RPA response functions in infinite nuclear matter

Marco Martini

Nuclear matter response function

- One of the main tools to study electron and neutrino scattering on nuclei in the quasielastic peak and beyond
- Recently employed to investigate the properties of effective nuclear forces
- Many application in nuclear astrophysics:
  EoS; Phase transitions (liquid-gas, magnetic…); neutrino mean free path

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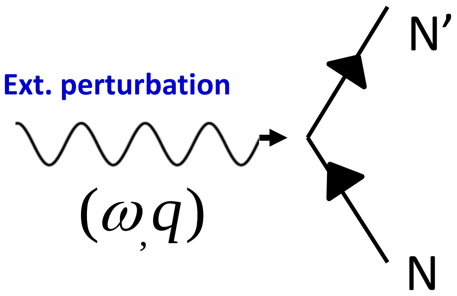
Nuclear response function



$$\mathcal{Q}^{(X)} = e^{-i\omega t} \sum_a e^{i\mathbf{q}\cdot\mathbf{r}_a} \Theta_a^{(X)}$$

Operators Θ_a	1	$\vec{\sigma}$	$\vec{\tau}$	$\vec{\sigma}\,\vec{\tau}$
$(X) = (S,T)$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$

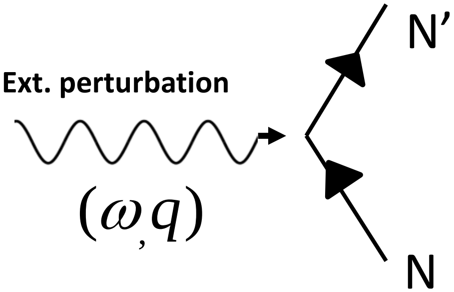
Response function:

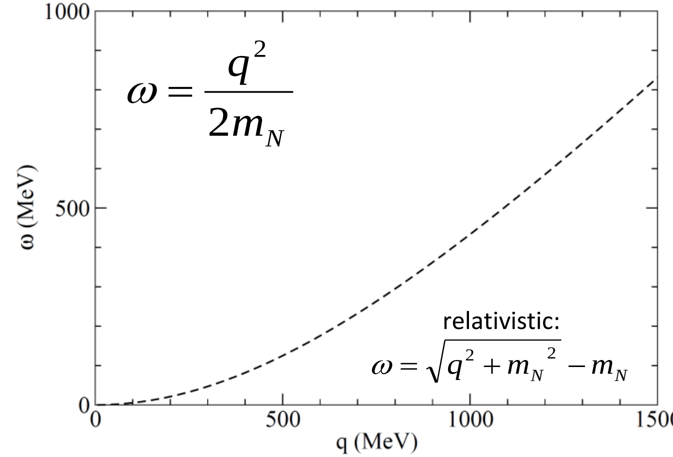
$$\Pi_X(q,\omega) = \frac{1}{V} \sum_n |\langle n|\mathcal{Q}^{(X)}|0\rangle|^2 \left(\frac{1}{\omega - E_{n0} + i\eta} - \frac{1}{\omega + E_{n0} - i\eta} \right)$$

Poles: energies of the excites states

$$R_X(q,\omega) = -\frac{V}{\pi} \mathrm{Im}\Pi_X(q,\omega)$$

Response of non interacting systems





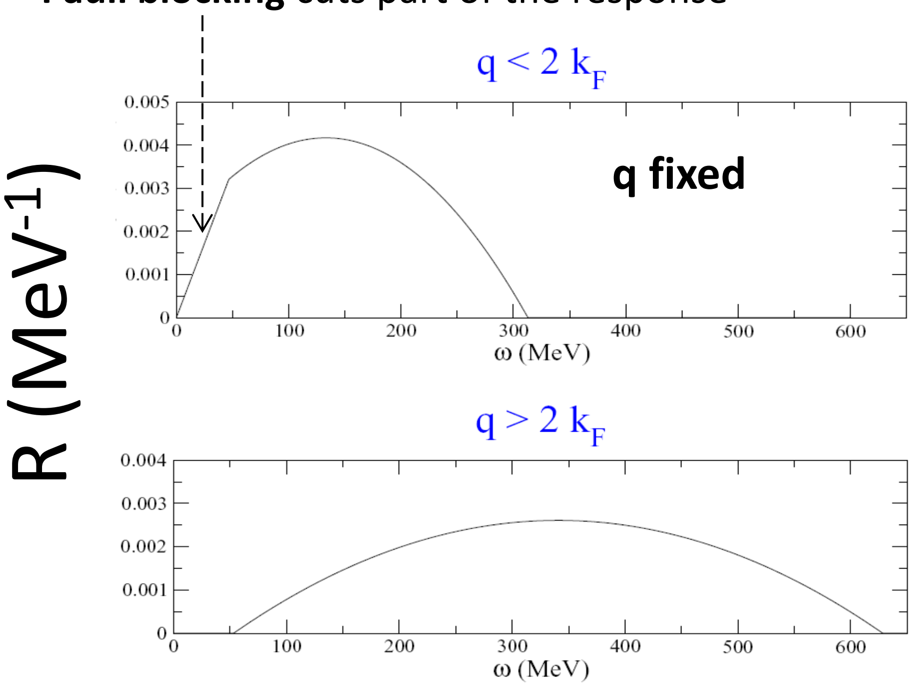
- **Free nucleon at rest:**

  Response function $\propto \delta(\omega - q^2/2m_N)$

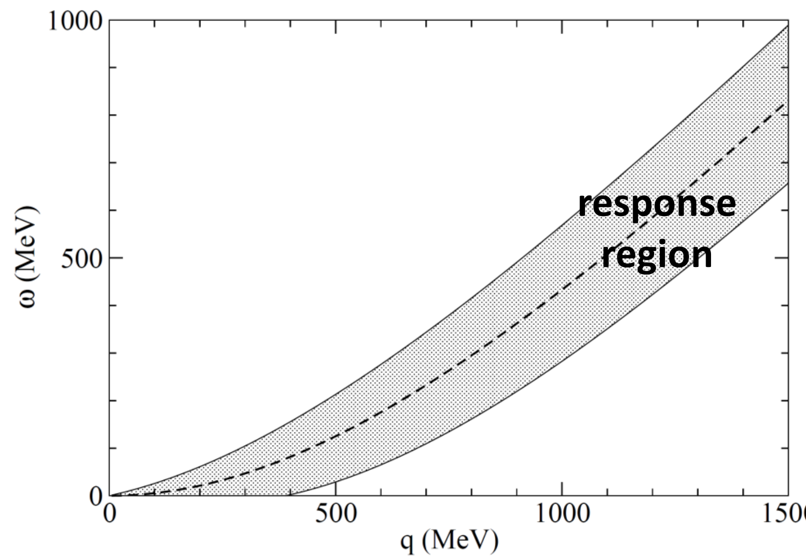
- **Non interacting nuclear matter**

Fermi momentum k_F spreads δ distribution (Fermi Gas)

Pauli blocking cuts part of the response

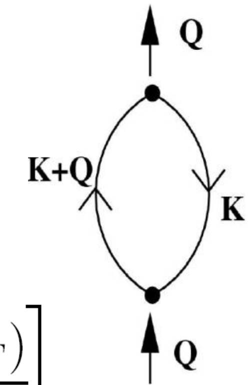


$$\frac{q^2}{2m_N} - \frac{qk_F}{m_N} \leq \omega \leq \frac{q^2}{2m_N} + \frac{qk_F}{m_N}$$



Non interacting nuclear matter: Fermi Gas

$$G_{\alpha\beta}(\vec{k},\omega) = \delta_{\alpha\beta}\left[\frac{\theta(k-k_F)}{\omega-\omega_k+i\eta}+\frac{\theta(k_F-k)}{\omega-\omega_k-i\eta}\right] \qquad \omega_k = \frac{k^2}{2m}$$

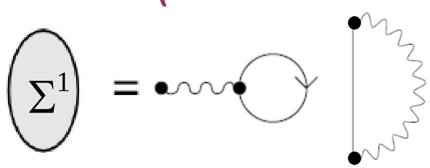


$$\Pi(Q) = -2\,i\int\frac{d^4K}{(2\pi)^4}[G(K)G(K+Q)]$$

$$\Pi^0(\vec{q},\omega) = g\int\frac{d\vec{k}}{(2\pi)^3}\left[\frac{\theta(|\vec{k}+\vec{q}|-k_F)\theta(k_F-k)}{\omega-(\omega_{\vec{k}+\vec{q}}-\omega_{\vec{k}})+i\eta}-\frac{\theta(k_F-|\vec{k}+\vec{q}|)\theta(k-k_F)}{\omega+(\omega_{\vec{k}}-\omega_{\vec{k}+\vec{q}})-i\eta}\right]$$

Mean Field Approximation (Hartree-Fock)

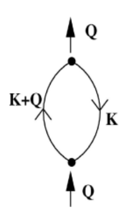
$$\varepsilon_k = \omega_k + \Sigma^1(k)$$



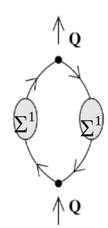
Effective mass

$$\frac{m^*}{m} = \left(1+\frac{m}{k_F}\frac{\partial\Sigma^{(1)}(k)}{\partial k}\bigg|_{k=k_F}\right)^{-1}$$

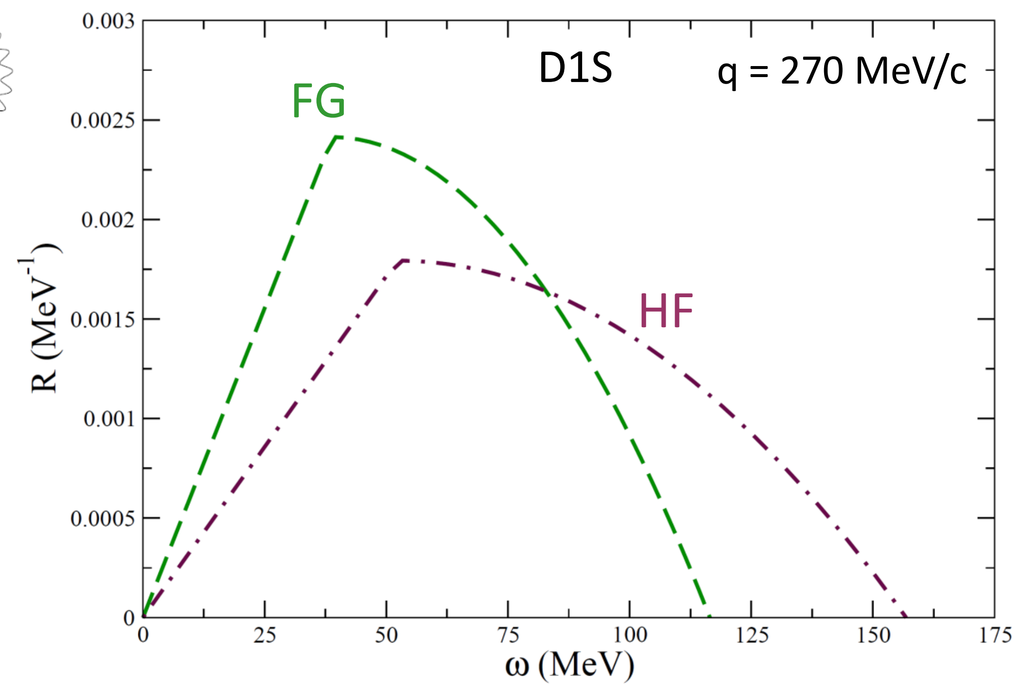
Fermi Gas $\longrightarrow$ Hartree-Fock



$\longrightarrow$



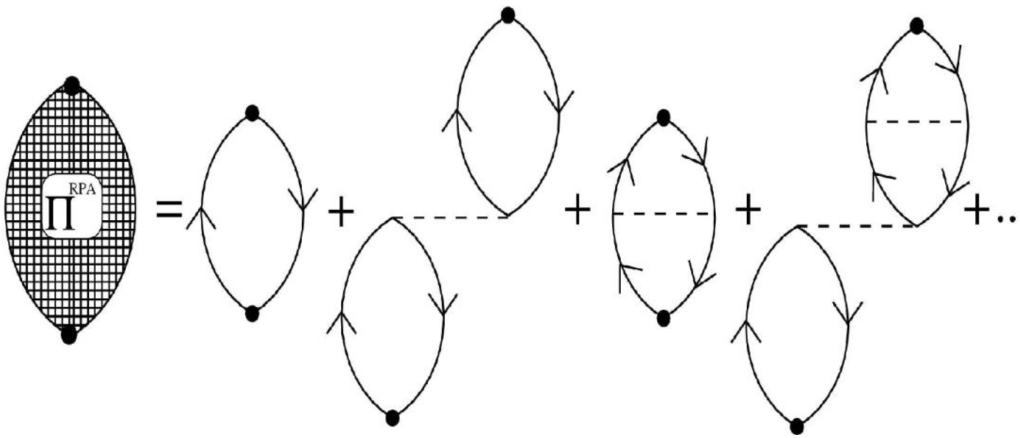
$$m \rightarrow m^*$$



Switching on the p-h interaction: the RPA equation

The external perturbation acting on one nucleon is transmitted

The response becomes collective



- For zero range (Skyrme) interactions the Π^{RPA} results are analytical

  *Garcia-Recio, Navarro, Van Giai, Salcedo, Ann. Phys. 214, 293 (1992) Central terms

  *Margueron, Van Giai, Navarro, Phys. Rev. C 74, 015805 (2006) + Spin-orbit

  *Davesne, Martini, Bennaceur, Meyer, Phys. Rev. C 80, 024314 (2009) + Tensor

- For finite range (Gogny, Yukawa,…) interactions fully analytical calculations for Π^{RPA} no longer possible

- Many papers with zero range forces and with finite range forces but in the Landau Migdal ($q \to 0$) limit for the exchange
- Few papers considering fully antisymmetrized RPA with finite range forces
- Only 2 papers considering fully antisymmetrized RPA with the Gogny force:

  *Margueron, Navarro, Van Giai, Schuck, Phys. Rev. C 77, 064306 (2008) central (D1)

  *De Pace, Martini, Phys. Rev. C 94, 024342 (2016) central (D1,D1S,D1N,D1M); + Tensor (many)

based on the continued fraction technique

Continued fraction

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_n + \cfrac{1}{\ddots}}}}} \qquad \alpha = [a_0, a_1, a_2, \dots, a_n, \dots]$$

Examples:

$$\pi = 3.14159265359\ldots \qquad \pi = [3,7,15,1,292,1,1,1,2,1,4,1,2,9,1,2]$$

$$3 + \frac{1}{7} = 3.\overline{142857} \qquad 3 + \cfrac{1}{7 + \cfrac{1}{15}} = 3.141509\ldots \qquad 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1}}} = 3.14159292\ldots$$

$$e = 2.71828182846\ldots \qquad e = [2,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,8,1,6,\ldots]$$

$$2 + \frac{1}{1} = 3 \qquad 2 + \cfrac{1}{1 + \cfrac{1}{2}} = 2.\overline{6} \qquad 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1}}} = 2.75$$

In the CF technique there is no general way of estimating the convergence of the series

Continued fraction expansion of the polarization propagator

$$\Pi_X^{\mathrm{RPA}} = \frac{\Pi^{(0)}}{1 - \Pi_X^{(1)\mathrm{d}}/\Pi^{(0)} - \Pi_X^{(1)\mathrm{ex}}/\Pi^{(0)} - \dfrac{\Pi_X^{(2)\mathrm{ex}}/\Pi^{(0)} - \left[\Pi_X^{(1)\mathrm{ex}}/\Pi^{(0)}\right]^2}{1 - \ldots}}$$

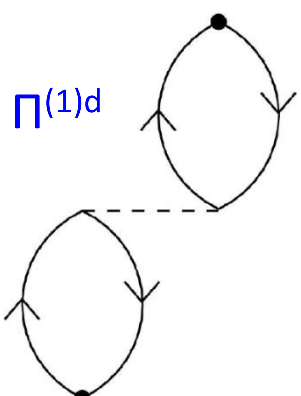
At infinite order the CF expansion gives the exact result as summation of the perturbative series. When truncated at finite order, it reproduces the standard perturbative series at the same order plus an approximation for each one of the infinite number of higher order contributions.

First order

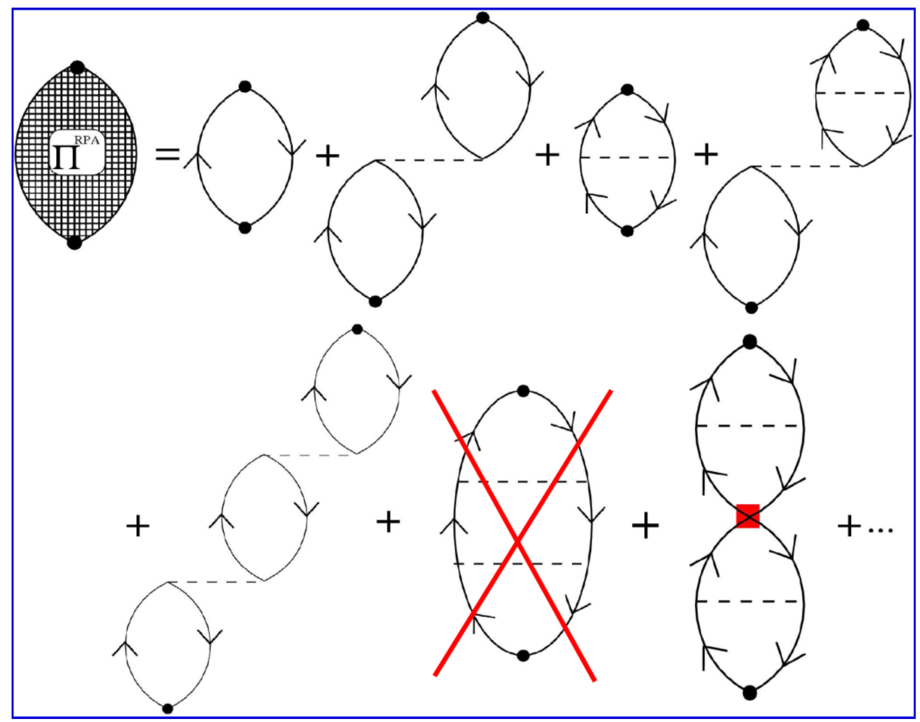
$$\Pi_{\mathrm{CF1}}^{\mathrm{RPA}} = \frac{\Pi^{(0)}}{1 - \Pi^{(1)}/\Pi^{(0)}}$$

Same structure as the ring approximation

$\Pi^{(1)\mathrm{d}}$

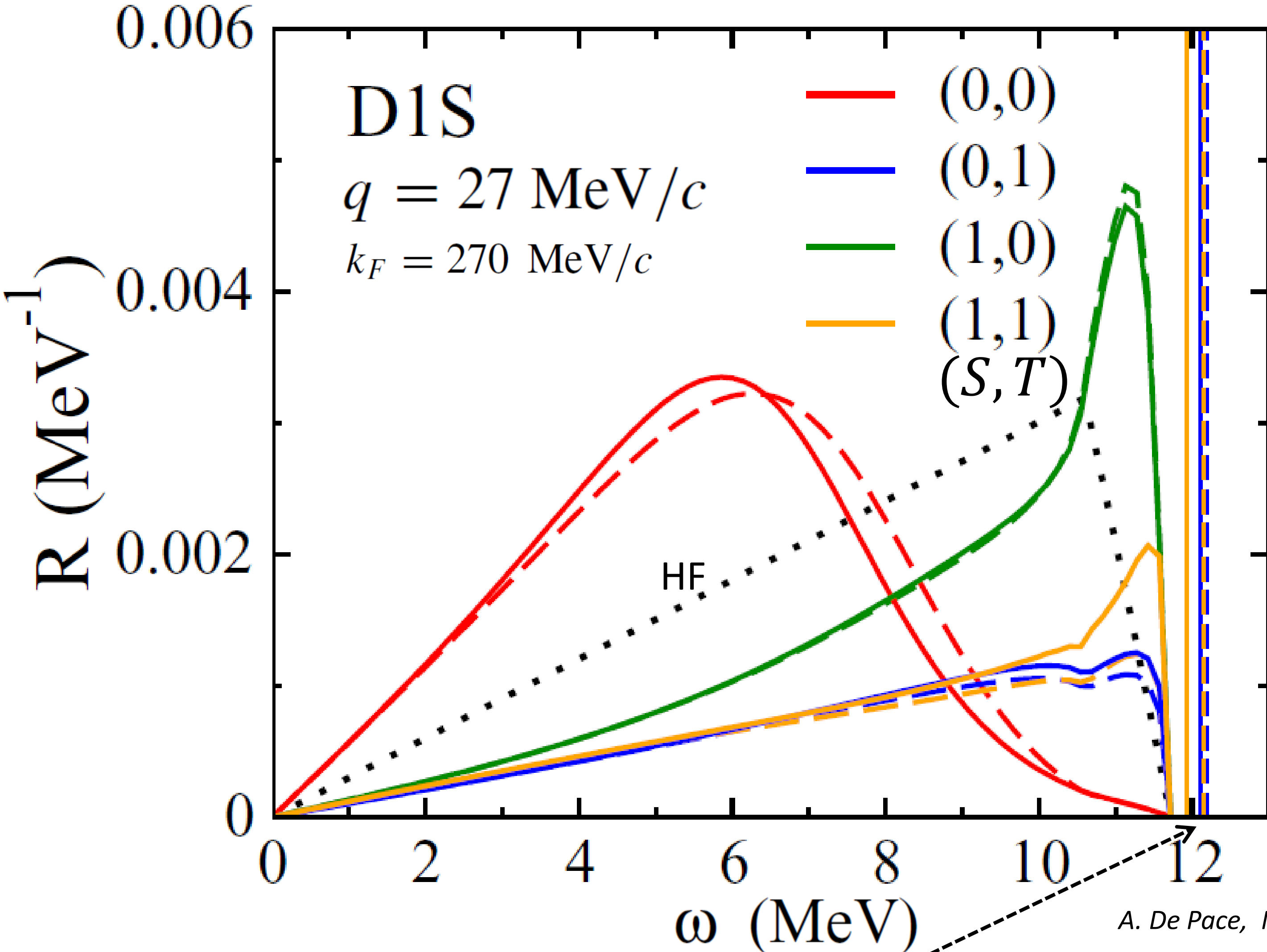


$\Pi^{(1)\mathrm{ex}}$



RPA response with Gogny at first and second order in CF

The 2nd order is the highest order so far reached in the context of finite-range forces



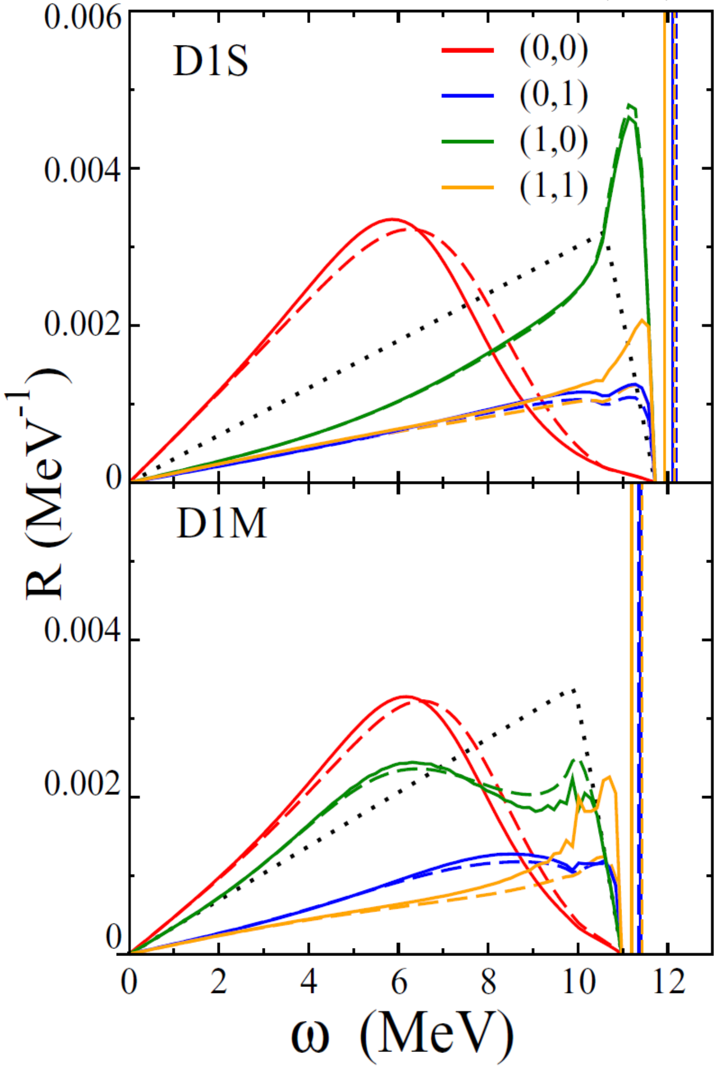
A. De Pace, M. Martini, PRC 94, 024342 (2016)

- **Reshaping** of the responses
- **Quenching** or **enhancement** depending on the repulsive or attractive character of the force
- Appearence of **collective modes** in **(0,1)** and **(1,1)**
- Differences between 1° and 2° CF order in **(0,0)** and **(1,1)**

Collective modes with D1S and D1M

Nuclear matter

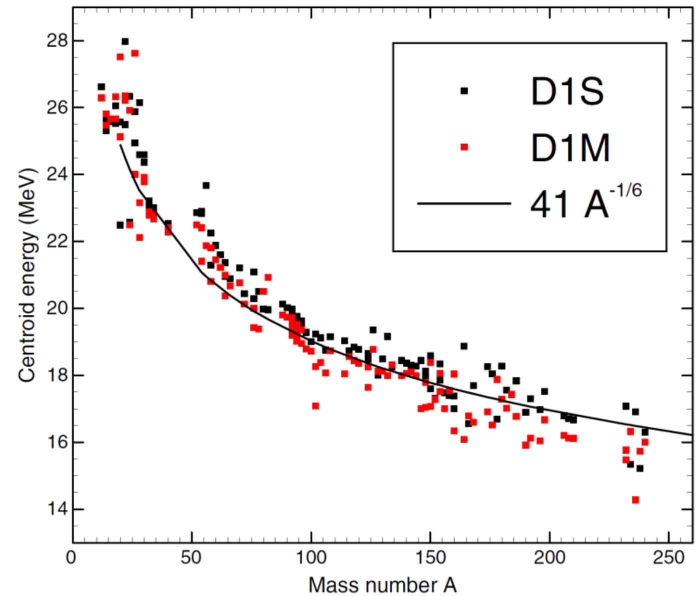
A. De Pace, M. Martini, PRC 94, 024342 (2016)



