Introduction to experimental shape coexistence studies

John L. Wood
School of Physics
Georgia Institute of Technology

Coexistence: where we have been

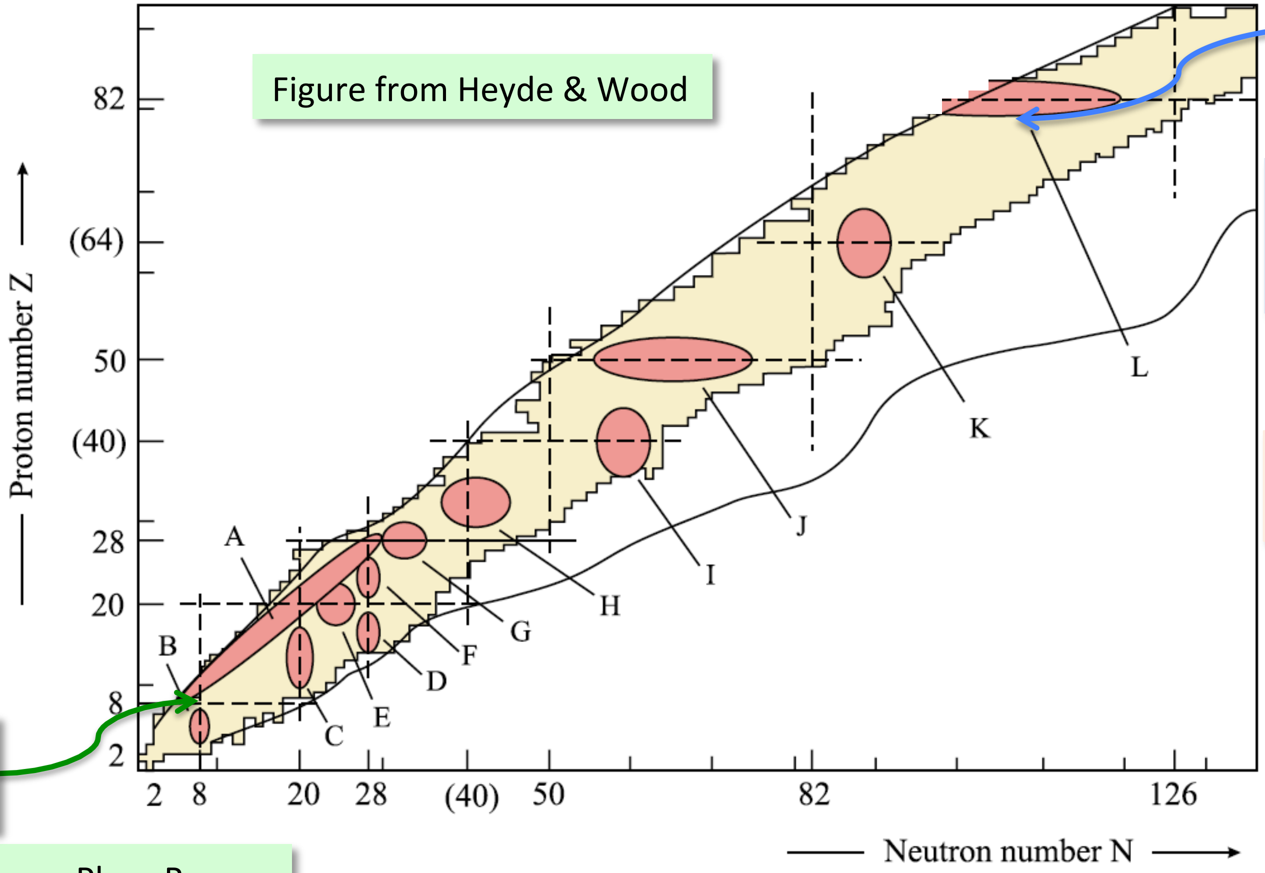


Figure from Heyde & Wood

^{185}Hg

J. Bonn et al., Phys. Lett. **38B** 308 (1972)

THE SURPRISE: 1972

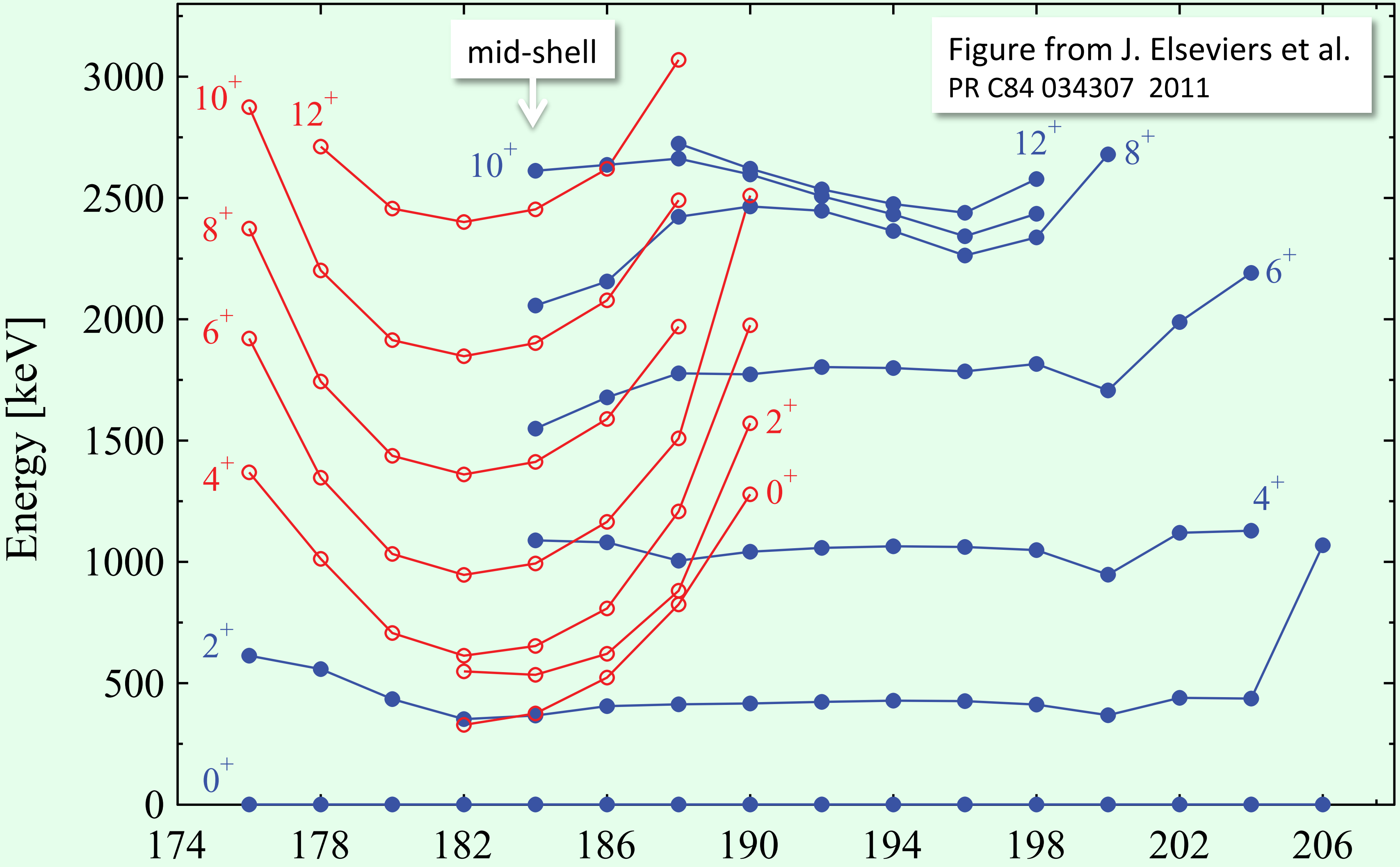
^{16}O

H. Morinaga, Phys. Rev. **101** 254 (1956)

THE BEGINNING: 1956

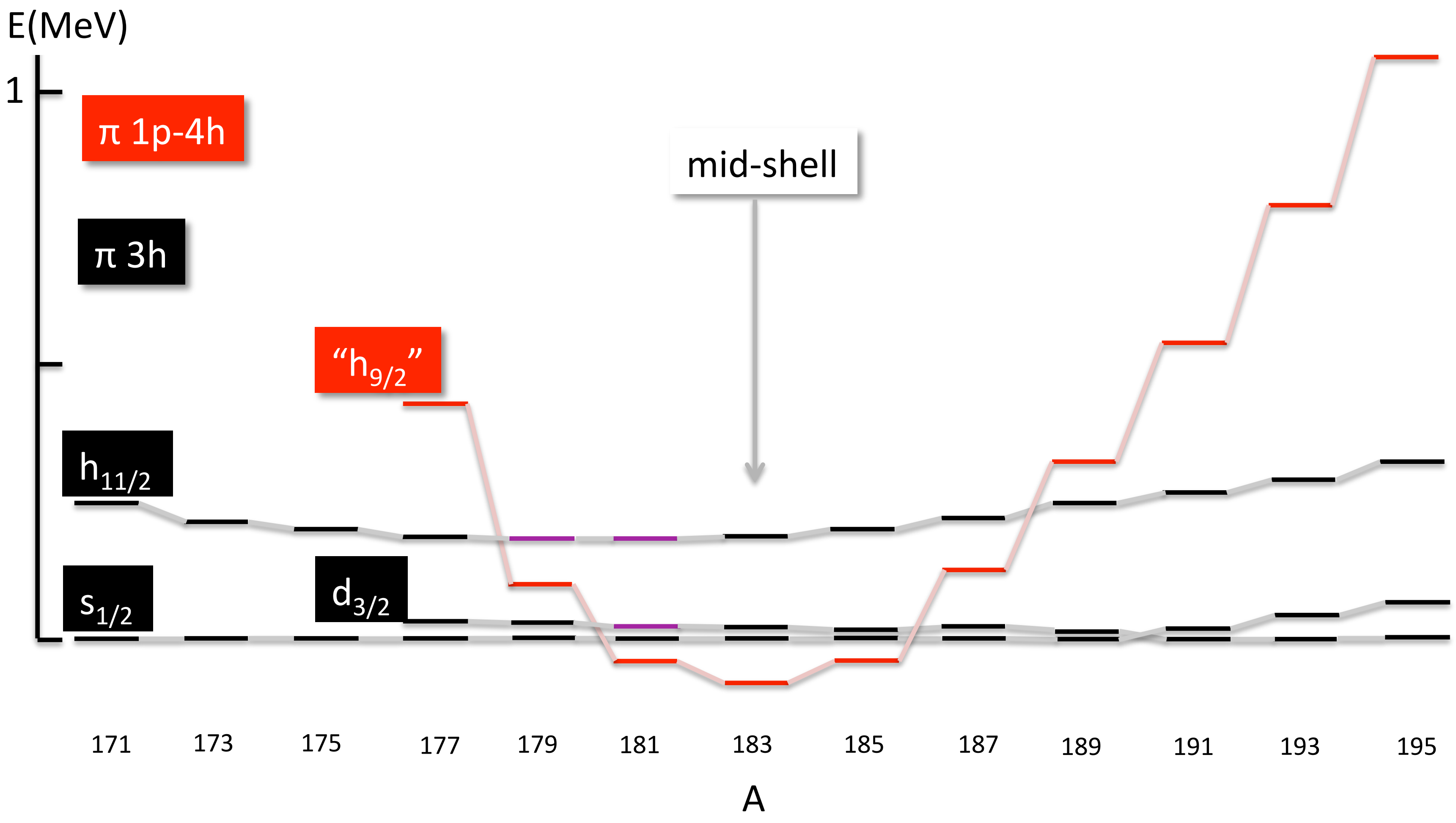
Coexistence in the even-Hg isotopes

NOTE: deformed states exhibit a characteristic “*parabolic*” energy pattern



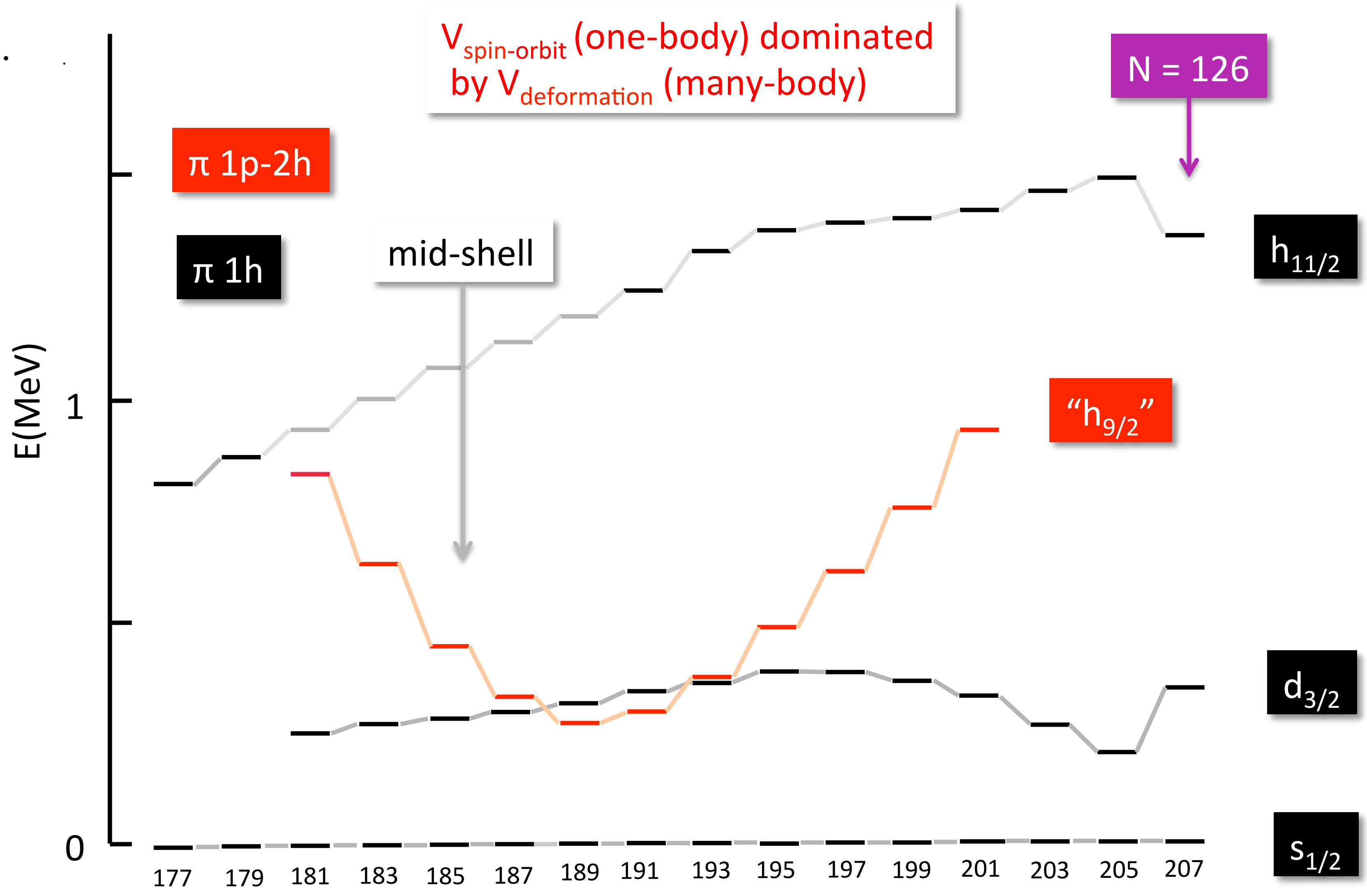
Odd-Au isotopes: $h_{9/2}$ "intruder" state

intruder states exhibit a characteristic *"parabolic"* energy pattern



Odd-Tl isotopes:

NOTE the “$h_{9/2}$” state lies *below* the $h_{11/2}$ state



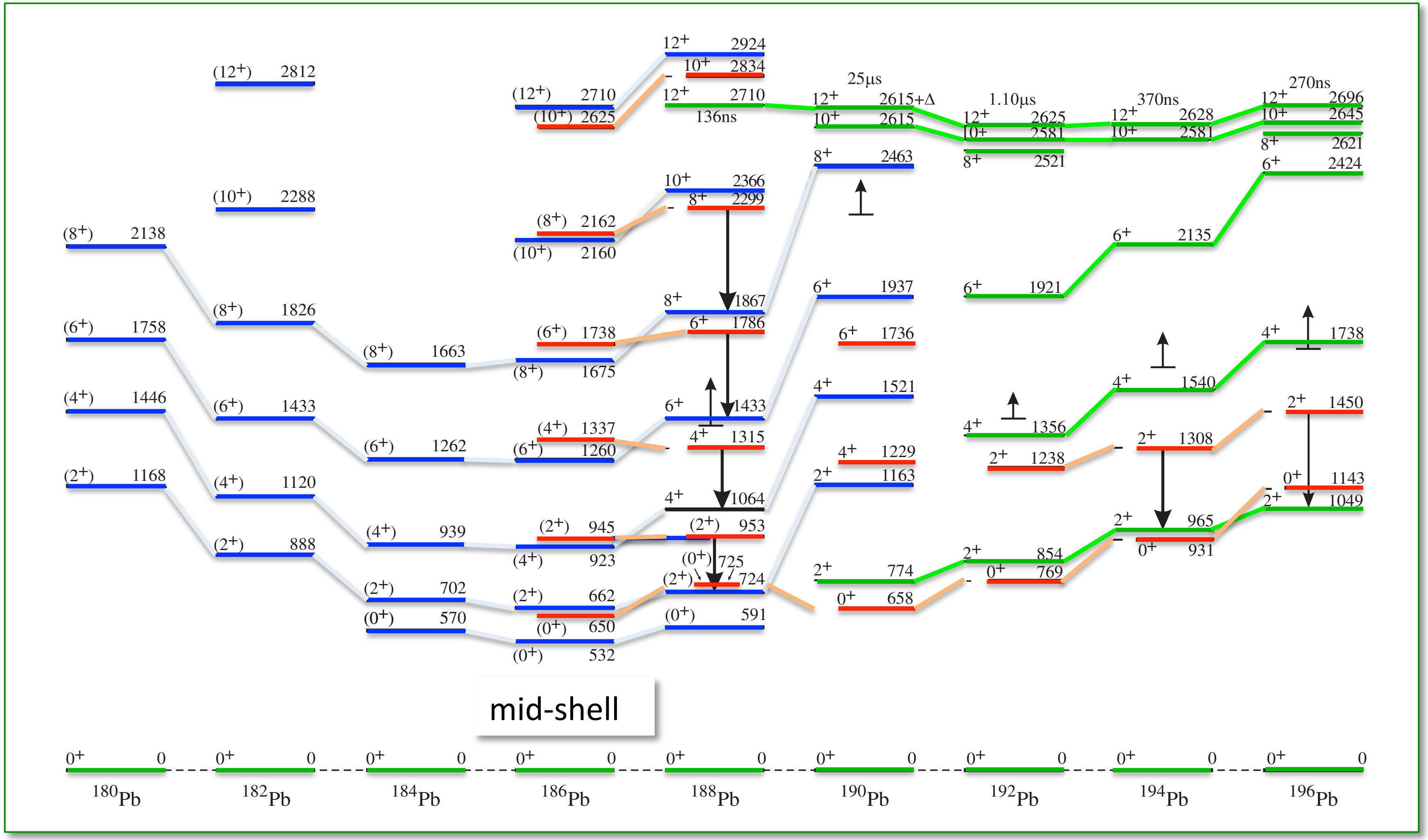
$V_{spin-orbit}$ (one-body) dominated by $V_{deformation}$ (many-body)

Coexistence in even-Pb isotopes:

multiple parabolas and spherical (seniority) structure

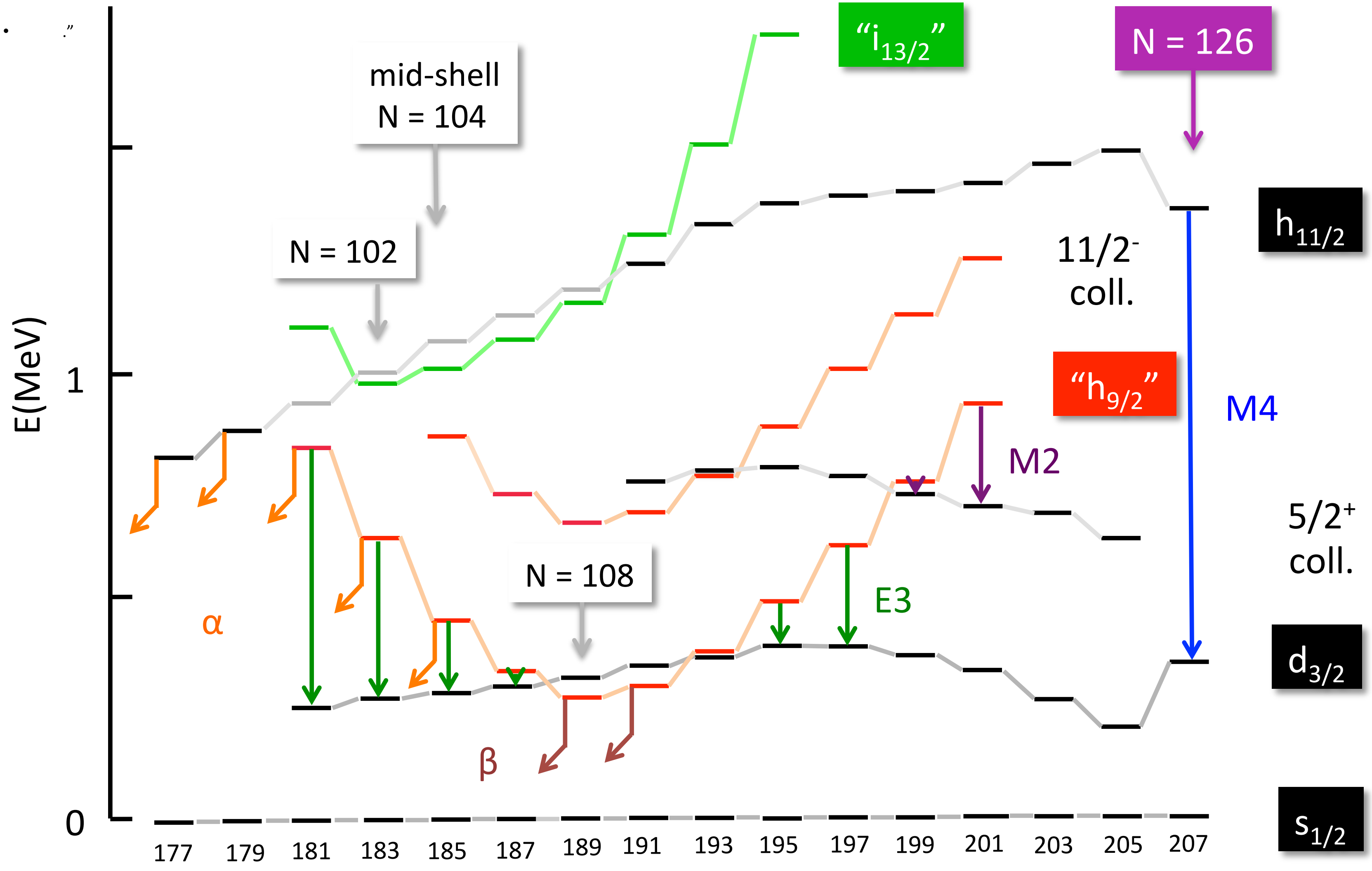
Figure: Heyde & Wood

Heavy arrows indicate E0+M1+E2 transitions



Odd-Tl isotopes: $h_{9/2}$ and $i_{13/2}$ intruder states

“displaced” parabolas



Coexistence in odd-Tl isotopes:
$h_{9/2}$ multiple parabolas

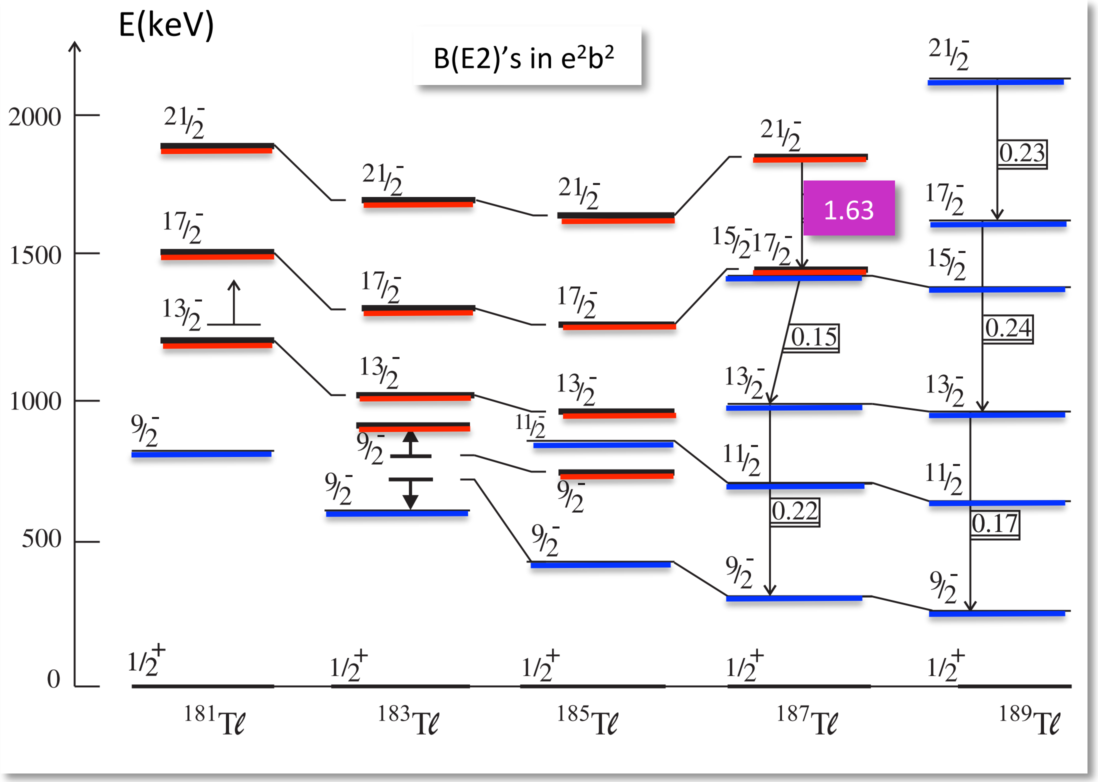


Figure from Heyde & Wood

$\pi h_{9/2}$ intruder bands

Note:

★ coexistence of strongly coupled, 9/2, 11/2, 13/2,… and decoupled bands, 9/2, 13/2, 17/2,… in ^{185,187}Tl

★ large B(E2) value

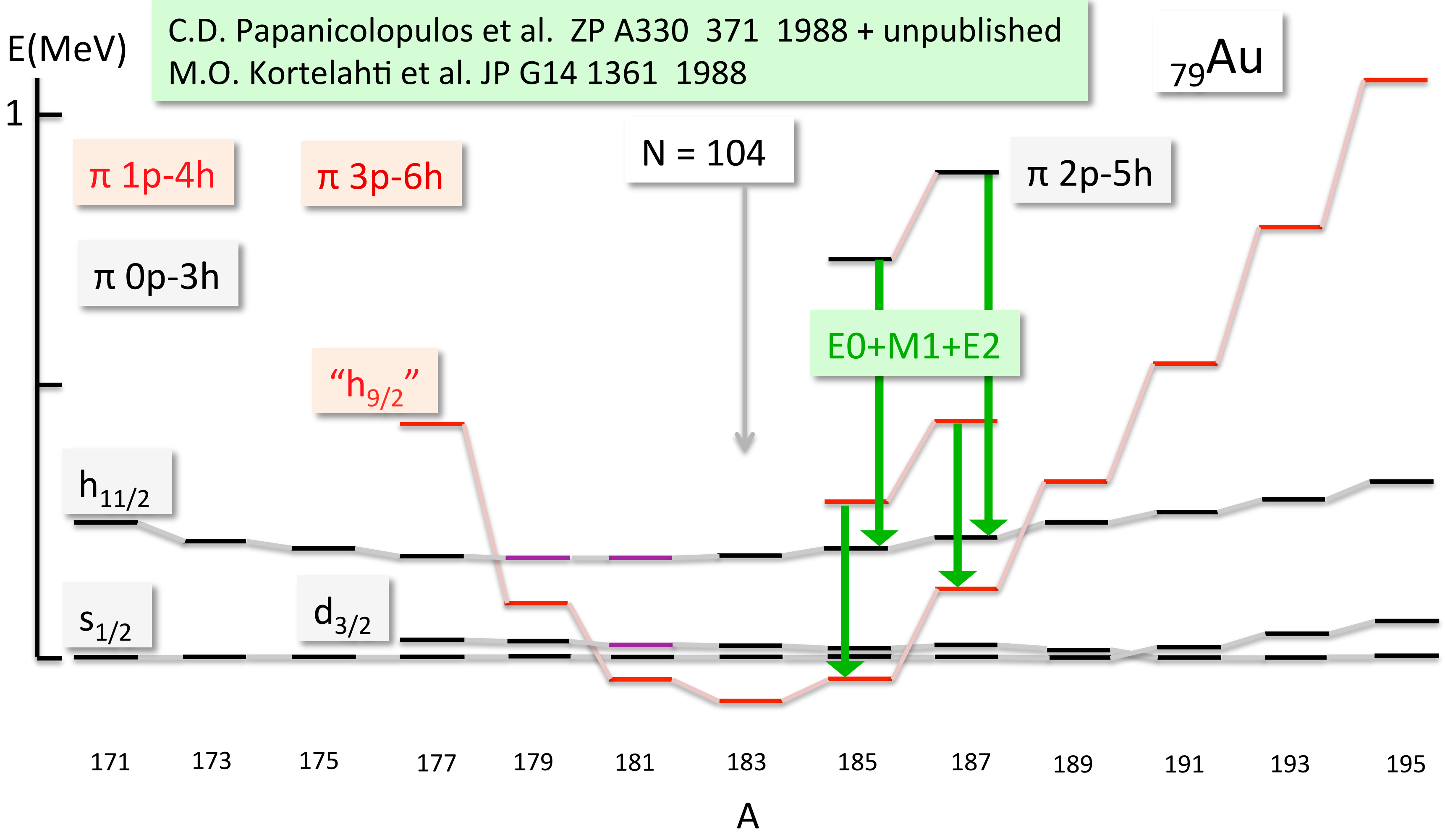
★ $h_{9/2}$ π(1p) x Hg(2h; weak oblate)

★ $h_{9/2}$ π(1p) x Hg(2p-4h; strong prolate)

Coexistence in odd-Au isotopes:

E0 transitions and multiple parabolas

C.D. Papanicolopulos et al. ZP A330 371 1988 + unpublished
M.O. Kortelahti et al. JP G14 1361 1988

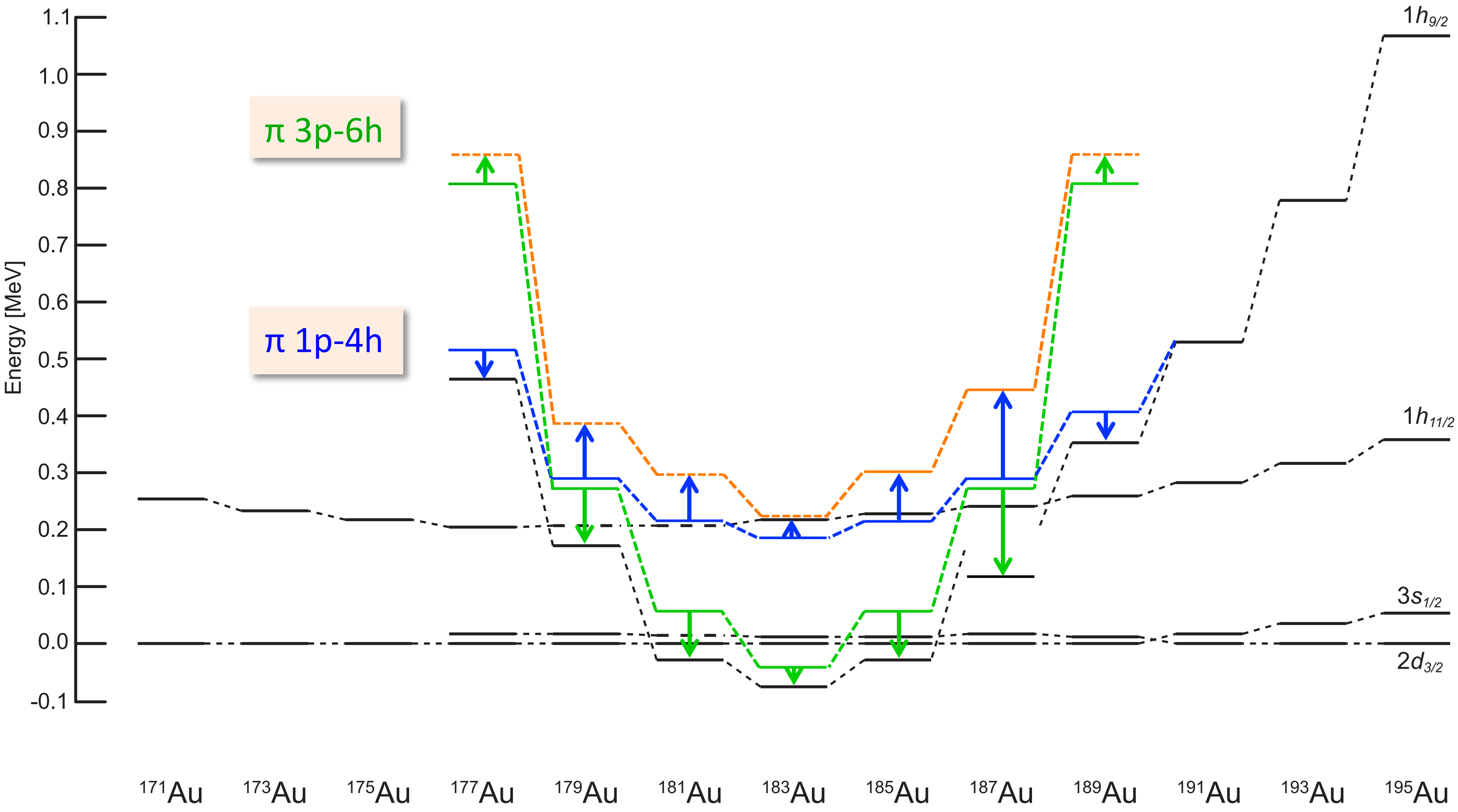


Dave Joss (Liverpool)

Martin Venhart

four coexisting shapes in ^{185,187}Au

Odd-Au "double-intruder" structure associated with the $h_{9/2}$ configuration: π 1p-4h and π 3p-6h



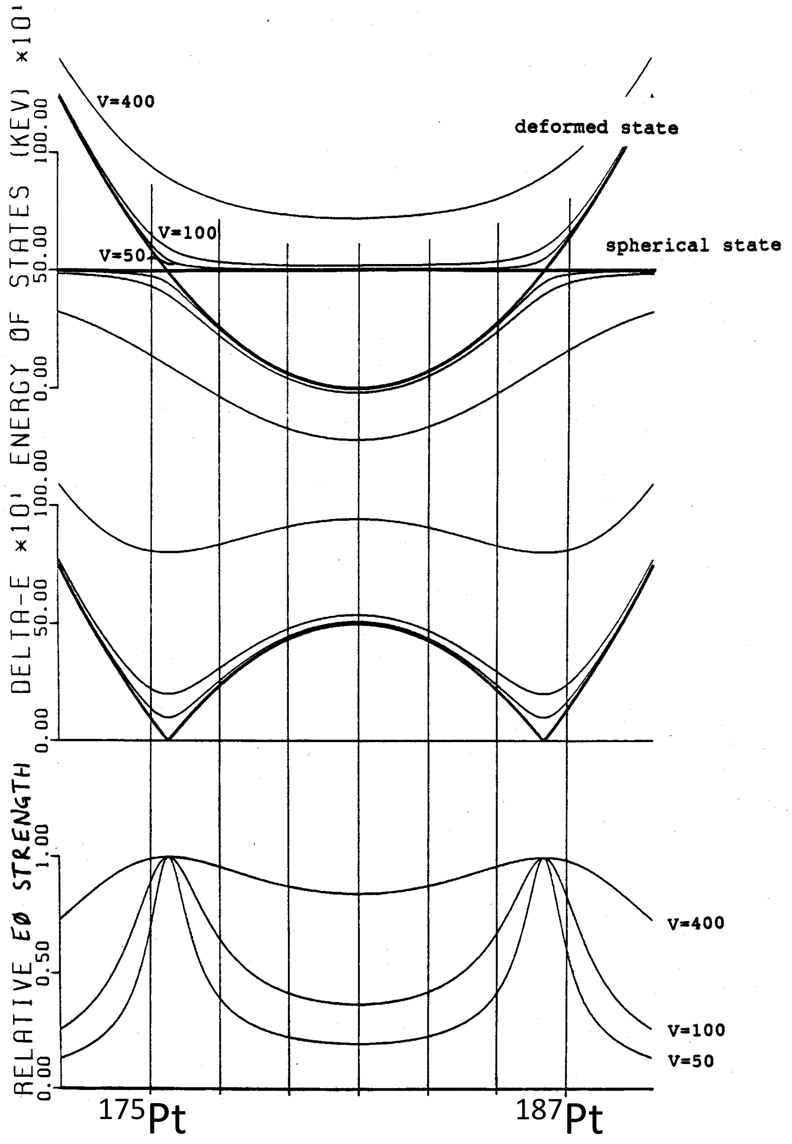
Coexistence in the even-Pt isotopes: mixing and E0 transition strength

