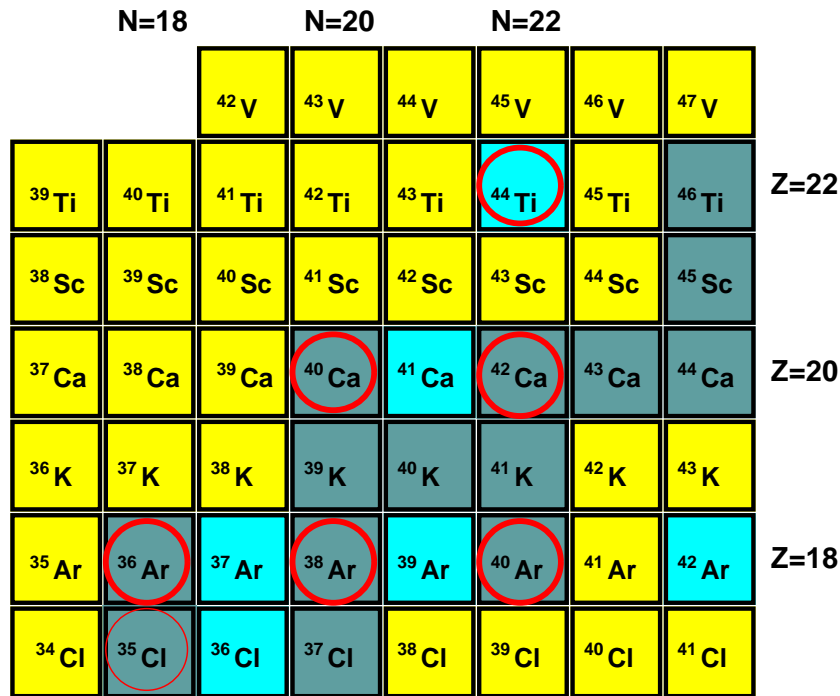


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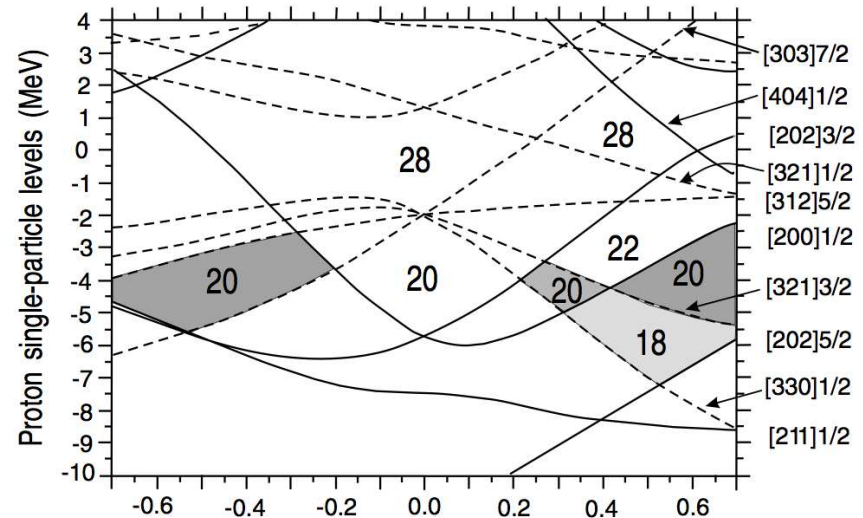
# Shape coexistence in the $A \sim 40$ region

