A symmetry-based semimicroscopic no-core approach to quarteting and clustering

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Introduction and summary

Semimicroscopic approach:
Microscopic model space + phenom. operators.

Symmetry-based: group-symm.: basis+operat.

Quartet model
Cluster model
Quadr. deform.
Unifying symmetry

I. Historical background: $SU^{ST}(4)$, $SU(3)$

II. Quartets

III. Clusters

IV. Quadrupole deformation

V. Unifying symmetry

VI. Conclusion

I. $SU^{ST}(4)$

Supermultiplet-theory

E.P. Wigner, Phys. Rev. 51, 106 (1937).

Unified description of the spin and isospin degrees of freedom.

I. SU(3)

J.P. Elliot, Proc. Roy. Soc. A245, 128; 562 (1958).

SU(3) shell model.
In fact $U^{ST}(4) \times SU(3)$ shell model.
Harmonic oscillator + QQ interaction.

Description of the spectra of light nuclei.
Shell picture of the rotation and deformation;
SU(3) symm. and quadr.def. are uniq. related.

Further SU(3) 1958

From shell model to cluster model:
Wildermuth-Kanellopoulos *Nucl. Phys. 7, 150 (1958)*
Harm. osc. appr. $H_{SM} = H_{CM}$
Bayman-Bohr: SU(3); *Nucl. Phys. 9, 596 (1958/59).*

A quadrupole collective or a cluster band is picked up from the spherical shell model basis by their special SU(3) symmetry.

Cluster-shell duality:

The GS of ^{16}O, ^{20}Ne etc. is a pure SM configuration and at the same time a pure cluster configuration.

Recent discoveries:

1. Other states (SD, HD,…) as well.
2. Rediscovery in a different formalism.
   (Renaissance of the shell-like clustering.)

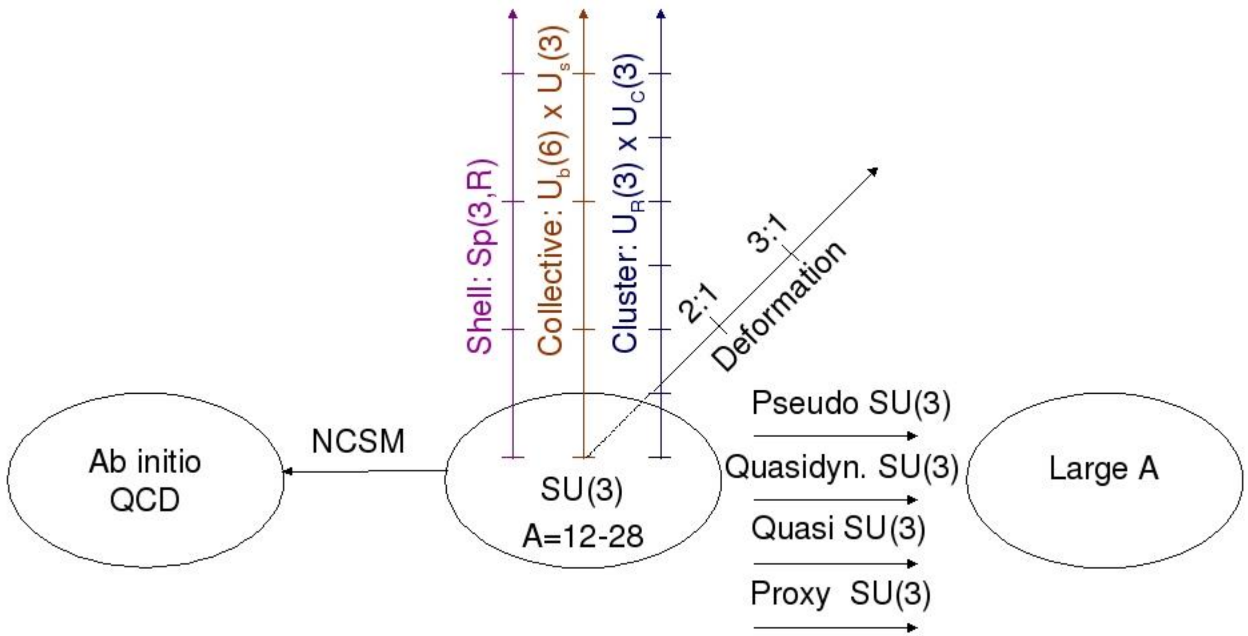
For a single major-shell problem the connection between the shell, collective and cluster models is provided by an SU(3) dynamical symmetry:

$$U(3) > SU(3) > SO(3)$$

Soon afterwords

The SU(3) symmetry breaks down.

Much effort from different aspects.



Shell: Sp(3,R)
Collective: $U_b(6) \times U_s(3)$
Cluster: $U_R(3) \times U_C(3)$
2:1
3:1
Deformation
Ab initio QCD
NCSM
SU(3)
A=12-28
Pseudo SU(3)
Quasidyn. SU(3)
Quasi SU(3)
Proxy SU(3)
Large A

Generalizations

- Many major shells
- No-core
- Mass number
- Large deformation
- Building blocks: pairs, quartets

II. Quartets

Importance of quarteting:

- short-range attractive nucl.-nucl. force: occupy the same single-particle orbital.
- Pauli-principle: 2p+2n.

Ground state, binding energy.

