



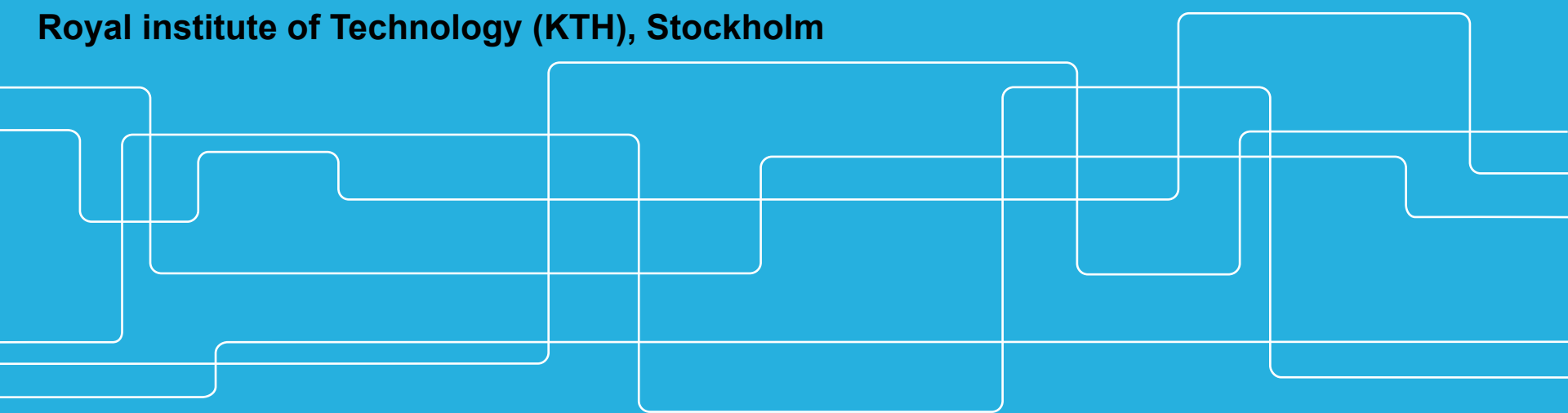
ESNT workshop 10-14 September 2018

KTH ROYAL INSTITUTE
OF TECHNOLOGY

Effective pn interactions in $N=Z$ nuclei from a shell model perspective

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The coupling of few nucleons

- Neutron-proton correlation from the binding energy

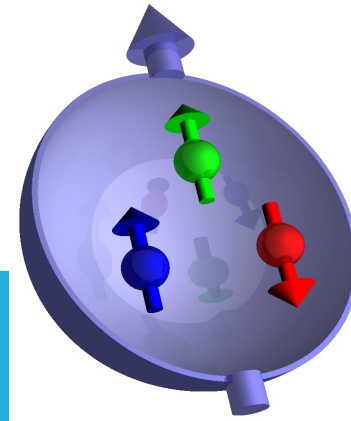
The residual np interaction

- Systems with identical particles in *single-j* orbitals

- Neutron-proton spin-aligned pair coupling

❖ *J=1 pairs*

❖ *Quartet*

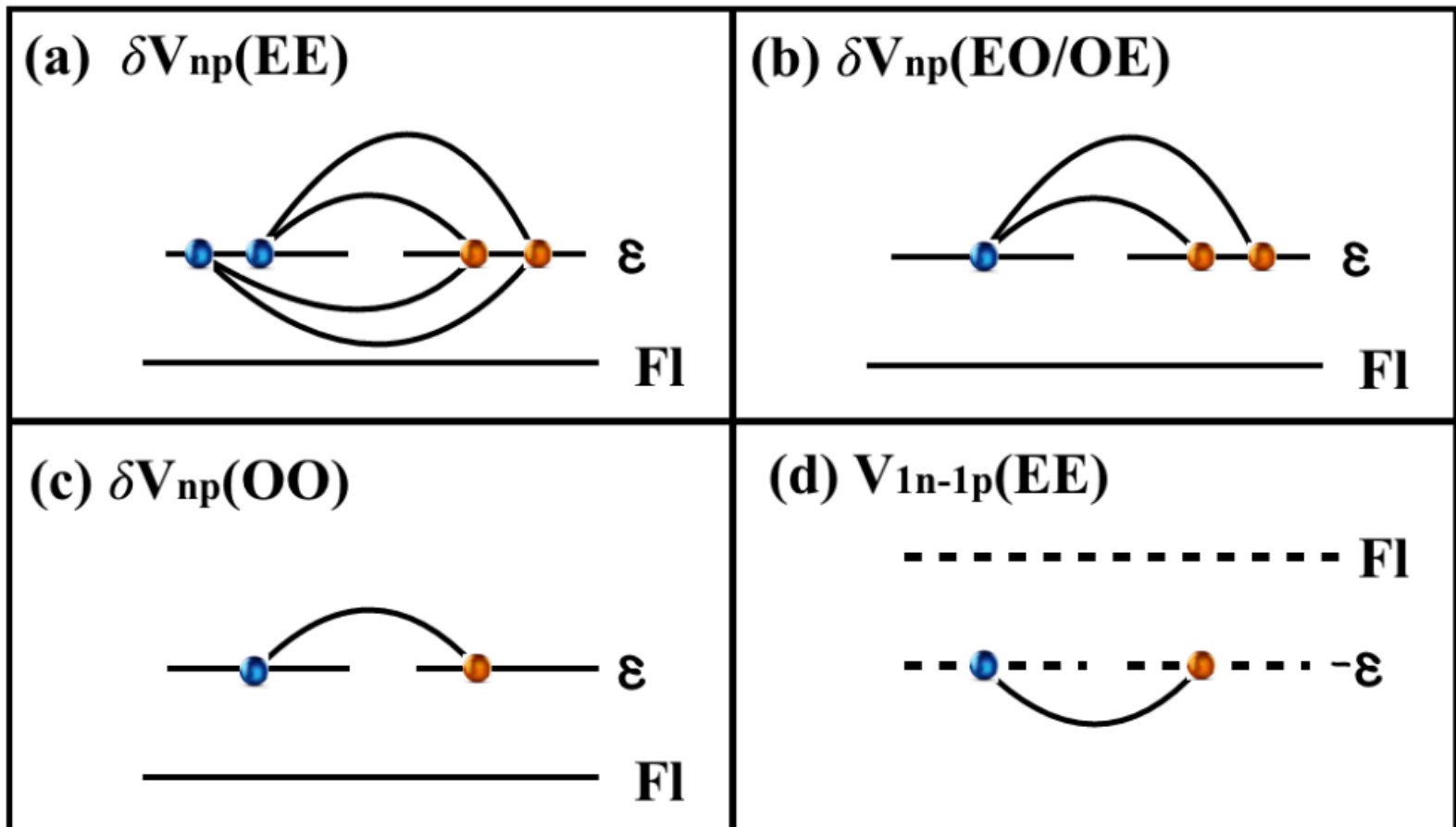


The average proton-neutron interaction and Wigner energy



$$V_{pn}(Z, N) = \frac{1}{4} [B(Z, N) + B(Z - 2, N - 2) - B(Z - 2, N) - B(Z, N - 2)],$$

J.-Y. Zhang, R.F. Casten, D.S. Brenner, Phys. Lett. B 227 (1989) 1.

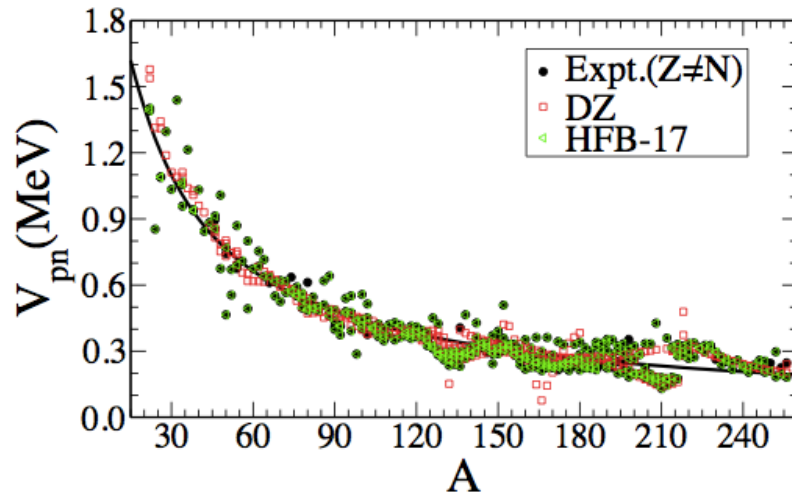


The average proton-neutron interaction and Wigner energy

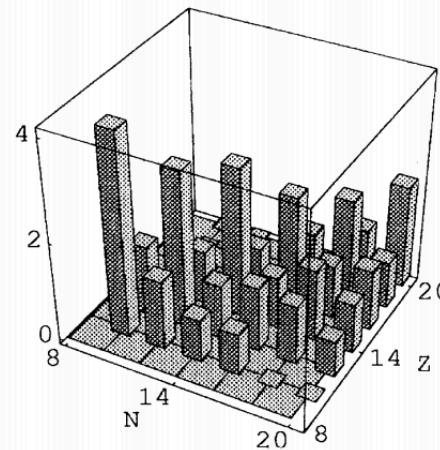


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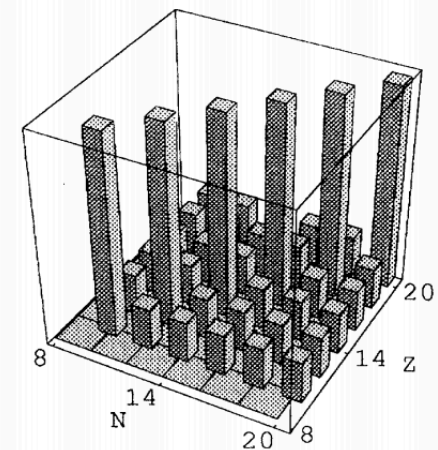
J.-Y. Zhang, R.F. Casten, D.S. Brenner, Phys. Lett. B 227 (1989) 1.



(a) sd shell (even-even)



(b) SU(4) (even-even)



P. Van Isacker, D. D. Warner, and D. S. Brenner
Phys. Rev. Lett. 74, 4607 (1995)



HFB-24 mass formula

The force used in the Hartree-Fock-Bogoliubov (HFB) mass model is an extended Skyrme force (containing t_4 and t_5 terms), along with a 4-parameter delta-function pairing force derived from realistic calculations of infinite nuclear and neutron matter (all details are given in Goriely et al., Phys. Rev. 88, 024308, 2013).

Pairing correlations are introduced in the framework of the Bogoliubov method. Deformations with axial and left-right symmetry are admitted.

The total binding energy is given by

$$E_{\text{tot}} = E_{\text{HFB}} + E_{\text{wigner}}$$

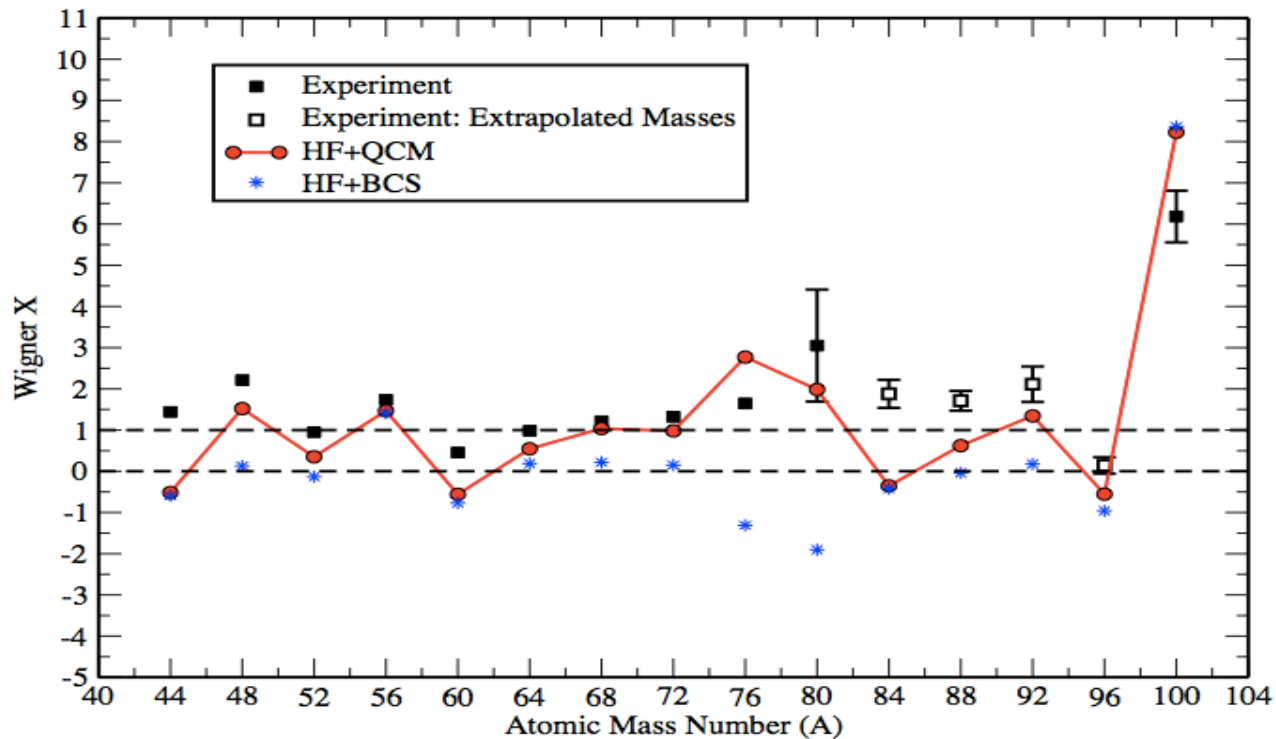
where

- E_{HFB} is the HFB binding energy including a cranking correction to the spurious rotational energy and a phenomenological vibration correction energy
- $E_{\text{wigner}} = V_W \exp(-\lambda ((N-Z)/A)^2) + V'_W |N-Z| \exp(-(A/A_0)^2)$ is a phenomenological correction for the Wigner energy.