Magda Zielińska  
IRFU/DPhN, CEA Saclay, France

# Highly-deformed structures in the $A \sim 40$ region

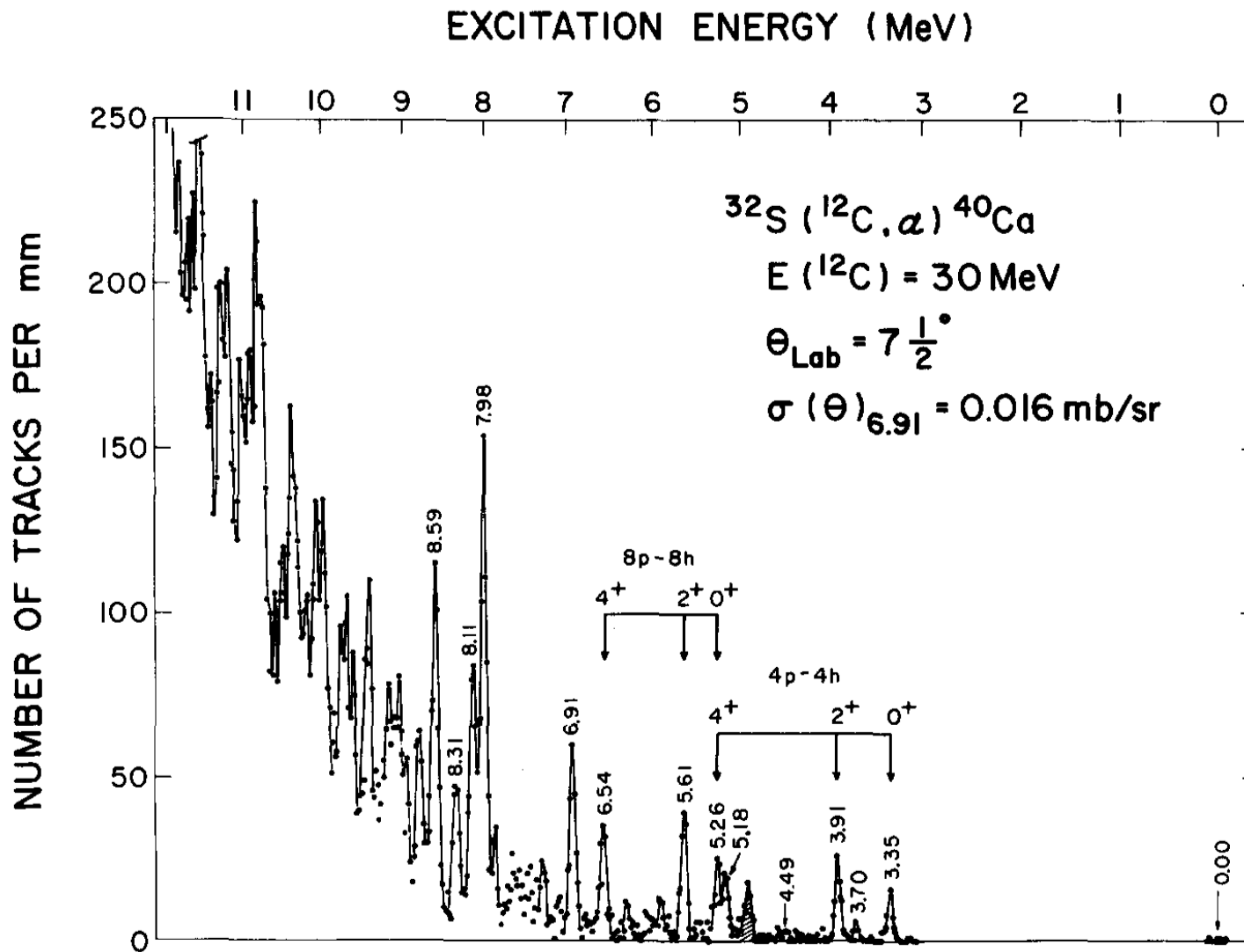


E. Ideguchi et al., PRL 81 (2001) 222501



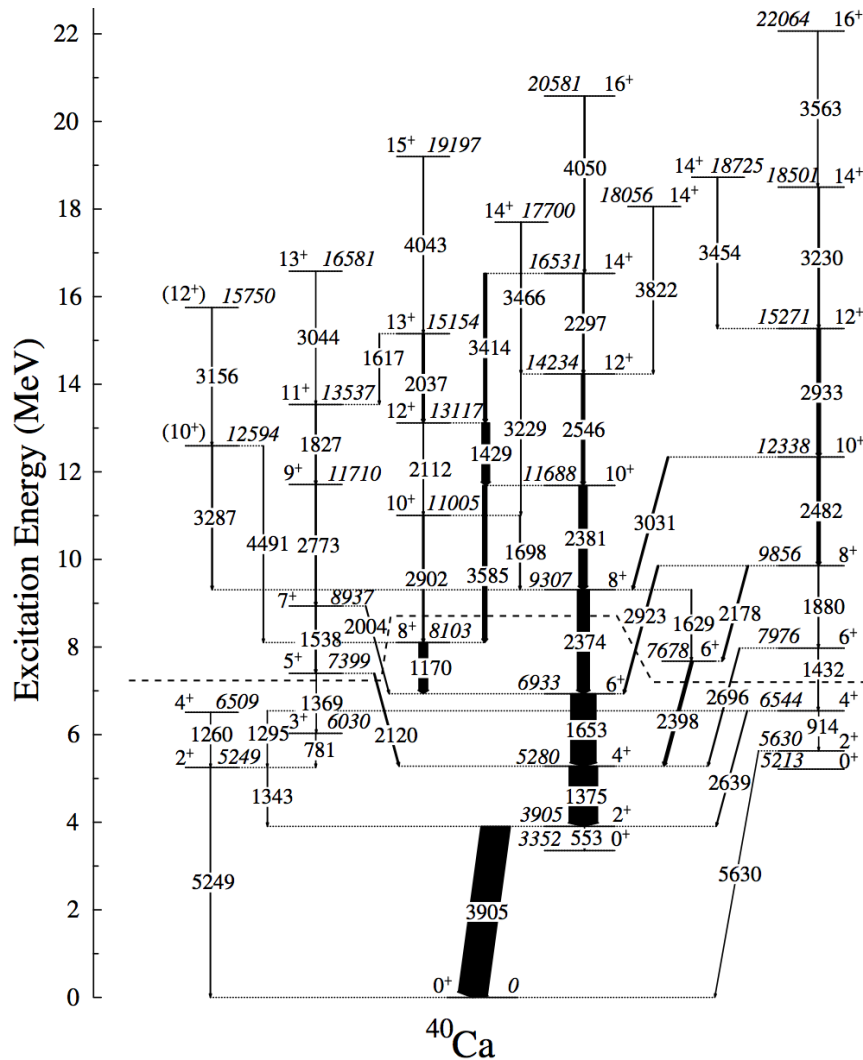
- spherical and highly-deformed magic numbers appear at similar particle numbers – dramatic shape coexistence
- intense transitions linking very deformed structures to ground-state bands – mixing of configurations

