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Observable and quasi-observable in the nucleon spin decomposition

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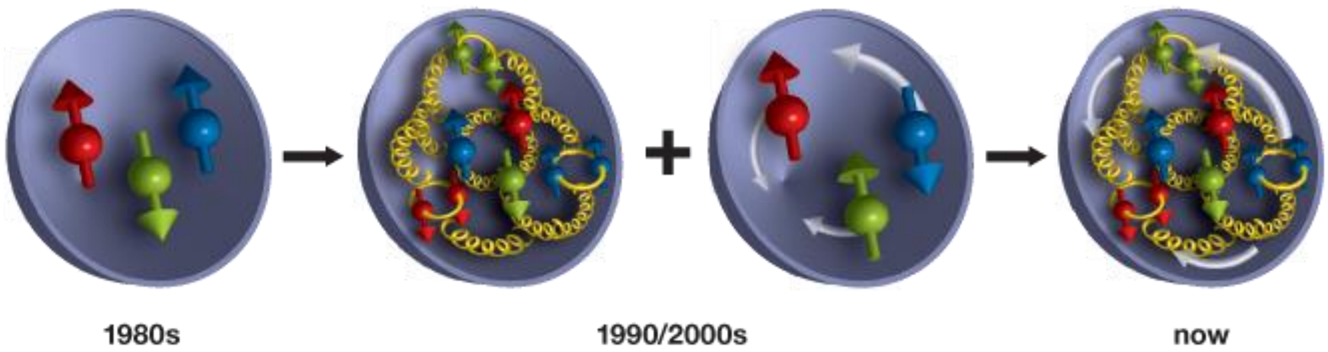
Outline

- Photon angular momentum
- Modern nucleon spin decomposition
- Active and passive transformations
- Background dependence

Structure of nucleons

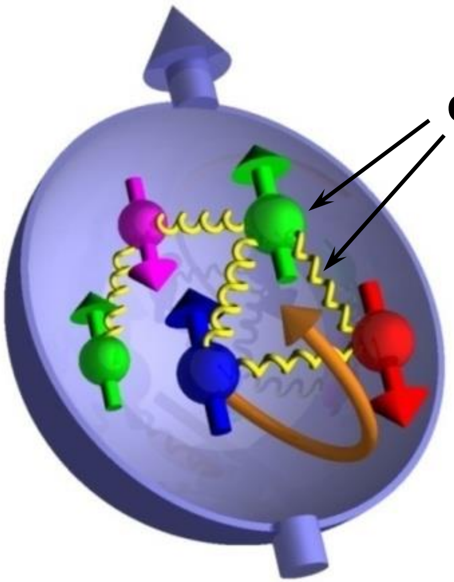
Our picture/understanding of the nucleon evolves !



But many questions remain unanswered …

In particular, where does the nucleon spin come from ?

Gauge theory



Gauge non-invariant

Gauge invariant

[…] in QCD we should make clear what a quark or gluon parton is in an interacting theory. The subtlety here is in the issue of gauge invariance: a pure quark field in one gauge is a superposition of quarks and gluons in another. Different ways of gluon field gauge fixing predetermine different decompositions of the coupled quark-gluon fields into quark and gluon degrees of freedom.

[Bashinsky, Jaffe (1998)]

A choice of gauge is akin to a choice of basis

Photon spin and OAM

Most textbooks claim that no gauge-invariant decomposition of photon AM exists

$$\vec{J}_\gamma = \int \mathrm{d}^3r\, \vec{r} \times (\vec{E} \times \vec{B})$$

But formally the following decomposition is gauge invariant

$$\vec{S}_\gamma = \int \mathrm{d}^3r\, \vec{E} \times \vec{A}_\perp$$

$$\vec{L}_\gamma = \int \mathrm{d}^3r\, E^i(\vec{r} \times \vec{\nabla})A^i_\perp$$

Helmoltz decomposition

$$\vec{A} = \vec{A}_\parallel + \vec{A}_\perp, \qquad \vec{A}_\parallel = \vec{\nabla}\,\frac{\vec{\nabla}\cdot\vec{A}}{\vec{\nabla}^2}$$

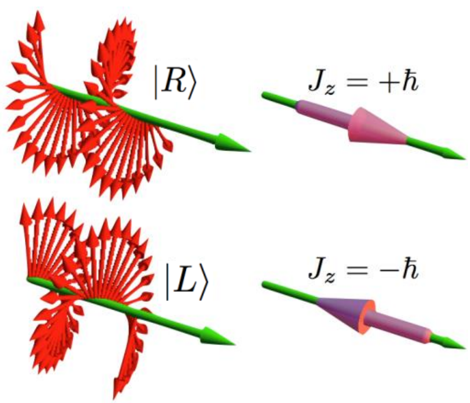
Non-local !

No contradiction because most textbooks implicitly refer to local expressions only !

Photon spin and OAM

Should we be happy with $\vec{J}_\gamma = \int \mathrm{d}^3r\, \vec{r} \times (\vec{E} \times \vec{B})$ **?**

Well… for a circularly polarized plane wave travelling along z



$$J^z_\gamma = \int \mathrm{d}^3r\, [\vec{r} \times \underbrace{(\vec{E} \times \vec{B})}_{\propto\, \vec{e}_z}]^z = 0 \;\; !$$

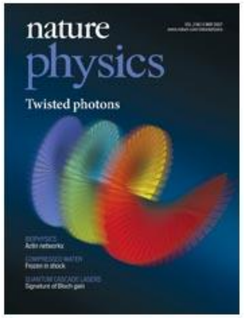
$$S^z_\gamma = \int \mathrm{d}^3r\, (\vec{E} \times \vec{A}_\perp)_z = V\, \frac{E_0^2}{\omega}$$

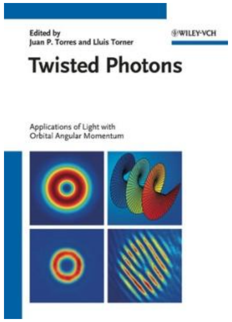
$$L^z_\gamma = \int \mathrm{d}^3r\, E^i(\vec{r} \times \underbrace{\vec{\nabla} A^i_\perp}_{\propto \vec{e}_z})_z = 0$$

These two descriptions are related by a non-zero surface term

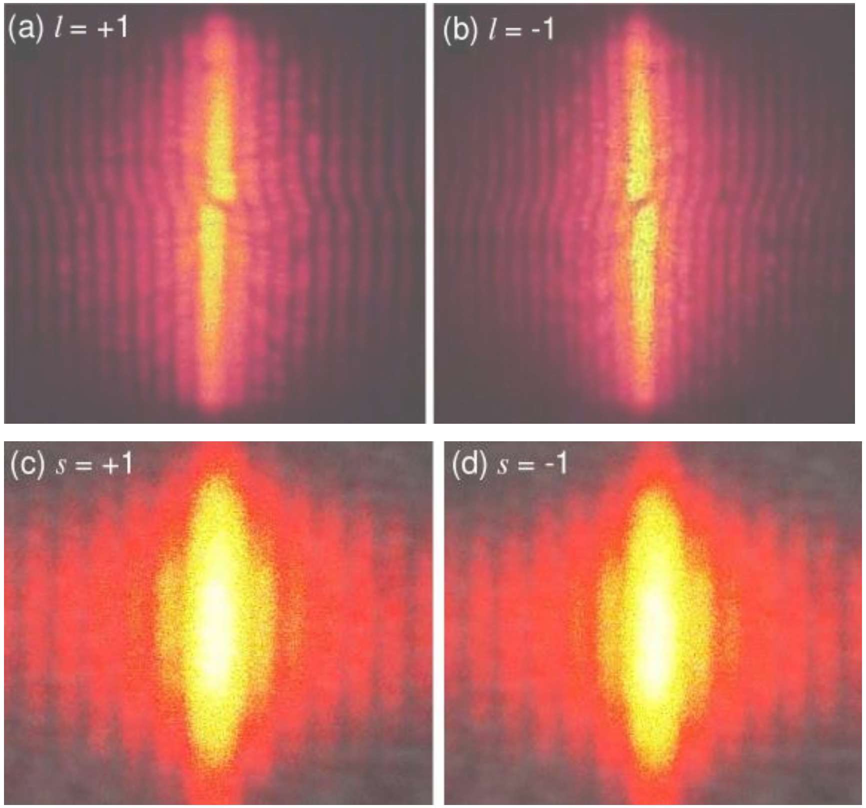
Photon spin and OAM

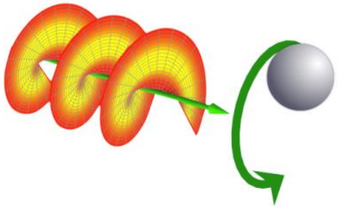
Should we be happy with $\vec{J}_\gamma = \int \mathrm{d}^3r\, \vec{r} \times (\vec{E} \times \vec{B})$ **?**



