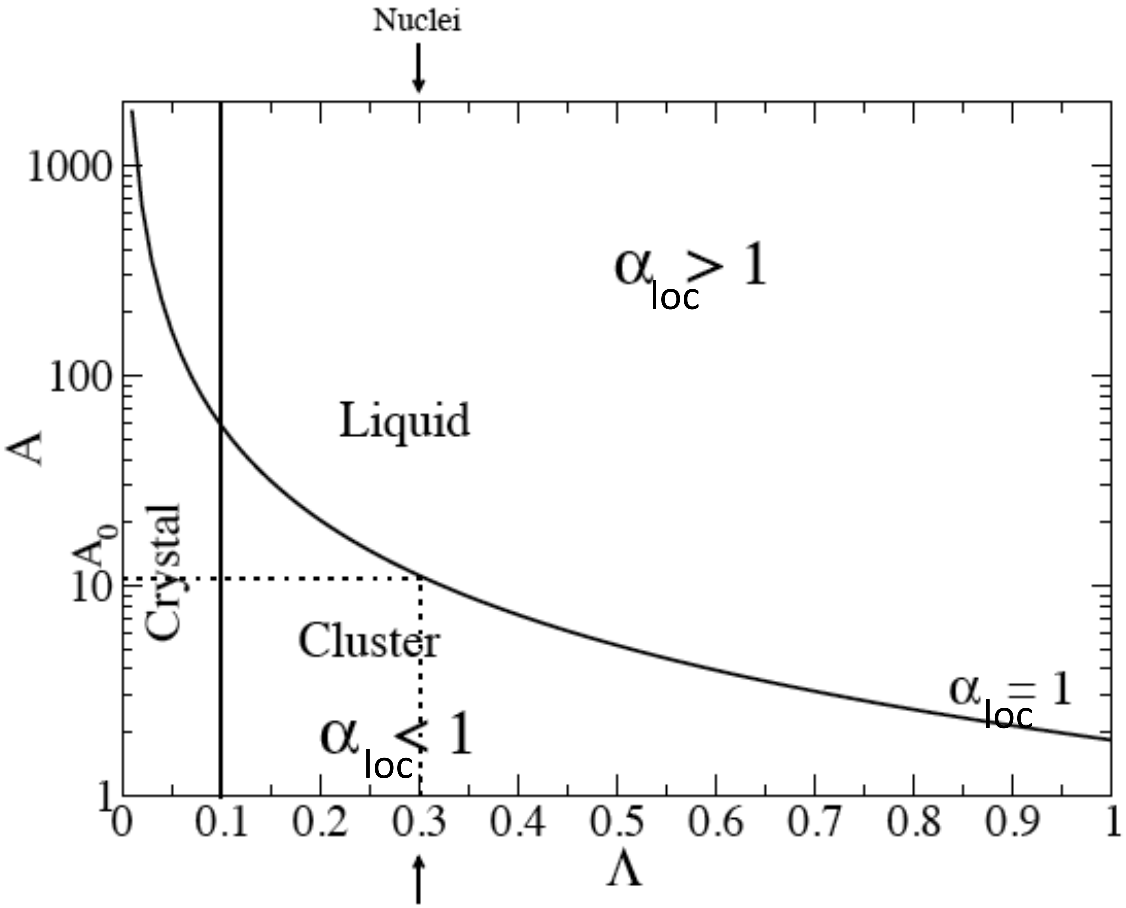
$$\Lambda \hat{=} \frac{\hbar^2}{m\bar{r}^2 V_0'}$$

$$= \frac{\text{zero-point kinetic energy of the confined particle}}{\text{potential energy}}$$

