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ESNT

Espace de Structure Nucléaire Théorique
DSM - DAM

ESNT Workshop “Shape coexistence and electric monopole transitions in atomic nuclei”
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Shape coexistence and quantum phase transition in the Monte-Carlo Shell Model

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JICFuS RIKEN NISHINA CENTER 東京大学 THE UNIVERSITY OF TOKYO KU LEUVEN MICHIGAN STATE UNIVERSITY Founded 1855

This work has been supported by MEXT and JICFuS as a priority issue (Elucidation of the fundamental laws and evolution of the universe) to be tackled by using Post ‘K’ Computer

Outline

I Introduction

II Presently used numerical methodology of many-body problems

III First application of MCSM to shape coexistence: Ni isotopes

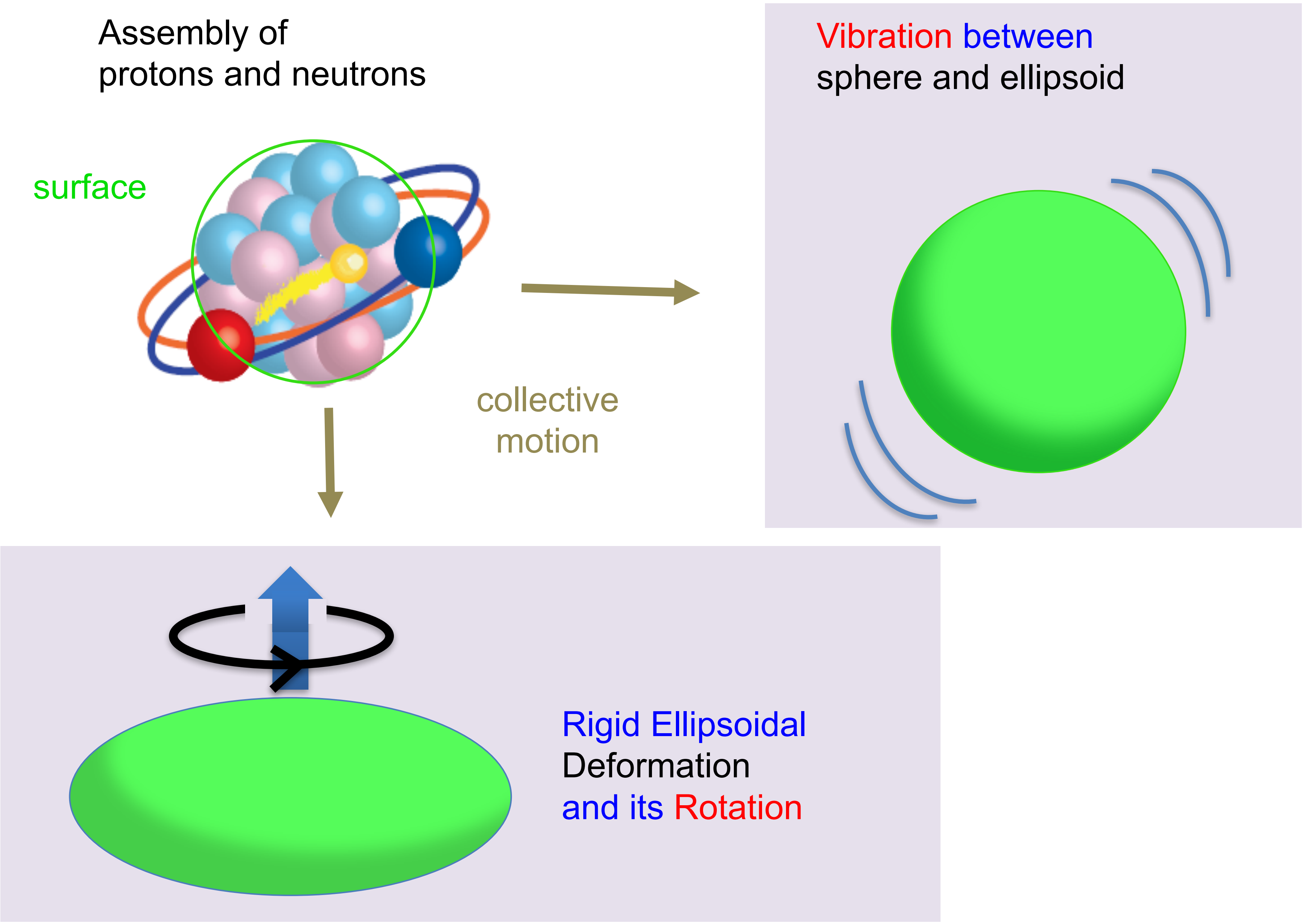
IV An example from Quantum Phase Transition in Zr isotopes

V Basic mechanism

VI Shape coexistence and/or critical phenomena in Hg/Pb isotopes

VII Remarks

Collective modes : various types → in the case of the quadrupole deformation



Assembly of
protons and neutrons

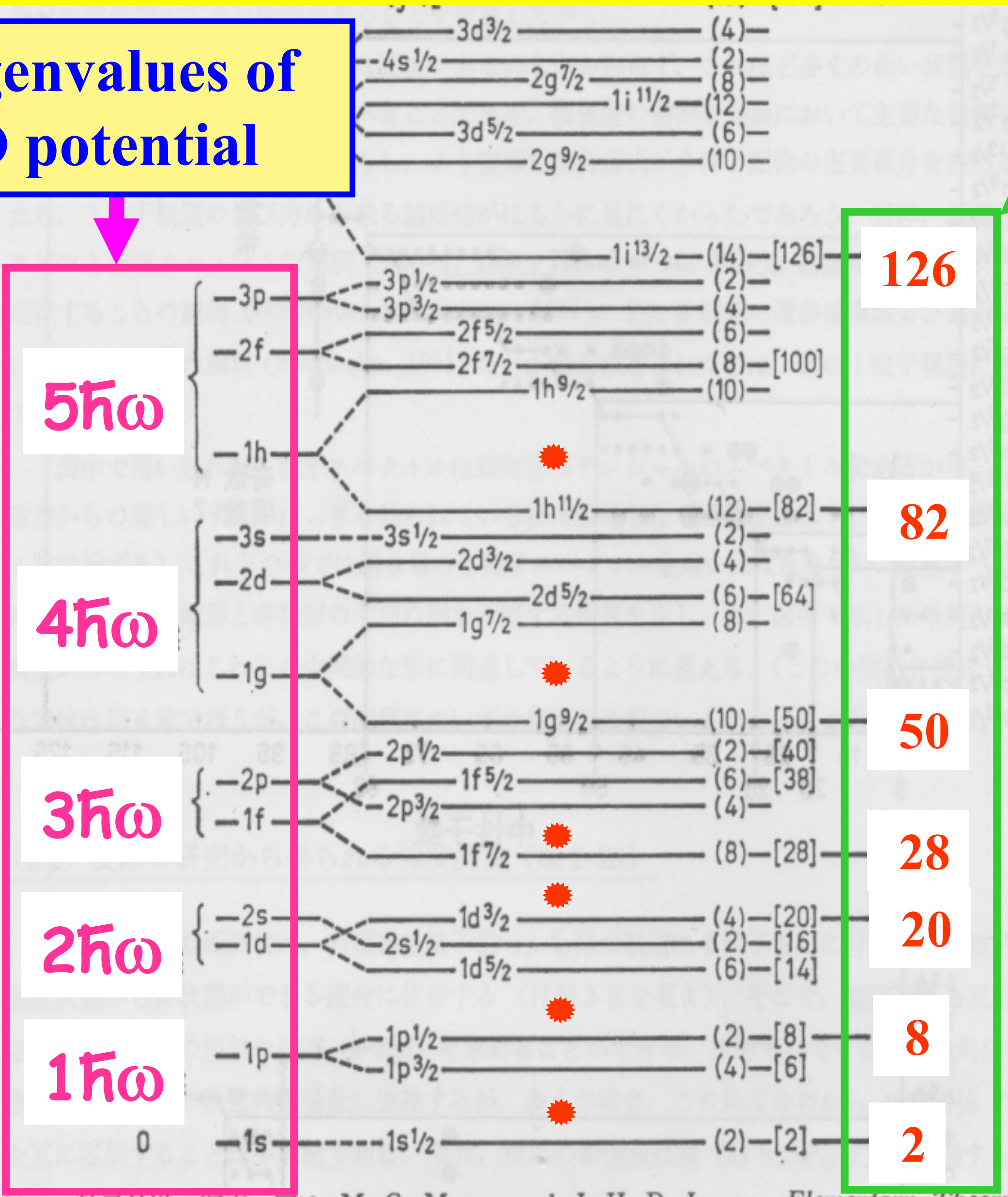
surface

collective
motion

Vibration between
sphere and ellipsoid

Rigid Ellipsoidal
Deformation
and its Rotation

Single-particle states : shell structure and magic numbers due to a “potential”



Eigenvalues of HO potential

5ħω

4ħω

3ħω

2ħω

1ħω

126

82

50

28

20

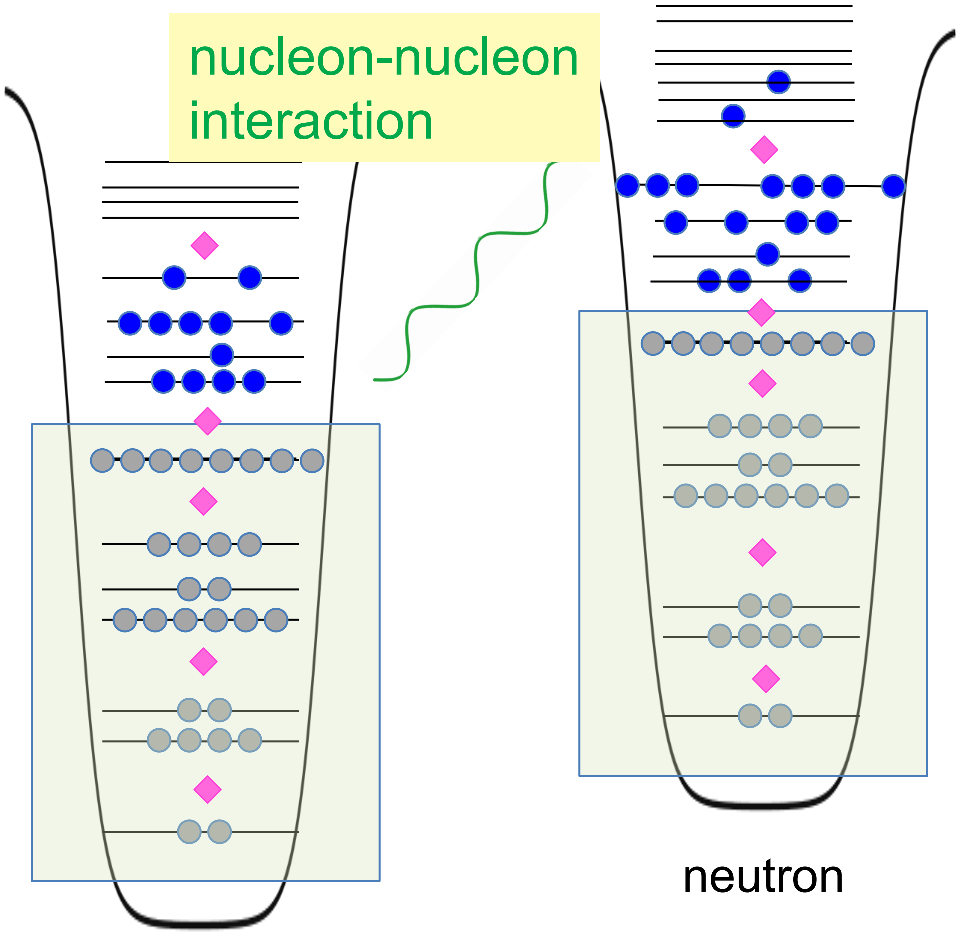
8

2

Magic numbers by Mayer and Jensen (1949)

図2-23 1粒子軌道の順序. 図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった.

single-particle states and correlations



Protons and neutrons are orbiting in the mean potential like a “vase”
→ single-particle states

Lower orbits form the inert core (or closed shell) (*shaded parts in the figure*)

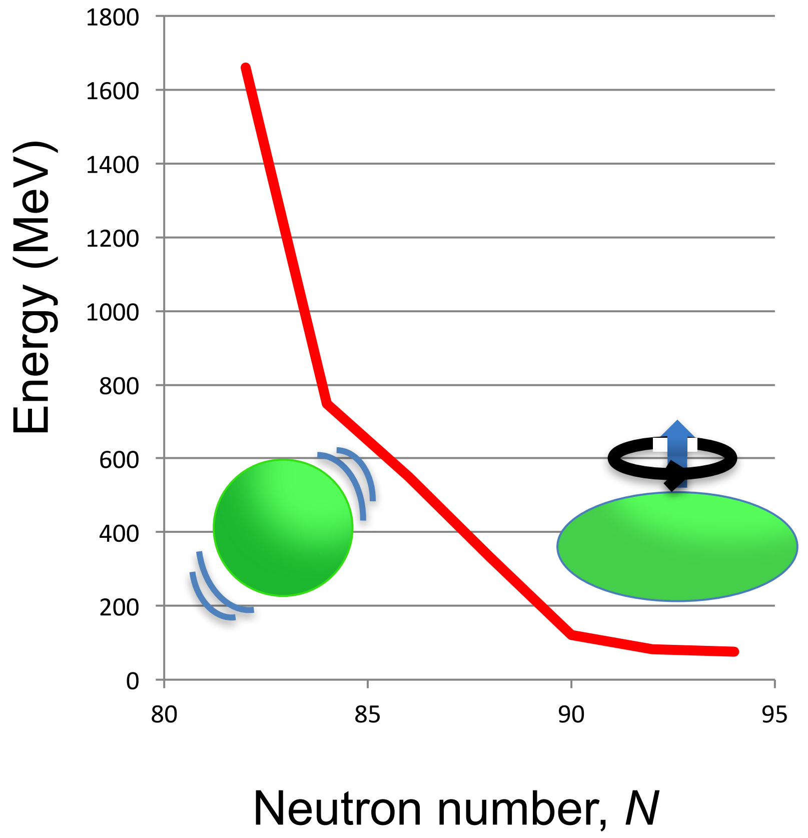
Upper orbits are partially occupied and nucleons are active (valence orbits and nucleons).

Correlations due to nucleon-nucleon interaction produce the mixing of various configurations of single-particle states.