70-ies: Quartet excitations

Quartet: 2p+2n in a single-particle orb. > excite

A. Arima, V. Gillet, J. Ginocchio,Phys. Rev. Lett. 25 (1970) 1043.

Quartet-symmetry: permut. [4], $U^{ST}(4)$ [1,1,1,1]

M. Harvey, Nucl. Phys. A202 (1973) 191.

2p+2n may sit in different shells;

any number of major shell excitation.

Interacting boson models

J. Dukelsky et al 1982:

U(6) spectrum generation: + parity

Phys. Lett. B115 (1982) 359.

F. Iachello, A.D. Jackson 1982:

Both parities

Phys. Lett. B108 (1982) 151.

Shell-like models: semi-algebraic: only S and T.

Interacting boson models:
Fully algebraic, without shell-model connection.

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Fully algebraic description (including spectrum generation) with well-defined shell content?

Algebraic models for shell-like quarteting

(J. Cseh, Phys. Lett. B743 (2015) 213.)

Quartet: as defined by Arima et al (no internal structure)

Formalism: U(3) as introduced by Elliott,

Phenomenologic Algebraic Quartet Model (PAQM).

Quartet: as defined by Harvey (in terms of nucleons)

Formalism: $U^{ST}(4) \otimes U(3)$

Semimicroscopic Algebraic Quartet Model (SAQM).

Quartet: 2p+2n Wigner-scalar.

No linear combination coefficients.

No condensate.

Algebra-chain:

$$U(3) \supset SU(3) \supset SO(3) \supset SO(2)$$

$$\left|[n_1, n_2, n_3], (\lambda, \mu), K, \quad L, \qquad M\right\rangle$$

1. Complete set of basis states
2. Dynamical symmetry:

   H in terms of invariant operators

   Eigenvalue-problem: analytical solution

$$\hat{H} = (h\omega)\hat{n} + a\hat{C}^{(2)}_{SU3} + b\hat{C}^{(3)}_{SU3} + d\,\frac{1}{2\theta}\hat{L}^2,$$

$$B(E2, I_i \to I_f) = \frac{2I_f + 1}{2I_i + 1}\alpha^2 \left|\left\langle(\lambda,\mu)KI_i, (1,1)2 \parallel (\lambda,\mu)KI_f\right\rangle\right| C^{(2)}(\lambda,\mu)$$

SAQM is an effective model:

1) ^{16}O: positive-parity spectrum: self-consistency argument, no-parameter model.
2) SAQM
3) Symmetry-adapted no-core shell model

   (SA-NCSM) T. Dytrych et al:

   work is in progress to test how far the effective model is in line with the microscopic content of the ab initio model.

III. Clusters

Quartet state: a special shell model configuration,
single-center problem.

Cluster state: a molecule-like configuration,
two- (or more) center problem.

Interrelation: symmetry.
Overlap: small, large, finite, sometimes 100%.

Semimicroscopic Algebraic Cluster Model

J. Cseh, Phys. Lett. B281 (1992) 173;

J. Cseh, G. Lévai, Ann. Phys. (NY) (1994) 165.

Internal structure of clusters:

Elliott-model with $U^{ST}_{C_i}(4) \otimes U_{c_i}(3)$ algebraic structure.

J.P. Elliott, Proc. Roy. Soc. 245 (1958) 128; 562.

Relative motion:

(truncated) vibron model with $U_R(4)$ algebraic structure

F. Iachello, Phys. Rev. C23 (1981) 2778;

F. Iachello, R.D. Levine, J. Chem. Phys. 77 (1982) 3046.

Binary clusterization:

$$U_{C_1}^{ST}(4)\otimes U_{c_1}(3)\otimes U_{C_2}^{ST}(4)\otimes U_{c_2}(3)\otimes U_R(4).$$

Operators: group generators, matrix elements, algebraic.

Model space: only Pauli-allowed states,

as in the Microscopic Cluster Model with U(3) basis.

(H. Horiuchi, T. Hecht, Y. Suzuki, K. Kato,…)