In finite nuclei: localisation α

$$\alpha \hat{=} \frac{b}{r_0} = \frac{\sqrt{\hbar R}}{r_0 (2mV_0)^{1/4}}$$

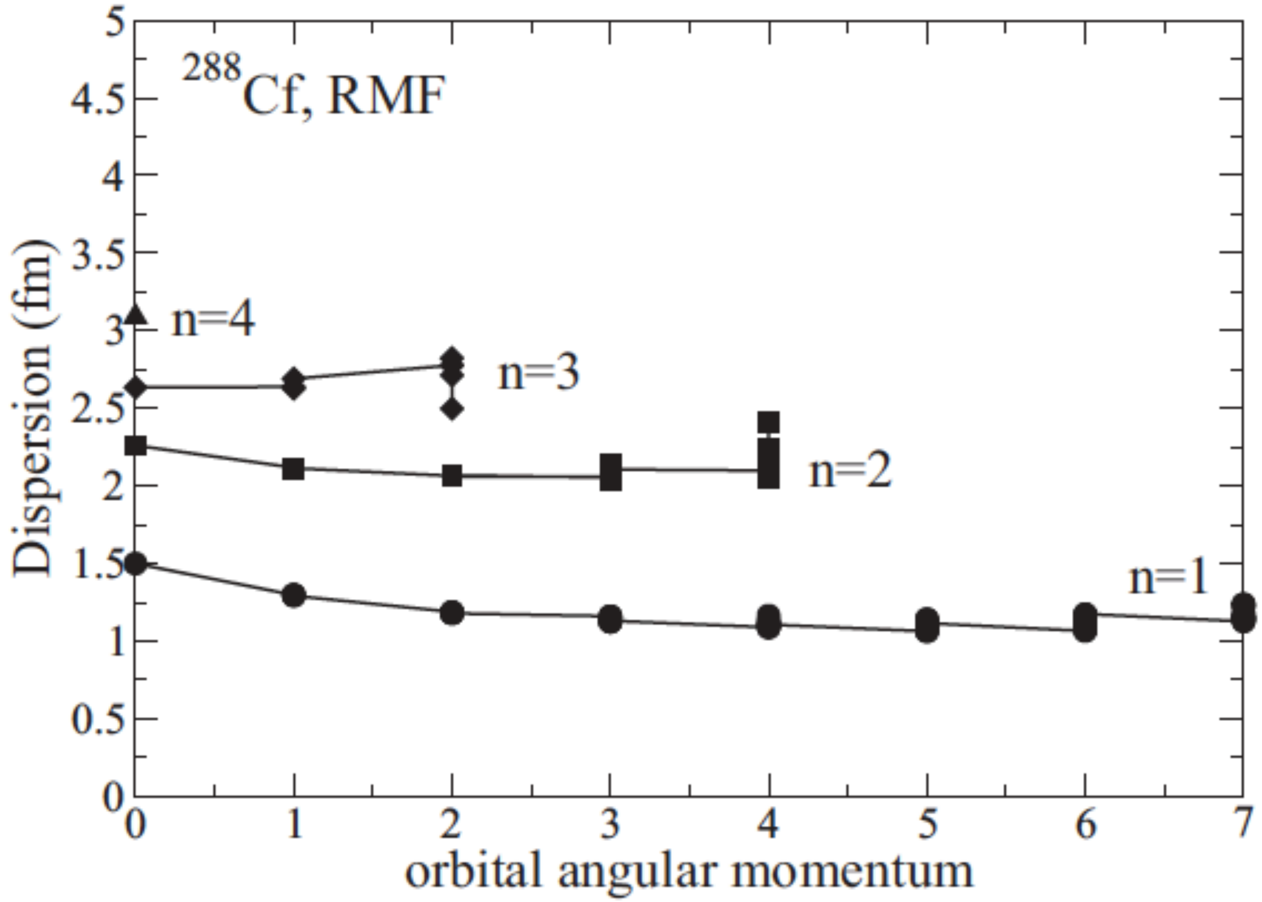
Saturation



Saturation ⟶ Light nuclei: confining potential vs. Quantum liquid delocalisation from the interaction

J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, PRC 89(2014)031303(R)