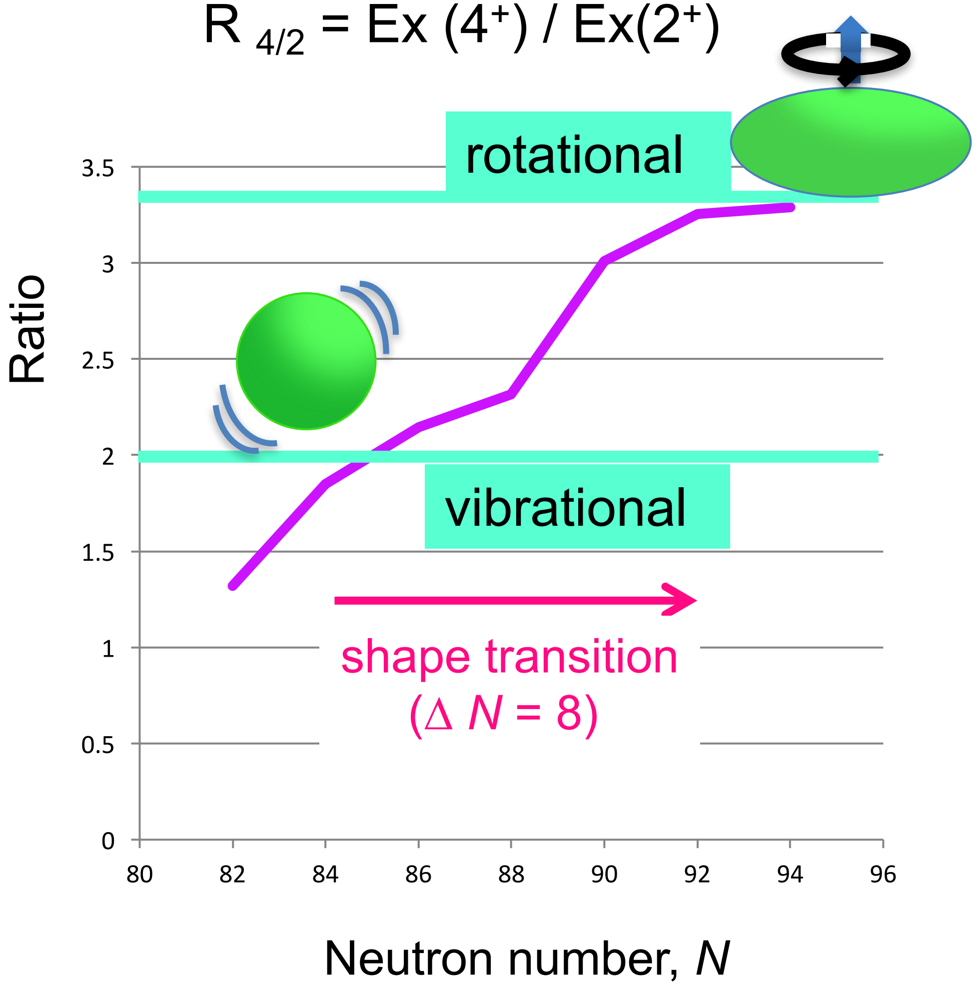
Various shapes appear as a function of *N* (or *Z*) : How can we describe it ?

2^+ and 4^+ level properties of Sm (Z=62) isotopes

Ex (2^+) :
excitation energy of first 2^+ state



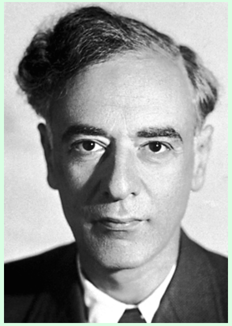
$R_{4/2} = Ex(4^+) / Ex(2^+)$



starting point of the microscopic description

Atomic nucleus is a quantum Fermi liquid :

The nucleus is composed of almost free nucleons interacting weakly via residual forces in a (solid) (mean) potential like a solid “vase”.



Landau

The shape of atomic nucleus can be described by the deformation of the “vase”, a la Nilsson model.

A. Bohr

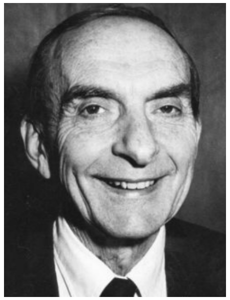
Mottelson

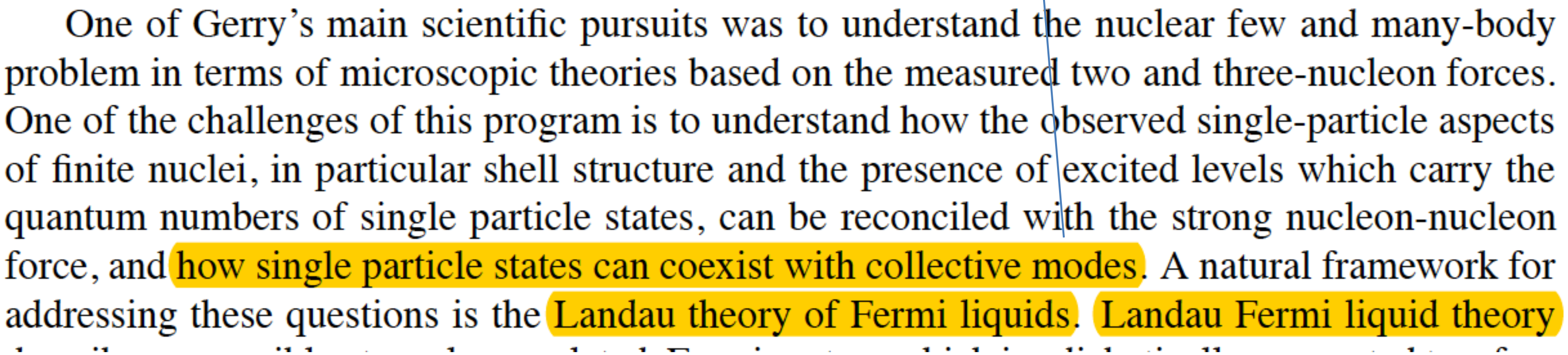
Nilsson

open question

T. Schaefer, Fermi Liquid theory: A brief survey in memory of Gerald E. Brown, NPA 2014)

G.E. Brown



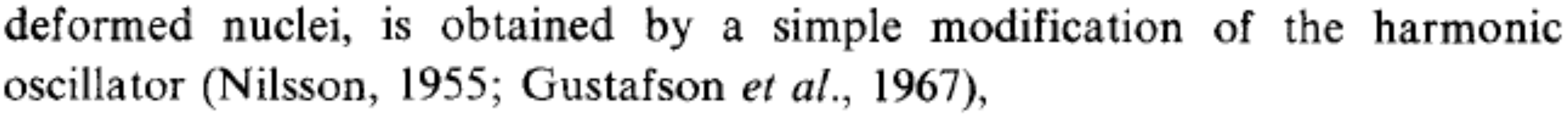


One of Gerry’s main scientific pursuits was to understand the nuclear few and many-body problem in terms of microscopic theories based on the measured two and three-nucleon forces. One of the challenges of this program is to understand how the observed single-particle aspects of finite nuclei, in particular shell structure and the presence of excited levels which carry the quantum numbers of single particle states, can be reconciled with the strong nucleon-nucleon force, and how single particle states can coexist with collective modes. A natural framework for addressing these questions is the Landau theory of Fermi liquids. Landau Fermi liquid theory

Additional deformed field : Nilsson model

Nilsson model Hamiltonian

"*Nuclear structure II*" by Bohr and Mottelson



deformed nuclei, is obtained by a simple modification of the harmonic oscillator (Nilsson, 1955; Gustafson *et al.*, 1967),

$$H = \frac{\mathbf{p}^2}{2M} + \boxed{\tfrac{1}{2}M\left(\omega_3^2 x_3^2 + \omega_\perp^2 (x_1^2 + x_2^2)\right)} + \boxed{v_{ll}\hbar\omega_0\left(\mathbf{l}^2 - \langle \mathbf{l}^2 \rangle_N\right) + v_{ls}\hbar\omega_0(\mathbf{l}\cdot\mathbf{s})} \tag{5-10}$$

quadrupole deformed field

$$\langle \mathbf{l}^2 \rangle_N = \tfrac{1}{2}N(N+3)$$

spherical field

constant within a region

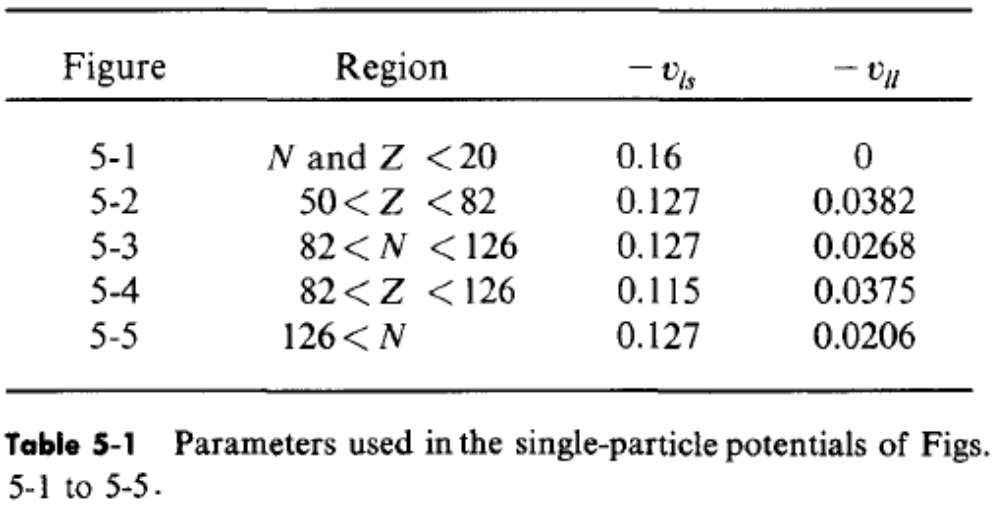


Figure	Region	$-v_{ls}$	$-v_{ll}$
5-1	N and Z <20	0.16	0
5-2	$50 < Z < 82$	0.127	0.0382
5-3	$82 < N < 126$	0.127	0.0268
5-4	$82 < Z < 126$	0.115	0.0375
5-5	$126 < N$	0.127	0.0206

Table 5-1 Parameters used in the single-particle potentials of Figs. 5-1 to 5-5.

Spin-orbit force

$$\left.\begin{array}{ll} A=\ 68 & 1.28 \\ A=100 & 1.12 \\ A=186 & 0.91 \end{array}\right\} (\mathbf{l}\cdot\mathbf{s})$$

Main subject of this talk

This effect becomes stronger as the nucleus moves away from the closed shell.

Intuitively speaking, the quadrupole deformation is determined by

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$

resistance power ← For instance, pairing force.

What else ?

- The quadrupole force is a part of nuclear forces :
  quadrupole-quadrupole component in the spin-tensor decomposition.
- Driving force for the rotational spectrum, for instance, in Elliott's SU(3)
- Its mean-field effect ➡ Nilsson model
- Pairing + QQ interaction model

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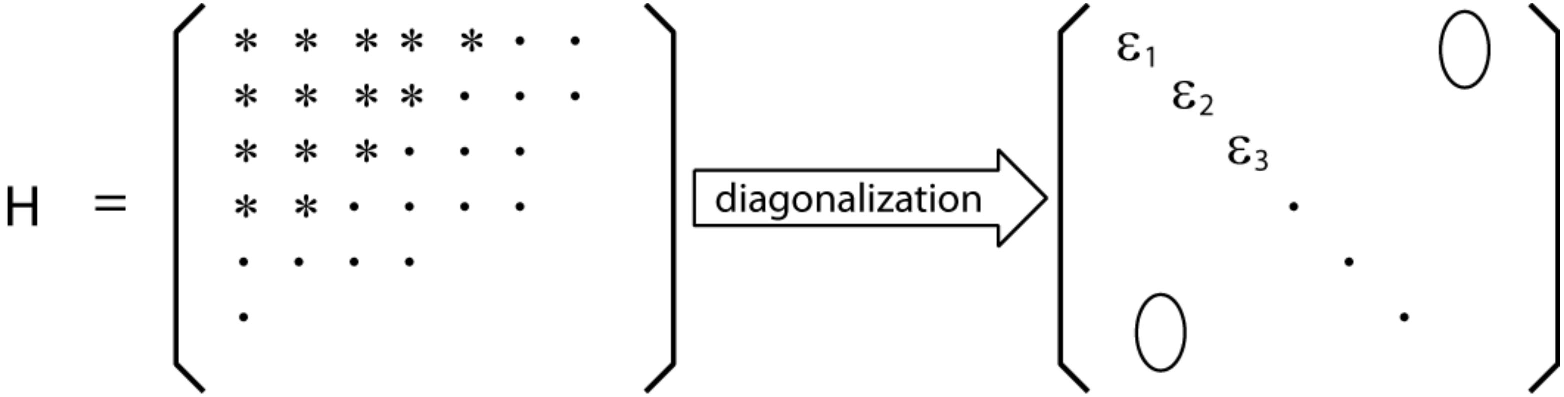
Two types of shell-model calculations

dim. <~ 10^{10}



Conventional Shell Model

all Slater determinants

Direct diagonalization

For even bigger problem,

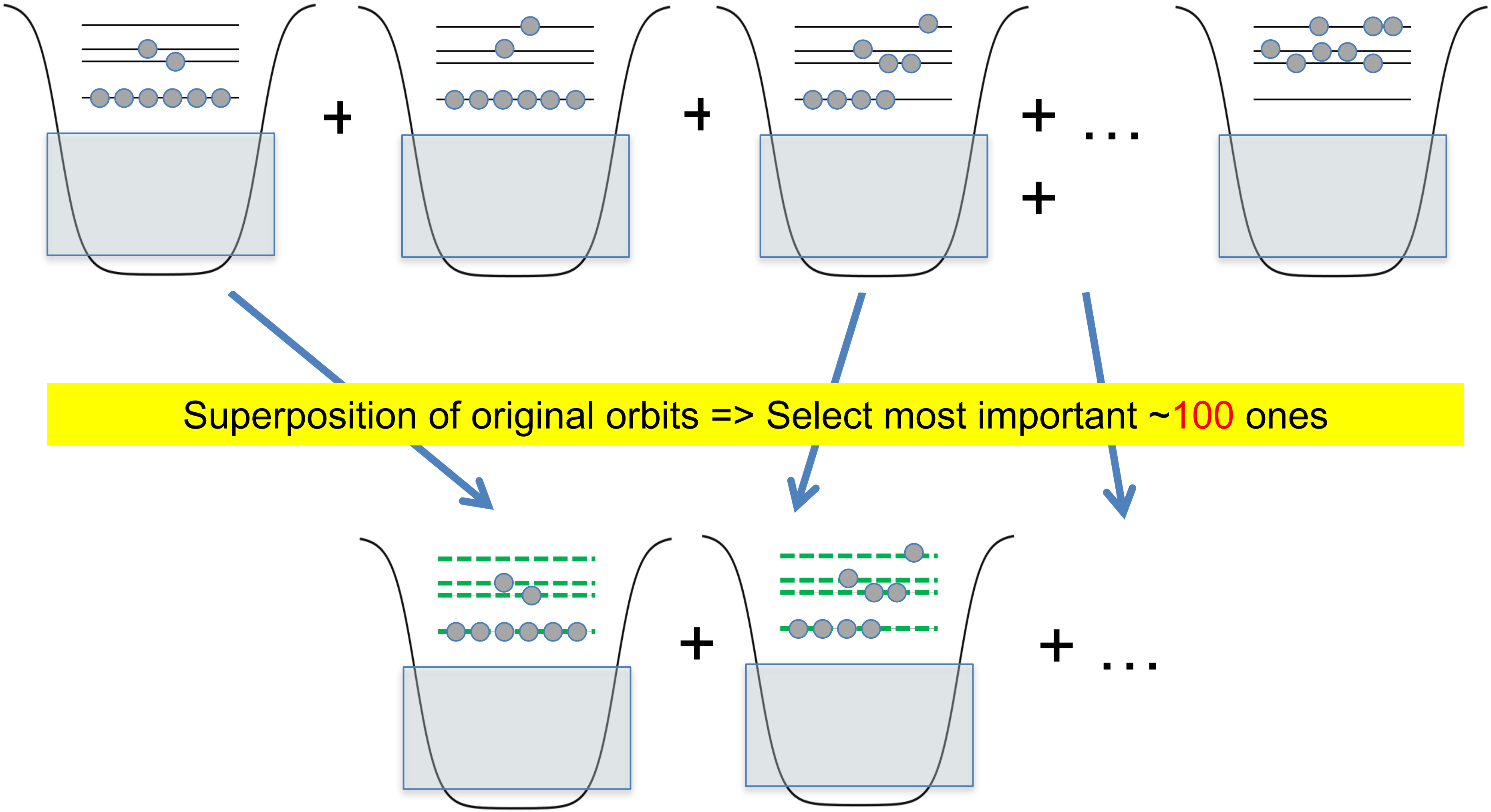
~100 dim.