Wigner energy from HF+QCM

$$E(N,Z) = E(N=Z) + a_s \frac{(N-Z)^2}{A} + a_w \frac{|N-Z|}{A} + \delta E_{shell} + \delta E_P \quad (\text{no Coulomb})$$

$$E(N,Z) = E(N=Z) + \frac{T(T+X)}{2\Theta} \quad T = T_z \quad T=0,2,4$$



D. Negrea and N. S, PRC (2014)

T=1 pairing has a significant contribution to Wigner energy !

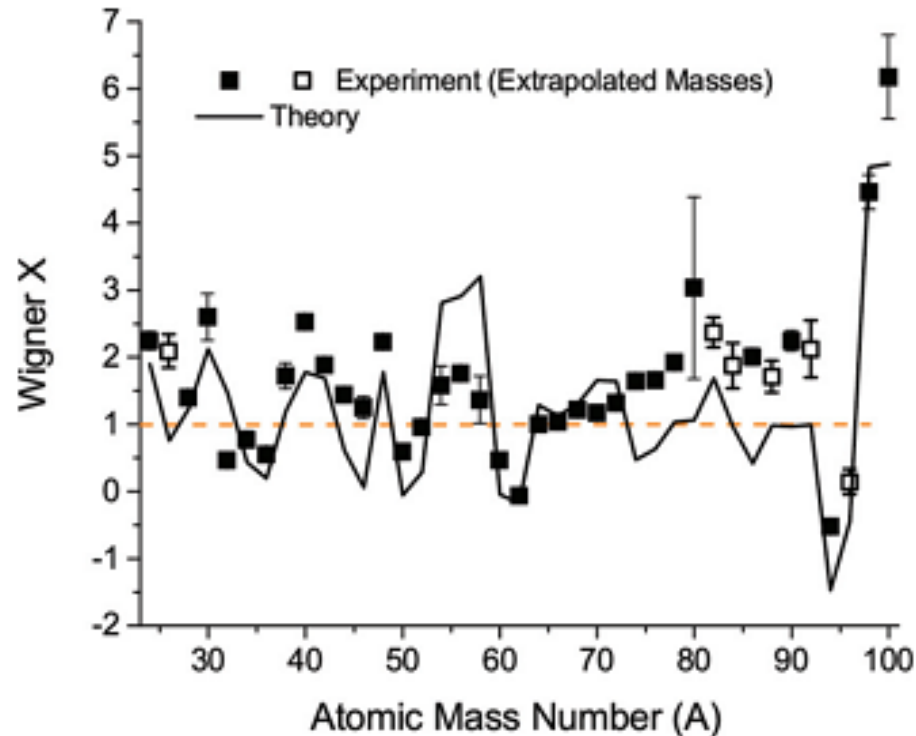
Relation between Wigner energy and proton-neutron pairing

I. Bentley^{1,2} and S. Frauendorf¹

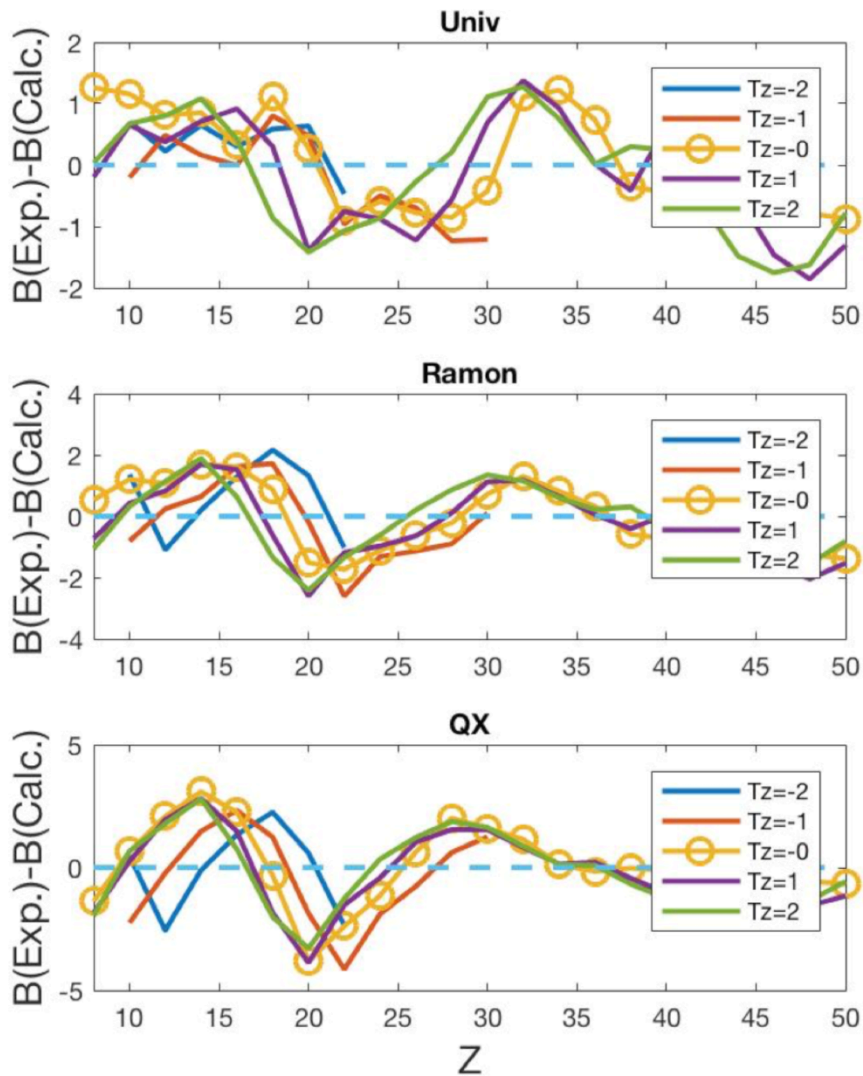
$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk', \tau} \hat{P}_{k, \tau}^+ \hat{P}_{k', \tau} + C \vec{T} \cdot \vec{T},$$

- empirical s.p energies splitting !
- pairing is treated exactly by diagonalisation

$$E(N, Z) = E(N = Z) + \frac{T(T + X)}{2\Theta}$$



Mac-mic calculation without np pairing



$$E_{\text{LDM}} = a_v A + a_s A^{2/3} + a_{\text{sym}} T(T + 1)/A$$

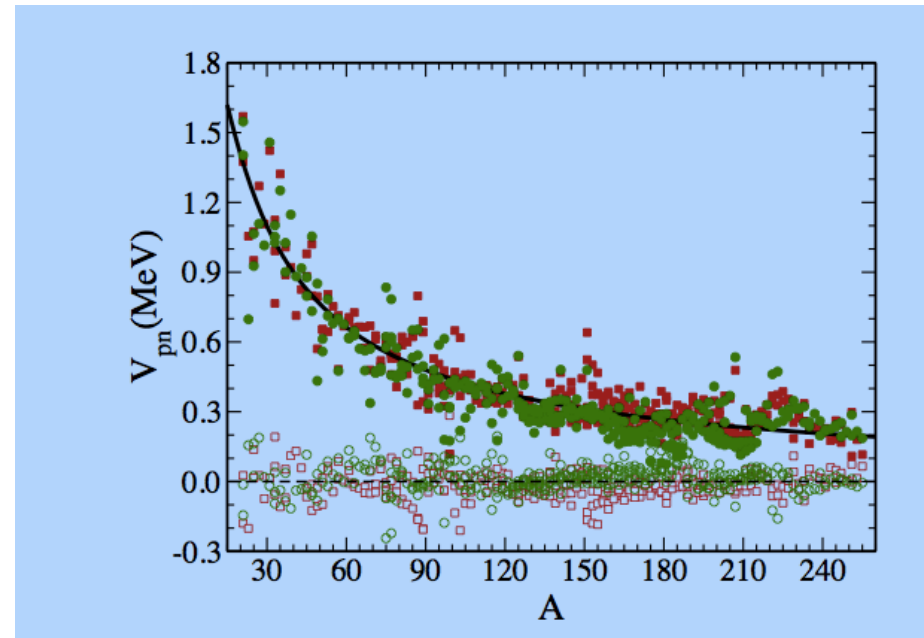
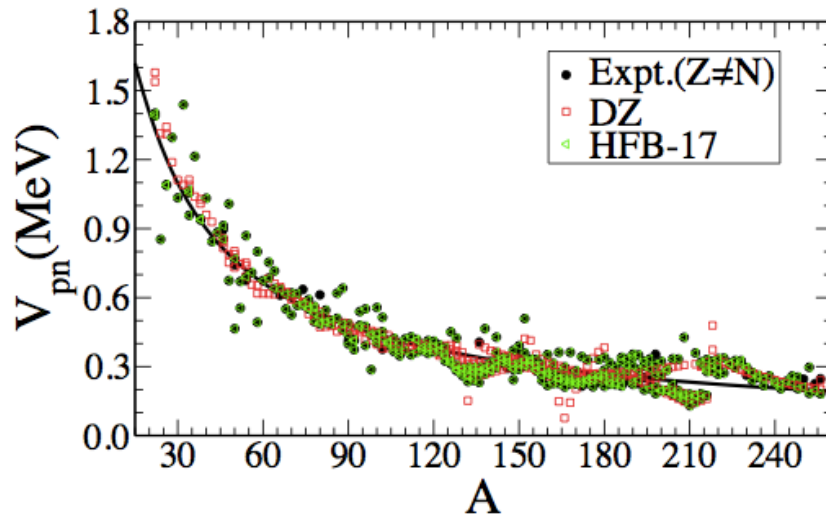
$$+ a_{\text{syms}} T(T + 1)/A^{4/3} + C \frac{Z^2}{A^{1/3}} + C_4 \frac{Z^2}{A},$$

The average proton-neutron interaction



$$V_{pn}(Z, N) = \frac{1}{4} [B(Z, N) + B(Z - 2, N - 2) - B(Z - 2, N) - B(Z, N - 2)],$$

J.-Y. Zhang, R.F. Casten, D.S. Brenner, Phys. Lett. B 227 (1989) 1.



$$V_{pn}(Z, N) = \frac{1}{2} [S_n(Z, N - 1) - S_n(Z - 2, N - 1)] + \frac{1}{4} [\Delta_n(Z, N) - \Delta_n(Z - 2, N)].$$

Seniority with isospin



$$E = \varepsilon n + \frac{a}{2}n(n-1) + \frac{b}{2} \left[\mathcal{T}(\mathcal{T}+1) - \frac{3n}{4} \right] + G \left[\frac{n-v}{4}(4j+8-n-v) - \mathcal{T}(\mathcal{T}+1) + s(s+1) \right],$$

Odd-even staggering

$$E = \varepsilon n + \frac{2a-G}{4}n(n-1) + \frac{b-2G}{2} \left[\mathcal{T}(\mathcal{T}+1) - \frac{3n}{4} \right] + (j+1)G(n-v) + G \left[\frac{v^2}{4} - v + s(s+1) \right],$$

Pairing energy in mass formula $E_p \propto 2 - v$,

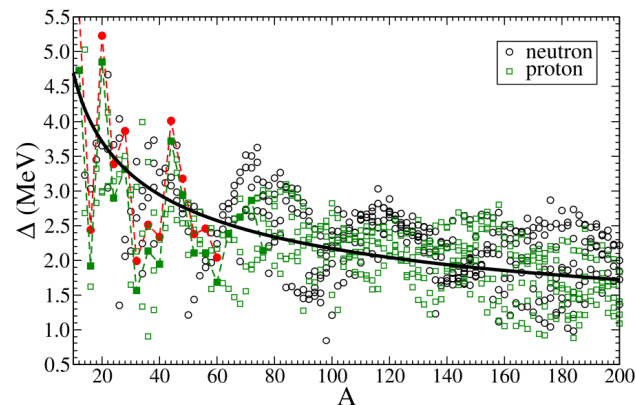


Fig. 2. (Color online.) Empirical proton–proton (squares) and neutron–neutron (circles) interactions in even–even nuclei extracted from experimental nuclear masses as a function of the mass number A [23]. The solid symbols denote those in the $N = Z$ nuclei.

Additional binding for N=Z nuclei

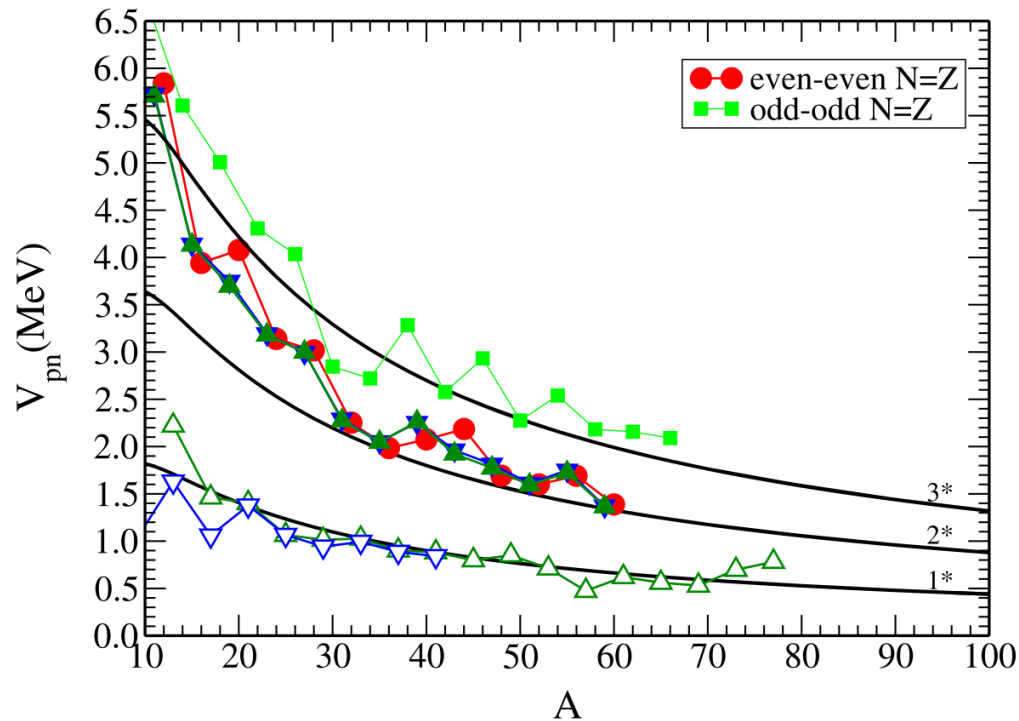


Fig. 4. (Color online.) Experimental V_{pn} values of even-even $N = Z$ nuclei (filled circles) and the adjacent odd-odd (squares) and odd- A nuclei (triangles). The filled and open triangles correspond to systems with one nucleon subtracted from and added to the even-even nuclei, respectively. The solid line labeled 1* describes the average behavior of V_{pn} in even-even $N \neq Z$ nuclei from Fig. 1. 2* and 3* denotes its twice and three time values.