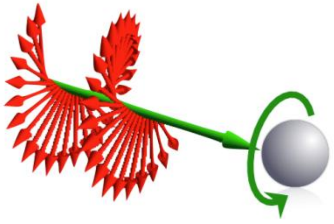


Single-slit experiment





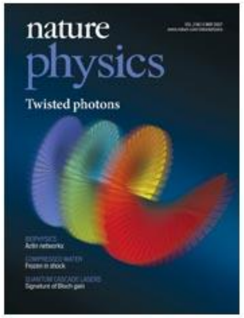
OAM interaction

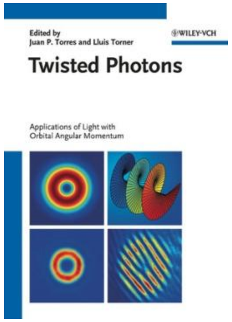


SAM interaction

[Ghai *et al.* (2009)]

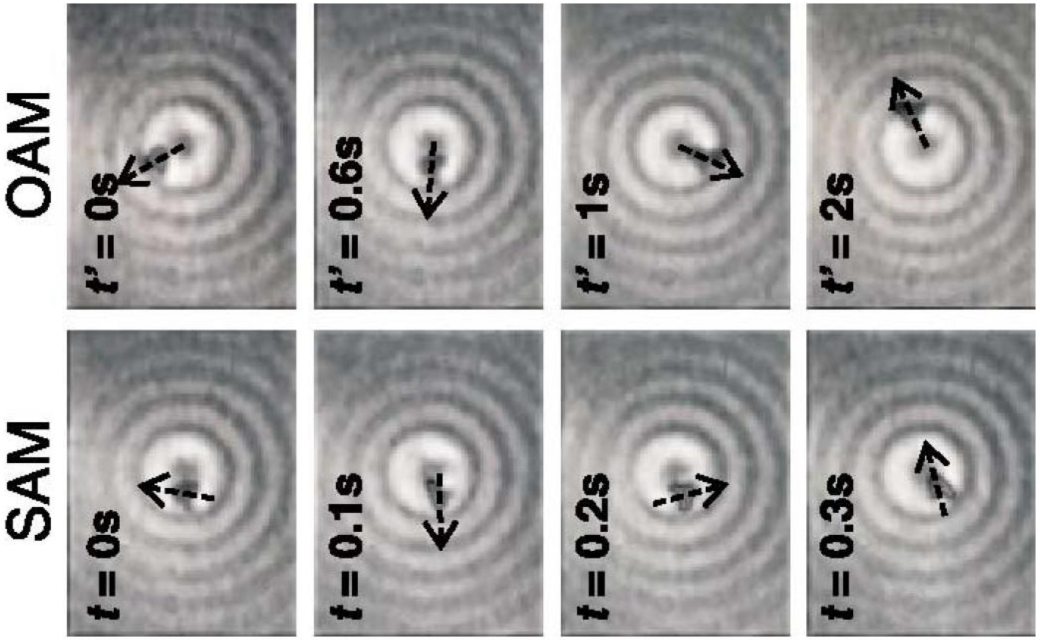
Photon spin and OAM

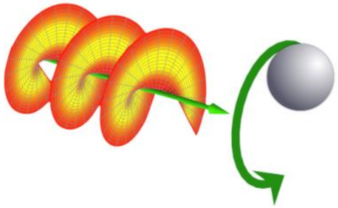




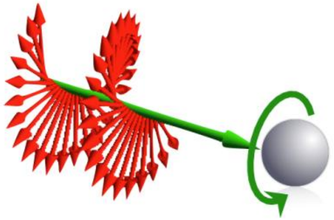
Should we be happy with $\vec{J}_\gamma = \int \mathrm{d}^3 r\, \vec{r} \times (\vec{E} \times \vec{B})$ **?**

Optically trapped microscopic particle





OAM interaction



SAM interaction

[O'Neil *et al.* (2002)]
[Garcés-Chavéz *et al.* (2003)]

Modern spin decomposition

[Leader, C.L. (2014)]

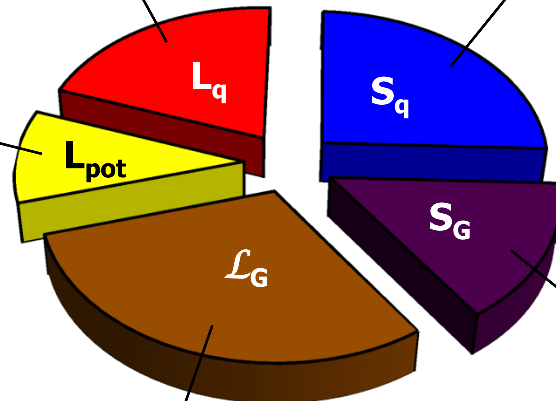
$$\vec{L}_q = \int \mathrm{d}^3r\, \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{S}_q = \int \mathrm{d}^3r\, \psi^\dagger \tfrac{1}{2}\vec{\Sigma}\psi$$

$$\vec{L}_{\text{pot}} = -\int \mathrm{d}^3r\, \rho^a \vec{r} \times \vec{A}^a_{\text{phys}}$$

$$\vec{S}_G = \int \mathrm{d}^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

$$\vec{\mathcal{L}}_G = -\int \mathrm{d}^3r\, E^{ai}(\vec{r} \times \vec{\mathcal{D}}^{ab}_{\text{pure}})A^{ai}_{\text{phys}}$$



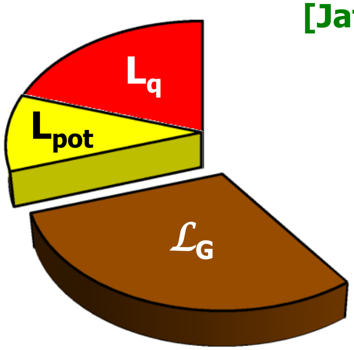
$$\boxed{\vec{A} = \vec{A}_{\text{pure}} + \vec{A}_{\text{phys}}}$$

Gauge-invariant !

Canonical

[Jaffe, Manohar (1990)]
[Chen *et al.* (2008)]

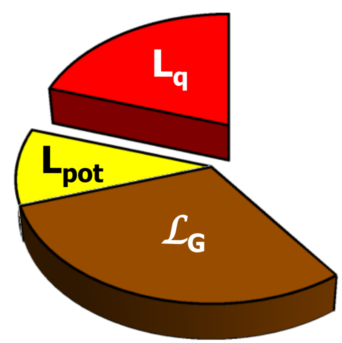
$\vec{\mathcal{L}}_q$



Kinetic

[Ji (1997)]
[Wakamatsu (2010)]

$\vec{L}_G$



Chen *et al*. approach

$$\boxed{A_\mu(x) = A_\mu^{\text{pure}}(x) + A_\mu^{\text{phys}}(x)}$$

Classical, background field

Quantum, dynamical field

[Chen *et al*. (2008,2009)]
[Wakamatsu (2010,2011)]

<u>NB:</u> old recurrent idea

Gauge transformation

$$A_\mu^{\text{pure}}(x) \mapsto U(x)\left[A_\mu^{\text{pure}}(x) + \tfrac{i}{g}\partial_\mu\right]U^{-1}(x)$$
$$A_\mu^{\text{phys}}(x) \mapsto U(x)A_\mu^{\text{phys}}(x)U^{-1}(x)$$