# Identification of 4p-4h and 8p-8h structures in $^{40}\text{Ca}$

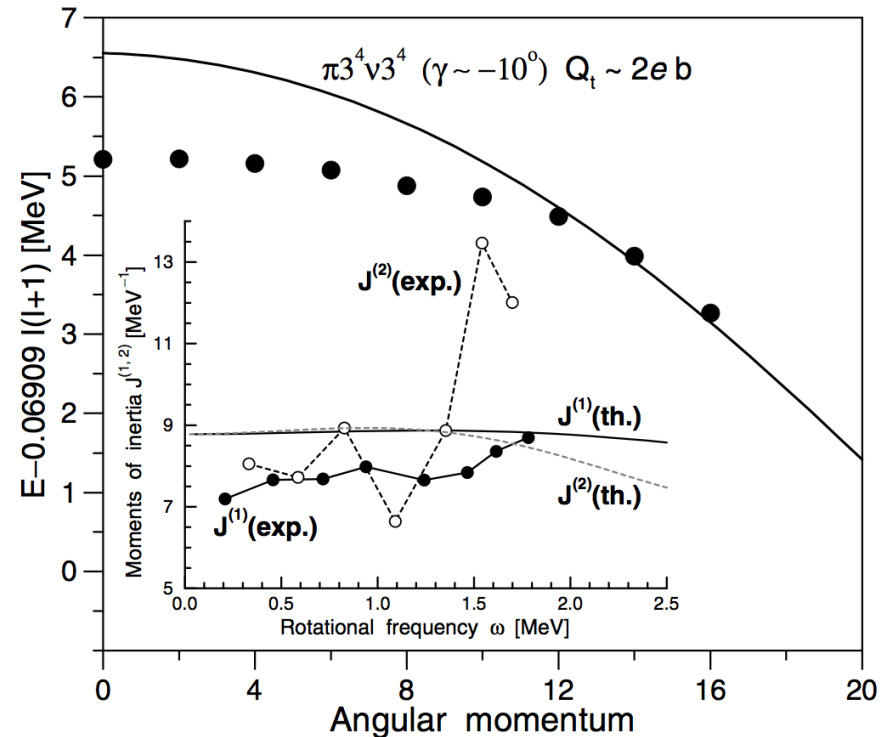


R. Middleton et al, Phys. Lett. 39B (1972) 339

# High-spin spectroscopy around $^{40}\text{Ca}$



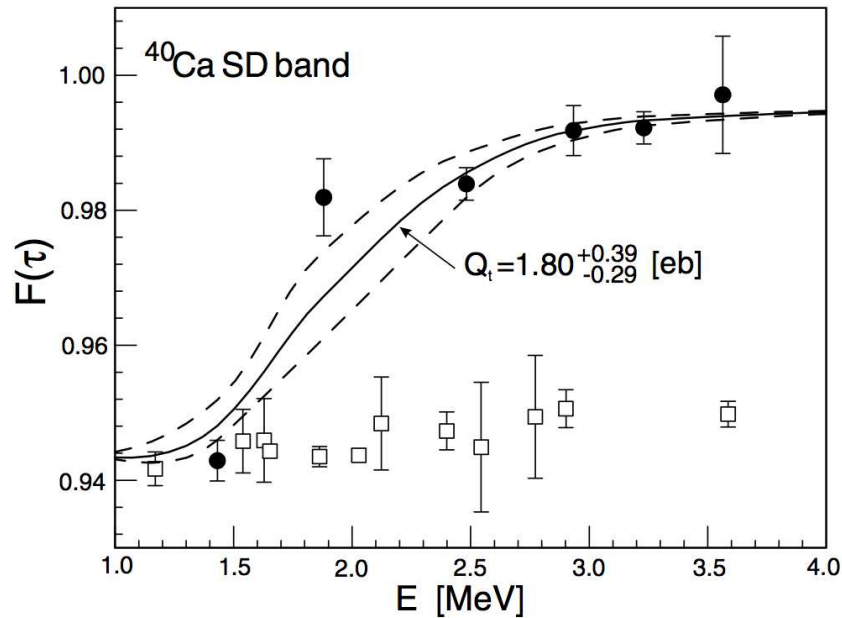
E. Ideguchi et al., PRL 81 (2001) 222501



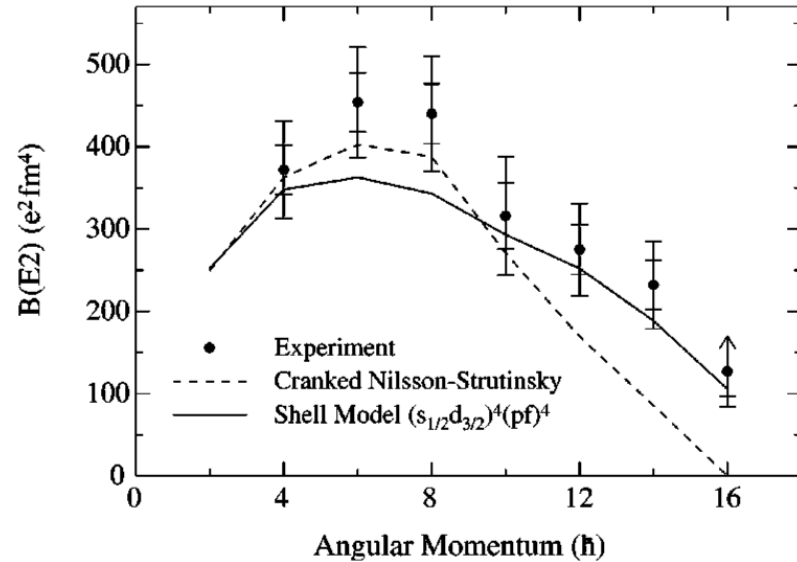
- regular rotational bands built on  $0^+$  states observed up to spin  $14^+ - 16^+$  in  $^{40}\text{Ca}$ ,  $^{38}\text{Ar}$ ,  $^{36}\text{Ar}$ ...