Monte Carlo Shell Model

bases important for a specific eigenstate

Selected
important basis vectors

Possible configurations: 10^{23} ways at maximum for Zr isotopes to be discussed



Superposition of original orbits => Select most important ~100 ones

Advanced Monte Carlo Shell Model

N_B : number of basis vectors (dimension)

$$\left|\Psi(D)\right\rangle = \sum_{n=1}^{N_B} c_i P^{J,\Pi} \left|\phi(D^{(n)})\right\rangle$$

amplitude

Projection op.

N_p : number of (active) particles

N_{sp} : number of single-particle states

$$\left|\phi(D^{(n)})\right\rangle = \prod_{\alpha=1}^{N_p} \underbrace{\left(\sum_{i=1}^{N_{sp}} a_i^{\dagger} D_{i\alpha}^{(n)} \right)}_{\text{Deformed single-particle state}} \left|-\right\rangle$$

n-th basis vector (Slater determinant)

$$E(D) = \left\langle \Psi(D) \middle| H \middle| \Psi(D) \right\rangle$$

Minimize $E(D)$ as a function of D utilizing qMC and conjugate gradient methods

Step 1 : quantum Monte Carlo type method

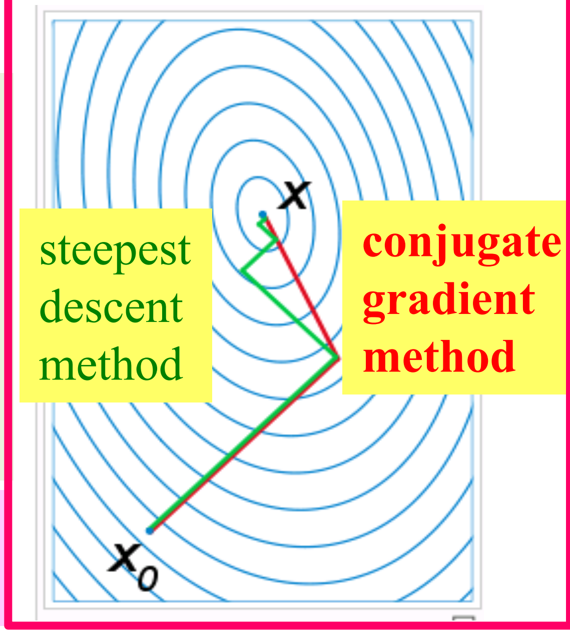
→ candidates of n-th basis vector (σ : set of random numbers)

$$\left|\phi(\sigma)\right\rangle = \prod e^{\Delta\beta \cdot h(\sigma)} \cdot \left|\phi^{(0)}\right\rangle$$

“ σ” can be represented by matrix D

Select the one producing the lowest *E(D)* (rate < 0.1 %)

Step 2 : polish *D* by means of the conjugate gradient method “variationally”.

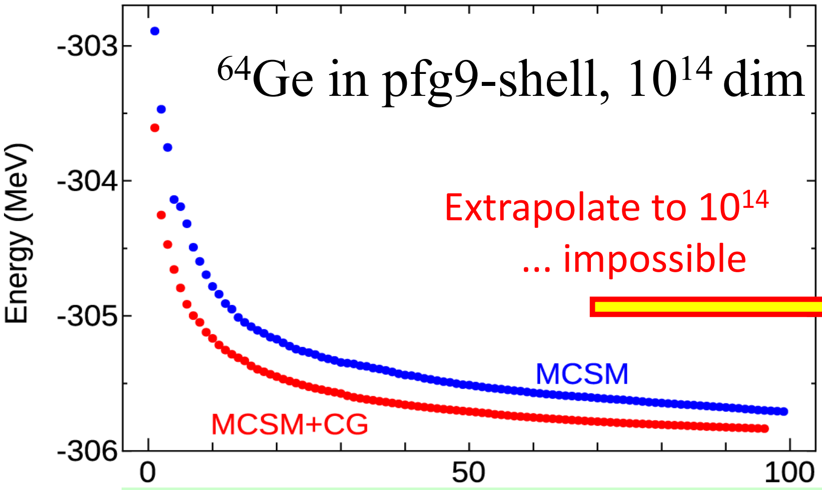


Step 3 : Energy variance extrapolation

Energy variance : $\langle \Delta H^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2$

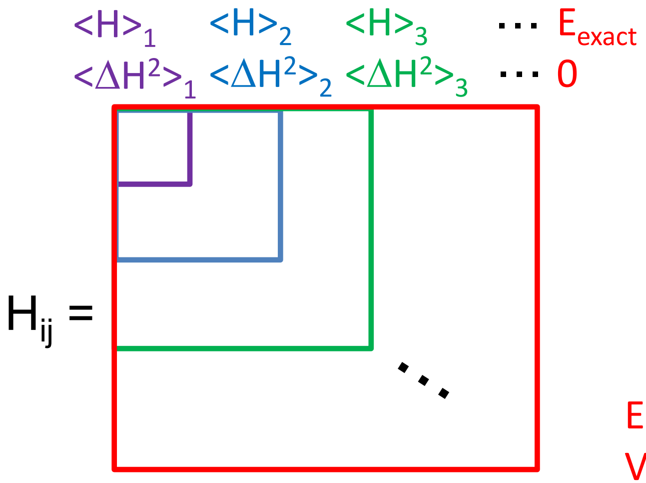
As the number of basis vectors increases, the approximated w.f. approaches the exact one and the energy variance approaches zero.

Extrapolate towards $\langle \Delta H^2 \rangle \rightarrow 0$

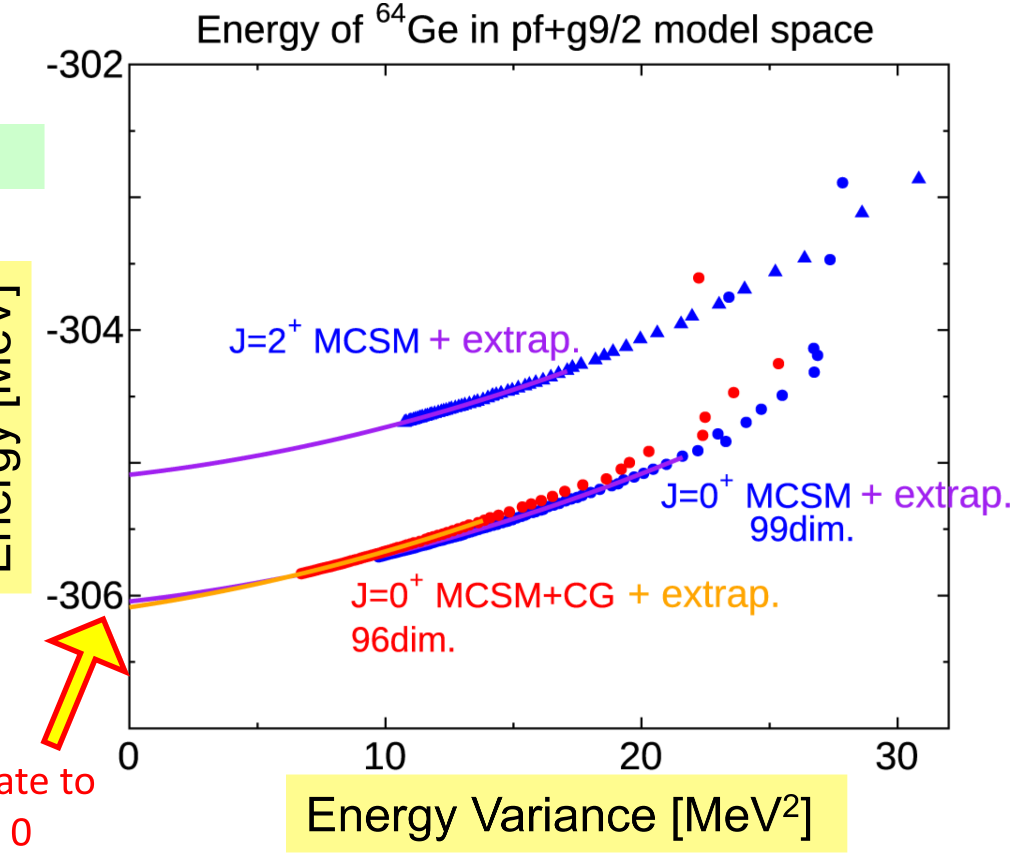


Extrapolate to 10^{14}
... impossible

Number of MCSM basis states (dimension)



Energy of ^{64}Ge in pf+g9/2 model space



Extrapolate to Variance 0

MCSM (Monte Carlo Shell Model *-Advanced version-*)

1. Selection of important many-body basis vectors by quantum Monte-Carlo + diagonalization methods

   basis vectors : about 100 selected Slater determinants composed of "deformed" single-particle states

2. Variational refinement of basis vectors

   conjugate gradient method

3. Variance extrapolation method -> exact eigenvalues

+ innovations in algorithm and code (=> now moving to GPU)

K computer (in Kobe) 10 peta flops machine

⇒ *Projection of basis vectors*

Rotation with three Euler angles

with about 50,000 mesh points

Example : 8^+ ^{68}Ni 7680 core x 14 h

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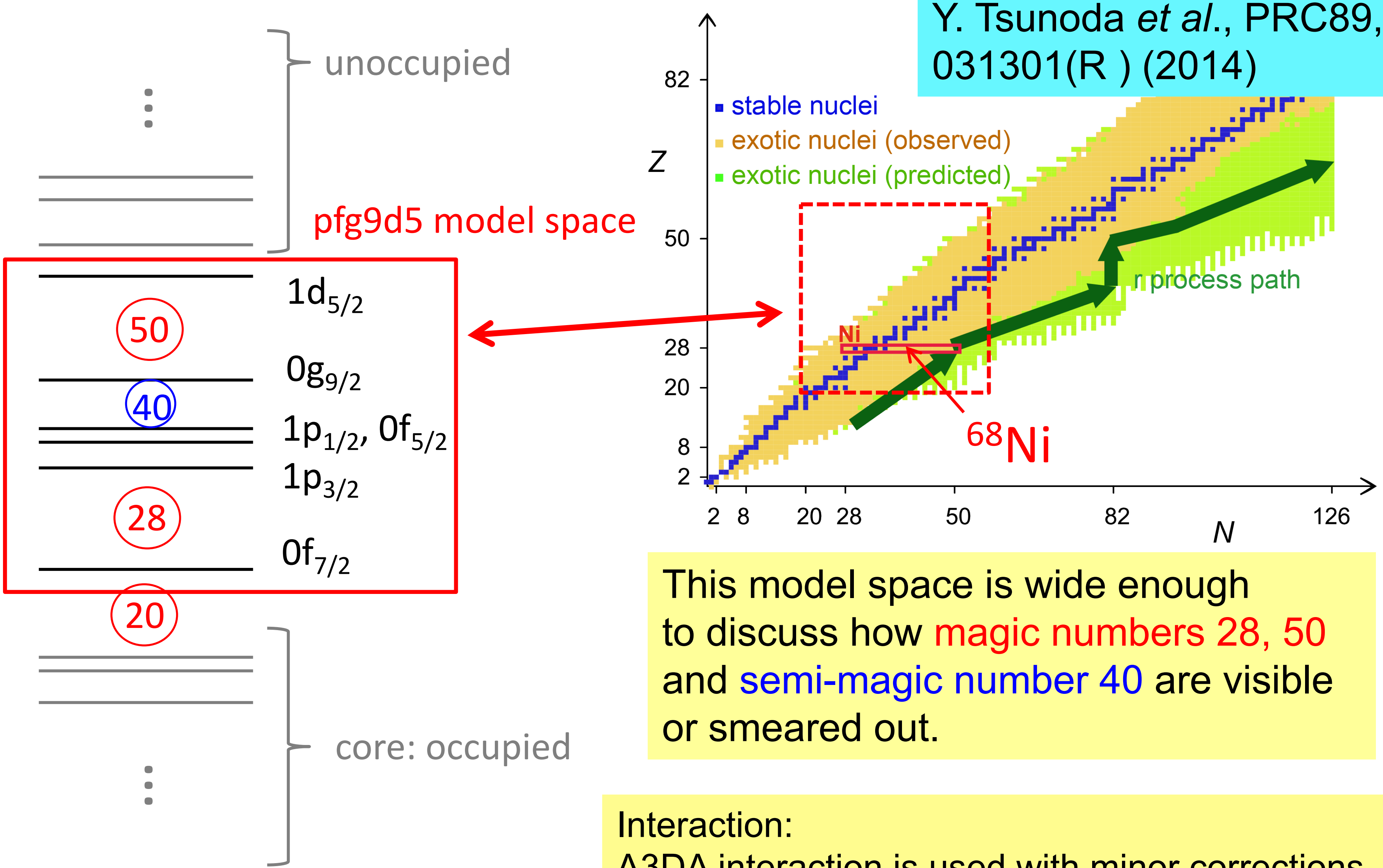
Monte Carlo Shell Model (MCSM) calculation on Ni isotopes

Y. Tsunoda *et al.*, PRC89, 031301(R) (2014)



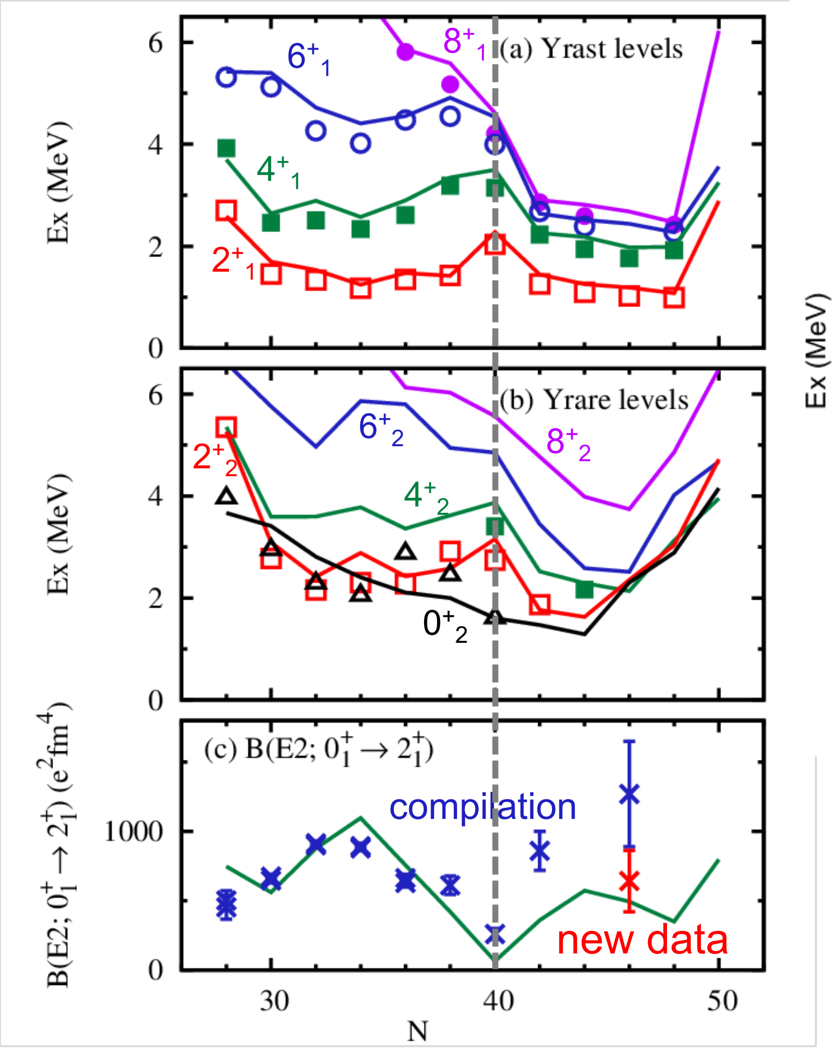
This model space is wide enough to discuss how magic numbers 28, 50 and semi-magic number 40 are visible or smeared out.