Pure-gauge covariant derivatives

$$D_\mu^{\text{pure}} = \partial_\mu - ig\,A_\mu^{\text{pure}}(x)$$
$$\mathcal{D}_\mu^{\text{pure}} = \partial_\mu - ig\left[A_\mu^{\text{pure}}(x),\quad\right]$$

Field strength

$$F_{\mu\nu}^{\text{pure}}(x) = \frac{i}{g}\left[D_\mu^{\text{pure}}, D_\nu^{\text{pure}}\right] = 0$$
$$F_{\mu\nu}(x) = \mathcal{D}_\mu^{\text{pure}}A_\nu^{\text{phys}}(x) - \mathcal{D}_\nu^{\text{pure}}A_\mu^{\text{phys}}(x) - ig\left[A_\mu^{\text{phys}}(x), A_\nu^{\text{phys}}(x)\right]$$

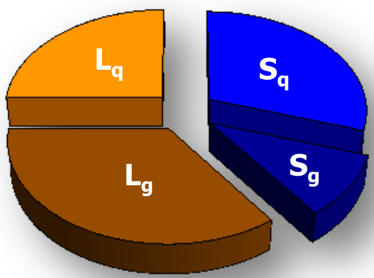
Background dependence

$$A_\mu = \underbrace{(A^{\text{pure}}_\mu + C_\mu)}_{\widetilde{A}^{\text{pure}}_\mu} + \underbrace{(A^{\text{phys}}_\mu - C_\mu)}_{\widetilde{A}^{\text{phys}}_\mu}$$

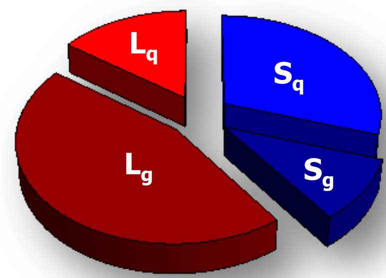
[Stoilov (2010)]
[C.L. (2013)]

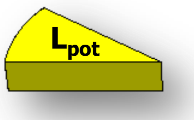
Infinitely many possibilities !

Coulomb condition $\vec{\mathcal{D}}_{\text{pure}} \cdot \vec{A}_{\text{phys}} = 0$



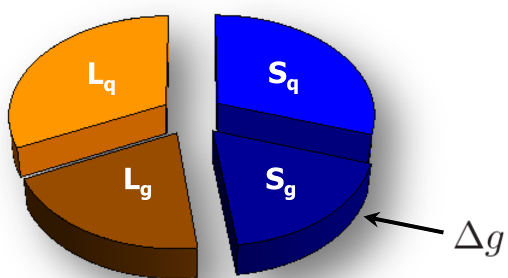
[Chen *et al.* (2008)]



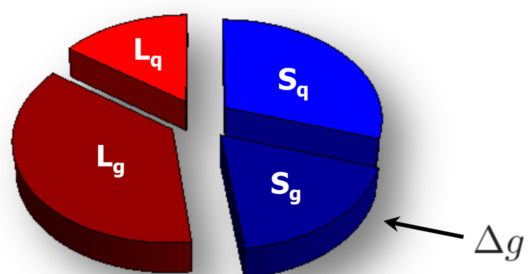


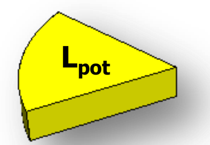
[Wakamatsu (2010)]

Light-front condition $A^+_{\text{phys}} = 0$



[Hatta (2011)]
[C.L. (2013)]





Background vs gauge dependence

What is the difference between fixing a background and fixing a gauge ?

Fixing a background amounts to deciding what are the physical degrees of freedom

Breaks locality but preserves symmetry

Fixing a gauge amounts to choosing a basis in internal space at each point

Breaks symmetry but preserves locality

Background vs gauge dependence

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Fixing a gauge amounts to choosing a basis in internal space at each point

Breaks symmetry but preserves locality

What do we mean by gauge invariance ?

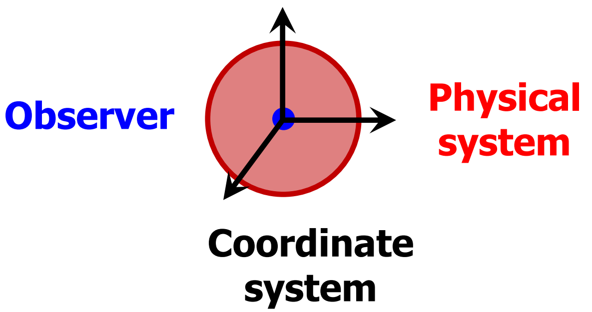
Fundamental laws are postulated to be both symmetric and local

Physical quantities are necessarily symmetric but not necessarily local

Similar story with Lorentz invariance …

Lorentz symmetry

Standard / textbook / non-covariant framework



$$p^0 = M$$

Mass **(rest-frame energy)**

Lorentz symmetry

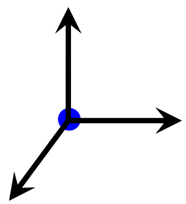
Standard / textbook / non-covariant framework

$$p^0 = \sqrt{\vec{p}^{\,2} + M^2}$$ **Energy**

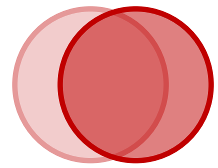
Lorentz symmetry

Standard / textbook / non-covariant framework

Observer



Coordinate system



Physical system

Active transformation

$$p'^0 = \sqrt{\vec{p}'^2 + M^2}$$ **Energy**

Lorentz symmetry

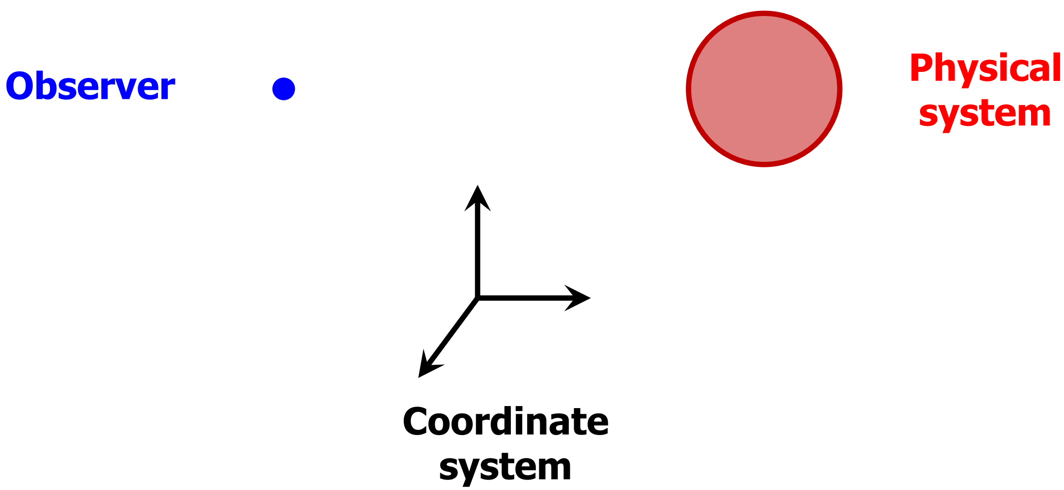
Standard / textbook / non-covariant framework