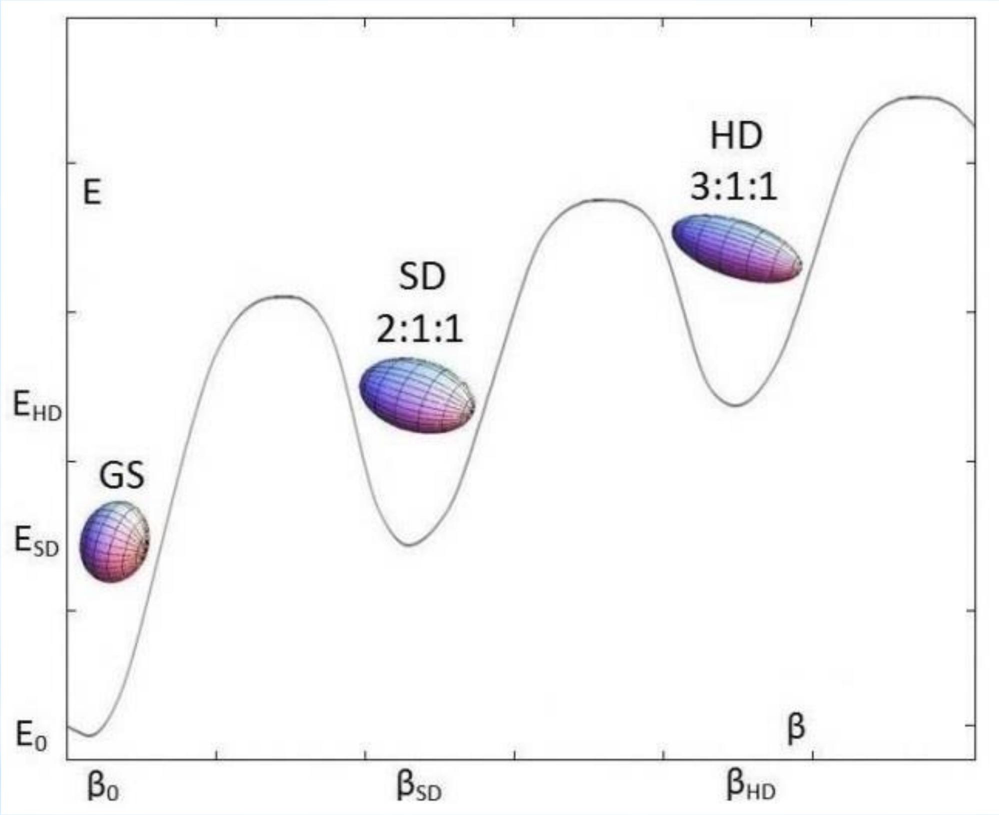
IV. Quadrupole deformation

Spherical, superdeformed, hyperdeformed,… shapes with ratios of major axes

1:1:1 , 2:1:1 , 3:1:1, …

turn out to be exceptianally stable.

One way of seeing it is to investigate the energy-surface as a function of the quadrupole deformation.



E
HD
3:1:1
SD
2:1:1
E_{HD}
GS
E_{SD}
E_0
β
$β_0$
$β_{SD}$
$β_{HD}$

An alternative way:

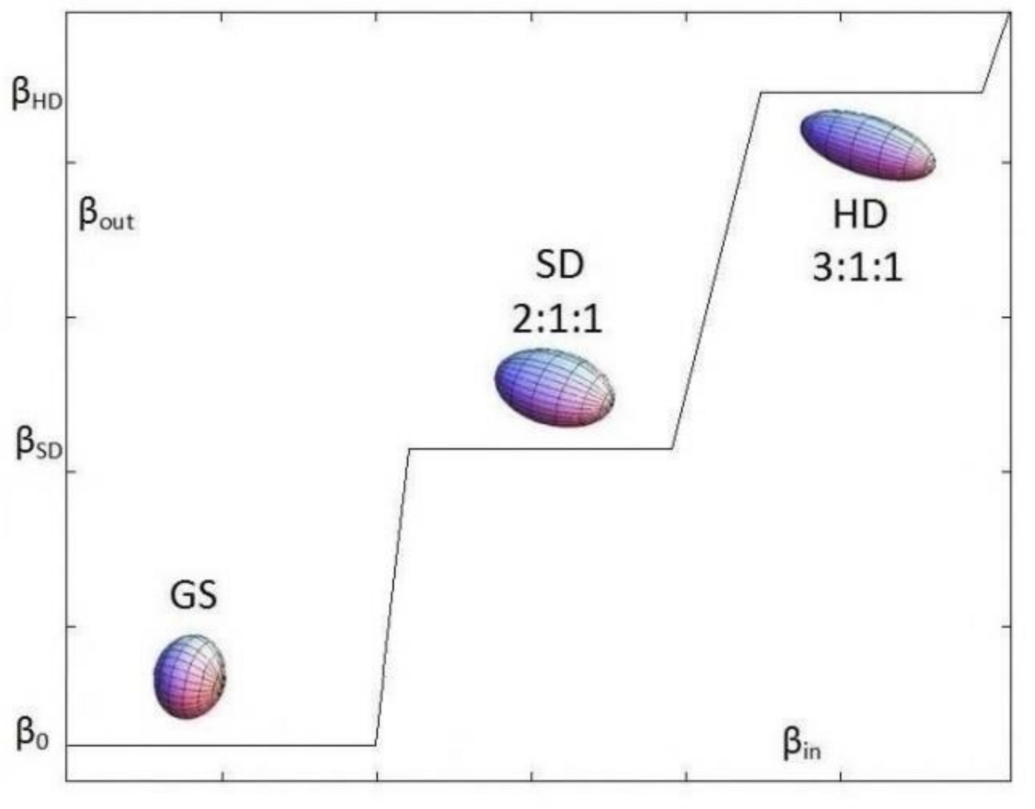
stability and self-consistency of the (quasidynamical) SU(3) symmetry (qudrupole deformation).

