Many-body observables and effectiveness in nuclear physics

Mehdi Drissi

ESNT Workshop « Qu'est ce que l'observable »

22/06/18

Supervisors:

T. Duguet (CEA-Saclay) and V. Somà (CEA-Saclay)

Collaborators:

U. Van Kolck (IPN Orsay), CJ. Yang (Washington University) , M. Pavón Valderrama (Beihang University)

Outline

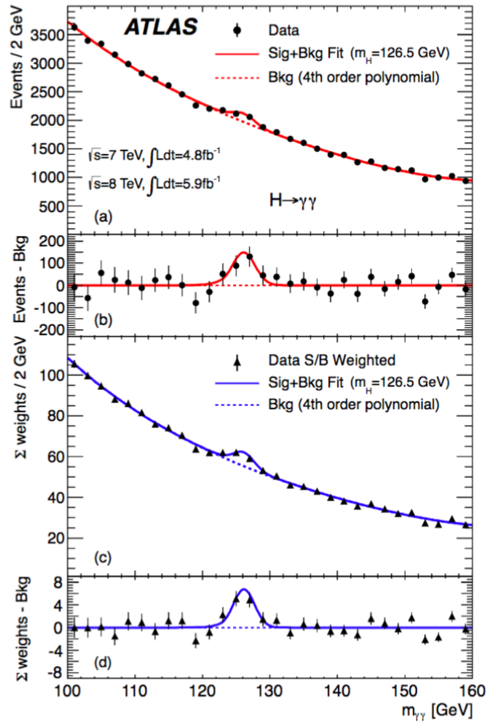
1. **Effectiveness of observable**
2. Effective field theories (EFTs)
3. Nuclear systems and EFTs
4. Extension to many-body observables

Effectiveness of observable

The need for a theory decoupling from UV physics

Observables in experiments

- Experimental observables result of a measurement process
- Extract number from experience
  - ➔ Already non-observable/extracted
  - ➔ Try to define a number least dependent on measurement process (e.g. length littoral)
- Measurements are limited by the resolution of our tools (accuracy)
- Improve resolution by taking explicitly into account known background processes
  - ➔ E.g. subtraction of background by PSD/ToF analysis
- But up to now our measurement capabilities remains finite
  - ➔ There is always an uncertainty on the result of an experiment



[Atlas Collaboration, Phys. Lett. B, 2012]

Experimental observables come in probability distribution of numbers

Theoretical prediction of observables

Old-fashion approach

1. Set a model
   - Degrees of freedom
   - Dynamical equations
   - Relation to observables

2. Make predictions
   - Expected value

3. Compare with experimental data
   Statistical tests e.g. χ^2

But one never observes an exact value

Effective approach

1. Set a model
   - Idem

   \+ Effective model ≠ elementary degree of freedoms
   - Scale of unresolved physics M_{hi}
   - Estimation of corrections on observables

2. Make predictions
   - Mean value
   - Standard deviation (+ higher-moments)

3. Compare with experimental data
   Statistical tests ← theoretical uncertainty

But predictions on what ?

Effective approach in Quantum Mechanic

Observables → Amplitude of transition associated to unitary evolution operator (S-matrix)

Observables → Eigenvalues of self-adjoint operator $\mathcal{O}$ → e.g. Hamiltonian for energy

of a system with particular symmetry → Eigenvalues in a **subspace** of the Fock space $\mathcal{F}$

General system

$$H = \sum_{p,A=0}^{+\infty} H_A^{(p)} \qquad \langle H \rangle = \left\langle \sum_{p,A=0}^{+\infty} H_A^{(p)} \right\rangle_{\mathcal{F}}$$

System with A particles

$$H^{\text{eff}} = \sum_{A_1=}^{A} \sum_{p=0}^{+\infty} H_{A_1}^{(p)} \qquad \langle H \rangle = \left\langle \sum_{A_1=}^{A} \sum_{p=0}^{+\infty} H_{A_1}^{(p)} \right\rangle_{\mathcal{F}_A \subset \mathcal{F}}$$

	2-body	3-body	4-body
LO			X
NLO		X	?
N²LO	X		?

How to estimate corrections ?