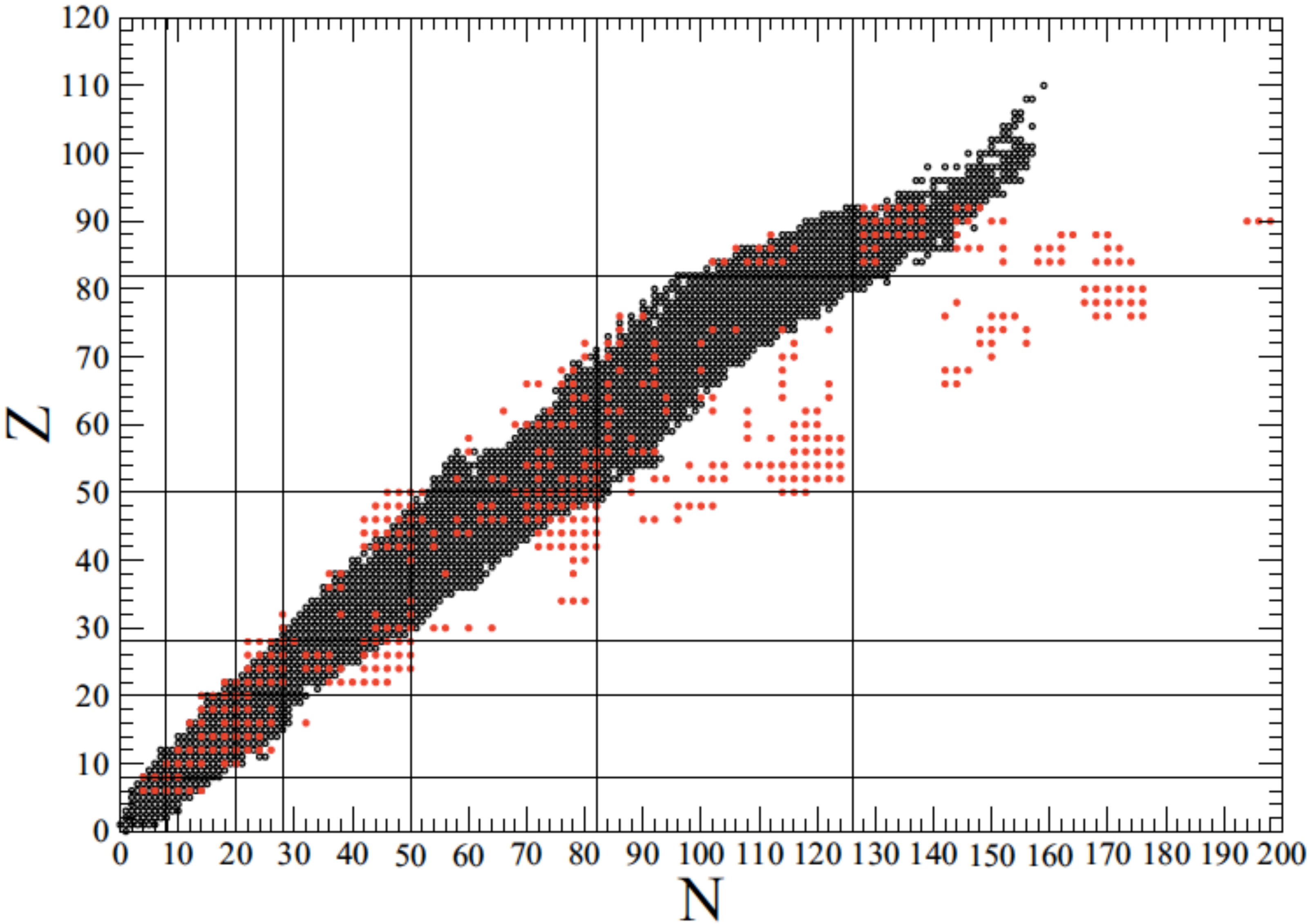
Dispersion in nuclei: a striking pattern



$$\alpha_{loc} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0}\sqrt{2n-1} = \frac{\sqrt{\hbar(2n-1)}}{\left(2mV_0r_0^2\right)^{1/4}}A^{1/6}$$