Passive transformation
+ change of perspective

$$p'^0 = \sqrt{\vec{p}\,'^2 + M^2}$$ **Energy**

Lorentz symmetry

Covariant framework



$$E_u = p \cdot u$$

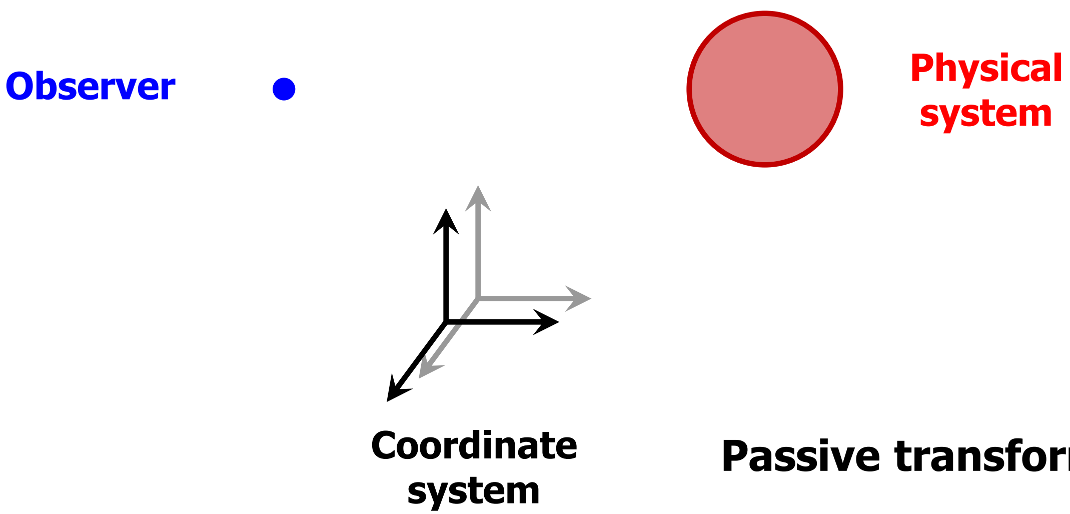
Invariant energy

System's four-momentum

Observer's four-velocity

Lorentz symmetry

Covariant framework



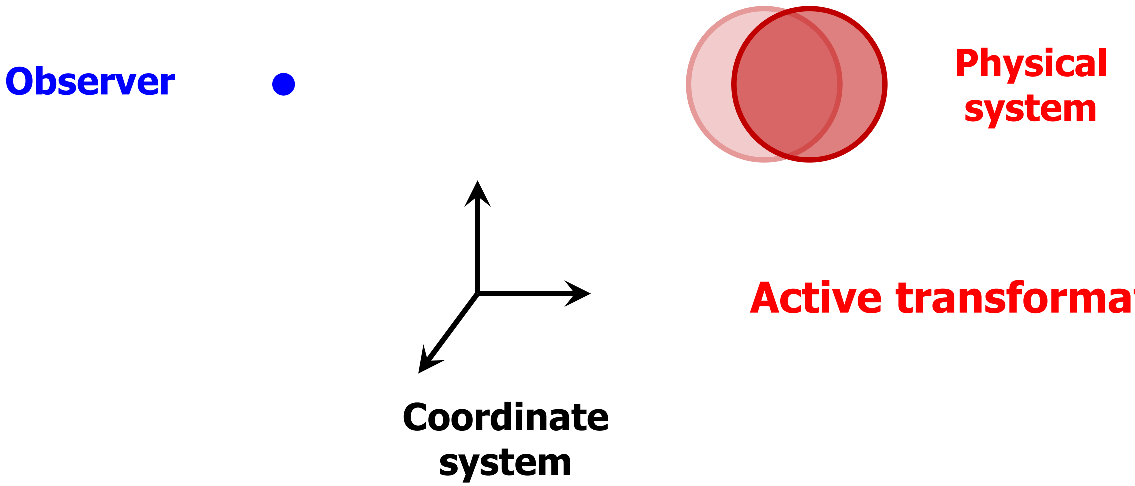
Passive transformation

$$\begin{aligned} E'_{u'} &= (\Lambda p) \cdot (\Lambda u) \\ &= p \cdot u = E_u \end{aligned}$$

Invariant energy

Lorentz symmetry

Covariant framework



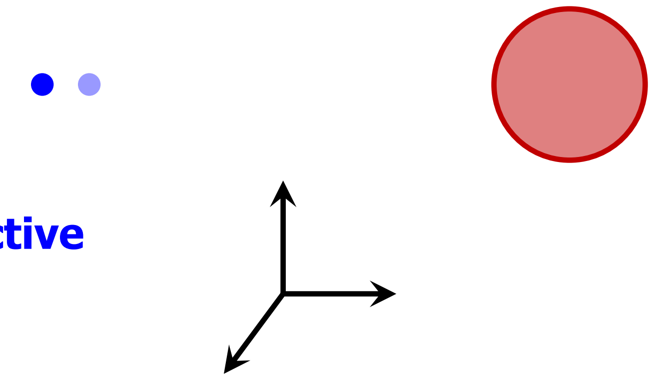
Active transformation

$$E'_u = (\Lambda p) \cdot u$$

Invariant energy

Lorentz symmetry

Covariant framework



Observer

Physical system

Change of perspective

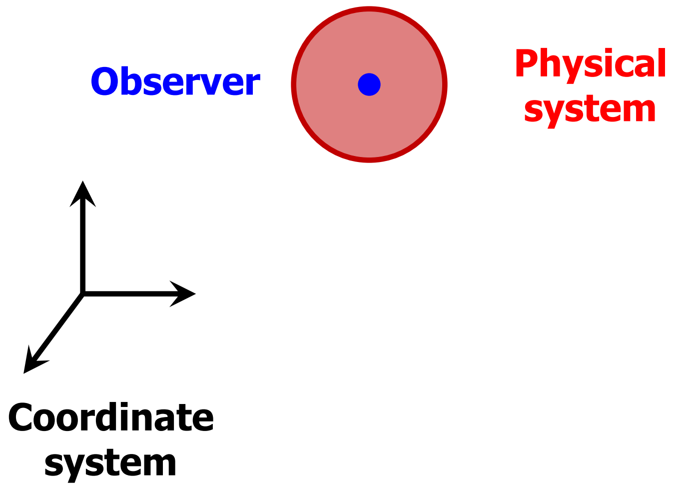
Coordinate system

$$\begin{aligned} E_{u'} &= p \cdot (\Lambda^{-1} u) \\ &= (\Lambda p) \cdot u = E'_u \end{aligned}$$

Invariant energy

Lorentz symmetry

Covariant framework

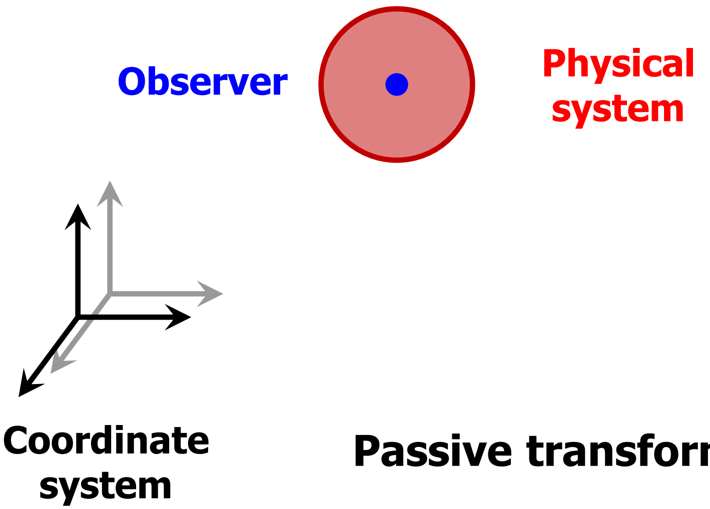


$$E_\star = p \cdot u_\star$$
$$= p^2/M = M$$

Invariant mass **(proper energy)**

Lorentz symmetry

Covariant framework

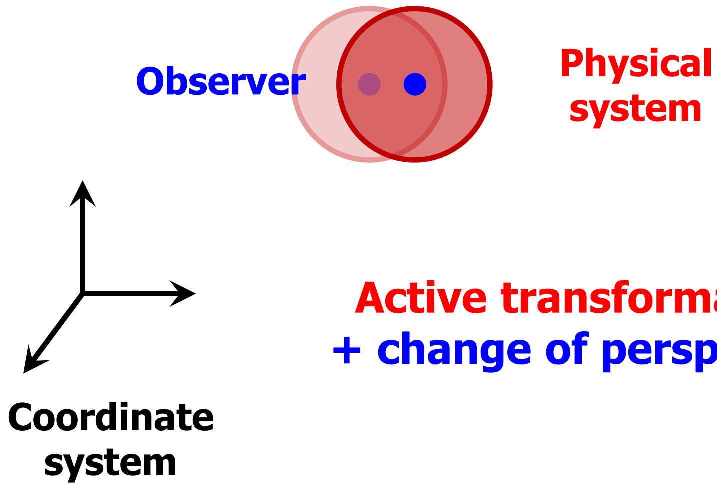


Passive transformation

$$E'_\star = (\Lambda p)\cdot(\Lambda u_\star) = p\cdot u_\star = E_\star$$

Invariant mass **(proper energy)**

Covariant framework



Active transformation
+ change of perspective

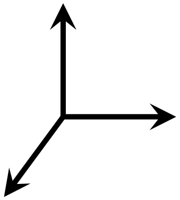
$$E'_\star = (\Lambda p)\cdot(\Lambda u_\star) = p\cdot u_\star = E_\star$$