The decoupling assumption

- To be able to make predictions, assume:

H: elt dof ~ ▪ Physics beyond M_{hi} (UV) can be neglected (strong)

OR

H: eff dof ~ ▪ UV can be taken into account effectively (weak)

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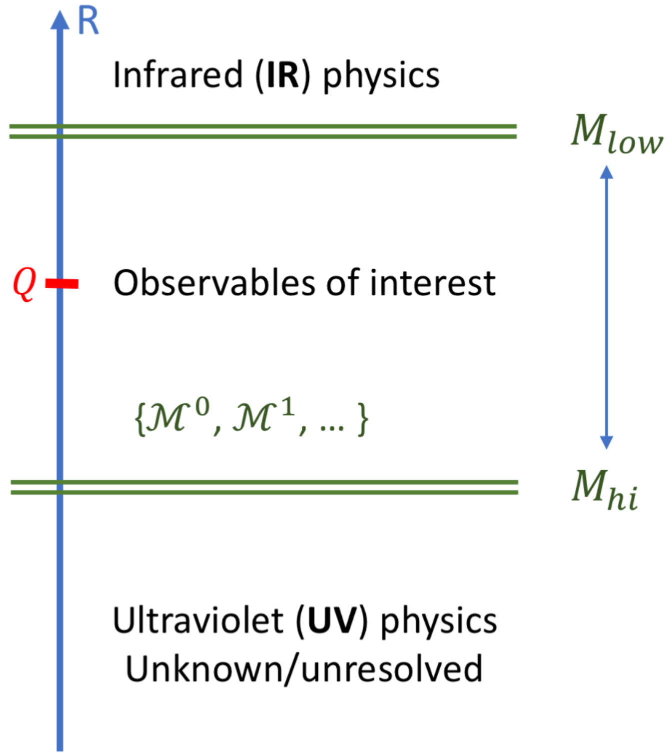
- From one model $\mathcal{M}$ to a class of models $\{\mathcal{M}^p, p \geq 0\}$:
  - $\mathcal{M}^0$ → Leading contribution
  - $\mathcal{M}^p$ → Refined models at order p
  - Convergence/divergence with $p \to +\infty$ $\Leftrightarrow$ Range of validity
  - UV effectively taken into account in parameters of $\mathcal{M}^p$
  - **But** each order needs more & more parameters !

In range of validity (applies to the last two items)



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Effective Field Theories

Observables and their decoupling from UV physics

Effective field theory implementation

- Folk theorem : [S. Weinberg 79]

  The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and the assumed symmetries.

- Inputs
  - Degrees of freedom

    → type of particles (e.g. nucleon, pion etc)
  - Symmetries

    → Internal + Space-time
  - Coupling constants

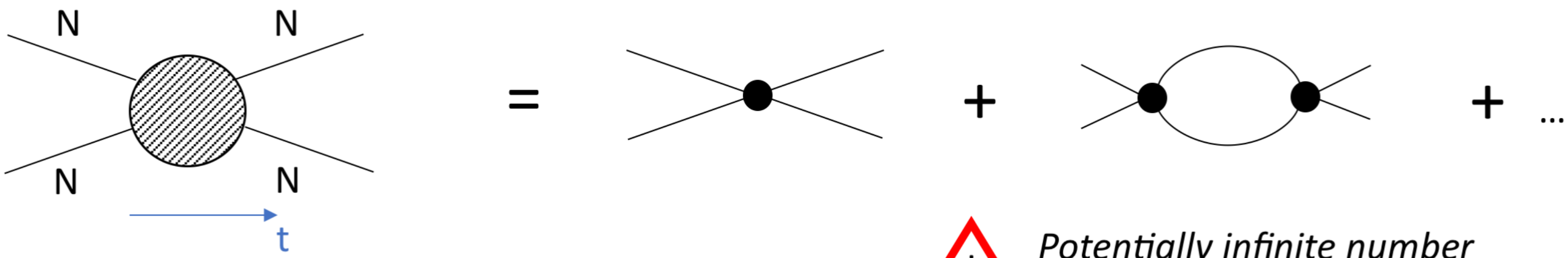
    → e.g. fitted to experimental data (or to an underlying theory)

    → Models already contains uncertainty through their fit

→ **Gives you unsolvable dynamical equations ! (nice but insufficient for observable predictions)**

Perturbation theory

- **Feynman diagrams.** Physical process decomposed as a sum of perturbations



Potentially infinite number of diagrams and type of vertices

- **Power-counting rules.** Which diagrams contribute to different order ?