V = 50, 100, 400 keV

E (sph.gs) = 0

E (gs) = 0

$\rho^2(E0)$

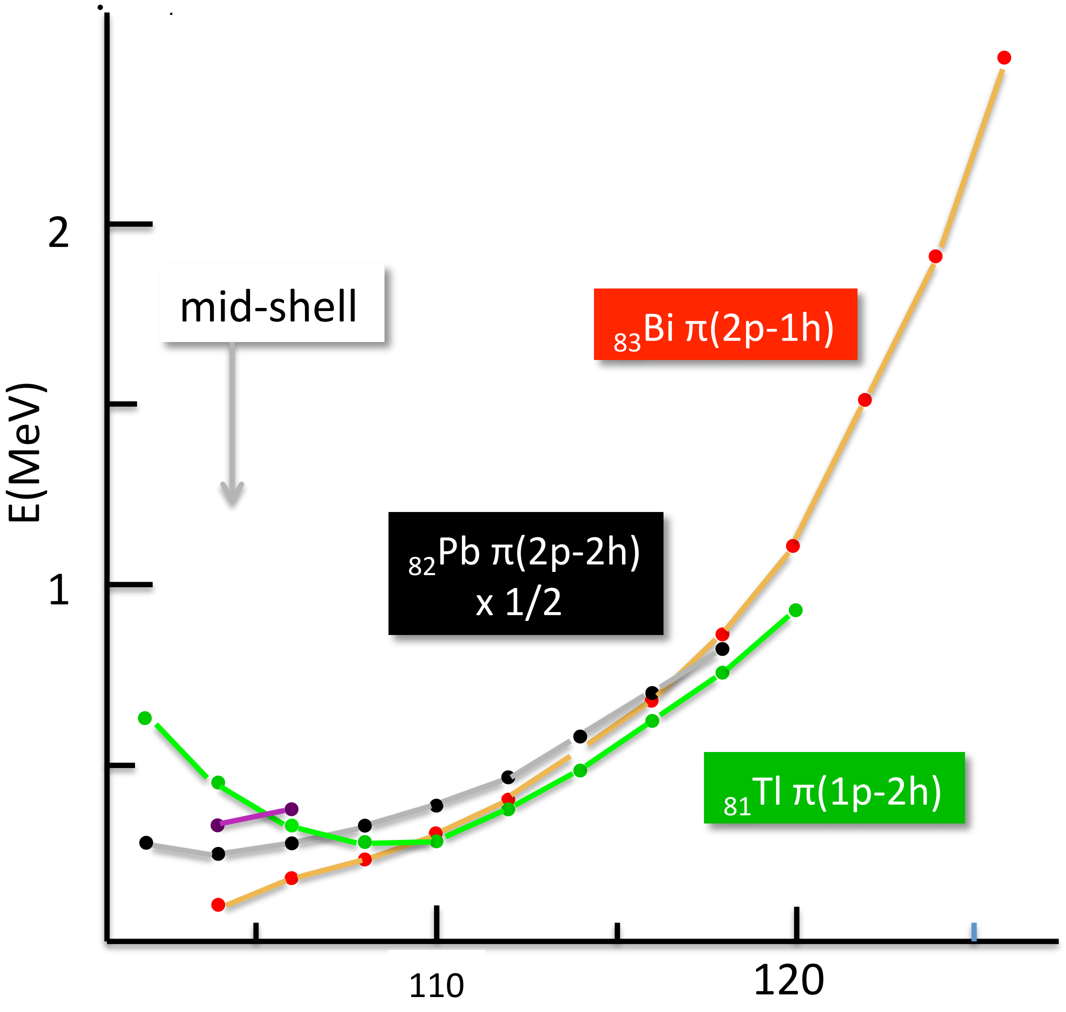


Two different views of the same energy spectra

From: J. von Schwarzenberg, PhD thesis, Ga Tech 1991

Relationship between energies of intruder states in odd-mass nuclei and coexistence in even-mass nuclei

Unpaired nucleons are not the “drivers” of deformation



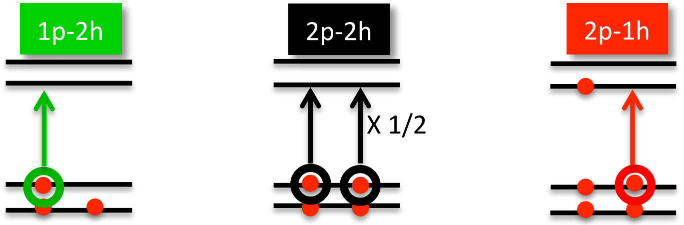
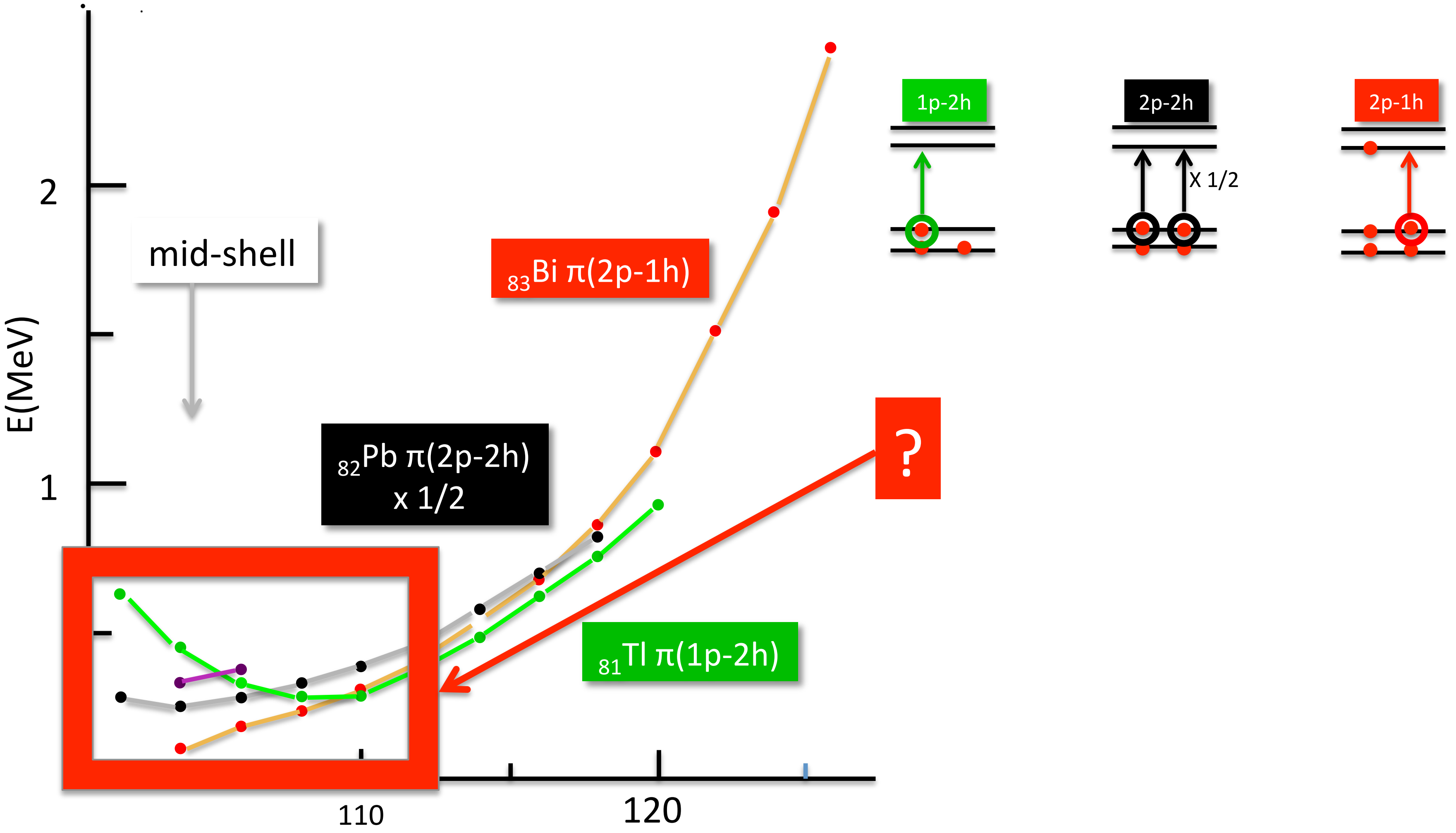


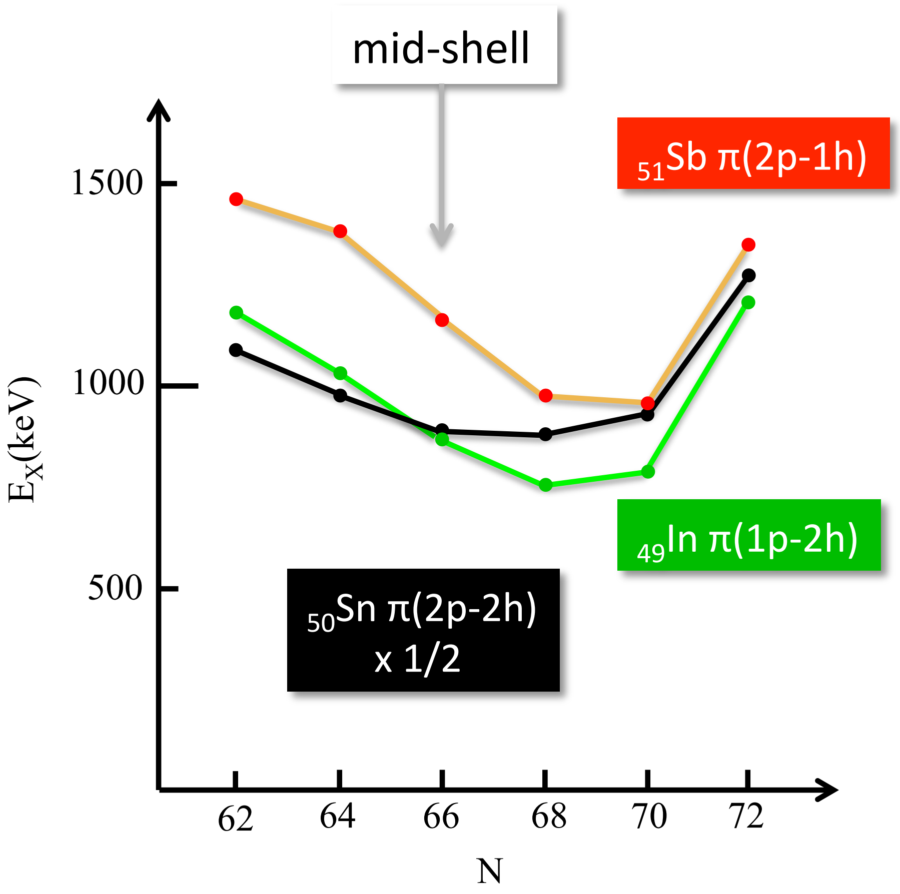
Figure adapted from Heyde & Wood

Relationship between energies of intruder states in odd-mass nuclei and coexistence in even-mass nuclei

multiple parabolas?



Relationship between energies of intruder states in odd-mass nuclei and coexistence in even-mass nuclei



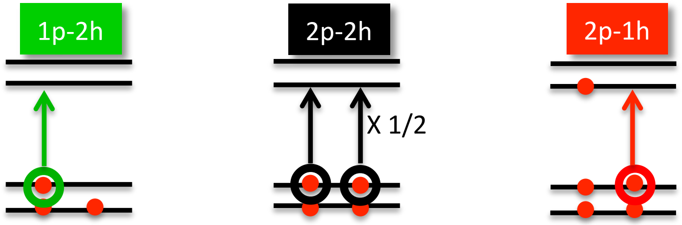


Figure adapted from Heyde & Wood

Odd-mass intruder states and their association with low-energy excited 0^+ states

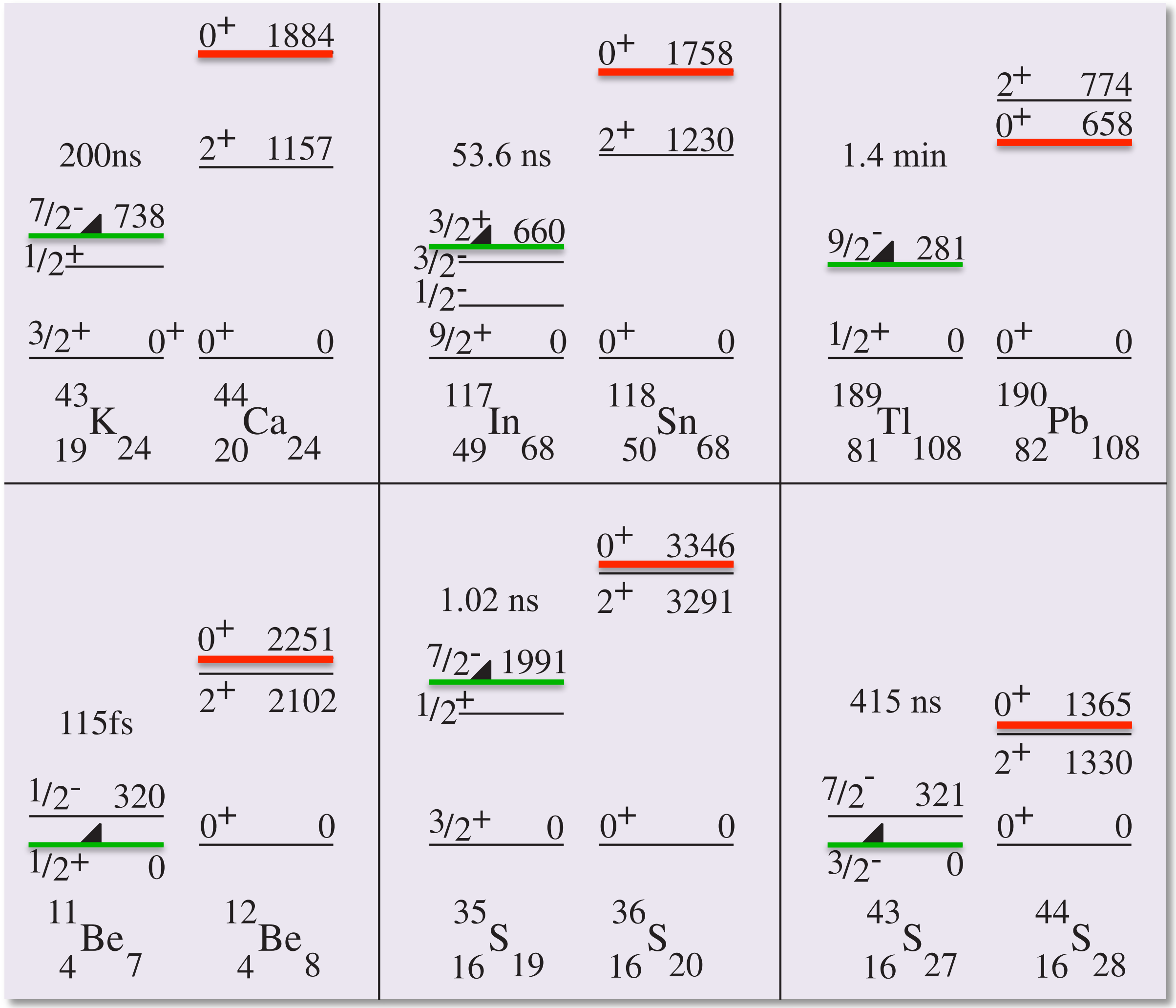
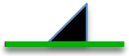


Figure: Heyde & Wood



$T_{1/2}$ isomeric state

intruder state

Schematic view of energy contributions to intruder state energies

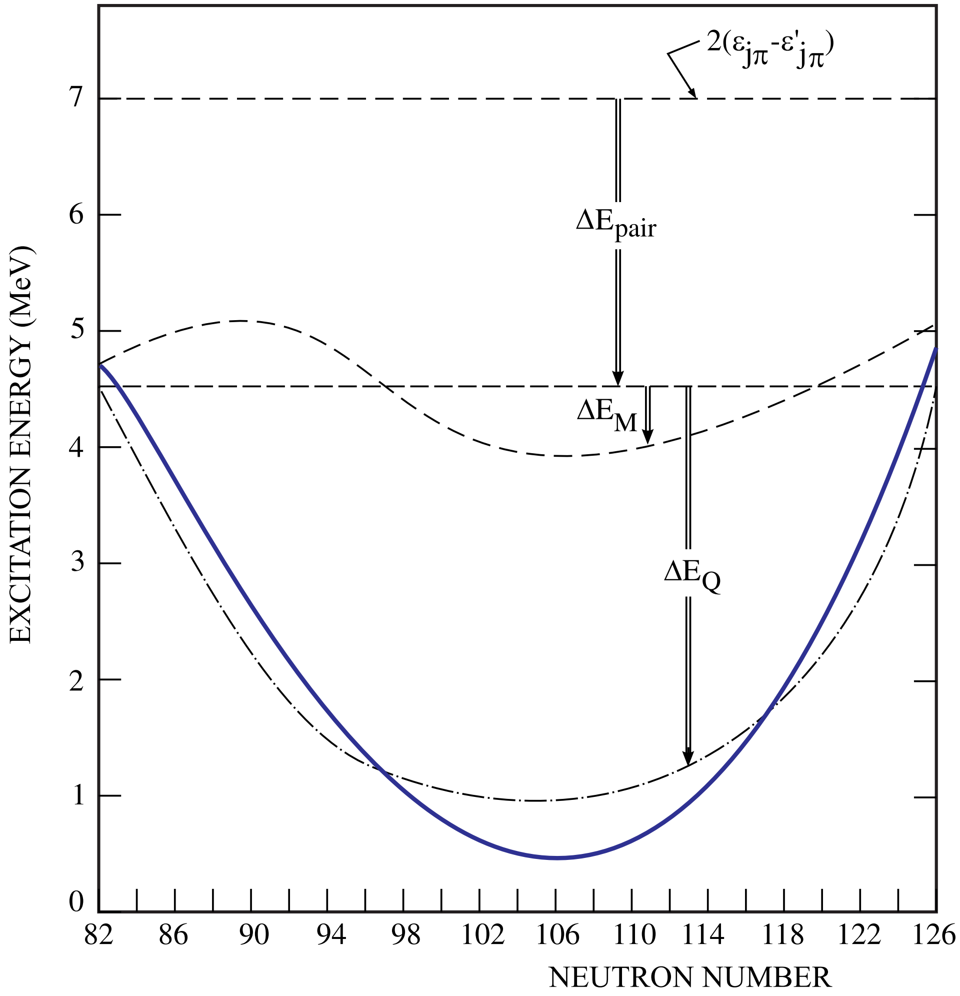
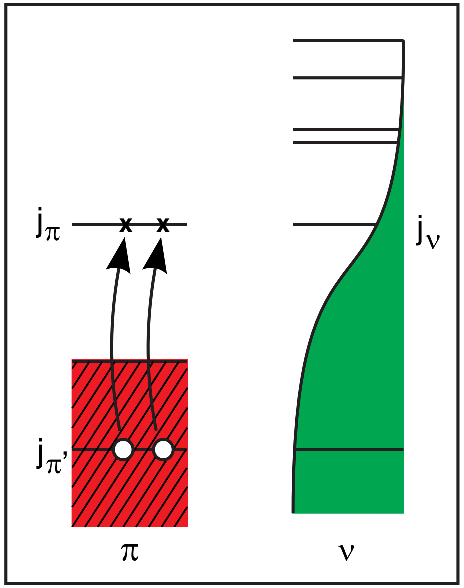
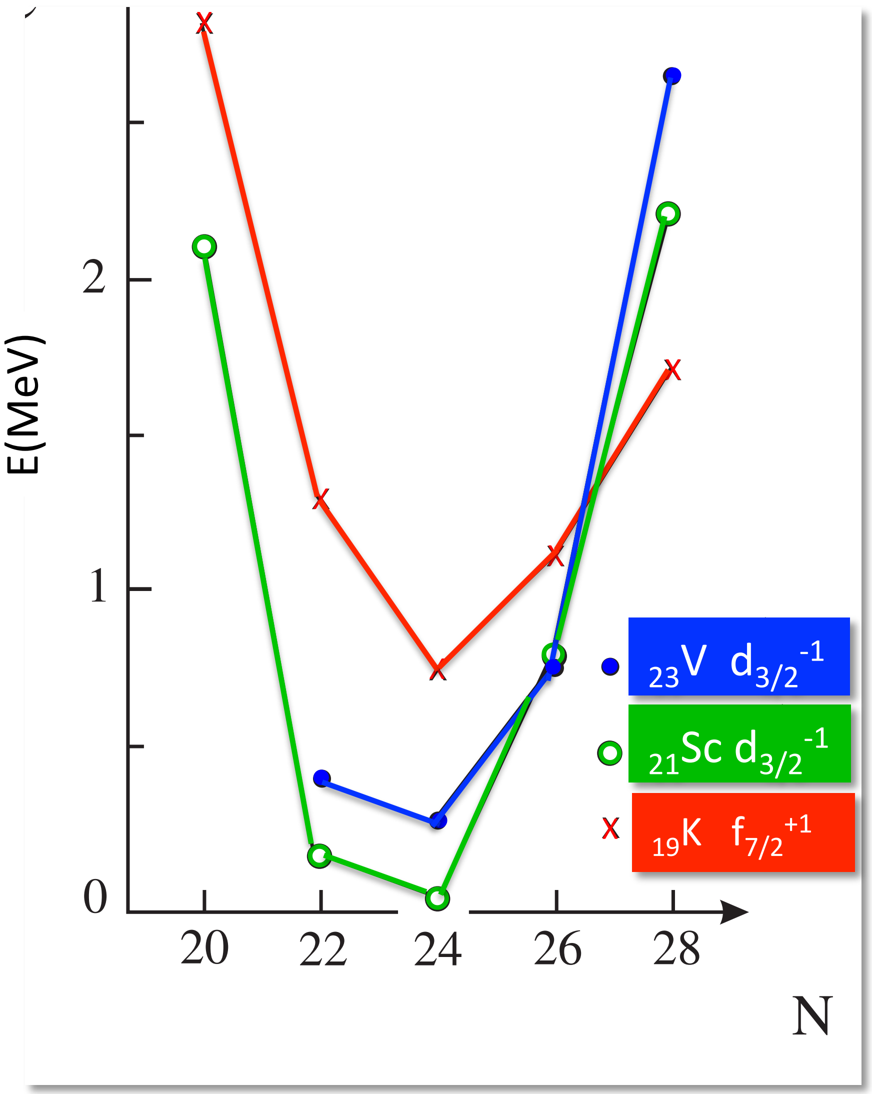


Figure: Heyde & Wood



Coexistence in the odd K, Sc, and V isotopes:

deformed intruder states exhibit a characteristic parabolic energy trend



NOTE:

★ Parabolas sharper in light nuclei than in heavy nuclei because shells more confining.

★ Ground state of $^{45}Sc_{24}$: almost an “island of inversion”.

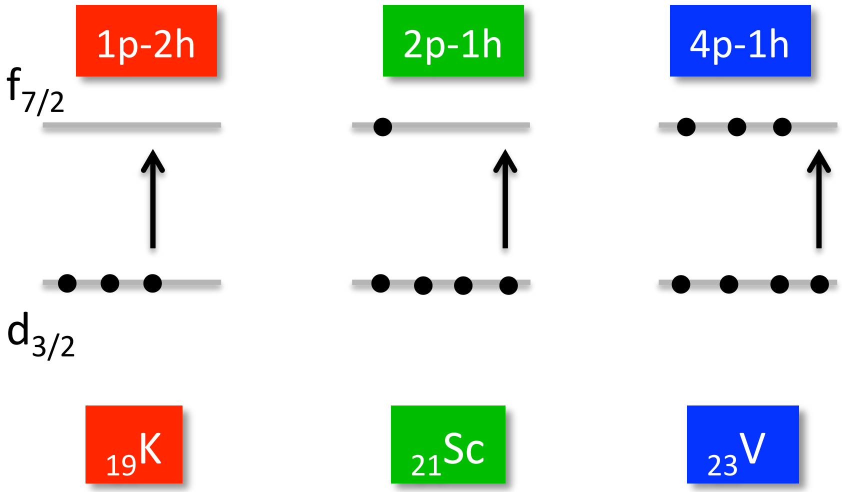


Figure: Heyde & Wood

Intruder state energies @ N = 20 have contributions from multiple sources which are not limited to the pure* shell model

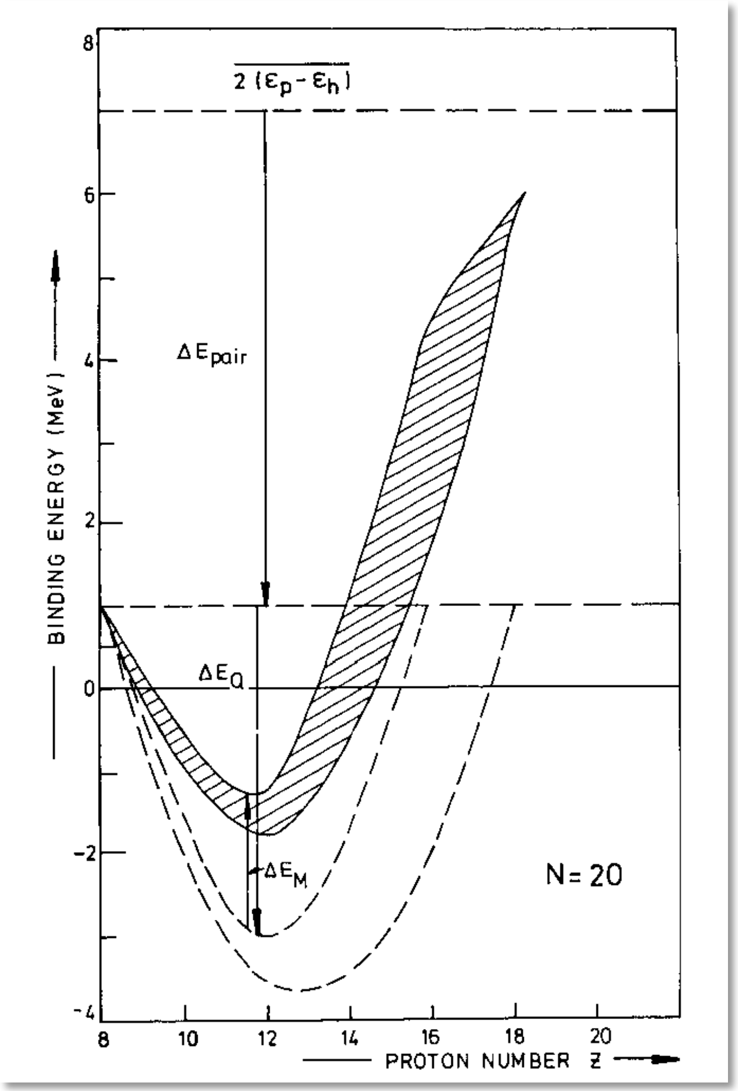


Figure from Heyde and Wood JP G17 135 1991

*the pure shell model is an independent-particle model:

intruder state energies have very large contributions from many-body *correlations:*

pairing--ΔE_{pair}

quadrupole--Δ_Q

monopole--Δ_M

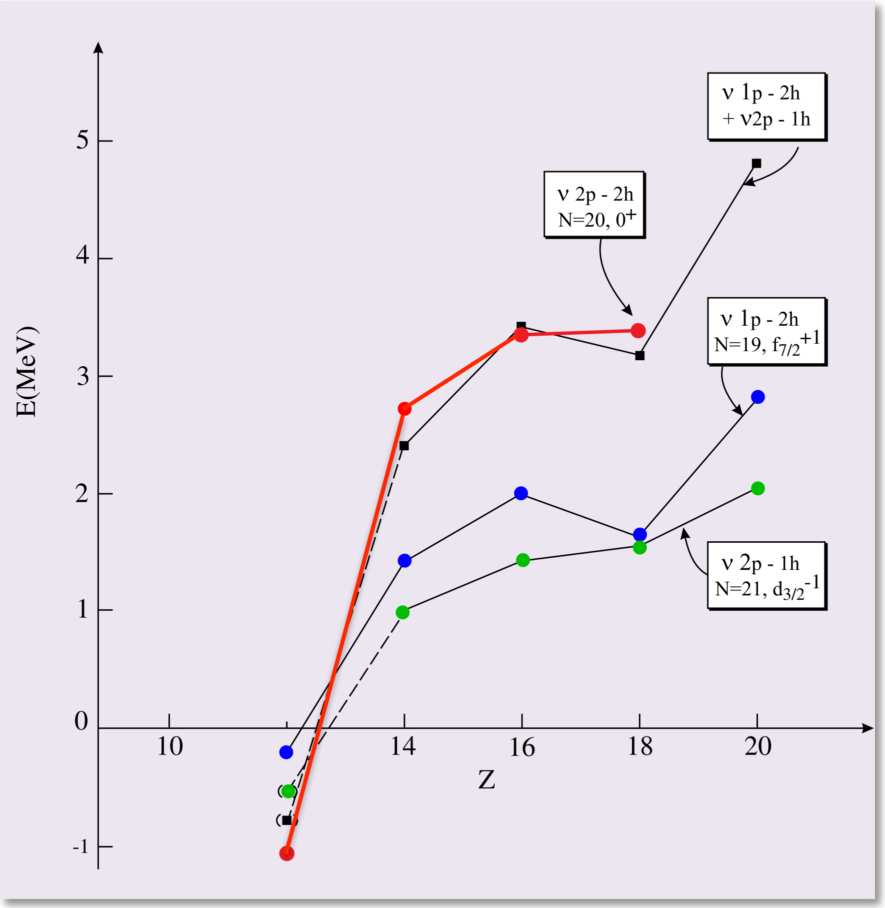
Mapping the borders of the Island of Inversion: not a profound structural issue—there is coexistence inside and outside of the borders.

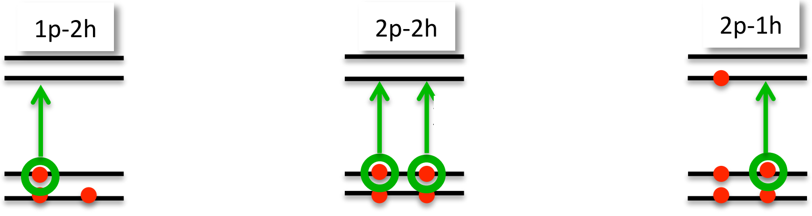
See also Poves and Retamosa PL B184 311 1987

0^+ ν(2p-2h) intruder state energies @ N=20:

estimates from ν (1p-2h) + ν (2p-1h) energies

Figure adapted from Heyde & Wood





N = 19

$f_{7/2}^{+1}$ x
Cooper
hole pair

N = 20

Cooper
particle pair x
Cooper
hole pair

N = 21

$d_{3/2}^{-1}$ x
Cooper
particle pair

Expt. ● ν 1p-2h

Expt. ● ν 2p-2h

Est. ■ ν 2p-2h

Expt. ● ν 2p-1h

See: G Neyens, PR **C 84** 064310 2011
JP **G 43** 024007 2016

Shape coexistence @ N=20

0_2^+ state identification:

^{32}Mg K. Wimmer et al. PRL 105 252501 2010

^{34}Si F. Rotaru et al. PRL 109 092503 2012

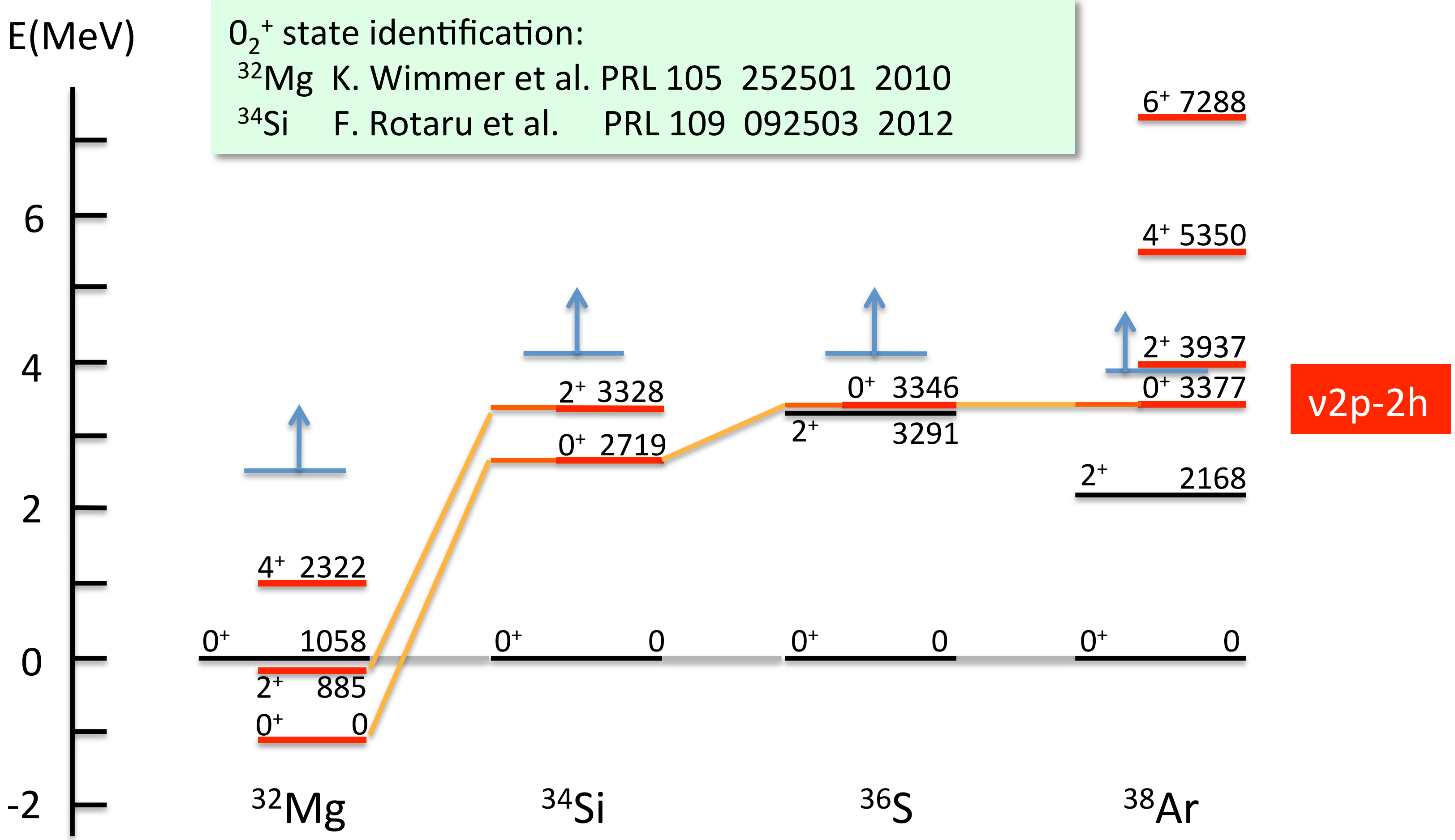
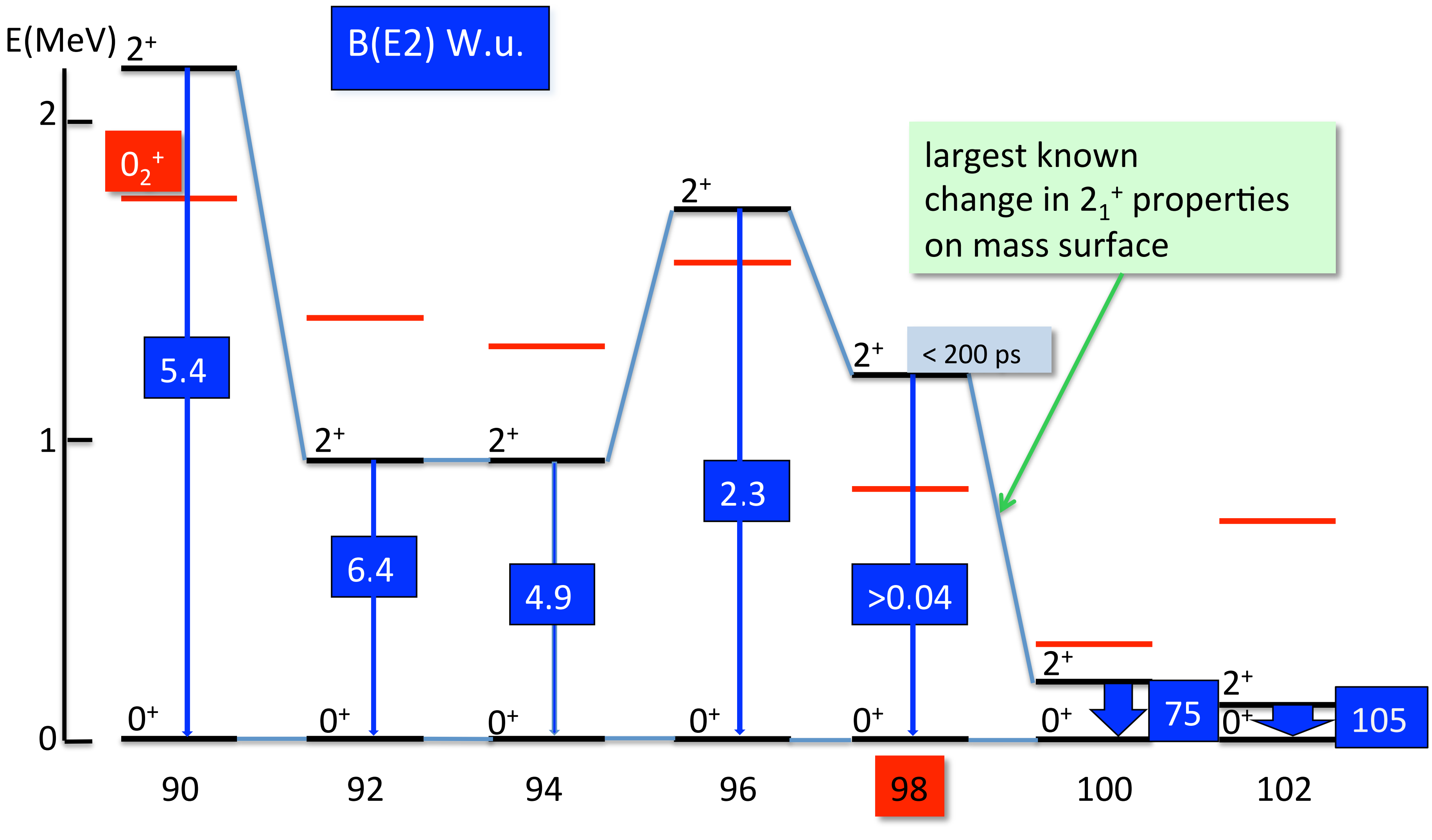


Figure adapted from Heyde & Wood

Sudden changes in Zr ground-state properties:

Intruder (shape coexistence) or critical point "phase" change?