# Lifetime measurements in deformed bands

E. Ideguchi et al., PRL 81 (2001) 222501



C. Svensson et al., PRL 85 (2000) 2693



- $^{40}\text{Ca}$ :  $B(E2; 4_{\text{SD}}^+ \rightarrow 2_{\text{SD}}^+) = 170 \text{ Wu}$ 
  - band built on  $0_2^+$ :  $Q_0 = 0.74 \pm 0.14 \text{ eb}$  ( $\beta = 0.27$ )
- $^{36}\text{Ar}$ : lifetimes of individual SD states determined,  $Q_0$  not constant in the band:
  - $4^+ \rightarrow 2^+$ :  $114(9) \text{ eb}$  ( $\beta = 0.46$ ),  $6^+ \rightarrow 4^+$ :  $121(9) \text{ eb}$ ,  $8^+ \rightarrow 6^+$ :  $116(9) \text{ eb}$ ,
  - $10^+ \rightarrow 8^+$ :  $97(11) \text{ eb}$ ,  $12^+ \rightarrow 10^+$ :  $89(9) \text{ eb}$  ( $\beta = 0.35$ )

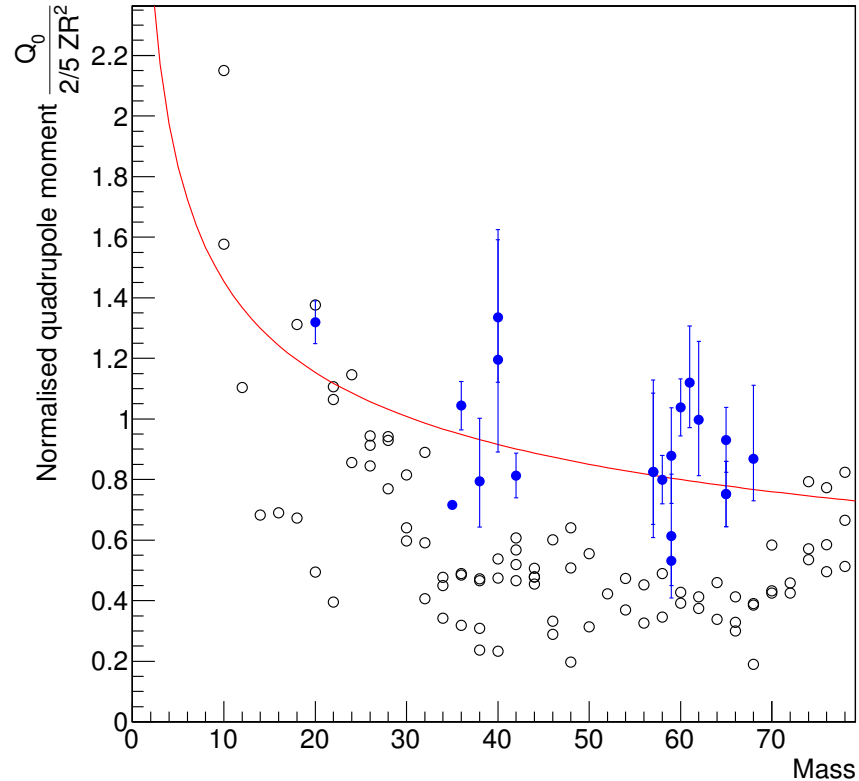
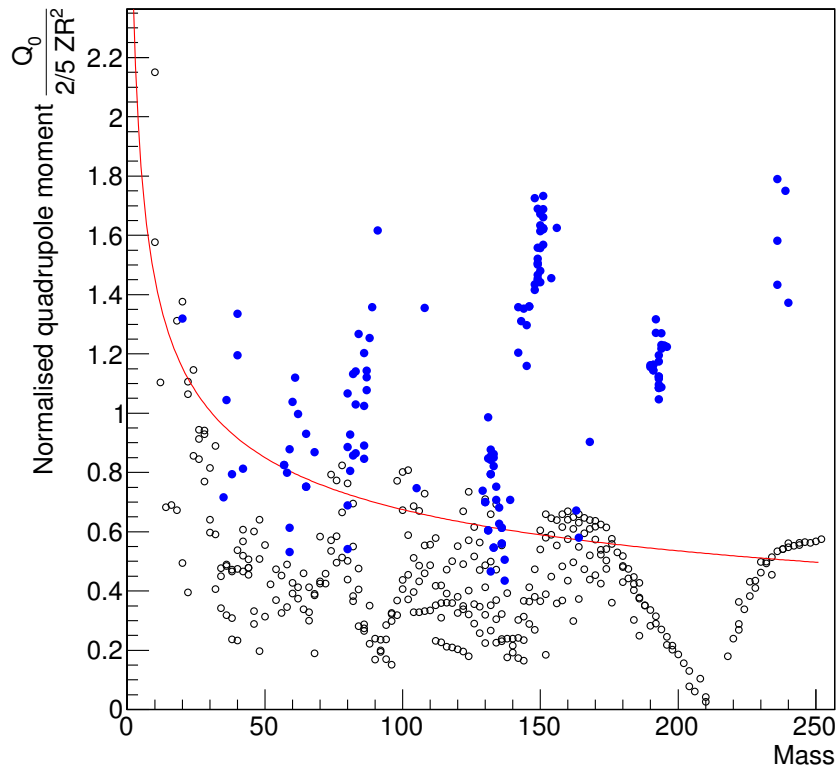
## Highly-deformed structures in the $A \sim 40$ region