Truncation of perturbation series to (hopefully) a computable, at each order, subpart

From UV divergences to UV independence

- **Problem**
  - Most diagrams are UV divergent
  - Ill-defined theory
- **But**
  - Feynman diagrams are not observable ! Only their sum gives observable estimation
  - EFT calculations should be independent of UV

    → We can introduce an arbitrary regularization procedure → Modify Hamiltonian in the UV sector

    → Observables should be independent of this regularization
- **Solution**
  1. Modify the UV arbitrarily → well-defined theory
  2. Modifying Hamiltonian $H = H_b + H_{ct}$ → recover independence of UV
  3. Fit the **total** Hamiltonian to experimental data

  } Renormalization procedure

→
- Arbitrariness of regularization compensated by regulator dependent Hamiltonian.
- In practice H_{ct} will be of the same form as H_b → Regulator dependence in coupling constants.
- In the end we should verify

$\partial_\Lambda \langle \mathcal{O}(\Lambda, C(\Lambda))\rangle \approx 0$ → Renormalization group (RG) equations

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Nuclear systems

Fundamental problem of current phenomenological descriptions

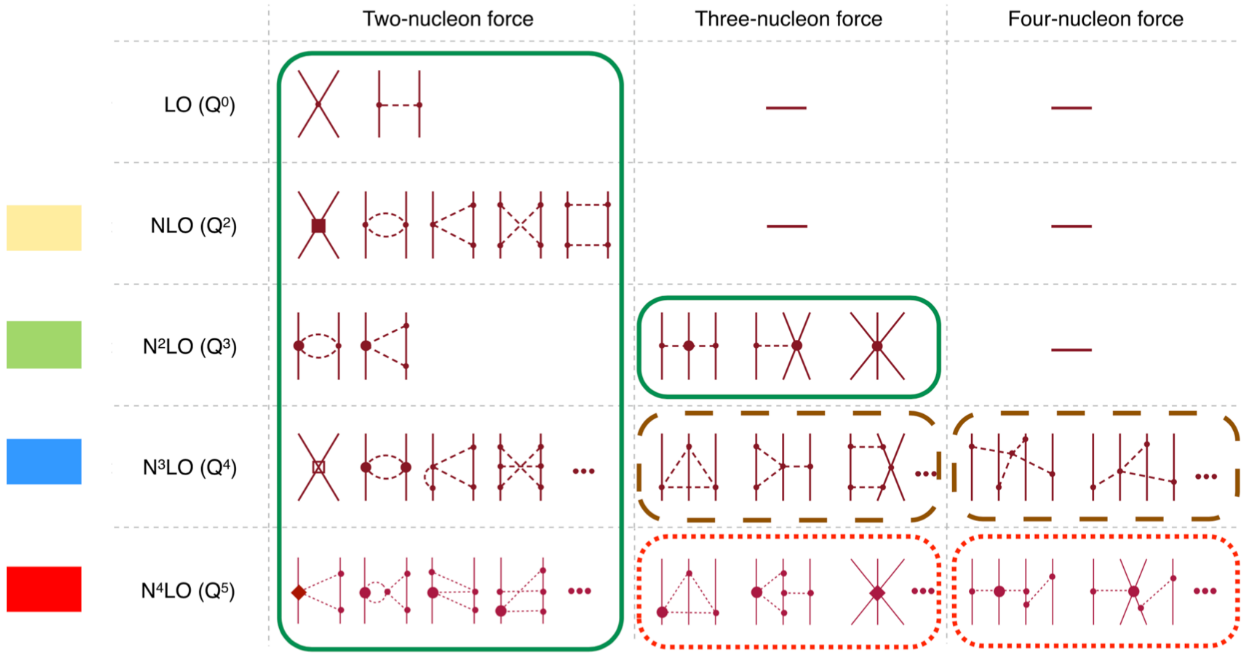
Historical perspective on NN interactions