$(0,1) \leftrightarrow GDR$

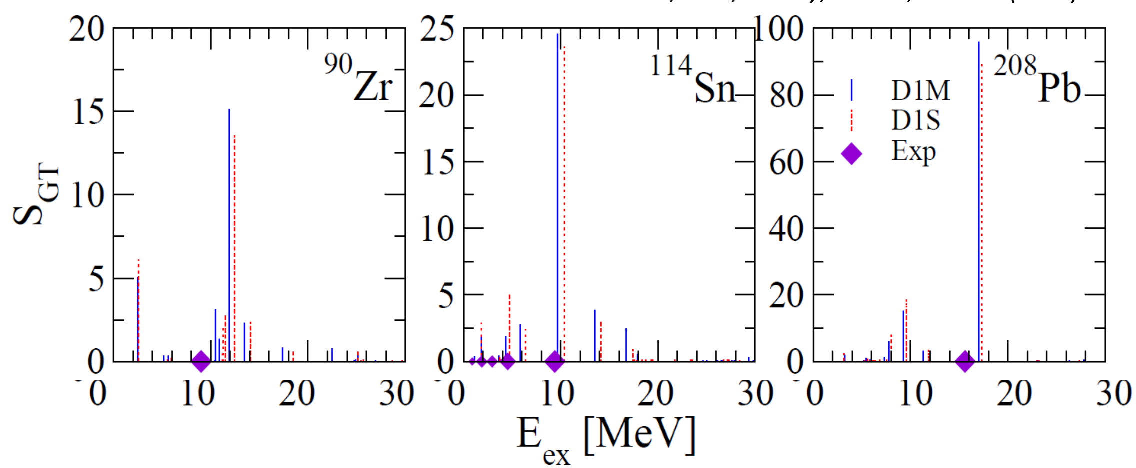
Finite nuclei

Martini, Péru, Hilaire, Goriely, Lechaftois, PRC 94, 014304 (2016)



$(1,1) \leftrightarrow GT$

Martini, Péru, Goriely, PRC 89, 044306 (2014)

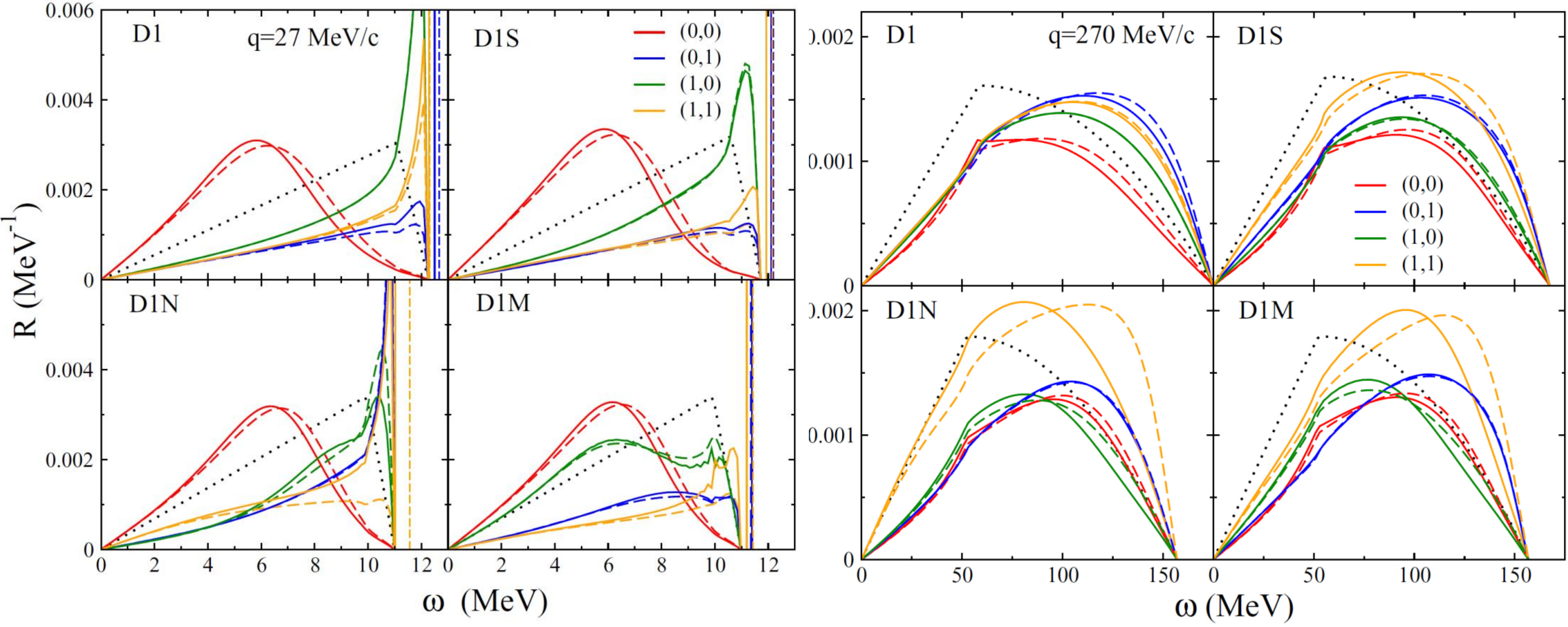


Lower energy collective modes for D1M when compared to D1S

Nuclear responses for Gogny D1, D1S, D1N and D1M

q=27 MeV/c　　　　　　　　　　q=270 MeV/c

A. De Pace, M. Martini, PRC 94, 024342 (2016)



- The responses calculated with the different parametrizations can show important differences
- The convergence of the CF expansion strongly depends on the force parameters in the different (S,T) channels

Including tensor terms

- The tensor terms of the effective nuclear interaction are usually neglected in MF calculations
- In Skyrme and Gogny interactions tensor terms have been considered only in the last years

Skyrme

Finite nuclei

G.Colo',H.Sagawa,S.Fracasso,andP.F.Bortignon,Phys.Lett. B646, 227 (2007)

T.Lesinski, M.Bender, K.Bennaceur, T. Duguet, and J .Meyer, Phys. Rev. C76, 014312 (2007)

Nuclear matter

D. Davesne, M. Martini, K. Bennaceur, J. Meyer, Phys. Rev. C 80, 024314 (2009)

A. Pastore, D. Davesne, Y. Lallouet, M. Martini, K. Bennaceur, and J. Meyer, Phys. Rev. C85, 054317 (2012)

A. Pastore, M. Martini, V. Buridon, D. Davesne, K. Bennaceur, and J. Meyer, Phys. Rev. C86, 044308 (2012)

D.Davesne, A.Pastore, and J.Navarro,Phys.Rev.C89,044302 (2014)

Gogny

Finite nuclei

- GT2: Gaussian Tensor-isovector; refitting of all the parameters T. Otsuka, T. Matsuo, D. Abe, PRL 97, 162501 (2006)
- Adding tensor component to D1S and D1M without changing the central parameters

- D1ST, D1MT: Tensor-isovector based on AV18; spin-orbit modified Anguiano, et al PRC83, 064306 (2011)
- D1ST2a, D1ST2b:Gaussian Tensor-isovector and Tensor-isoscalar Anguiano et al. PRC86, 054302 (2012)
- D1ST2c, D1MT2c: Gaussian Tensor-isovector and Tensor-isoscalar; S.O modified De Donno et al. PRC 90 (2014)