isotope	experimental $\beta_2$	configuration	$0^+$ energy	
$^{40}\text{Ca}$	$0.59^{+0.11}_{-0.07}$	8p-8h	5.2 MeV	E. Ideguchi et al., PRL 81 (2001) 222501
	$0.27 \pm 0.05$	4p-4h	3.4 MeV	
$^{36}\text{Ar}$	$0.46 \pm 0.03$	4p-8h	4.3 MeV	C. Svensson et al., PRL 85 (2000) 2693
$^{38}\text{Ar}$	$(0.42^{+0.11}_{-0.08})$	4p-6h	3.4 MeV	R. Austin, PhD thesis (2004)
	$>0.68$	4p-6h	4.7 MeV	
$^{40}\text{Ar}$	$0.48^{+0.16}_{-0.10} \pm 0.05$	4p-4h	2.1 MeV	E. Ideguchi et al., PLB 686 (2010) 18
$^{44}\text{Ti}$	not known	8p-4h	3.1 MeV	D. O'Leary et al., PRC 61 (2000) 064314
$^{42}\text{Ca}$	$0.43(4) (0_2^+)$	6p-4h	1.8 MeV	K. Hadyńska-Klęk, PRL 117 (2016) 062501
	$0.45(4) (2_2^+)$	6p-4h		

- $^{40}\text{Ca}$ ,  $^{40}\text{Ar}$ : fit of one  $Q_0$  to the entire band
- $^{36}\text{Ar}$ : individual lifetimes determined,  $\beta_2$  from those of  $4^+$ ,  $6^+$  and  $8^+$
- $^{38}\text{Ar}$ : various fit methods tested, results strongly depend on assumptions
- $^{42}\text{Ca}$ : Coulomb excitation

# SD bands throughout the nuclear chart

K. Hadyńska-Klęk, PRC submitted

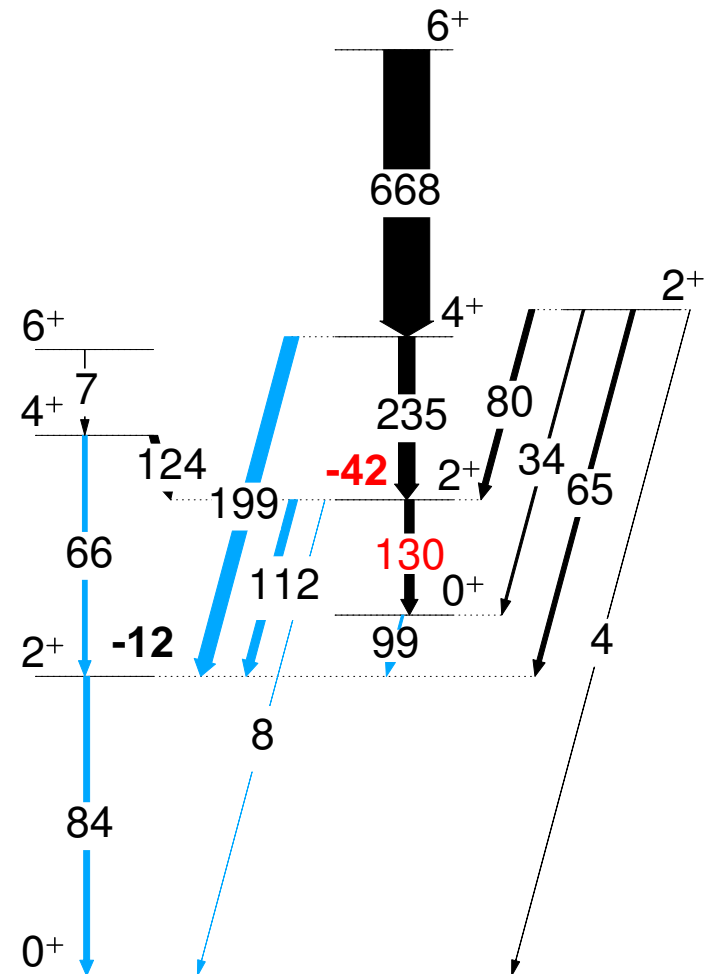


- SD bands in the  $A \sim 150$ ,  $A \sim 190$  and  $A \sim 230$  regions clearly separated from normal-deformed states
- those for  $A < 150$  span a broad range of deformations and are closer to the  $A^{-1/3}$  line

# Transition probabilities in $^{42}\text{Ca}$

K. Hadyńska-Klęk, PRL 117 (2016) 062501

	$\langle I_i    E2    I_f \rangle$ [e·fm <sup>2</sup> ]	$B(E2 \downarrow; I_i^+ \rightarrow I_f^+)$ [W.u.]
$2_1^+ \rightarrow 0_1^+$	$20.5^{+0.6}_{-0.6}$	$9.7^{+0.6}_{-0.6}$
$4_1^+ \rightarrow 2_1^+$	$24.3^{+1.2}_{-1.2}$	$7.6^{+0.7}_{-0.7}$
$0_2^+ \rightarrow 2_1^+$	$22.2^{+1.1}_{-1.1}$	$57^{+6}_{-6}$
$2_2^+ \rightarrow 0_1^+$	$-6.4^{+0.3}_{-0.3}$	$1.0^{+0.1}_{-0.1}$
$2_2^+ \rightarrow 2_1^+$	$-23.7^{+2.3}_{-2.7}$	$12.9^{+2.5}_{-2.5}$
$4_2^+ \rightarrow 2_1^+$	$42^{+3}_{-4}$	$23^{+3}_{-4}$
$2_2^+ \rightarrow 0_2^+$	$26^{+5}_{-3}$	$15^{+6}_{-4}$
$4_2^+ \rightarrow 2_2^+$	$46^{+3}_{-6}$	$27^{+4}_{-6}$
	$\langle I_i    E2    I_i \rangle$ [e·fm <sup>2</sup> ]	$Q_{sp}$ [e·fm <sup>2</sup> ]
$2_1^+$	$-16^{+9}_{-3}$	$-12^{+7}_{-2}$
$2_2^+$	$-55^{+15}_{-15}$	$-42^{+12}_{-12}$