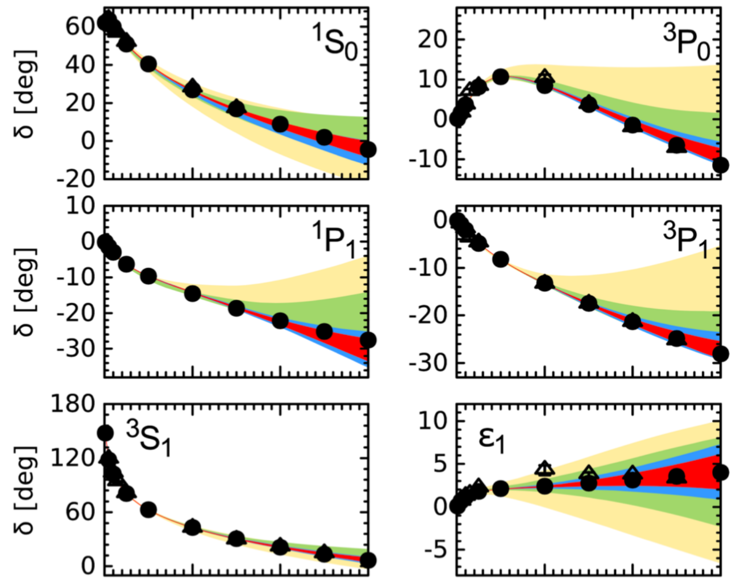
- 1935 : Yukawa potential
  → Birth of meson theory
- 40s-50s : Pion theories
  → Troubles with multi-pion, anti-nucleons diagrams
- 60s-00s : One-Boson-Exchange Model (ρ, σ, ω, ...)
  → Good data fitting
  → But no systematic
- 90s – today : EFT based interactions [S.Weinberg 90 91]
  → Chiral potentials → Try to estimate errors from breaking of RG
  [Entem, Machleidt 03] [Epelbaum, Glöckle, Meissner 05]

  → EFTs → Take RG invariance as guiding constraint
  [Kaplan, Savage, Wise 96] [Nogga, Timmermans, van Kolck 05]

No RG invariance

Chiral potentials





Generally UV dependent observables → problems needs to be addressed

EFTs program in nuclear physics

Start simple

→ Pionless theory
→ few-body $A \leq 4$ (here only $^1S_0$ partial wave)

} Gives your Hamiltonian

	2-body	3-body	4-body
LO			X
NLO		X	?
N²LO	X		?

Power-counting rules

→ Naïve Dimensional Analysis ?
→ Resum any diagram with LO vertices ?

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Back to unsolvable problem in many-body sector

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Needs to investigate further truncation scheme many-body sector

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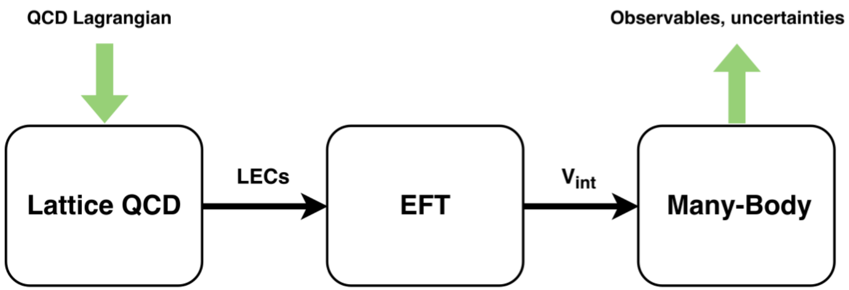
Problems with many-body observables in nuclear systems

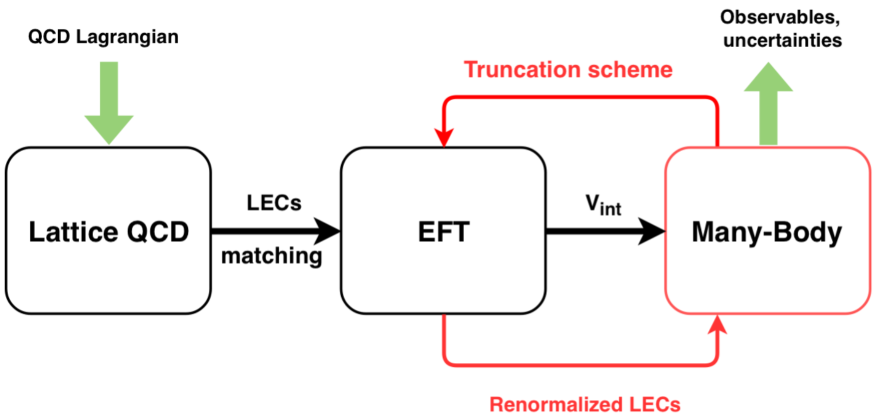
Extending EFT requirements to many-body schemes

EFTs and many-body observables

- Traditional view : V_{int} as a black box



- Interplay using EFT interactions



Many many-body schemes

- Focus on Many-Body Perturbation Theories

Fock space decomposition

$$\mathcal{F} \equiv \bigoplus_{N=0}^{+\infty} (\mathcal{H}'_1)^{\otimes N} \quad \Longleftrightarrow \quad \begin{array}{c} \forall \mu,\ \beta^\dagger_\mu |\Phi\rangle = |\Phi^\mu\rangle \\ \beta_\mu |\Phi\rangle = 0\ . \end{array} \quad \Longleftrightarrow \quad \begin{array}{l} \{\beta^\dagger_\mu, \beta_\nu\} = \delta_{\mu\nu} \\ \{\beta^\dagger_\mu, \beta^\dagger_\nu\} = 0 \\ \{\beta_\mu, \beta_\nu\} = 0\ . \end{array}$$