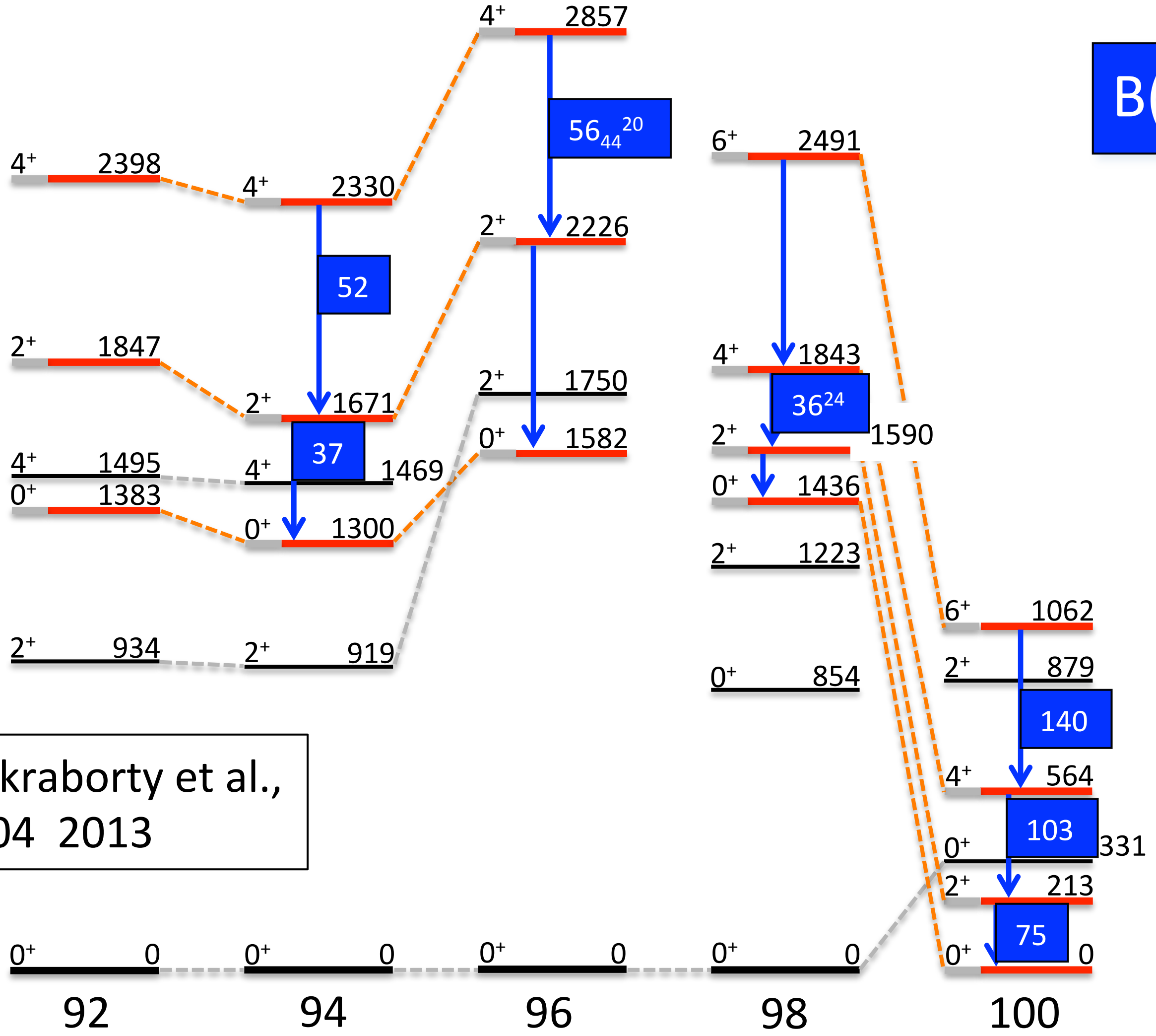


largest known change in 2_1^+ properties on mass surface

0_2^+ states and deformation in Zr isotopes, 50 ≤ N ≤ 62: electric quadrupole transition strengths

Zr

B(E2) W.u.



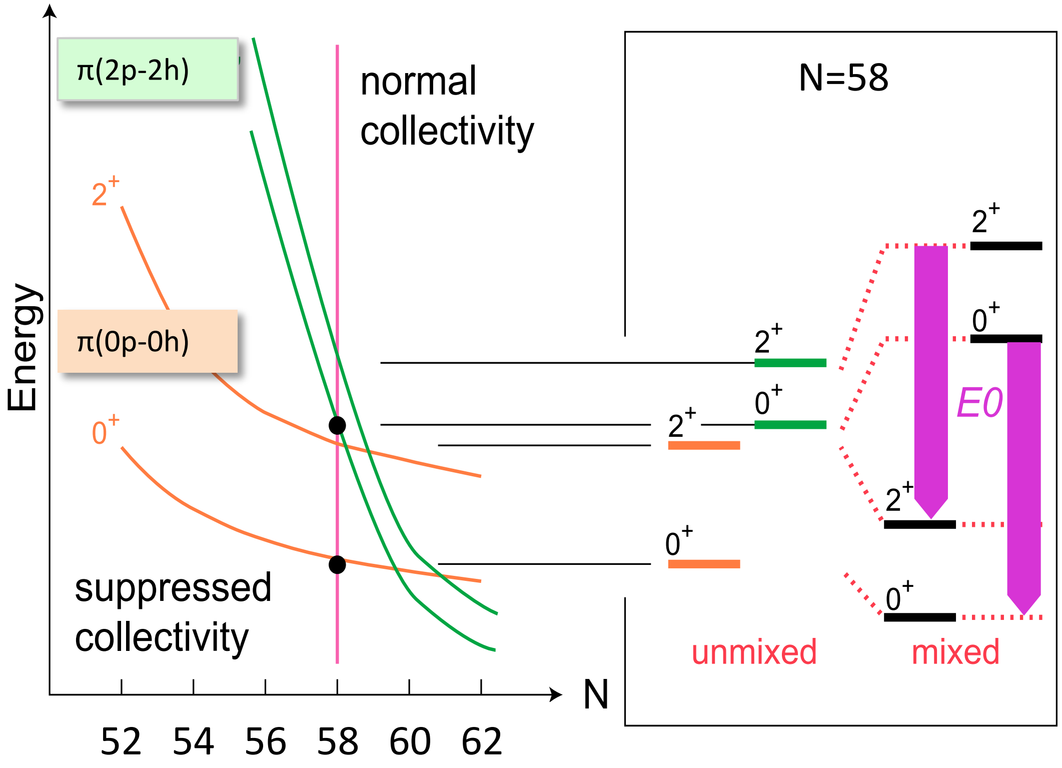
^{94}Zr: see A Chakraborty et al., PRL 110 022504 2013

A deformed structure can intrude to become a ground state:

appears to produce a “collective phase change”

Nuclei are manifestations of coexisting structures
that may invert by addition of a few nucleons, and may mix.

Proton pair excitations with respect to the Z = 40 subshell

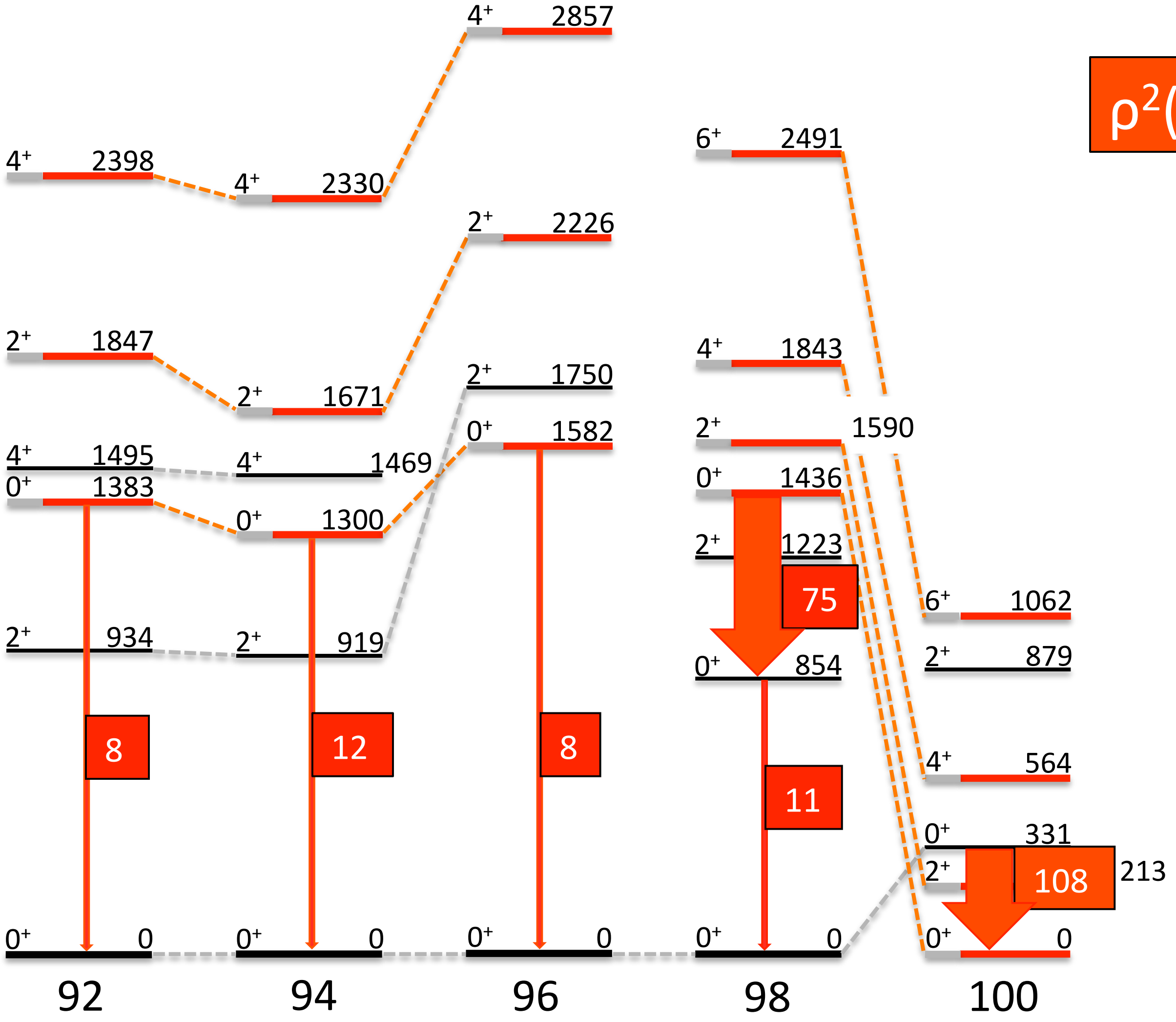


0_2^+ states and deformation in Zr isotopes, 50 ≤ N ≤ 62:

electric monopole transition strengths

Zr

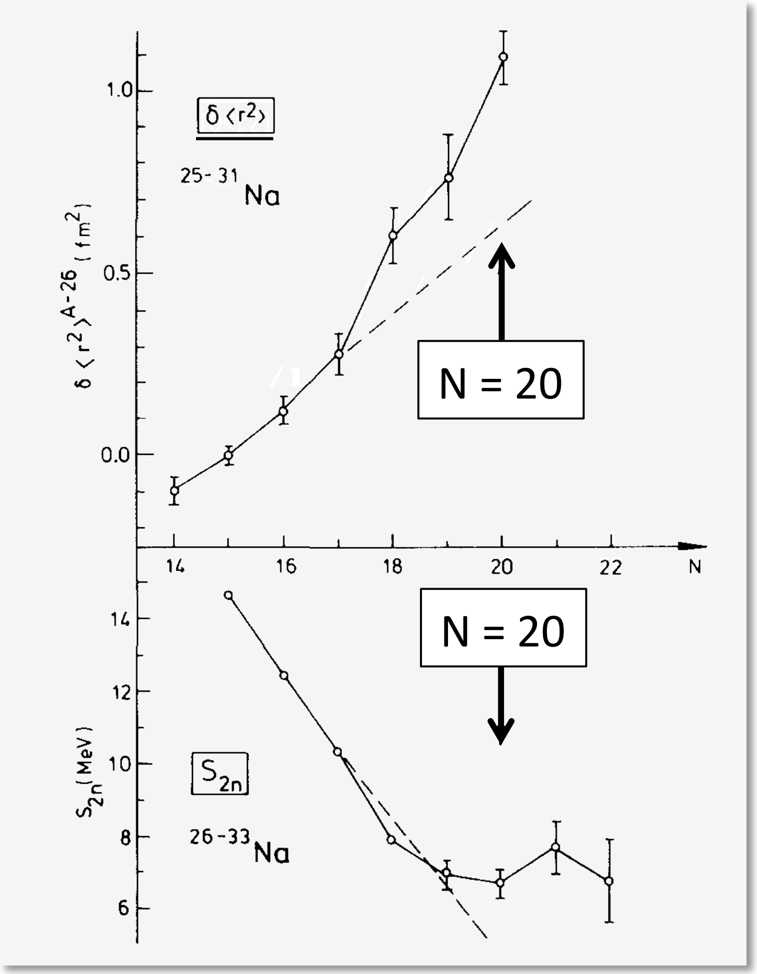
$\rho^2(E0) \bullet 10^3$



Isotope shifts and two-neutron separation energies

- A direct view of ground-state properties through atomic hyperfine spectroscopy and mass measurements
- Does not require decay of a parent isotope (further from stability, production-rate limitations)

N=20: sudden onset of deformation in the Na isotopes revealed by ground-state isotope shift and mass data



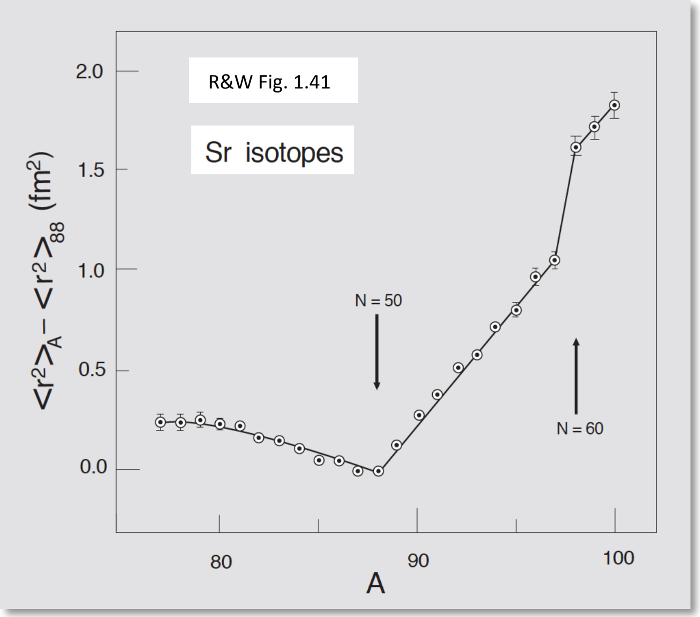
Na isotope-shifts determined by:

G. Huber et al., PRL 34, 1209 (1975); PR C18, 2342 (1978)

Na two-neutron separation energies deduced from masses determined by:

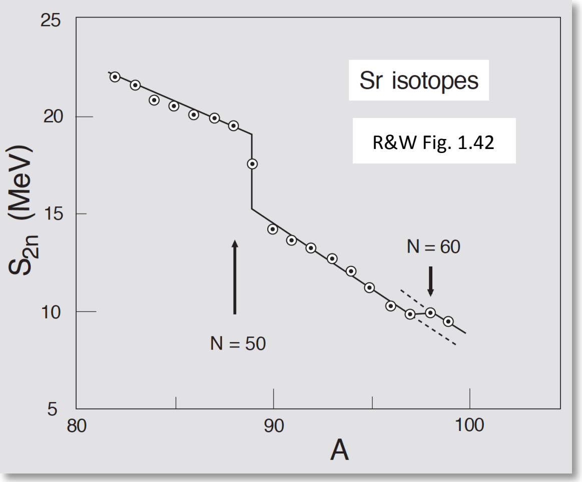
C. Thibault et al., PR C12, 644 (1975)

Ground-state properties are a direct signature of shell and deformation structures



Differences in mean-square charge radii (isotope shifts) determined by:

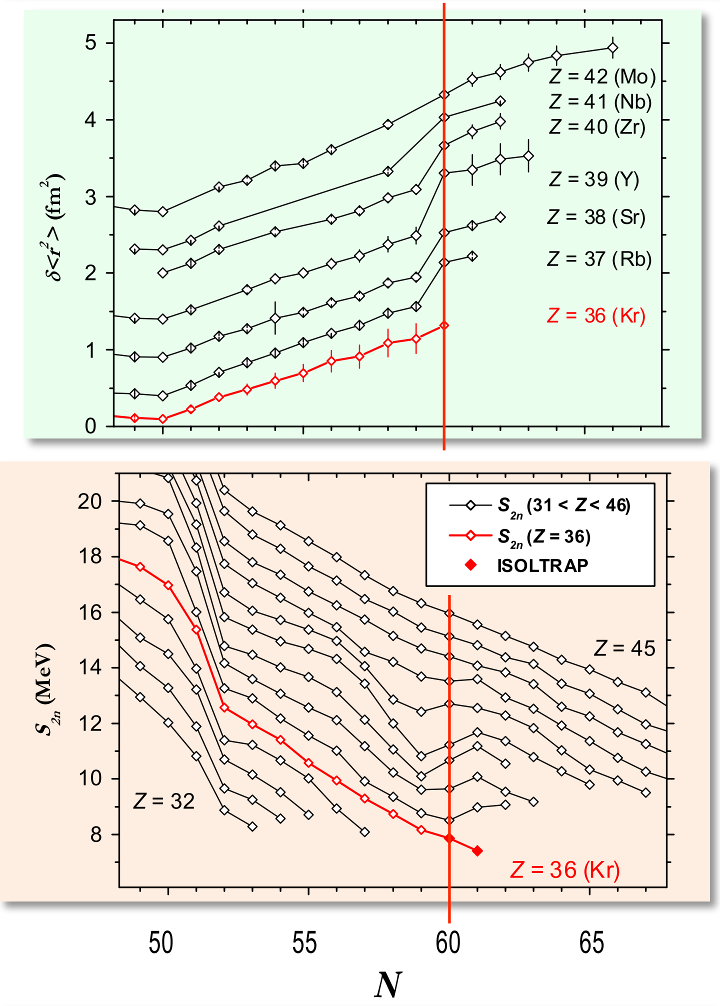
optical hyperfine spectroscopy using lasers



Two-neutron separation energies deduced from nuclear masses determined by:

direct mass measurements

Ground state properties, S_{2n} and $\delta\langle r^2 \rangle$, in the regions of N = 60, 90 are very similar



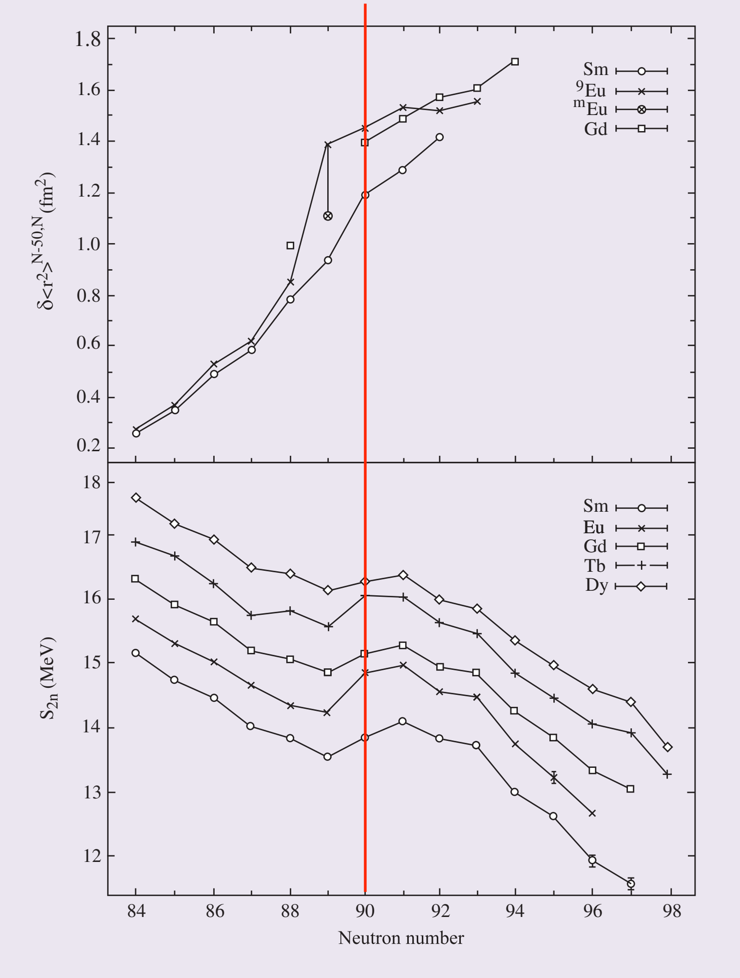
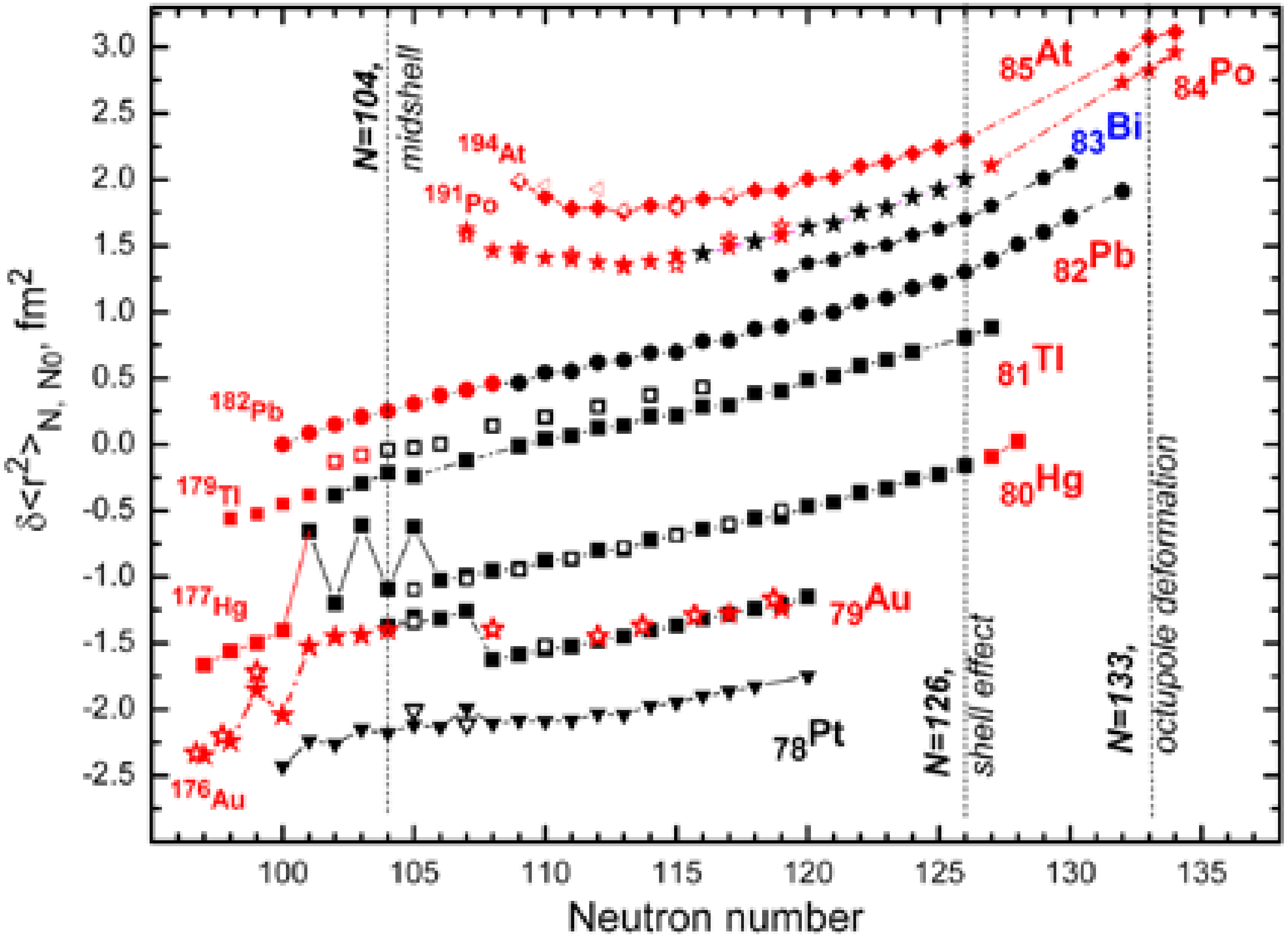


Figure from S. Naimi et al. Phys. Rev. Lett. 105 032502 (2010)

Figure from Heyde & Wood

Isotope shifts: Pt, Au, Hg, Tl, Pb, Bi, Po, At

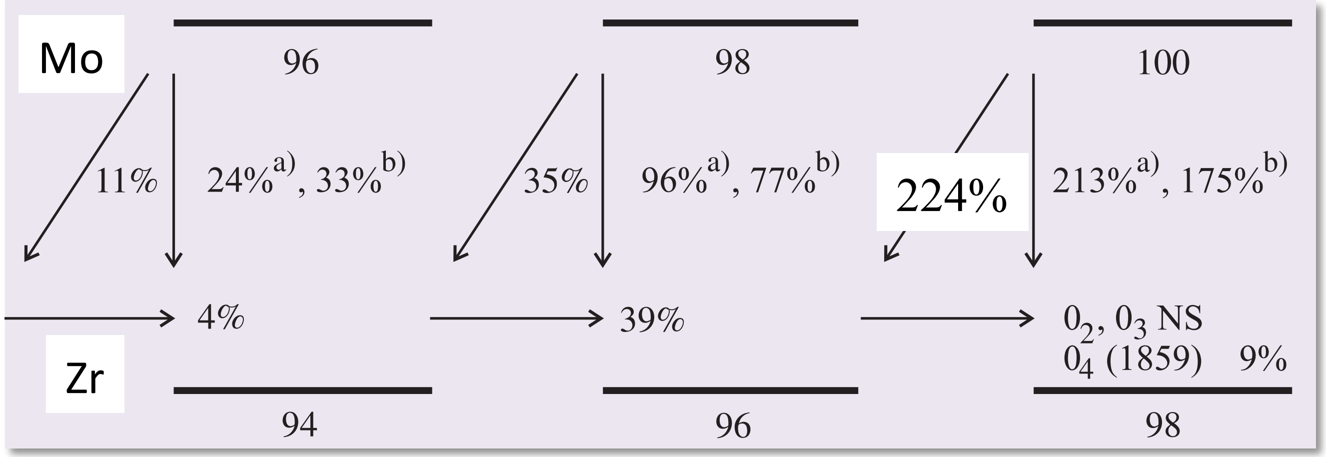
ISOLDE collaboration--red



Two- and four-nucleon transfer reactions

- Reveal distinct pairing condensates

Two- and four-nucleon transfer reactions: Mo ➔ Zr



Mo

96

11%

24%$^{a)}$, 33%$^{b)}$

4%

94

98

35%

96%$^{a)}$, 77%$^{b)}$

39%

96

100

224%

213%$^{a)}$, 175%$^{b)}$

0_2, 0_3 NS

0_4 (1859) 9%

98

Zr

Figure: Heyde & Wood

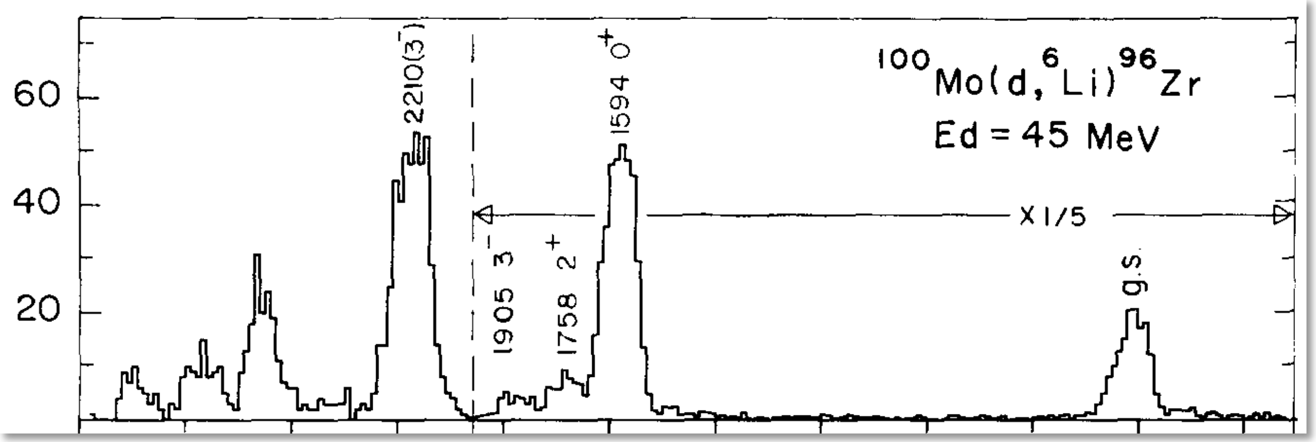


Figure from A. Saha et al. PL B82 208 1979

(^{6}Li,^{8}B)--a)

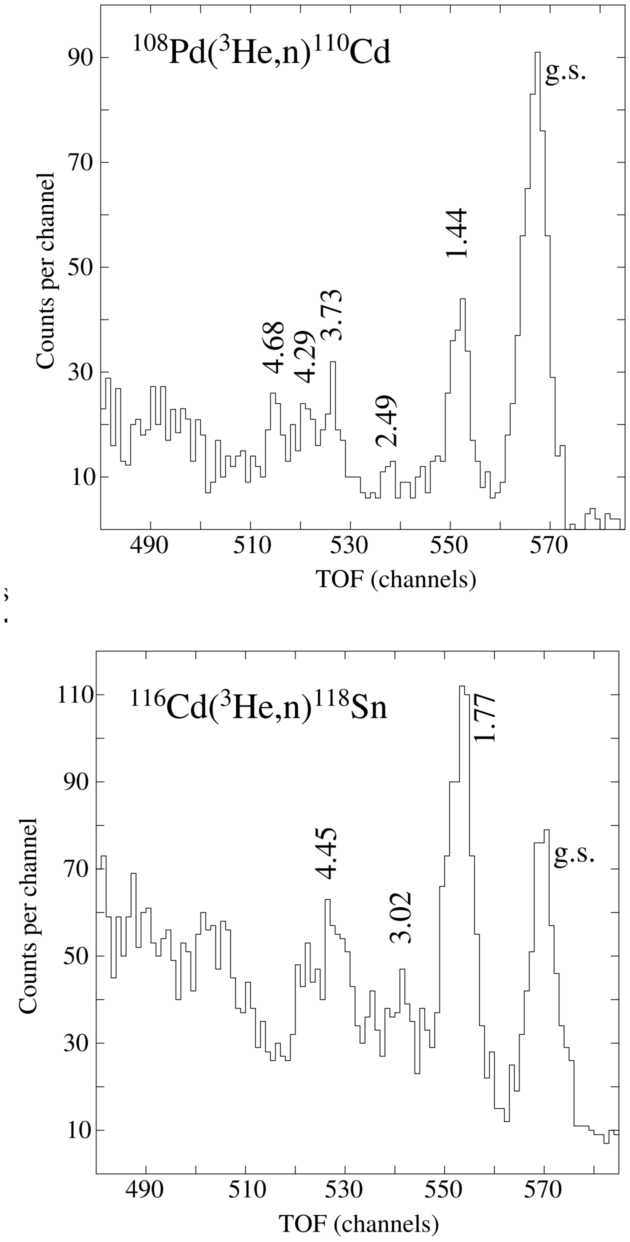
(^{14}C,^{16}O)--b)

(d,^{6}Li)

(t,p)

gs ➔0_2^+ %

gs ➔gs 100%



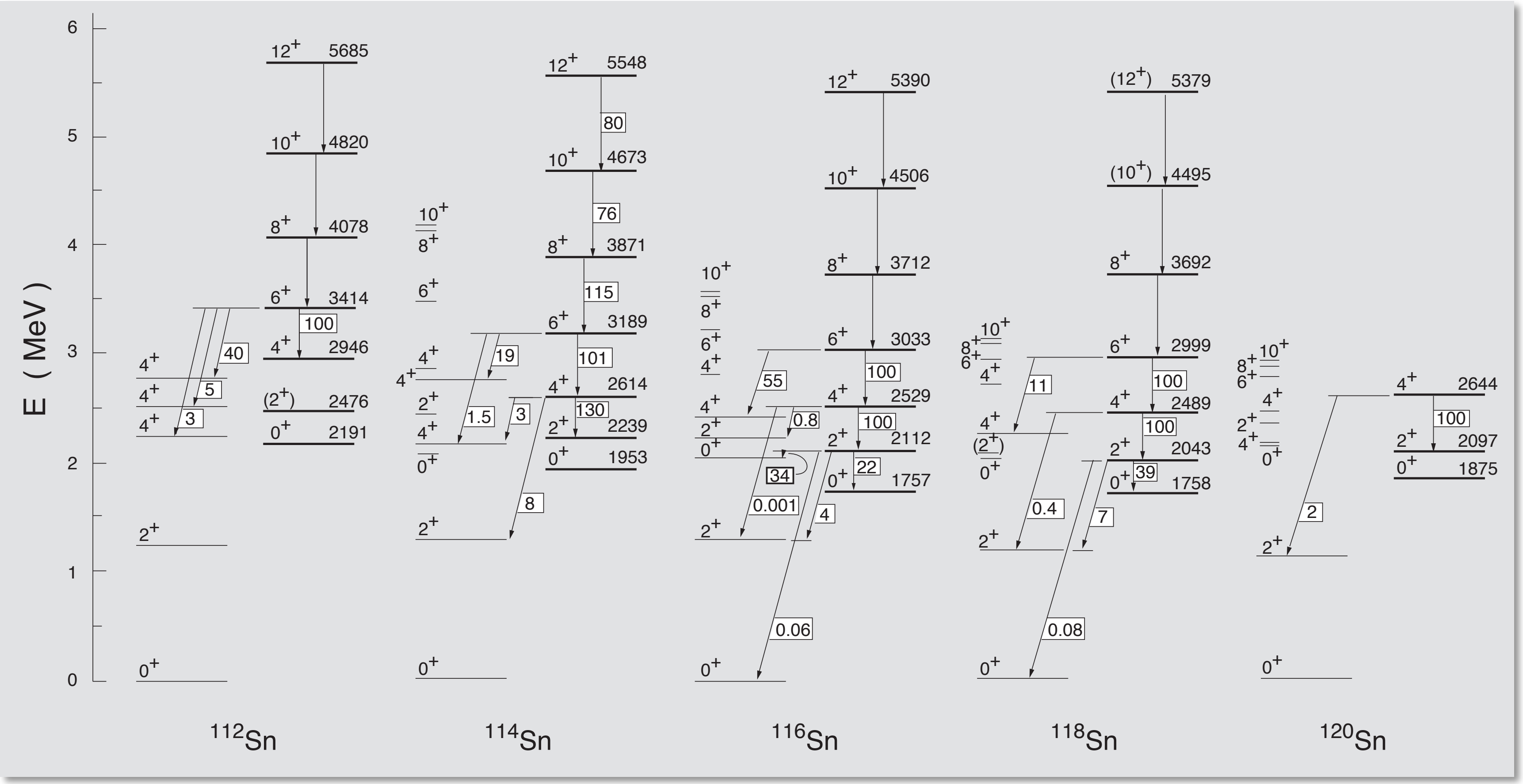
Time-of-flight (TOF) spectra for neutrons in the two-neutron transfer reactions, a). the ^{108}Pd(^{3}He,n)^{110}Cd and b). ^{116}Cd(^{3}He,n)^{118}Sn.The events corresponding to the ground states are marked as g.s. All of the strong peaks correspond to 0^+ states. The peaks at 1.44 MeV in ^{110}Cd And 1.77 MeV in ^{118}Sn match the excitation energies of the lowest energy deformed states in these isotopes. The spectra are taken from

H. W. Fielding et al., Nucl. Phys. A281, 389 (1977).

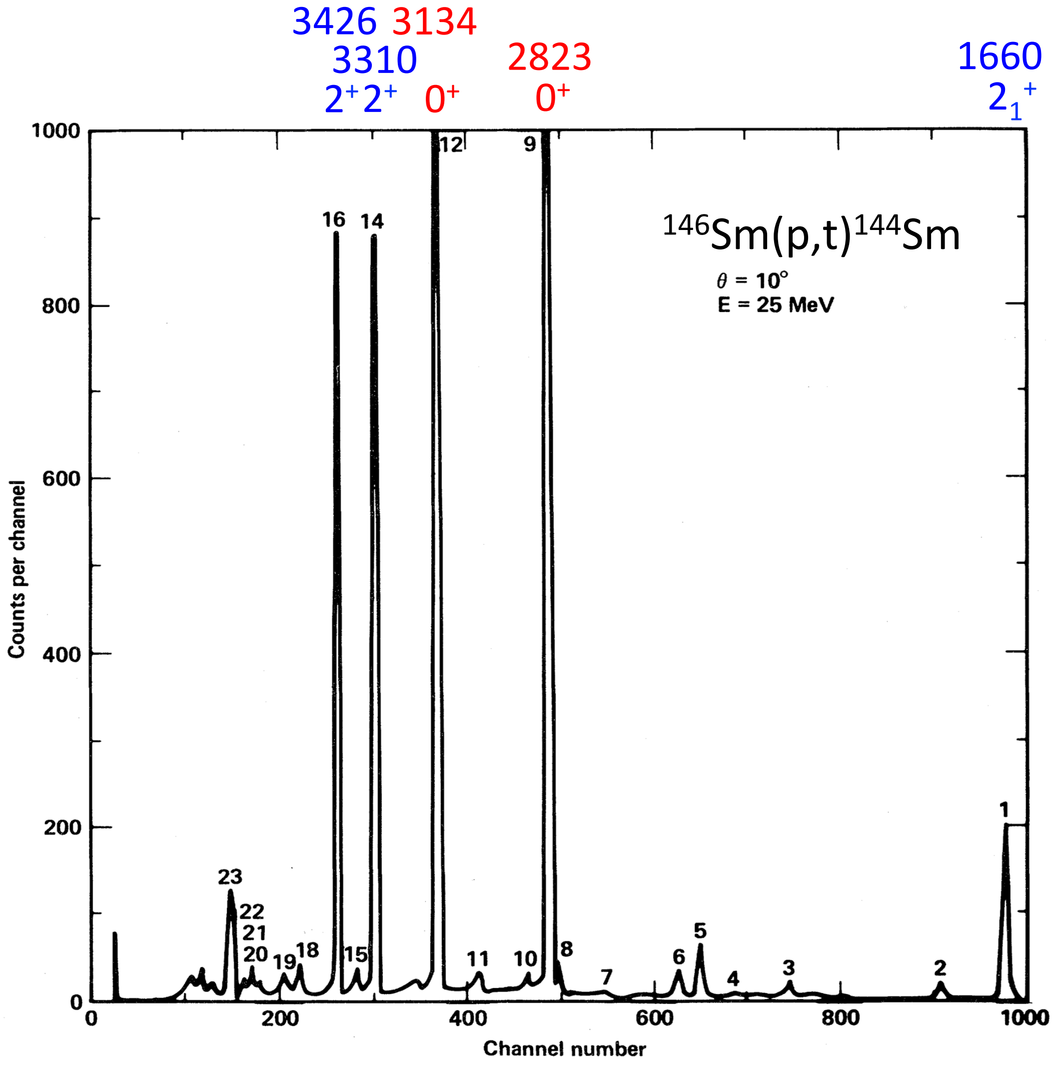
Rotational bands in (closed shell) $^{112\text{-}120}$Sn: built on excited 0^+ states