Invariant mass **(proper energy)**

Background vs gauge dependence (cont'd)

Fixing a background amounts to choosing an internal observer

Fixing a gauge amounts to choosing an internal coordinate system

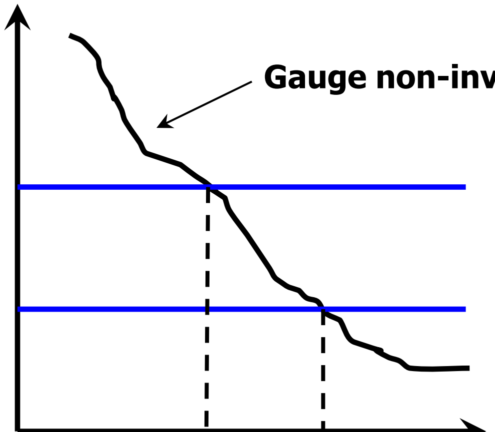


To each background/observer is associated a natural gauge/coordinate system

When fixing a gauge/coordinate system, one often implicitly fixes the background/observer

Gauge-invariant extension (GIE)

[Hoodbhoy, Ji (1999)]



(*e.g.* $\psi_D(x) = \exp\left[-ie\frac{\vec{\nabla}\cdot\vec{A}}{\vec{\nabla}^2}\right]\psi(x)$)

[Dirac (1955)]

Practical approaches

A

- **Consider only simple (local) gauge-invariant quantities**
- **Relate these quantities to observables**
- **Try to find an interpretation (optional)**

Does not account for non-local aspects like Aharonov-Bohm effect and gluon spin

Practical approaches

A

- **Consider only simple (local) gauge-invariant quantities**
- **Relate these quantities to observables**
- **Try to find an interpretation (optional)**

⚠ **Does not account for non-local aspects like Aharonov-Bohm effect and gluon spin**

B

- **Fix the gauge**
- **Consider quantities with simple interpretation**
- **Try to find the corresponding observables**

⚠ **Gauge invariance is lost, and so the question of measurability is unclear**

Practical approaches

A

- **Consider only simple (local) gauge-invariant quantities**
- **Relate these quantities to observables**
- **Try to find an interpretation (optional)**

Does not account for non-local aspects like Aharonov-Bohm effect and gluon spin

B

- **Fix the gauge**
- **Consider quantities with simple interpretation**
- **Try to find the corresponding observables**

Gauge invariance is lost, and so the question of measurability is unclear

C

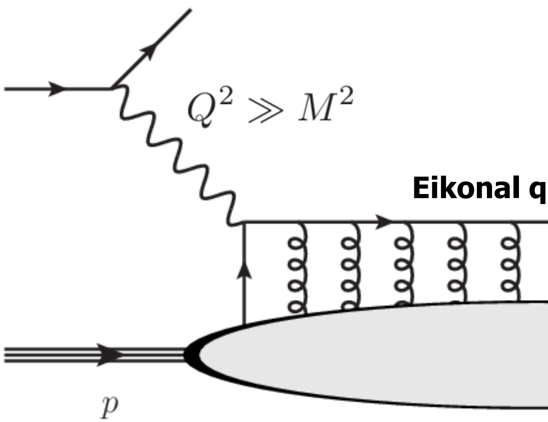
- **Define new complicated (non-local) gauge-invariant quantities**
- **Consider quantities with simple interpretation**
- **Try to find the corresponding observables**

Proper background has to be identified

Nucleon structure

What is the natural background/observer in the nucleon spin decomposition ?

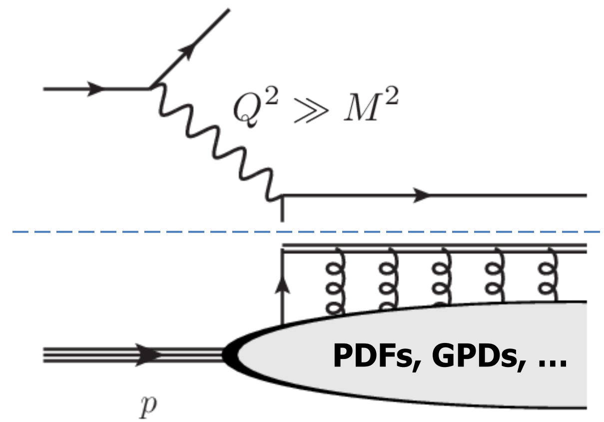
System



Local

$\sim$

« Factorized » system



Non-local

Wilson line

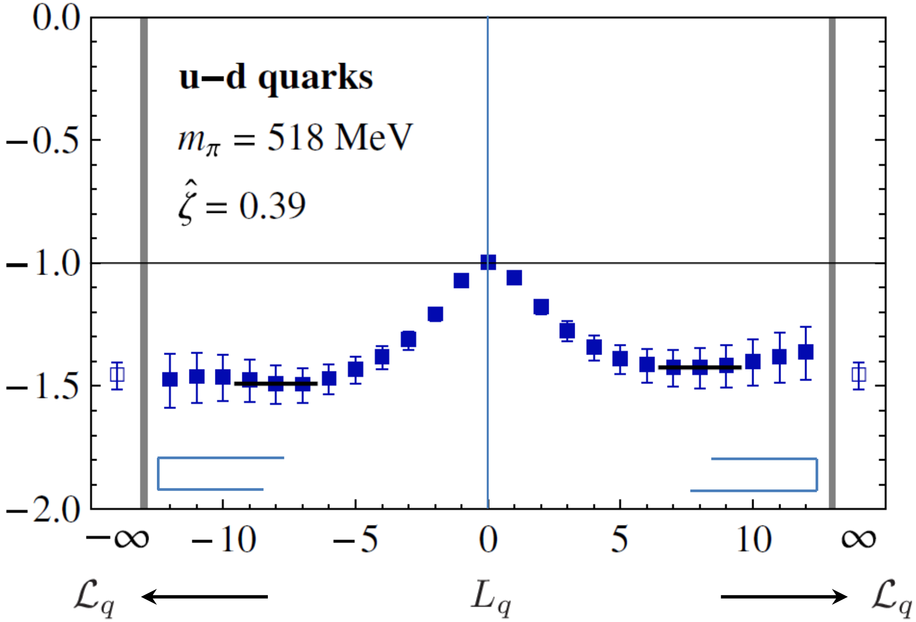
$$\mathcal{W} = \mathcal{P}\left[e^{ig\int_0^\infty A^+ \, \mathrm{d}z^-}\right]$$

background field

$$A_\mu^{\text{pure}} = \mathcal{W}\frac{i}{g}\partial_\mu\mathcal{W}^{-1}$$

Accessing the contributions with quasi-distributions

[Engelhardt (2017)]



