

# Nuclear Shapes: A monopole guided tour into the quadrupole realm

**Alfredo Poves**

**Department of Theoretical Physics and IFT, UAM-CSIC,  
Madrid**

**ESNT-Saclay, October 2017**



**In collaboration with  
F. Nowacki, E. Caurier, A. P. Zuker,  
K. Sieja, S. M. Lenzi and B. Bounseng**

# Outline

- **Basic Monopole Tools**
- **The Quadrupole Interaction: Rotors and Shapes**
- **Islands of Inversion: N=20 and N=28**
- **$^{68}\text{Ni}$ : The Portal to the N=40 Iol**
- **$^{78}\text{Ni}$ : The Portal to the N=50 Iol**
- **Mergers**
- **Conclusions**

# The Plot

- **The two basic players in the nuclear dynamics are the spherical mean field and the multipole hamiltonian:**  
$$H = \mathcal{H}_m + \mathcal{H}_M$$
- **Magic numbers are associated to large energy gaps in the spherical mean field. Therefore, to promote particles above the Fermi level costs a large amount of energy.**
- **The Multipole Hamiltonian is responsible for the very strong nuclear correlations**
- **It is proper to the nucleus that, quite often, certain highly correlated configurations (dubbed "intruders") overwhelm their loss of mean field energy with their huge gains in correlation energy.**

## The Spherical Mean Field (Monopole Hamiltonian)

$$\mathcal{H}_m = \sum n_i \epsilon_i + \sum \frac{1}{(1 + \delta_{ij})} \bar{V}_{ij} n_i (n_j - \delta_{ij})$$

the coefficients  $\bar{V}$  are angular averages of the two body matrix elements, or centroids of the two body interaction:

$$\bar{V}_{ij} = \frac{\sum_J V_{ijj}^J[J]}{\sum_J [J]}$$

the sums run over Pauli allowed values.

It can be written as well as:

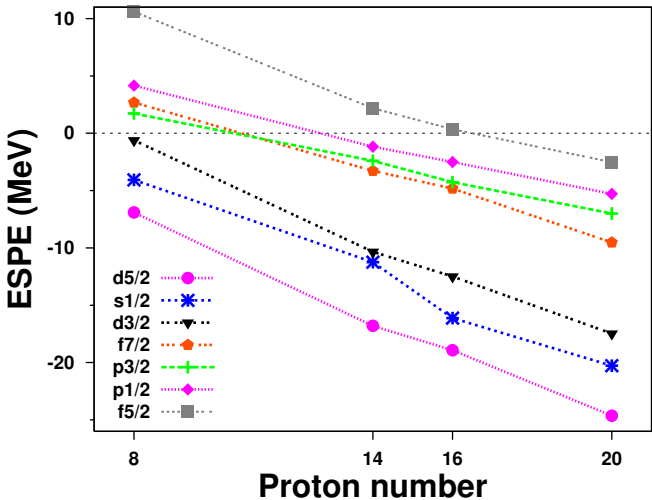
$$\mathcal{H}_m = \sum_i n_i \left[ \epsilon_i + \sum_j \frac{1}{(1 + \delta_{ij})} \bar{V}_{ij} (n_j - \delta_{ij}) \right]$$

Thus

$$\mathcal{H}_m = \sum_i n_i \hat{\epsilon}_i([n_j])$$

**We call these  $\hat{\epsilon}_i([n_j])$  effective single particle energies (ESPE). It is seen that the monopole hamiltonian determines the evolution of the underlying (non observable) spherical mean field (aka, shell evolution) as we add particles in the valence space.**

# Shell Evolution: ESPE's



## Item more ....

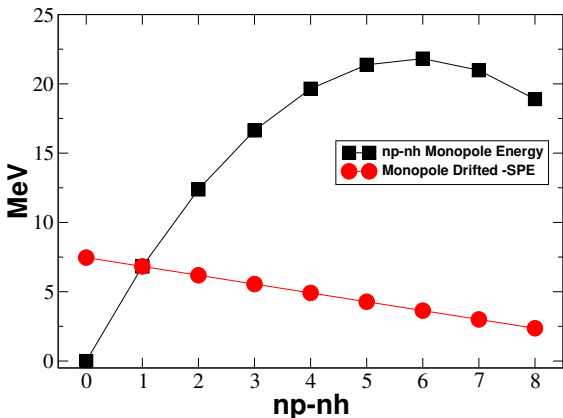
The monopole hamiltonian produces variations of the spherical mean field in a single nucleus for different np-nh configurations with respect to the normal filling (MCD; Monopole Configuration Drift, which T. Otsuka has dubbed type II shell evolution).