Scenario:

quadrupole deformation > Nilsson-calculation >

quasi-dynamical SU(3) > quadrupole deformation

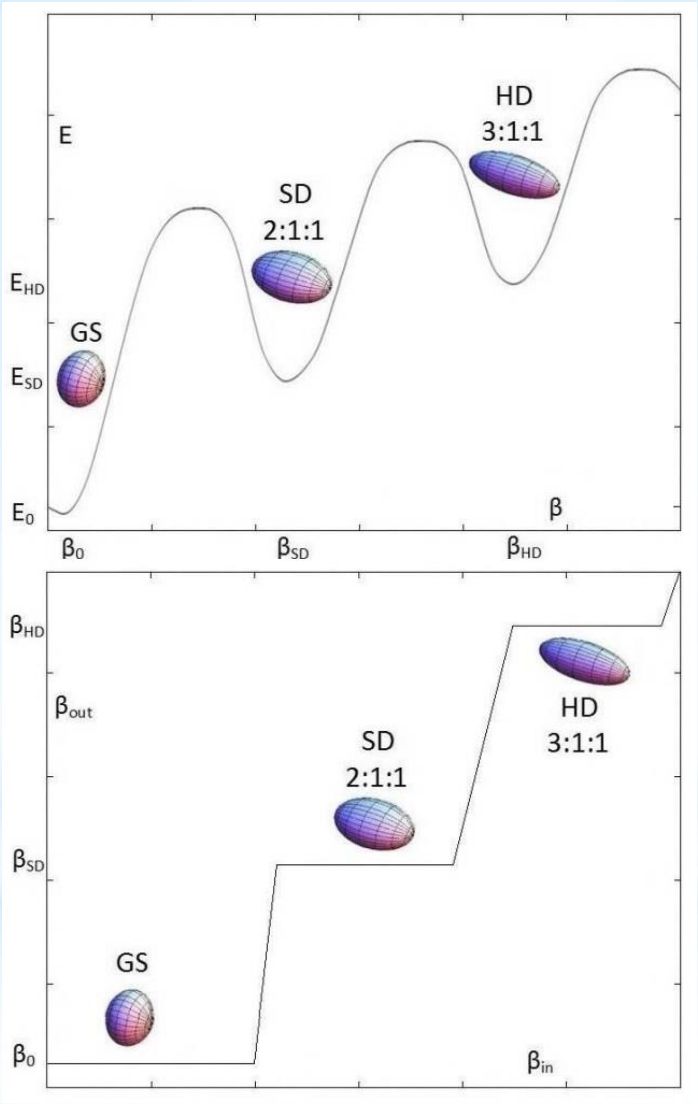
Self-consistency + stability.



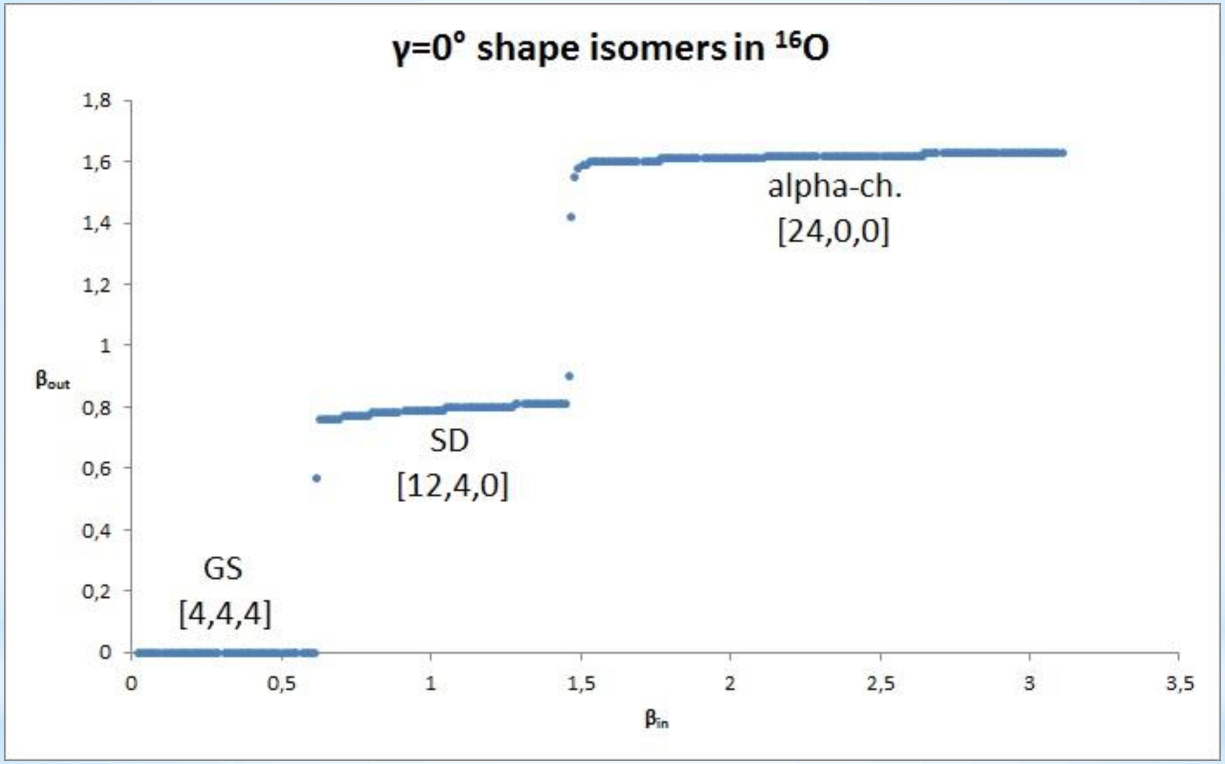
βHD
βout
SD
2:1:1
HD
3:1:1
βSD
GS
β0
βin

Energy-minima and stabil deformation (symmetry) are in good agreement. (Nice to see that different methods give the same results.)

Symmetry: selection rule, connection to cluster-configuration, connection to reaction channels.