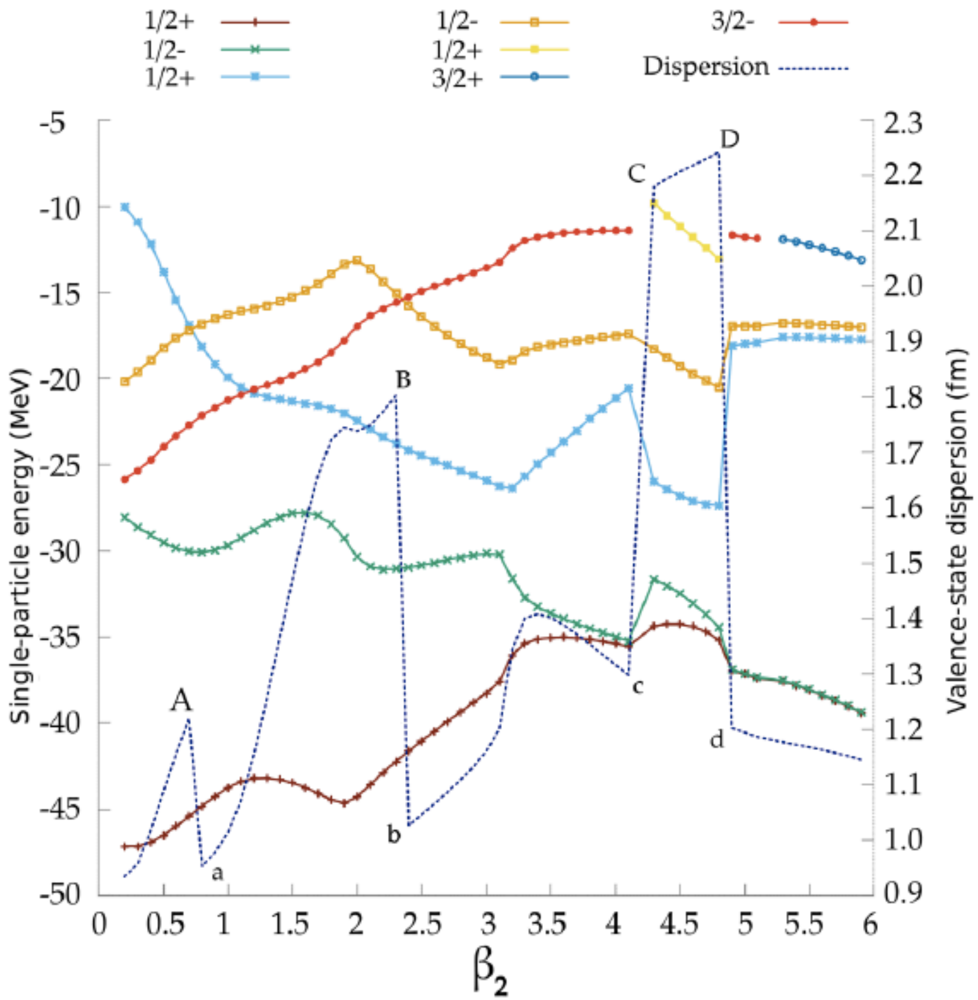
J.-P. Ebran, E. Khan, R.-D. Lasseri, and D. Vretenar. PRC 97, 061301(R) (2018))

α-valence localisation over the nuclear chart

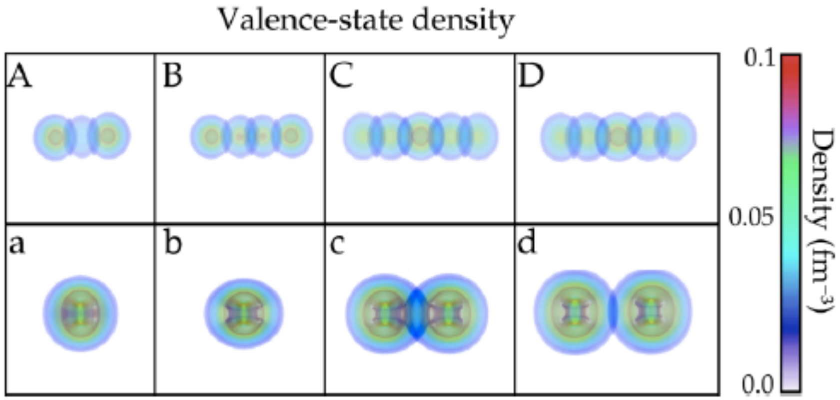


Axially symmetric RHB DD-ME2 calc.

Dispersion and s.p. states



^{20}Ne



Before concluding

« The nature of the transition from independent-particle motion to the crystalline state and the associated value of the characteristic parameter present significant unsolved problems »

Bohr & Mottelson Vol I

⟶ Clusters in nuclei