**All in all, it provides the control parameters for the nuclear dynamics, given the universality of the nuclear correlators; in A. Zuker's words:**

**"Multipole Proposes and Monopole Disposes"**

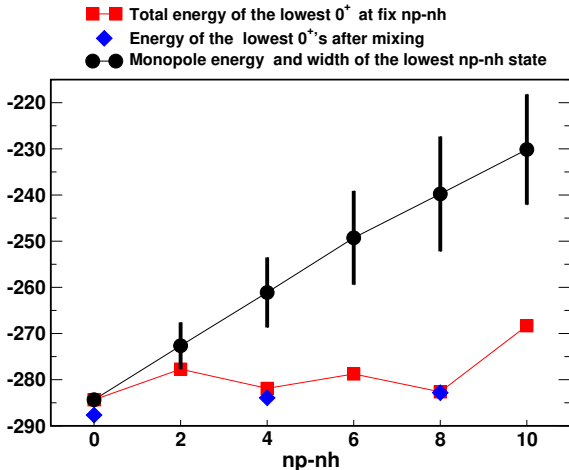


# Monopole Configuration Drift



Monopole energy of the  $(0d_{3/2})^{-n} (1p_{3/2})^n$  configurations with respect to doubly magic  $^{40}\text{Ca}$

# Intruder States; MCD or Correlations



Spherical, Deformed and Superdeformed States in  $^{40}\text{Ca}$

# Monopole anomalies of the realistic NN interactions

They are the more blatant in the neutron-neutron interaction; for instance not producing neither a magic  $^{48}\text{Ca}$ , nor the right location of the neutron drip line in the Oxygen isotopes

On the contrary their monopole neutron proton tensor part is correct, and the spin orbit splittings well accounted for.

Nowadays we put the blame in the missing residual three body effects, and the "ab initio" practitioners are making lots of efforts to cure them.

# The Multipole Hamiltonian

- **The multipole hamiltonian is responsible for the collective nuclear behavior. It is universal and well given by the realistic NN interactions. Its main components are:**
- **BCS-like isovector and isoscalar pairing. When pairing dominates, as in the case of nuclei with only neutrons (or only protons) on top of a doubly magic nucleus, it produces nuclear superfluids.**
- **Quadrupole-Quadrupole and Octupole-Octupole terms of very simple nature ( $r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$ ) which tend to make the nucleus deformed. In this limit, the pairing correlations mainly show up as responsible for the moment of inertia of the nuclear rotors.**

## Multipole Universality and NNN Forces

	pp(JT)			ph( $\lambda\tau$ )			
	10	01	21	20	40	10	11
USD-A	-5.62	-5.50	-3.17	-3.24	-1.60	+1.56	+2.99
KUO	-5.83	-4.96	-3.21	-3.53	-1.38	+1.61	+3.00
CCEI	-6.79	-4.68	-2.93	-3.40	-1.39	+1.21	+2.83
NN	-6.06	-4.38	-2.92	-3.35	-1.31	+1.03	+2.49
NN+NNN	-6.40	-4.36	-2.91	-3.28	-1.23	+1.10	+2.43

**DZ analysis, coherent particle-particle  
and particle-hole terms**

# Why and When do the quadrupole correlations thrive in the nucleus?

- **The fact that the spherical nuclear mean field is close to the HO has profound consequences, because the dynamical symmetry of the HO, responsible for the accidental degeneracies of its spectrum, is  $SU(3)$ , among whose generators it is the quadrupole operator.**
- **When valence protons and neutrons occupy the degenerate orbits of a major oscillator shell, and for an attractive Q·Q interaction, the many body problem has an analytical solution in which the ground state of the nucleus is maximally deformed (Elliott's model)**

# Why and When do the quadrupole correlations thrive in the nucleus?

- In cases when both valence neutrons and protons occupy quasi-degenerate orbits with  $\Delta j= 2$  and  $\Delta j=2$ , including  $j=p+1/2$  (Quasi-SU3), or quasi-spin multiplets (Pseudo-SU3)
- For example,  $0f_{7/2}$  and  $1p_{3/2}$ , or  $0g_{9/2}$   $1d_{5/2}$  and  $2s_{1/2}$  form Quasi-SU3 multiplets and  $0f_{5/2}$ ,  $1p_{3/2}$  and  $1p_{1/2}$  a Pseudo-SU3 triplet