E
HD
3:1:1
SD
2:1:1
E_{HD}
GS
E_{SD}
E_0
β
β_0
β_{SD}
β_{HD}
β_{HD}
β_{out}
HD
3:1:1
SD
2:1:1
β_{SD}
GS
β_0
β_{in}


γ=0° shape isomers in 16O
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
βout
0
0,5
1
1,5
2
2,5
3
3,5
βin
GS
[4,4,4]
SD
[12,4,0]
alpha-ch.
[24,0,0]

Horizontal platos:

A) Appearance of shape isomers.

B) Validity of SU(3) symmetry.

Work in progress:

How much the distribution of the wavefunction of the SA-NCSM with realistic interaction (ab initio method) is in line withe the results of the symmetry-based semimicroscopic approach.

GS, SD, alpha-chain states.

V. Multichannel dynamical symmetry (MUSY)

It connects different cluster configurations.
A composite symmetry of a composite system.
(J. Cseh, Phys. Rev. C 50 (1994) 2240.)

Formulated within the semimicroscopic algebraic cluster model (SACM).
(J. Cseh, Phys. Lett. B 281 (1992) 173.)

The (no-core) shell (or quartet) configuration is a special 1-cluster configuration: MUSY.
(J. Cseh, Phys. Lett. B743 (2015) 213.)

Theoretical aspect of MUSY

Connection between the shell, collective and cluster models for multi major-shell problems:

(No-core) Symplectic shell model
(G. Rosensteel, D. Rowe, PRL 38 (1977) 10;
T. Dytrych et al. J. Phys. G 35 (2008) 123101.)

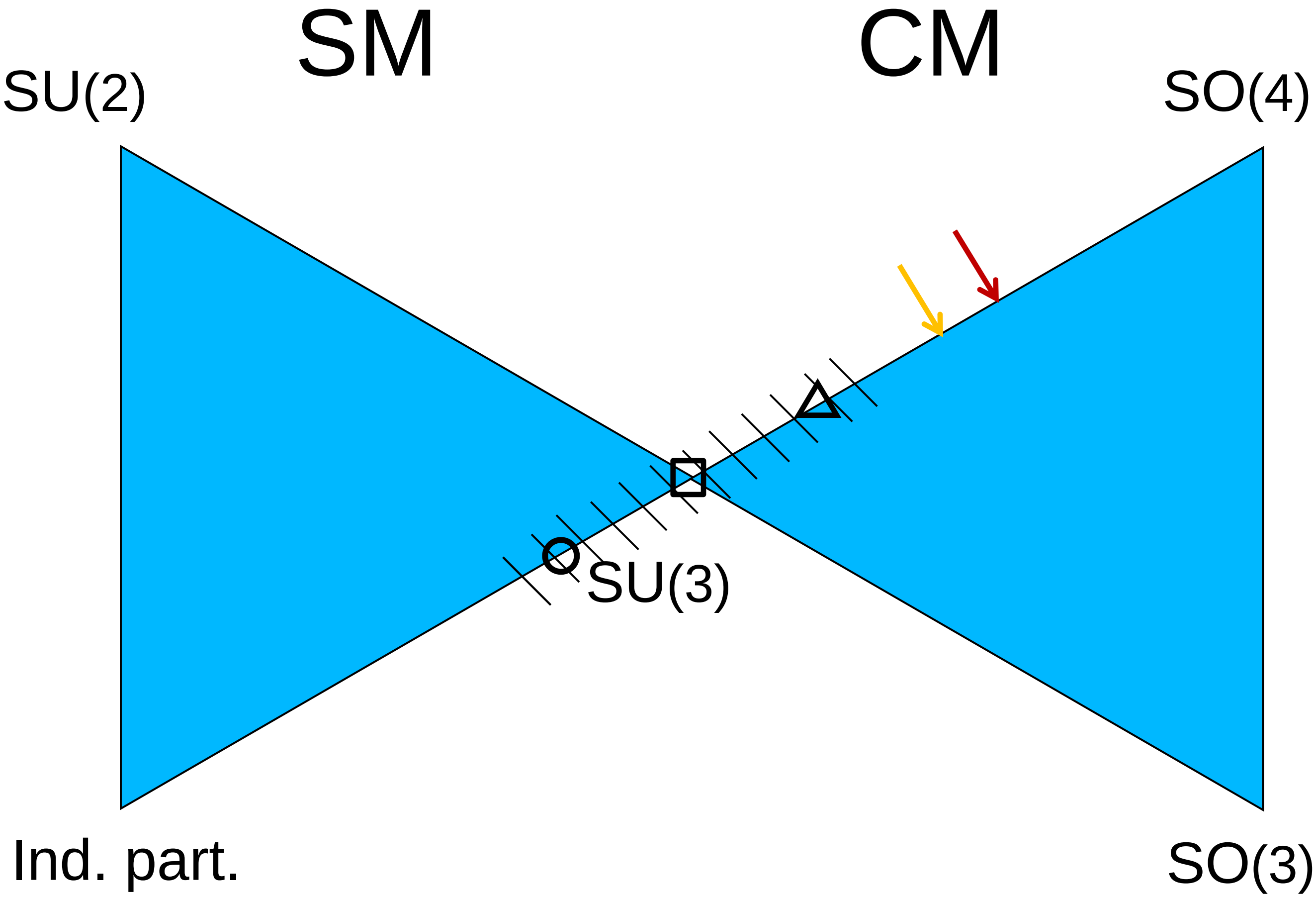
Contracted symplectic model
(D.J. Rowe, G. Rosensteel, Phys. Rev. C 25 (1982) 3236(R);
O. Castanos, J. P. Draayer, Nucl. Phys. A 491 (1989) 349.)