$$\hat{V} = a + bt_1 \cdot t_2 + GP_0,$$

For even-even nuclei with $n_\pi \neq n_\nu$,

$$V_{pn} = -\frac{4V_{m;T=1} + 2(V_{m;T=0} - V_{m;T=1})}{4} = \frac{b}{4} - a.$$

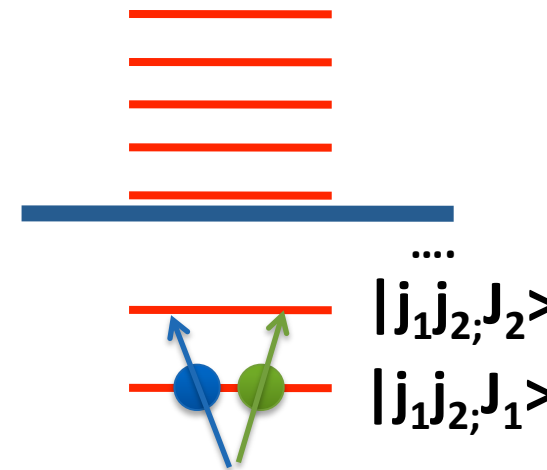
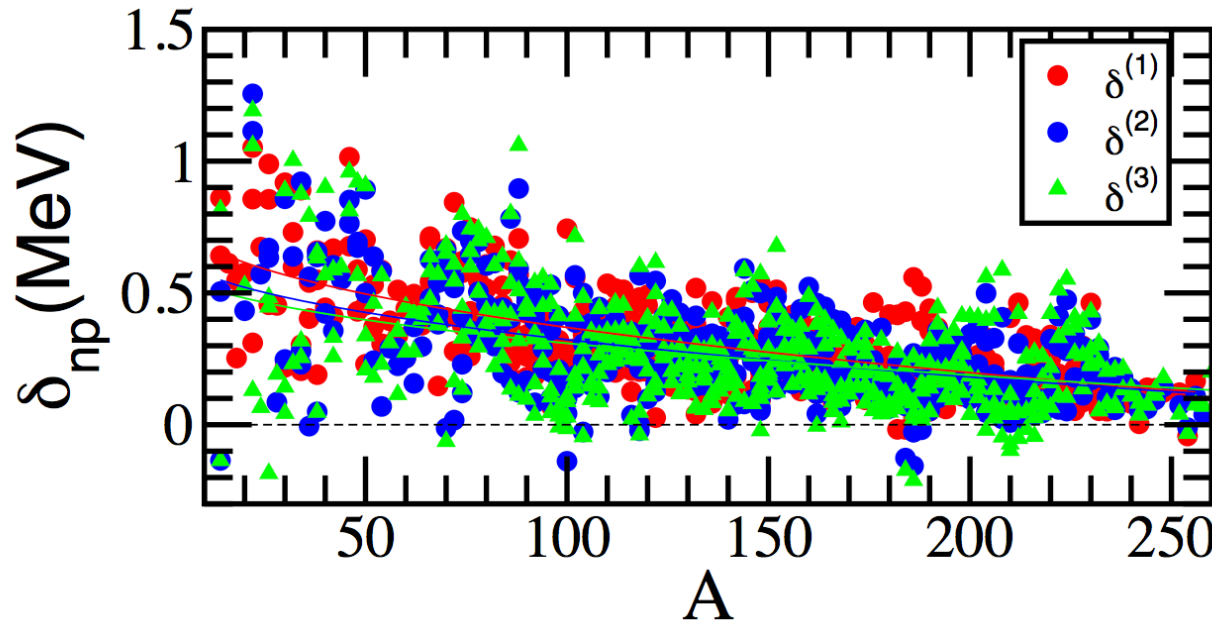
in the case of $n_\pi = n_\nu$ (i.e., $N = Z$),

$$\begin{aligned} V_{pn} &= -\frac{4V_{m;T=1} + 3(V_{m;T=0} - V_{m;T=1})}{4} - \frac{G}{2} \\ &= \frac{b}{2} - a - \frac{G}{2}. \end{aligned}$$

odd-odd $N = Z$

$$\begin{aligned} V_{pn}(Z-1, Z-1) &= B(Z-1, Z-1) + B(Z-2, Z-2) \\ &\quad - B(Z-1, Z-2) - B(Z-2, Z-1) \\ &= \frac{3b}{4} - a. \end{aligned}$$

The residual np interaction in odd-odd nuclei from binding energy systematics: Two quasi-particles in the even-even sea

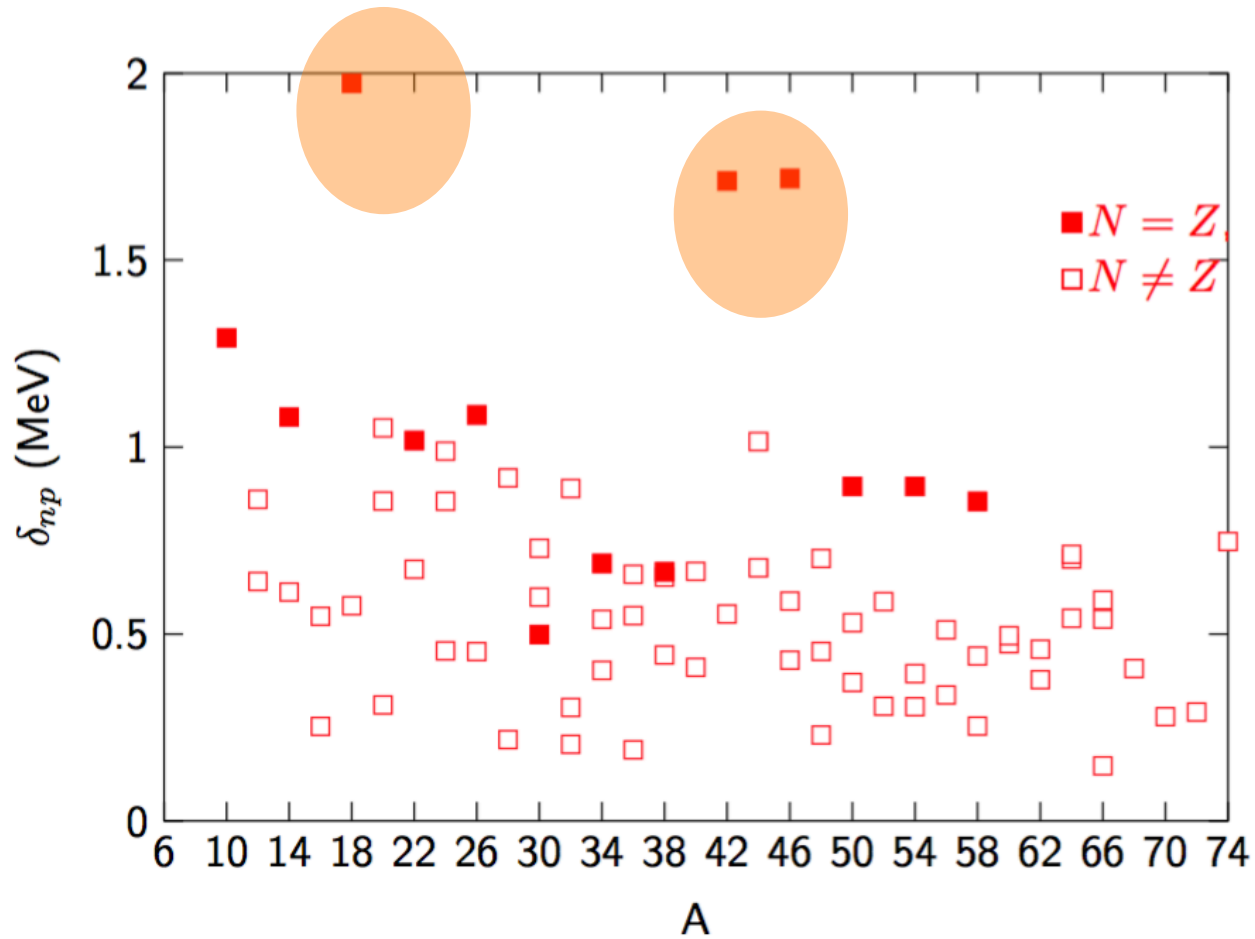


$$\delta_{np}^{(1)} = \Delta_n^{(3)}(Z, N) - \Delta_n^{(3)}(Z - 1, N)$$

$$\delta_{np}^{(2)} = V_{oo} - V_{ee},$$

$$\delta_{np}^{(3)} = V_{oo} - V_{smooth}.$$

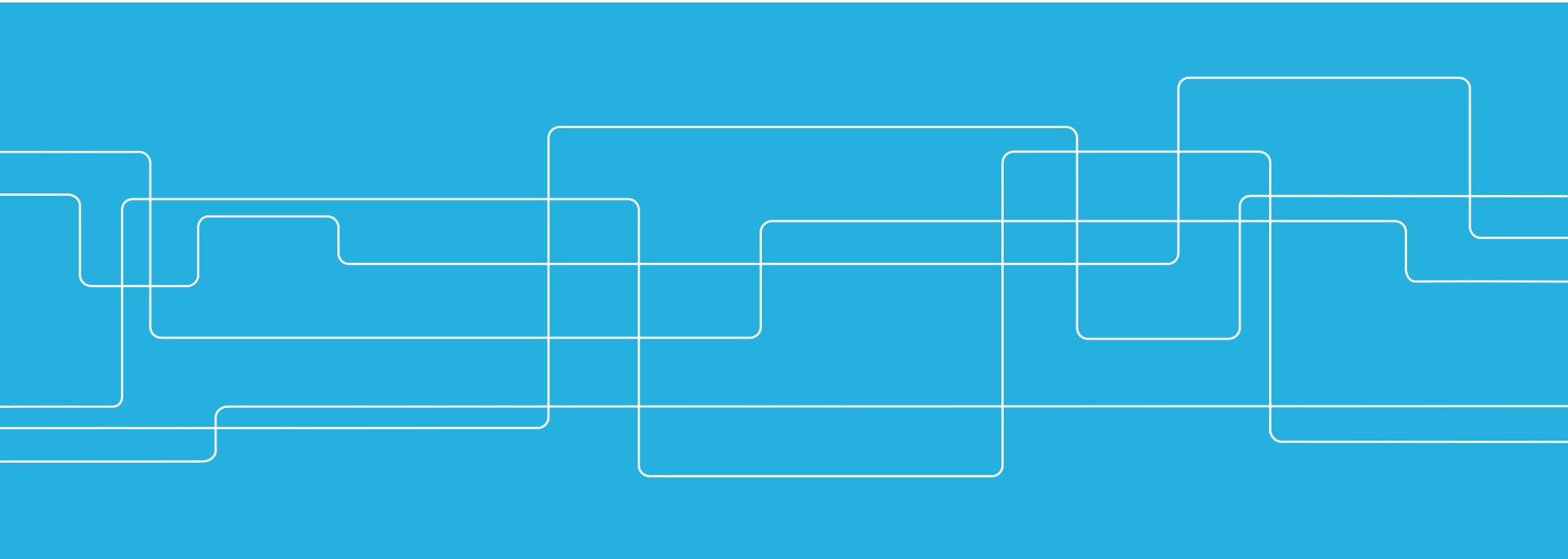
N=Z odd-odd nuclei





More is different

Neutron-proton pairing correlation





Trivial examples:

The spin trap isomers:

The 12⁺ spin trap in ⁵²Fe

$$E_{12}(^{52}\text{Fe}) = \frac{6}{13} \bar{V}_5 + 3\bar{V}_6 + \frac{33}{13} \bar{V}_7,$$

$$E_{10_1^+}(^{52}\text{Fe}) = 0.310\bar{V}_3 + 1.429\bar{V}_4 + 0.497\bar{V}_5 + 1.571\bar{V}_6 + 2.193\bar{V}_7,$$

CQ, *Phys. Rev. C* 81, 034318 (2010).

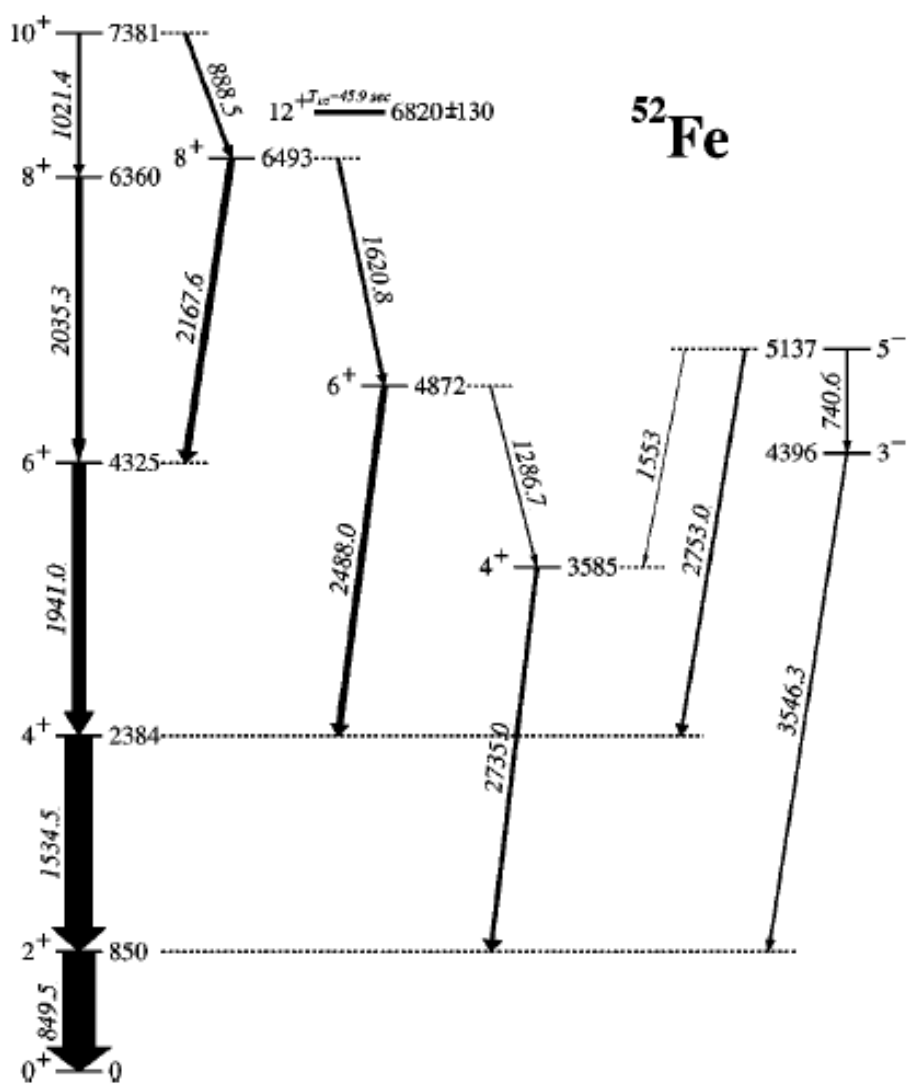
The predicted 16⁺ spin trap in ⁹⁶Cd

$$E_{16}(^{96}\text{Cd}) = \frac{8}{17} \bar{V}_7 + 3\bar{V}_8 + \frac{43}{17} \bar{V}_9.$$

$$E_{14_1^+}(^{96}\text{Cd}) = 0.307\bar{V}_5 + 1.428\bar{V}_6 + 0.493\bar{V}_7 + 1.572\bar{V}_8 + 2.200\bar{V}_9.$$

K. Ogawa, *Phys. Rev. C* 28, 958 (1983).

CQ, *Phys. Rev. C* 81, 034318 (2010).