Figure from Rowe & Wood

B(E2)'s in W.u. [100 = rel. value]



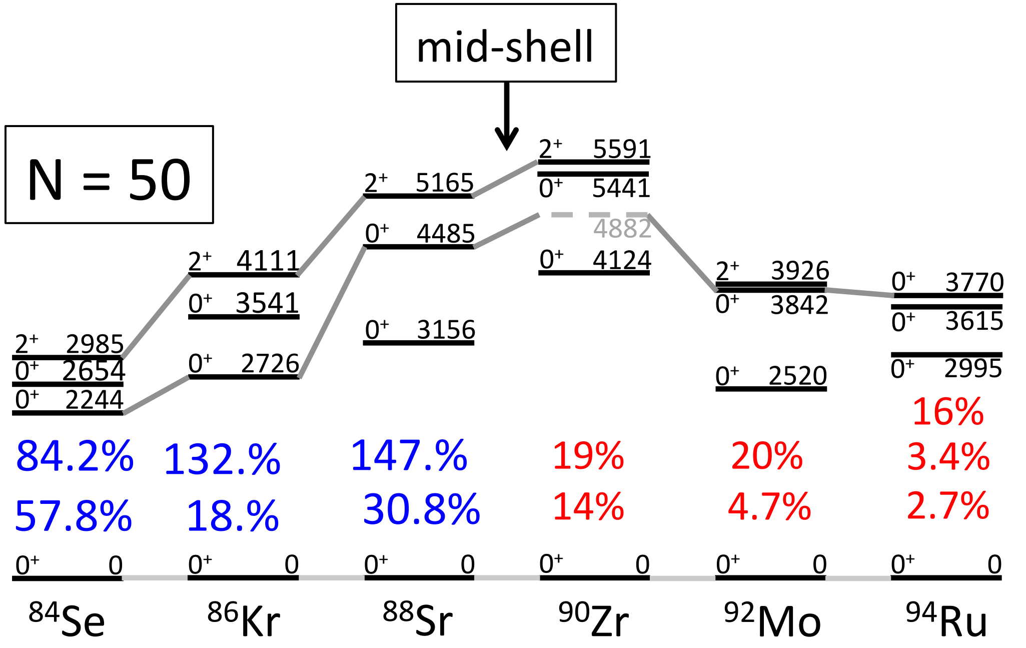
N = 82: ν (2p-2h) 0^+, 2^+ ^{144}Sm



Flynn et al.,
PR C28 97 (1983)

N = 50, 82: ν (2p-2h) 0^+, 2^+

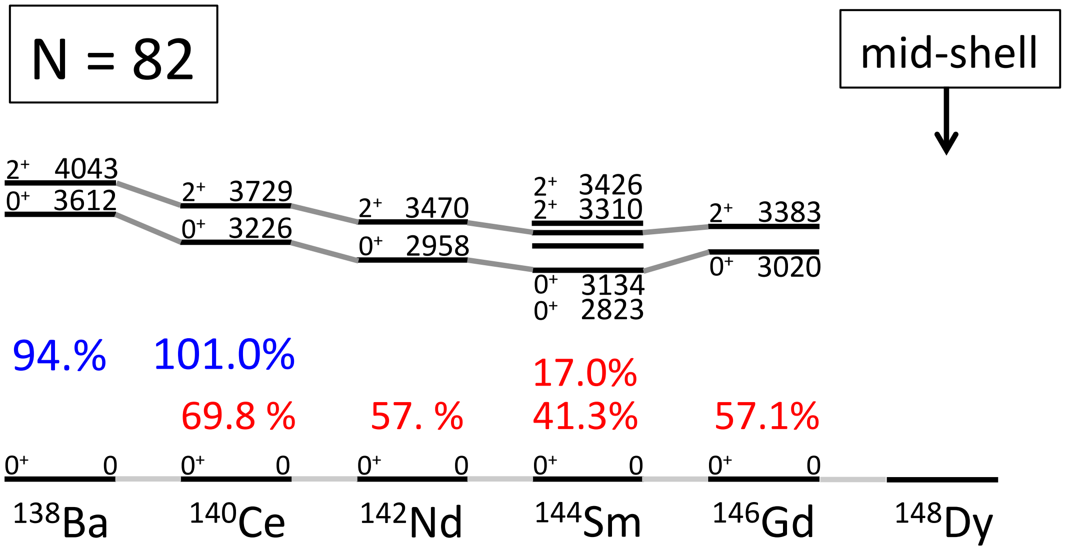
PR C37 587
PR C13 568
PR C2 1020



PR C4 196
PRL 29 1014
NP A207 425

t,p

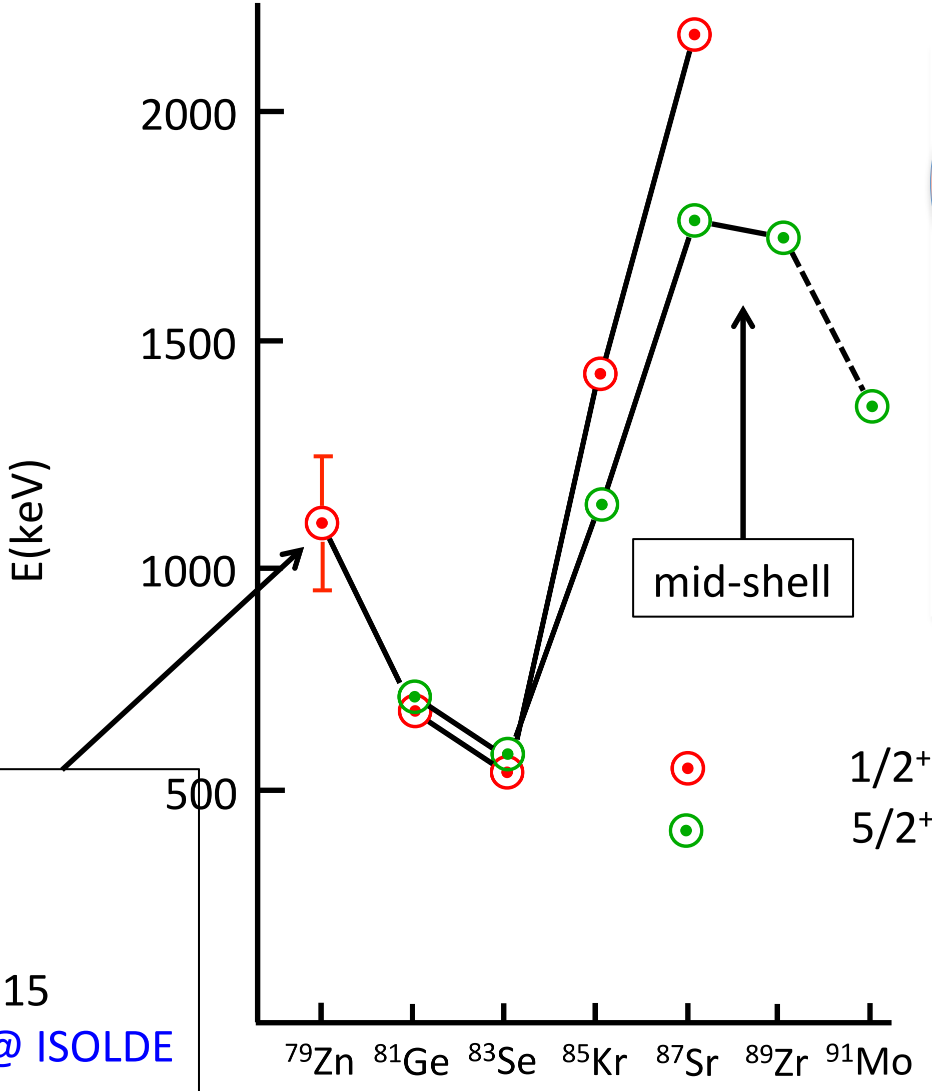
p,t

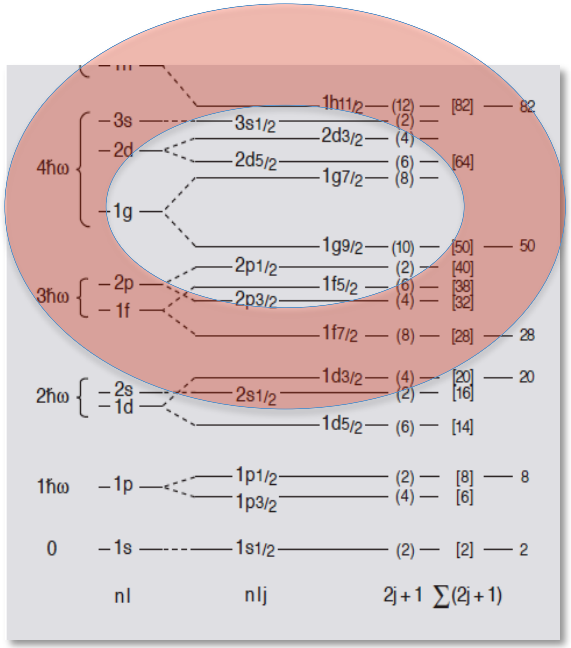


PL B98 166
PR C6 1802

PR C6 1802
PL B30 533
PR C28 97
- do -

Intruder ν(1p-2h) “$2d_{5/2}$, $3s_{1/2}$” structure in N = 49 isotones





^{79}Zn:

d,pγ @ISOLDE

R. Orlandi et al.,

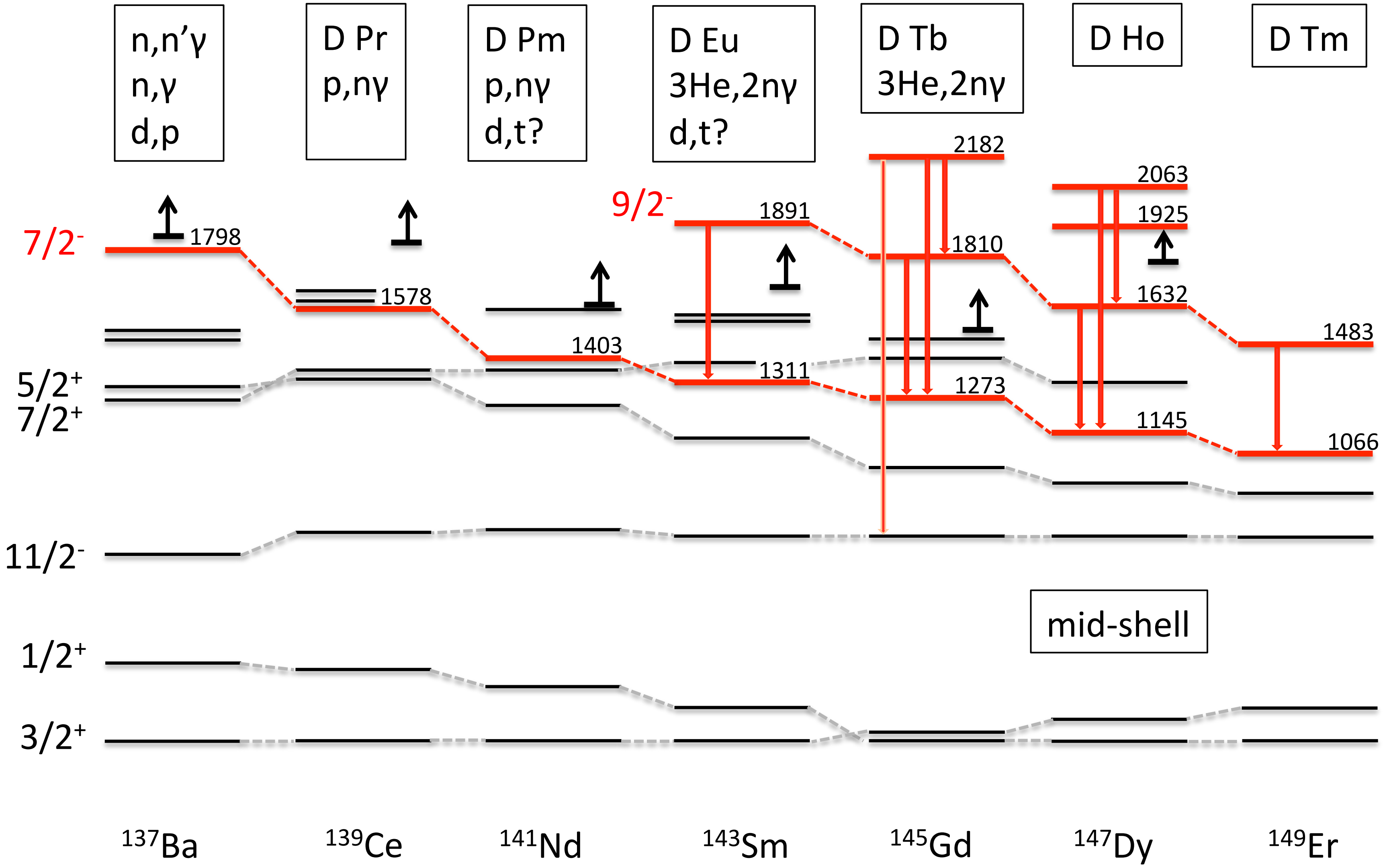
PL **B740** 298 2015

hfs laser spect. @ ISOLDE

X.F. Yang et al.,

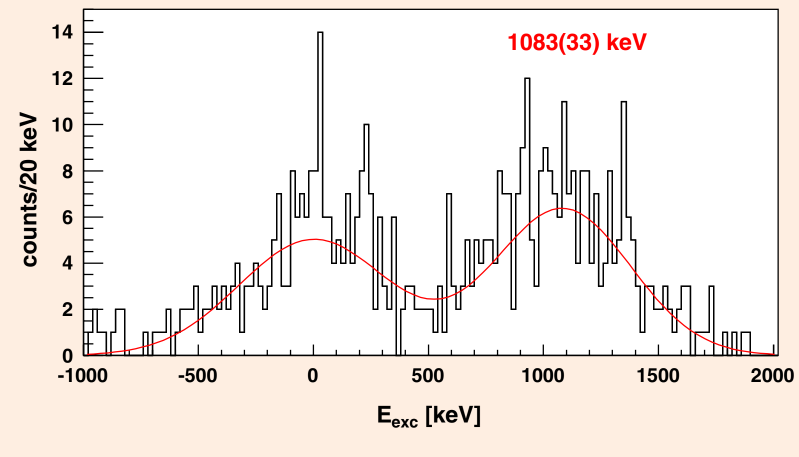
PRL **116** 182502 2016

Intruder ν(1p-2h) “$2f_{7/2}$” structure in N = 81 isotones

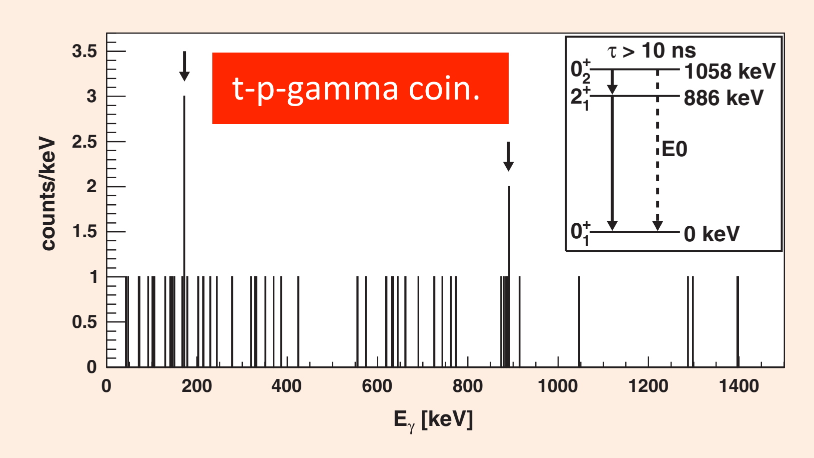


All decay branches shown

^{32}Mg: 0_2^+ state observed by (t,p) via inverse kinematics with a ^{30}Mg beam



K. Wimmer et al., PRL 105, 252501 (2010)
--REX-ISOLDE



N=20 systematic showing the ν(2p-2h) 0^+ bands @ Z=14-18

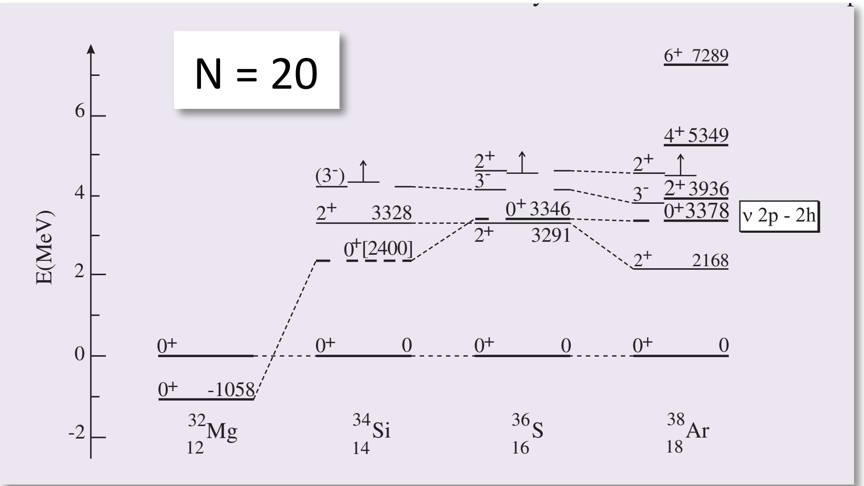


Figure from Heyde & Wood

Excited 0^+ states in the Ca isotopes:

multi-particle-multi-hole states, and...?

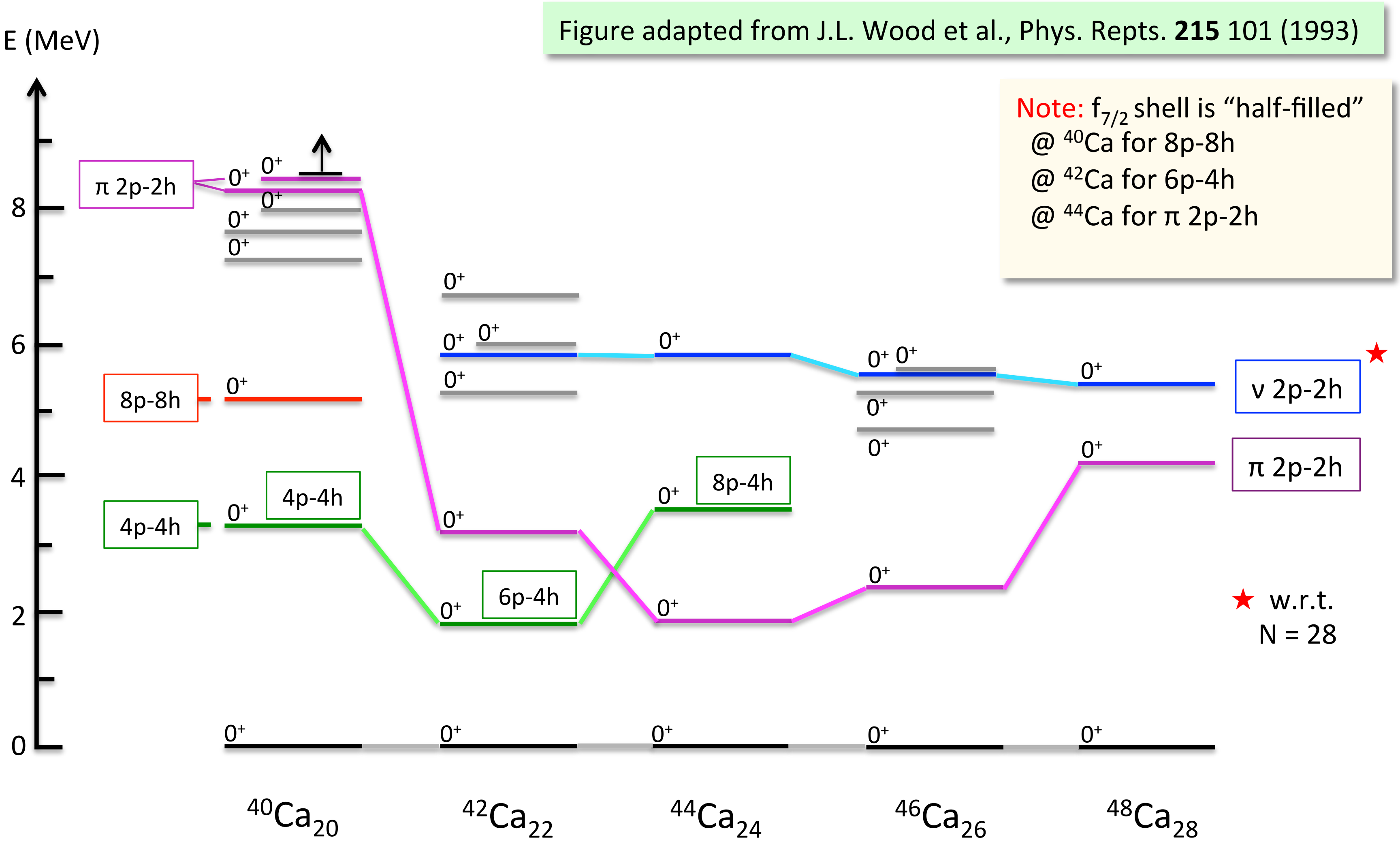


Figure adapted from J.L. Wood et al., Phys. Repts. **215** 101 (1993)

Note: $f_{7/2}$ shell is "half-filled"
@ ^{40}Ca for 8p-8h
@ ^{42}Ca for 6p-4h
@ ^{44}Ca for π 2p-2h

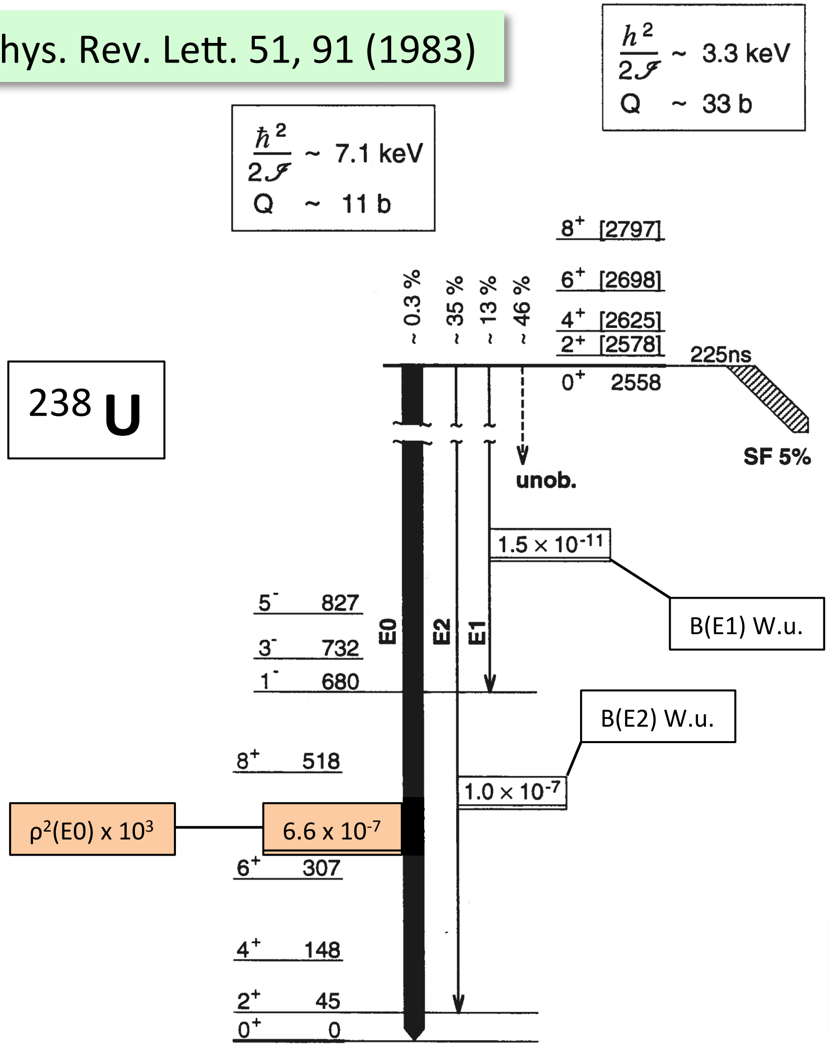
★ w.r.t. N = 28

E0’s and $\rho^2(E0)$’s

- Reveal mixing of configurations with different mean-square charge radii / different deformations

E0 transition between states with very different deformations and mean-square charge radii

J. Kantele et al., Phys. Rev. Lett. 51, 91 (1983)



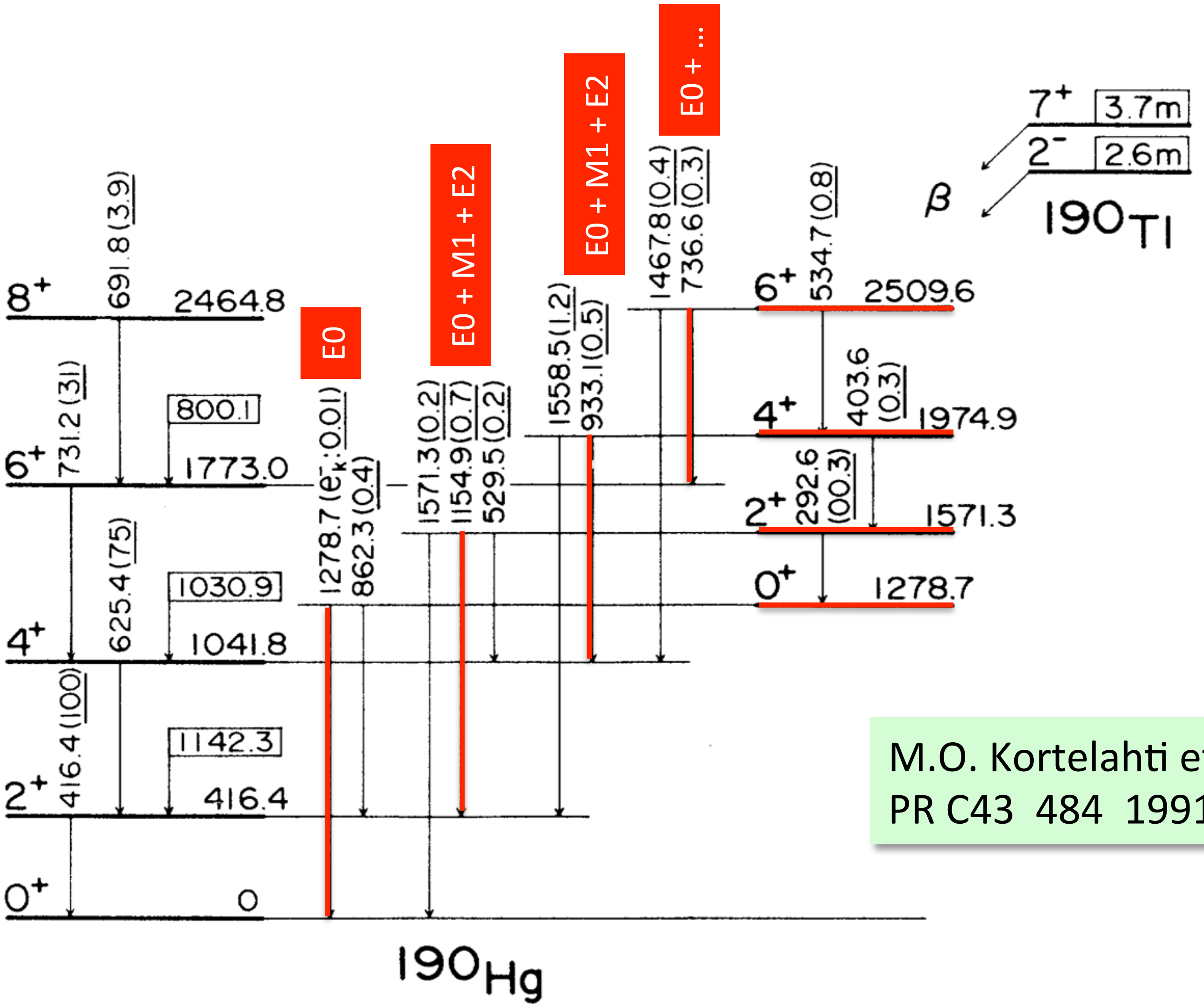
The E0 strength from the ^{238}U "fission" isomer is the weakest known

T. Kibédi and R.H. Spear, ADNDT 89 77 2005

Figure from JLW et al., Nucl. Phys. A651 323 1999

Conversion electron spectroscopy:

uniquely sensitive to E0 transitions, identifies shape coexistence

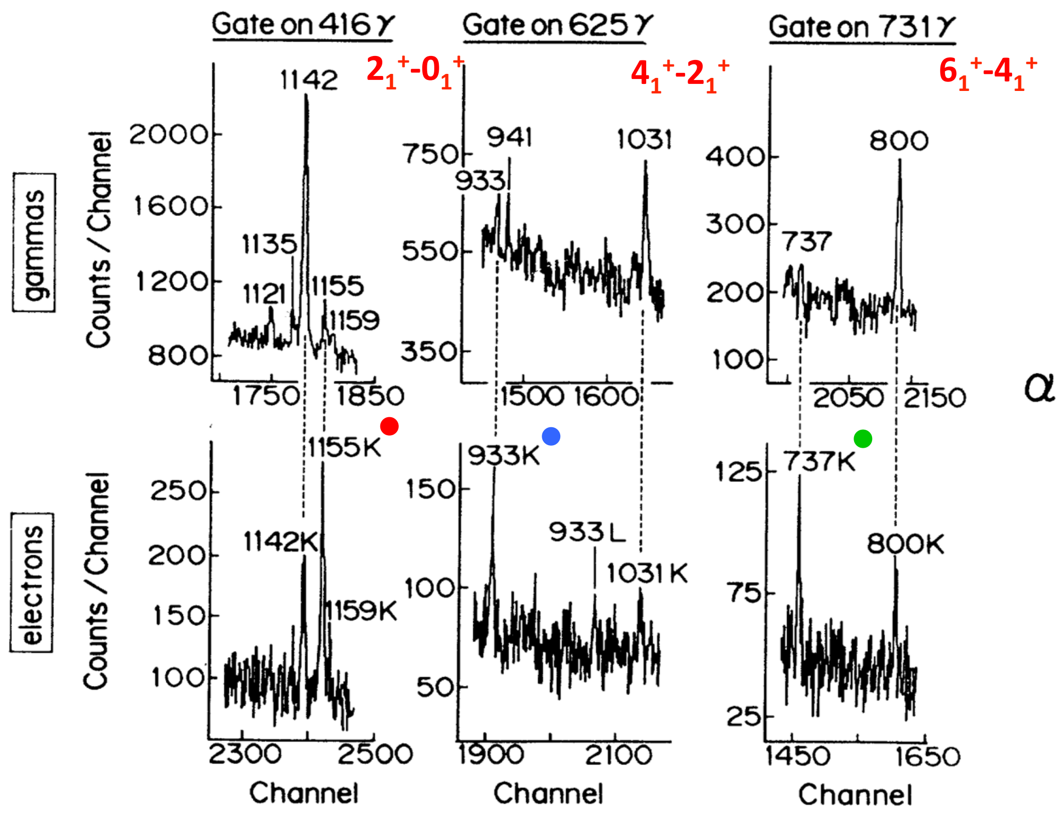


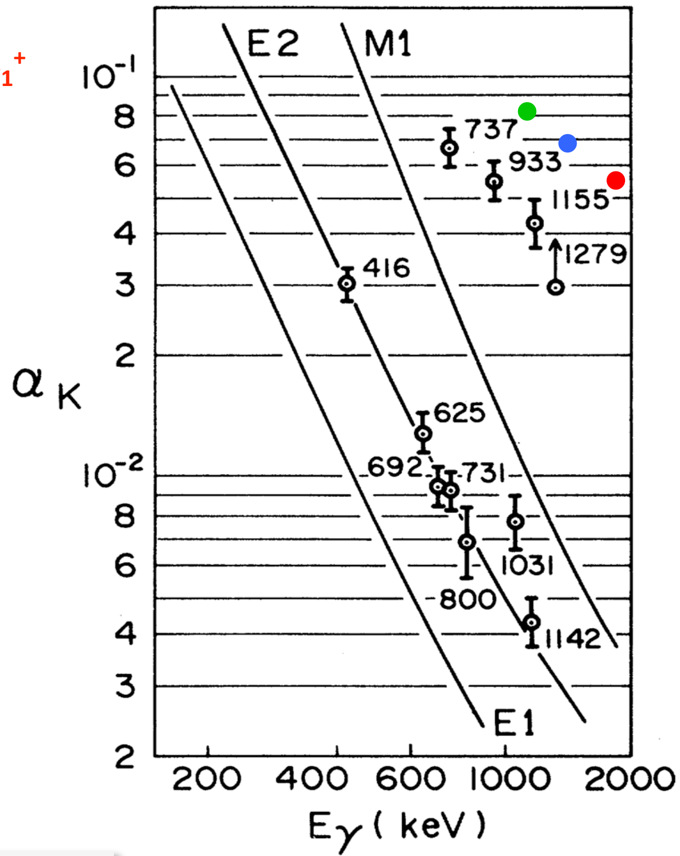
M.O. Kortelahti et al.,
PR C43 484 1991 UNISOR

E0 transitions--$\alpha_K > \alpha_K(M1)$:

complex decay schemes require γ-e coincidences

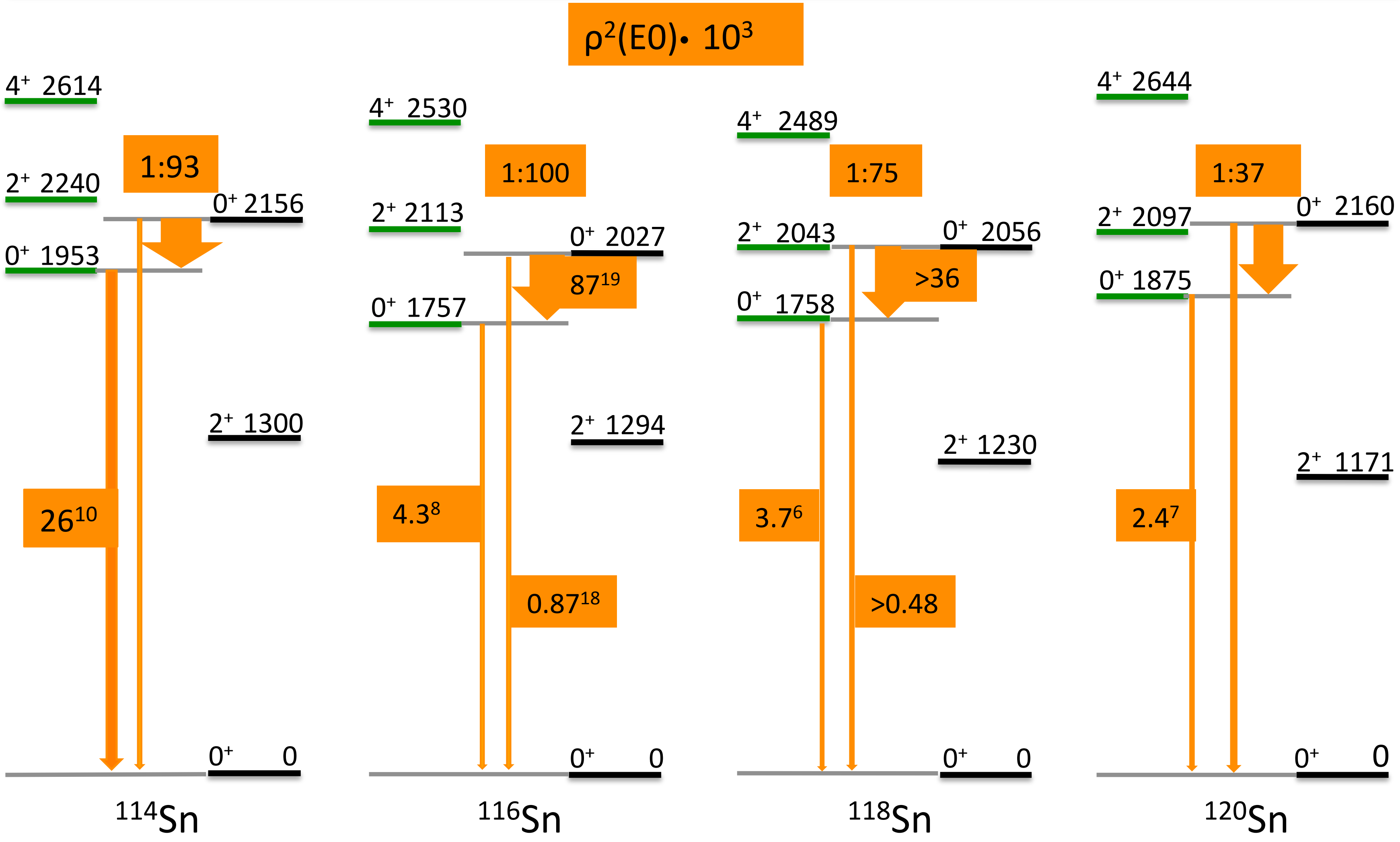
M.O. Kortelahti et al., PR C43 484 1991





^{190}Hg

E0 transitions between shape coexisting states in the Sn isotopes



E0: transition operator and matrix element --a model *independent* description

E0 transition strengths are a measure of the off-diagonal matrix elements of the mean-square charge radius operator.