Semimicroscopic algebraic cluster model
(J. Cseh, G. Lévai, Ann. Phys. 230 (1994) 165.)

L-S coupling, Wigner's $U^{ST}(4)$

space part:

$$U_s(3) \otimes U_x(3) \supset U(3) \supset O(3)$$


SM
CM
SU(2)
SO(4)
SU(3)
Ind. part.
SO(3)

3 different kinds of clustering:
- shell like,
- rigid molecule-like,
- weak coupled.

Like:
- rotation,
- vibration,
- gamma-unstable,

are all quadrupole collective motions.

Yet another formulation of quarteting and quartet-cluster interplay

THSR wavefunctions of alpha-condansate

e.g. Hoyle-state in ^{12}C

Surprise: a single THSR (delocalized) wf has cca 100% overlap with the (localized) RGM wf in ^{20}Ne.

(Re)discovery of the shell-like quarteting.

No self-contradiction any more.

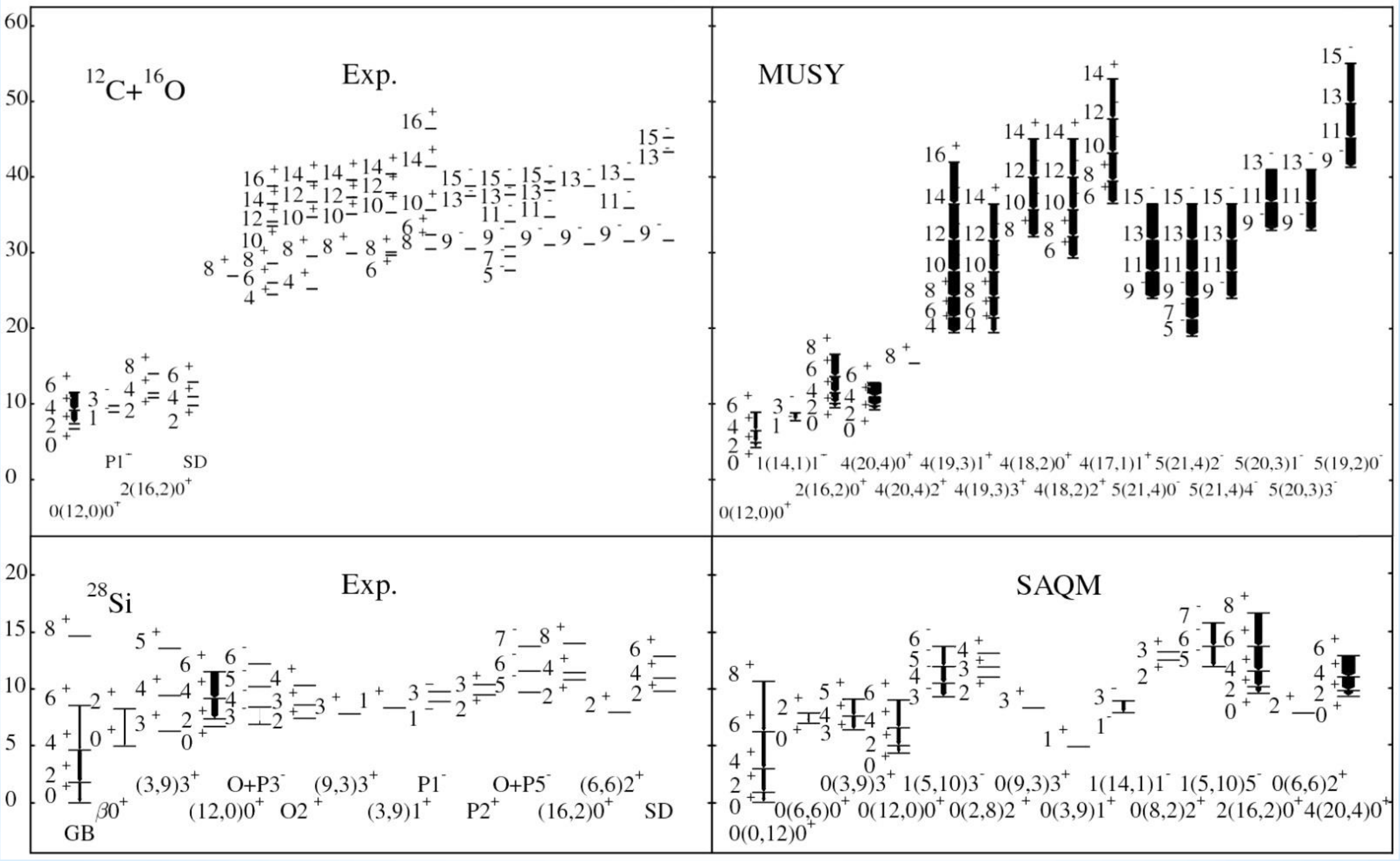
Quartet-wf for the microscopic calculation of the alpha-preformation factor in heavy nuclei.

Ch-Jp-Fr-D collaboration.