C. A. Ur *et al.*, *Phys. Rev. C* 58, 3163 (1998).

The relative positions of these spin traps are sensitive to the strength of the interaction $V_{J=2j}$

The energy expression



$$E_I = C_J^I V_J,$$

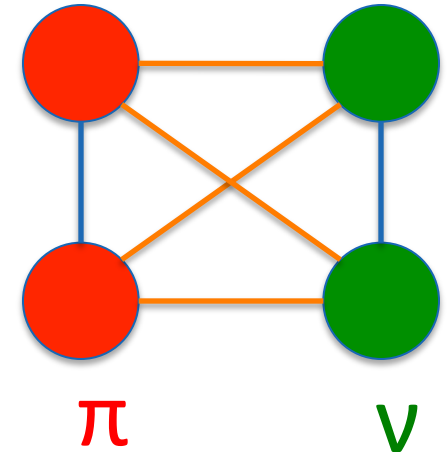
$$C_J^I = \frac{1}{2} \langle \left[(a^\dagger a^\dagger)^J (a a)^J \right]^0 | | \rangle$$

The total number of pairs with all spins J is given by

$$\sum_J C_J^I = n(n-1)/2,$$

and

$$\sum_{J, \text{odd}} C_J^I = \frac{1}{2} \left[\frac{n}{2} \left(\frac{n}{2} + 1 \right) - T(T+1) \right],$$



✧ For systems with $n=4$ and isospin $T=0$, there are three isoscalar pairs and three isovector pairs;

✧ For those with $T=2$ (four identical nucleons), we have six isovector pairs.

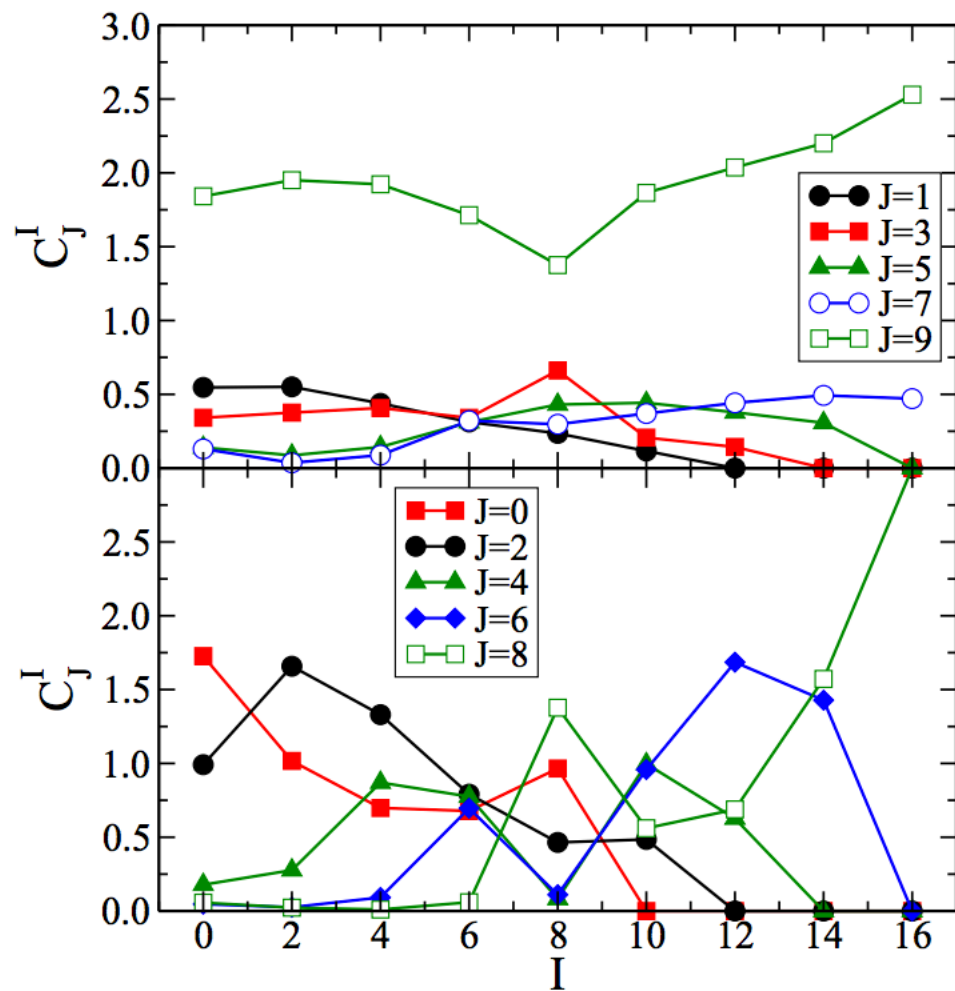
E. Moya de Guerra, A. A. Raduta, L. Zamick, and P. Sarriguren, Nucl. Phys. A 727, 3 (2003).

CQ, Phys. Rev. C 81, 034318 (2010).

I. Talmi, Nucl. Phys. A 846 (2010) 31.

Average "number of pairs"

$$\langle \Psi_N | | ((a_i^\dagger a_j^\dagger)_{J\pi} \times (a_i a_j)_{J\pi})_0 | | \Psi_N \rangle \quad E_I = C_J^I V_J.$$



$$C_{J=0}^I(nn) = C_{J=0}^I(pp) = C_{J=0}^I(np)$$



Off-diagonal long range order (ODLRO)

$$\lambda = \frac{1}{M(1 - M/L)} \sum_{k, k'=1}^L \langle c_k^\dagger c_{\bar{k}'}^\dagger c_{\bar{k}} c_k \rangle - \langle c_k^\dagger c_k \rangle \langle c_{\bar{k}'}^\dagger c_{\bar{k}'} \rangle, \quad (9)$$

where L is the total number of doubly-degenerate, canonically conjugate pair states k, \bar{k} .

In BCS approximation, the modified Yang prescription leads to a condensate fraction

$$\lambda_{BCS} = \frac{1}{M(1 - M/L)} \sum_{k=1}^L u_k^2 v_{\bar{k}}^2. \quad (10)$$

General properties of the effective interaction



➤ **Isovector (T=1):** $J=0,2,\dots,2J-1$, $J=0$ term attractive (*pairing*), others close to zero

➤ **Isoscalar (T=0):** $J=1,3,\dots,2j$, strongly attractive (mean field)

✧ The $J=1$ and $2j$ terms are the most attractive ones.

✧ $L=0, J=1$ pairing

✧ The aligned pair was not much studied

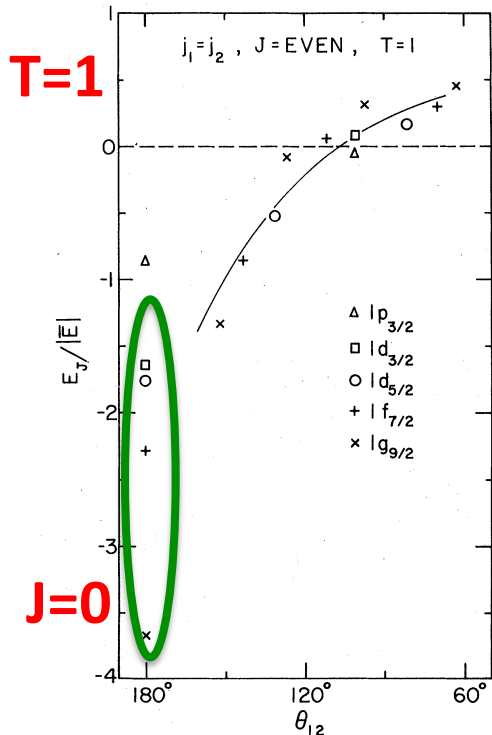
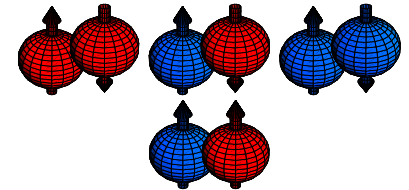


FIG. 3. Comparison of data from various multiplets with $j_1=j_2$ and $T=1$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

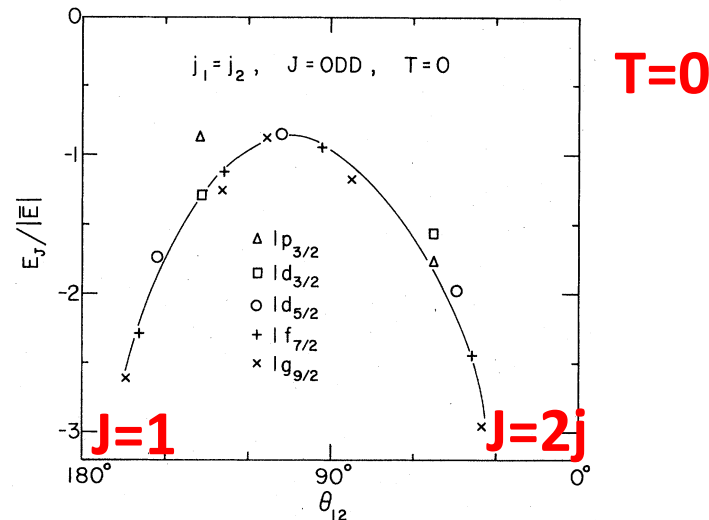


FIG. 2. Comparison of data from various multiplets with $j_1=j_2$ and $T=0$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

$$\cos \theta_{12} = \frac{J(J+1)}{2j(j+1)} - 1$$

J.P. Schiffer and W.W. True, Rev.Mod.Phys. 48,191 (1976)



Monopole Hamiltonian

Determines average energy of eigenstates in a given configuration.

- Important for binding energies, shell gaps

$$H_m = \sum_a \varepsilon_a n_a + \sum_{a \leq b} \frac{1}{1 + \delta_{ab}} \left[\frac{3V_{ab}^1 + V_{ab}^0}{4} n_a (n_a - \delta_{ab}) + (V_{ab}^1 - V_{ab}^0) (T_a \cdot T_b - \frac{3}{4} n_a \delta_{ab}) \right]$$

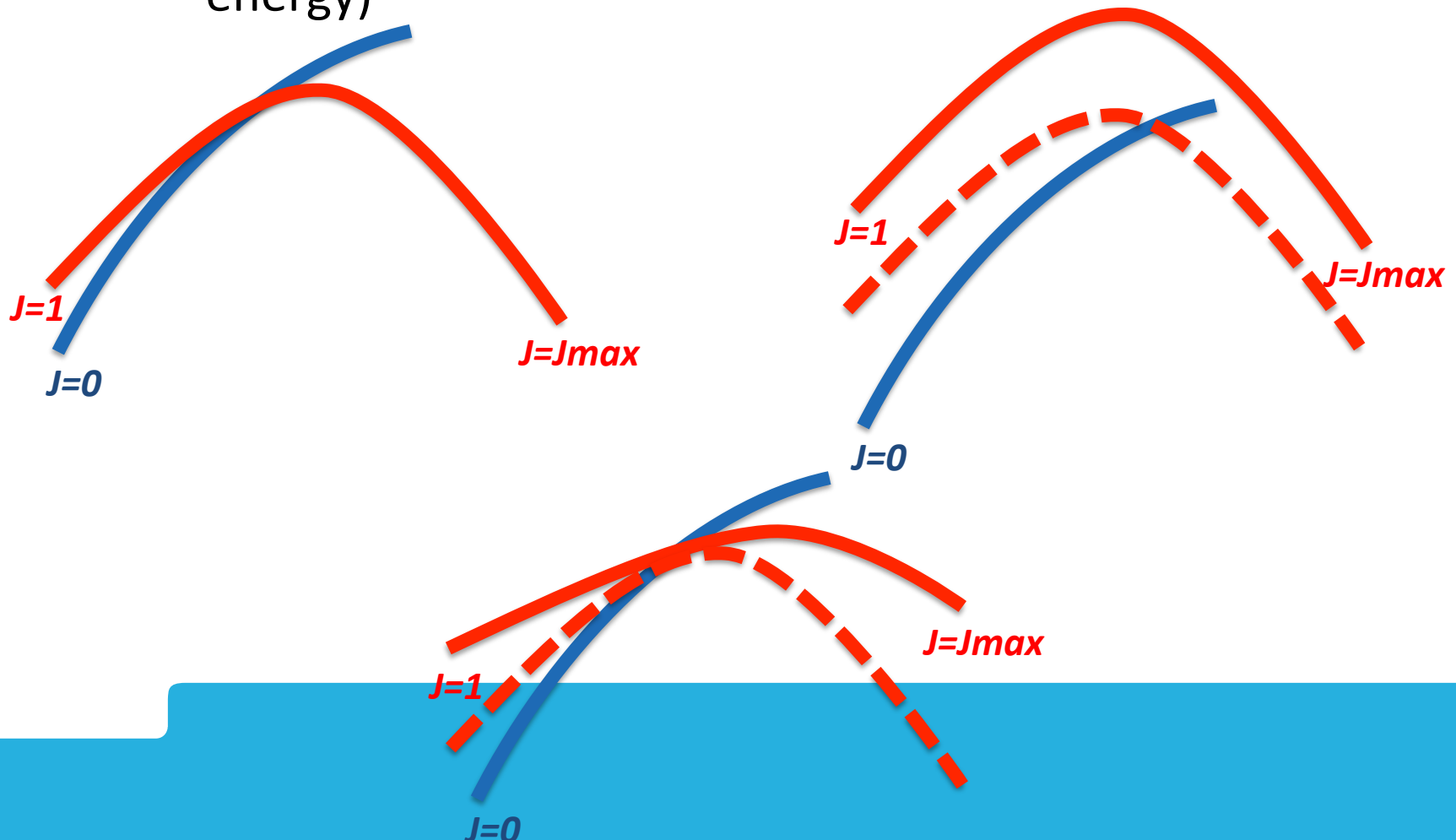
n_a, T_a ... number, isospin operators of orbit a