Interaction:
A3DA interaction is used with minor corrections

Energy levels and B(E2) values of Ni isotopes

Description by the same Hamiltonian

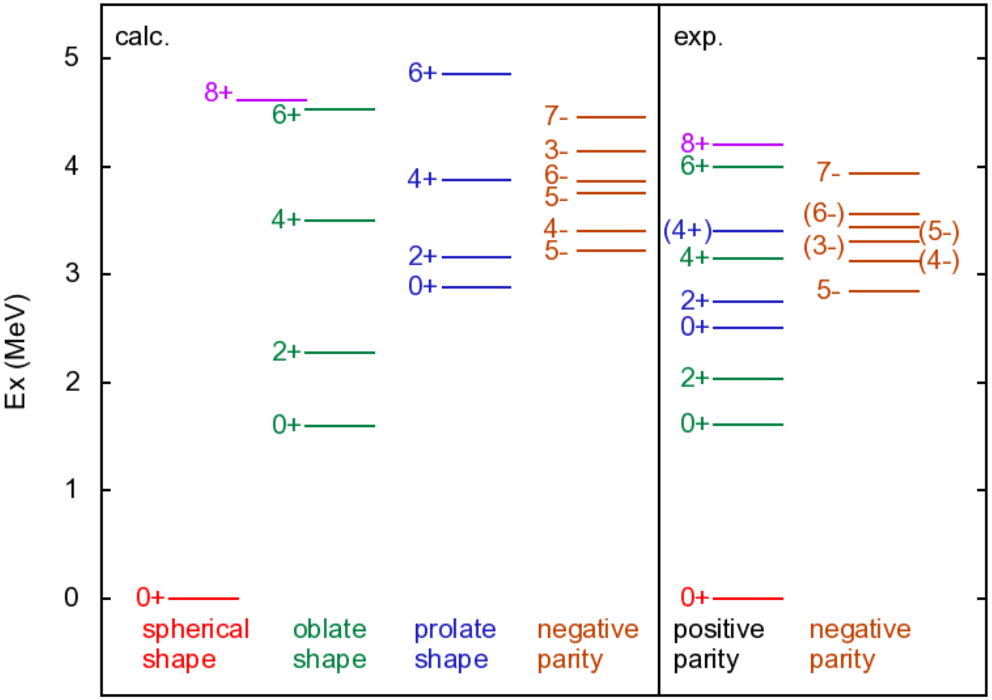
× exp.
— calc.



Shape coexistence in ^{68}Ni

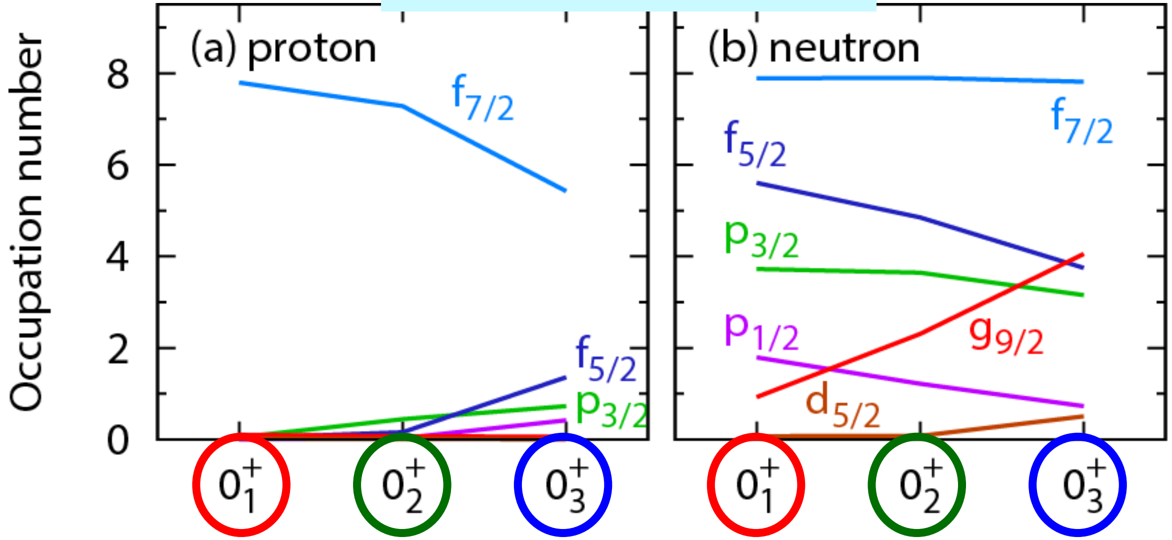
^{68}Ni

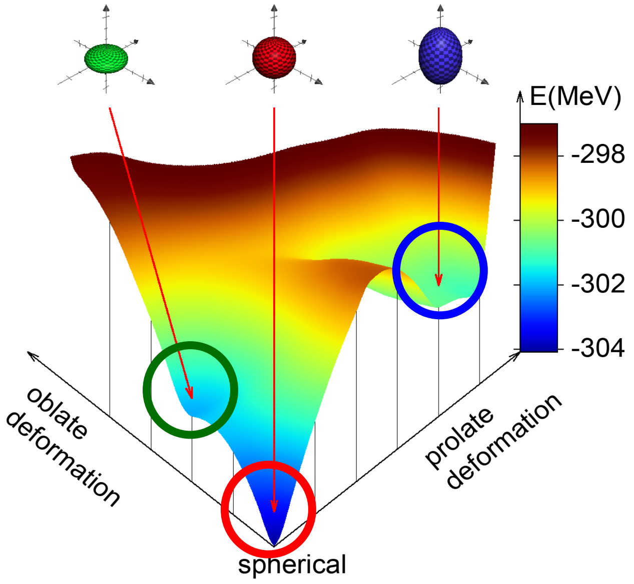
calc. exp.



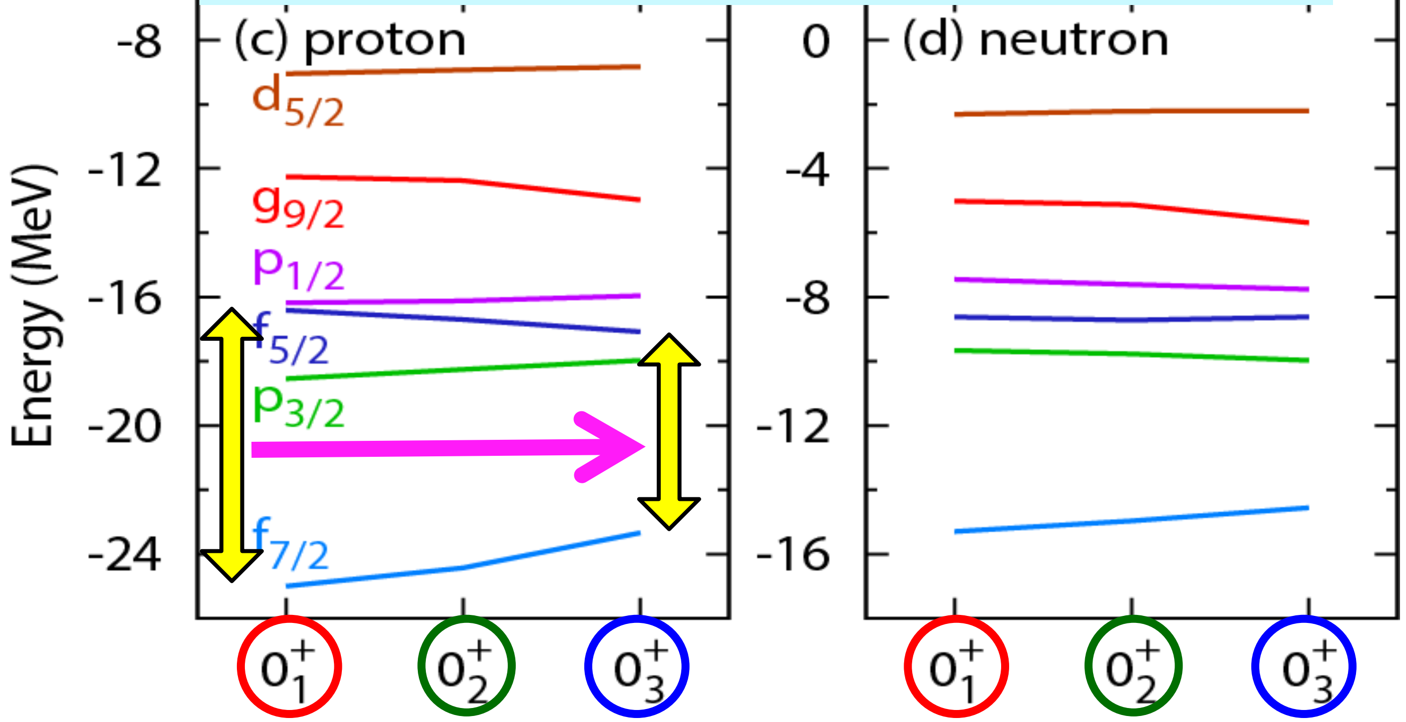
Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)

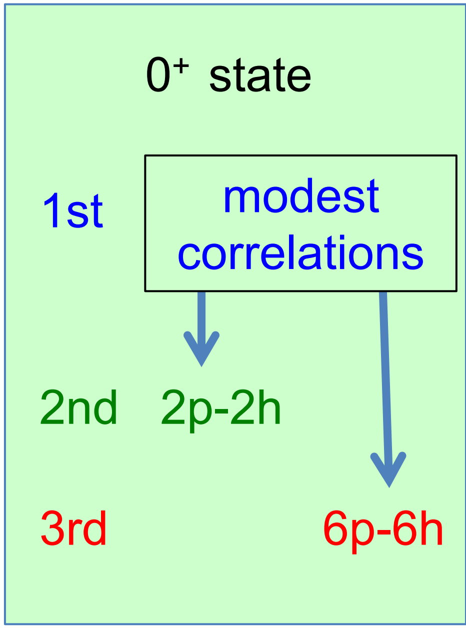
Occupation numbers





Effective s.p.e. by actual occupation numbers





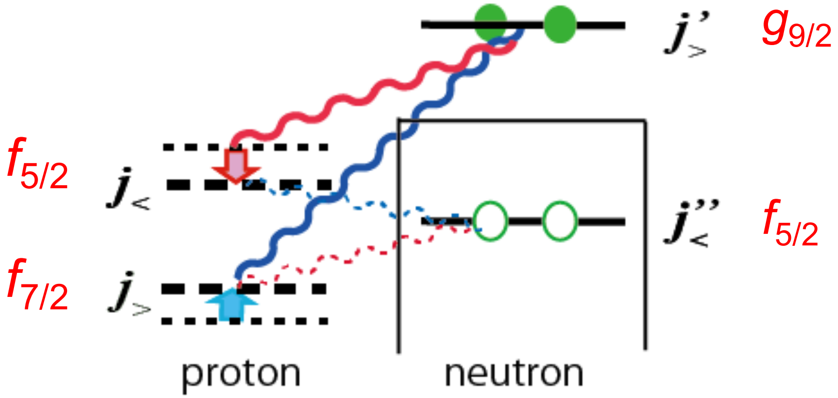
0^+ state

1st modest correlations

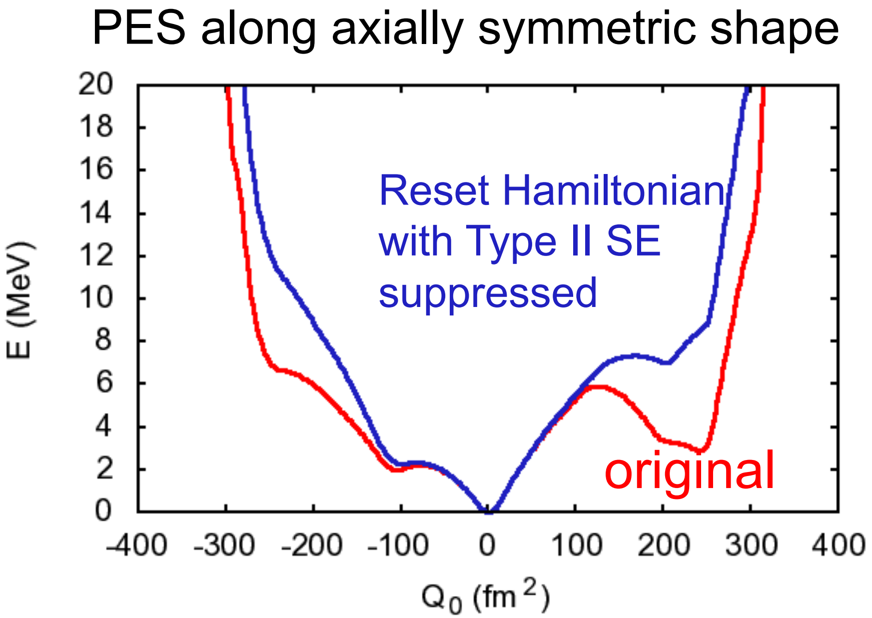
2nd 2p-2h

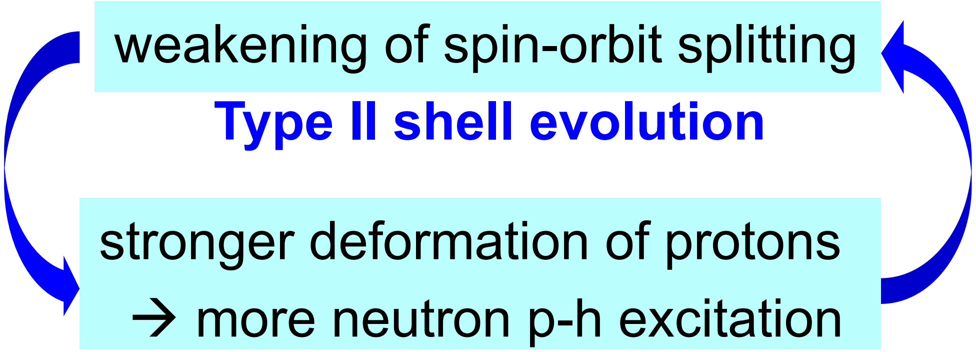
3rd 6p-6h

Type II Shell Evolution in ^{68}Ni (Z=28, N=40)



Spin-orbit splitting works against quadrupole deformation (*cf. Elliott's SU(3)*).


PES along axially symmetric shape



weakening of spin-orbit splitting

Type II shell evolution

stronger deformation of protons

→ more neutron p-h excitation

Type II shell evolution is suppressed by resetting monopole interactions as

$$\pi f_{7/2} - \nu g_{9/2} = \pi f_{5/2} - \nu g_{9/2}$$

$$\pi f_{7/2} - \nu f_{5/2} = \pi f_{5/2} - \nu f_{5/2}$$

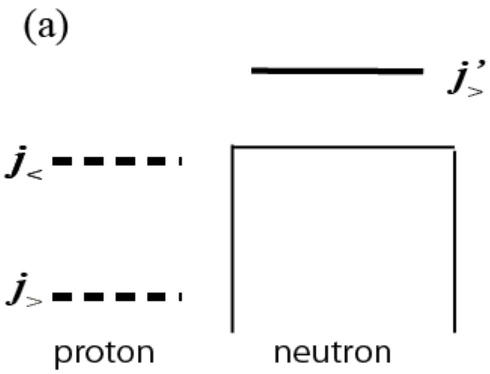
The local minima become much less pronounced.

Shape coexistence is enhanced by type II shell evolution because the same quadrupole interaction can work more efficiently.