Practical aspect of MUSY

Unified description of different configurations.
J. Cseh, G. Riczu, Phys. Lett. B 757 (2016) 312.

Experimental spectra, predictions.

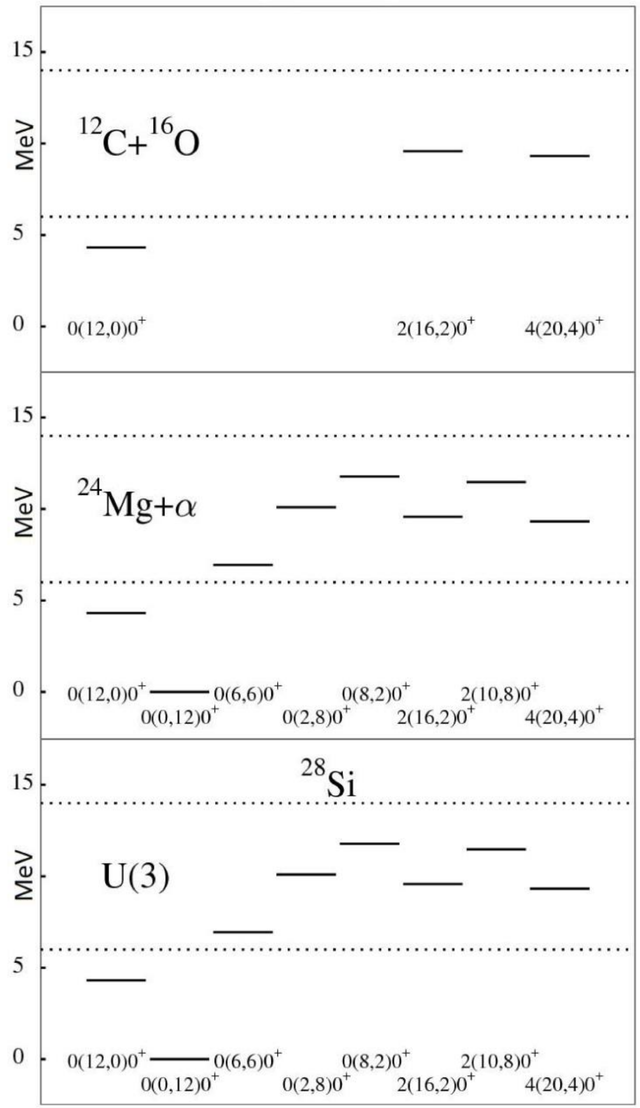


(J. Cseh, G. Riczu, Phys. Lett. B 757 (2016) 312.)

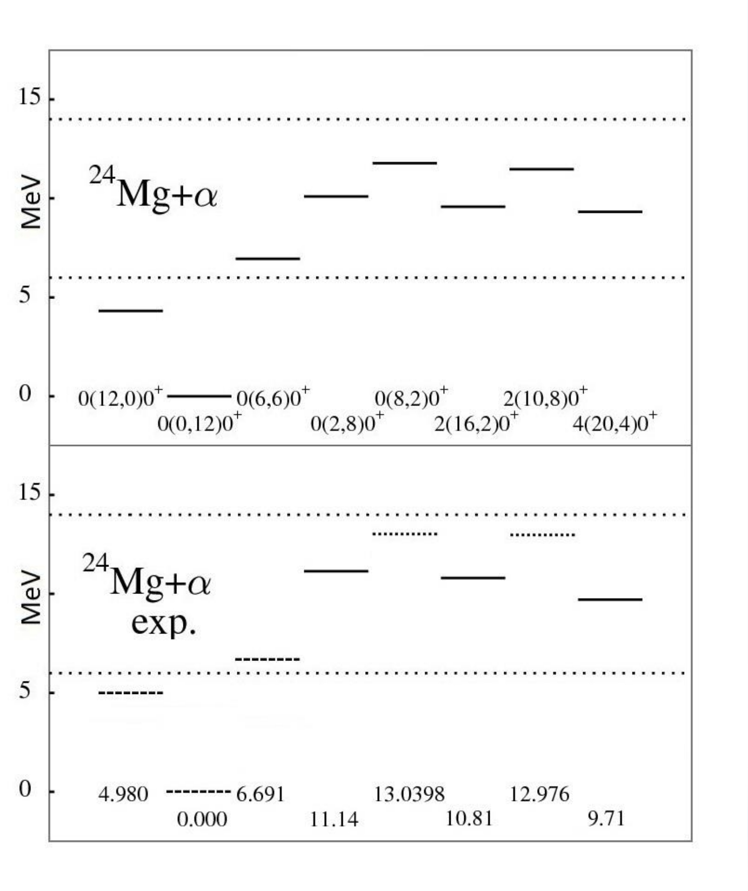
100% overlap of shell (quartet) and cluster states:

0(0,12) 0^+, GS
0(12,0) 0^+,
1(14,1) 0^- ,
2(16,0) 2^+,
4(20,4) 0^+, SD.

$$\hat{H} = (h\omega)\hat{n} + a\hat{C}^{(2)}_{SU3} + b\hat{C}^{(3)}_{SU3} + d\,\frac{1}{2\theta}\hat{L}^2$$