Monopole centroids

- Angular-momentum averaged effects of two-body interaction
- **The monopole interaction itself does not induce mixing between different configurations.**
- **Strong mixture of the wave function is mainly induced by the residual $J=0$ pairing and QQ np interaction**

$$V_{ab}^T = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

- What matter for the wave functions are the relative values between different two-body matrix elements with same isospin (the multipole channel)
- The monopole interactions determine the relative positions of states with different total isospin (and the symmetry energy)





The coupling of few nucleons

The $\nu=0$ state is uniquely defined, but ...

$$|\text{g.s.}\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^{n/2} |\Phi_0\rangle$$
$$|\nu = 2; JM\rangle = (P_j^+)^{(n-2)/2} A^+(j^2 JM) |\Phi_0\rangle$$



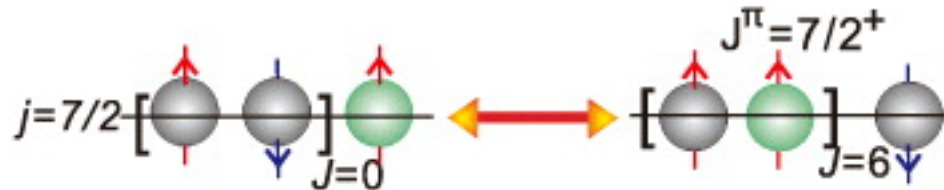
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Three identical particles



This can be tested by knock out/pair transfer reactions

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry.

Weinberg's third law of progress in theoretical physics

$$|J_1 J_2 J\rangle = - \sum_{J_1' J_2'} \hat{J}_1 \hat{J}_2 \hat{J}_1' \hat{J}_2' X(jjJ_1; jjJ_2; J_1' J_2' J) |J_1' J_2' J\rangle,$$

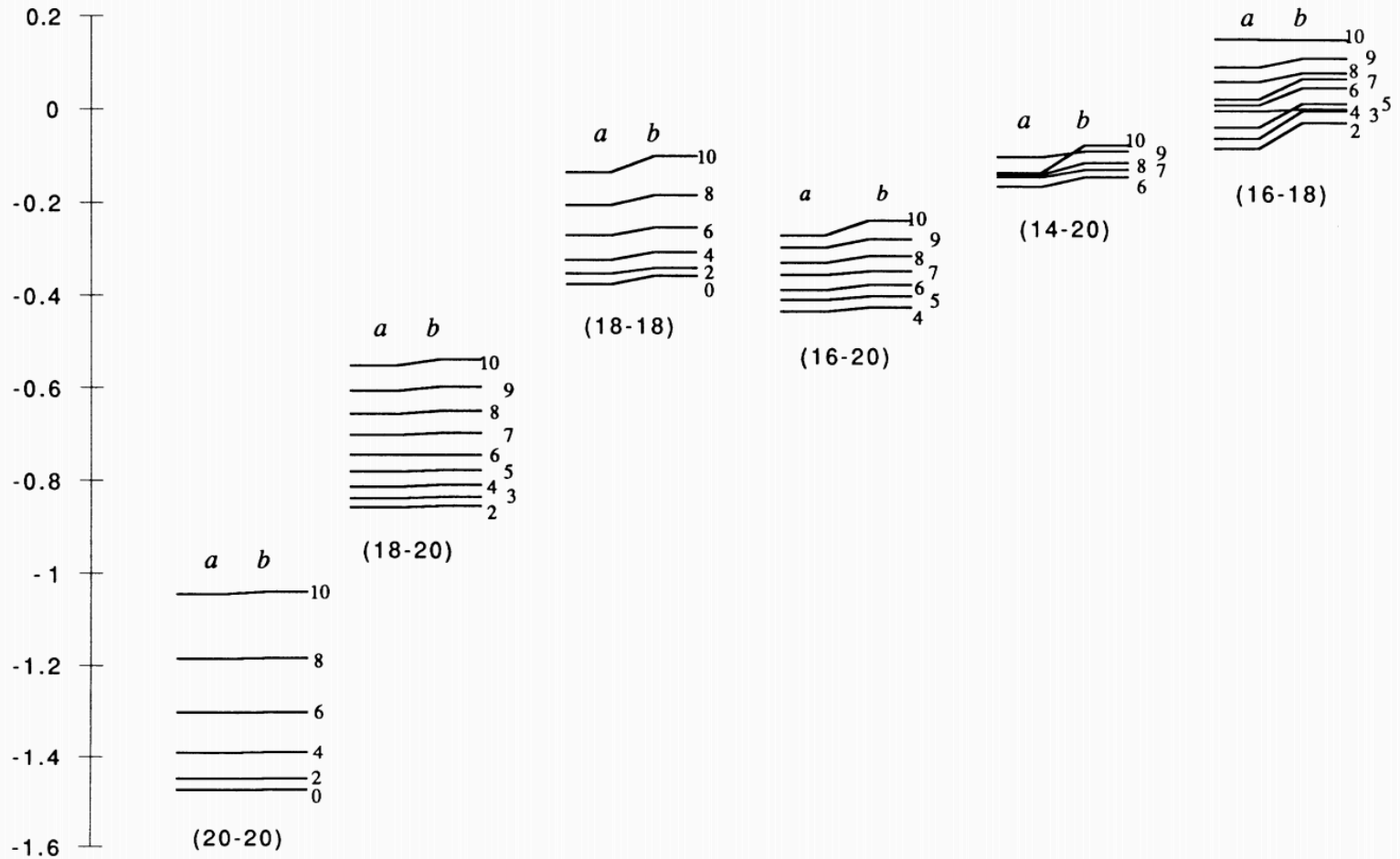


FIG. 1. The spectrum of four particles in a single- j shell ($j = \frac{21}{2}$, $H = -Q \cdot Q$, energies are in arbitrary units). Part a , the shell-model calculation; b , the GPFM calculation.

The coupling of few neutrons and protons



In 'shell model'

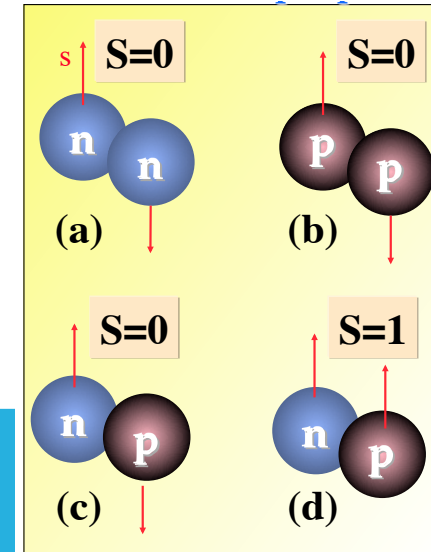
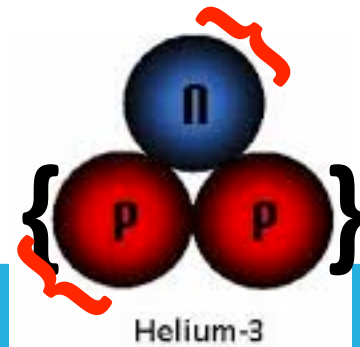
$$|\Psi_{\pi} \otimes \Psi_{\nu}\rangle$$

Or we can do like these

$$|J_1 \otimes J_2 \dots\rangle_I$$

$$|[[[J_1 \otimes J_2]_{I_{12}} \otimes J_3]_{I_{123}} \dots]\rangle_I$$

$$|[J_1 \otimes J_2]_{I_{12}} \otimes [J_3 \otimes J_4]_{I_{34}} \dots\rangle_I$$



$^{96}\text{Cd} (2n-2p)$: A simple example to show the pair content

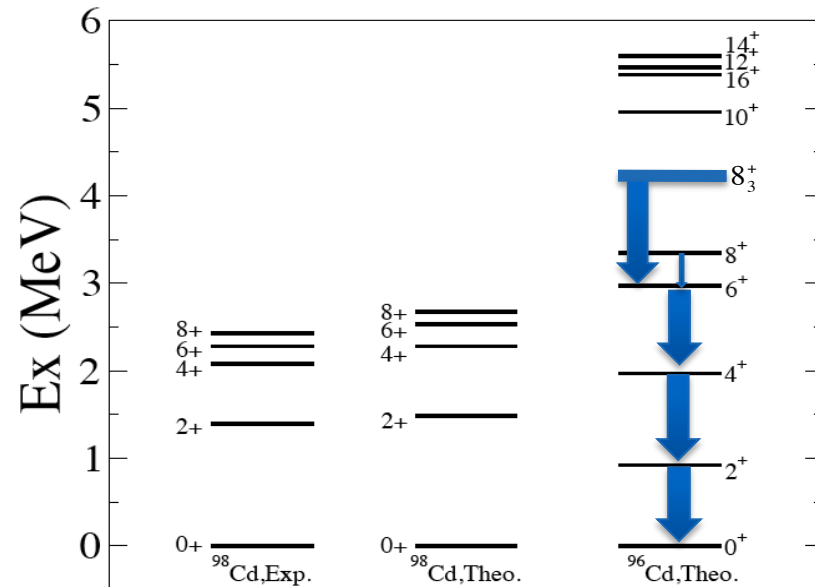


Usually the wave function can be expanded as

$$|\Psi_I\rangle = \sum_{J_p, J_n} X_I(J_p J_n) |j_\pi^2(J_p) j_\nu^2(J_n); I\rangle,$$

The thus obtained wave function is a mixture of many component as a result of the np interaction

$$\begin{aligned} |\Psi_0(\text{gs})\rangle &= 0.76|[\pi^2(0)\nu^2(0)]_I\rangle + 0.57|[\pi^2(2)\nu^2(2)]_I\rangle \\ &+ 0.24|[\pi^2(4)\nu^2(4)]_I\rangle + 0.13|[\pi^2(6)\nu^2(6)]_I\rangle \\ &+ 0.14|[\pi^2(8)\nu^2(8)]_I\rangle. \end{aligned}$$



The striking feature is that if we project it on to np coupled terms, the wave function can be represented by a single term $(\nu\pi)_9 \otimes (\nu\pi)_9$

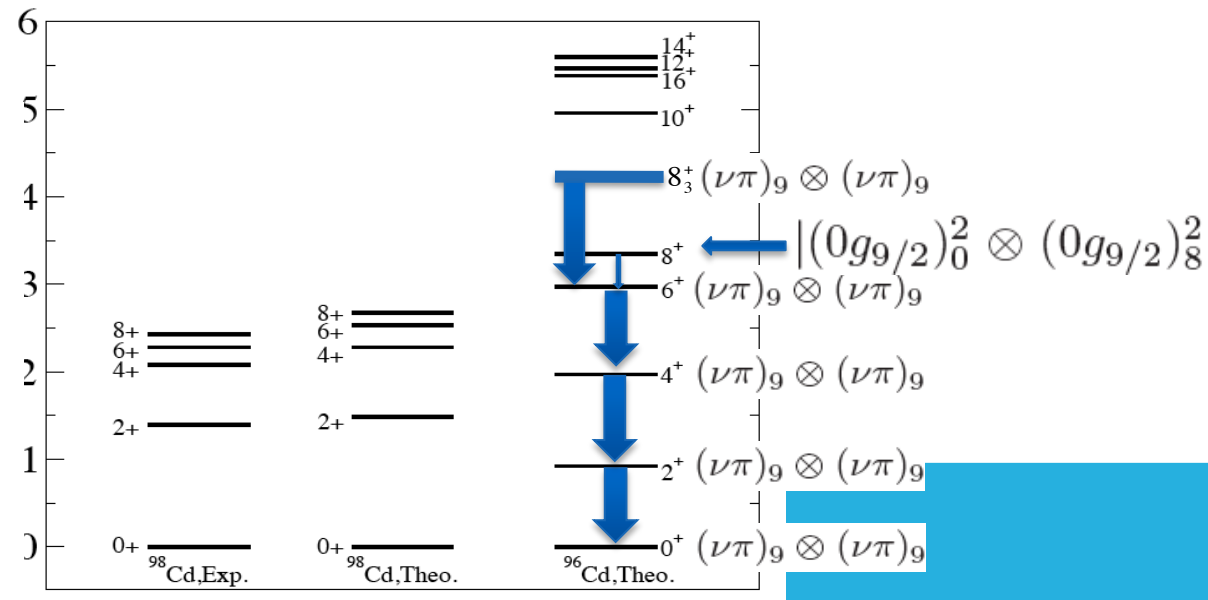
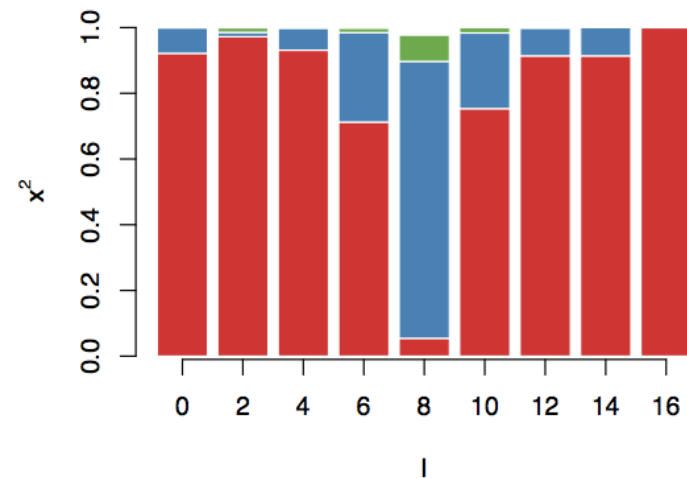
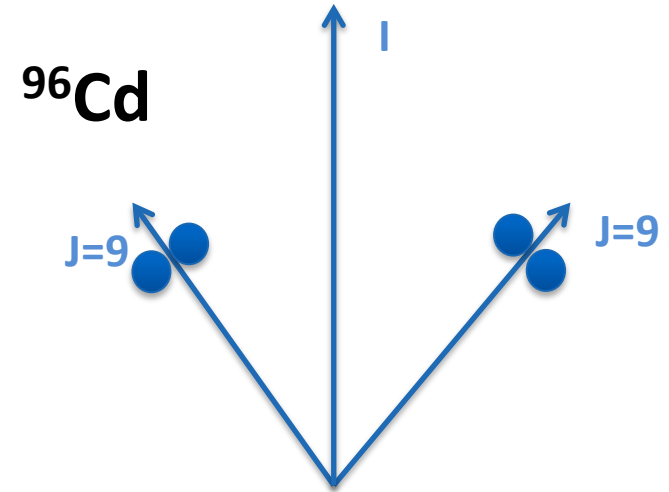
$$\langle [j_p j_n(J_1) j_p j_n(J_2)]_J | [j_p^2(J_p) j_n^2(J_n)]_J \rangle = -2\hat{J}_1 \hat{J}_2 \hat{J}_p \hat{J}_n \left\{ \begin{matrix} j & j & J_p \\ j & j & J_n \\ J_1 & J_2 & J \end{matrix} \right\}$$