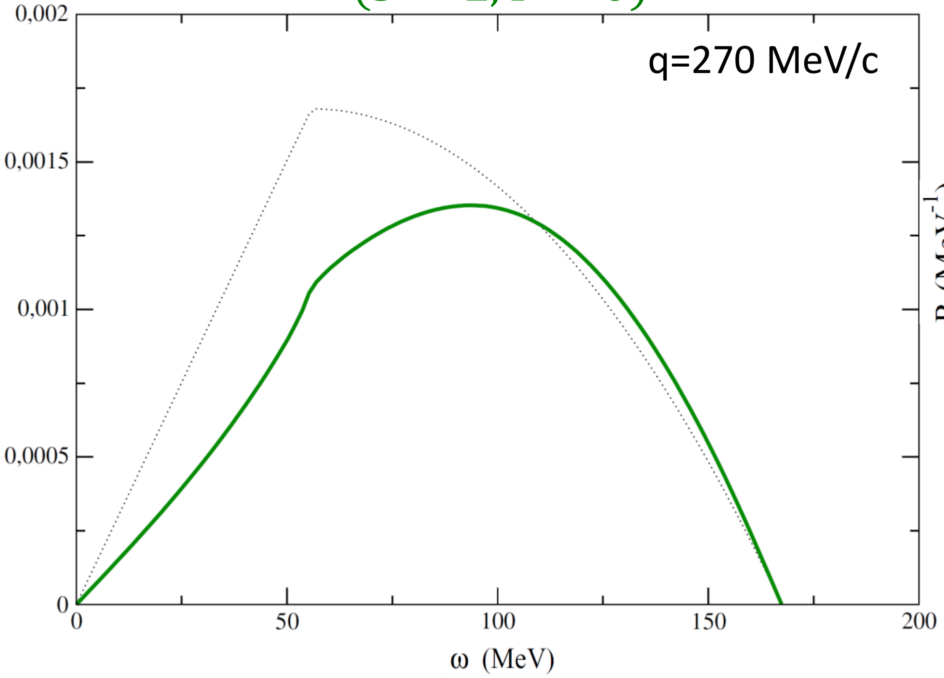
Nuclear matter

A. De Pace, M. Martini, Phys. Rev. C 94, 024342 (2016)

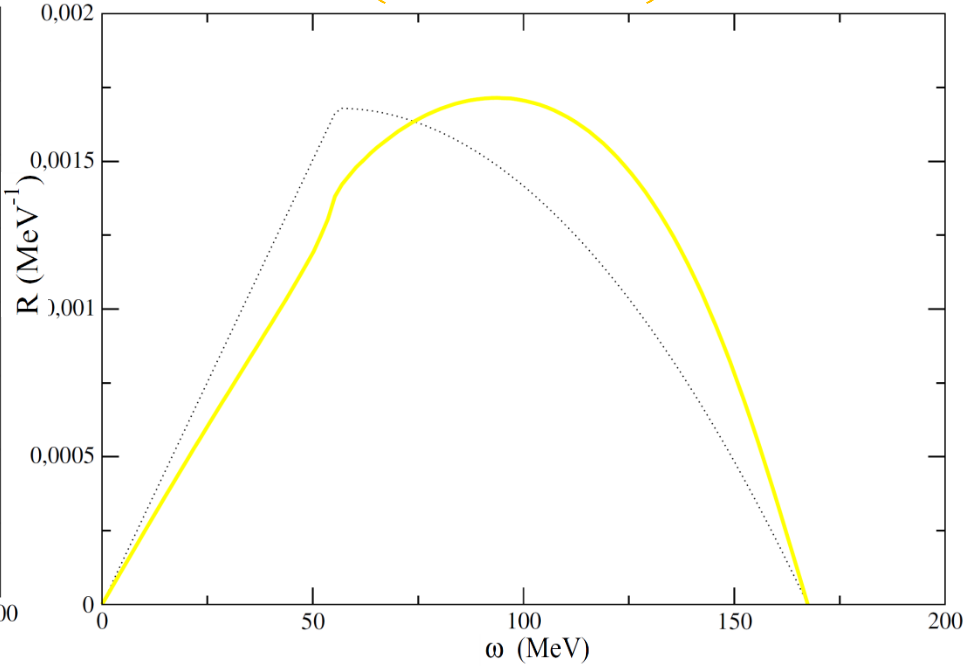
From the (S,T) results without tensor…

D1S

$(\boldsymbol{S=1,T=0})$

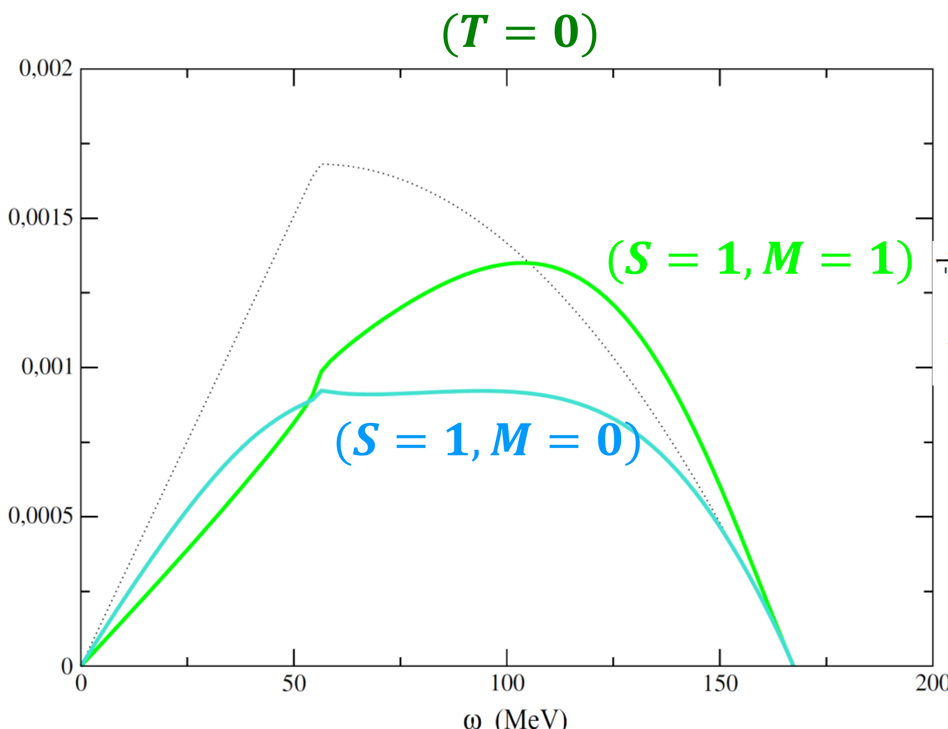


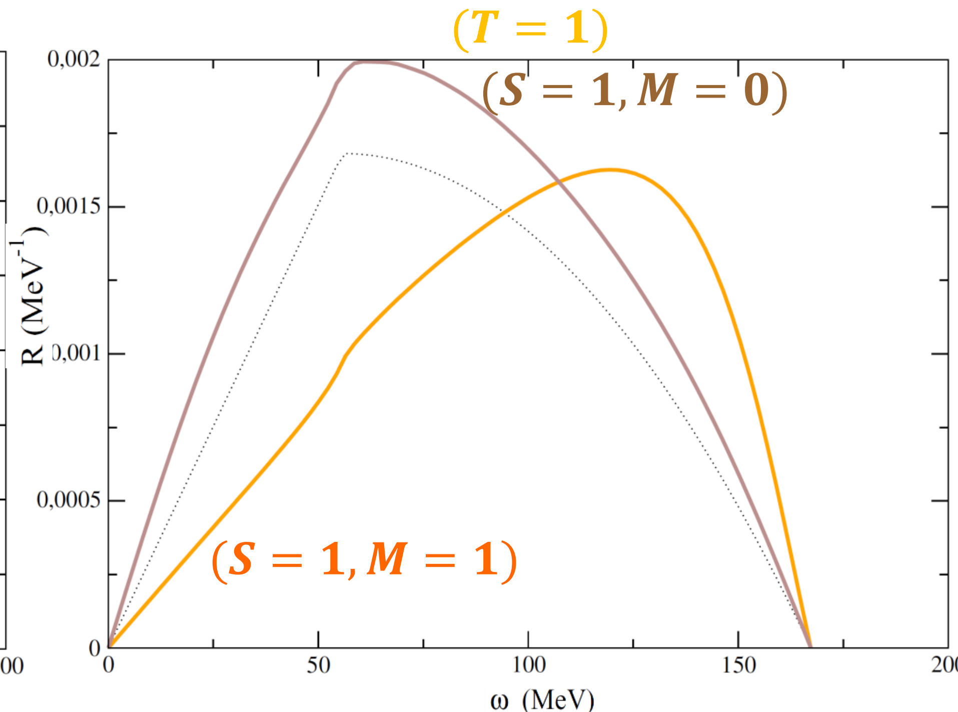
$(\boldsymbol{S=1,T=1})$



… to the (S,M,T) results with tensor

D1ST

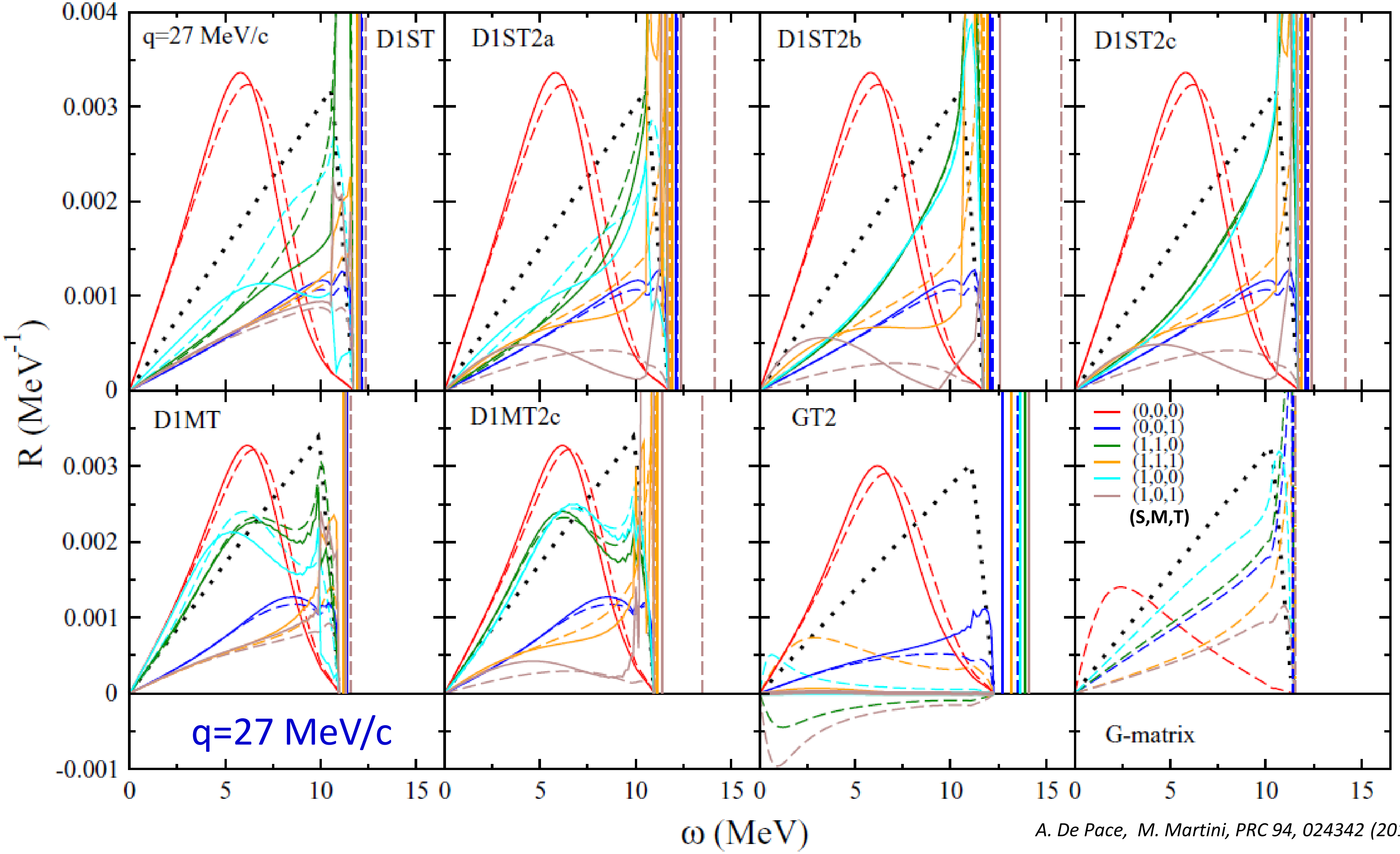




M=0:
Spin Longitudinal $(\boldsymbol{\sigma}_j \cdot \widehat{q})$

M=1:
Spin Transverse $(\boldsymbol{\sigma}_j \times \widehat{q})$

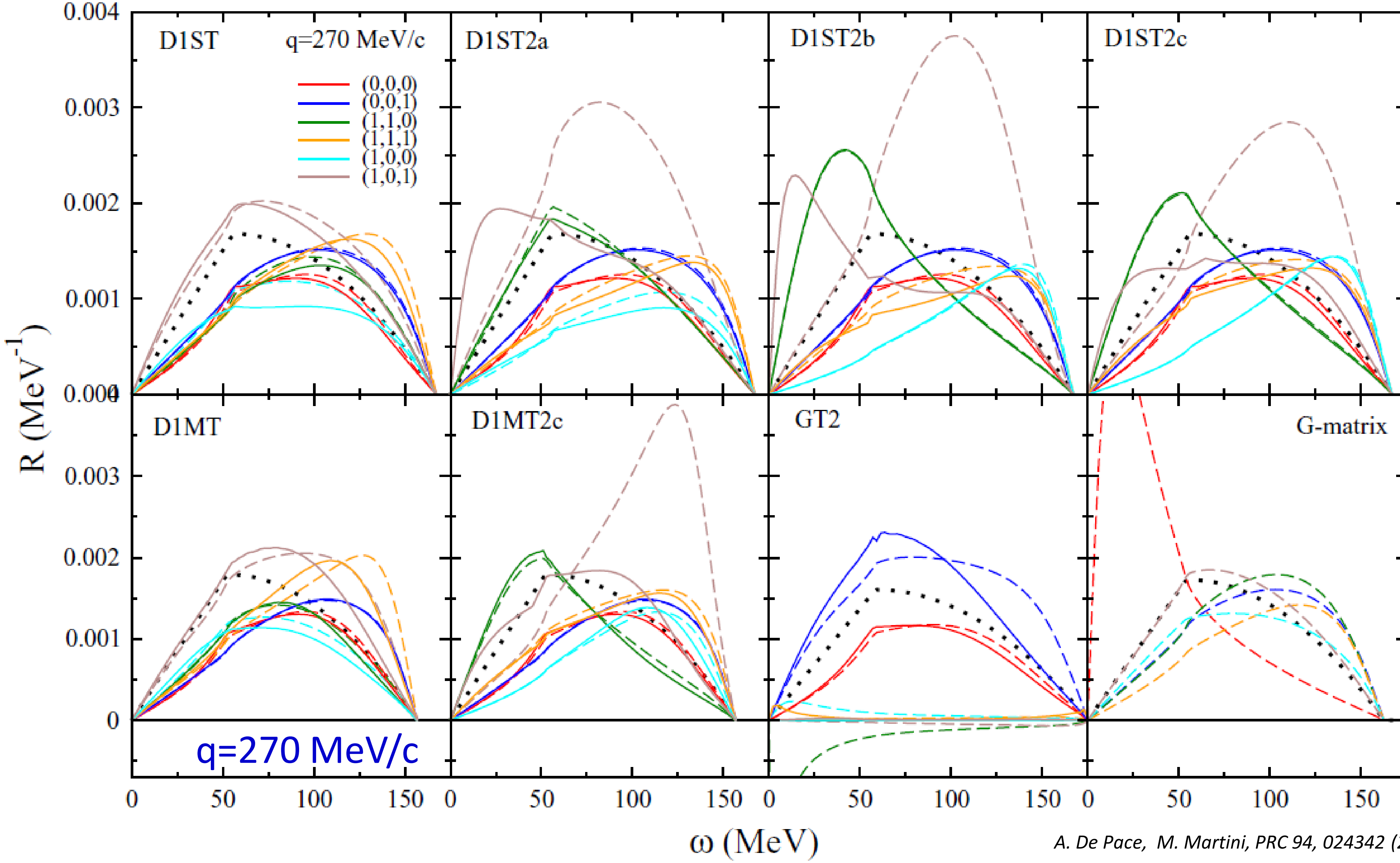
RPA responses with Gogny+tensor at first and second order in CF



A. De Pace, M. Martini, PRC 94, 024342 (2016)

- Homogeneity of results in the S =0 channels and heterogeneity in the S =1 ones
- The differences between the results obtained at 1st and 2nd order in the CF expansion are more pronounced for the forces including tensor terms
- Some unphysical results (R<0) with GT2 ↔ lack of convergence of the CF method with GT2

RPA responses with Gogny+tensor at first and second order in CF



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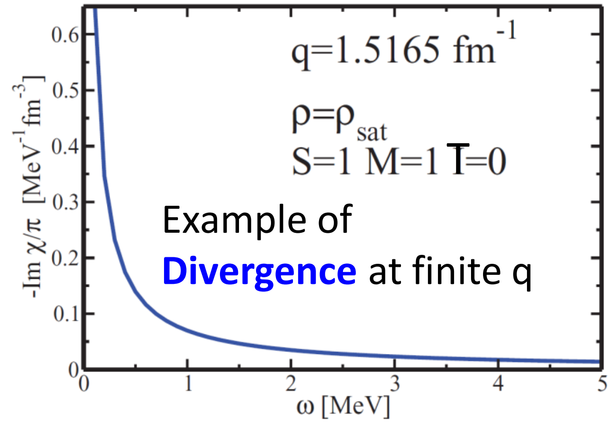
RPA Response functions as tools to detect finite-size instabilities

$$\Pi_X^{\mathrm{RPA}} = \frac{\Pi^{(0)}}{1 - \Pi_X^{(1)\mathrm{d}}/\Pi^{(0)} - \Pi_X^{(1)\mathrm{ex}}/\Pi^{(0)} - \dfrac{\Pi_X^{(2)\mathrm{ex}}/\Pi^{(0)} - \left[\Pi_X^{(1)\mathrm{ex}}/\Pi^{(0)}\right]^2}{1 - \dots}}$$

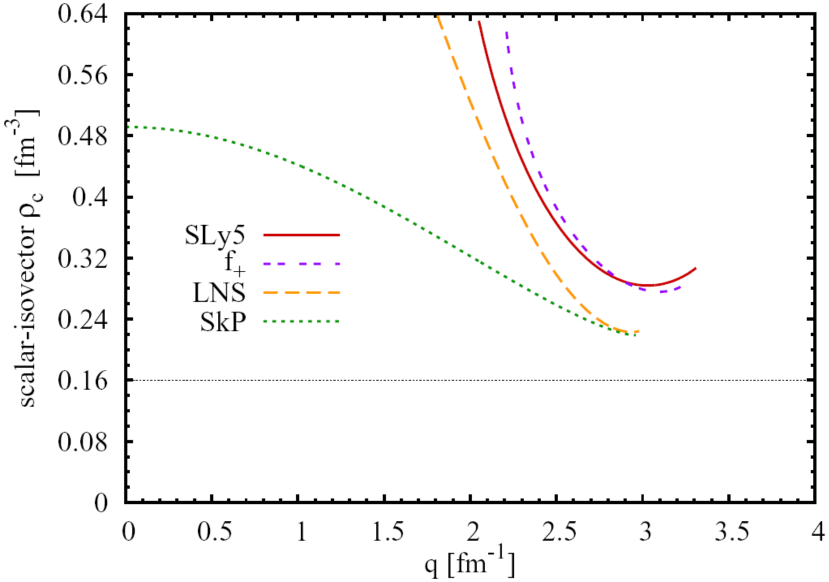


$\Pi_{\mathbf{RPA}}$ depends on: q, ω, interaction, density (k_F)

$\Pi_{\mathbf{RPA}}$ can diverge

Critical density ρ_C:

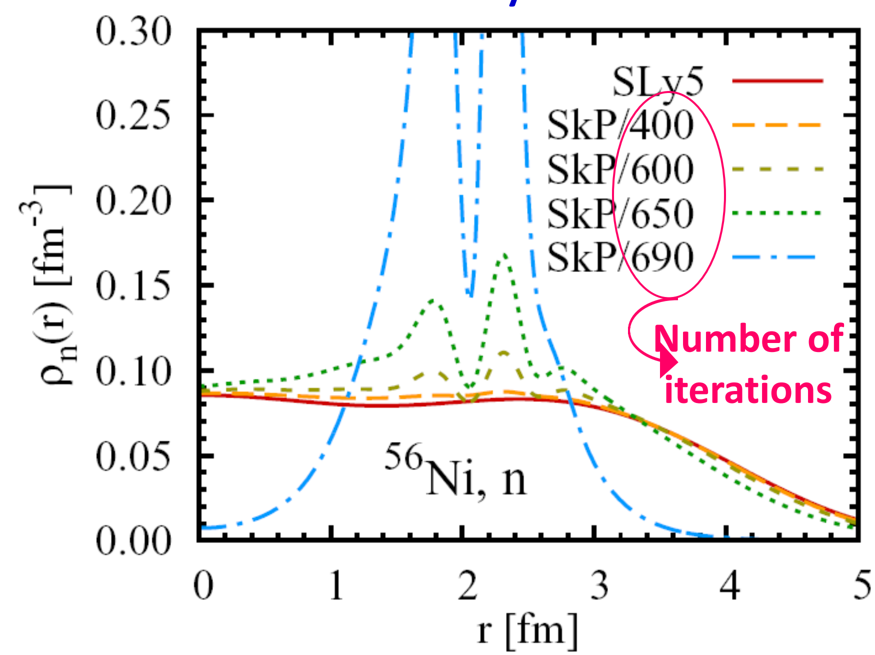
density where $\Pi_{\mathbf{RPA}}(q,\omega=0)$ diverges



At finite q

ρ_C can be close to ρ_{SAT}

MF density distribution



Impossibility to converge for MF calculations

Qualitative link proposed in T. Lesinski, K. Bennaceur, T. Duguet, J. Meyer, PRC 74, 044315(2006)

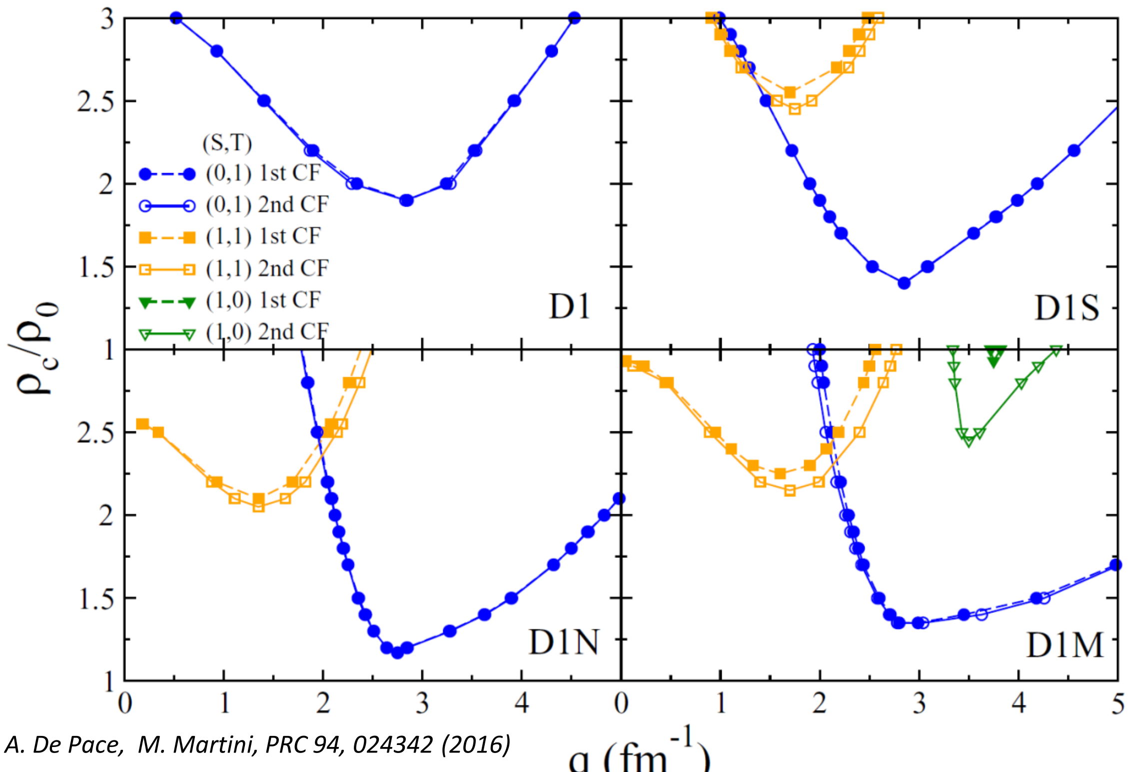
Finite nuclei and nuclear matter instabilities

Quantitative analysis of the connection between finite nuclei and nuclear matter instabilities:

- (S=0, T=1): *Hellemans,Pastore,Duguet,Bennaceur,Davesne,Meyer,Bender,Heenen,PRC88,064323 (2013)*
- S=1: *Pastore, Tarpanov, Davesne, Navarro,PRC 92, 024305 (2015)*

1. **A functional is stable if the lowest critical density at which a pole occurs in nuclear matter calculations is larger than the central density of ^{40}Ca (~1.2 ρ_{sat})**
2. **One has also to verify that this pole represents a distinct global minimum in the (ρ_c,q) plane**

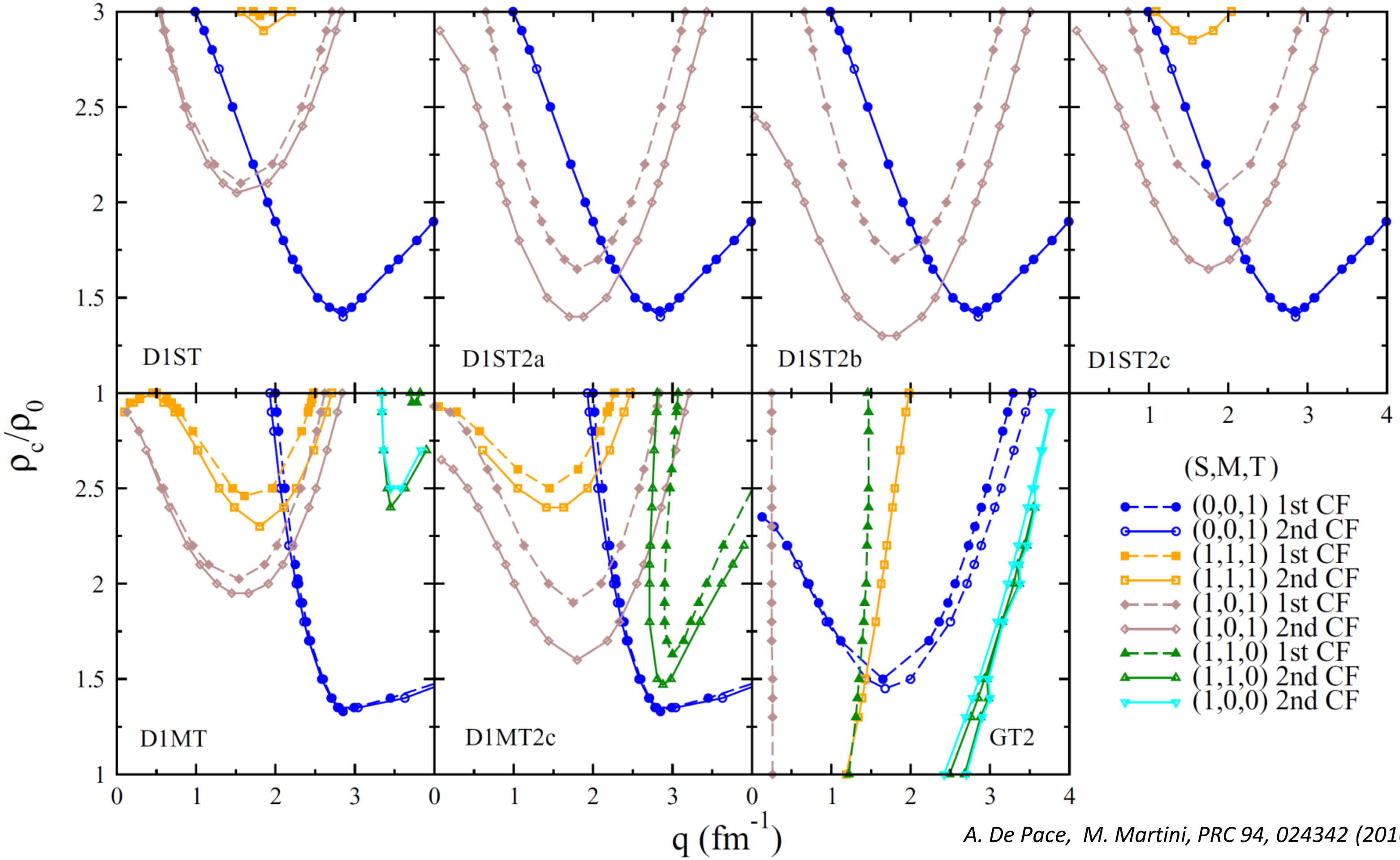
Critical densities for the most commonly used Gogny forces



A. De Pace, M. Martini, PRC 94, 024342 (2016)

- D1, D1S, and D1M satisfy the stability criteria in all the channels hence they are free of spurious finite-size instabilities
- D1N should be treated with some caution
- Calculation at 1st order in the CF expansion can be considered enough for instabilities studies

Critical densities for the Gogny forces with tensor terms



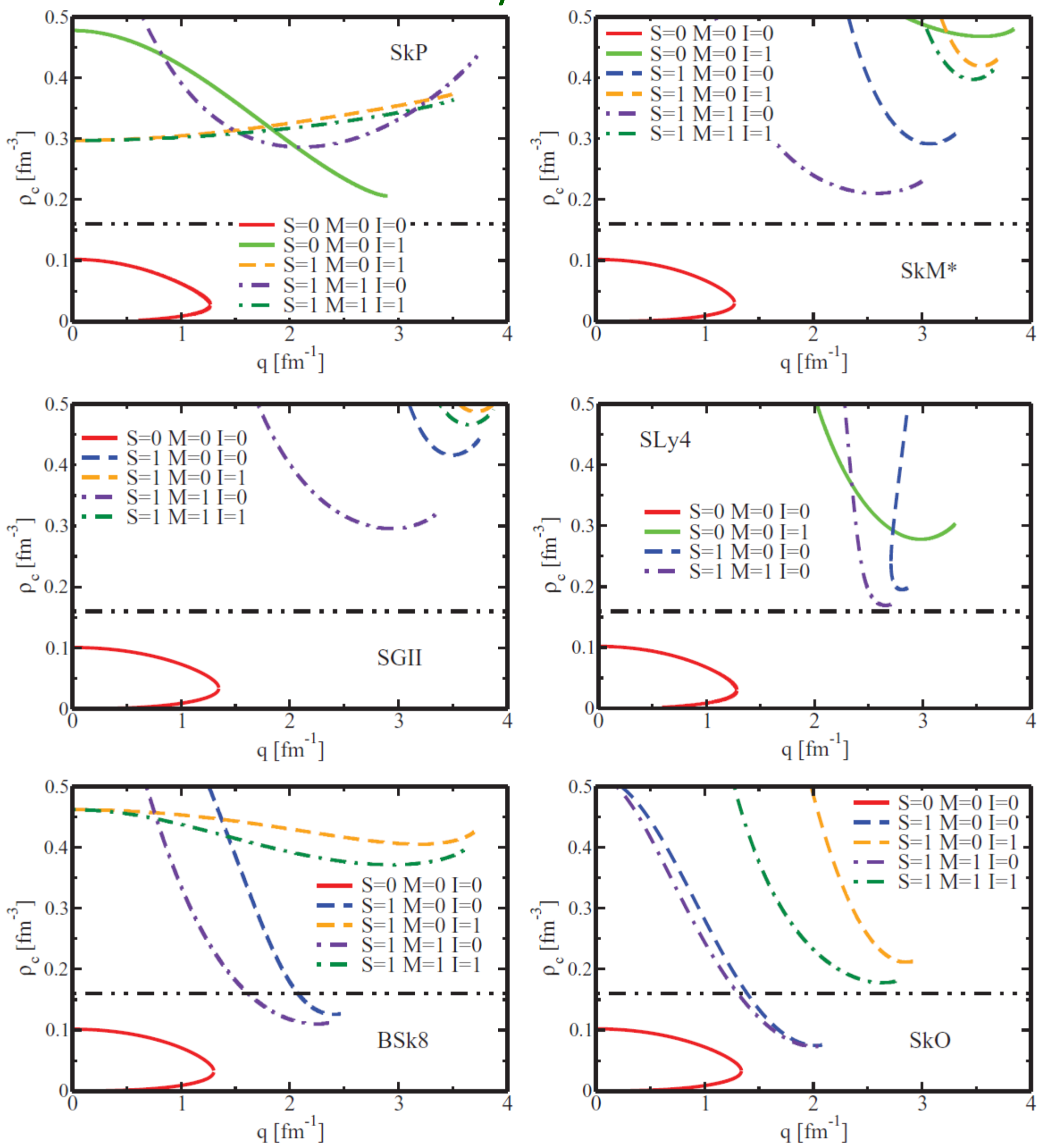
A. De Pace, M. Martini, PRC 94, 024342 (2016)

- In the S =1 channels differences between the results at 1st and 2nd order in the CF expansion appear
- The stability criteria are satisfied in all the (S,M,T) channels by all the Gogny forces including tensor terms of type D1MT* and D1ST*
- At least at 1st and 2nd order in the CF expansion, the GT2 force is unstable in all the S =1 channels.