Hamiltonian decomposition

$$H = H_0 + H_1 \quad \Longleftrightarrow \quad \begin{array}{l} H_0 \equiv H^{00} + \bar{H}^{11} \\ H_1 \equiv H^{20} + \breve{H}^{11} + H^{02} \end{array} \quad \Longleftrightarrow \quad \bar{H}^{11} = \sum_\mu E^0_\mu \beta^\dagger_\mu \beta_\mu$$

(both decompositions) $\rightarrow$ $\boldsymbol{G^0_{\mu\nu}}$

- What EFT in many-body sector ? → Need to test many different truncation schemes

How renormalization procedure translates into MBPTs ?

Fermi-sea MBPT

- Fermi-sea decomposition

Fermi-sea vacuum

$$|\Phi\rangle = \prod_{p<k_f} a_p^\dagger \, |0\rangle$$

Kinetic basis

$$|p\rangle$$

Kinetic spectrum

$$\{E_p = \frac{p^2}{2m}\}$$

- Bare propagator

$$iG^0_{\alpha\beta}(p,\omega) = \delta_{\alpha\beta}\left[\frac{\theta(p-k_f)}{\omega - E_p + i\epsilon} + \frac{\theta(k_f - p)}{\omega - E_p - i\epsilon}\right]$$

- Rewrite it in a medium-insertion approach

$$iG^0_{\alpha\beta}(p,\omega) = \delta_{\alpha\beta}\left[\frac{1}{\omega - E_p + i\epsilon} + 2i\pi\delta(\omega - E_p)\theta(k_f - p)\right]$$

Importance of renormalization consistency

Hartree-Fock : (expanded in $\frac{2\sqrt{2}k_F}{\Lambda}$ for convenience)

$$\frac{E^{HF}}{A}(\Lambda; k_F) = \frac{k_F^2}{2m}\left\{\frac{3}{5} - \frac{mC_0(\Lambda)}{(2\pi)^2} 8k_F \left[\frac{1}{12} + O\left(\left(\frac{2\sqrt{2}k_F}{\Lambda}\right)^2\right)\right]\right\}$$

Ladder pp/hh resummation :

$$\frac{E^{Ld}}{A}(\Lambda; k_F) = \frac{k_F^2}{2m}\left\{\frac{3}{5} - \frac{48}{\pi}\int_0^1 \mathrm{d}s\ s^2 \int_0^{\sqrt{1-s^2}} \mathrm{d}\kappa\ \kappa \arctan\left(\frac{k_F I_{\frac{k_F}{\Lambda}}(s,\kappa)}{\frac{4\pi}{mC_0(\Lambda)} + \frac{k_F}{\pi}\tilde{R}_{\frac{k_F}{\Lambda}}(s,\kappa)}\right)\right\}$$

+

$C_0(\Lambda)$ matched at first order to a_0

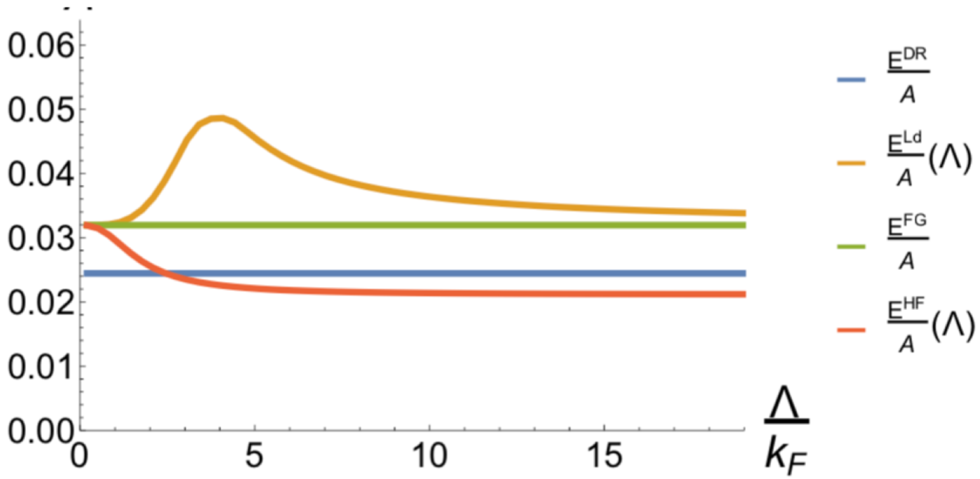
$$C_0(\Lambda) = \frac{4\pi}{M} a_0$$

$C_0(\Lambda)$ exactly matched to a_0

$$C_0(\Lambda) = \frac{4\pi}{M}\frac{1}{\frac{1}{a_0} - \frac{\Lambda}{\sqrt{2\pi}}}$$