# Why Nuclear Shape?

- Because we are still heirs of the semiclassical liquid-drop like models
- **The very concept of shape requires to break the rotational (and reflection) invariance, or, equivalently to define an intrinsic reference frame. But even if the symmetry is broken, we need to rely on semiclassical models, liquid-drop like, to define a vocabulary which describes properties akin to the concept of shape.**
- The surface of a drop can be expressed in the basis of the spherical harmonics  $Y_{\lambda,\mu}(\theta, \phi)$ . The coefficients of the development,  $\alpha_{\lambda,\mu}$ , are the shape parameters. To speak about nuclear shape, we need a protocol to extract the best information about these intrinsic shape parameters from the nuclear wave functions in the laboratory frame.



# Shape Parameters; Quadrupole Deformation

- From the values of  $Q_{spec}(J)$  and the  $B(E2)$ 's in a rotational band one can get  $Q_0$ , the intrinsic quadrupole moment using the BM formulas for the axial rotor. The deformation parameter  $\beta$  can be extracted using different recipes, for instance:

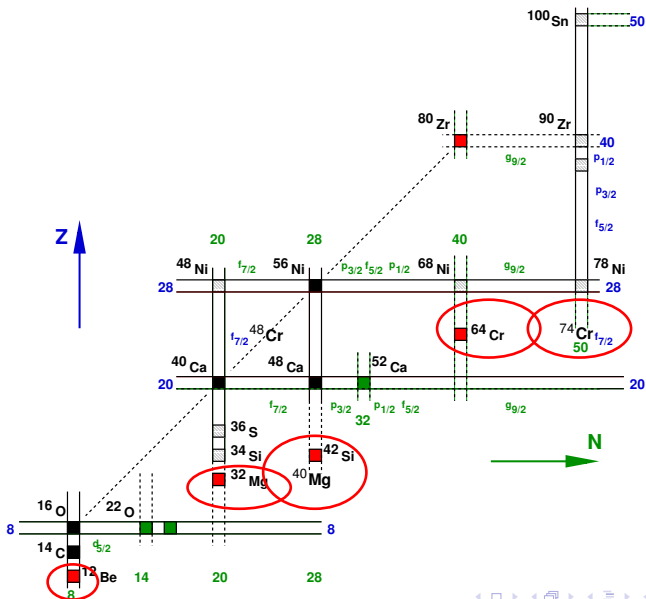
$$Q_0 = \frac{3}{\sqrt{5\pi}} R^2 Z (1 + 0.16 \beta) \beta \quad (1)$$

- If the nucleus is not axially symmetric, the situation becomes more convoluted, because now we need to recover two shape parameters,  $\beta$  and  $\gamma$ . The former can in most cases be extracted from the  $B(E2)$ 's as in the axial case, but for  $\gamma$  we have to resort to other expediciencies. Davidov and Filipov use the collective model to extract the values of  $\gamma$  from the  $B(E2)$  values of the transitions between the yrast and the  $\gamma$  bands.

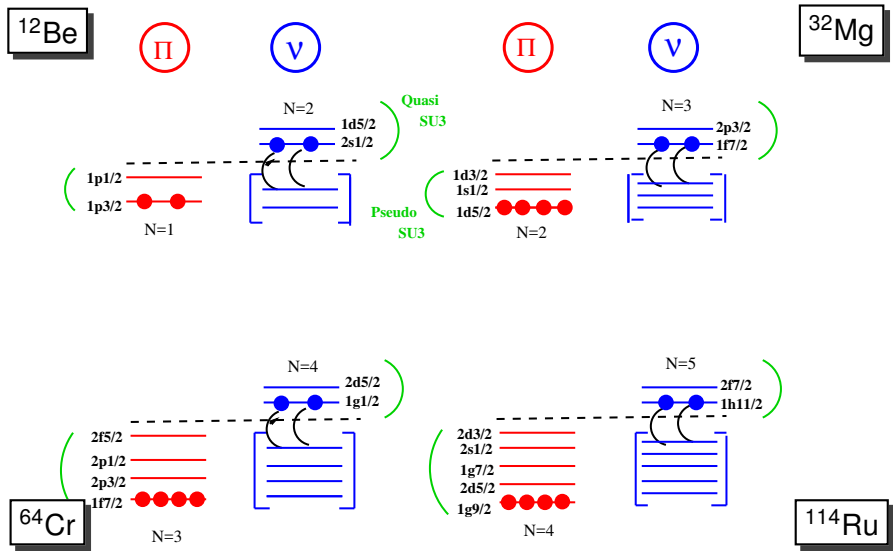
# Shape Parameters; Quadrupole Deformation