15
MeV
5
0
$^{12}C+^{16}O$
0(12,0)0+
2(16,2)0+
4(20,4)0+
$^{24}Mg+\alpha$
0(12,0)0+
0(0,12)0+
0(6,6)0+
0(2,8)0+
0(8,2)0+
2(16,2)0+
2(10,8)0+
4(20,4)0+
^{28}Si
U(3)
0(12,0)0+
0(0,12)0+
0(6,6)0+
0(2,8)0+
0(8,2)0+
2(16,2)0+
2(10,8)0+
4(20,4)0+



P. Adsley et al, Phys. Rev. C **95**, 024319 (2017)

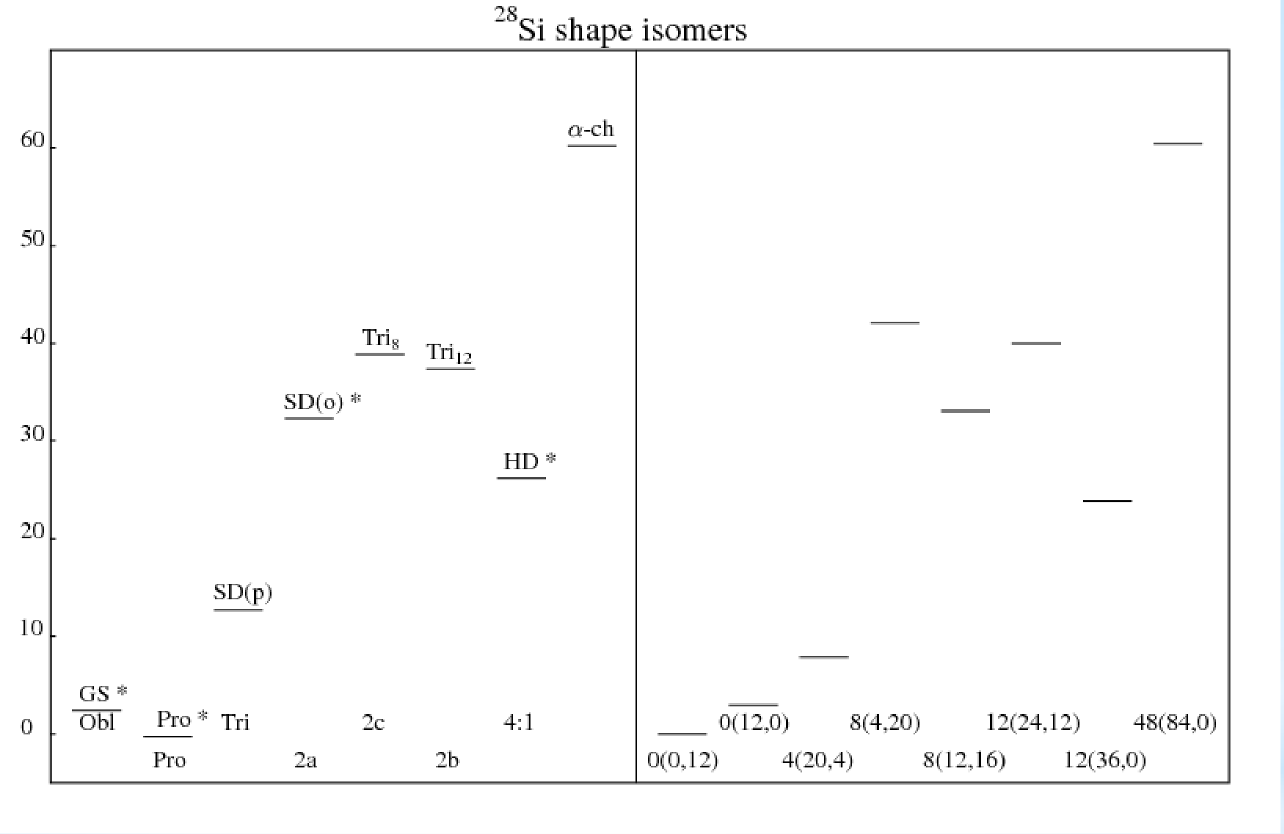
Detailed spectrum in two local minima.
(Long-standing question:
Are the ^{12}C+^{16}O resonances in the SD valley?
Yes, they are.)

If MUSY describes the spectra in two minima,
what about the other minima?

Is a simple, dynamically symmetric Hamiltonian
able to produce several local minima?

If yes, how it compares to other results?

Effective model for all the shape isomers?
Shell configurations of several coexistig shapes?
(D.J. Rowe et al, Phys. Rev. Lett. 97 (2006) 202501.)



^{28}Si shape isomers
0
10
20
30
40
50
60
GS *
Obl
Pro *
Pro
Tri
SD(p)
2a
SD(o) *
Tri$_8$
2c
Tri$_{12}$
2b
HD *
4:1
α-ch
0(0,12)
0(12,0)
4(20,4)
8(4,20)
8(12,16)
12(24,12)
12(36,0)
48(84,0)

VI. Conclusions

Semimicroscopic Algebraic

Quartet Model (SAQM)

Cluster Model (SACM)

Method for shape isomers

Unifying symmetry

VI. Conclusions

The multichannel dynamical symmetry (MUSY)

connects the shell (quartet), collective and cluster models of multi major-shell excitations.

Describes the detailed spectra of different configuartions in different energy window in a unified way.

It gives several shape isomers.

It has a considerable predictive power.

Thank you for your attention!