Staple length

Large momentum effective theory

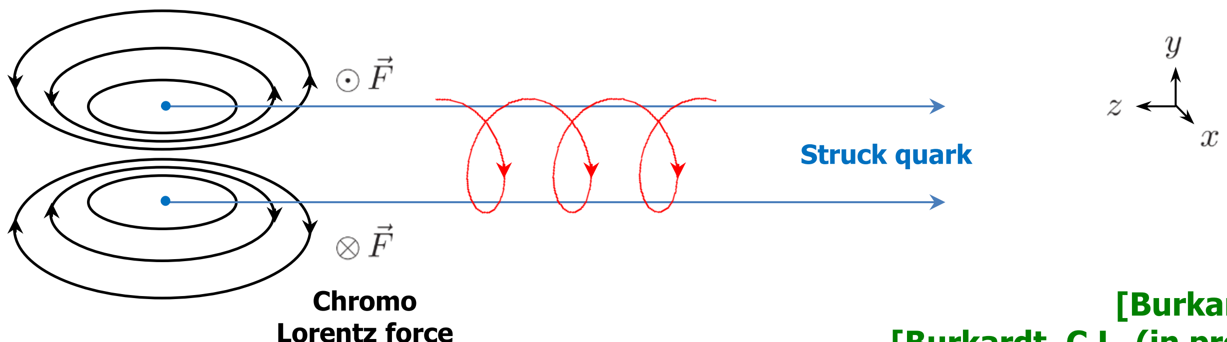
$$f(\cdots) = \lim_{P_z \to \infty} f(\cdots; P_z)$$

[Ji (2013)]
[Ji, Zhang, Zhao (2015)]
[Zhao, Liu, Yang (2016)]

« Inside » the nucleon

« Outside » the nucleon

$$L_{\text{pot}} = L_q - \mathcal{L}_q < 0$$



[Burkardt (2013)]
[Burkardt, C.L. (in preparation)]

Summary

« physical » **« measurable »**

« gauge invariant »

Observables — **Measurable, physical, gauge invariant (*active* and *passive*)**

E.g. **cross-sections**

Expansion scheme — **Background-dependent**

E.g. **collinear factorization**

***Quasi*-observables** — **« Measurable », « physical », « gauge invariant » (only *passive*)**

E.g. **parton distributions**

Backup slides

Angular momentum

Quantum mechanics

$$\vec{J} = \vec{L} + \vec{S} \qquad \vec{L} = \vec{r} \times \vec{p}$$

Quantum field theory

$$J^{\mu\alpha\beta}(x) = L^{\mu\alpha\beta}(x) + S^{\mu\alpha\beta}(x) \qquad L^{\mu\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x)$$

Poincaré covariance

$$\partial_\mu J^{\mu\alpha\beta}(x) = 0, \quad \partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad \boxed{T^{[\alpha\beta]}(x) = -\partial_\mu S^{\mu\alpha\beta}(x)}$$

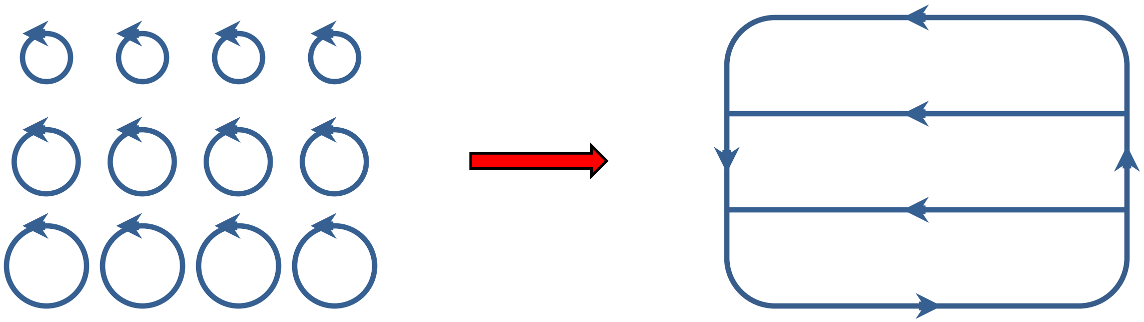
Absent in GR

Energy-momentum tensor

In presence of spin density $T^{0i} \neq T^{i0}$

Belinfante « improvement »

$$\begin{aligned} T_B^{\mu\nu} &\equiv T^{\mu\nu} + \tfrac{1}{2}\partial_\lambda[S^{\lambda\mu\nu} + S^{\mu\nu\lambda} + S^{\nu\mu\lambda}] \\ &= T_B^{\nu\mu} \end{aligned}$$



Spin density gradient **Four-momentum circulation**

In rest frame

$$M = \int \mathrm{d}^3 r\, T_B^{00}(\vec{r})$$

$$J^i = \int \mathrm{d}^3 r\, \epsilon^{ijk} r^j T_B^{0k}(\vec{r})$$

No « spin » contribution !

Energy-momentum form factors

Mellin moment of twist-2 vector GPDs $\langle p', s'|T^{++}(0)|p, s\rangle$

$$\int \mathrm{d}x\, x\, H(x,\xi,t) = A(t) + 4\xi^2 C(t)$$

$$\int \mathrm{d}x\, x\, E(x,\xi,t) = B(t) - 4\xi^2 C(t)$$

[Ji (1996)]

Poincaré covariance $\langle p', s'|T_q^{[\alpha\beta]}(0)|p, s\rangle = -i\Delta_\mu \langle p', s'|S_q^{\mu\alpha\beta}(0)|p, s\rangle$

$$D_q(t) = -G_A^q(t)$$

[C.L., Mantovani, Pasquini (2018)]

No contribution to AM from $\bar{C}(t)$

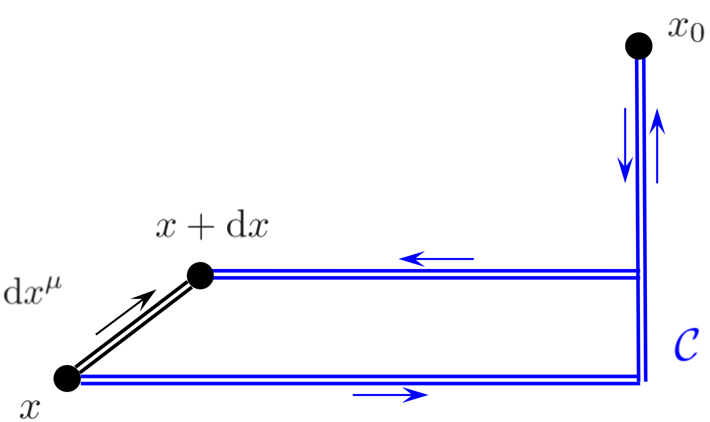
(which contributes to pressure ...)

Background dependence

[Hatta (2012)]
[C.L. (2013)]

Geometrical interpretation

Fixed reference point

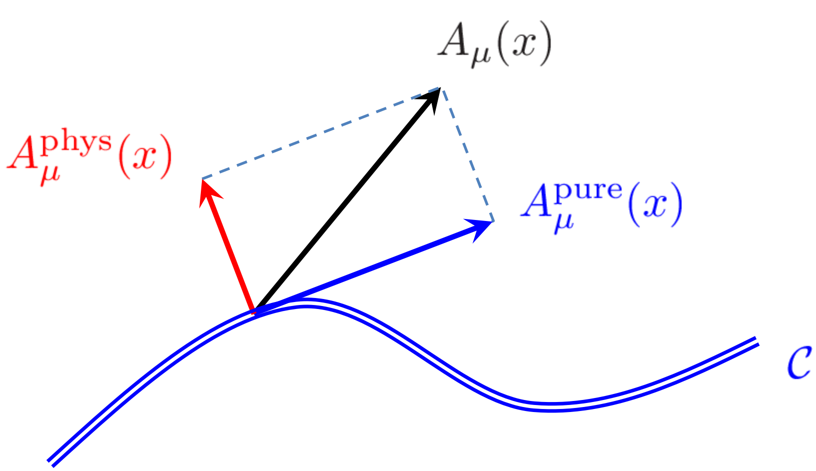


$$\mathcal{W}(x+\mathrm{d}x, x) = 1 + igA_\mu(x)\mathrm{d}x^\mu$$

Wilson line

$$\mathcal{W}_\mathcal{C}(y,x) = \mathcal{P}\left[e^{ig\int_x^y A_\mu(s)\,\mathrm{d}s^\mu}\right]$$

$$\mathcal{W}_\mathcal{C}(x+\mathrm{d}x, x_0)\mathcal{W}_\mathcal{C}(x_0,x) = 1 + igA_\mu^{\mathrm{pure}}(x)\mathrm{d}x^\mu$$