- measured quadrupole moment of  $2_2^+$  corresponds to  $\beta = 0.48(14)$

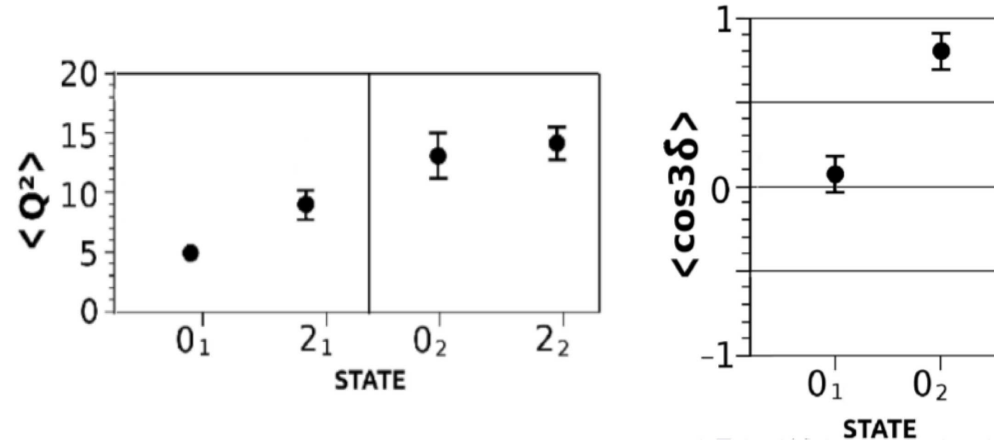




# Shape parameters of $0^+$ and $2^+$ states in $^{42}\text{Ca}$

K. Hadyńska-Klęk, PRL 117 (2016) 062501

state	$\langle Q^2 \rangle_{\text{exp}}$ [ $\text{e}^2\text{fm}^4$ ]
$0_1^+$	500 (20)
$2_1^+$	900 (100)
$0_2^+$	1300 (230)
$2_2^+$	1400 (250)
state	$\langle \cos(3\delta) \rangle_{\text{exp}}$
$0_1^+$	0.06 (10)
$0_2^+$	0.79 (13)



$$\bar{\beta} = \sqrt{\langle \beta^2 \rangle} = \sqrt{\frac{\langle Q^2 \rangle}{q_0^2}}$$

$$\bar{\gamma} = \arccos \langle \cos(3\delta) \rangle$$

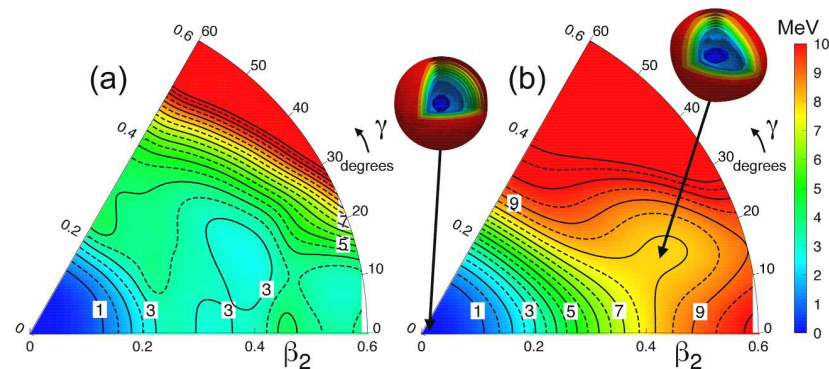
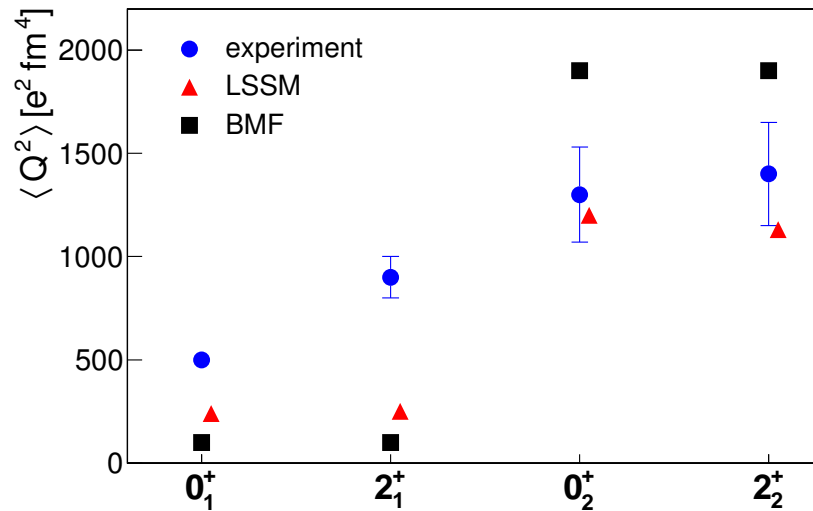
deformation in the side band:  $\bar{\beta}=0.43(4)$ ,  $\bar{\gamma}=13(6)^\circ$

ground-state band:  $\bar{\beta}=0.26(2)$ ,  $\bar{\gamma}=29(2)^\circ$  (?)

are these static deformations, or fluctuations?

$$\text{softness in } \beta: \sigma(Q^2) = \sqrt{\langle Q^4 \rangle - \langle Q^2 \rangle^2}$$