Mixing of configurations with different mean-square charge radii produces E0 transition strength.

$$\rho^2(E0) = \frac{1}{\Omega\,\tau(E0)}$$

"Electronic factor"

$$\Omega = \Omega(Z,\Delta E) = \Omega_K + \Omega_{L_1} + \dots + \Omega_{e^+e^-}$$

Monopole strength parameter

$$\rho_{if}(E0) = \frac{\langle f|\sum_j e_j r_j^2|i\rangle}{eR^2} \equiv \frac{\langle f|m(E0)|i\rangle}{eR^2} \equiv \frac{M_{if}(E0)}{eR^2}$$

$$|i\rangle = \alpha|1\rangle + \beta|2\rangle\,, \quad |f\rangle = -\beta|1\rangle + \alpha|2\rangle$$

$$M_{if}(E0) = \alpha\beta\left\{\langle 2|m(E0)|2\rangle - \langle 1|m(E0)|1\rangle\right\} + (\alpha^2-\beta^2)\langle 1|m(E0)|2\rangle$$

$$\boxed{M_{if}(E0) \simeq \alpha\beta\,\Delta\langle r^2\rangle}$$

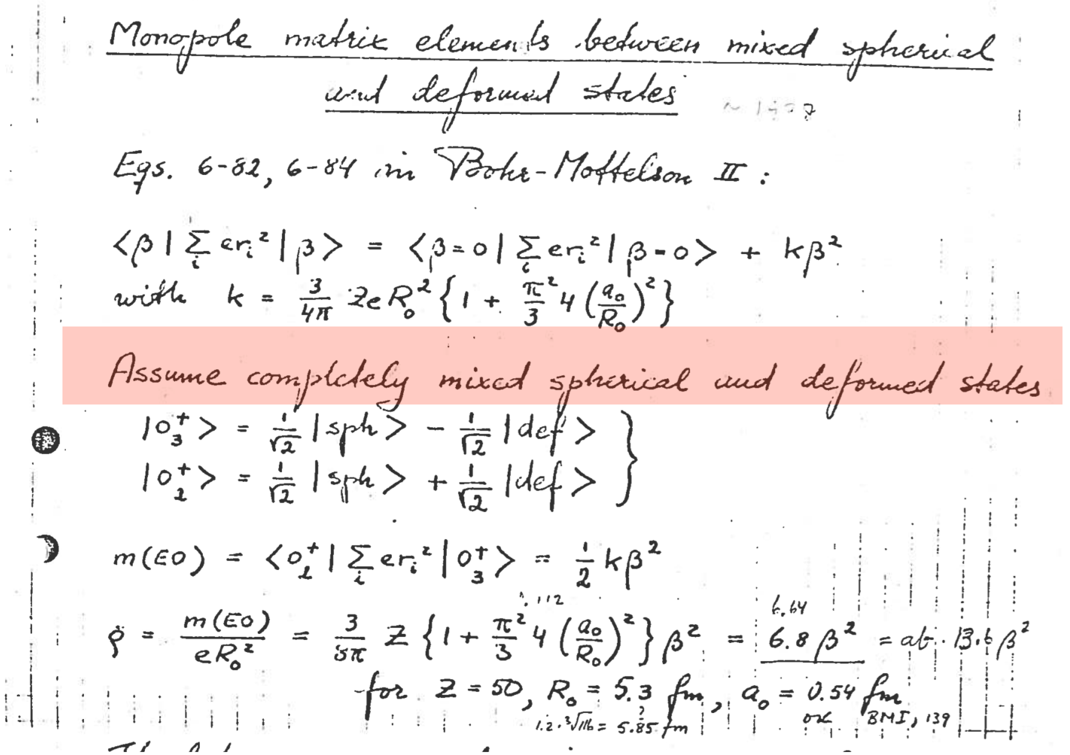
Ω values: http://bricc.anu.edu.au

τ: partial lifetime for E0 decay branch

J. Kantele et al. Z. Phys. A289 157 1979
and see
JLW et al. Nucl. Phys. A651 323 1999

Origin of idea that E0 strength heralds shape coexistence is due to Jan Blomqvist
(priv. comm. to JLW from Rauno Julin)

Jan Blomqvist ~1978



Monopole matrix elements between mixed spherical and deformed states

~1977

Eqs. 6-82, 6-84 in Bohr-Mottelson II :

$$\langle\beta|\sum_i e r_i^2|\beta\rangle = \langle\beta=0|\sum_i e r_i^2|\beta=0\rangle + k\beta^2$$

with $k = \frac{3}{4\pi} Z e R_0^2 \left\{1 + \frac{\pi^2}{3} 4 \left(\frac{a_0}{R_0}\right)^2\right\}$

Assume completely mixed spherical and deformed states

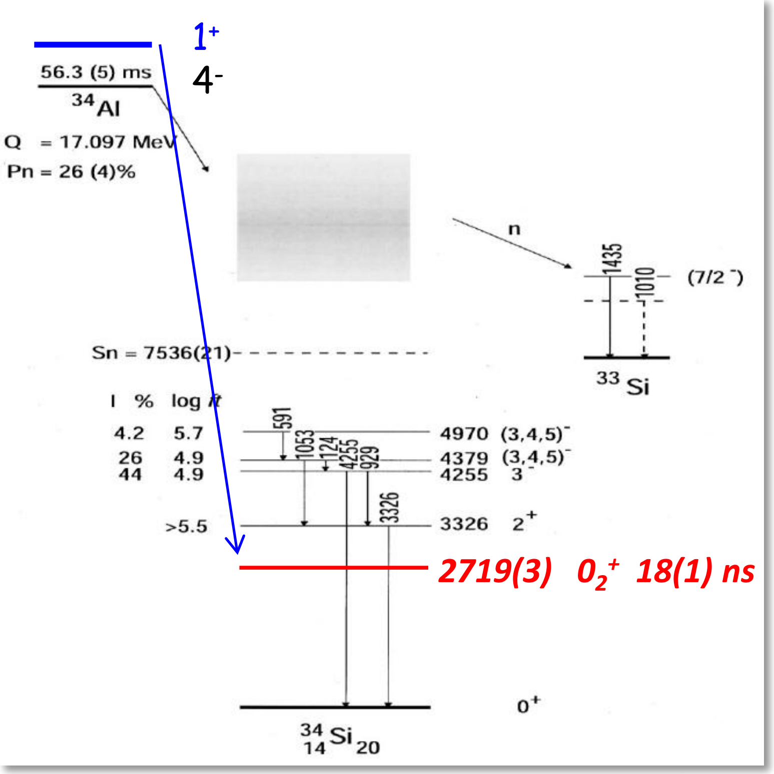
$$\left.\begin{aligned} |0_3^+\rangle &= \tfrac{1}{\sqrt{2}}|sph\rangle - \tfrac{1}{\sqrt{2}}|def\rangle \\ |0_2^+\rangle &= \tfrac{1}{\sqrt{2}}|sph\rangle + \tfrac{1}{\sqrt{2}}|def\rangle \end{aligned}\right\}$$

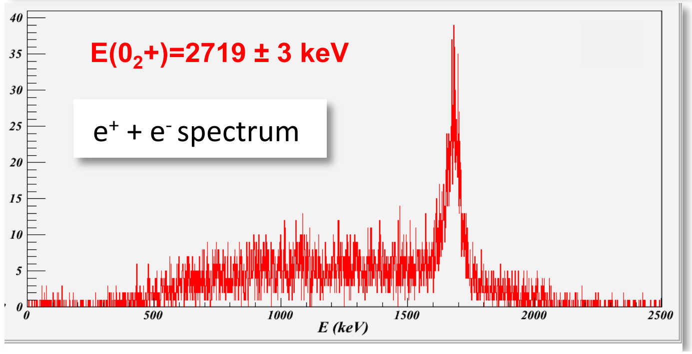
$$m(E0) = \langle 0_2^+|\sum_i e r_i^2|0_3^+\rangle = \frac{1}{2}k\beta^2$$

$$\rho = \frac{m(E0)}{eR_0^2} = \frac{3}{8\pi} Z \left\{1 + \frac{\pi^2}{3} \overset{.112}{4} \left(\frac{a_0}{R_0}\right)^2\right\}\beta^2 = \underline{\overset{6.64}{6.8\,\beta^2}} = ab.\ 13.6\,\beta^2$$

for $Z = 50$, $R_0 = 5.3$ fm, $a_0 = 0.54$ fm
$1.2 \cdot \sqrt[3]{116} = 5.85$ fm ? ok BMI, 139

^{34}Si: 0_2^+ state observed by internal-pair (electron) spectroscopy via β decay of ^{34m}Al





F. Rotaru et al. PRL 109 092503 (2012) --LISE3@GANIL

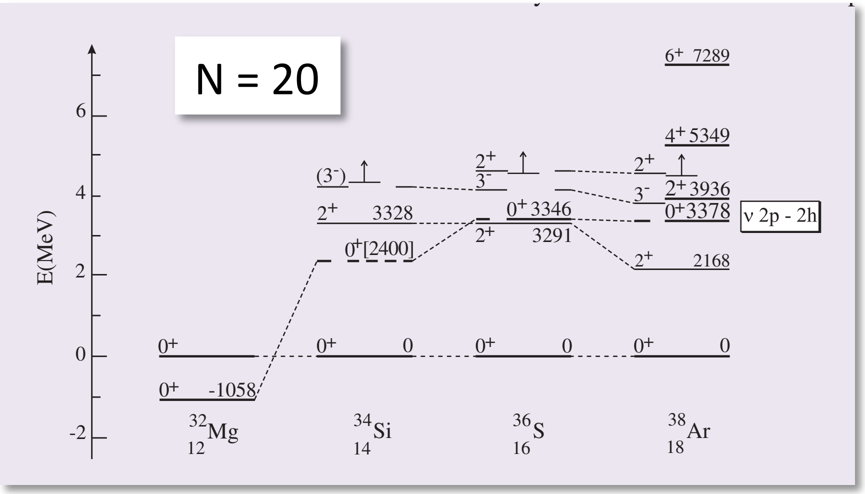
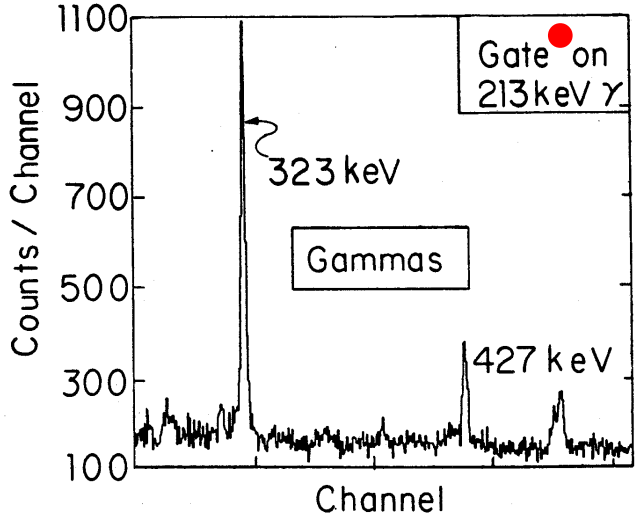
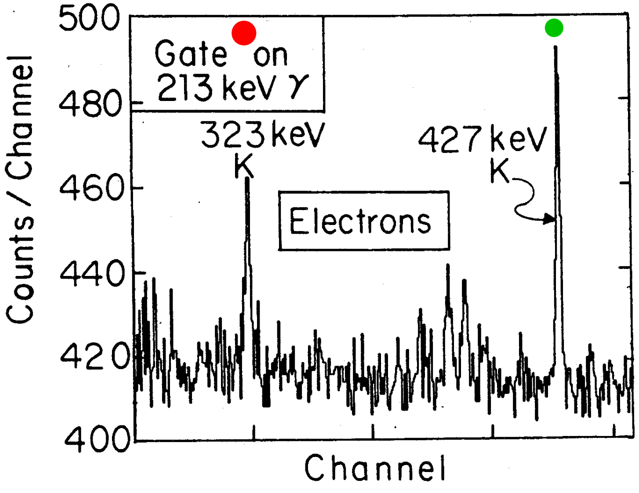


Figure from Heyde & Wood

E0 transitions between “single”and “double” intruder states in ^{185}Au

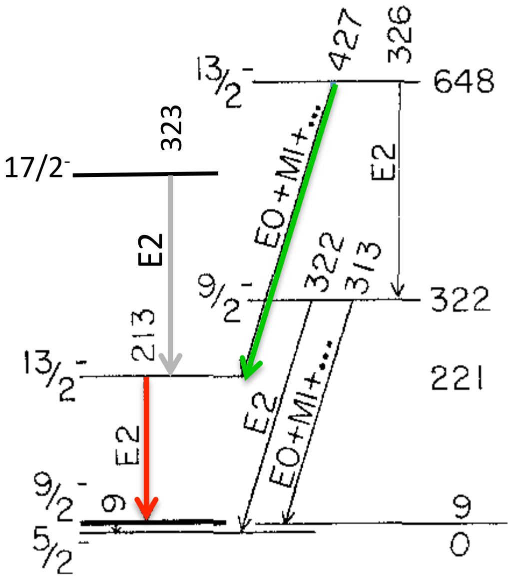




C.D. Papanicolopulos PhD thesis Ga Tech 1987 and ZP A330 371 1988 UNISOR

Z = 79 427 keV

mult	α_K
E1	0.010
E2	0.027
M1	0.11
expt.	0.33



9/2$^-$ state @ 9 keV: “double” intruder state: $\pi h_{9/2}$ (1p) × ^{184}Pt [π(2p-6h)] = π(3p-6h)

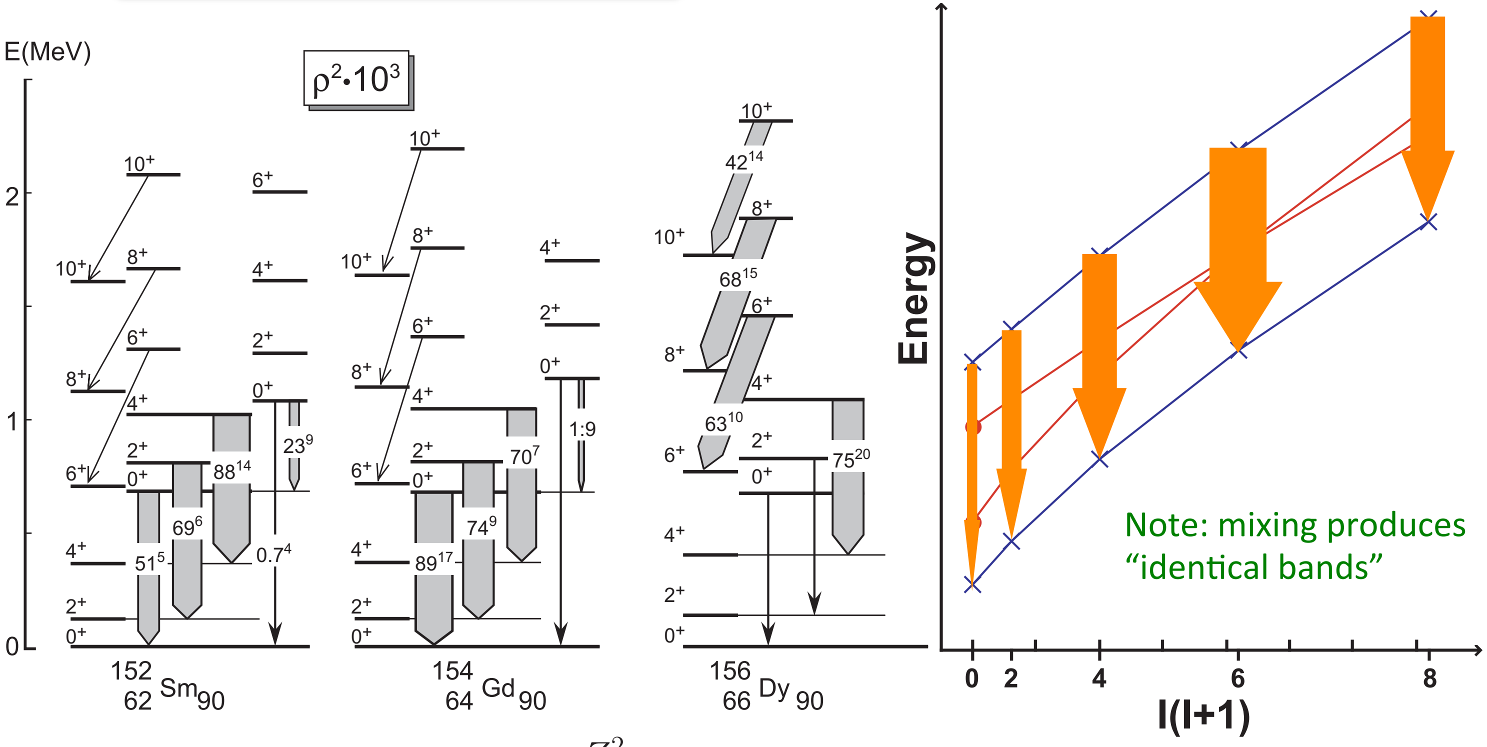
9/2$^-$ state @ 322 keV: “single” intruder state: $\pi h_{9/2}$ (1p) × ^{184}Pt [π(4h)] = π(1p-4h)

Shape coexistence in the N = 90 isotones:

revealed by E0 transition strengths

Strong mixing of coexisting shapes produces strong electric monopole (E0) transitions and identical bands.

Data from Heyde & Wood



Note: mixing produces “identical bands”

$$\rho^2 \cdot 10^3 = \alpha^2\beta^2 \left(\Delta\langle r^2\rangle\right)^2 \cdot 10^3 \, \frac{Z^2}{R_0^4}$$

$$R_0 = 1.2A^{1/3}\ \text{fm}$$

→ *E0* strength is a function of mixing.

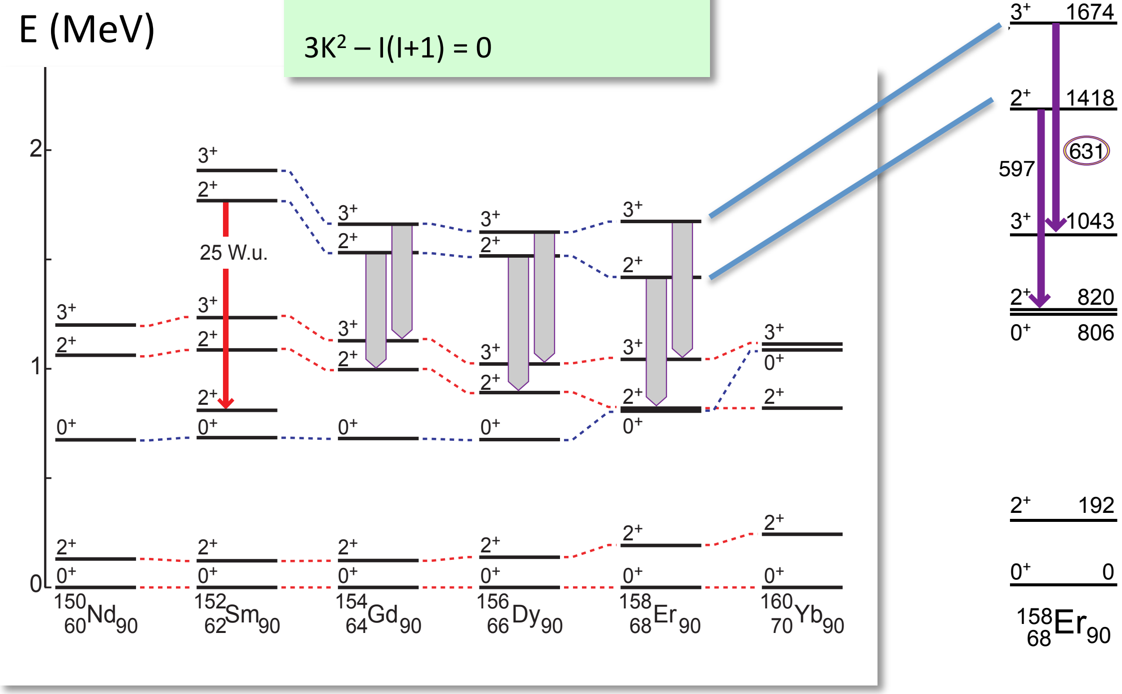
Coexistence in the N = 90 isotones:

K = 0 and K = 2 bands

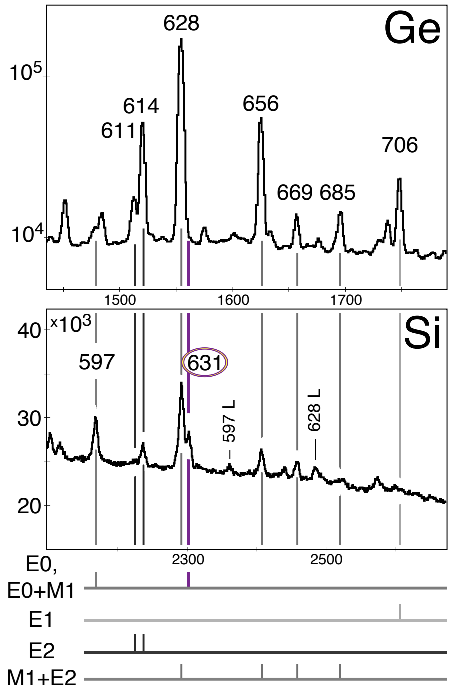
E0 transitions between pairs of K = 2 bands

$3^+,K=2 \rightarrow 3^+,K=2$: no γ

$3K^2 - I(I+1) = 0$



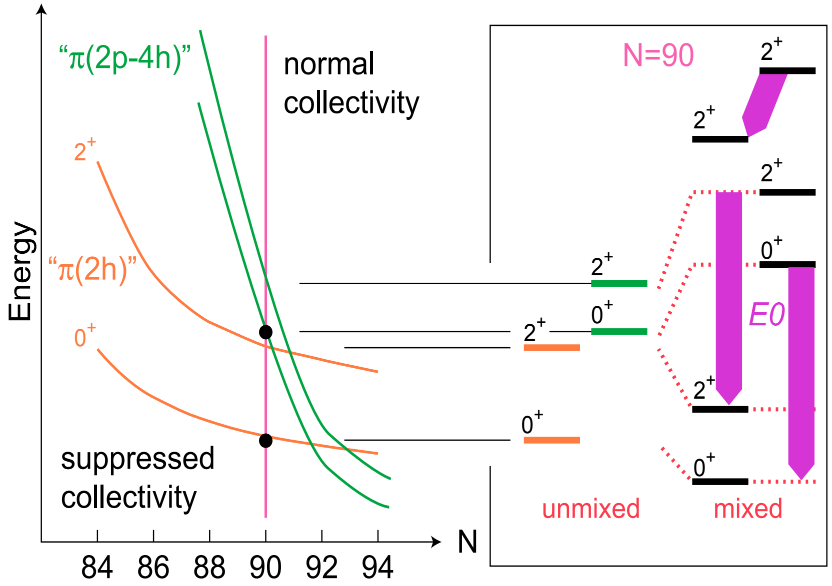
8π and PACES at TRIUMF/ISAC.



Kulp, Wood, Garrett, Zganjar and others

^{152}Sm and the neighboring N = 90 isotones are a manifestation of shape coexistence

Proton particle-hole excitations across the *Z* = 64 gap may be the source of the coexisting shapes.



Less-deformed 2h and more-deformed 2p-4h structures coexist at low energy at *N*=90.

Strong mixing obscures the energy differences that are indicative of different shapes.

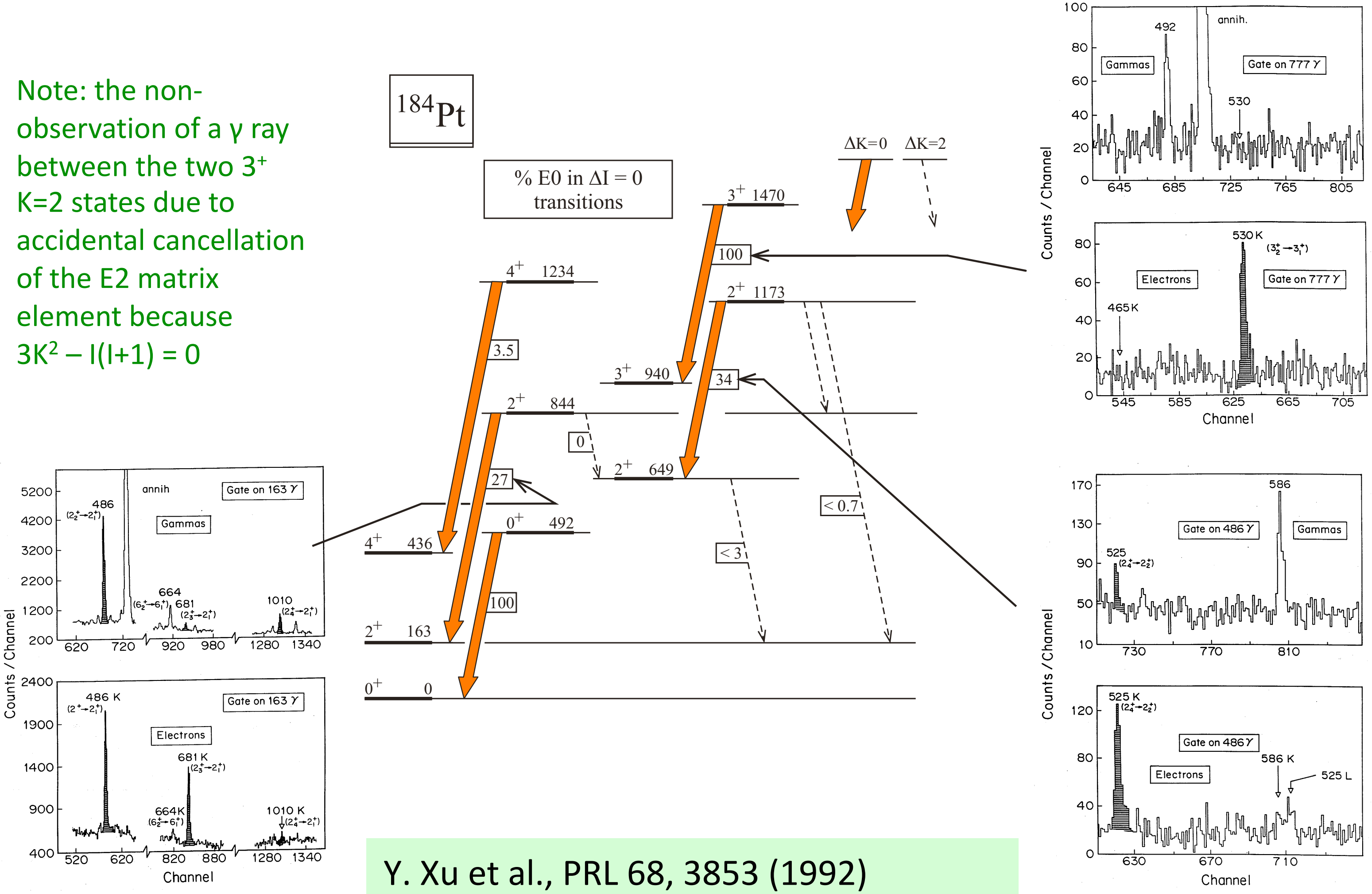
Strong *E0* transitions are a key signature of the mixing of coexisting structures.

As observed, the *K*=2 bands will also mix strongly, resulting in *E0* transitions.

Coexistence in the even-Pt isotopes:

coexistence of K = 0 and K = 2 bands in ^{184}Pt

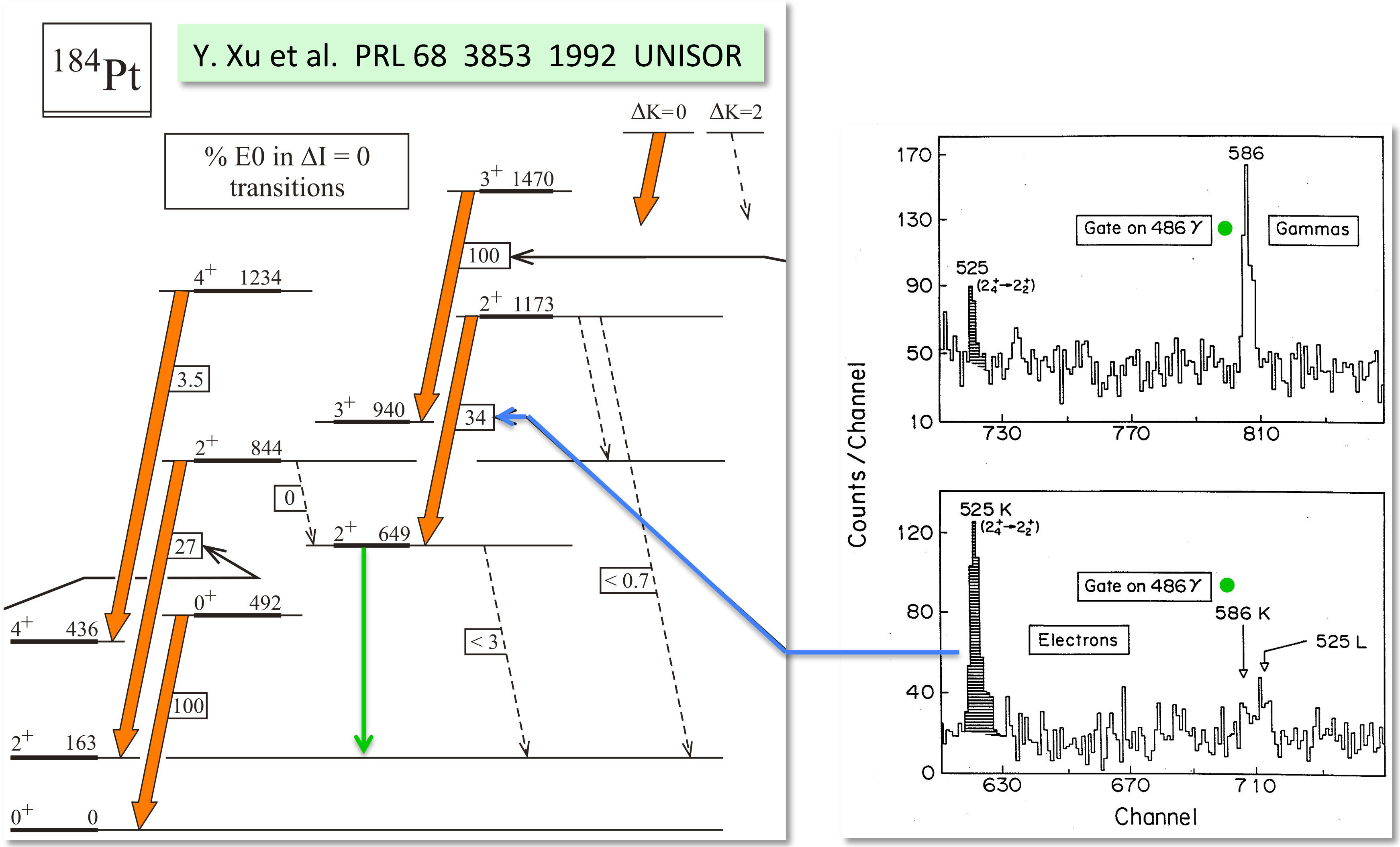
Note: the non-observation of a γ ray between the two 3^+ K=2 states due to accidental cancellation of the E2 matrix element because $3K^2 - I(I+1) = 0$



Y. Xu et al., PRL 68, 3853 (1992)

Coexistence in the even-Pt isotopes:

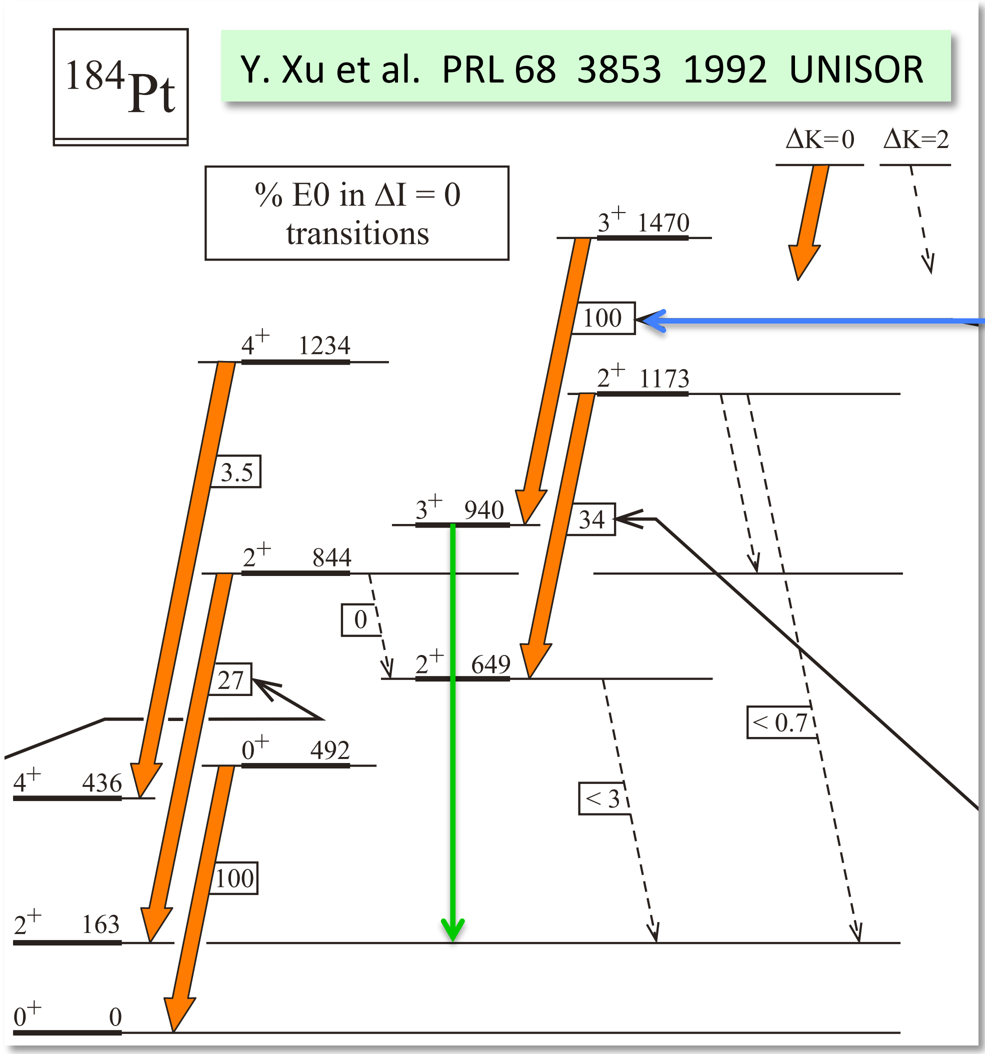
K = 0 and K = 2 bands



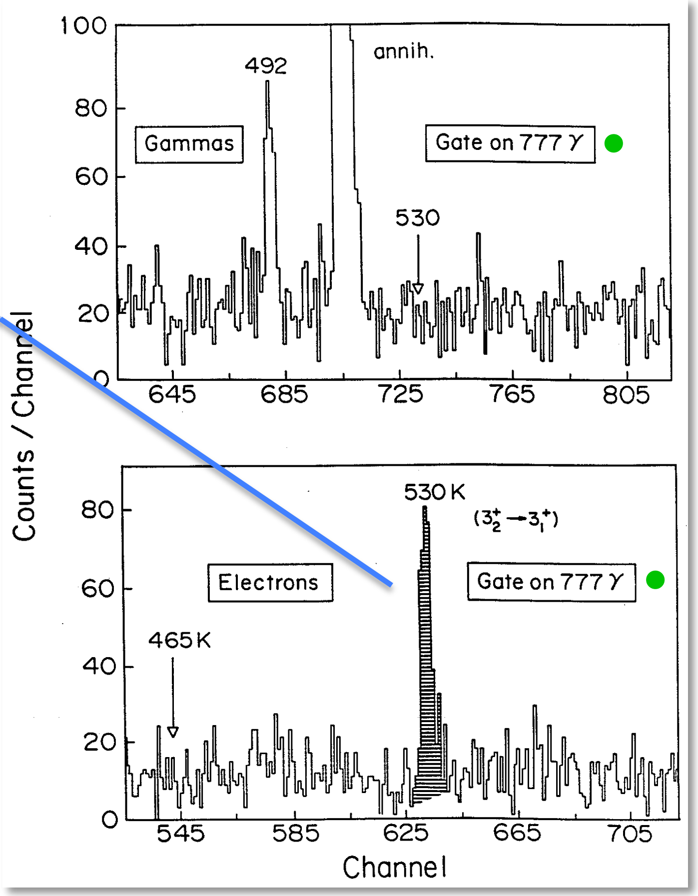
Y. Xu et al. PRL 68 3853 1992 UNISOR

Coexistence in the even-Pt isotopes:

K = 0 and K = 2 bands



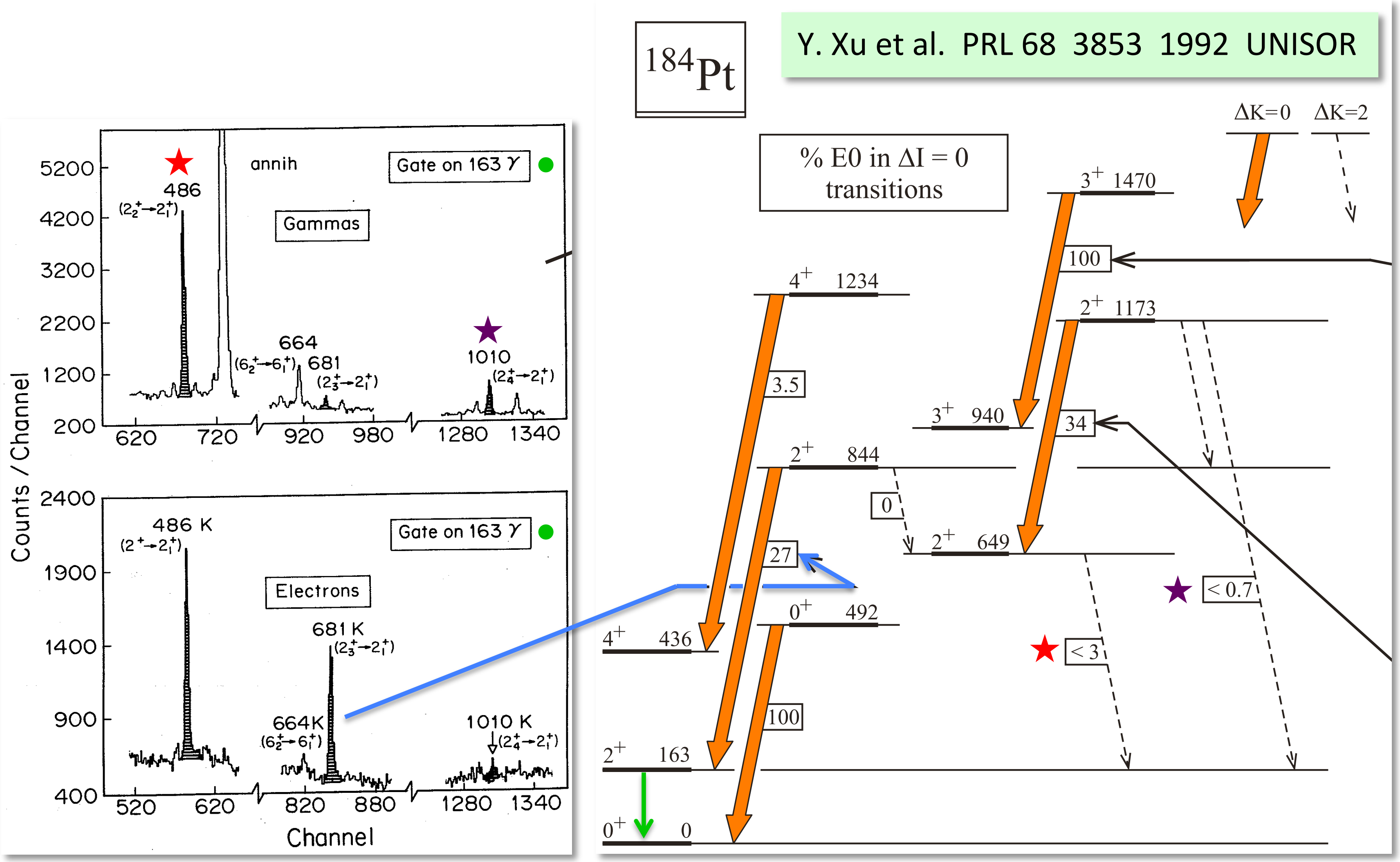
Y. Xu et al. PRL 68 3853 1992 UNISOR



E2 / M1 from low-temperature nuclear orientation on-line

Coexistence in the even-Pt isotopes:

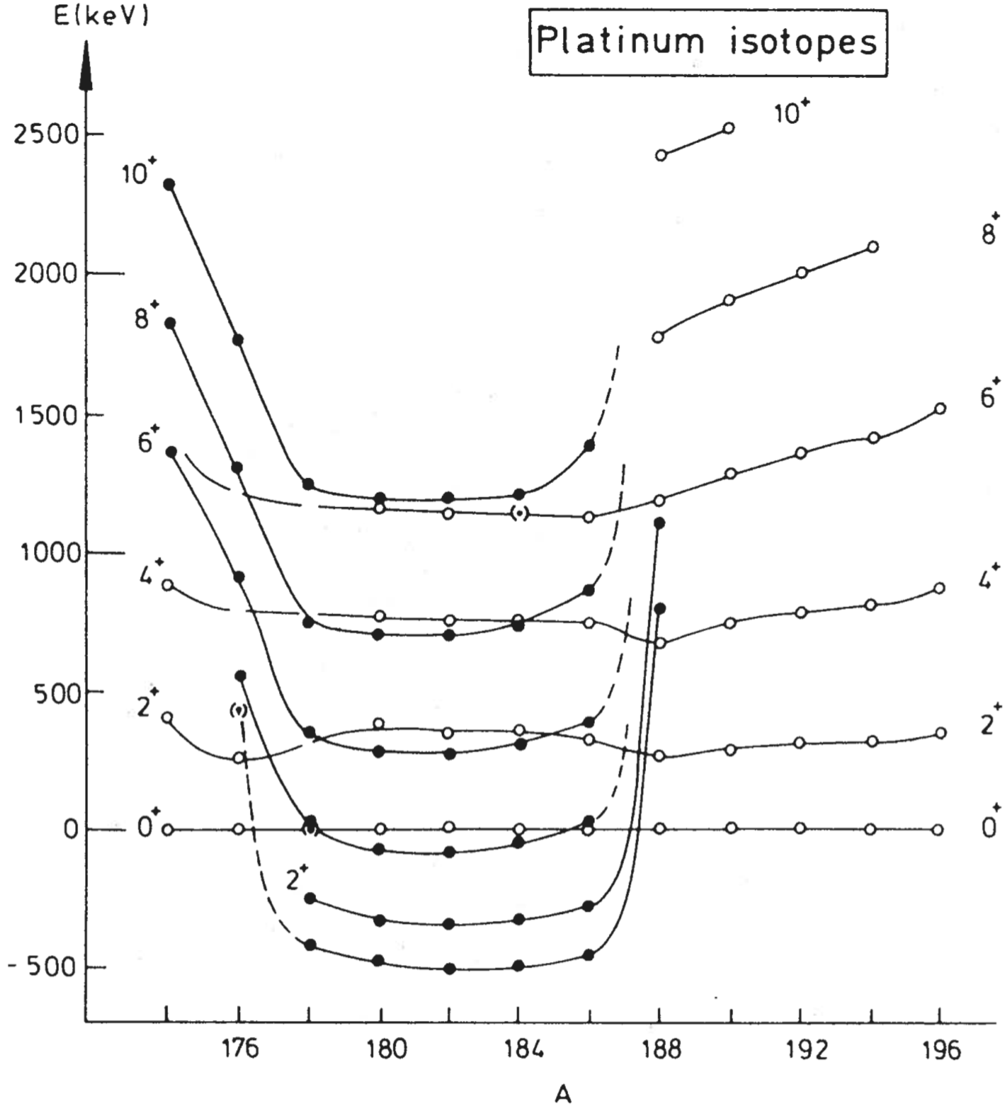
K = 0 and K = 2 bands



Y. Xu et al. PRL 68 3853 1992 UNISOR

Coexistence in the even-Pt isotopes:

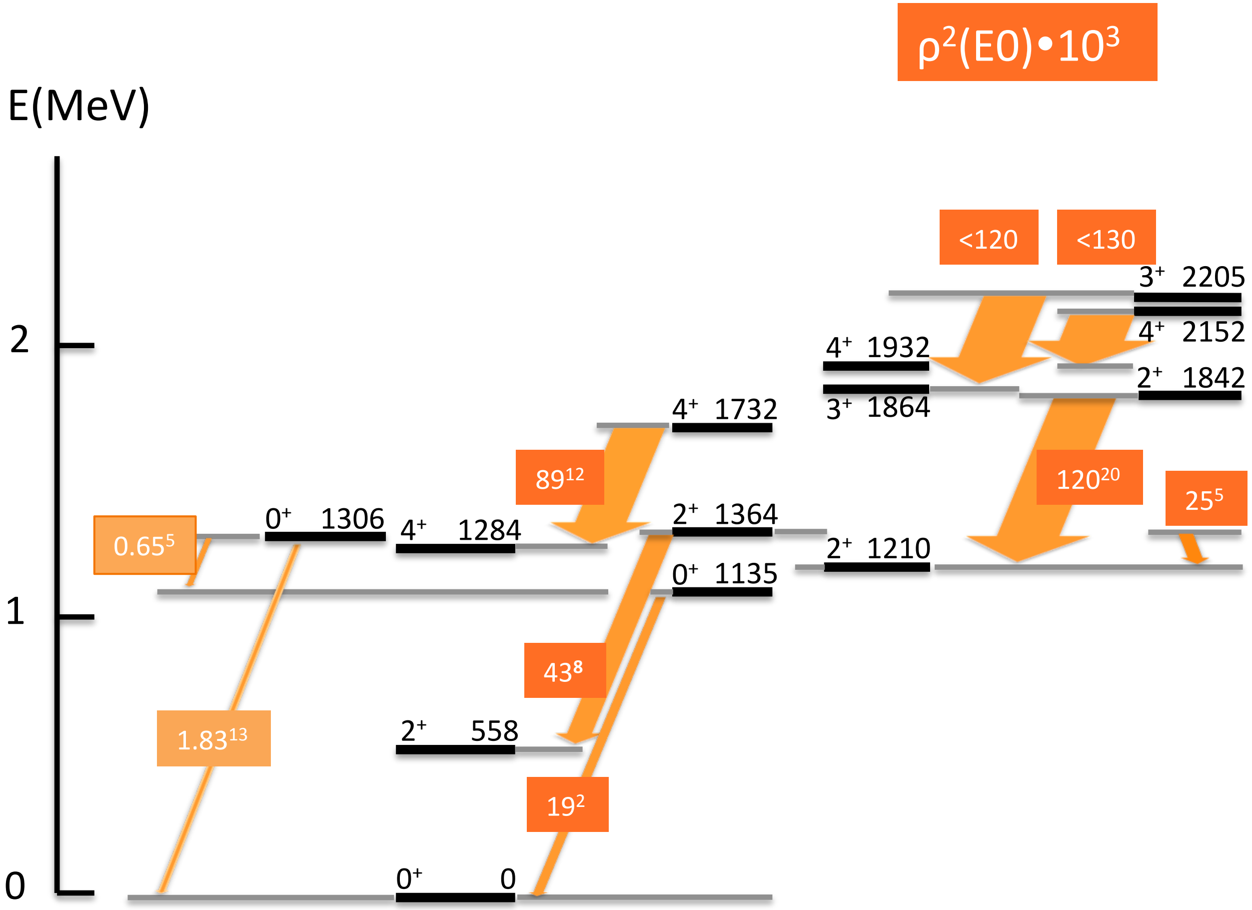
intruder ground-state interpretation



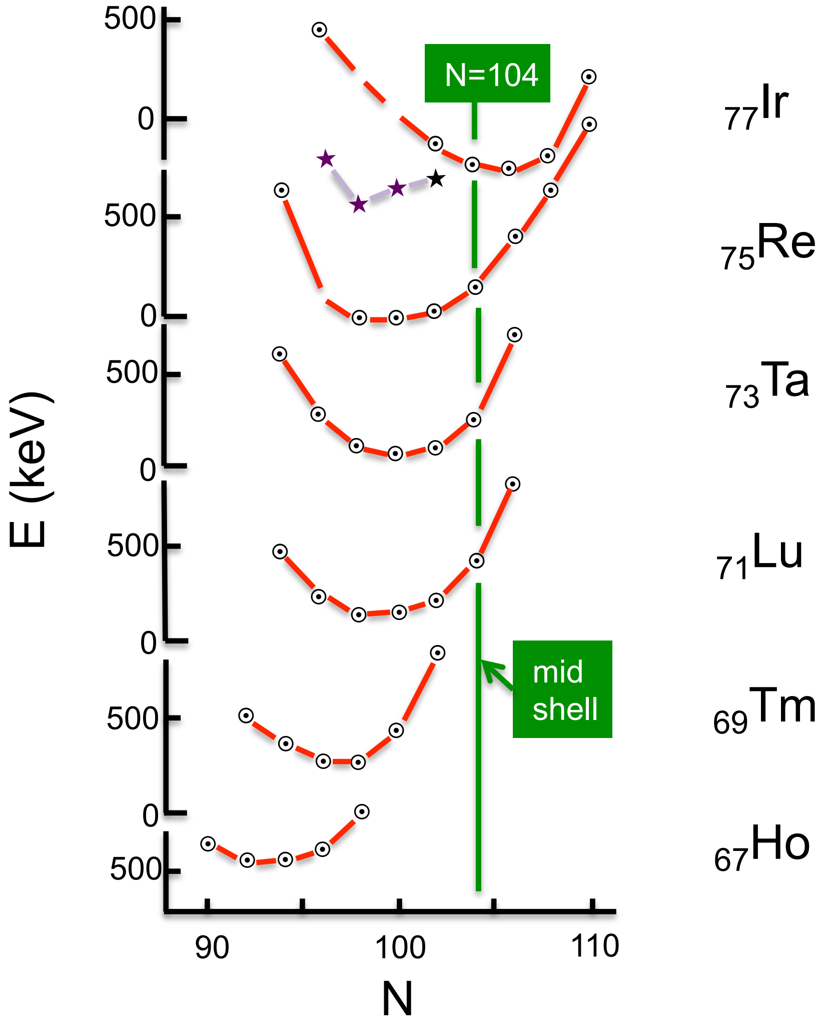
Coexistence in the even-Cd isotopes:

K = 0 and K = 2 bands

^{114}Cd



Nilsson 1/2$^-$ 541 odd-proton (h$_{9/2}$ intruder) energy systematics



Energies of the
π 9/2, 1/2$^-$ [541] state

NOTE: the minimum energy shifts by approximately ΔN = 2 for each change of ΔZ = 2

The shapes of the “parabolas” are not understood

Os 0_2^+ energies--★

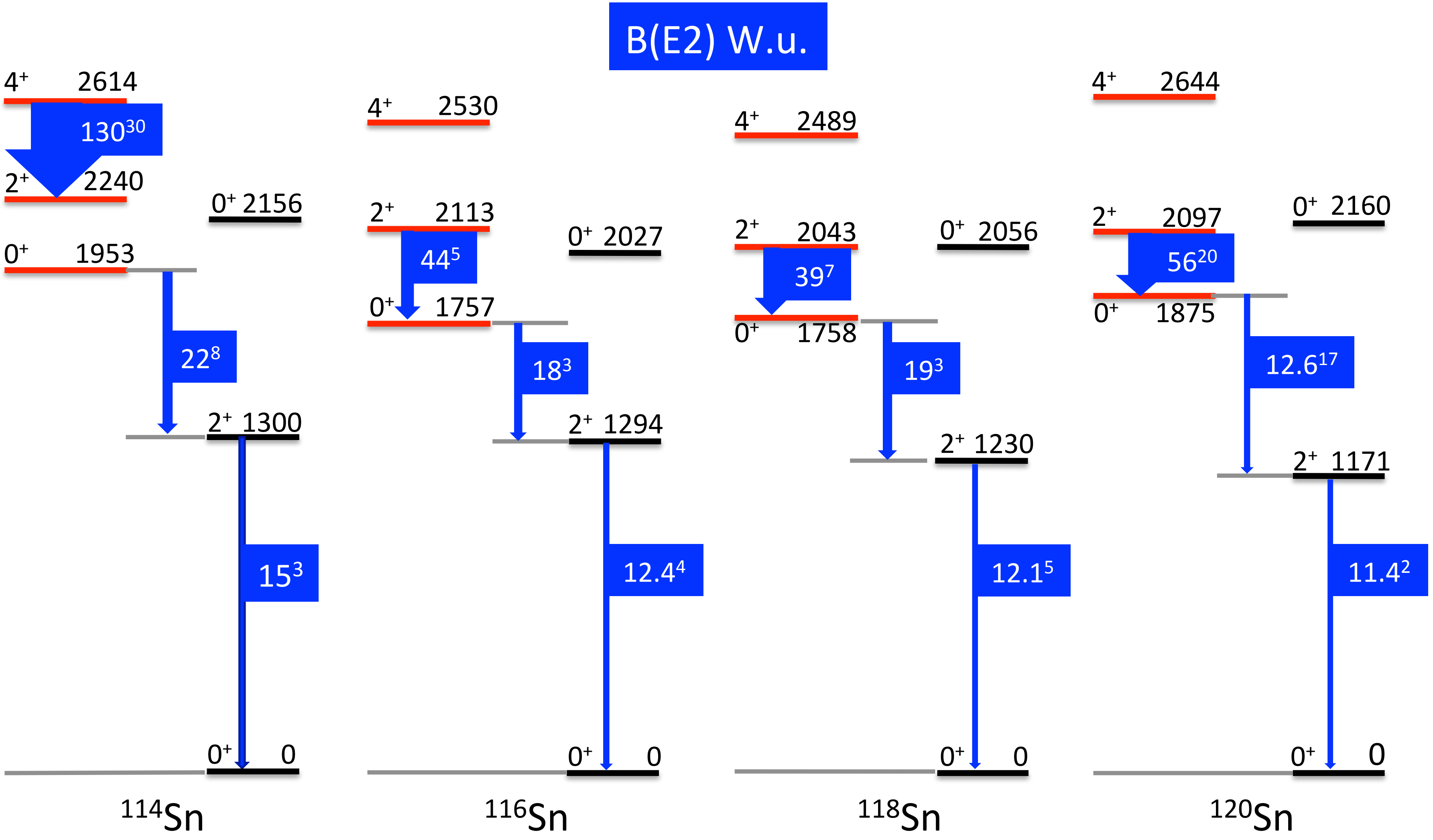
B(E2)'s and <|E2|>'s

- Reveal directly the quadrupole collectivity (deformation)

OR

- Reveal mixing of configurations with different intrinsic quadrupole moments

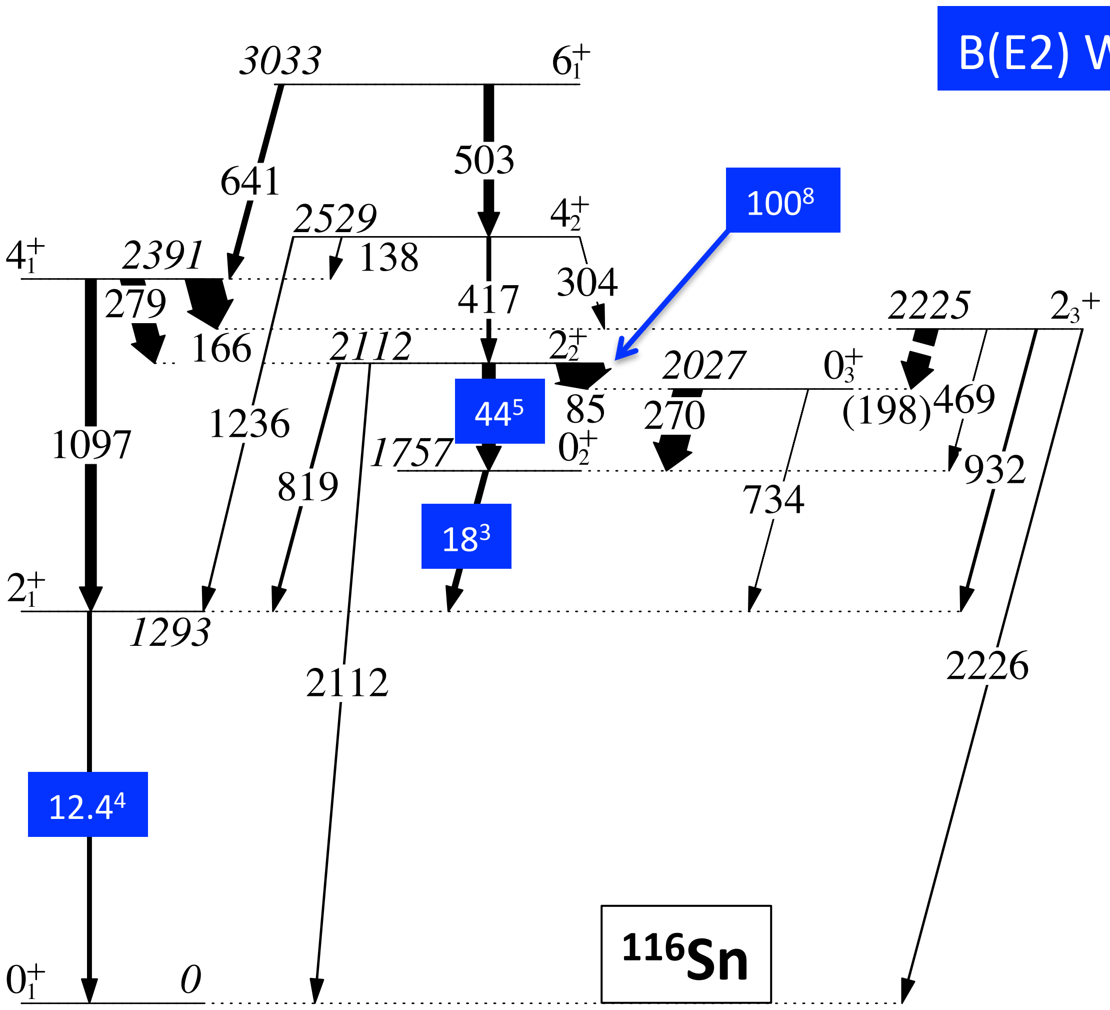
E2 transitions associated with shape coexisting states in the Sn isotopes



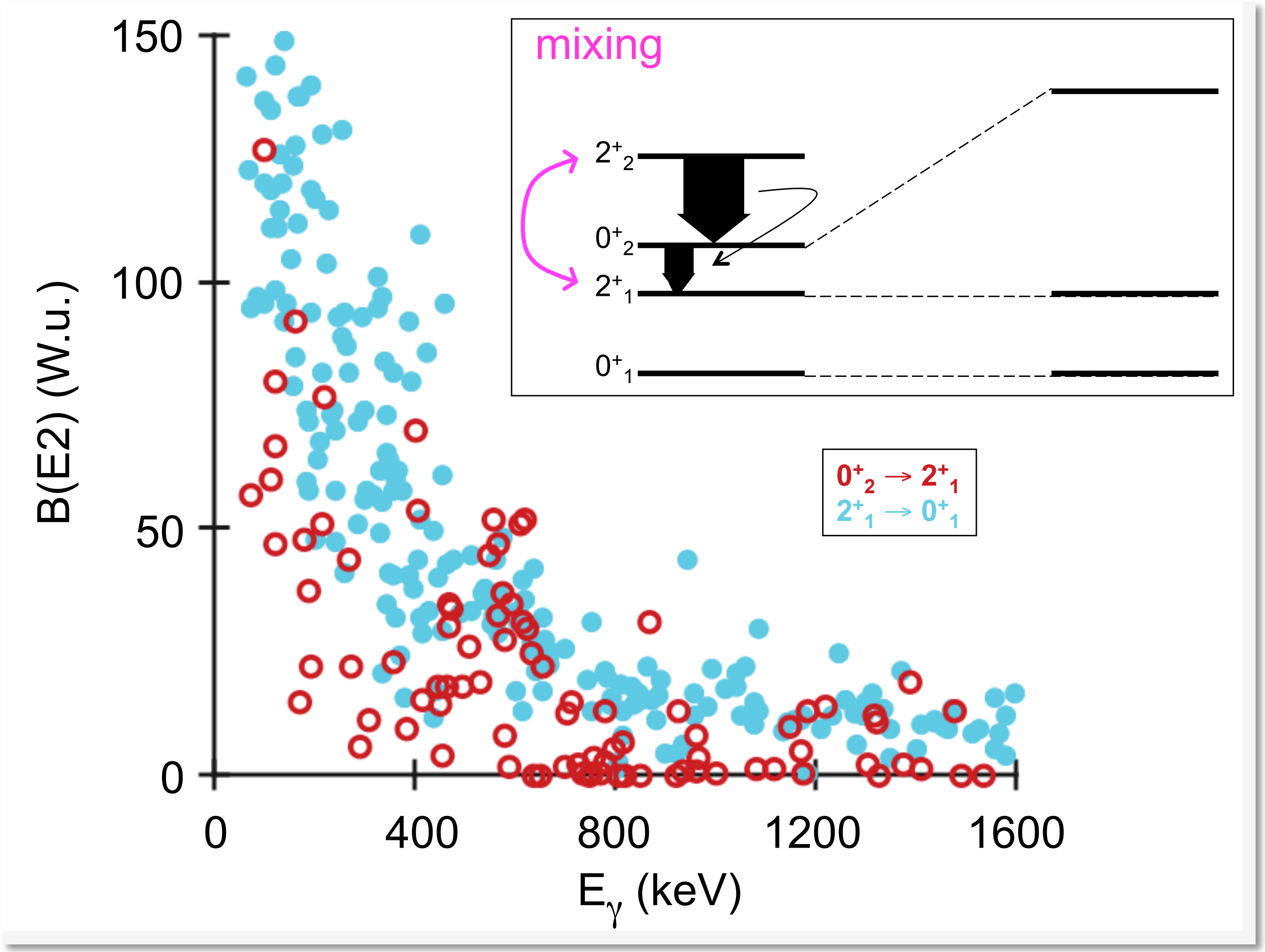
Evidence for mixing of 0_2^+ and 0_3^+ configurations in ^{116}Sn

B(E2) W.u.

JL Pore et al.
EPJ A53 27 2017

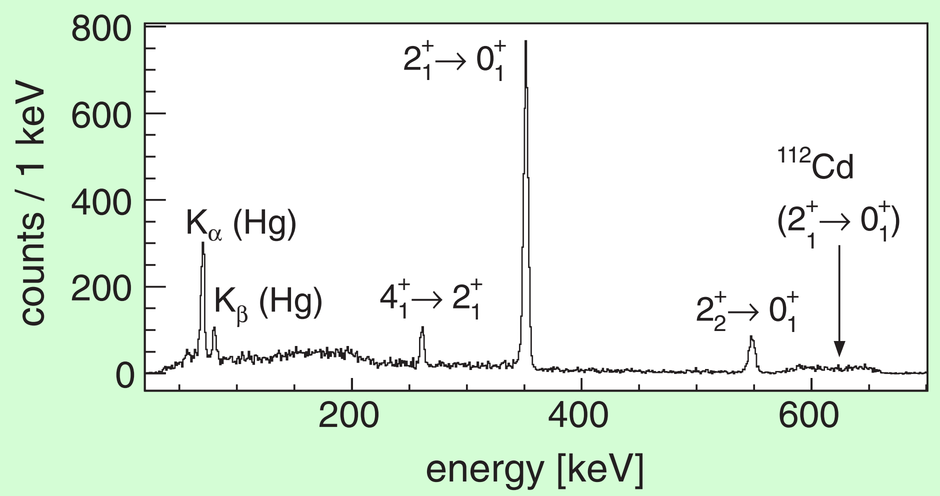


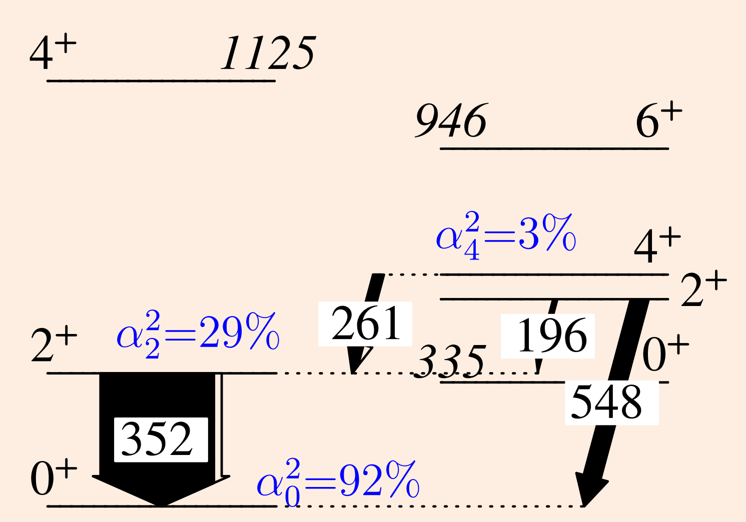
$B(E2; 0_2^+ \rightarrow 2_1^+)$ vs. $E(0_2^+) - E(2_1^+)$: coexistence and mixing yields $B(E2; 0_2^+ \rightarrow 2_1^+) \sim \alpha^2 \beta^2 (\Delta Q)^2$



Recall:
$B_{02} = 5 \times B_{20}$

Coulomb excitation of radioactive beams (^{182}Hg)





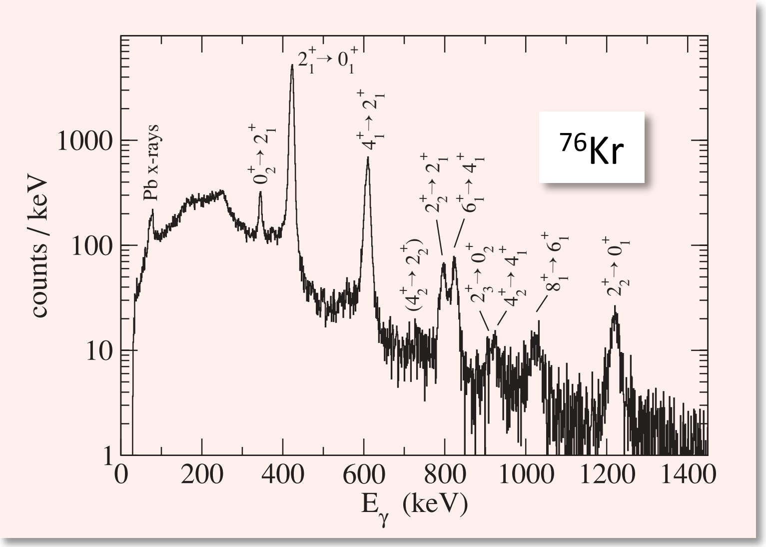
$\langle I_i \| E2 \| I_f \rangle$ (eb)	^{182}Hg
$\langle 0_1^+ \| E2 \| 2_1^+ \rangle$	$1.29^{+0.04}_{-0.03}$
$\langle 2_1^+ \| E2 \| 4_1^+ \rangle$	3.71 (6)
$\langle 0_1^+ \| E2 \| 2_2^+ \rangle$	−0.61 (3)
$\langle 0_2^+ \| E2 \| 2_1^+ \rangle$	$-2.68^{+0.15}_{-0.13}$
$\langle 0_2^+ \| E2 \| 2_2^+ \rangle$	−1.7 (2)
$\langle 2_1^+ \| E2 \| 2_2^+ \rangle$	−2.2 (4)
$\langle 2_2^+ \| E2 \| 4_1^+ \rangle$	3.1 (3)
$\langle 2_1^+ \| E2 \| 2_1^+ \rangle$	$-0.04^{+1.30}_{-1.40}$
$\langle 2_2^+ \| E2 \| 2_2^+ \rangle$	$0.8^{+1.0}_{-0.6}$

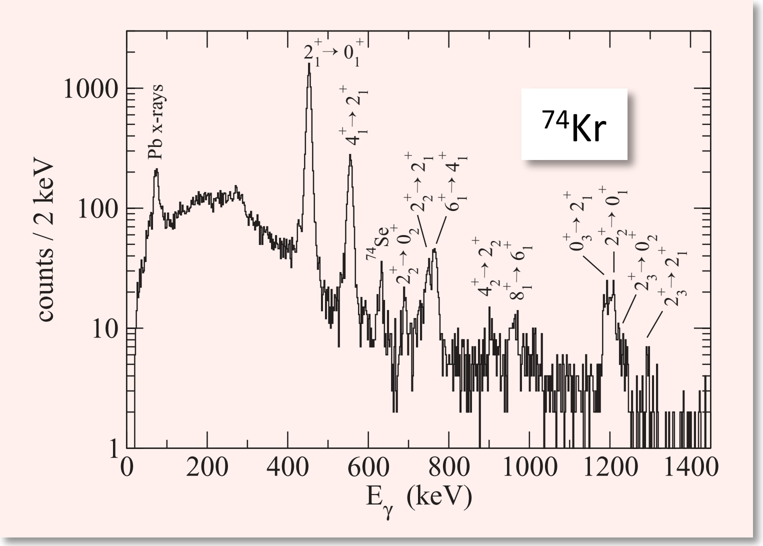
<|E2|>'s

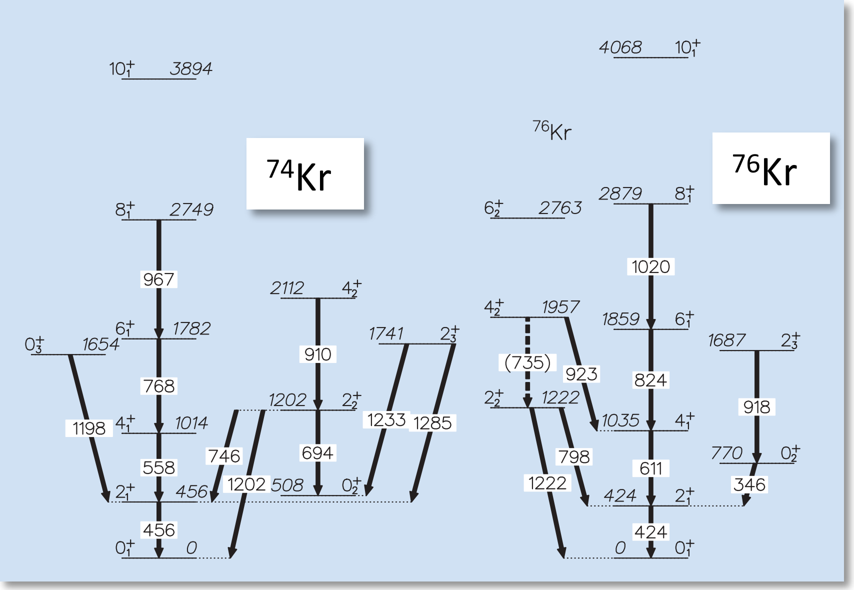
N. Bree, K. Wrzosek-Lipska et al. PRL 112 162701 2014,

Multistep Coulomb excitation of ^{74,76}Kr using radioactive beams of Kr on a ^{208}Pb target

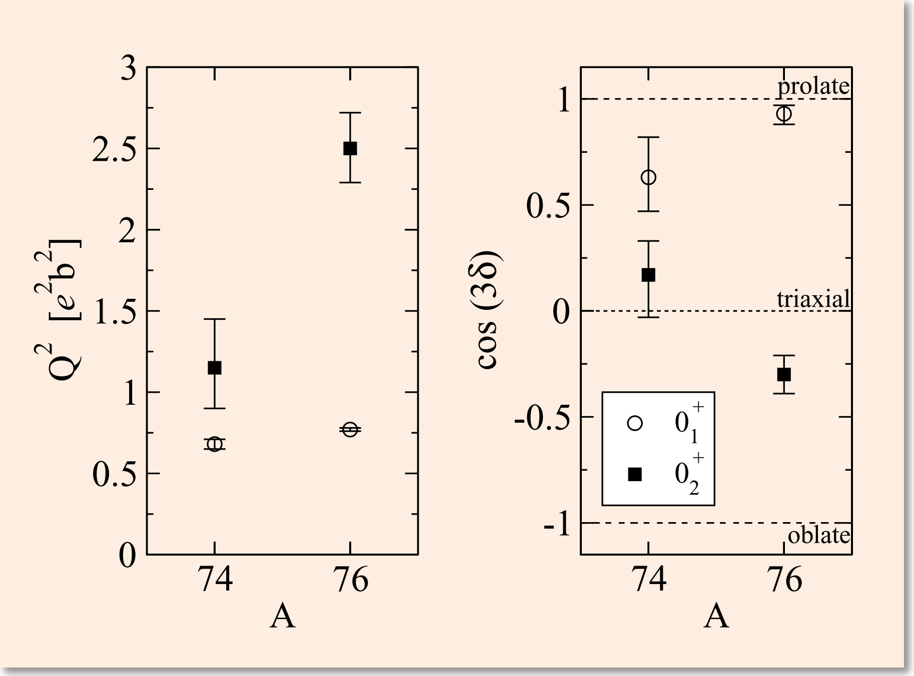
E. Clement et al., Phys. Rev. C75 054313 (2007)







Quadrupole shape invariants constructed from E2 matrix elements for ^{74,76}Kr



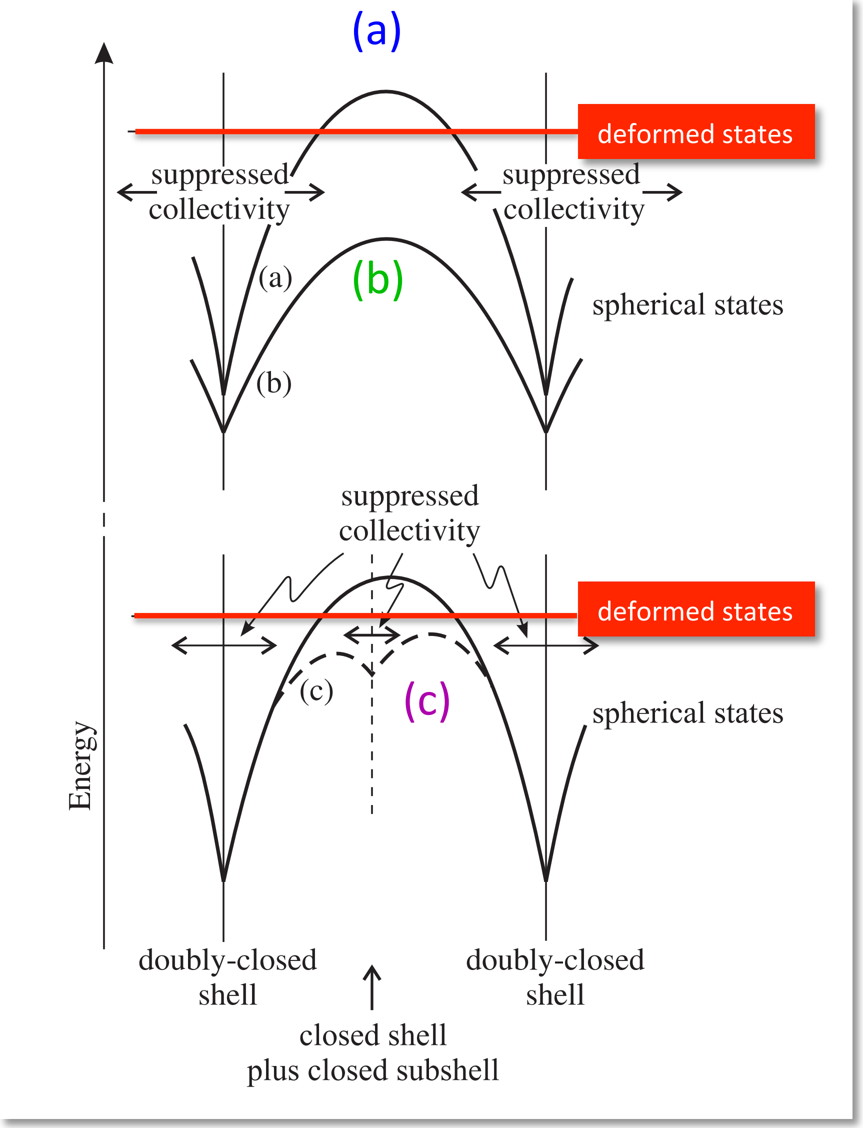
E. Clement et al., Phys. Rev. C75 054313 (2007)

$$\langle q^2\rangle \equiv \langle 0_1^+\|\hat{Q}\|2_1^+\rangle\langle 2_1^+\|\hat{Q}\|0_1^+\rangle + \langle 0_1^+\|\hat{Q}\|2_2^+\rangle\langle 2_2^+\|\hat{Q}\|0_1^+\rangle$$

for the ground state

$$\langle q^3 \cos 3\delta\rangle \equiv \sum_{r,s=1,2} \langle 0_1^+\|\hat{Q}\|2_r^+\rangle\langle 2_r^+\|\hat{Q}\|2_s^+\rangle\langle 2_s^+\|\hat{Q}\|0_1^+\rangle .$$