$^{96}\text{Cd} (2n-2p)$: A simple example to show the pair content



The calculated spectrum show a equidistant pattern along the yrast line up to $l=6$;

A naive picture is that the angular momenta of the states are generated by the rearrangement of the angular momentum vectors of the aligned np pairs.





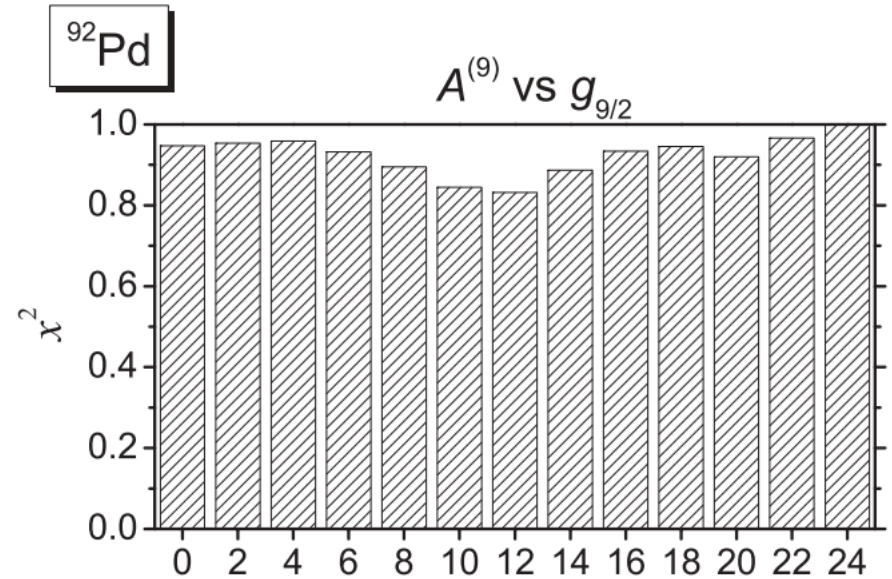
Interacting Boson models with aligned np pair

TABLE IV. Overlaps of the $(1g_{9/2})^4$ yrast eigenstates of the SLGT0 interaction with angular momentum J and isospin $T = 0$ with various two-pair states, expressed in percentages.

J	B^2	SP_J	D^2	DG	DI	DK	G^2	I^2	K^2
0	91	80	35				18	7.4	1.9
2	97	85	17	22			1.5	0.0	0.4
4	89	64	42	11	11		0.2	0.2	0.0
6	55	70		43	0.2	4.3	0.0	0.2	0.0
8	5.3	83			7.4	24	1.8	0.2	0.1
10	42					58		6.1	0.5
12	88							57	1.5
14	96								31.4
16	100								100

S. Zerguine and P. Van Isacker, *Phys. Rev. C* 83, 064314 (2011).

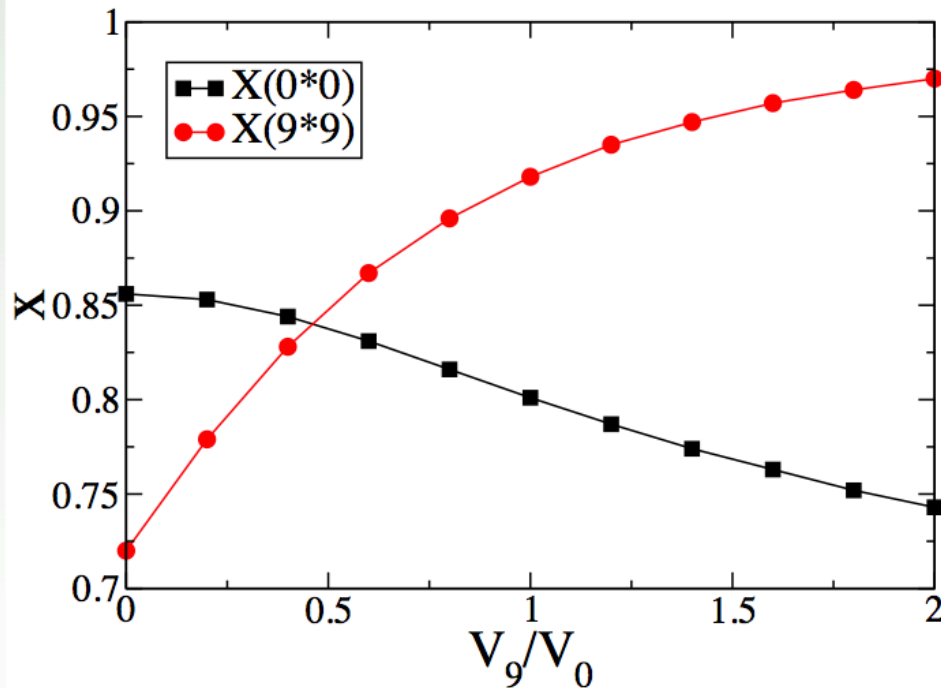
Pair truncation shell model approaches



G. J. Fu, J. J. Shen, Y. M. Zhao, and A. Arima *Phys. Rev. C* 87, 044312 (2013)

Wave function of ^{96}Cd calculated with a Hamiltonian containing $J = 0$ and 9 terms only.

- The $J=9$ term V_9 generates a states with pure aligned np coupling $|j_9^2 \otimes j_9^2\rangle$
- The inclusion of normal pairing is crucially important for reproducing the group state spin
- The $J=9$ term does not necessary to be stronger than the $J = 0$ term. It should be relatively stronger than other $T = 0$ terms. [For a simple single-j system, the relative position of $T = 0$ and 1 monopole terms does not play any effect on the wave functions.]

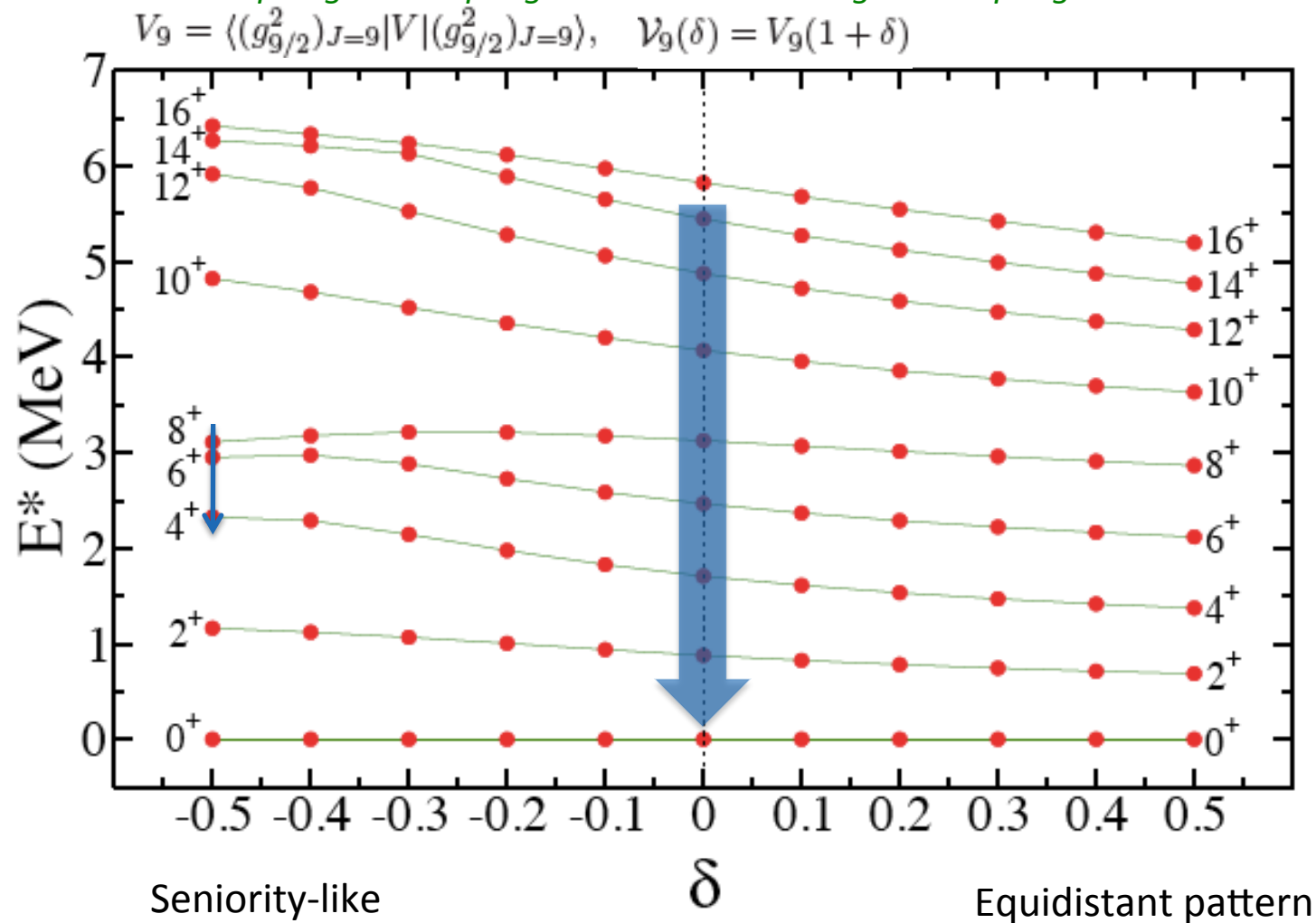


The spin-aligned pair plays a crucial role



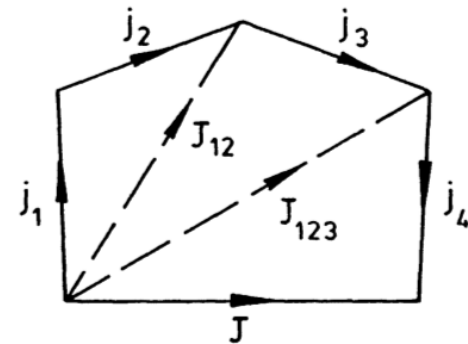
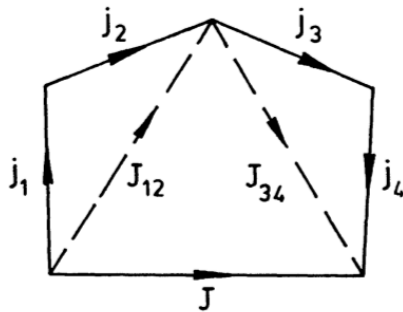
It is strongly attractive since this maximally aligned configuration has maximal overlap between the proton and neutron wave functions

Competition between the np aligned coupling and like nucleon aligned coupling?



Non-unique ways to define a many-pair state

- The four $J = 9$ np pairs in ^{92}Pd can couple in various ways. With the help of two-particle cfp one may express the wave function in terms of $((((\nu\pi)_9 \otimes (\nu\pi)_9)_{I'} \otimes (\nu\pi)_9)_{I''} \otimes (\nu\pi)_9)_I$.
- It is thus found that, among the various aligned np pair configurations, the dominating components can be well represented by a single configuration $((((\nu\pi)_9 \otimes (\nu\pi)_9)_{I'=16} \otimes (\nu\pi)_9)_{I''=9} \otimes (\nu\pi)_9)_I$. In the $0g_{9/2}$ shell, this configuration is calculated to occupy around 66% of the ground state wave function of ^{92}Pd , i.e., with amplitude $X(0_1^+) = 0.81$.
- The $J = 9$ term is not the generator for the full aligned np coupling. It generates the stretch configuration. The stretch component increase along the yrast line. The maximal $I = 24$ state corresponds to a pure stretch configuration.



CQ et al, PRC 84, 021301(R) (2011).

Z.X. Xu, C. Qi, J. Blomqvist, R.J. Liotta, R. Wyss Nucl. Phys. A 877, 51 (2012).

C. Qi, Prog. Theor. Suppl. 196, 414 (2012).

Quartet-like coupling



The four $J = 9$ np pairs in ^{92}Pd can couple in various ways. With the help of two-particle cfp one may express the wave function in terms of $((((\nu\pi)_9 \otimes (\nu\pi)_9)_{I'} \otimes (\nu\pi)_9)_{I''} \otimes (\nu\pi)_9)_{I}$.