- **Another possibility is to rely on the use of the expectation values of scalars constructed with the quadrupole operator like  $(Q_2 \times Q_2)^0$  or  $(Q_2 \times Q_2 \times Q_2)^0$  as proposed by Kumar. These expectation values can be written in terms of the shape parameters. Rowe and Rosensteel have dealt with this issues as well.**
- **Finally, one can use a basis in the intrinsic frame to perform laboratory frame calculations as in the MCSM (Monte Carlo Shell Model), and keep track of the shape parameters content of the physical solutions.**

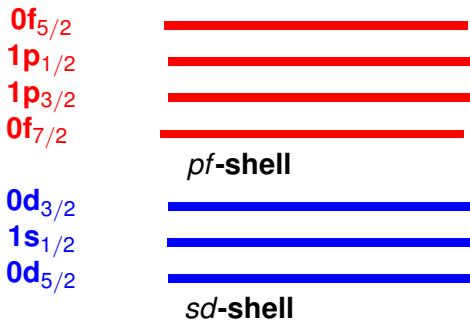
# Landscape of medium mass nuclei, featuring the Archipelago of Islands of Inversion



# How deformation sets in at N=8, 20, 40, 70.



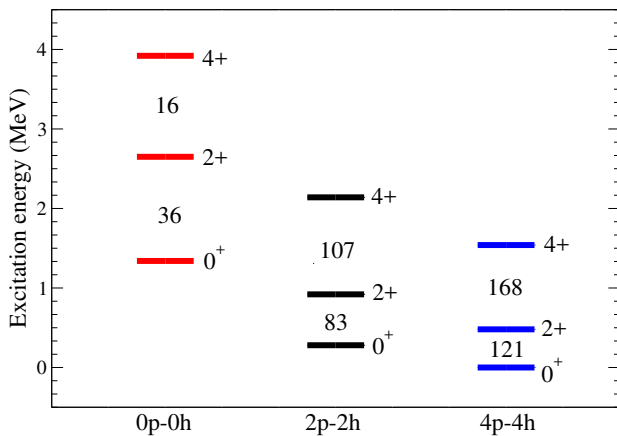
# N=20 to N=28. The Valence Space; *sd-pf*