Underlying mechanism of the appearance of low-lying deformed states : Type II Shell Evolution

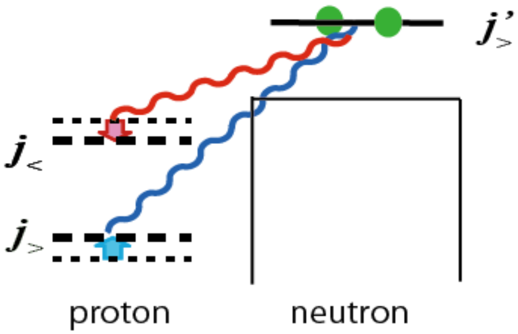
TO and Y. Tsunoda, J. Phys. G: Nucl. Part. Phys. 43 (2016) 024009

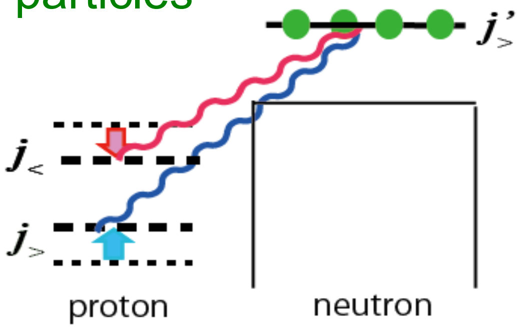
Monopole effects on the shell structure from the tensor interaction



Type I Shell Evolution : different isotopes

● : particles

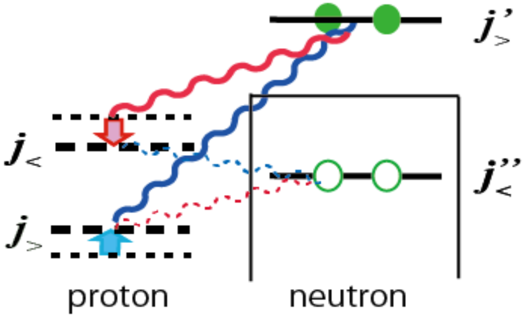


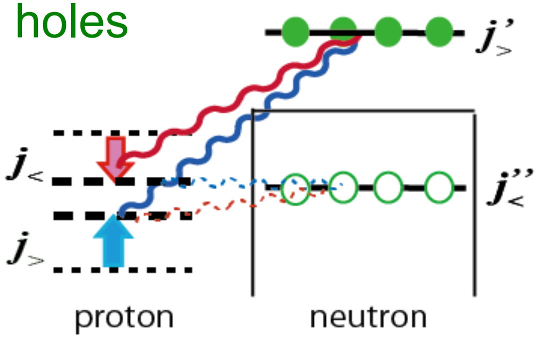


Type II Shell Evolution : within the same nucleus

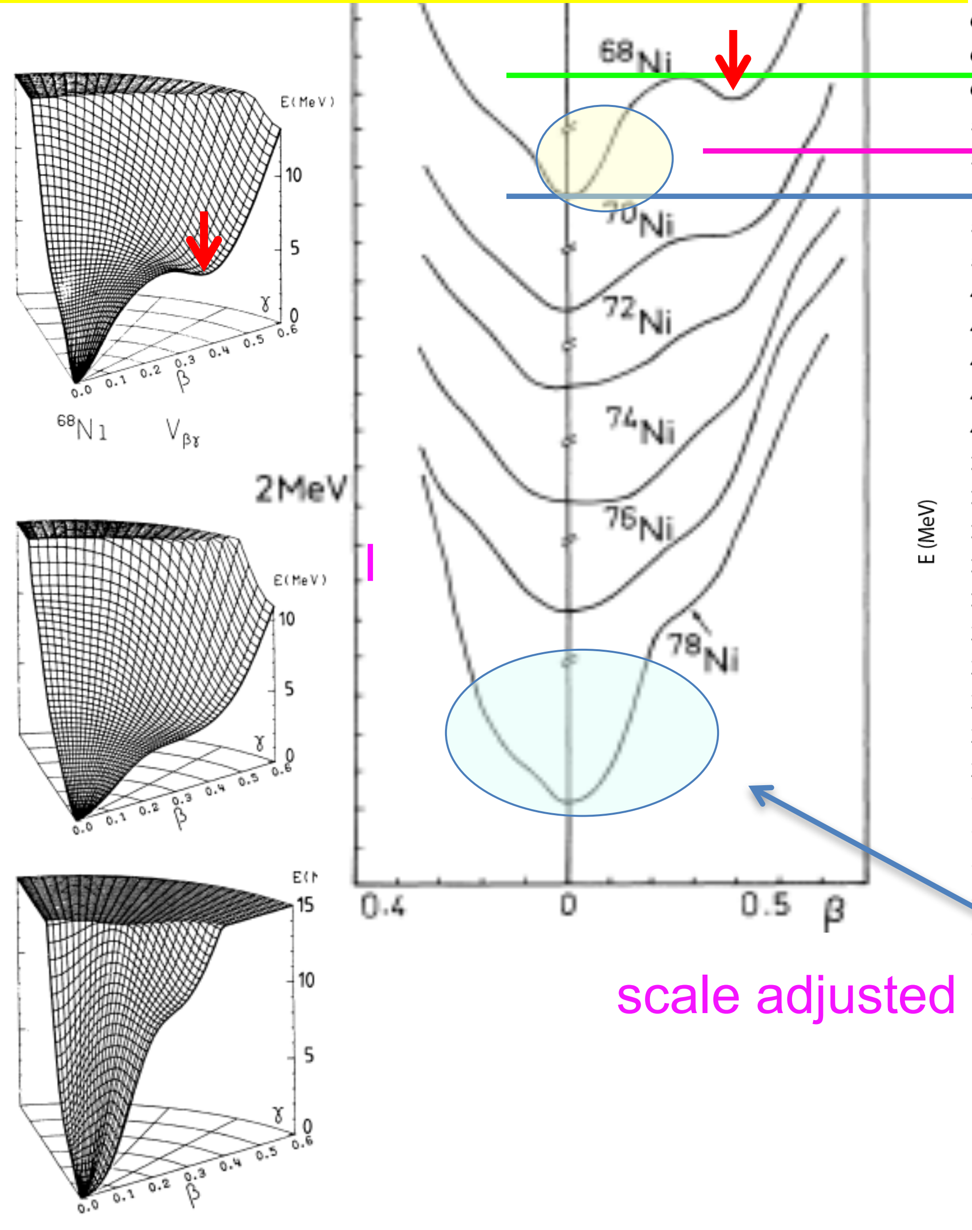
● : particles

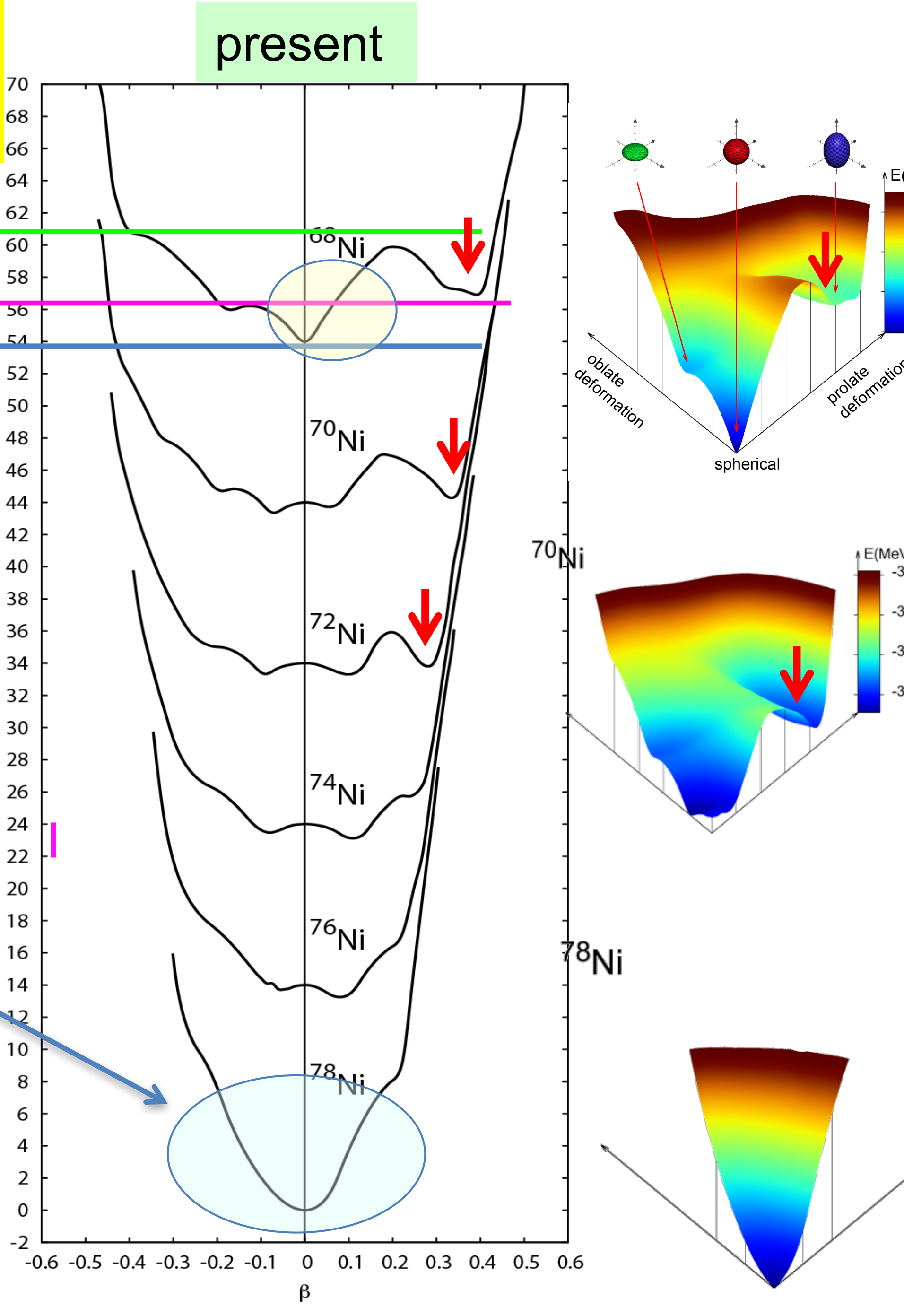
○ : holes



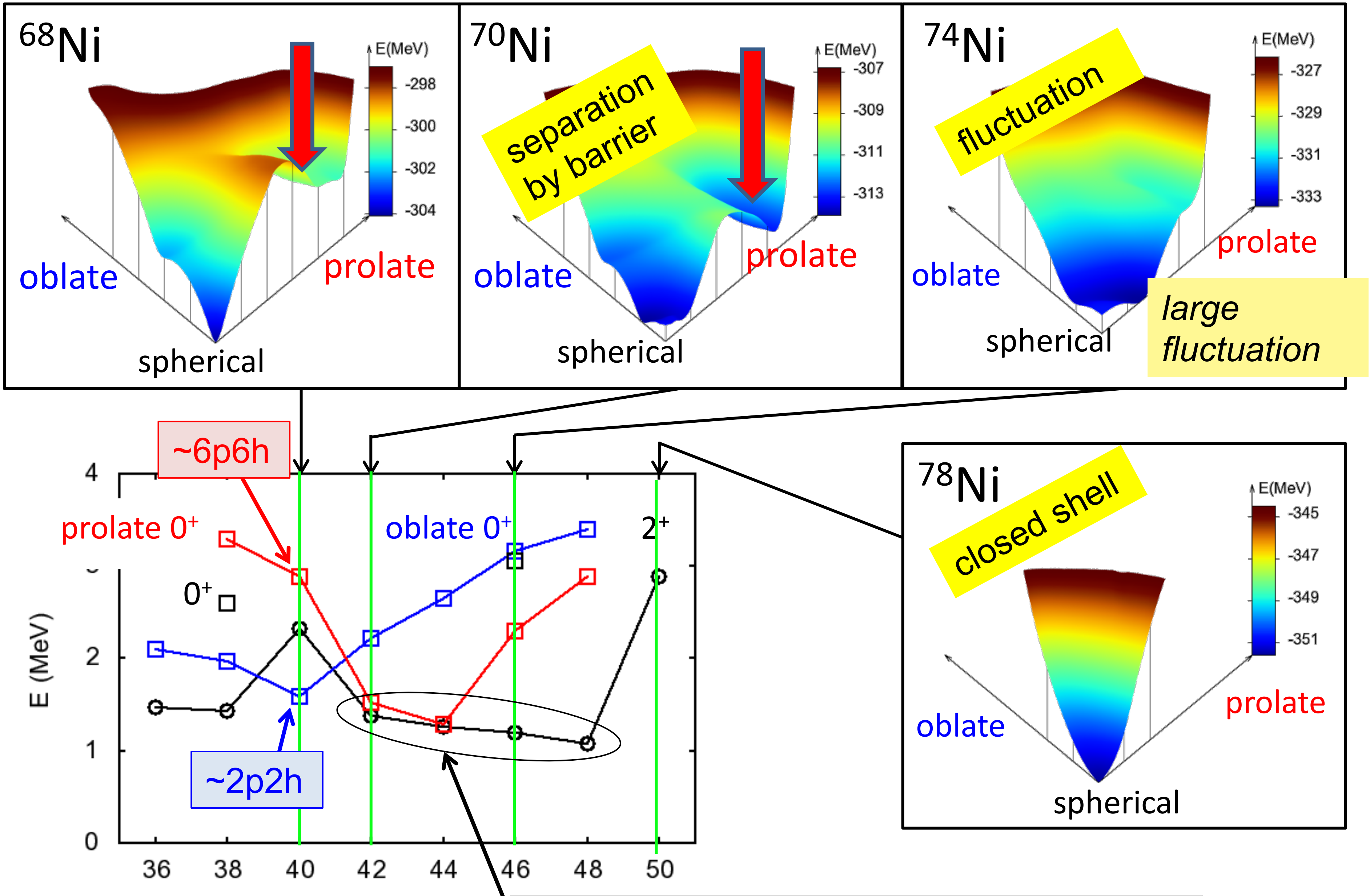


Bohr-model calc. by HFB with **Gogny** force, Girod, Dessagne, Bernes, Langevin, Pougheon and Roussel, PRC 37,2600 (1988)





Shape or structure evolution of Ni isotopes

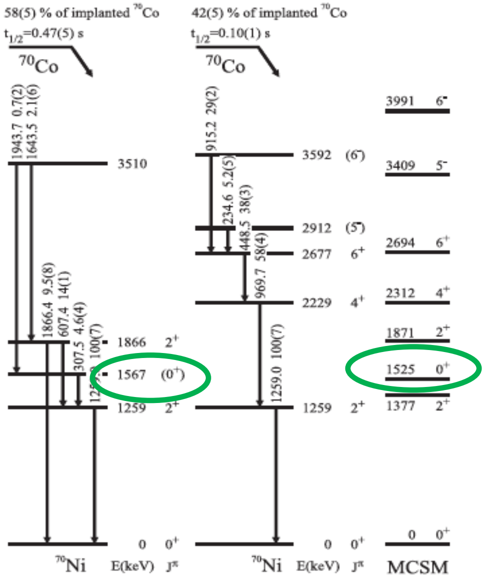


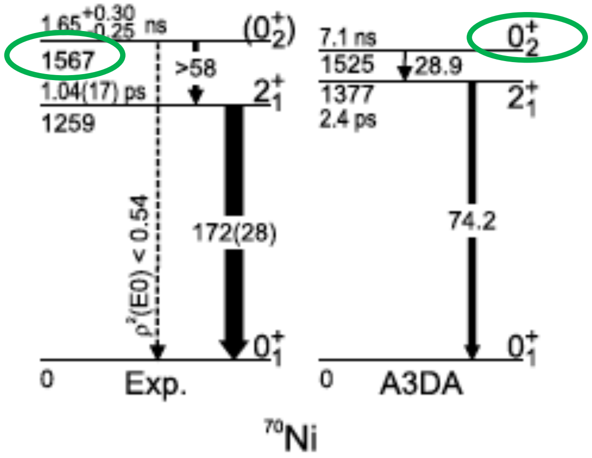
⇔ 2^+ states in the $g_{9/2}$ seniority scheme