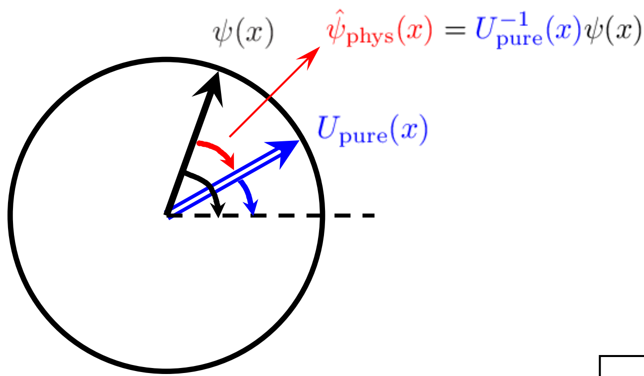
$$\boxed{\begin{aligned} A_\mu^{\mathrm{pure}}(x) &= \tfrac{i}{g}\,\mathcal{W}_\mathcal{C}(x,x_0)\,\tfrac{\partial}{\partial x^\mu}\mathcal{W}_\mathcal{C}(x_0,x) \\ A_\mu^{\mathrm{phys}}(x) &= -\int_{x_0}^{x}\mathcal{W}_\mathcal{C}(x,s)\,F_{\alpha\beta}(s)\,\mathcal{W}_\mathcal{C}(s,x)\,\tfrac{\partial s^\alpha}{\partial x^\mu}\,\mathrm{d}s^\beta \end{aligned}}$$

Non-local !

Stueckelberg symmetry

Quantum Electrodynamics

Phase in internal space

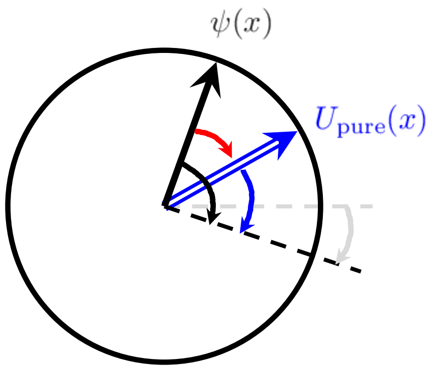


« Physical »

« Background »

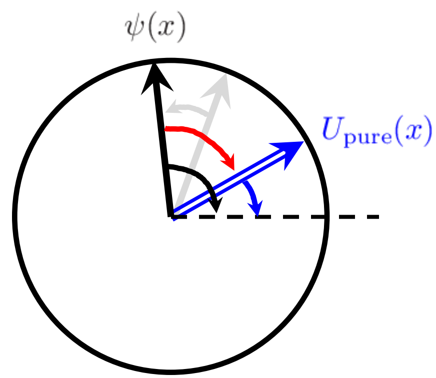
Stueckelberg

Passive



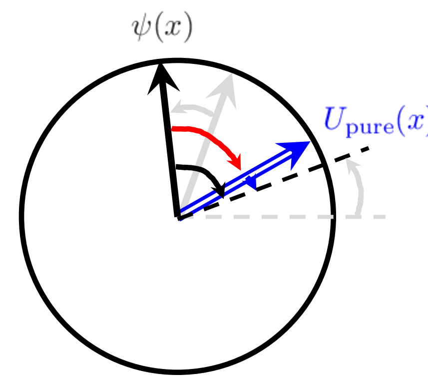
$$\psi(x) \mapsto U(x)\psi(x)$$
$$U_{\text{pure}}(x) \mapsto U(x)U_{\text{pure}}(x)$$
$$\hat{\psi}_{\text{phys}}(x) \mapsto \hat{\psi}_{\text{phys}}(x)$$

Active



$$\psi(x) \mapsto U(x)\psi(x)$$
$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)$$
$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

Active **x** **(*Passive*)$^{-1}$**



$$\psi(x) \mapsto \psi(x)$$
$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)U^{-1}(x)$$
$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

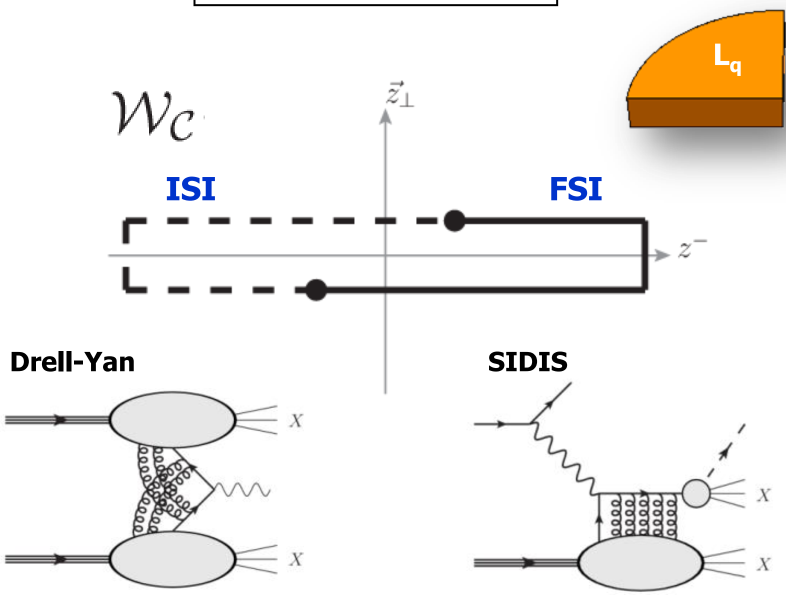
OAM and path dependence

[Ji, Xiong, Yuan (2012)]
[Hatta (2012)]
[C.L. (2013)]

Quark generalized OAM operator

$$\boxed{L^{\mu\nu\rho}_{q,\mathcal{C}}(x)=\overline{\psi}(x)\gamma^\mu(x^\nu\,\tfrac{i}{2}\overleftrightarrow{D}^\rho_{\text{pure}}-x^\rho\,\tfrac{i}{2}\overleftrightarrow{D}^\nu_{\text{pure}})\psi(x)}$$

Light-front

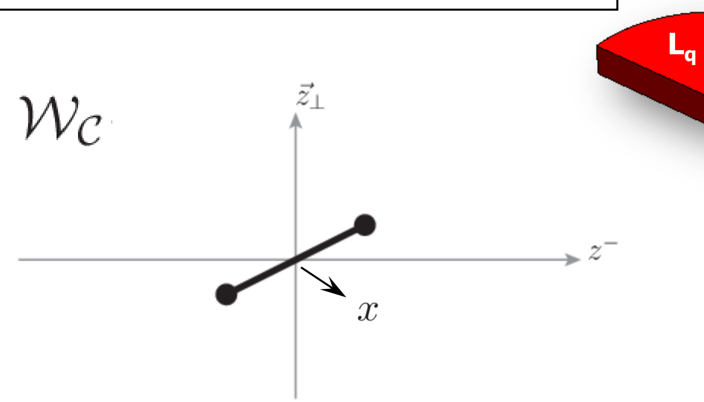


$D^{\text{pure}}_\mu \overset{A^+=0}{=} \partial_\mu$

$$\boxed{\ell_z^{\text{DY}}=\ell_z^{\text{SIDIS}}}$$

Naive T-even

x-based Fock-Schwinger

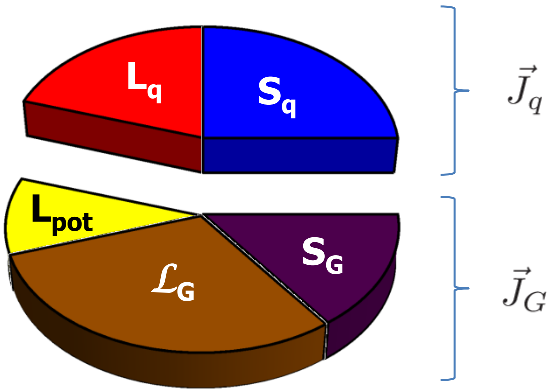


Coincides *locally* with kinetic quark OAM

$$A_\mu(x)=A^{\text{pure}}_\mu(x)$$
$$A_\mu(y)\neq A^{\text{pure}}_\mu(y)\qquad y\neq x$$

$$L^{\mu\nu\rho}_q(x)=\overline{\psi}(x)\gamma^\mu(x^\nu iD^\rho-x^\rho iD^\nu)\psi(x)$$