Shape coexistence at shell and subshell gaps: the suppression of collectivity



Shape coexistence in regions such as:

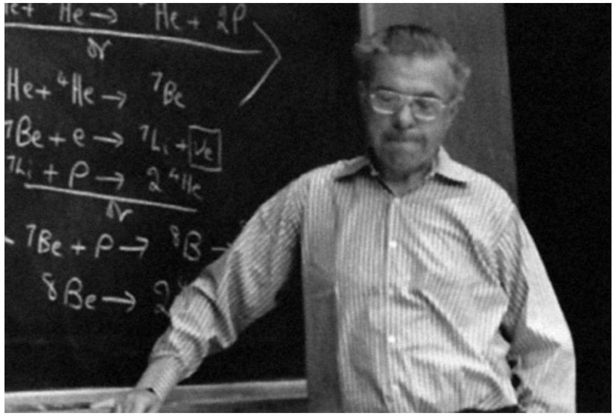
(a) ^{32}Mg

(b) $^{180-196}$Pb

(c) $^{90-98}$Zr

Figure from Heyde & Wood

The Hoyle state (7.65 MeV state in ^{12}C)

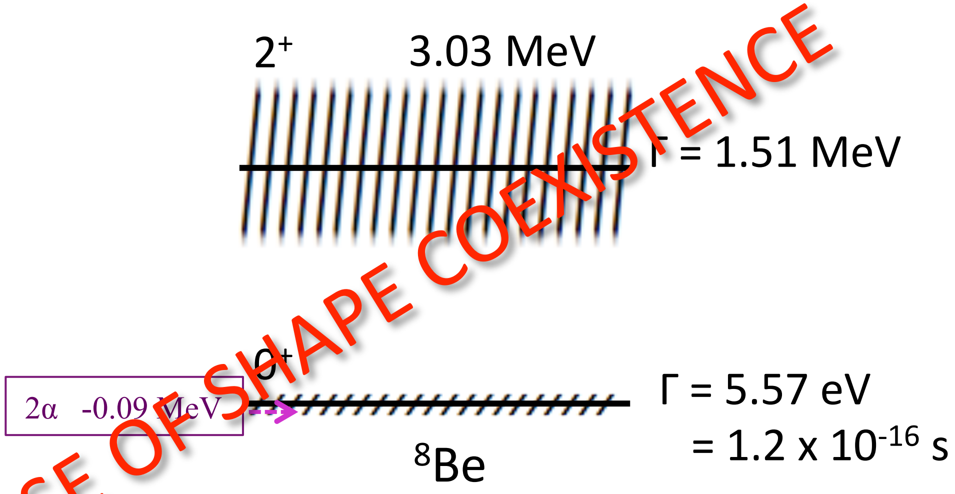


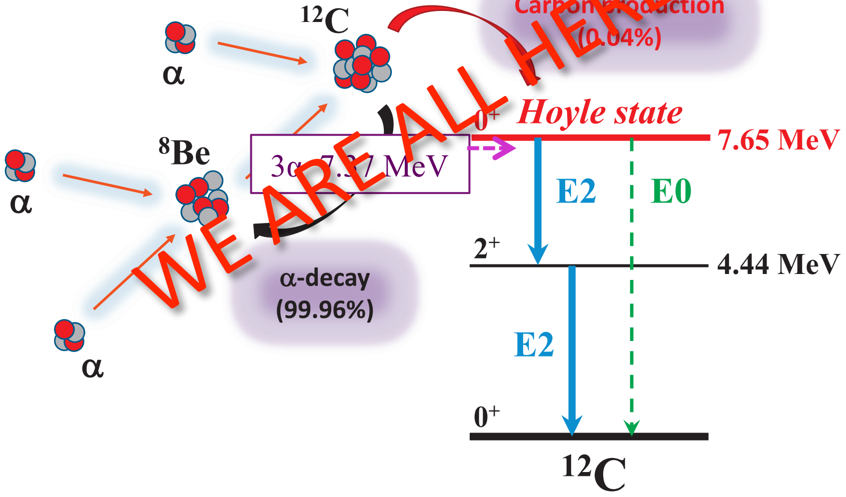
Sir Fred Hoyle (1915-2001)

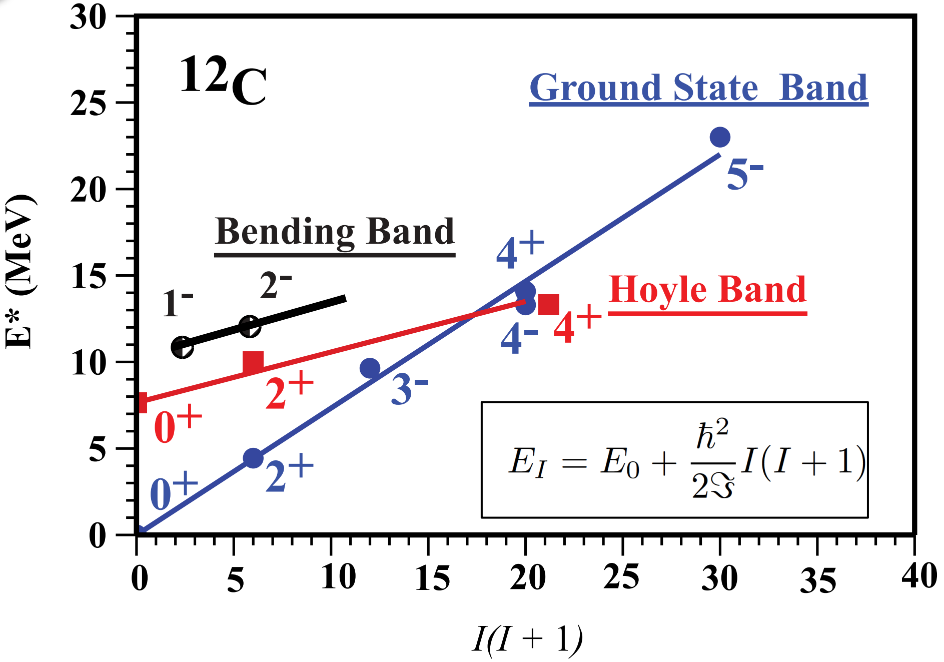
Helium fusion in stars

F. Hoyle, Astrophysical J. Suppl. Ser. **1** 121 1954

WE ARE ALL HERE BECAUSE OF SHAPE COEXISTENCE



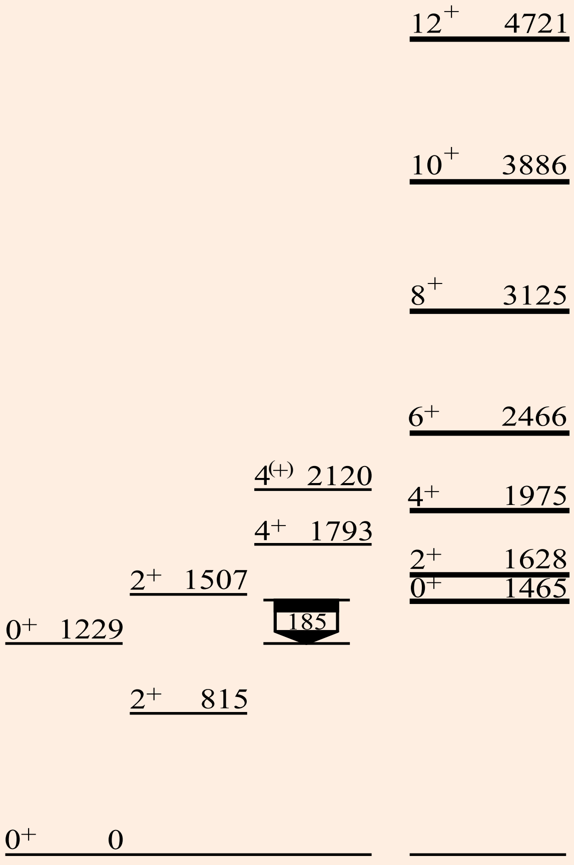




$$E_I = E_0 + \frac{\hbar^2}{2\Im} I(I+1)$$

Shape coexistence and subshells:
^{96}Sr and ^{98}Zr

Figure: Heyde & Wood

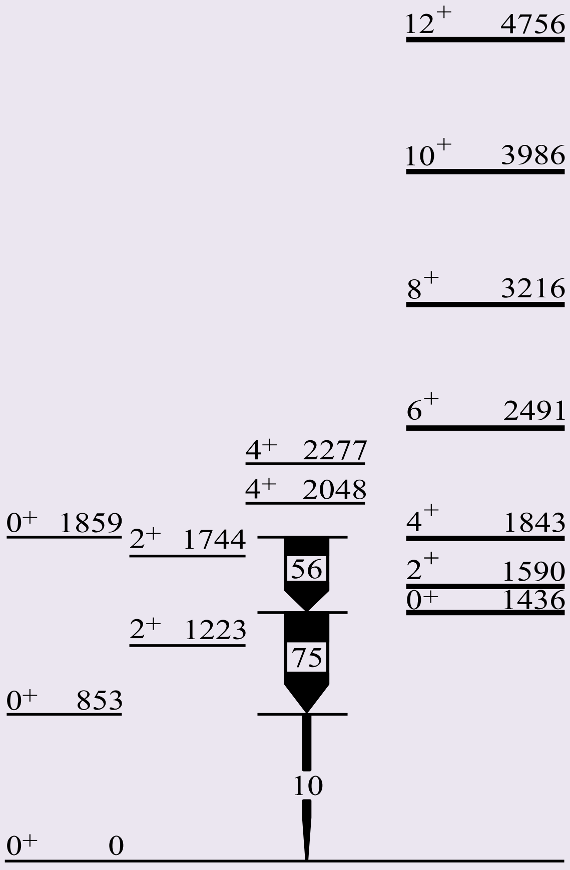


^{96}Sr$_{58}$

E0 transitions
$\rho^2(E0) \cdot 10^3$ values

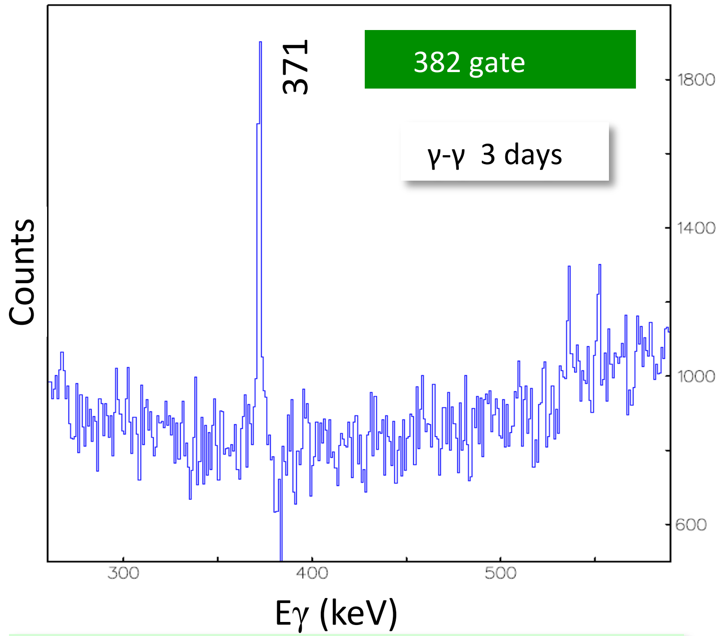
G. Lhersonneau et al.
PR C49 1379 1994

C.Y. Wu et al.
PR C70 064312 2004



^{98}Zr$_{58}$

^{94}Zr from two structural perspectives: vibrator OR coexisting seniority and deformed structures



gamma branches from the 2^+ 2p-2h state to:

0_1^+ state 58

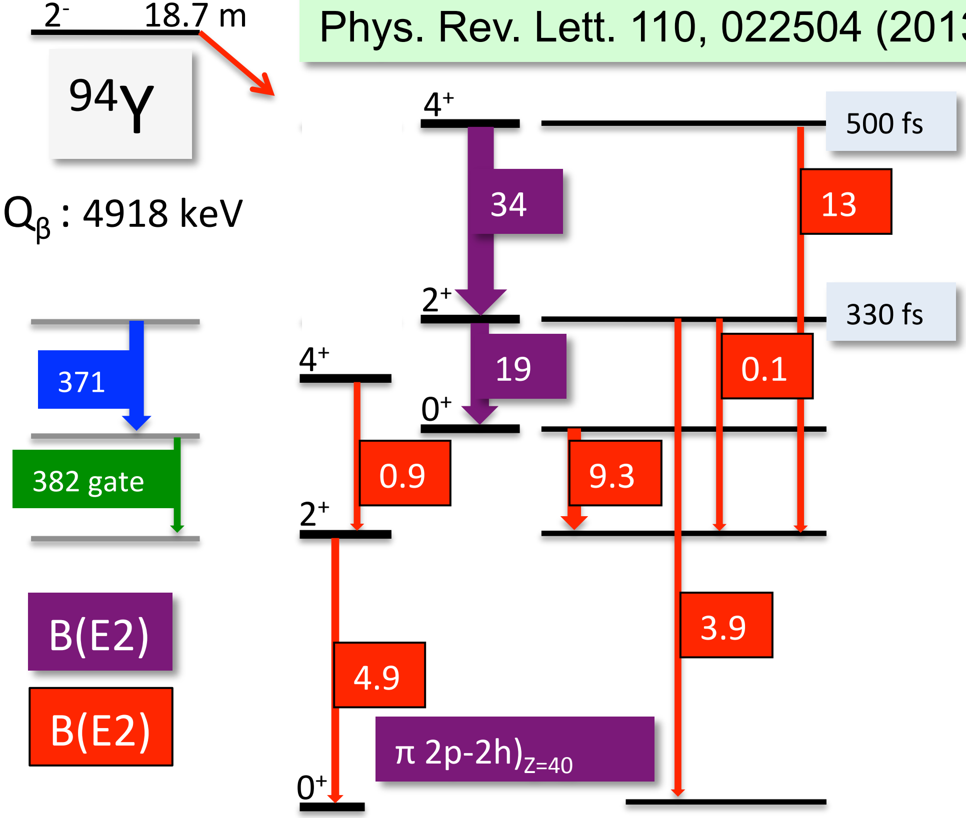
2_1^+ state 42 (δ = 0.02: 0.04% E2)

0^+ 2p-2h state 0.15

E. Elhami et al., PR C78, 054303 (2008)

8Pi expt.: S.W. Yates et al.,S1286 (5 days)

A. Chakraborty et al.,
Phys. Rev. Lett. 110, 022504 (2013)



^{94}Zr

$\nu\, d_{5/2}$
seniority

lifetime data from (n,n’ γ)
Doppler shifts–
U. Kentucky

0^+ states in ^{68}Ni:

0_2^+ ν=0 state? 0_3^+ π 2p-2h deformed state?

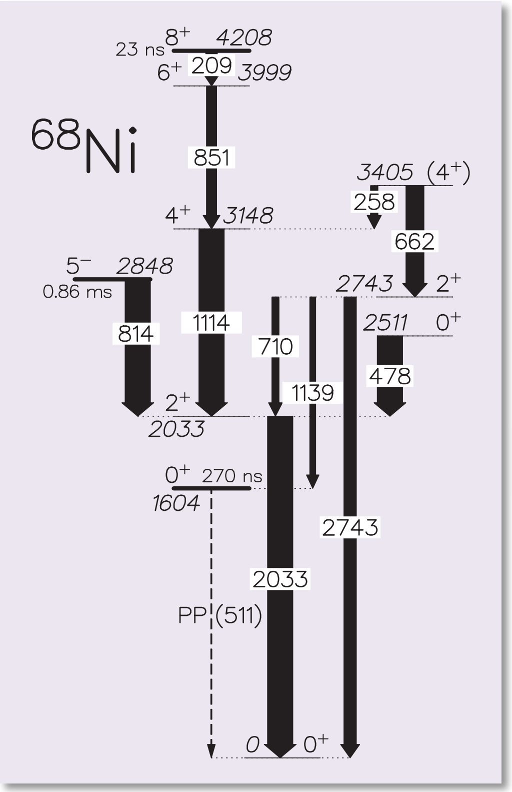
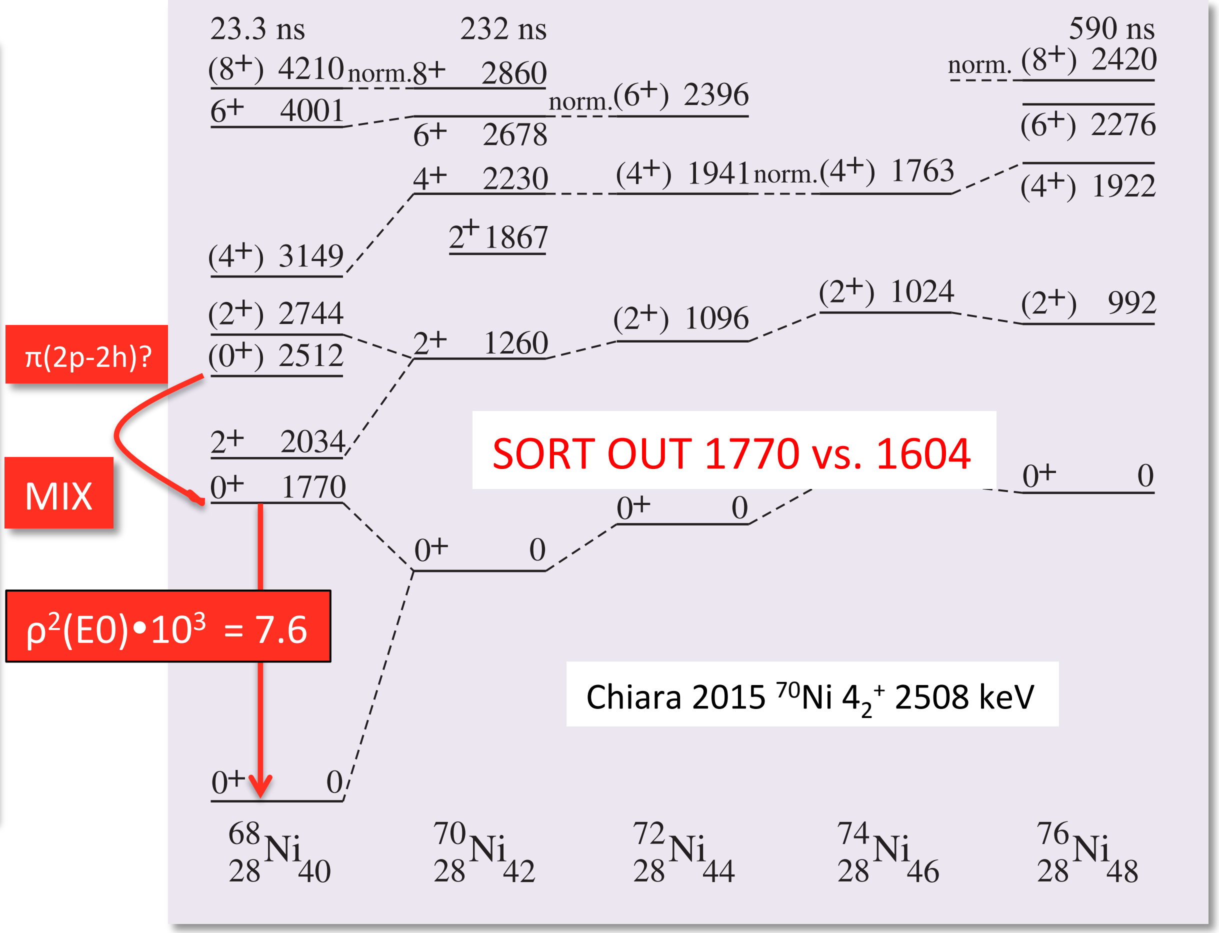


Figure taken from F. Recchia et al., PR C88 041302 2013



π(2p-2h)?

MIX

$\rho^2(E0) \bullet 10^3 = 7.6$

SORT OUT 1770 vs. 1604

Chiara 2015 ^{70}Ni 4_2^+ 2508 keV

S. Suchyta et al., PR C89 021301 2014

See also: D. Pauwels et al., PR C82 027304 2010

$0^+ \rightarrow 0^+$ decays are pure E0: no γ's (^{190}Hg)

M.O. Kortelahti et al. PR C43 484 1991

1279 keV pure E0 evidence

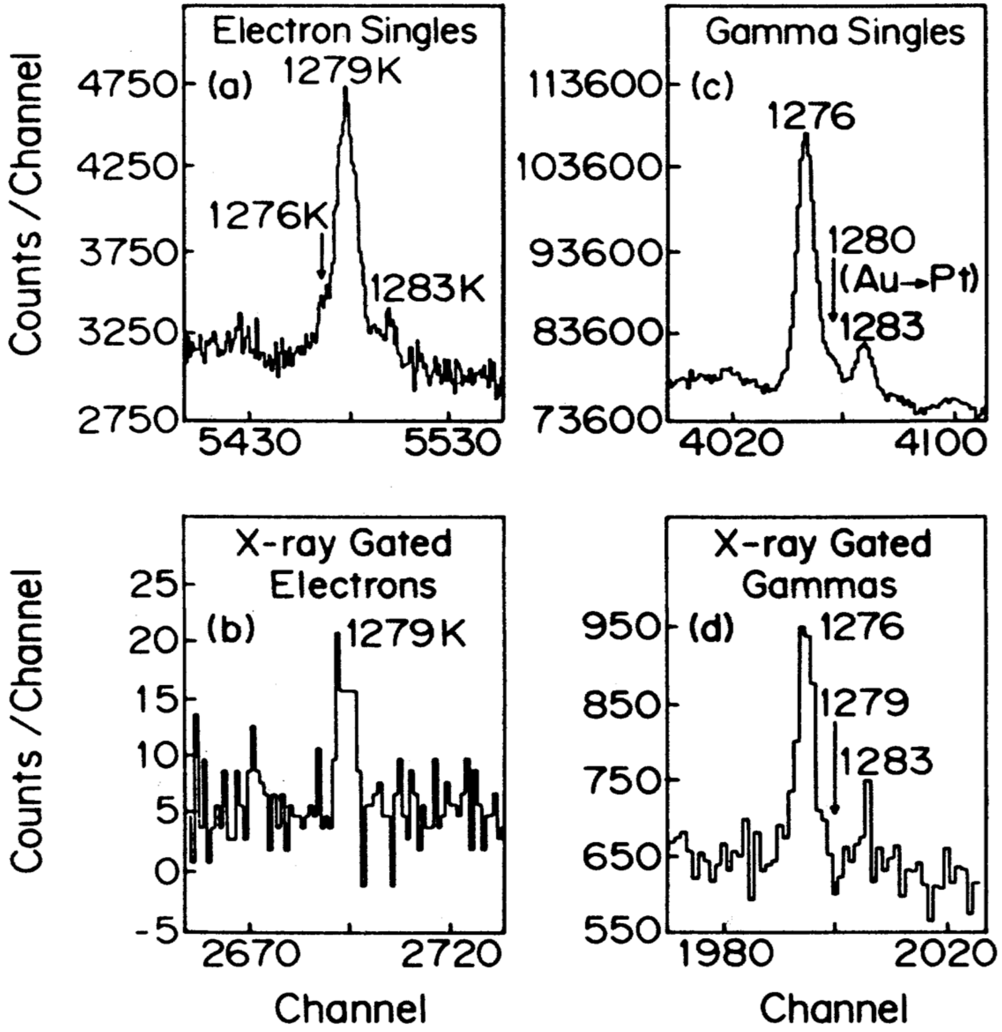
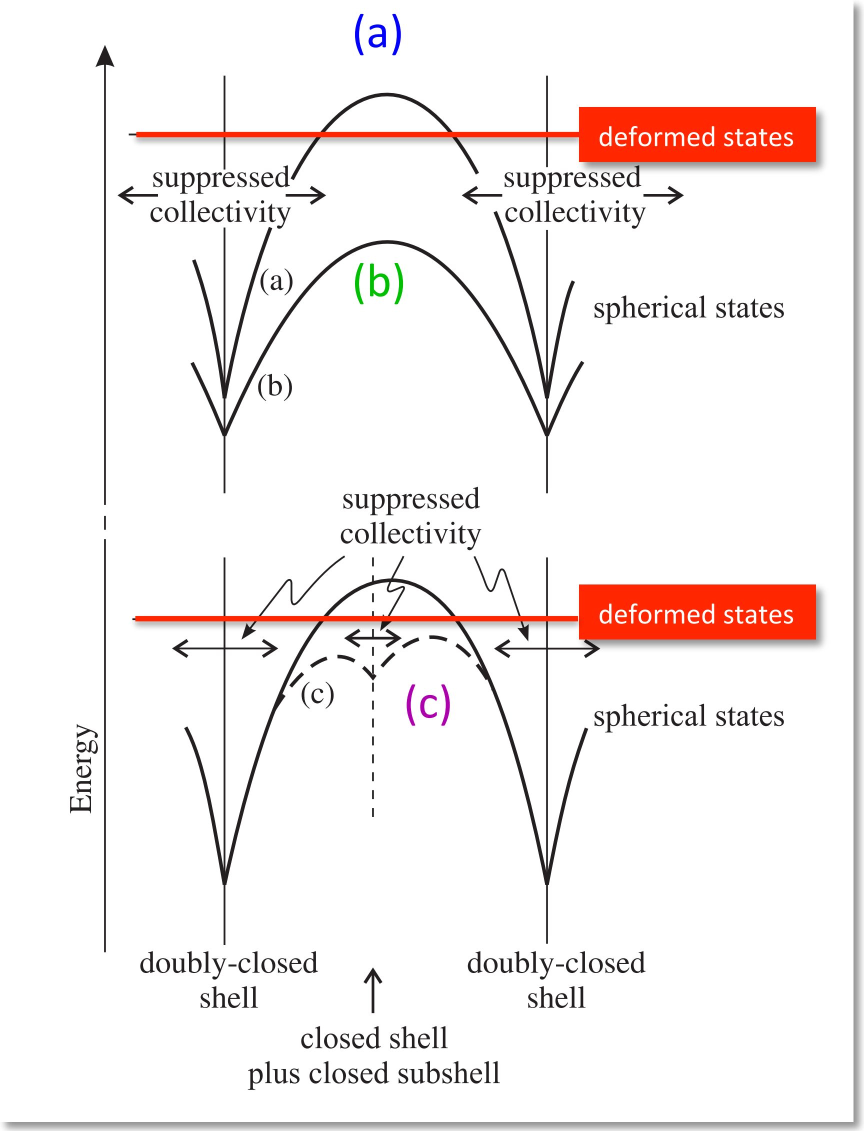


Figure 2.68a,b



Shape coexistence in
regions such as:

(a) ^{32}Mg

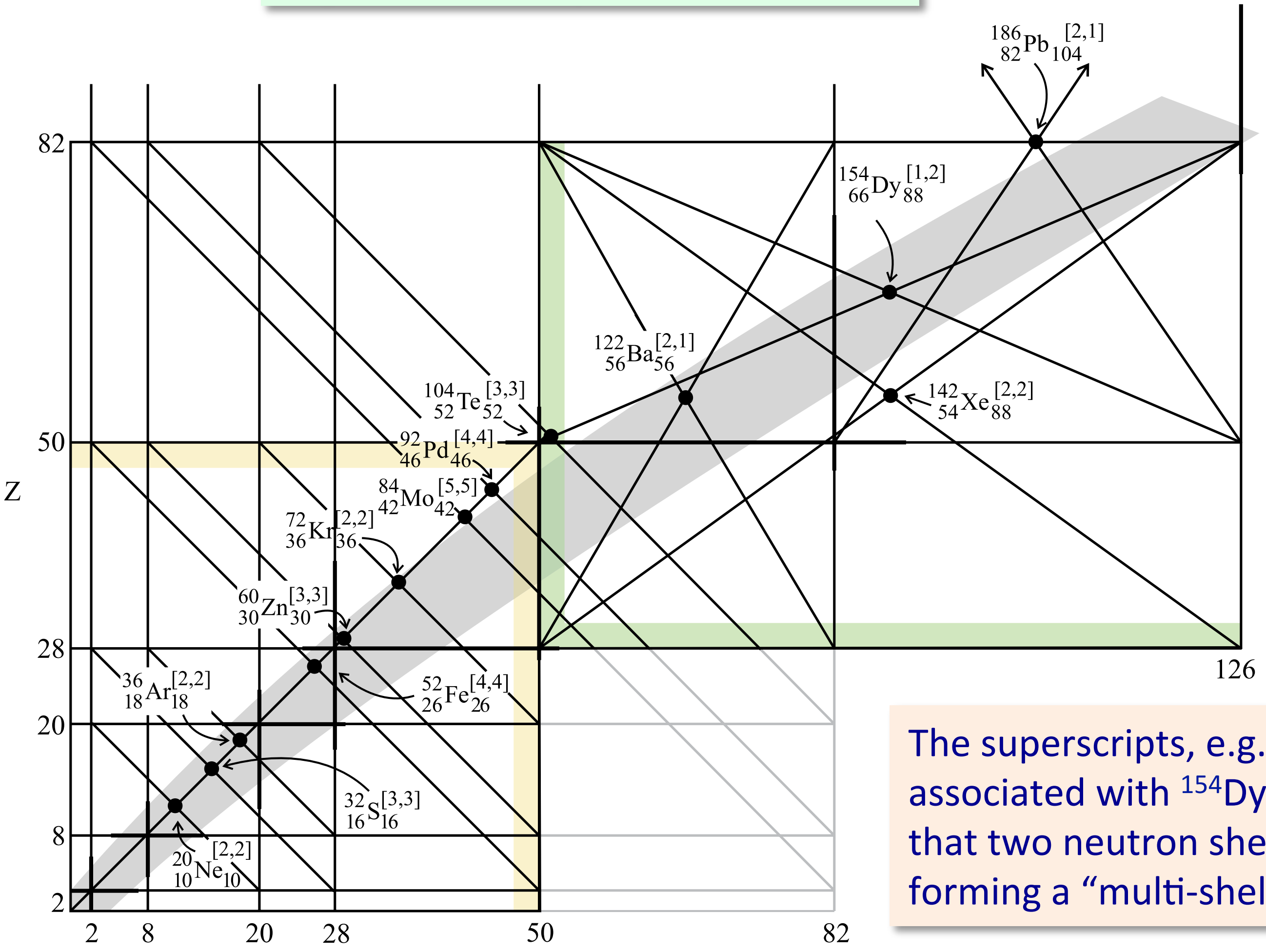
(b) $^{180-196}$Pb

(c) $^{90-98}$Zr

Figure from Heyde & Wood

Figure 2.67

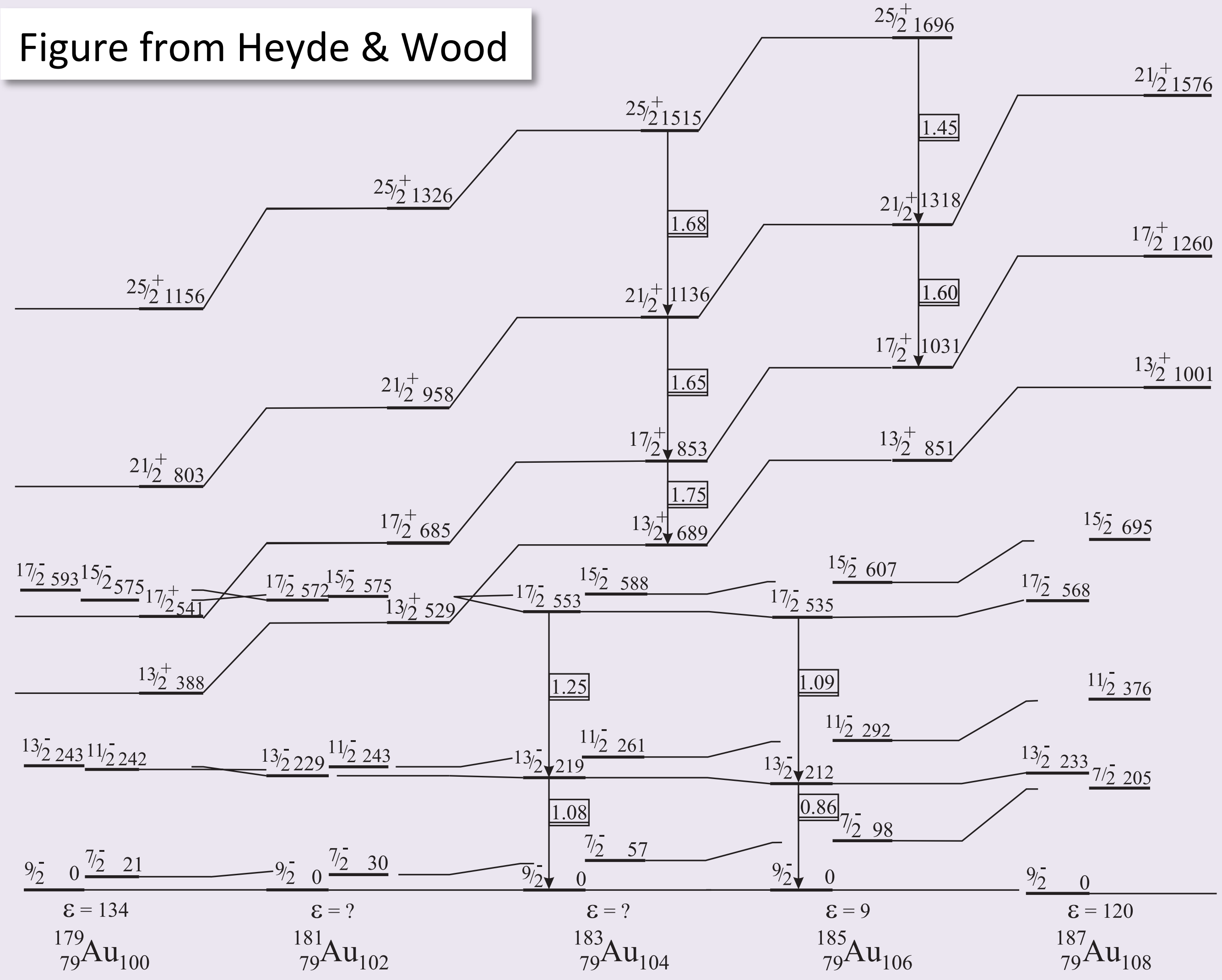
Figure from Heyde and Wood



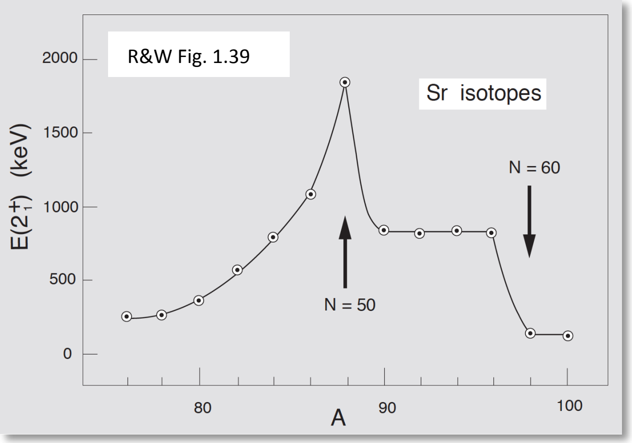
The superscripts, e.g., “[1,2]” associated with ^{154}Dy, designate that two neutron shells are forming a “multi-shell”

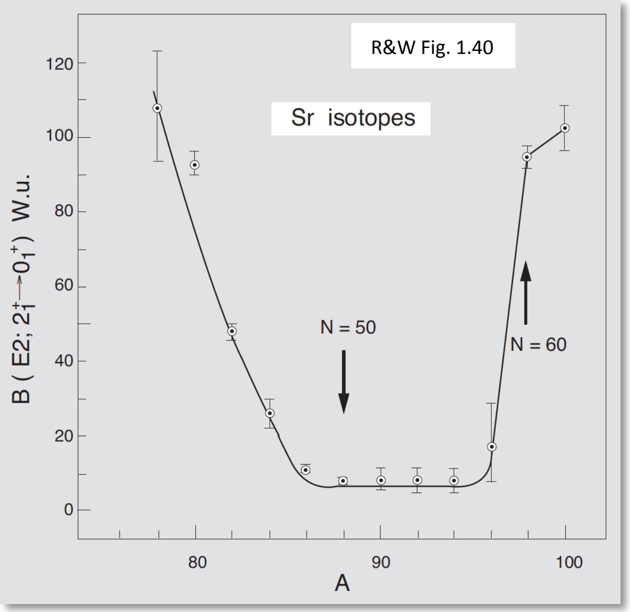
Coexistence in odd-Au isotopes: multiple parabolas