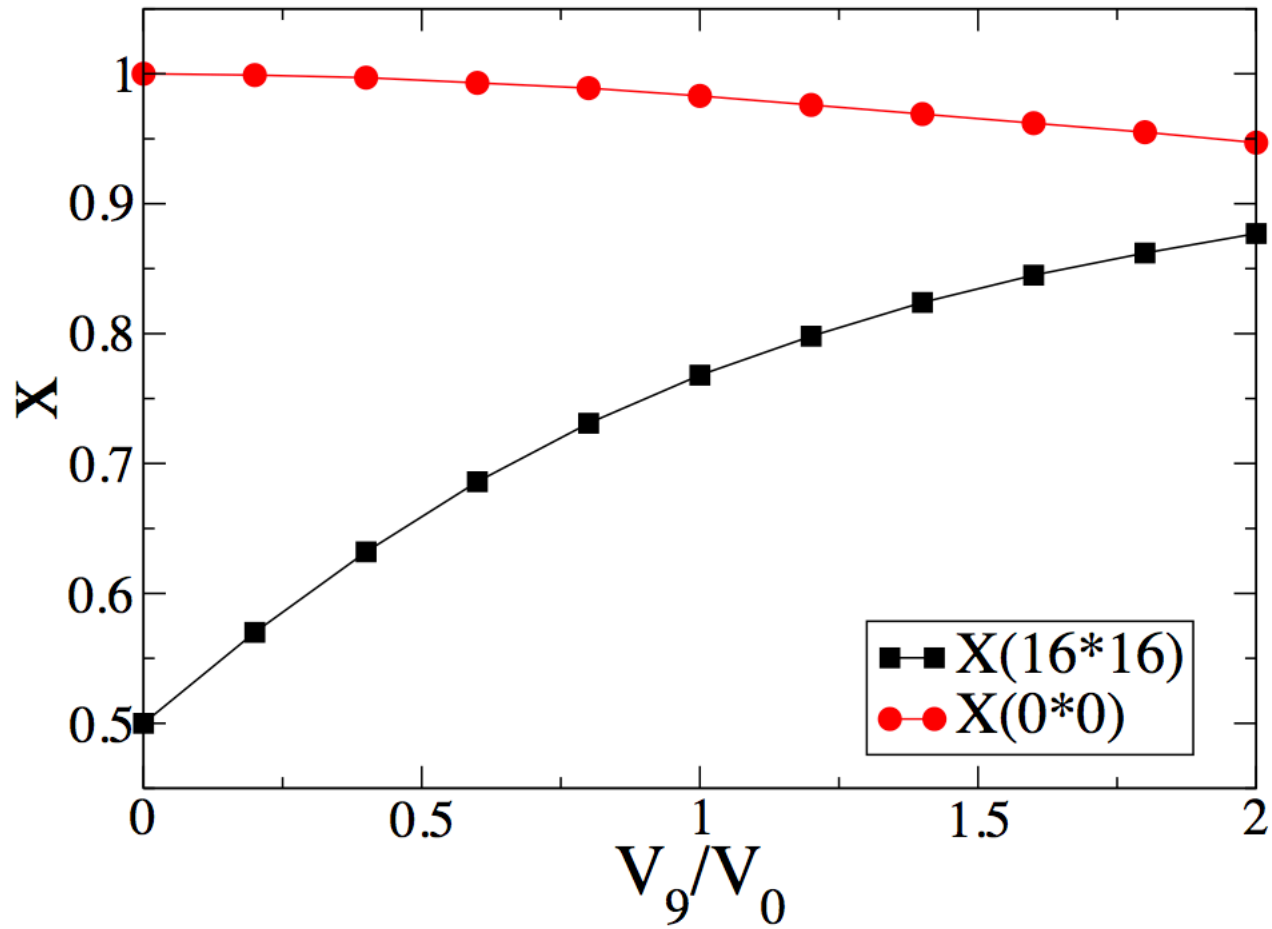
e I. Configurations with the largest probabilities for the state $^{92}\text{Pd}(0_1^+)$ corresponding to the tensorial products of different two-particle states (upper) and four-particle states (lower).

Configuration	x^2
$ \gamma_2 = 9^+ \gamma'_2 = 9^+ \gamma''_2 = 9^+ \gamma'''_2 = 9^+\rangle$	0.85
$ \gamma_2 = 9^+ \gamma'_2 = 9^+ \alpha_2 = 0^+ \beta_2 = 0^+\rangle$	0.76
$ \gamma_2 = 8^+ \gamma'_2 = 1^+ \alpha_2 = 0^+ \beta_2 = 8^+\rangle$	0.56
$ \gamma_2 = 8^+ \gamma'_2 = 1^+ \alpha_2 = 8^+ \beta_2 = 0^+\rangle$	0.56
$ \gamma_2 = 1^+ \gamma'_2 = 1^+ \alpha_2 = 0^+ \beta_2 = 0^+\rangle$	0.52
$ \gamma_4 = 0_1^+ \gamma'_4 = 0_1^+\rangle$	0.98
$ \gamma_4 = 8_1^+ \gamma'_4 = 8_1^+\rangle$	0.94
$ \gamma_4 = 8_2^+ \gamma'_4 = 8_2^+\rangle$	0.92
$ \gamma_4 = 16_1^+ \gamma'_4 = 16_1^+\rangle$	0.81

Quartet-like coupling

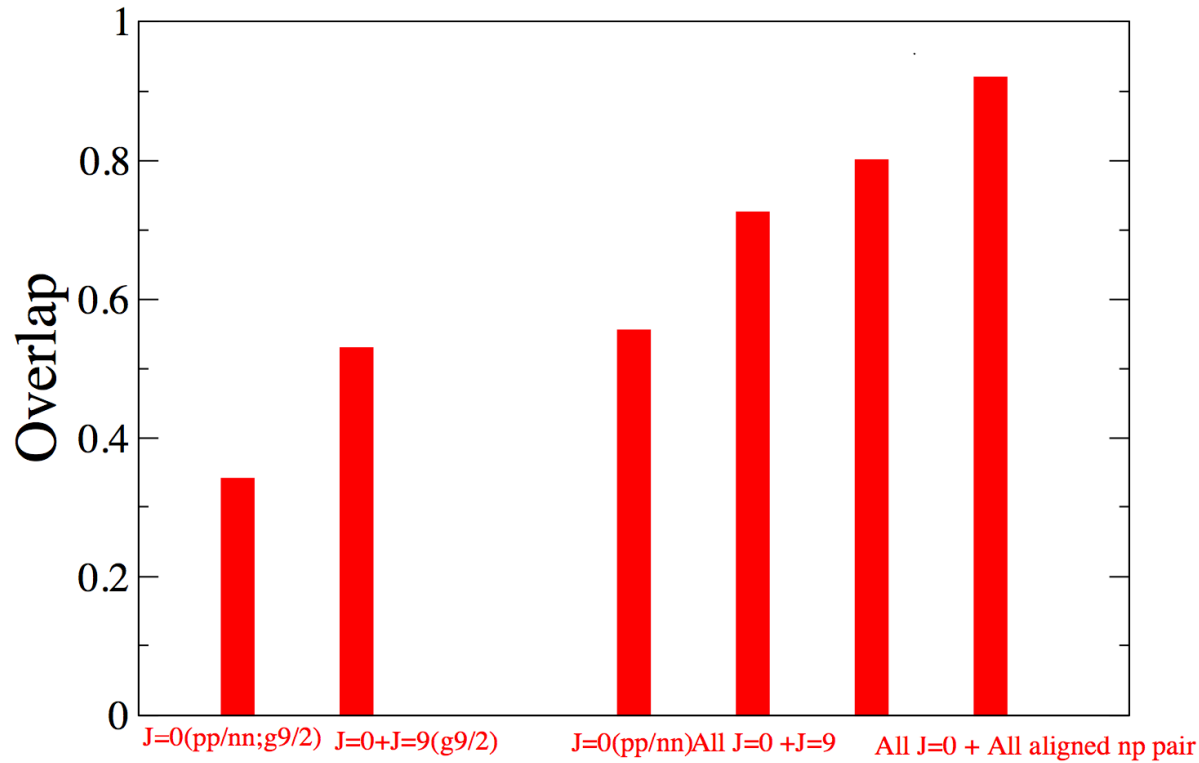


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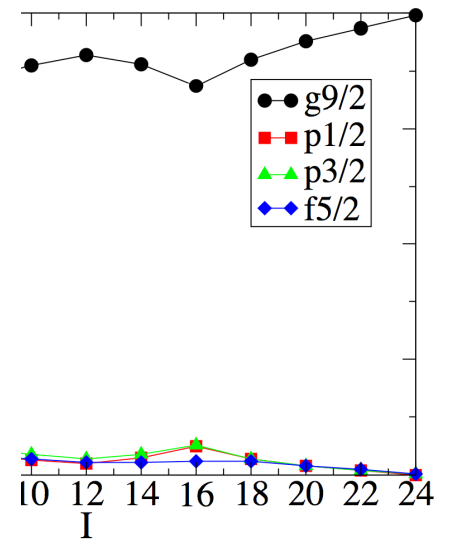


he

Extending to a space with several orbitals



ticles in different shells for ^{92}Pd



Discovery of ^{109}Xe and ^{105}Te : Superaligned α Decay near Doubly Magic ^{100}Sn

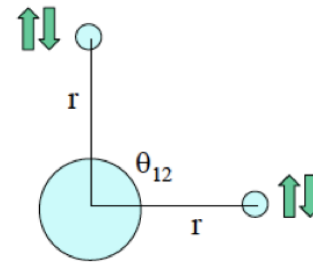
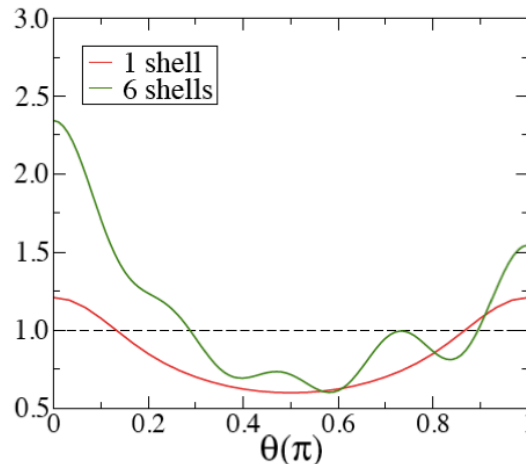
S. N. Liddick,¹ R. Grzywacz,^{2,3} C. Mazzocchi,² R. D. Page,⁴ K. P. Rykaczewski,³ J. C. Batchelder,¹ C. R. Bingham,^{2,3} I. G. Darby,⁴ G. Drafta,² C. Goodin,⁵ C. J. Gross,³ J. H. Hamilton,⁵ A. A. Hecht,⁶ J. K. Hwang,⁵ S. Ilyushkin,⁷ D. T. Joss,⁴ A. Korgul,^{2,5,8,9} W. Królás,^{9,10} K. Lagergren,⁹ K. Li,⁵ M. N. Tantawy,² J. Thomson,⁴ and J. A. Winger^{1,7,9}

The four-body (alpha) wave function can be written as

$$|\gamma_4\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2; \gamma_4) |\alpha_2 \otimes \beta_2\rangle,$$

where α_2 and β_2 denote proton and neutron wave functions, respectively.

Shell model
calculations on the
alpha formation
amplitude in N=Z
nuclei.



Relative angular distribution of four-particle wave function with (solid lines) and without (dashed line) neutron-proton interactions.