# Comparison with theoretical calculations



- coexistence of two very different structures reproduced by both theories, slightly triaxial SD minimum present in both potential maps
- deformation in the ground-state band increases with spin contrary to theoretical predictions  $\rightarrow$  mixing seems to be underestimated by calculations

# Configurations of ND and SD structures

$^{42}\text{Ca}$

$^{40}\text{Ca}$

F.Nowacki, H.Naidja, B.Bounthong  
Universit de Strasbourg, France

E.Caurier, J.Menendez, F.Nowacki and A.Poves,  
Phys. Rev. C 75, 054317 (2007)

state	2p-0h	4p-2h	6p-4h	8p-6h
$0_1^+$	40	40	17	3
$2_1^+$	45	36	16	3
$4_1^+$	55	35	9	1
$6_1^+$	55	35	9	1
$0_2^+$	10	18	49	23
$2_2^+$	12	13	50	24
$4_2^+$	1	15	62	22
$6_2^+$	1	24	61	14
$2_3^+$	0	14	59	26
$3_1^+$	0	4	66	30

Table V: np-nh structure of the superdeformed band of  $^{40}\text{Ca}$ .

J	0p-0h	2p-2h	4p-4h	6p-6h	8p-8h
0	-	-	9	4	87
2	-	-	11	4	85
4	-	-	8	5	87
6	-	-	3	5	91
8	-	-	2	6	91
10	-	-	1	12	87
12	-	-	2	29	69
14	-	-	11	27	63
16	-	-	0	40	60

TABLE IV. Percentage of np-nh components and energy of the first three  $0^+$  states (GS, ND, and SD) of  $^{40}\text{Ca}$ .

	0p-0h	2p-2h	4p-4h	6p-6h	8p-8h	$E(\text{th})$	$E(\text{exp})$
$0_{\text{GS}}^+$	65	29	5	-	-	0	0
$0_{\text{ND}}^+$	1	1	64	25	9	3.49	3.35
$0_{\text{SD}}^+$	-	-	9	4	87	4.80	5.21

## Shape parameters of $0^+$ and $2^+$ states in $^{42}\text{Ca}$

state	$\langle Q^2 \rangle_{\text{exp}}$	$\langle Q^2 \rangle_{\text{SM}}$	$\sigma(Q^2)_{\text{SM}}$	$\langle Q^2 \rangle_{\text{BMF}}$	$\sigma(Q^2)_{\text{BMF}}$
$0_1^+$	500 (20)	240	470	100	250
$2_1^+$	900 (100)	250	490	100	310
$0_2^+$	1300 (230)	1200	500	1900	520
$2_2^+$	1400 (250)	1130	500	1900	300

state	$\langle \cos(3\delta) \rangle_{\text{exp}}$	$\langle \cos(3\delta) \rangle_{\text{SM}}$	$\langle \cos(3\delta) \rangle_{\text{BMF}}$
$0_1^+$	0.06 (10)	0.34	0.34
$0_2^+$	0.79 (13)	0.67	0.49

$\sigma(Q^2)$  comparable with  $\langle Q^2 \rangle$  for the ground-state band

→ fluctuations about a spherical shape

excited band:  $\sigma(Q^2)$  few times lower than  $\langle Q^2 \rangle$

→ static deformation

## Two-states mixing model applied to $^{42}\text{Ca}$

- we assume that physical states are linear combinations of pure spherical and deformed configurations:

$$|I_1^+\rangle = +\cos\theta_1 \times |I_p^+\rangle + \sin\theta_1 \times |I_s^+\rangle$$

$$|I_2^+\rangle = -\sin\theta_1 \times |I_p^+\rangle + \cos\theta_1 \times |I_s^+\rangle$$

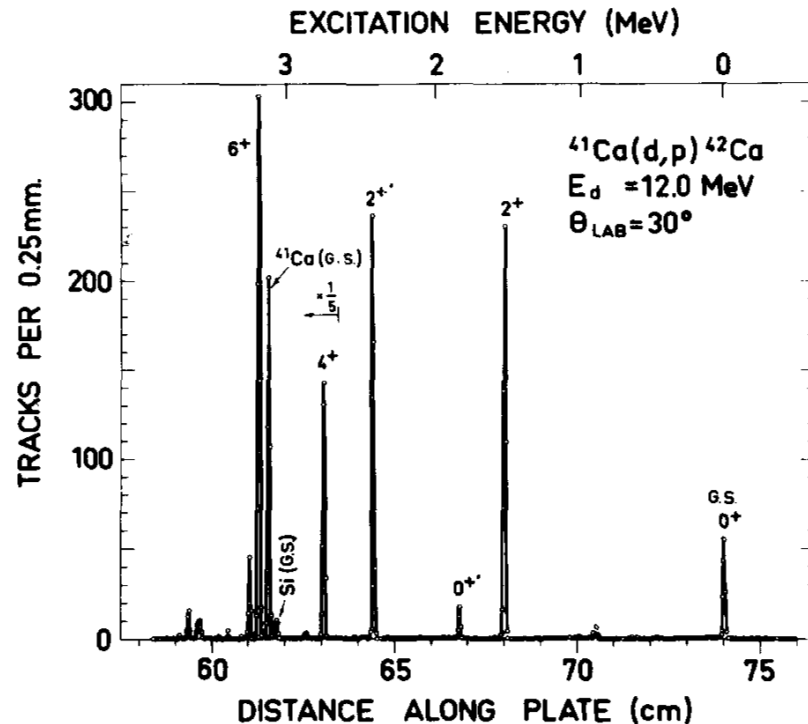
	Experiment	SM	BMF
$\cos^2(\theta_0)$	0.88(4)	0.85	0.96
$\cos^2(\theta_2)$	0.39(8)	0.83	0.97