Figure from Heyde & Wood



2_1^+ state properties are a strong signature of shell and deformed structures





Energies of 2_1^+ states determined by:

gamma-ray spectroscopy following β decay

problem—β-decaying parent is further from stability and yield will be (much) lower than nucleus of interest

gamma-ray spectroscopy following Coulomb excitation

Reduced E2 transition rates, B(E2) from 2_1^+ states determined by:

lifetime measurements using fast β-γ timing following β decay

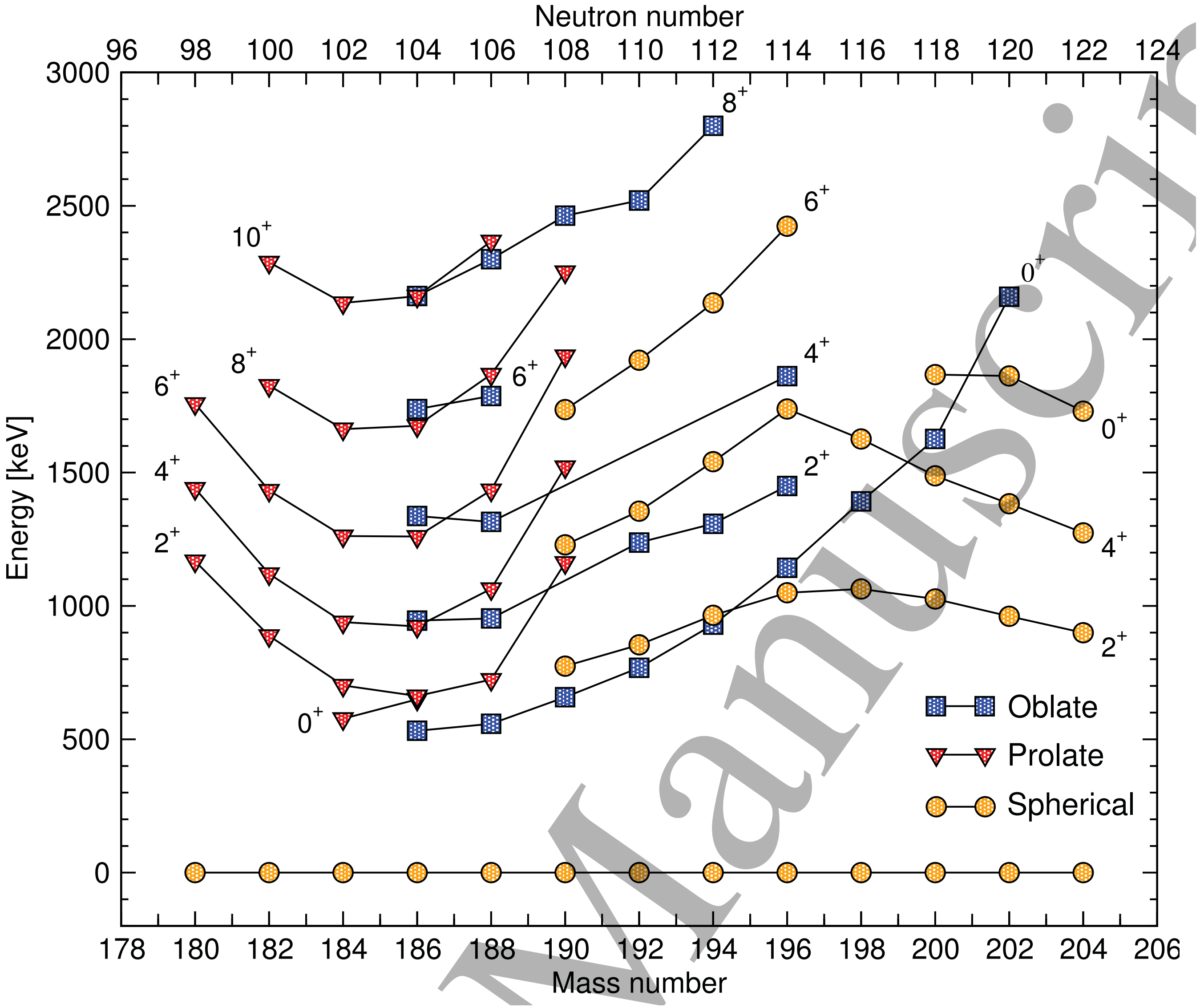
problem--see above

gamma-ray yields following Coulomb excitation

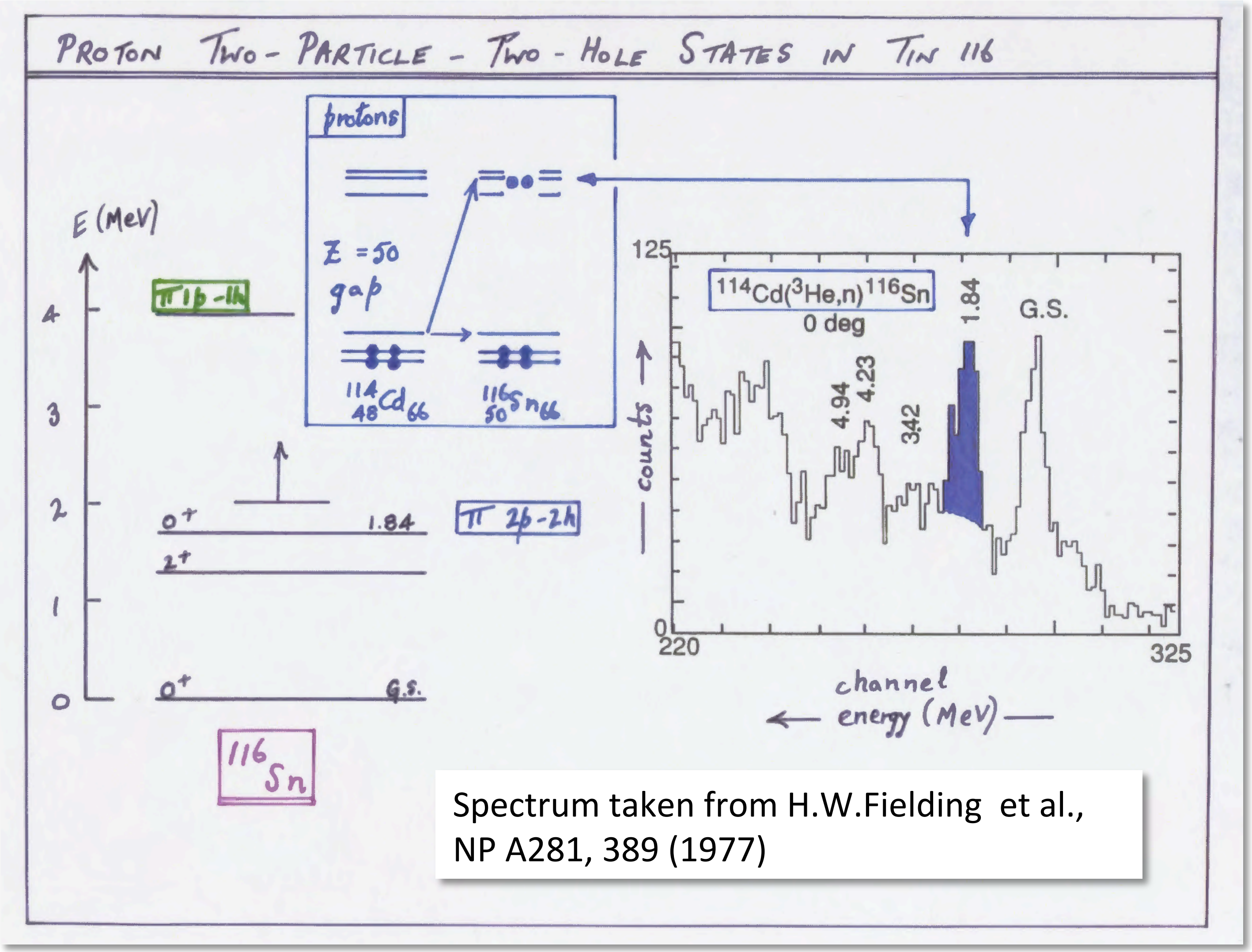
Coexistence in even-Pb isotopes:

multiple parabolas and spherical (seniority) structure

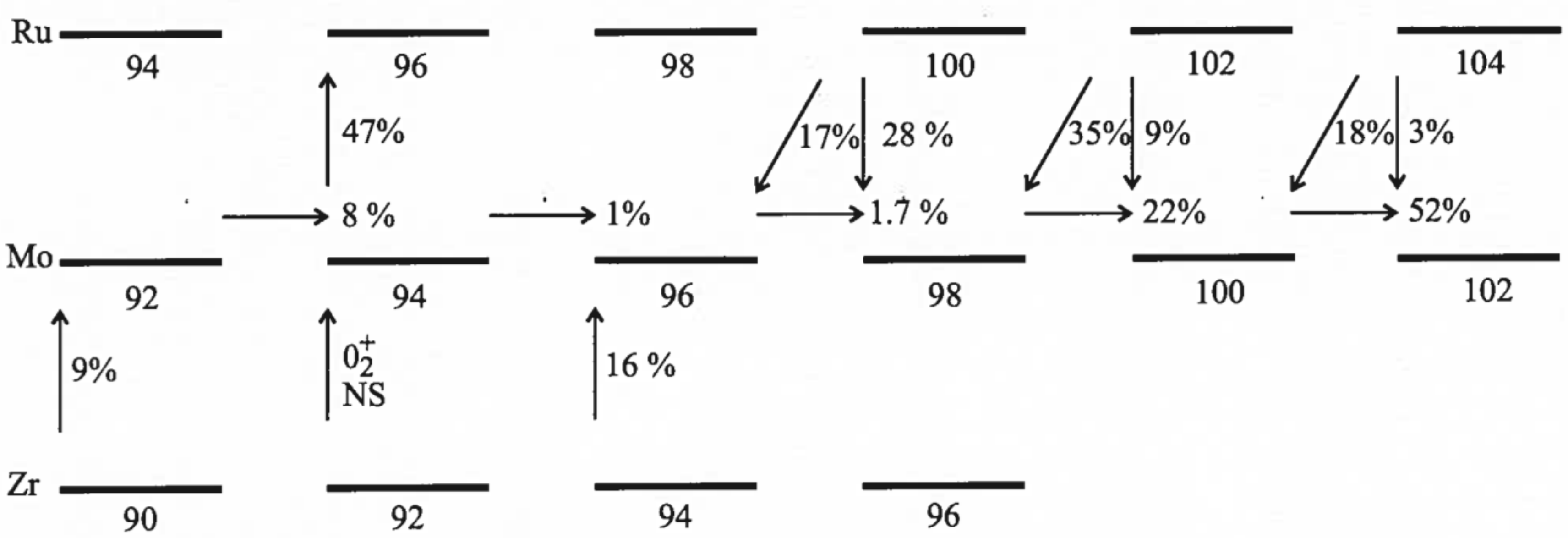
Pakkarinen et al., preprint 2017

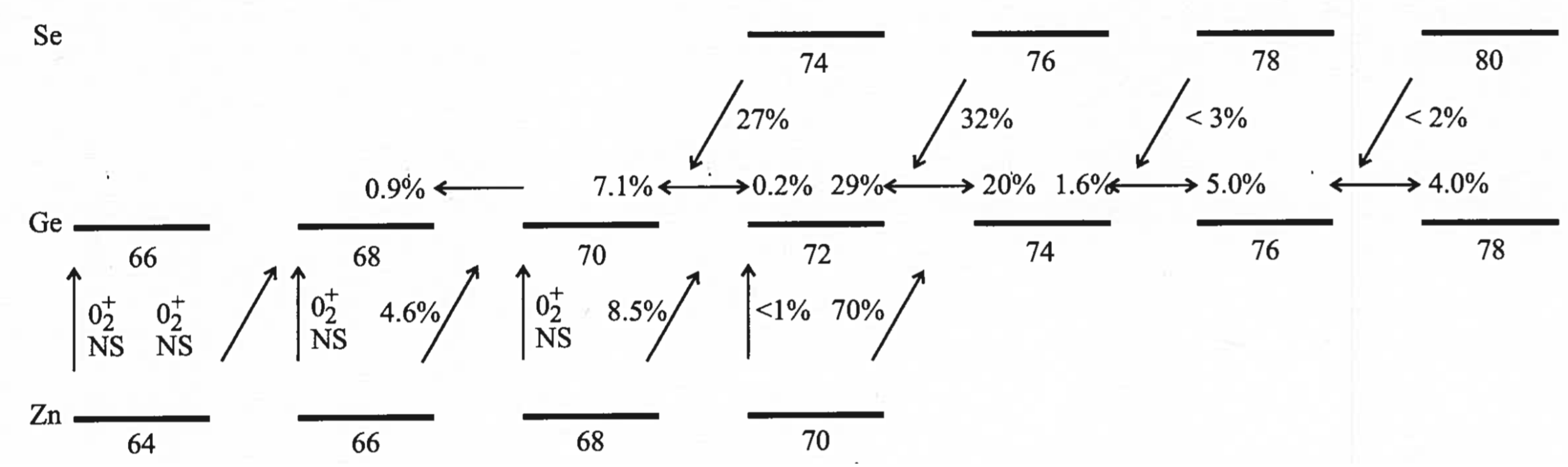


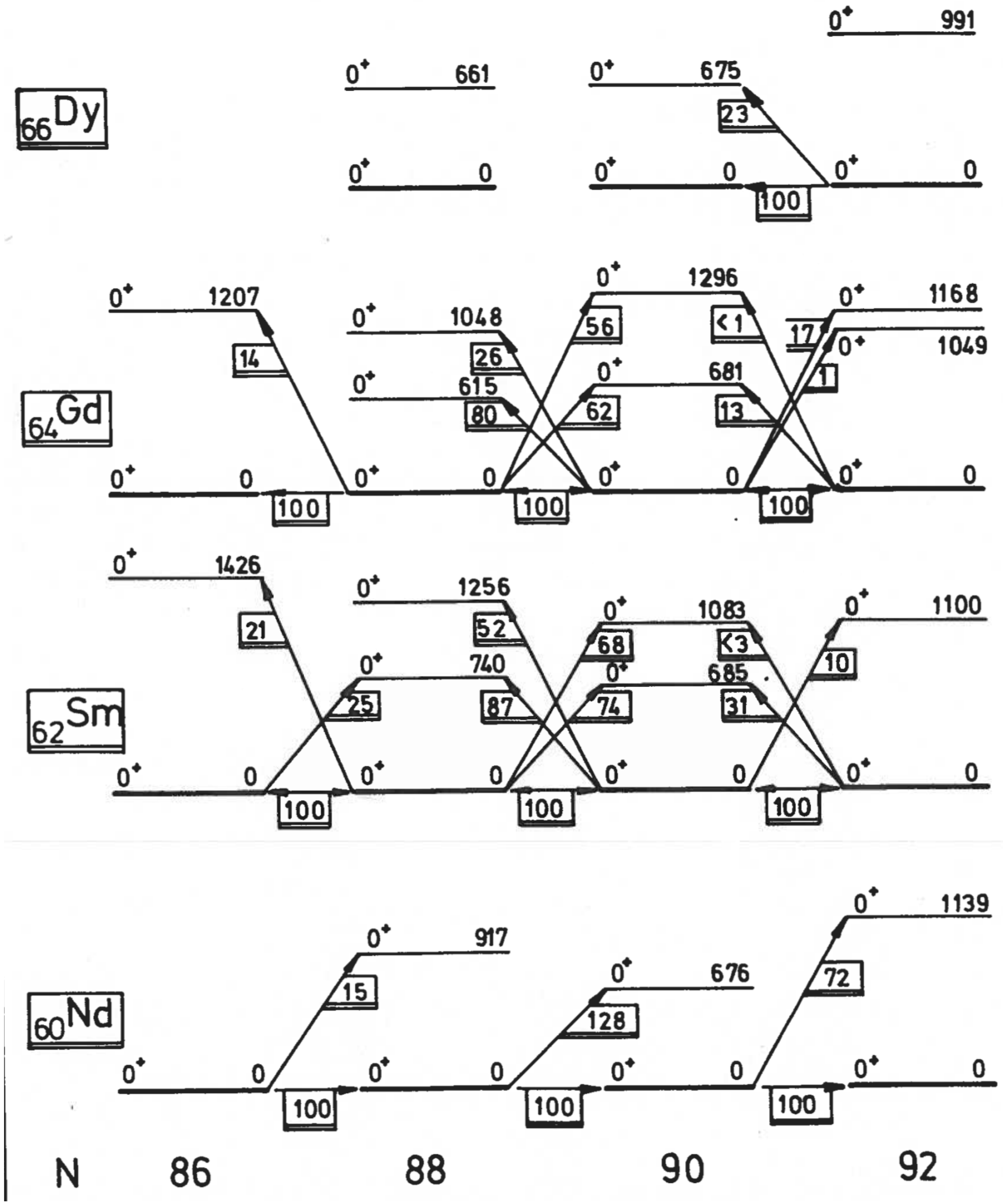
The nature of the shape coexisting state in ^{116}Sn revealed by (^{3}He,n) transfer reaction spectroscopy



Spectrum taken from H.W.Fielding et al.,
NP A281, 389 (1977)

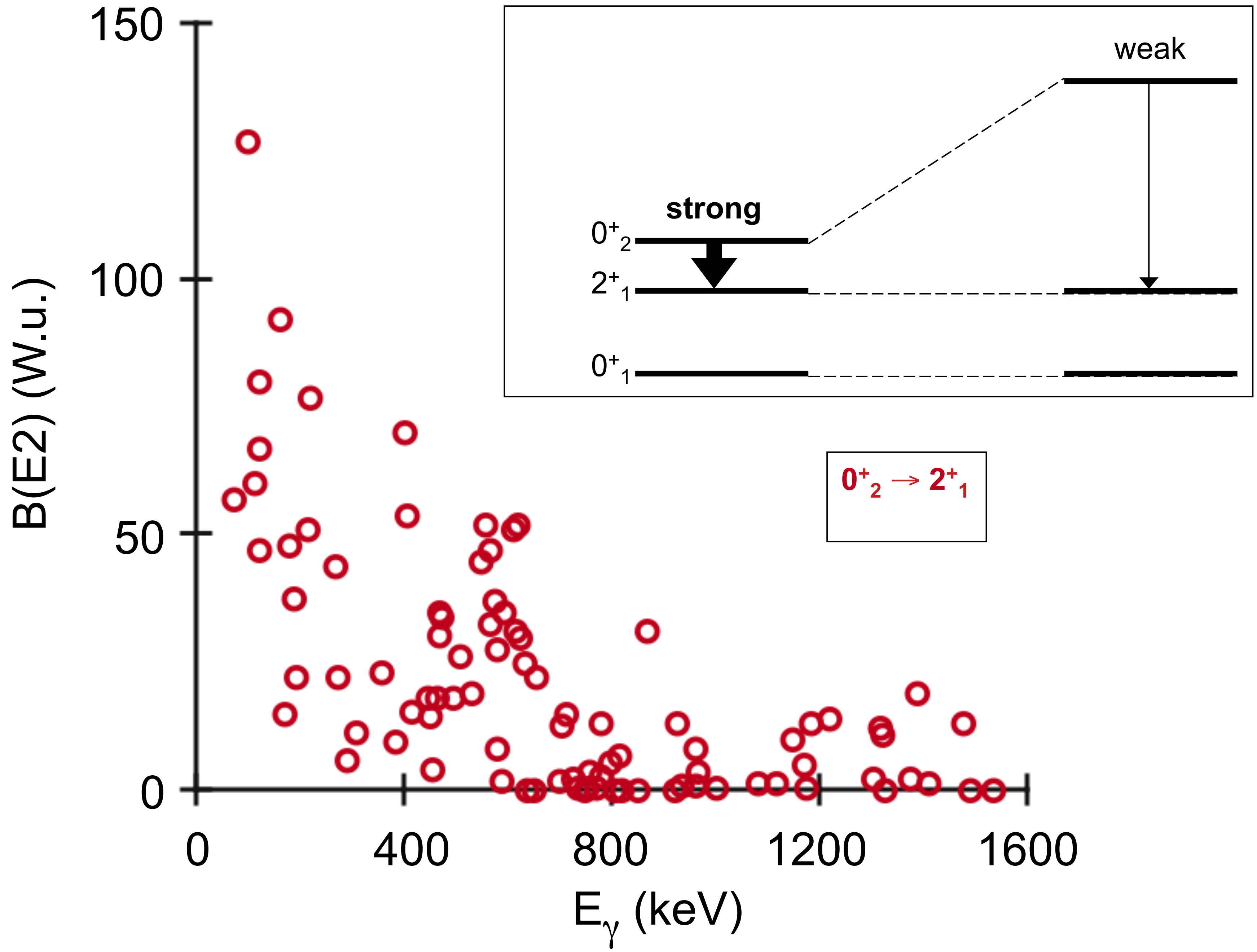

Ru
94
96
98
100
102
104
47%
17%
28 %
35%
9%
18%
3%
8 %
1%
1.7 %
22%
52%
Mo
92
94
96
98
100
102
9%
0_2^+
NS
16 %
Zr
90
92
94
96


Se
74
76
78
80
27%
32%
< 3%
< 2%
0.9%
7.1%
0.2%
29%
20%
1.6%
5.0%
4.0%
Ge
66
68
70
72
74
76
78
0_2^+ NS
0_2^+ NS
0_2^+ NS
4.6%
0_2^+ NS
8.5%
<1%
70%
Zn
64
66
68
70

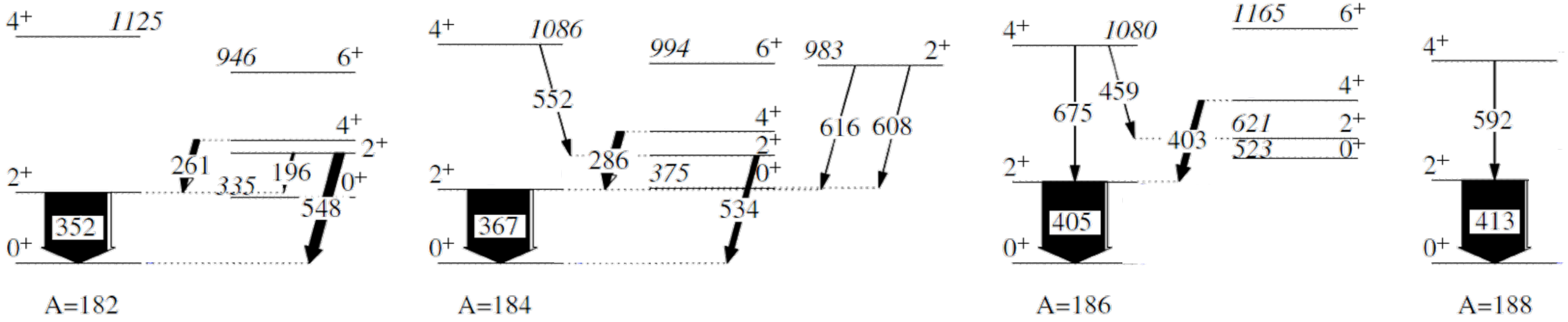


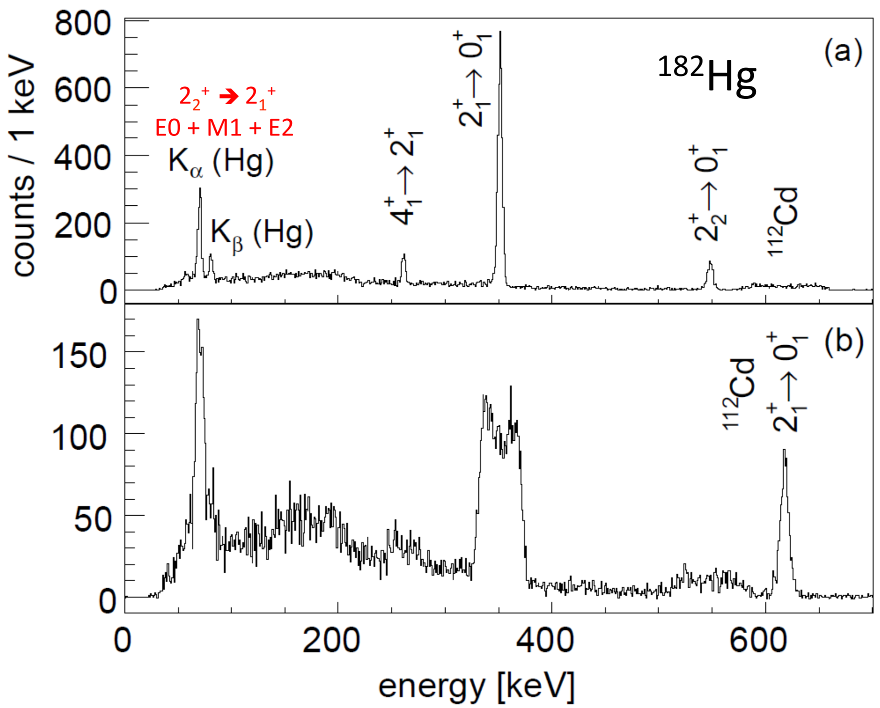
66Dy
0+ 991
0+ 661
0+ 675
23
0+ 0
0+ 0
0+ 0
100
64Gd
0+ 1207
14
0+ 1048
26
0+ 615
80
0+ 1296
56
<1
0+ 681
62
13
0+ 1168
17
0+ 1049
1
0+ 0
0+ 0
0+ 0
0+ 0
100
100
100
62Sm
0+ 1426
21
0+ 1256
52
0+ 740
25
87
0+ 1083
68
<3
0+ 685
74
31
0+ 1100
10
0+ 0
0+ 0
0+ 0
0+ 0
100
100
100
60Nd
0+ 917
15
0+ 676
128
0+ 1139
72
0+ 0
0+ 0
0+ 0
0+ 0
100
100
100
N
86
88
90
92

Systematics of B(E2; $0^+_2 \rightarrow 2^+_1$) vs. E_γ ($0^+_2 - 2^+_1$)



Hg (Z = 80) isotopes: detailed spectroscopy--Coulex





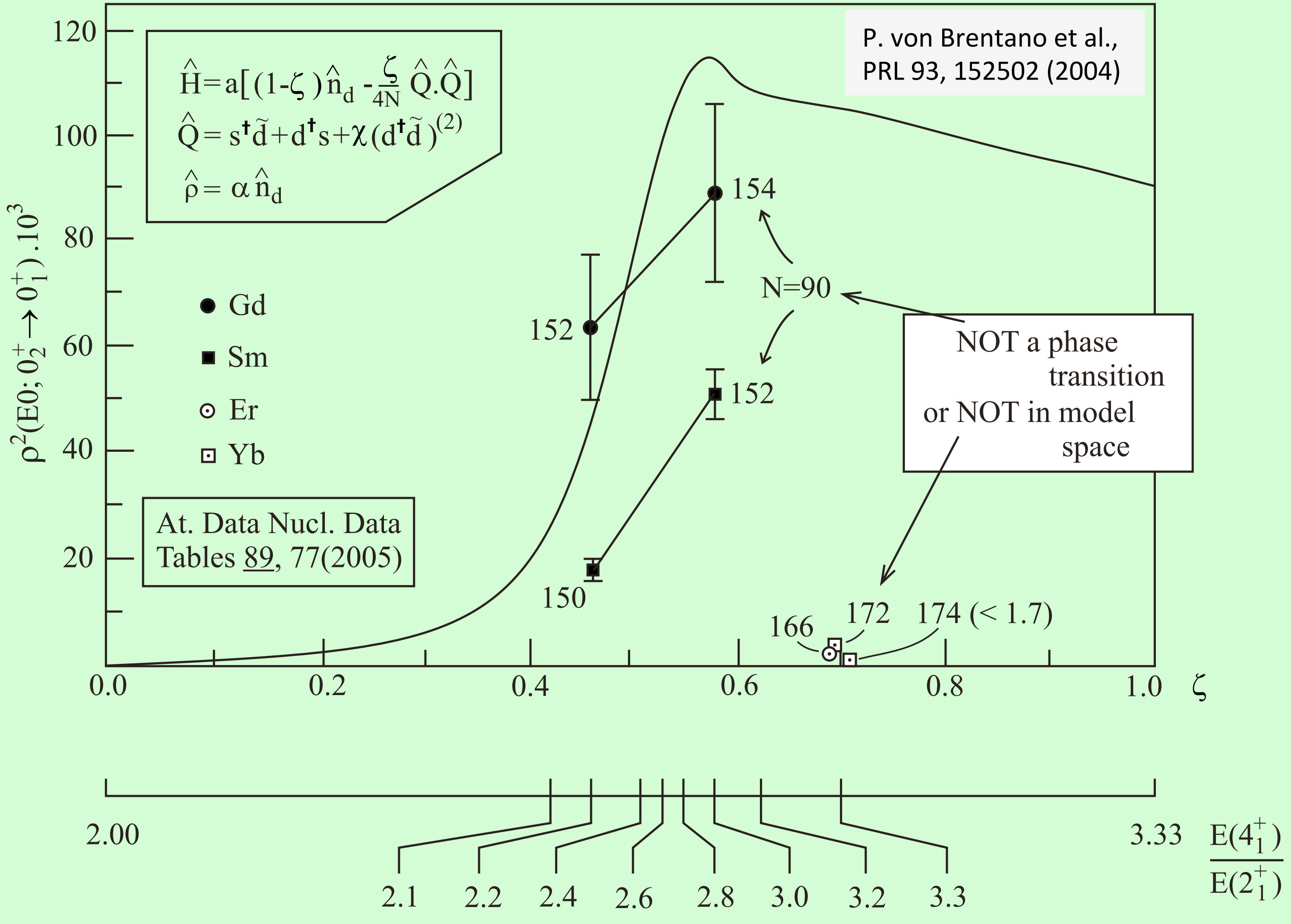
Multi-step Coulex @ REX-ISOLDE:
Hg beam (2.85 MeV/A) / ^{112}Cd target;
GOSIA analysis of γ-ray yield(angle)

N. Bree et al. PRL 112 162701 2014
figure courtesy of Liam Gaffney

“Shape Coexistence in Atomic Nuclei”--Kris Heyde and JLW,
Reviews of Modern Physics Vol. **83** 1467 2011
“Coexistence in Even-Mass Nuclei”--JLW, K Heyde, W Nazarewicz, M Huyse,
and P Van Duppen, Physics Reports **215** 101 1992
“Coexistence in Odd-Mass Nuclei”—K Heyde, P Van Isacker, M Waroquier, JLW,
and RA Meyer, Physics Reports **102** 291 1983
“Electric Monopole Transitions from Low-Energy Excitations in Nuclei”—JLW,
EF Zganjar, C De Coster, and K Heyde, Nucl. Phys. **A651** 323 1999

Electric monopole transition strengths: critical test of phase transition models

Fit of a phase transition model to ρ^2_{21} (E0).10^3 at N=88, 90



$$\hat{H} = a[(1-\zeta)\hat{n}_d - \frac{\zeta}{4N}\hat{Q}.\hat{Q}]$$

$$\hat{Q} = s^\dagger\tilde{d} + d^\dagger s + \chi(d^\dagger\tilde{d})^{(2)}$$

$$\hat{\rho} = \alpha\,\hat{n}_d$$

P. von Brentano et al., PRL 93, 152502 (2004)

At. Data Nucl. Data Tables 89, 77(2005)

NOT a phase transition or NOT in model space

Cadmium isotopes: systematics (selected states)
N = 50 to N = 82 positive parity, $^{98\text{-}130}$Cd

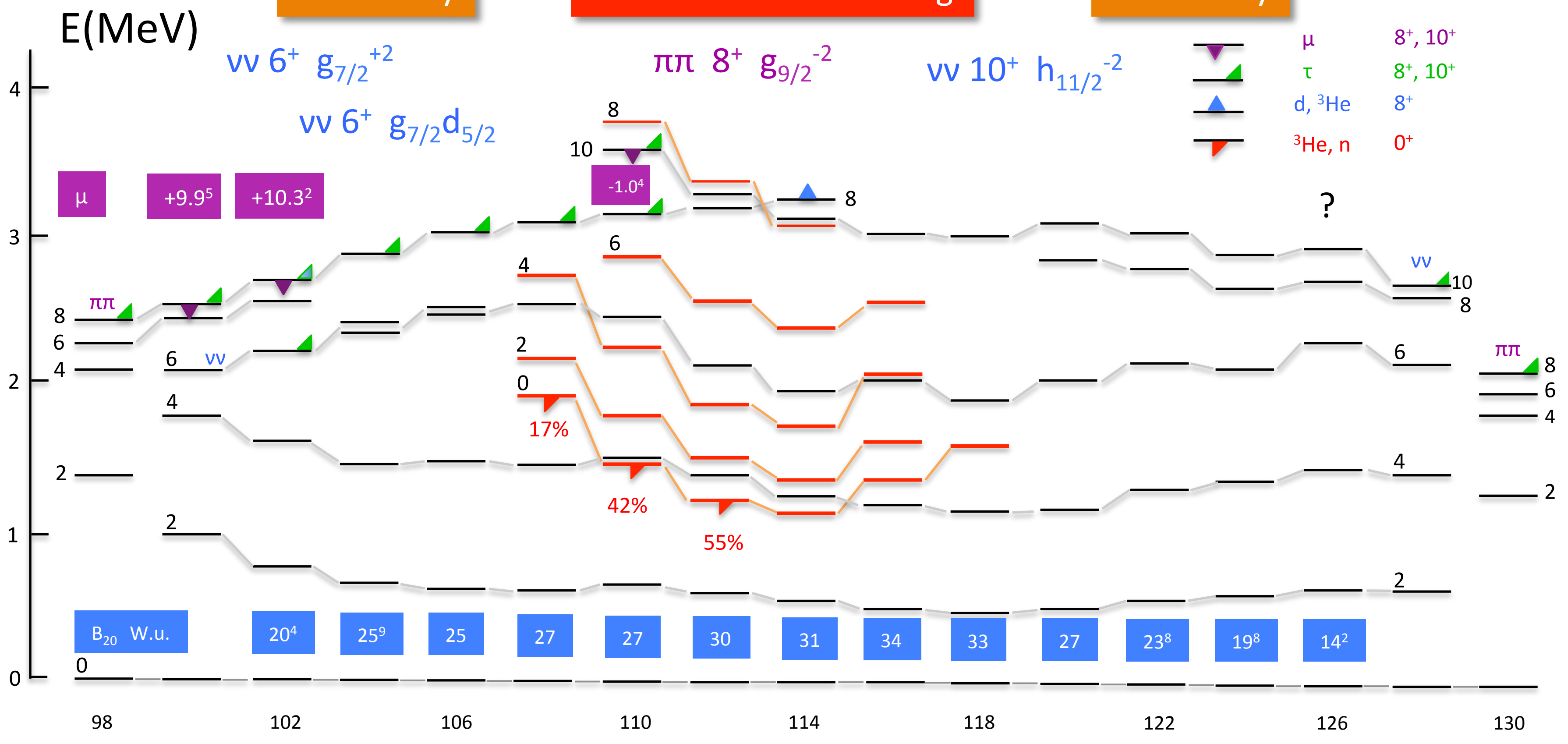
seniority

emerging collectivity

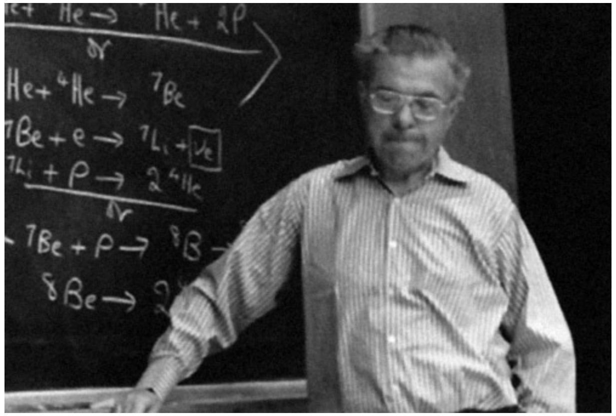
coexisting collective structures AND mixing

emerging collectivity

seniority



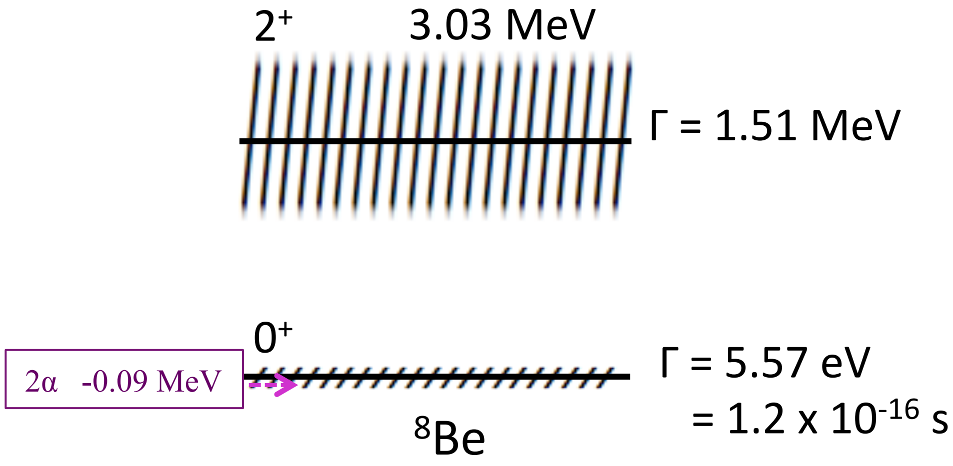
The Hoyle state (7.65 MeV state in ^{12}C)



Sir Fred Hoyle (1915-2001)

Helium fusion in stars

F. Hoyle, Astrophysical J. Suppl. Ser. **1** 121 1954



2^+ 3.03 MeV

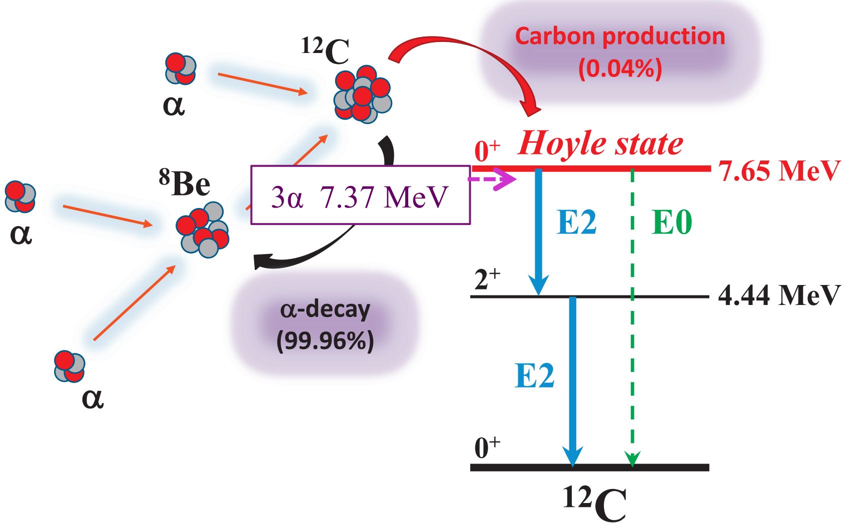
Γ = 1.51 MeV

0^+

2α -0.09 MeV

^{8}Be

Γ = 5.57 eV
= 1.2 x 10^{-16} s



^{12}C

α

^{8}Be

α

α

Carbon production (0.04%)

0^+ *Hoyle state*

7.65 MeV

3α 7.37 MeV

E2

E0

2^+

4.44 MeV

α-decay (99.96%)

E2

0^+

^{12}C

EVERYTHING YOU SEE, EXCEPT HYDROGEN, *i.e.*, MOST OF YOU AND ME CAME INTO EXISTENCE THROUGH THE HOYLE STATE

He nucleus = α

	Z	A
Helium	2	4
Beryllium	4	8
Carbon	6	12