Low-lying 0^+_2 in ^{70}Ni : prediction and verification

PHYSICAL REVIEW C **92**, 061302(R) (2015)

C. J. Prokop,[1,2,*] B. P. Crider,[1] S. N. Liddick,[1,2] *et al.*





Physics Letters B 763 (2016) 108–113

Shape coexistence from lifetime and branching-ratio measurements in ^{68,70}Ni

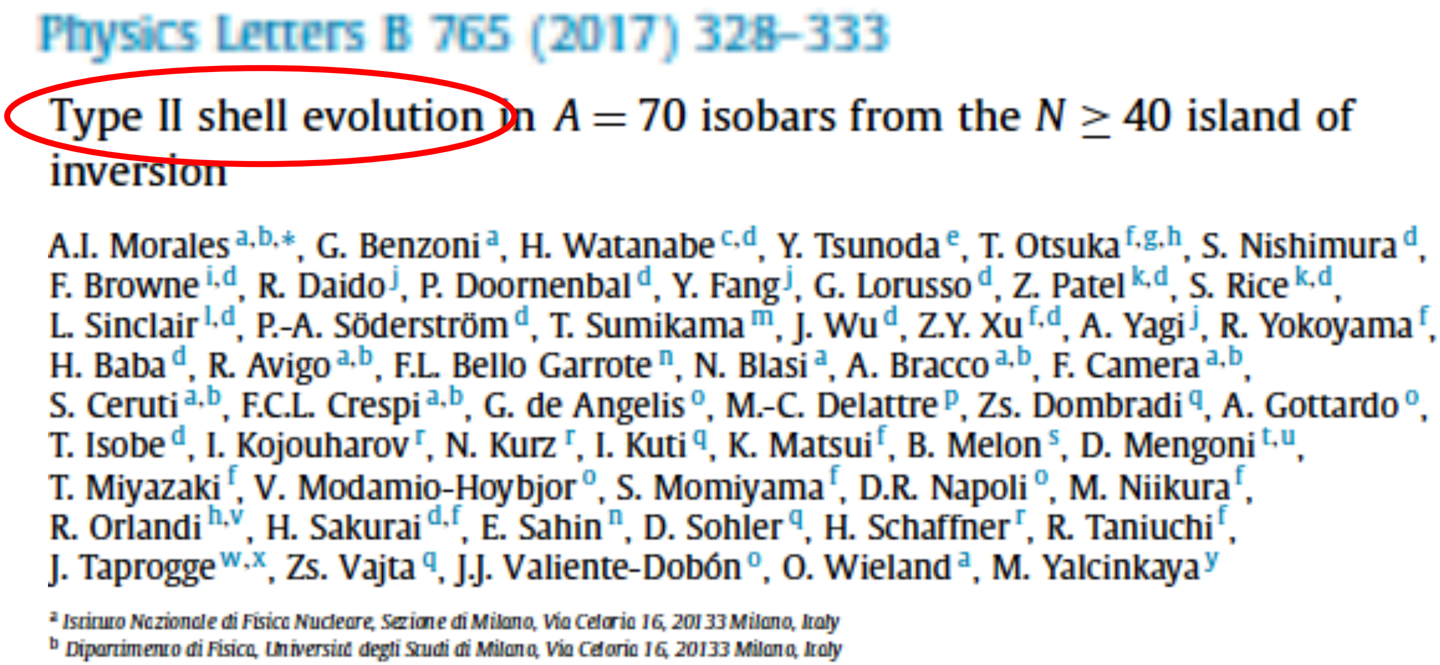
B.P. Crider[a,*], C.J. Prokop[a,b], S.N. Liddick[a,b], M. Al-Shudifat[c], A.D. Ayangeakaa[d], M.P. Carpenter[d], J.J. Carroll[e], J. Chen[a], C.J. Chiara[f], H.M. David[d,1], A.C. Dombos[a,g], S. Go[c], R. Grzywacz[c,h], J. Harker[d,i], R.V.F. Janssens[d], N. Larson[a,b], T. Lauritsen[d], R. Lewis[a,b], S.J. Quinn[a,g], F. Recchia[j], A. Spyrou[a,g], S. Suchyta[k], W.B. Walters[i], S. Zhu[d]

Upon addition of just two neutrons leading to ^{70}Ni, the expectations for shape coexistence differ. Some models predict spherical-prolate shape coexistence [10,19,16] while others predict no shape coexistence at all [23–25]. The recent observation of a tentative 0+ state at 1567 keV in ^{70}Ni [11] suggested a drop in excitation energy of the prolate potential minimum, in line with theoretical expectations for the neutron-rich, even-Ni isotopes. The measure-

[10] S. Suchyta, S.N. Liddick, Y. Tsunoda, T. Otsuka, M.B. Bennett, A. Chemey, M. Honma, N. Larson, C.J. Prokop, S.J. Quinn, N. Shimizu, A. Simon, A. Spyrou, V. Tripathi, Y. Utsuno, J.M. VonMoss, Shape coexistence in ^{68}Ni, Phys. Rev. C 89 (2014) 021301, http://dx.doi.org/10.1103/PhysRevC.89.021301.

[19] Y. Tsunoda, T. Otsuka, N. Shimizu, M. Honma, Y. Utsuno, Novel shape evolution in exotic Ni isotopes and configuration-dependent shell structure, Phys. Rev. C 89 (2014) 031301, http://dx.doi.org/10.1103/PhysRevC.89.031301.

[16] F. Flavigny, D. Pauwels, D. Radulov, I.J. Darby, H. De Witte, J. Diriken, D.V. Fedorov, V.N. Fedosseev, L.M. Fraile, M. Huyse, V.S. Ivanov, U. Köster, B.A. Marsh, T. Otsuka, L. Popescu, R. Raabe, M.D. Seliverstov, N. Shimizu, A.M. Sjödin, Y. Tsunoda, P. Van den Bergh, P. Van Duppen, J. Van de Walle, M. Venhart, W.B. Walters, K. Wimmer, Characterization of the low-lying 0+ and 2+ states in ^{68}Ni via β decay of the low-spin ^{68}Co isomer, Phys. Rev. C 91 (2015) 034310, http://dx.doi.org/10.1103/PhysRevC.91.034310.



Physics Letters B 765 (2017) 328–333

Type II shell evolution in $A = 70$ isobars from the $N \geq 40$ island of inversion

A.I. Morales [a,b,*], G. Benzoni [a], H. Watanabe [c,d], Y. Tsunoda [e], T. Otsuka [f,g,h], S. Nishimura [d], F. Browne [i,d], R. Daido [j], P. Doornenbal [d], Y. Fang [j], G. Lorusso [d], Z. Patel [k,d], S. Rice [k,d], L. Sinclair [l,d], P.-A. Söderström [d], T. Sumikama [m], J. Wu [d], Z.Y. Xu [f,d], A. Yagi [j], R. Yokoyama [f], H. Baba [d], R. Avigo [a,b], F.L. Bello Garrote [n], N. Blasi [a], A. Bracco [a,b], F. Camera [a,b], S. Ceruti [a,b], F.C.L. Crespi [a,b], G. de Angelis [o], M.-C. Delattre [p], Zs. Dombradi [q], A. Gottardo [o], T. Isobe [d], I. Kojouharov [r], N. Kurz [r], I. Kuti [q], K. Matsui [f], B. Melon [s], D. Mengoni [t,u], T. Miyazaki [f], V. Modamio-Hoybjor [o], S. Momiyama [f], D.R. Napoli [o], M. Niikura [f], R. Orlandi [h,v], H. Sakurai [d,f], E. Sahin [n], D. Sohler [q], H. Schaffner [r], R. Taniuchi [f], J. Taprogge [w,x], Zs. Vajta [q], J.J. Valiente-Dobón [o], O. Wieland [a], M. Yalcinkaya [y]

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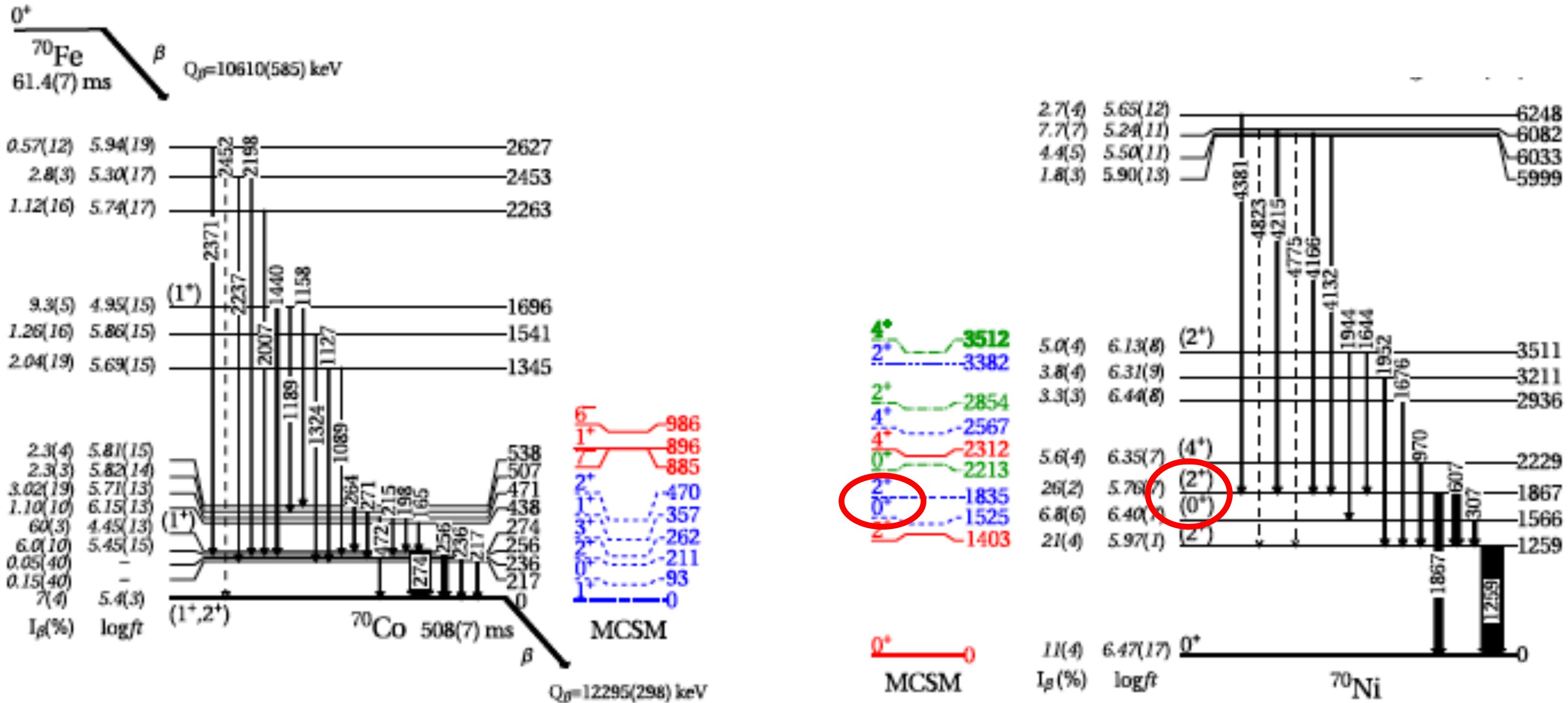


MCSM basis vectors on Potential Energy Surface

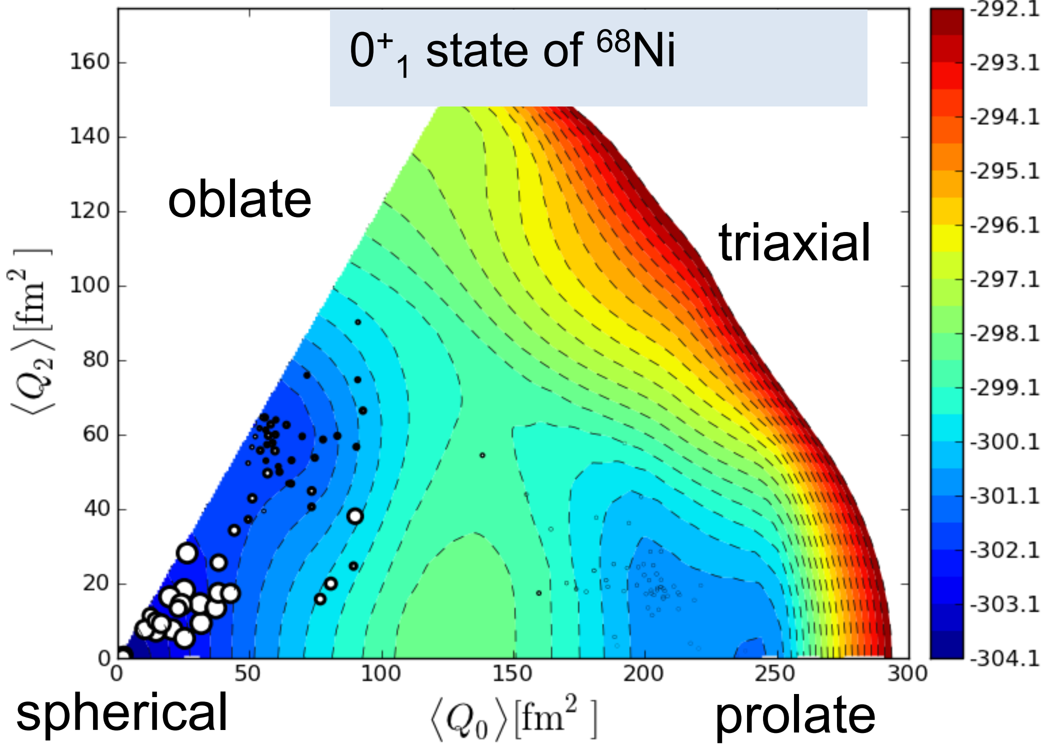
eigenstate $\Psi = \sum_i c_i P[J^\pi] \Phi_i$

Slater determinant of deformed s. p. states → intrinsic shape

amplitude

projection onto J^π

- PES is calculated by CHF for the shell-model Hamiltonian
- Location of circle : quadrupole deformation of unprojected MCSM basis vectors
- Area of circle : overlap probability between each projected basis and eigen wave function



Called ***T-plot*** in reference to

Y. Tsunoda, *et al.*

PRC 89, 031301 (R) (2014)