**EFFECTIVE INTERACTION**

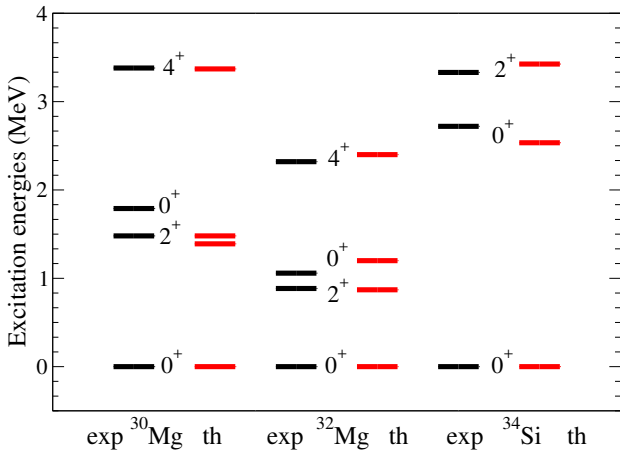
**SDPF-U-MIX**

# Deformed and Superdeformed states in $^{32}\text{Mg}$

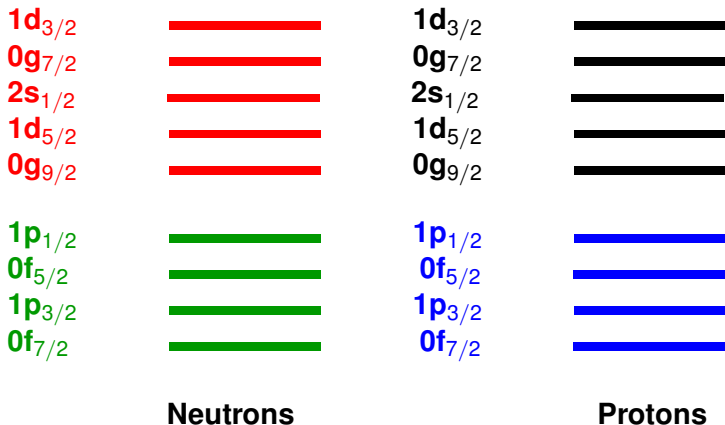


# Shape Coexistence in $^{30}\text{Mg}$ and $^{34}\text{Si}$

## The Portal to the N=20 Isot



# The pf-sdg valence space



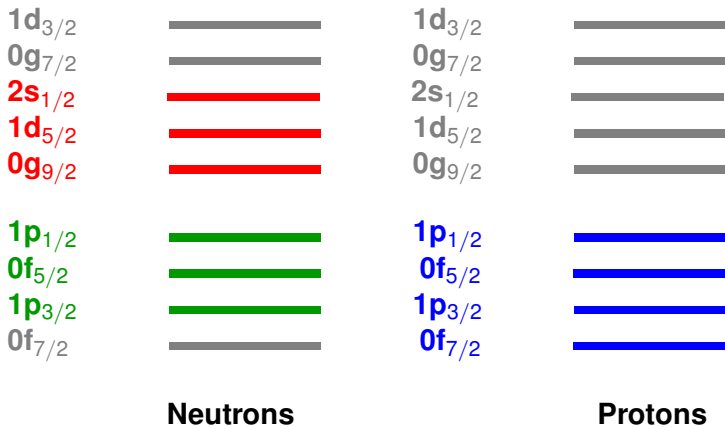
For this valence space we propose the PFSDG-U interaction



# The pf-sdg valence space

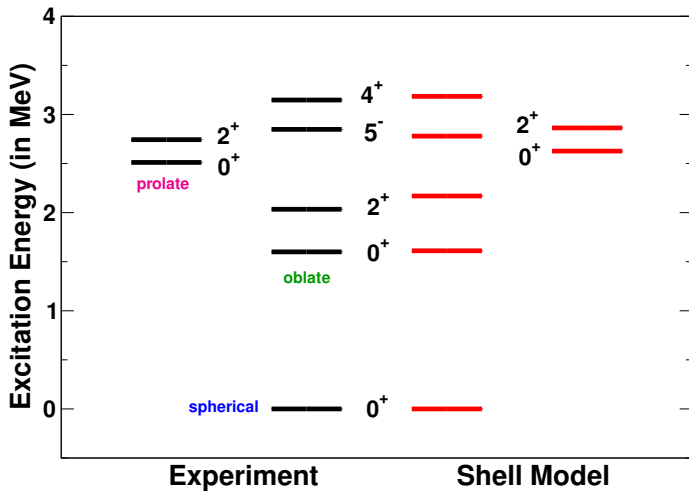
- Few (if any) nuclei would demand the full *pfsgd* space, which however contains a wealth of sub valence spaces full of physical insight, both at the neutron rich and at the neutron deficient side.
- For instance, the *gds* valence space for protons and neutrons can cope with the physics around  $^{100}\text{Sn}$  including its recently measured hyper allowed  $\beta$  decay
- $^{56}\text{Ni}$ , instead can be understood in the *pf* sub valence space, with a little help of the the *0g<sub>9/2</sub>* orbit for Dirk Rudolf's super deformed band
- Other valence spaces at the neutron rich edge will be discussed next

# The LNPS valence space; N=40



gray orbits are blocked

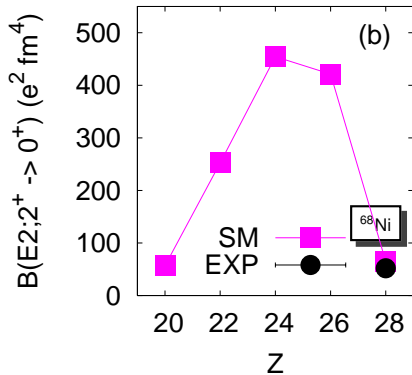
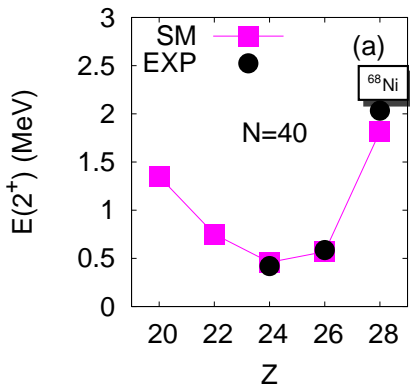
# The Portal to the N=40 Isot; $^{68}\text{Ni}$