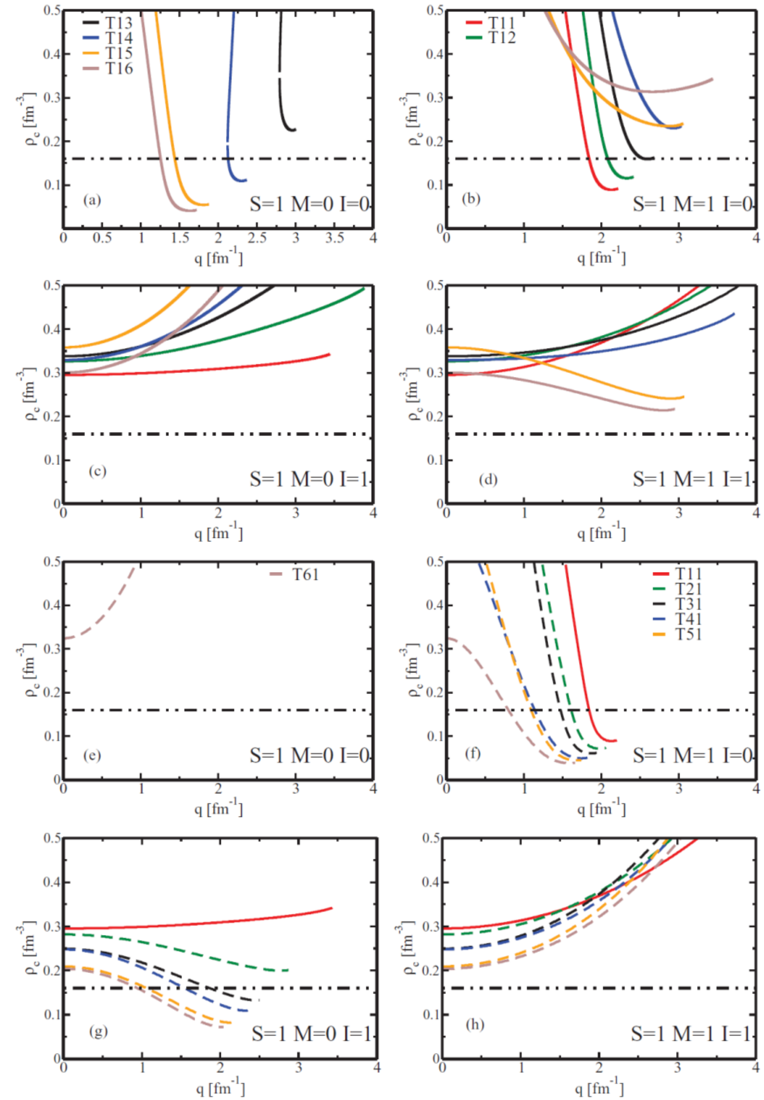
Perspective: Insert this tool in the fitting procedure to construct new Gogny type forces

Critical densities for some Skyrme functionals

Commonly used forces

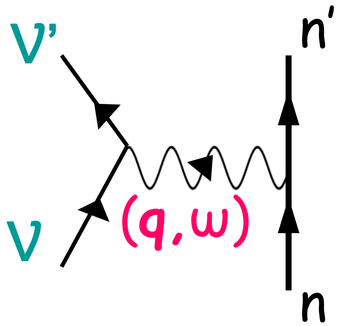


Tensor terms

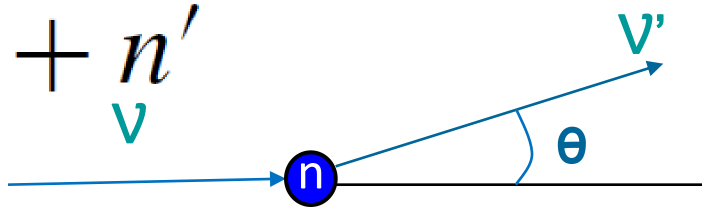


Pastore, Davesne, Lallouet, Martini, Bennaceur, Meyer, Phys. Rev. C 85, 054317 (2012)

Neutrino mean free path in neutron matter



$$\nu + n \longrightarrow \nu' + n'$$



$$\lambda = (\sigma\rho)^{-1}$$

Double differential cross section per neutron:

$$\frac{d^2\sigma\,(E_\nu)}{d\Omega_{k'}d\omega} = \frac{G_F^2 E_\nu'^2}{16\pi^2}\left[(1+\cos\theta)\,R^V(q,\omega) + g_A^2\,(3-\cos\theta)\,R^A(q,\omega)\right]$$

Nuclear response functions:

$$R^V(q,\omega) = -\frac{1}{\pi\rho}\mathrm{Im}\chi^{(S=0)}(q,\omega)$$

Response to density perturbation, S=0

Vector

$$R^A(q,\omega) = -\frac{1}{\pi\rho}\mathrm{Im}\chi^{(S=1)}(q,\omega)$$

Response to spin perturbation, S=1

Axial

ρ: neutron matter density

p.s: zero Temperature calculations

Tensor terms and neutrino mean free path

$$\frac{d^2\sigma(E_\nu)}{d\Omega_{k'}d\omega} = \frac{G_F^2 E_\nu'^2}{16\pi^2}\left\{(1+\cos\theta)\boxed{R^V} + g_A^2\left[\frac{2(E_\nu'\cos\theta - E_\nu)(E_\nu' - E_\nu\cos\theta)}{q^2} + 1 - \cos\theta\right]\boxed{R_L^A}\right.$$

$$\left. + g_A^2 2\left[\frac{E_\nu E_\nu'}{q^2}\sin^2\theta + 1 - \cos\theta\right]\boxed{R_T^A}\right\}.$$

$$R^V = -\frac{1}{\pi\rho}\mathrm{Im}\chi^{(S=0)}$$

Vector

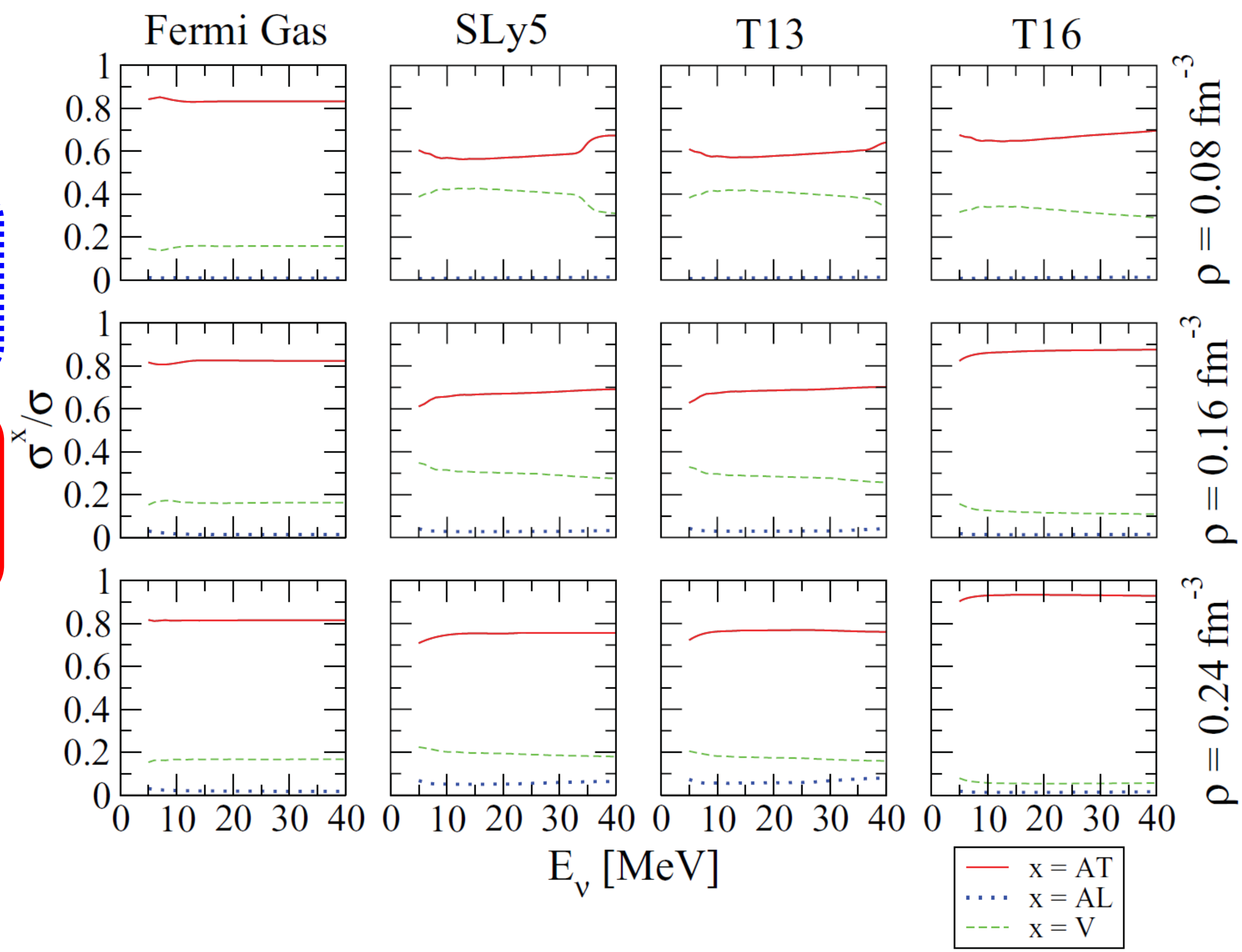
$$R_L^A = -\frac{1}{\pi\rho}\mathrm{Im}\chi^{(S=1,M=0)}$$