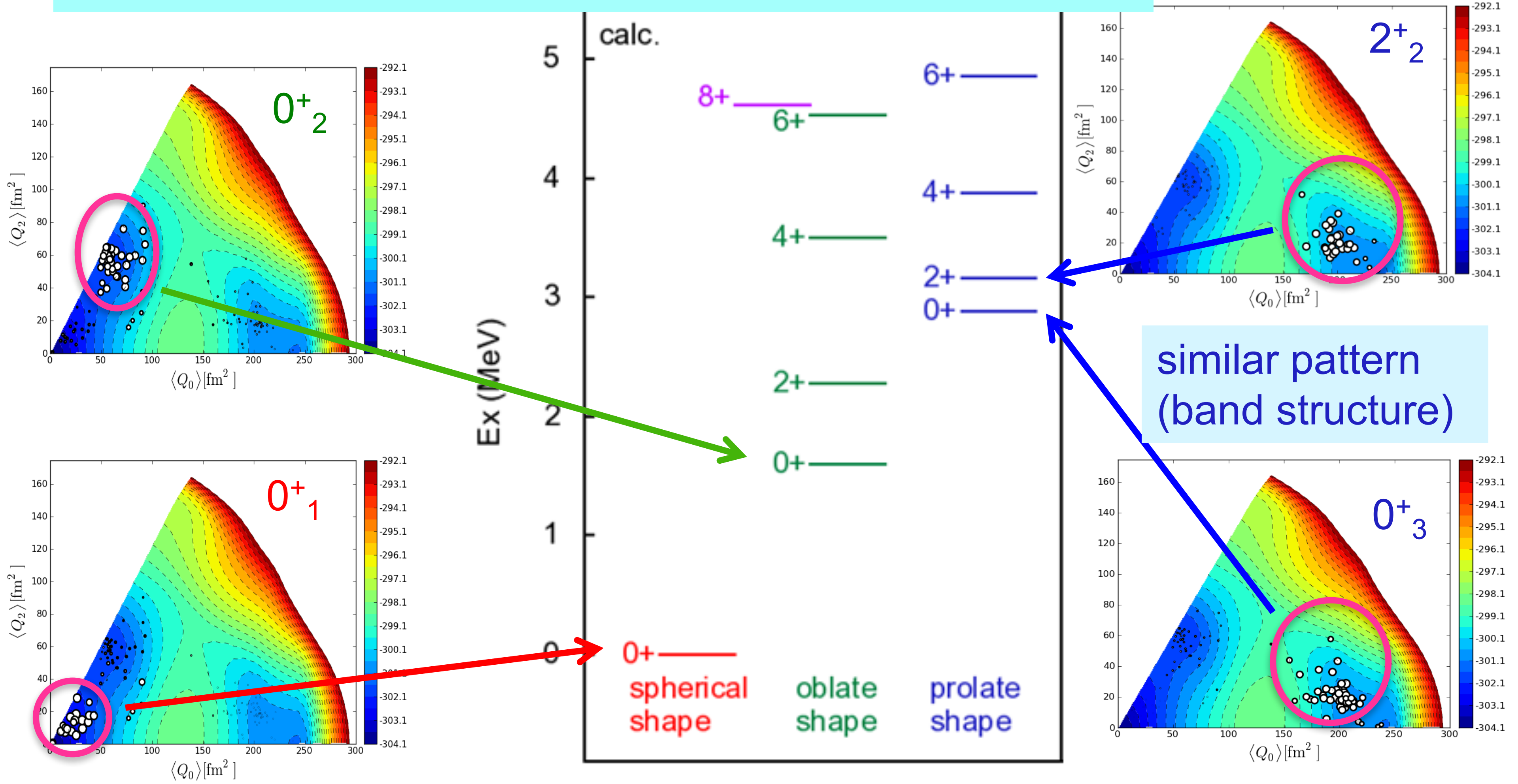
General properties of T-plot :

Certain number of large circles in a small region of PES

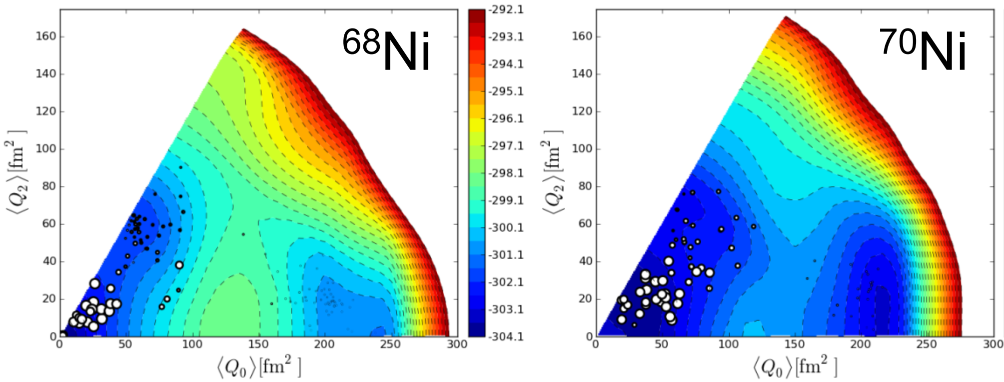
⇔ pairing correlations

Spreading beyond this can be due to shape fluctuation

Example : shape assignment to various 0^+ states of ^{68}Ni



similar pattern (band structure)



Energy of prolate state comes down.
Barrier becomes low.

T-plot

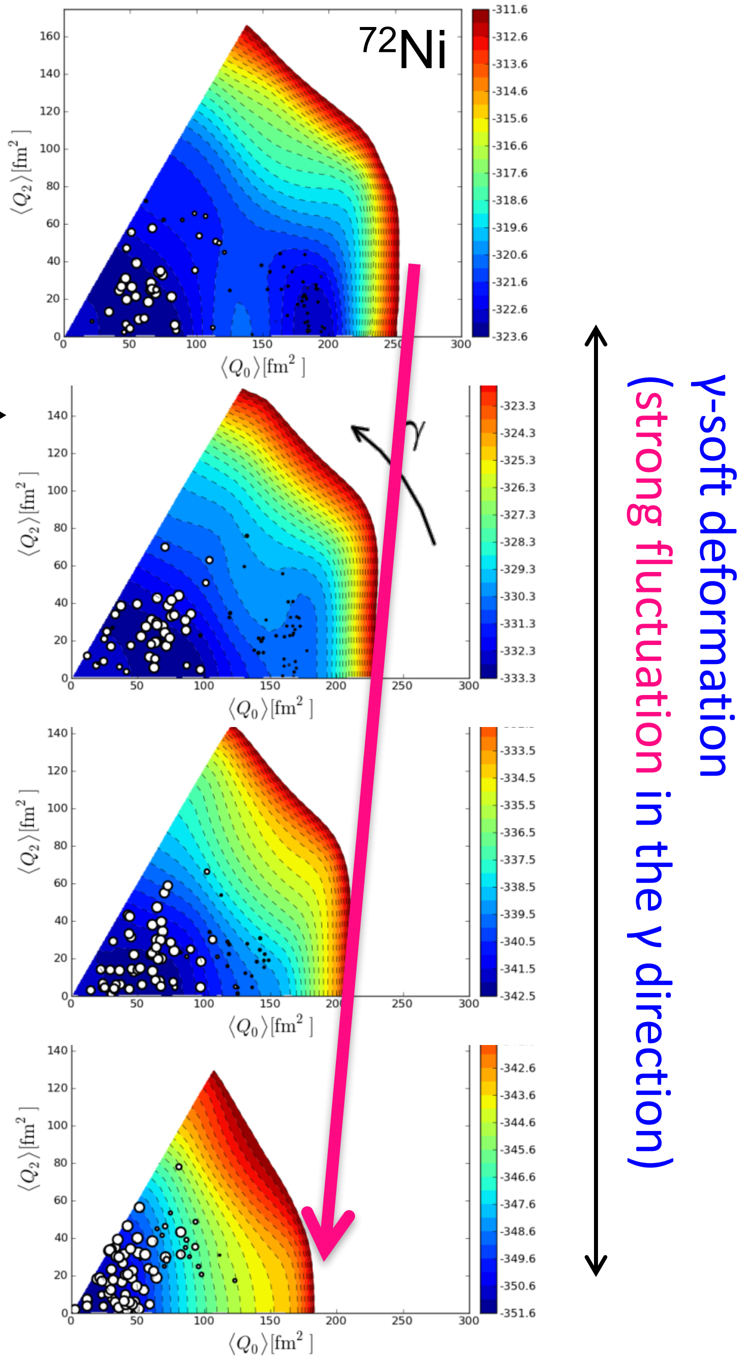
for 0^+_1 states of $^{68-78}$Ni

The ground state is always like seniority-zero (BCS-type) spherical state.

^{74}Ni

^{76}Ni

^{78}Ni

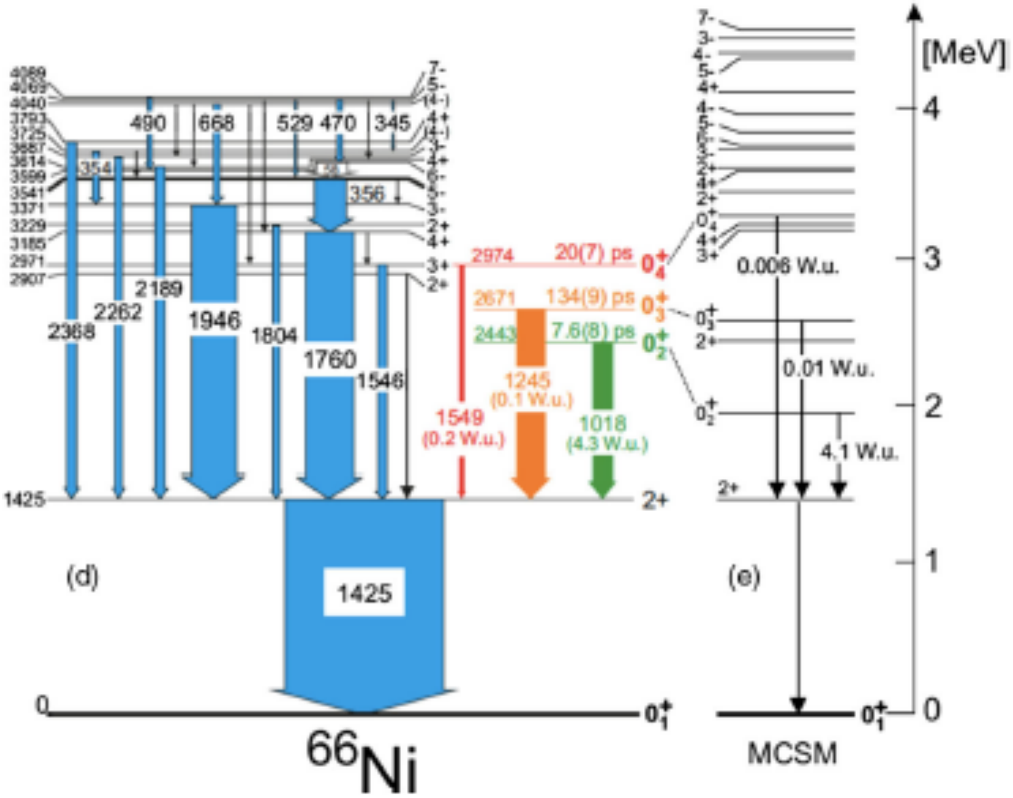


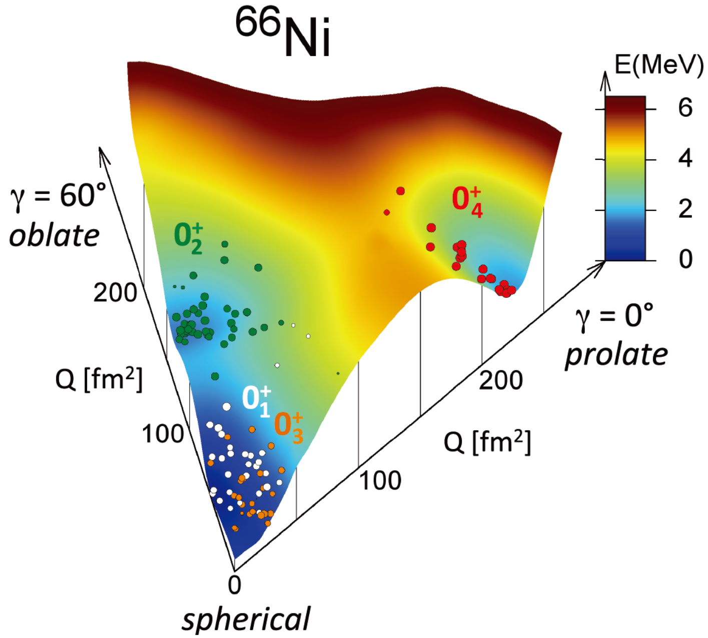
PRL 118, 162502 (2017) PHYSICAL REVIEW LETTERS week ending 21 APRIL 2017

Multifaceted Quadruplet of Low-Lying Spin-Zero States in ^{66}Ni: Emergence of Shape Isomerism in Light Nuclei

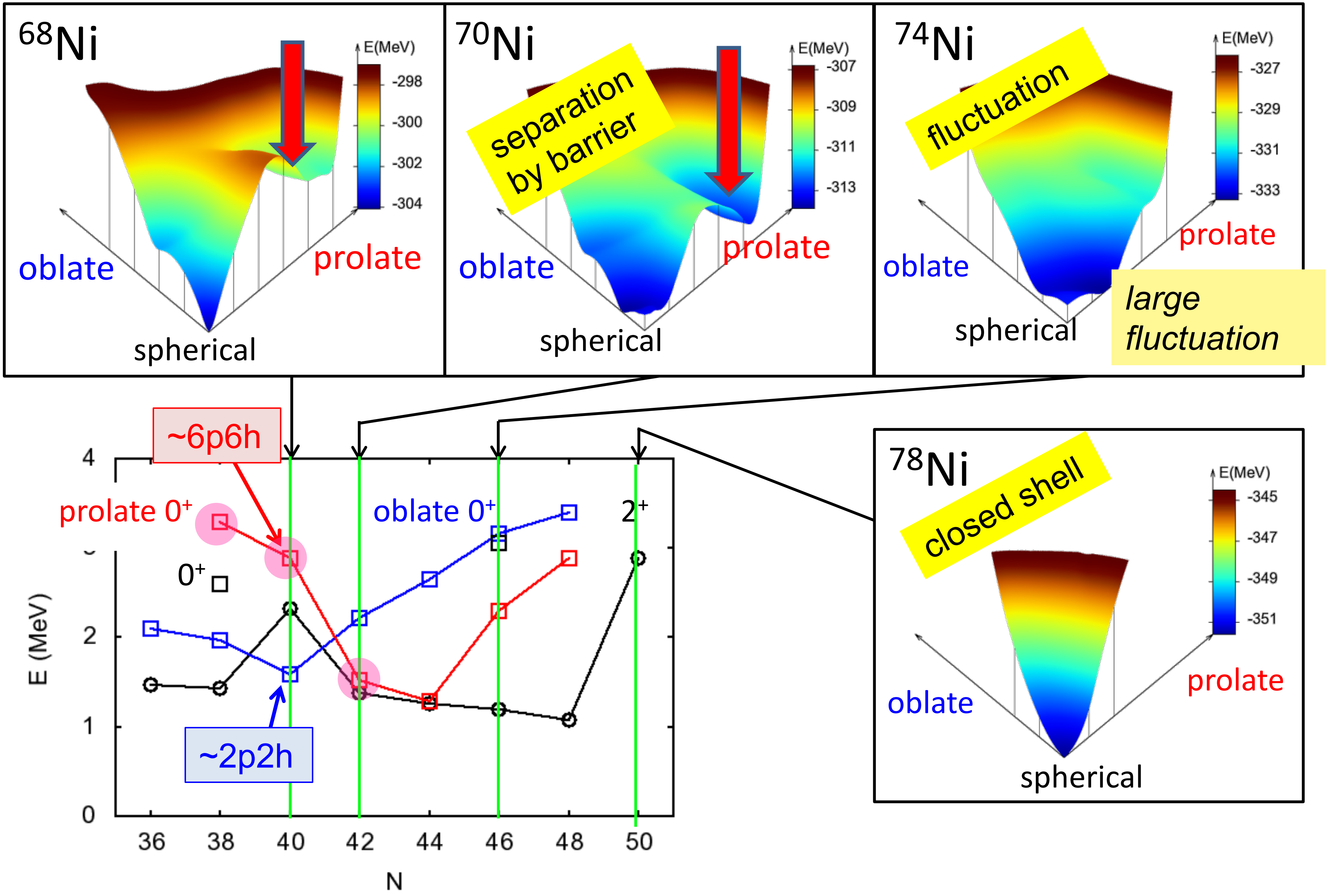
S. Leoni,[1,2,*] B. Fornal,[3] N. Mărginean,[4] M. Sferrazza,[5] Y. Tsunoda,[6] T. Otsuka,[6,7,8,9] G. Bocchi,[1,2] F. C. L. Crespi,[1,2] A. Bracco,[1,2] S. Aydin,[10] M. Boromiza,[4,11] D. Bucurescu,[4] N. Cieplicka-Oryńczak,[2,3] C. Costache,[4] S. Călinescu,[4] N. Florea,[4] D. G. Ghiță,[4] T. Glodariu,[4] A. Ionescu,[4,11] Ł. W. Iskra,[3] M. Krzysiek,[3] R. Mărginean,[4] C. Mihai,[4] R. E. Mihai,[4] A. Mitu,[4] A. Negreț,[4] C. R. Niță,[4] A. Olăcel,[4] A. Oprea,[4] S. Pascu,[4] P. Petkov,[4] C. Petrone,[4] G. Porzio,[1,2] A. Șerban,[4,11] C. Sotty,[4] L. Stan,[4] I. Știru,[4] L. Stroe,[4] R. Șuvăilă,[4] S. Toma,[4] A. Turturică,[4] S. Ujeniuc,[4] and C. A. Ur[12]