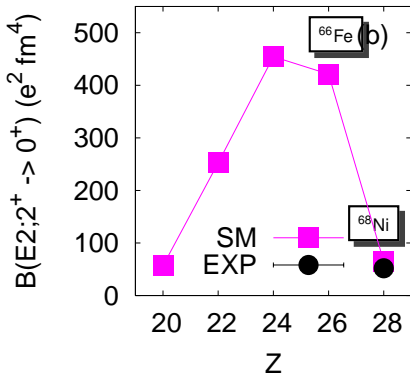
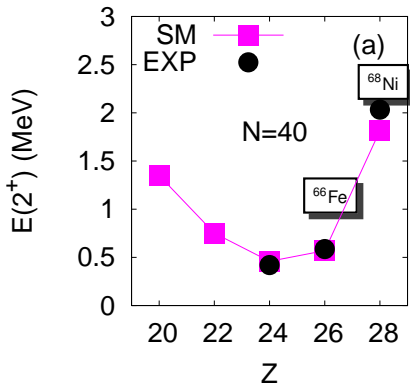
# The island of inversion south of $^{68}\text{Ni}$

- Removing protons from the  $0f_{7/2}$  orbit, activates the quadrupole collectivity, which, in turn, favors the np-nh neutron configurations across  $N=40$ , that take advantage of the quasi-SU3 coherence of the doublet  $0g_{9/2}$ -  $1d_{5/2}$ .
- Large scale SM calculations in the valence space of the full  $pf$ -shell for the protons and the  $0f_{5/2}$   $1p_{3/2}$   $1p_{1/2}$   $0g_{9/2}$  and  $1d_{5/2}$  orbits for the neutrons, predict a new region of deformation centered at  $^{64}\text{Cr}$ .

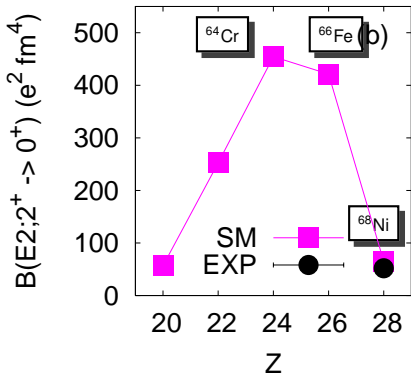
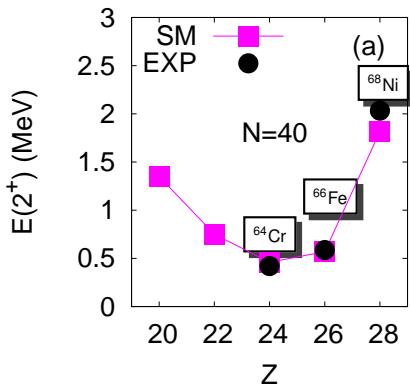
# Shape transition at N=40



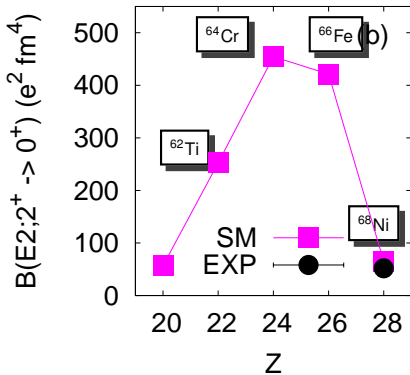
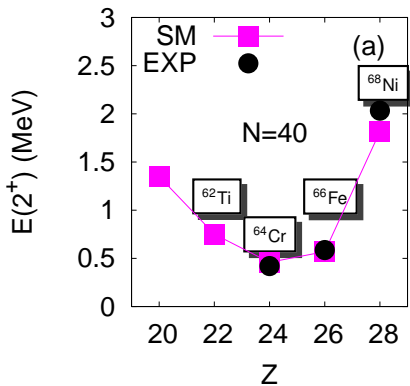
# Shape transition at N=40