Spin Longitudinal

$$R_T^A = -\frac{1}{\pi\rho}\mathrm{Im}\chi^{(S=1,M=\pm1)}$$

Spin Transverse

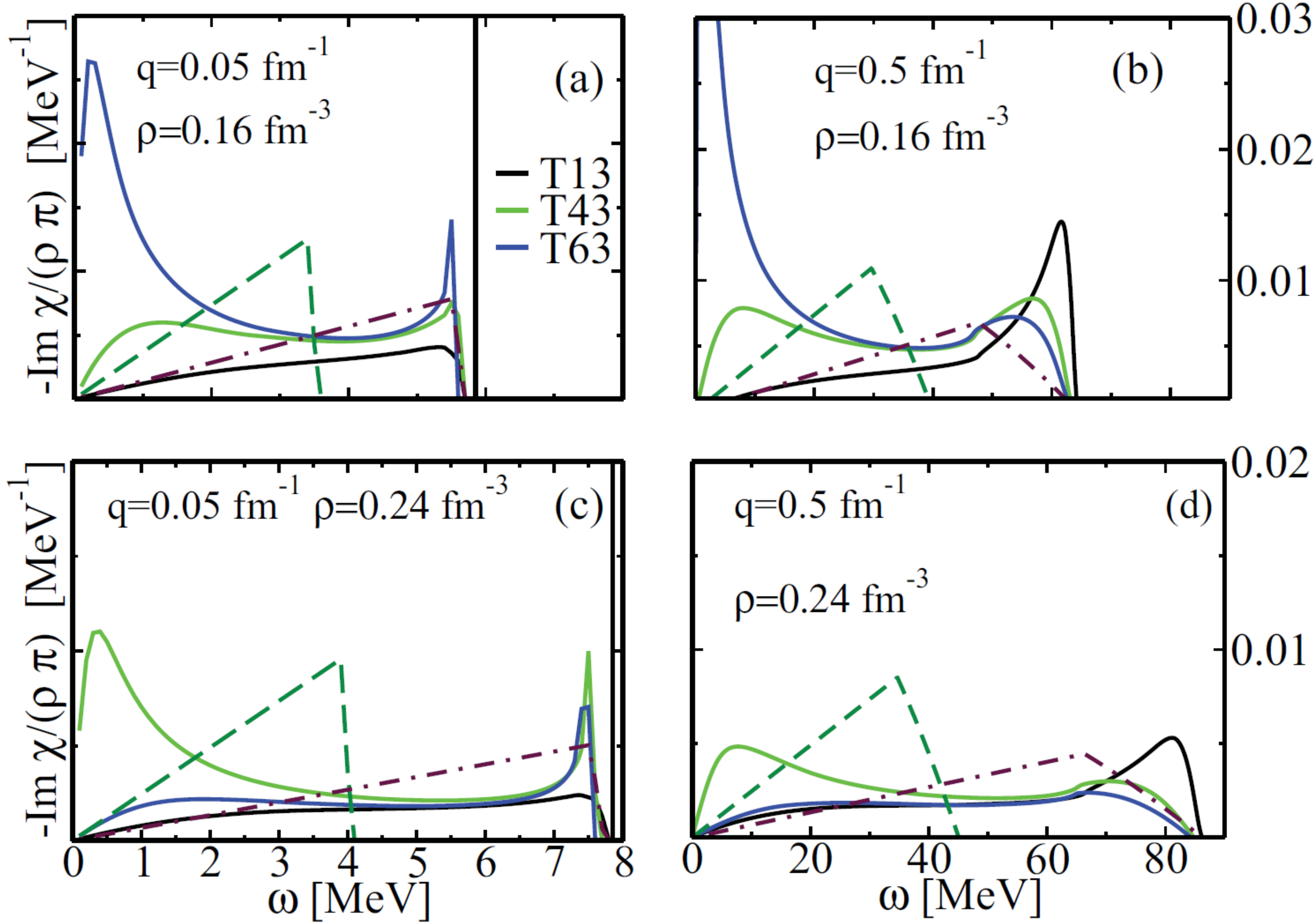
2 different sets of tensor parameters



Cross section dominated by Spin Transverse response

Variation of contributions reflects the effects of the interaction

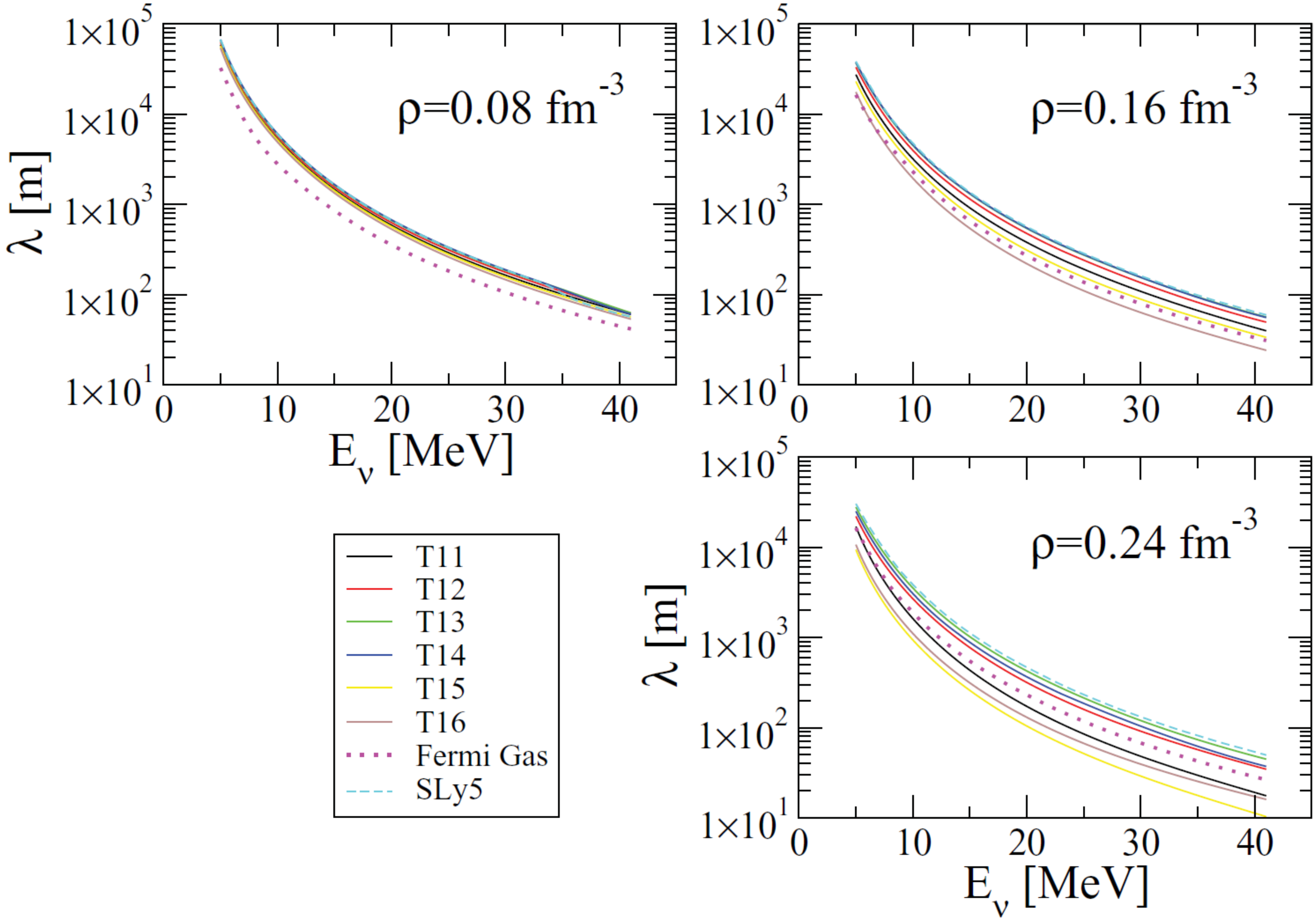
Spin transverse response for 3 set of tensor parameters



Different behaviors:

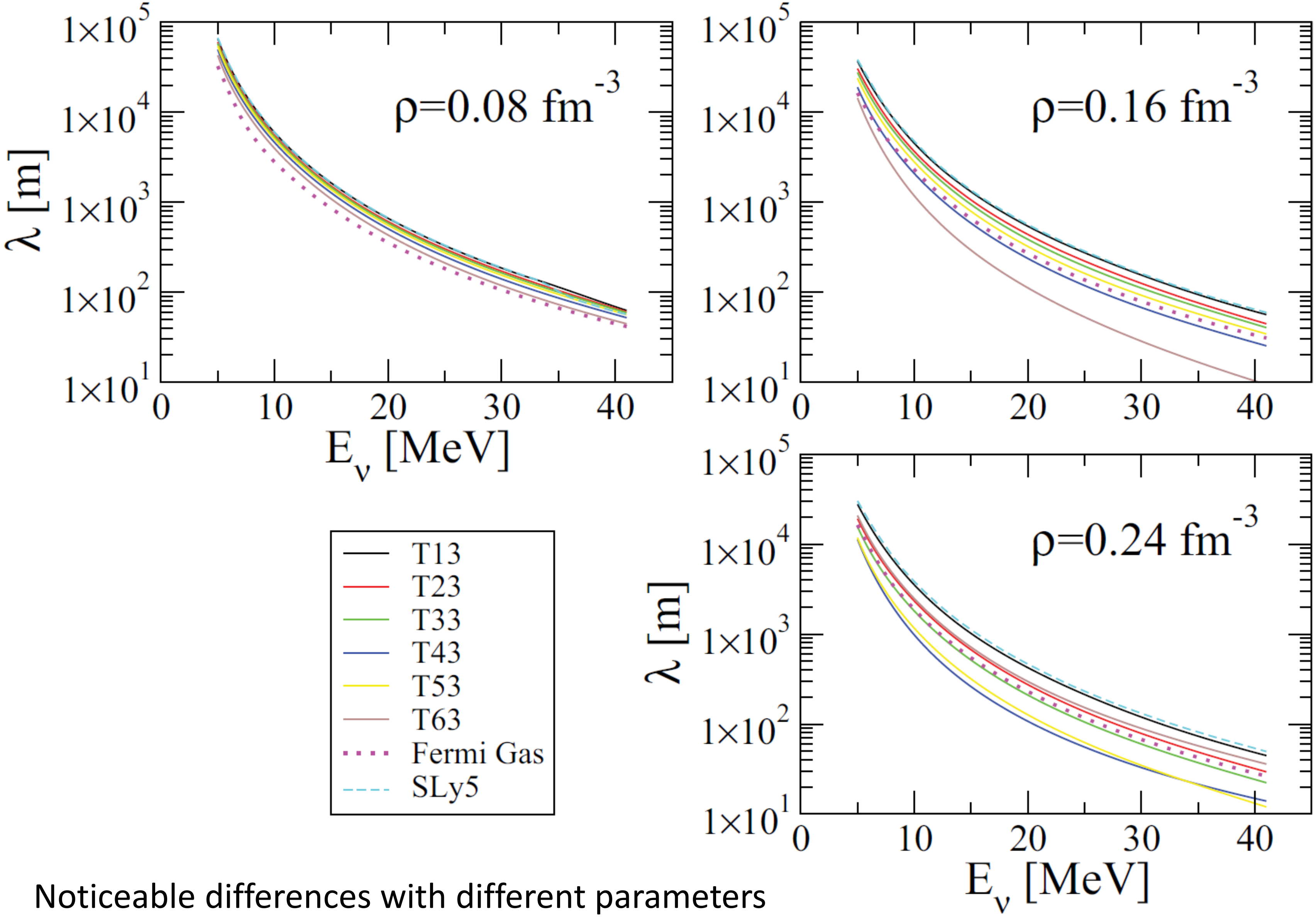
quenching, enhancement, divergences, collective mode

Impact of tensor terms on neutrino mean-free path (I)



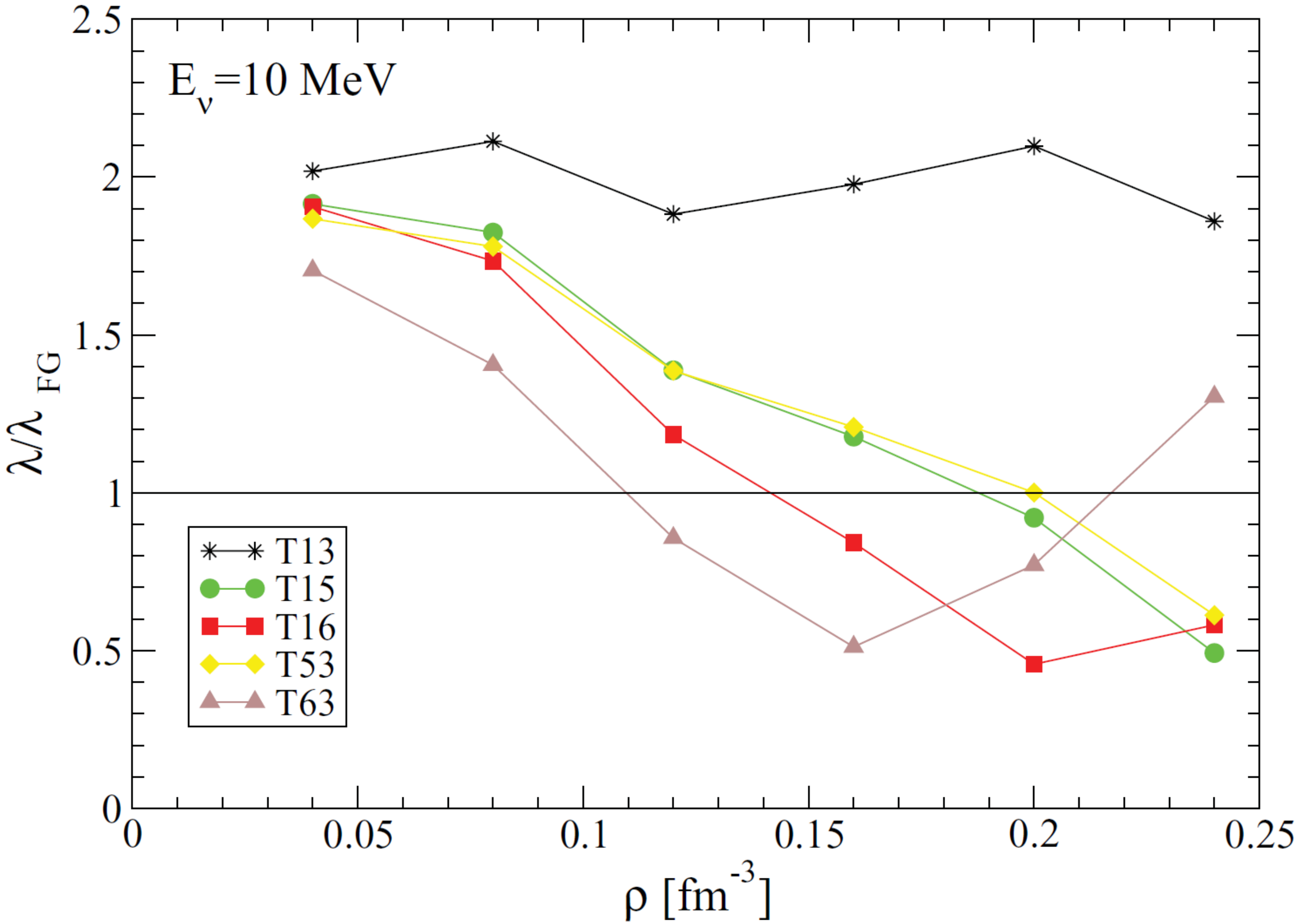
Pastore, Martini, Buridon, Davesne, Bennaceur, Meyer, Phys. Rev. C 86, 044308 (2012)

Impact of tensor terms on neutrino mean-free path (II)



- Noticeable differences with different parameters
- The spread increases with the density
- Density behavior not so trivial

Density behavior of neutrino mean-free path



Extra density dependent terms in Skyrme functionals

$$+\frac{1}{2}t_4(1+x_4\hat{P}_\sigma)\left[\mathbf{k}'^2\rho^\beta(\mathbf{R})\delta_\mathbf{r}+\delta_\mathbf{r}\rho^\beta(\mathbf{R})\mathbf{k}^2\right]$$

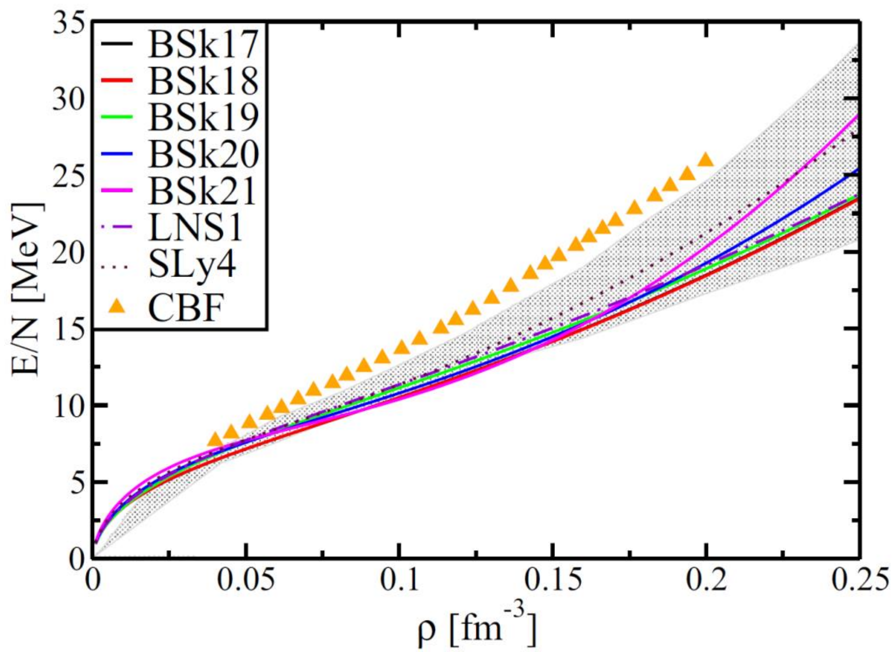
$$+t_5(1+x_5\hat{P}_\sigma)\mathbf{k}'\rho^\gamma(\mathbf{R})\cdot\delta_\mathbf{r}\mathbf{k}$$

Goriely, Chamel, Pearson, PRC 82, 035804 (2010)