Four particle spatial correlation vs alpha

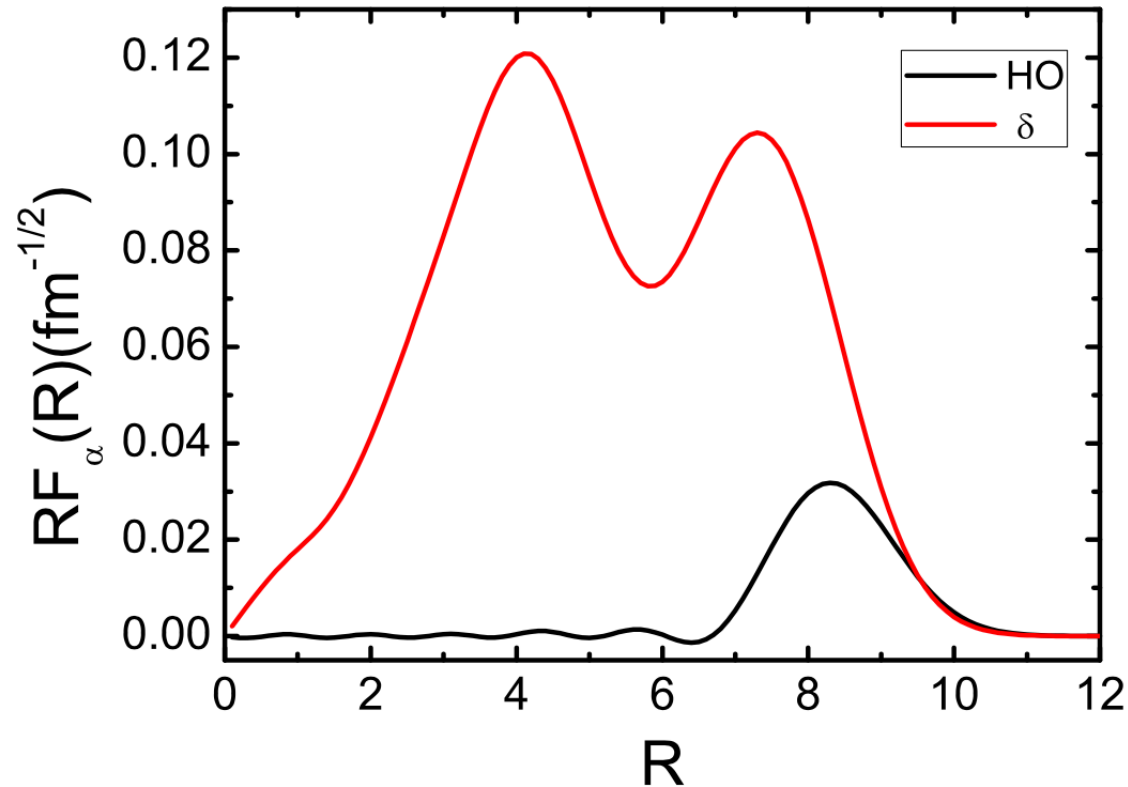


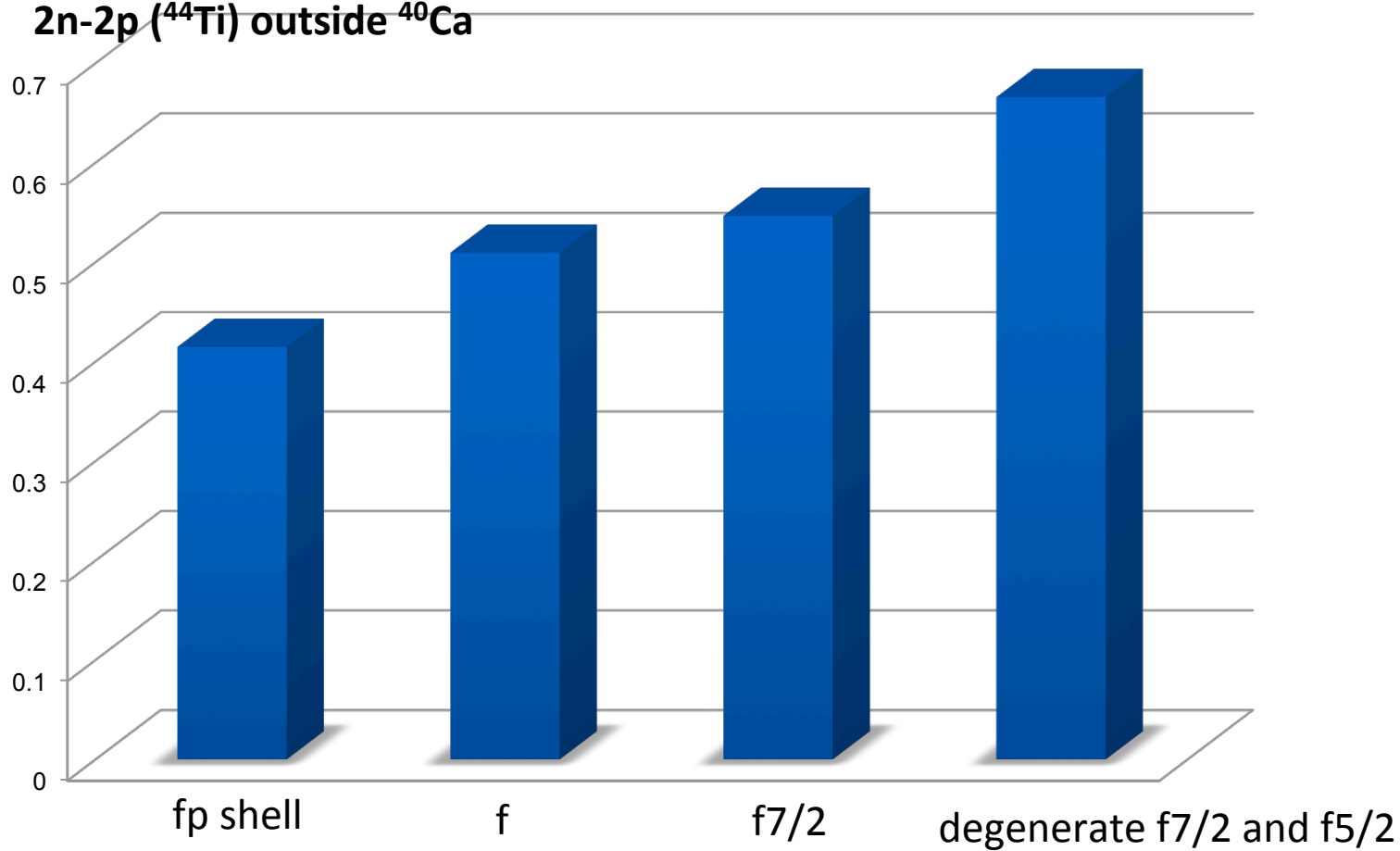
FIG. 10. (Color online) Comparison of the α formation amplitude $RF_\alpha(R)$ of ^{212}Po calculated from the δ function approximation and the Gaussian form of the α intrinsic wave function.



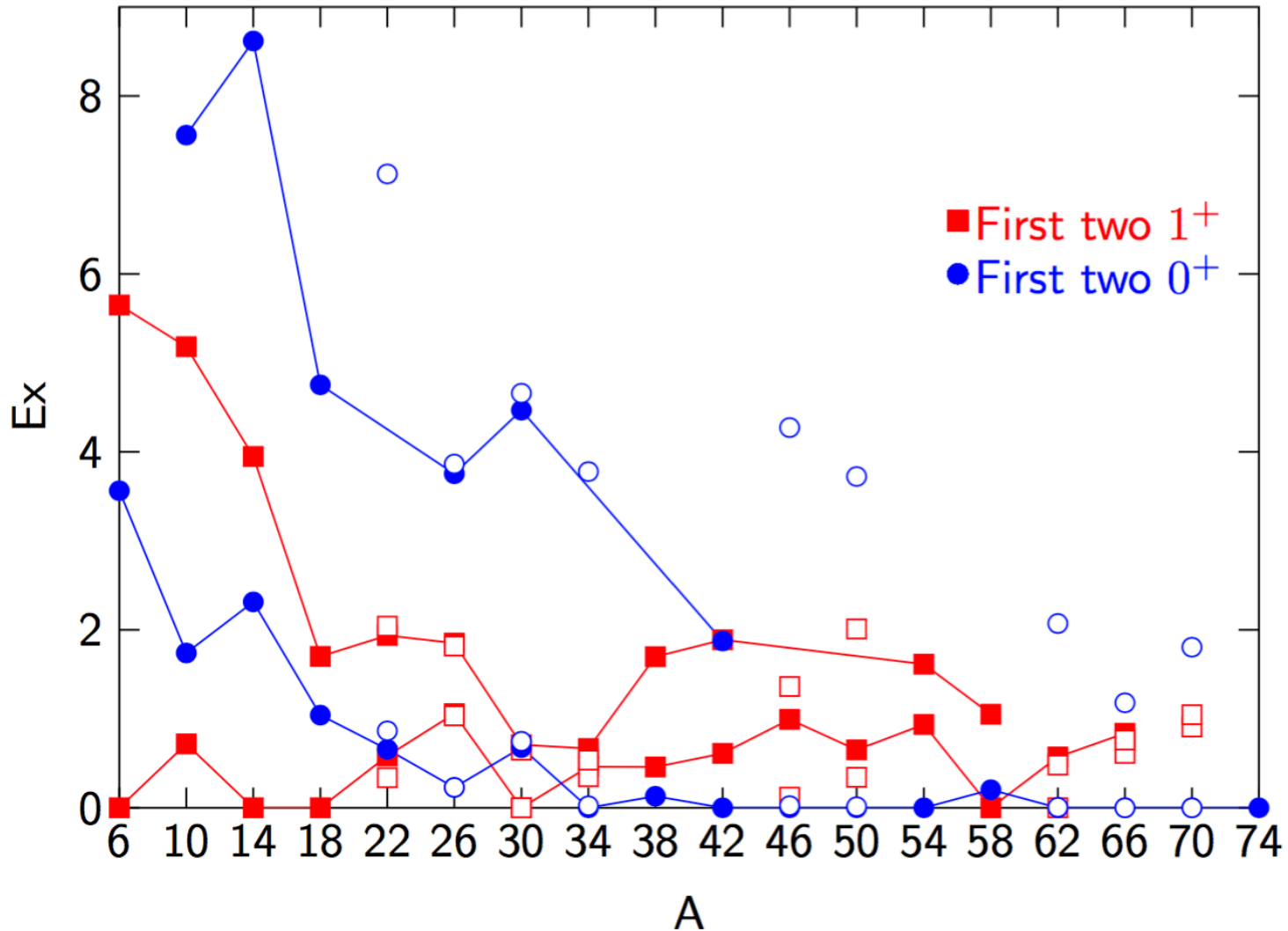
The J=1 pair

Overlap between the full wave function and that generated by the J=1 pair Hamiltonian

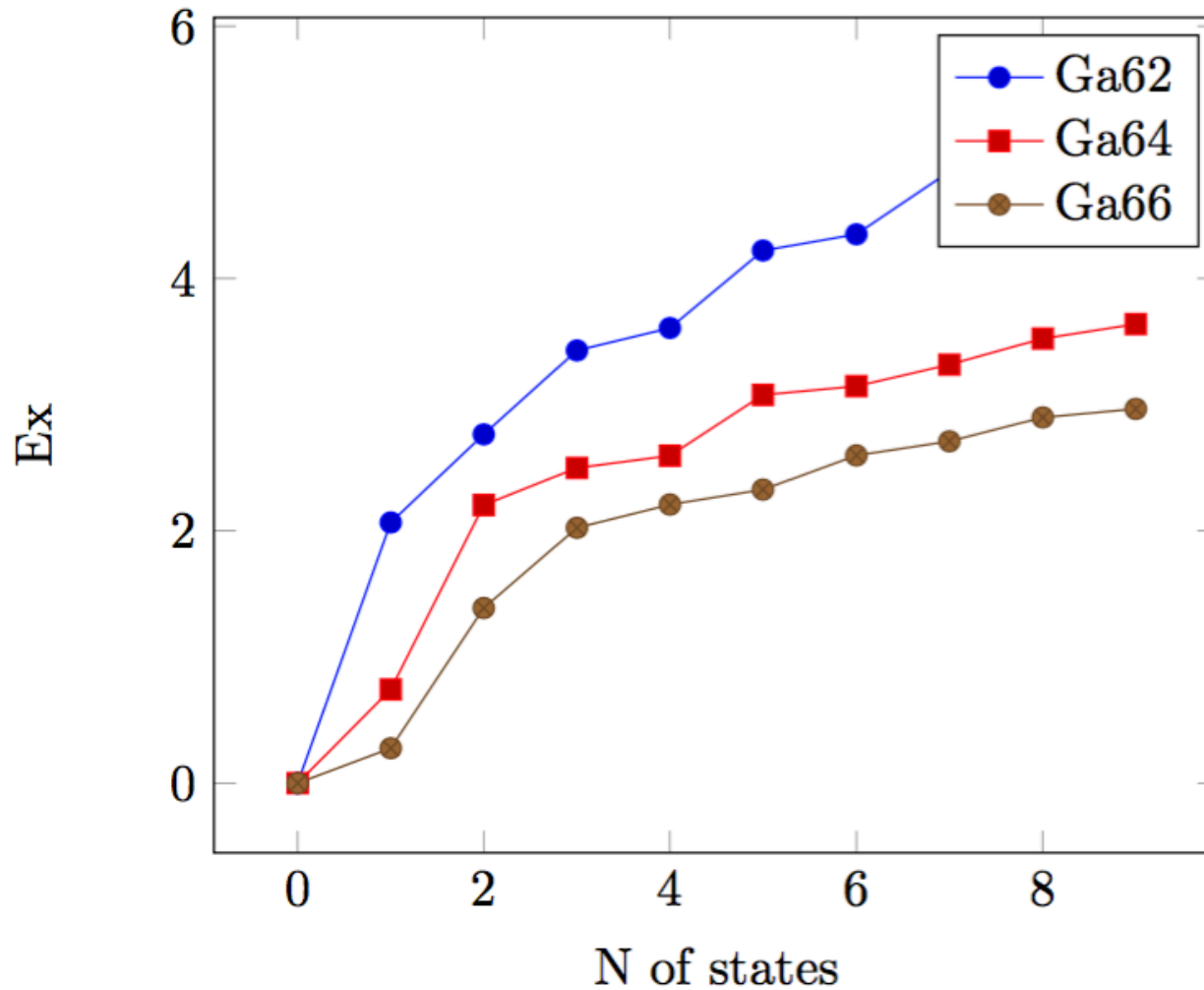
2n-2p (^{44}Ti) outside ^{40}Ca



Experimental and calculated 1+ and 0+ in odd-odd N=Z



Calculated low-lying 0^+ states





Summary

- The np pair correlation from the binding energy difference
- Neutron-proton spin-aligned pair coupling in $N=Z$ nuclei
 - ❖ *Crucial building blocks of the nuclear many-body wave function.*
 - ❖ *Important part of the effective two-body interaction.*
 - ❖ *Exhibiting regular low-lying spectra and $E2$ transition probabilities.*

Calculations in different spaces for ^{94}Ag (3p-3n)

	1^+	1142	
	9^+	1109	
	8^+	1064	
		<u> </u>	
9^+		976	
8^+		925	
		<u> </u>	
2^+		843	
	2^+	870	
		<u> </u>	
	7^+	760	
		<u> </u>	
7^+		614	
		<u> </u>	

^{94}Ag	^{94}Ag
fpg	pg
0^+	0^+
<u> </u>	<u> </u>
0	0

The ground state spin is calculated to be 0^+ .

The lowest $T = 0$ state is 7^+ .

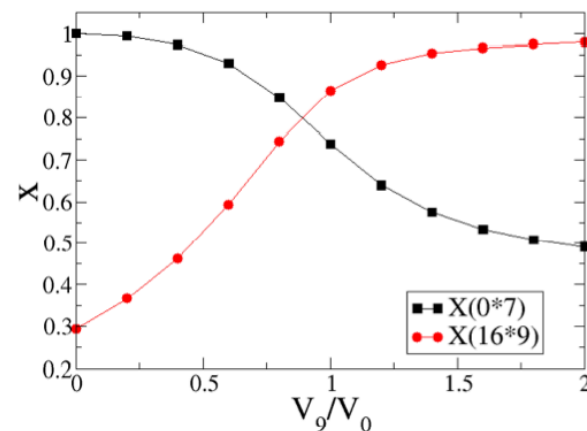
The wave function is dominated by

$$|[(j^2)_9(j^2)_9]_{16}(j^2)_9\rangle_{I=7}$$

A stretch isomer?

$0g_{9/2}$ -shell description of ^{94}Ag (3p-3n): 7_1^+

For simplicity, the Hamiltonian only contain the two matrix elements V_0 and V_9 . The wave function is dominated by the configuration of $|[(j^2)_9(j^2)_9]_{16}(j^2)_9\rangle_{I=7}$. Calculation with a realistic Hamiltonian gives a even larger value.



The odd system of ^{97}Cd (^{97}In)

- The odd system (2p-1n) exhibits similarity with the correspondingly 2p-2n system
- A natural outcome of the aligned coupling scheme is that the $9/2_1^+$ (The calculated ground state) is dominated by the coupling $|(j^2)_9 \otimes j\rangle$.