# Shape transition at N=40

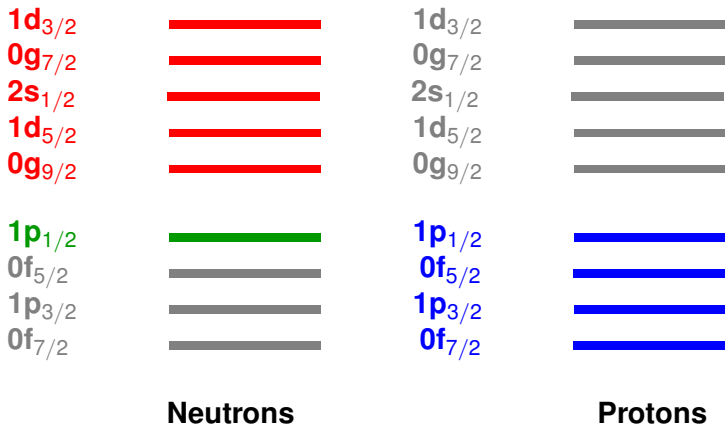


# Shape transition at N=40

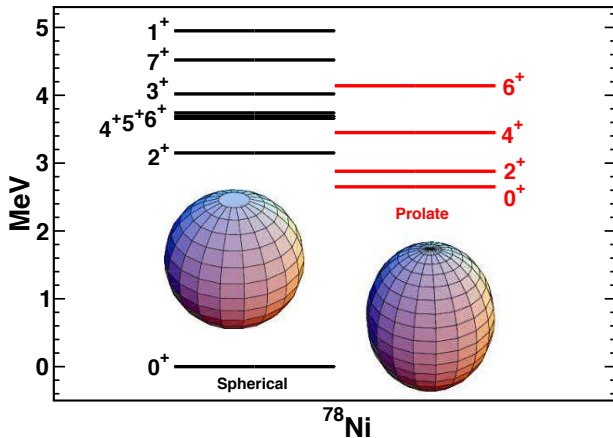




# The pf-sdg valence space at N=50

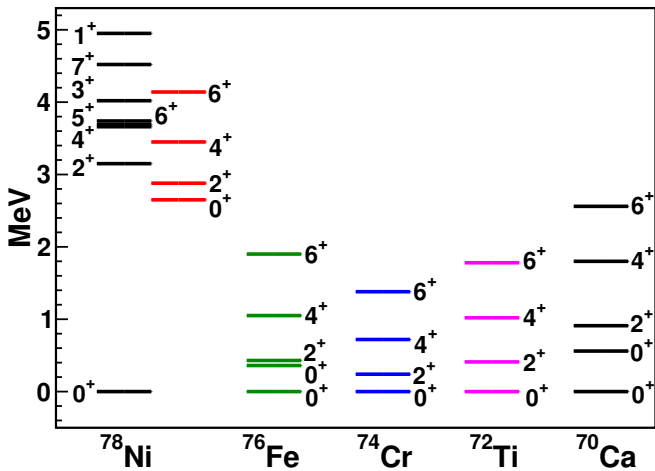


## Shape Coexistence, the Portal to the 5th Iol

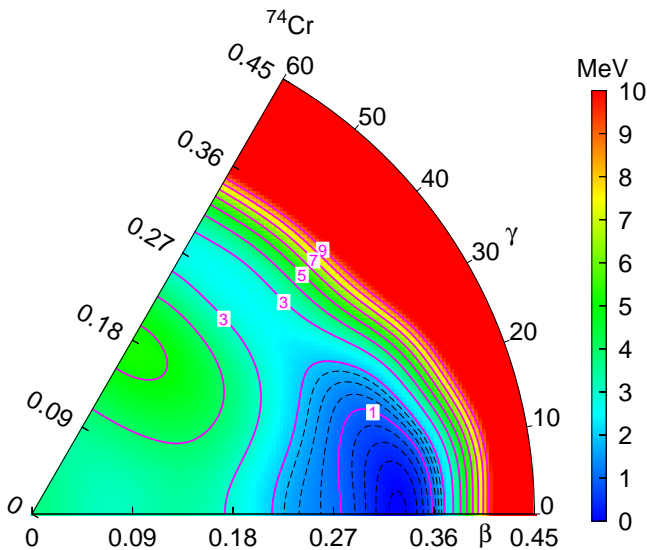


$^{78}\text{Ni}$  is doubly magic in its ground state

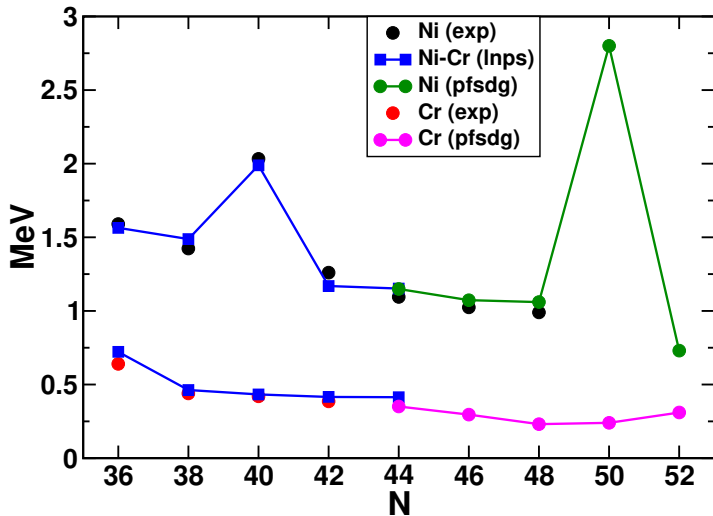
# The Iol surrounding $^{74}\text{Cr}$



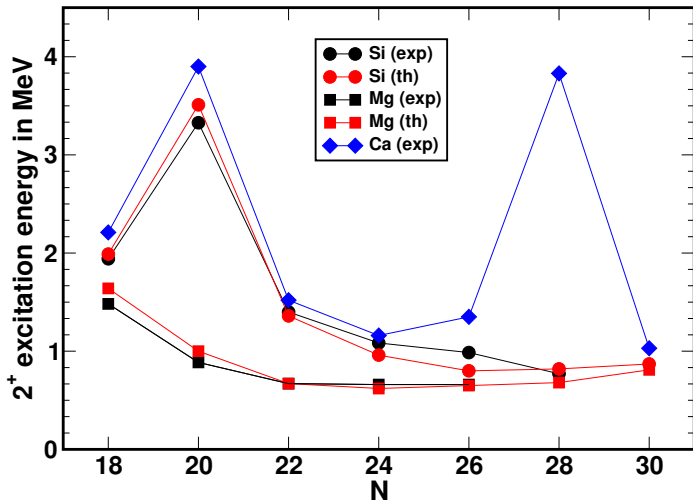
# The Intrinsic Frame View



# The N=40 and N=50 Iol's Merge

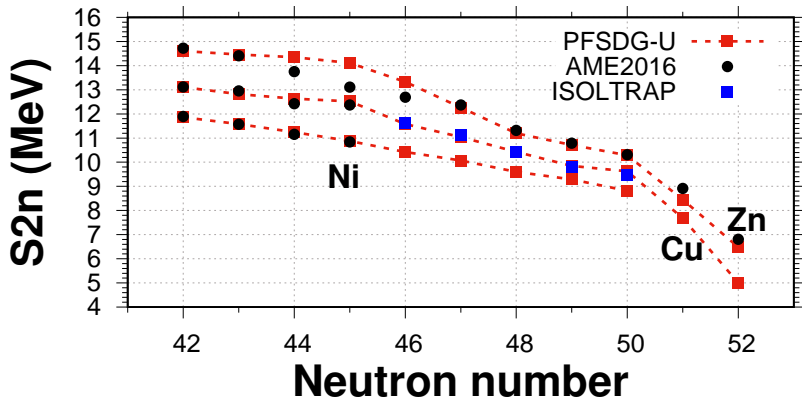


# Like the N=20 and N=28 lol's did



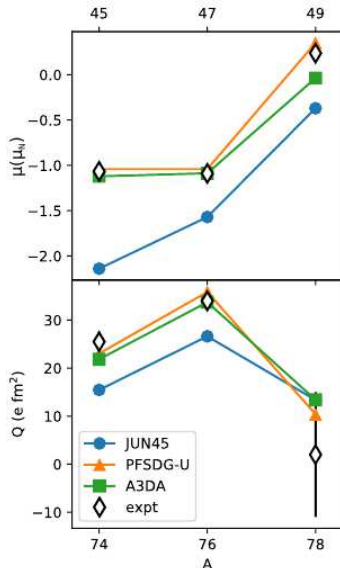
## Other observables: $S_{2N}$

A. Welker et al. PRL in press



# Copper Magnetic and Quadrupole moments

RP de Groote et al. PRC96 041302(R) (2017)





# Conclusions

- **The physics around magic or semi-magic closures depends of subtle balances between the spherical mean field and the (very large) correlation energies of the open shell configurations at play**
- **There is a common mechanism explaining the appearance of "islands of inversion/deformation" (lol's) in nuclei with large neutron excess, and shape coexistence usually shows up at their portals**
- **The lol's at N=20 and N=28 merge in the Magnesium isotopes.**
- **$^{68}\text{Ni}$  is a case of triple coexistence, precursor of the N=40 lol**
- **Shape coexistence in  $^{78}\text{Ni}$  is the portal to a new lol at N=50**
- **The lol's at N=40 and N=50 merge in the Chromium isotopes.**