Ji relation



$$\boxed{J_{q,G} = \int \mathrm{d}x\, \tfrac{x}{2}\left[H^{q,G}(x,0,0) + E^{q,G}(x,0,0)\right]}$$

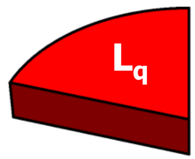
[Ji (1997)]

Genuine AM sum rule → **Vanishing total anomalous gravitomagnetic moment**

$$\sum_{a=q,G} \int \mathrm{d}x\, x\, E^a(x,0,0) = 0$$

[Teryaev (1999)]
[Brodsky, Hwang, Ma, Schmidt (2001)]

Penttinen-Polyakov-Shuvaev-Strikman relation



Twist-3 GPDs

$$\boxed{\begin{aligned} L_q &= -\int \mathrm{d}x\, x\, G_2^q(x,0,0) \\ &= \int \mathrm{d}x\, x \left[H^q(x,0,0) + E^q(x,0,0) + \tilde{E}_{2T}^q(x,0,0) \right] \end{aligned}}$$

[Penttinen *et al.* (2000)]
[Kiptily, Polyakov (2002)]
[Hatta, Yoshida (2012)]

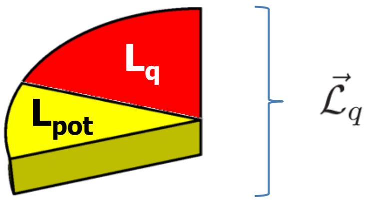
Quark AM sum rule

$$\int \mathrm{d}x\, x\, [H^q(x,0,0) + E^q(x,0,0) + 2\, G_2^q(x,0,0)] - \int \mathrm{d}x\, \tilde{H}^q(x,0,0) = 0$$

or equivalently

$$\int \mathrm{d}x\, x \left[H^q(x,0,0) + E^q(x,0,0) + 2\, \tilde{E}_{2T}^q(x,0,0) \right] + \int \mathrm{d}x\, \tilde{H}^q(x,0,0) = 0$$

Accessing the contributions with TMDs



Transversity relation **Model-dependent !**

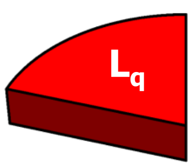
$$\mathcal{L}_q = \int \mathrm{d}x\, \mathrm{d}^2k_\perp \left[h_1^q(x, \vec{k}_\perp^2) - g_{1L}^q(x, \vec{k}_\perp^2) \right]$$

[Ma, Schmidt (1998)]

Pretzelosity relation **Model-dependent !**

$$\mathcal{L}_q = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \, \frac{\vec{k}_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x, \vec{k}_\perp^2)$$

[She, Zhu, Ma (2009)]
[Avakian *et al.* (2010)]
[Efremov *et al.* (2010)]
[C.L., Pasquini (2012)]

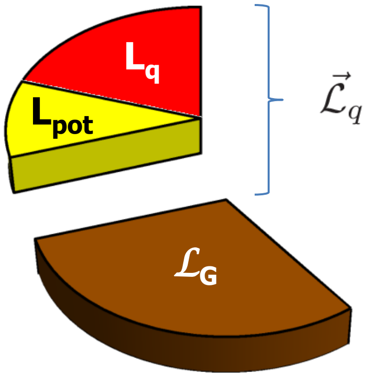


Lensing relation **Model-dependent !**

$$\int \mathrm{d}^2k_\perp \, f_{1T}^{\perp q}(x, \vec{k}_\perp^2) = -I^q(x)\, E^q(x, 0, 0)$$

[Burkardt (2004)]
[Bacchetta, Radici (2011)]

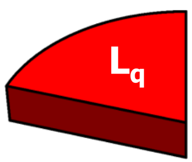
Accessing the contributions with GTMDs



F_{14} relation **(staple gauge link)**

$$\mathcal{L}_{q,G} = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\,F_{14}^{q,G}(x,0,\vec{k}_\perp,\vec{0}_\perp;\mathcal{W}_\sqsupset)$$

[C.L., Pasquini (2011)]
[Hatta (2011)]



F_{14} relation **(straight gauge link)**

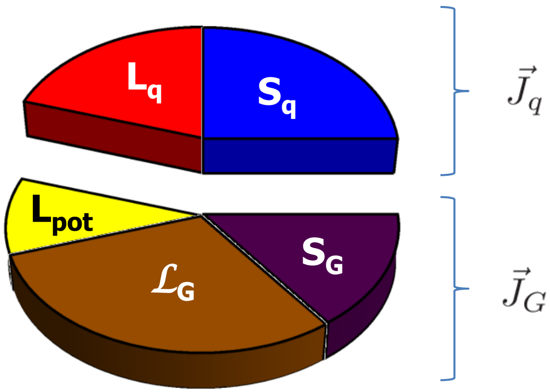
$$L_q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\,F_{14}^{q}(x,0,\vec{k}_\perp,\vec{0}_\perp;\mathcal{W}_|)$$

[Ji, Xiong, Yuan (2012)]
[C.L. (2012)]

$$L_G \neq -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\,F_{14}^{G}(x,0,\vec{k}_\perp,\vec{0}_\perp;\mathcal{W}_|)$$

[Liu, C.L. (2015)]

Accessing the contributions with EMFFs

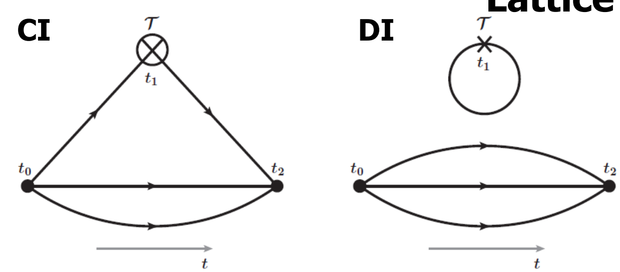


Ji relation

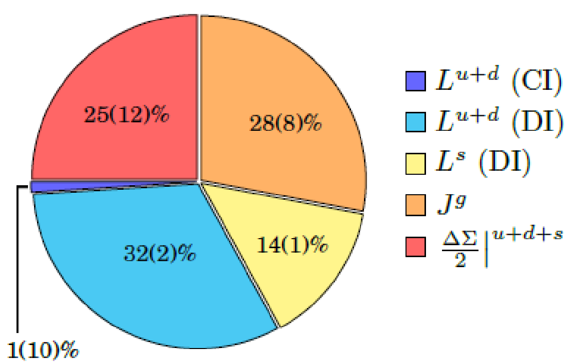
$$J_{q,G} = \frac{1}{2}\left[A^{q,G}(0) + B^{q,G}(0)\right]$$

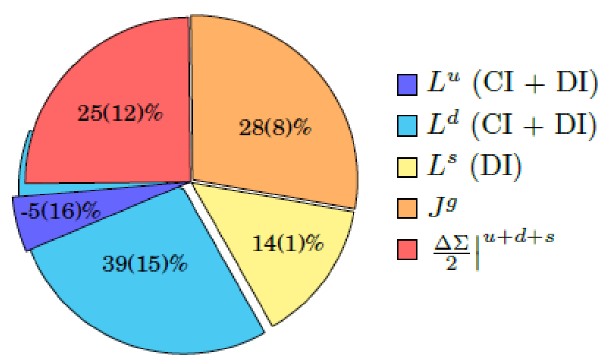
[Ji (1997)]

Lattice



χQCD collaboration





[Deka *et al.* (2015)]