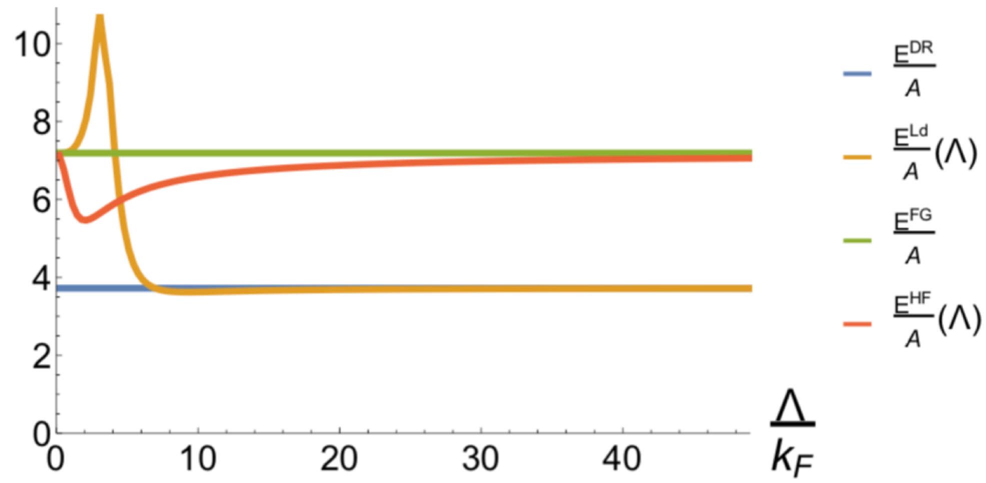
Importance of renormalization consistency

Hartree-Fock : (expanded in $\frac{2\sqrt{2}k_F}{\Lambda}$ for convenience)



$k_F = 60$ MeV

Ladder pp/hh resummation :



$k_F = 150$ MeV

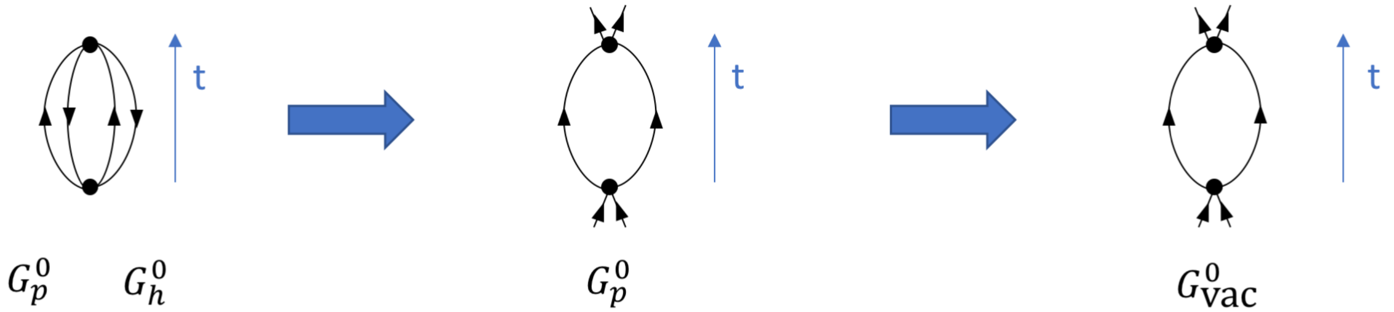
$\frac{E^{DR}}{A}$ from [Kaiser 11]

Toward general rules for renormalizing MBPTs

- Weinberg's asymptotic theorem

  UV divergences of Feynman diagrams depends only on asymptotic coefficient of the used bare propagator.

- Fermi-sea MBPT revisited
  - Hole propagator → No UV contributions → factorize out of integral
  - Particle propagator → Same UV behaviour as propagator in particle vacuum



- On going generalization to more general MBPTs

Merci !