- $\cos^2(\theta_0)$  can be also extracted from E0 transition strength:

$$\rho^2(E0) = \left(\frac{3Z}{4\pi}\right)^2 \cos^2(\theta_0)\sin^2(\theta_0) \cdot \left[ (\beta_1^2 - \beta_2^2) + \frac{5\sqrt{5}}{21\sqrt{\pi}} (\beta_1^3 \cos\gamma_1 - \beta_2^3 \cos\gamma_2) \right]^2$$

- obtained value of 0.84(4) is in perfect agreement with that extracted from E2 matrix elements
- $\cos^2(\theta_2) < 0.5$ : two-state mixing model cannot be applied to  $2^+$  states in  $^{42}\text{Ca}$

# Population of the deformed structure in one-neutron transfer

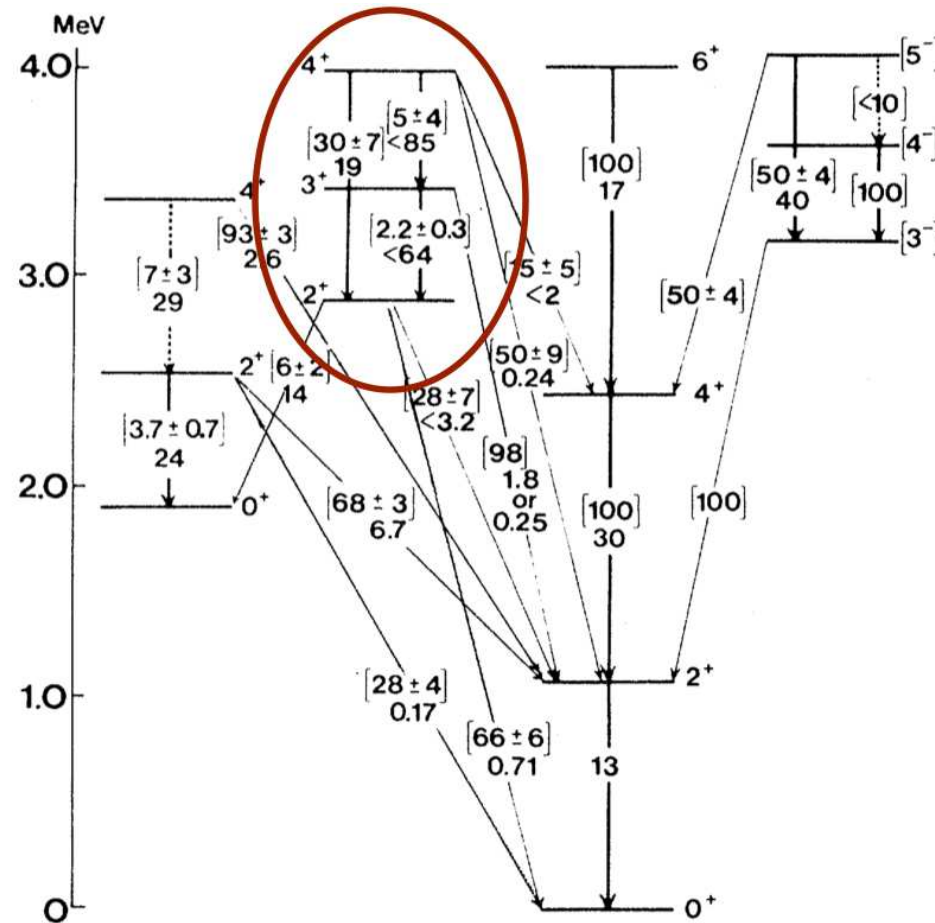


C. Ellegaard et al, Phys. Lett. 40B (1972) 641

- equal population of  $2_1^+$  and  $2_2^+$  in (d,p) – configuration mixing
- $4_1^+$  populated in one-nucleon transfer – it is not a two-phonon state, so the increase of  $\langle Q^2 \rangle$  in the g.s. cannot be interpreted as resulting from the vibrational character of  $2_1^+$

# Gamma deformation

J. Simpson et al, PRL 31 (1973) 947



- K=2 band identified in (p,t) and ( $\alpha$ ,  $\gamma$ ) studies of  $^{44}\text{Ti}$
- is there a similar band built on  $2_3^+$  in  $^{42}\text{Ca}$ ?



# Outlook

		N=18		N=20		N=22			
				42 V	43 V	44 V	45 V	46 V	47 V
39 Ti	40 Ti	41 Ti	42 Ti	43 Ti	44 Ti	45 Ti	46 Ti		
38 Sc	39 Sc	40 Sc	41 Sc	42 Sc	43 Sc	44 Sc	45 Sc		
37 Ca	38 Ca	39 Ca	40 Ca	41 Ca	42 Ca	43 Ca	44 Ca		
36 K	37 K	38 K	39 K	40 K	41 K	42 K	43 K		
35 Ar	36 Ar	37 Ar	38 Ar	39 Ar	40 Ar	41 Ar	42 Ar		
34 Cl	35 Cl	36 Cl	37 Cl	38 Cl	39 Cl	40 Cl	41 Cl		

Z=22 • are there similar structures in  $^{40,42}\text{Ti}$  and  $^{38}\text{Ca}$ ?

Z=20 • what about odd-mass nuclei?

•  $^{38}\text{Ar}$  seems not well understood

Z=18 • role of triaxiality?