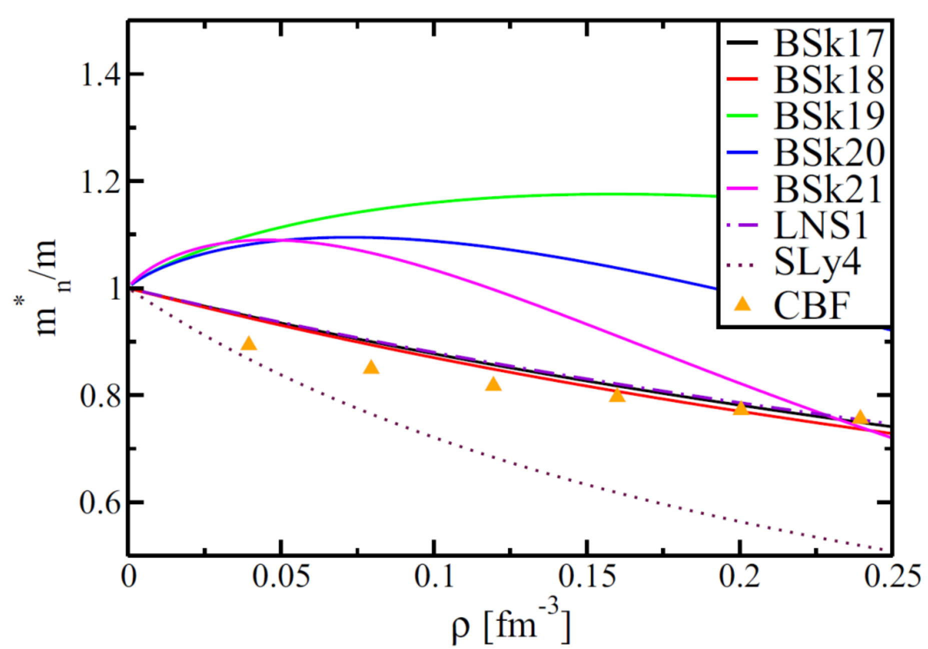
Brussels-Montreal Skyrme functionals (BSk)

- Fitted to ground state properties (masses and radii) of ≈ 2000 measured nuclei
- Constrained to reproduce properties of infinite nuclear matter obtained from *ab-initio* calc.
- Constrained to avoid ferromagnetic instabilities in infinite matter

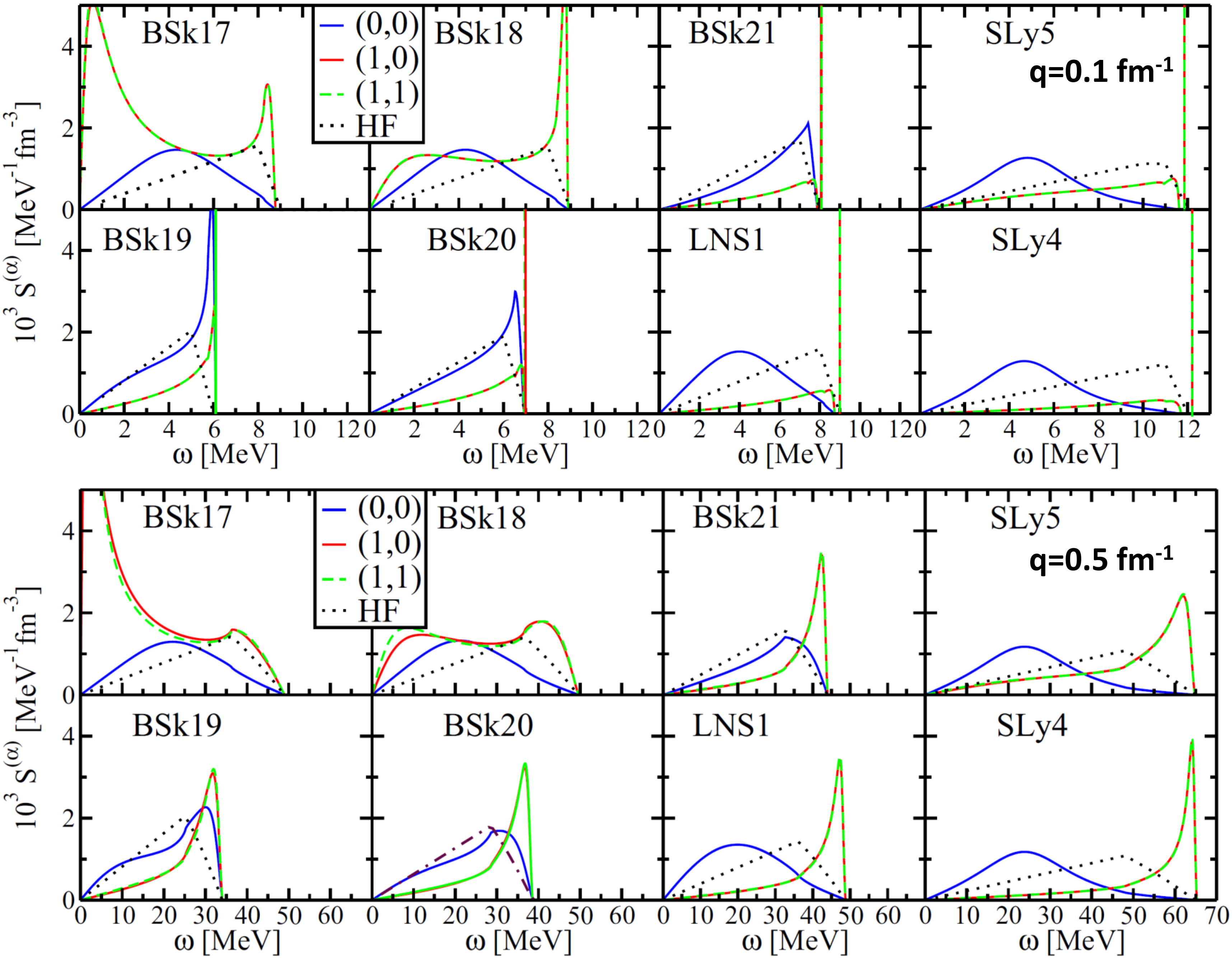
Neutron Matter Equation of State



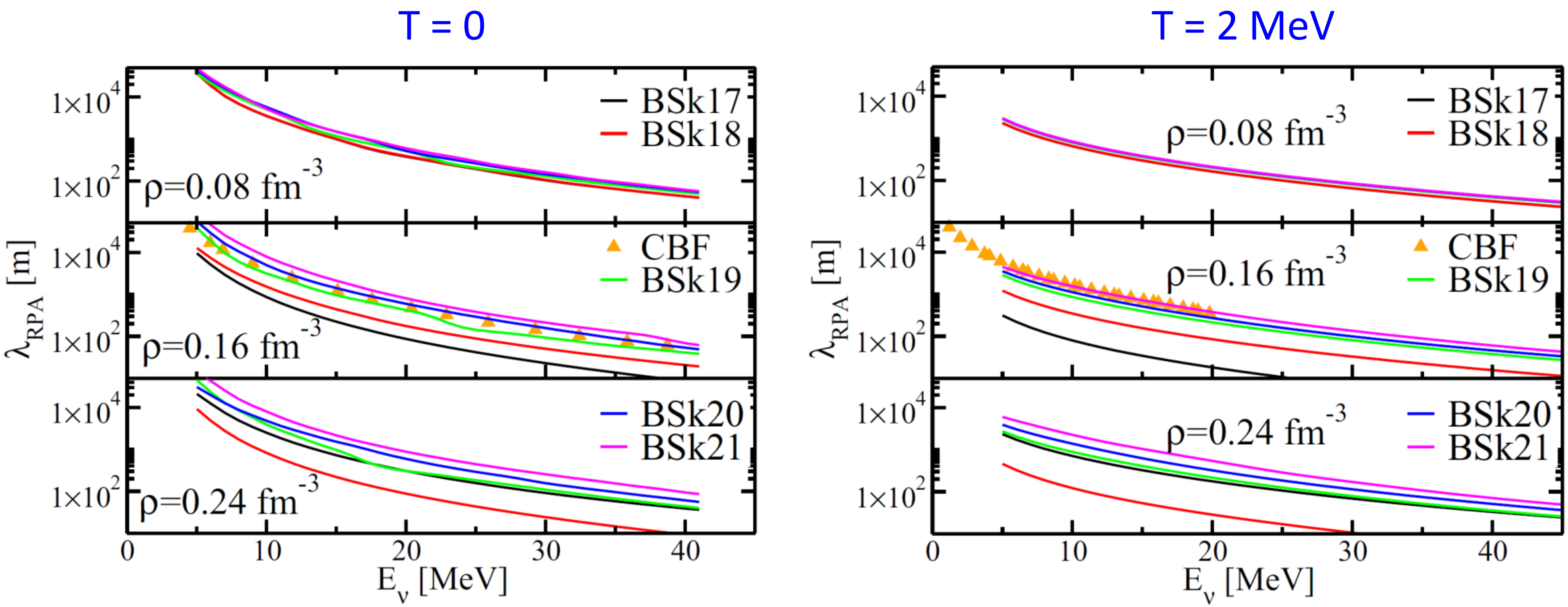
Effective neutron mass



Response functions



Neutrino mean-free path with Brussels-Montreal Skyrme functionals



BSk20 and BSk21 able to reproduce the rather complicated calculations based on CBF
---> remarkable advantage in terms of computational time

Pastore, Martini, Davesne, Navarro, Goriely, Chamel, Phys. Rev. C 90, 025804 (2014)

$$\rho(\mathbf{r}) = \rho_0 + \int d^3\mathbf{r}'\chi(\mathbf{r}-\mathbf{r}')V_{\text{ext}}(\mathbf{r}') \qquad \chi(\mathbf{r}-\mathbf{r}') = \left(\frac{\delta\rho(\mathbf{r})}{\delta V_{\text{ext}}(\mathbf{r}')}\right)$$

$$E(\mathbf{q}) = E_0 + \frac{\chi(q)}{\rho_0}v_q^2 + \cdots \qquad \lim_{q\to 0}\chi(q) = -\rho^2\kappa$$

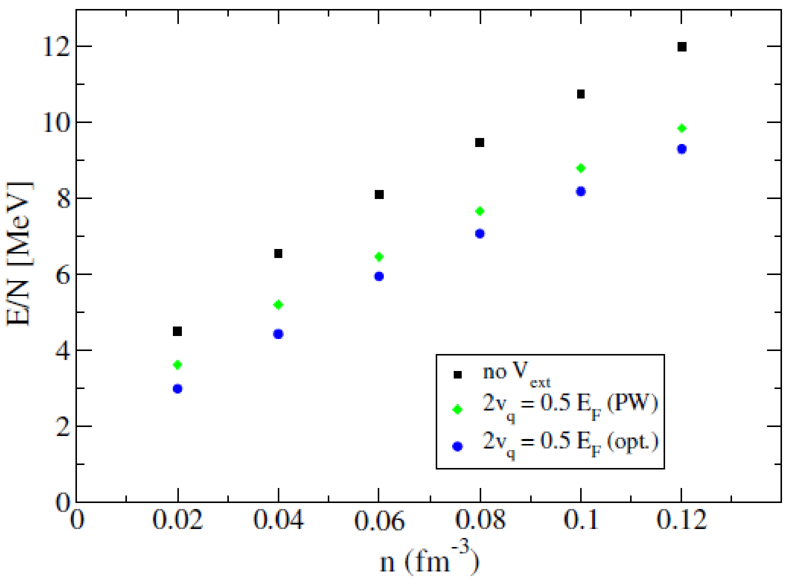


FIG. 1. (color online) Neutron-matter energy per particle as a function of density using NN interactions and AFDMC. Squares correspond to the case without a one-body potential; diamonds to a one-body potential of fixed strength $2v_q = 0.5E_F$, periodicity $q = 4\pi/L$, and plane-wave single-particle orbitals (PW); circles to a one-body potential of fixed strength $2v_q = 0.5E_F$, periodicity $q = 4\pi/L$, and optimized single-particle orbitals (opt.).

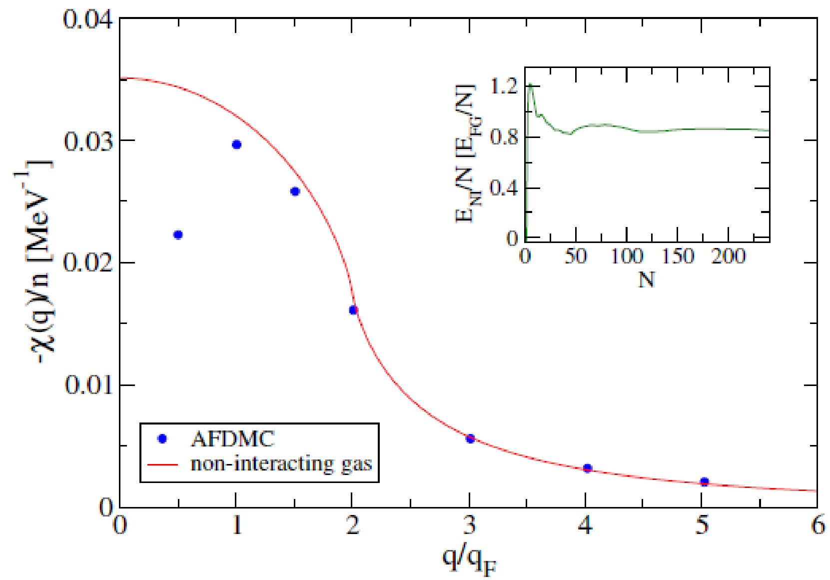


FIG. 3. Static-response function of neutron matter at a density of $n = 0.10\,\text{fm}^{-3}$. Points follow from AFDMC results using NN+NNN interactions, as well as several one-body strengths and periodicities. The line is the Lindhard function describing the response of a non-interacting Fermi gas. Inset: finite-size dependence of a non-interacting Fermi gas in the presence of a one-body potential of fixed strength $2v_q = 0.5E_F$ and periodicity $q = 4\pi/L$.

Buraczynski, Gezerlis. Phys.Rev.Lett. 116 (2016), 152501