[1]*Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy*
[2]*INFN sezione di Milano via Celoria 16, 20133, Milano, Italy*
[3]*Institute of Nuclear Physics, PAN, 31-342 Kraków, Poland*





Shape or structure evolution of Ni isotopes



Outline

I Introduction

II Presently used numerical methodology of many-body problems

III First application of MCSM to shape coexistence: Ni isotopes

IV An example from Quantum Phase Transition in Zr isotopes

V Basic mechanism

VI Shape coexistence and/or critical phenomena in Hg/Pb isotopes

VII Remarks

An example : shapes of Zr isotopes by Monte Carlo Shell Model

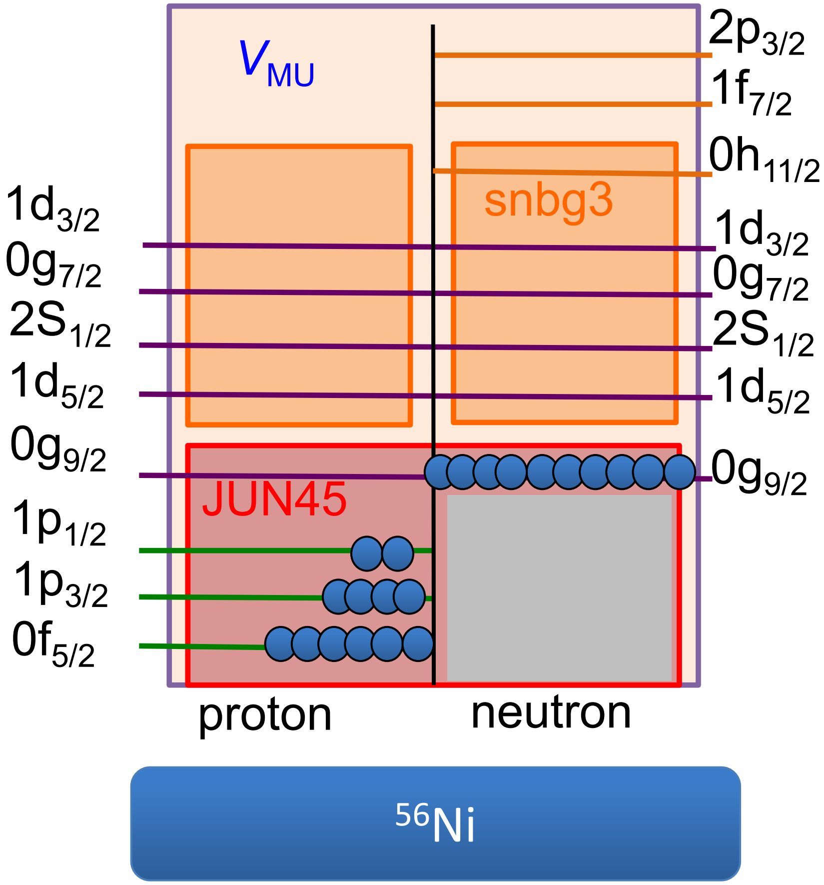
- Effective interaction:
  JUN45 + snbg3 + V_{MU}

  known effective interactions

  + minor fit for a part of T=1 TBME's

  Nucleons are excited fully within this model space (no truncation)

We performed Monte Carlo Shell Model (MCSM) calculations, where the largest case corresponds to the diagonalization of 3.7×10^{23} dimension matrix.



Togashi, Tsunoda, TO *et al*. PRL 117, 172502 (2016)

From earlier shell-model works …

PHYSICAL REVIEW C VOLUME 20, NUMBER 2 AUGUST 1979

Unified shell-model description of nuclear deformation

P. Federman

Instituto de Fisica, Universidad Nacional Autonoma de Mexico, Apartado Postal 20-364, Mexico 20, D. F.

S. Pittel

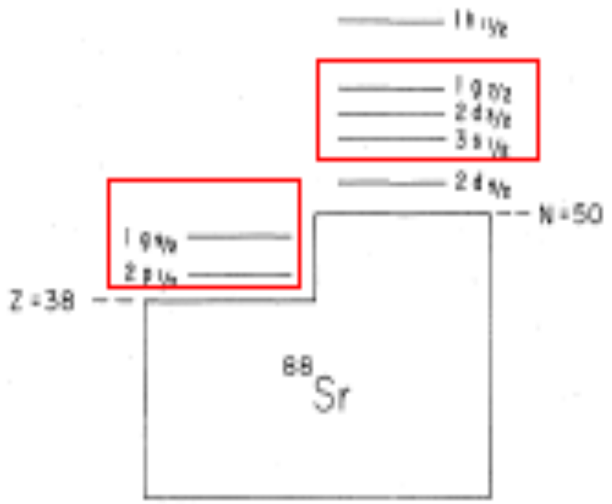


FIG. 3. Single-particle levels appropriate to a description of nuclei in the Zr-Mo region. An ^{88}Sr core is assumed.

PHYSICAL REVIEW C **79**, 064310 (2009)

Shell model description of zirconium isotopes

K. Sieja,[1,2] F. Nowacki,[3] K. Langanke,[2,4] and G. Martínez-Pinedo[1]

In this paper, we perform for the first time a SM study of Zr isotopes in an extended model space ($1f_{5/2}$, $2p_{1/2}$, $2p_{3/2}$, $1g_{9/2}$) for protons and ($2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1g_{7/2}$, $1h_{11/2}$) for neutrons, dubbed hereafter $\pi(r3-g)$, $\nu(r4-h)$.

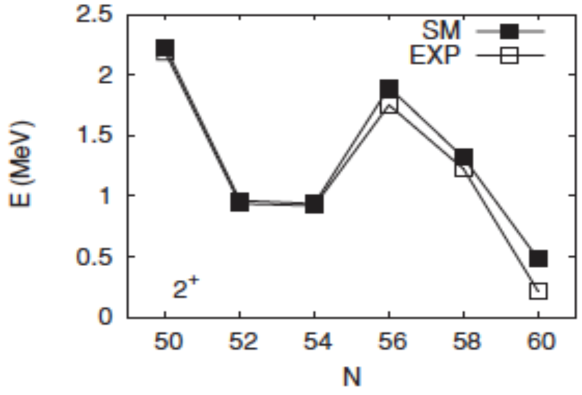
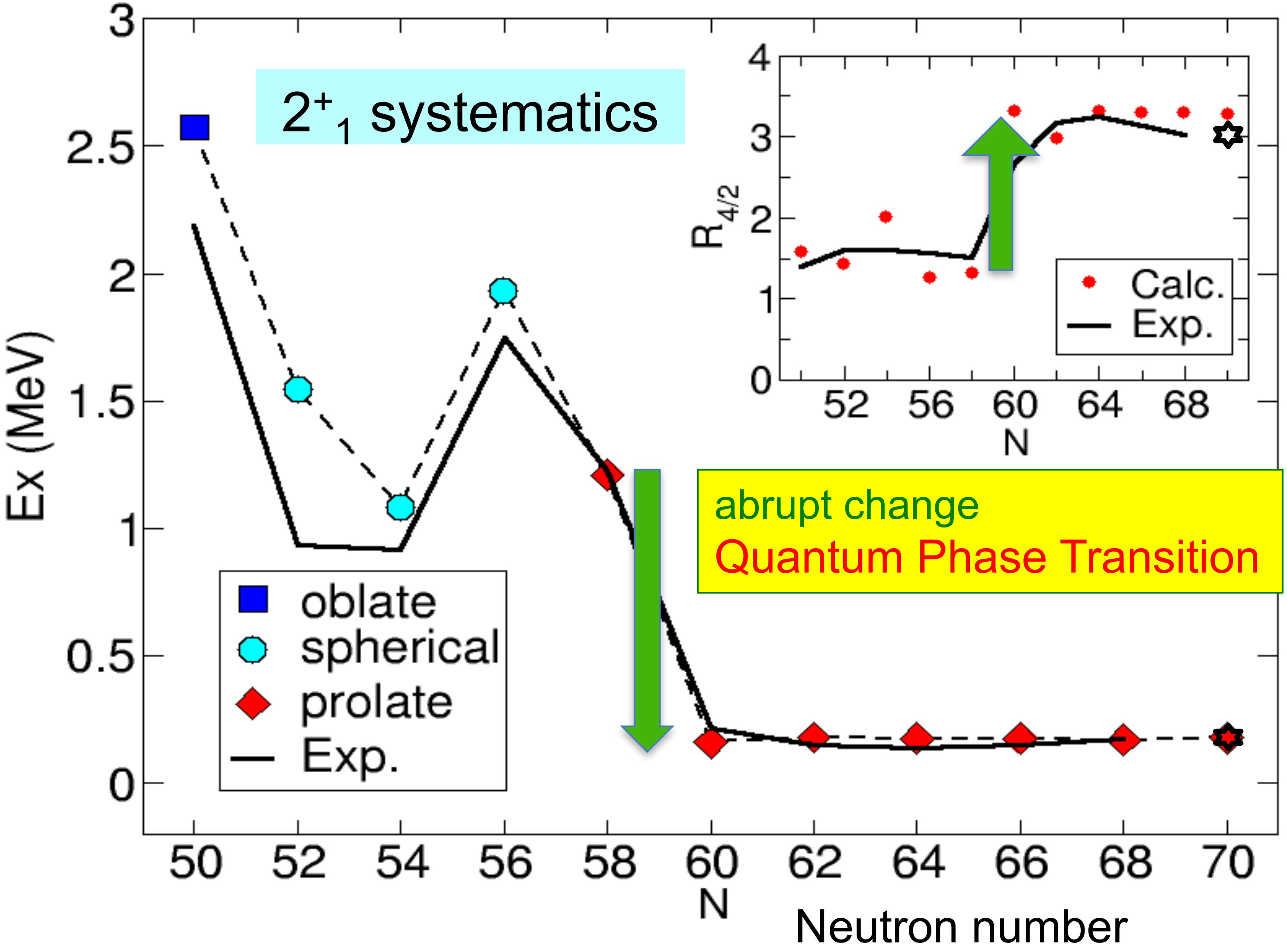


FIG. 12. Systematics of the experimental and theoretical first excited 2^+ states along the zirconium chain.

PRL **117,** 172502 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

Quantum Phase Transition in the Shape of Zr isotopes

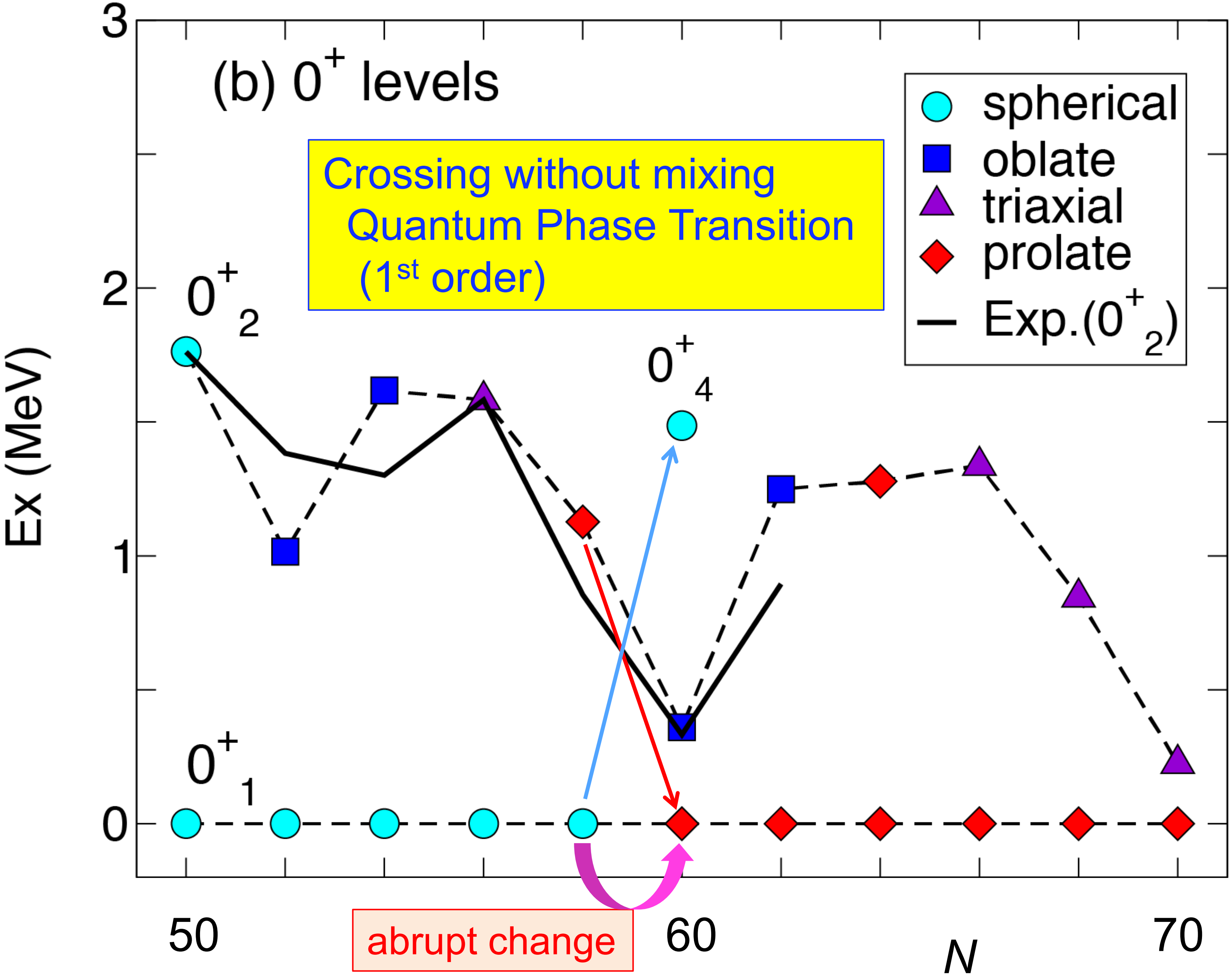
Tomoaki Togashi,[1] Yusuke Tsunoda,[1] Takaharu Otsuka,[1,2,3,4] and Noritaka Shimizu[1]



2^+_1 systematics

abrupt change

Quantum Phase Transition



Crossing without mixing
Quantum Phase Transition
(1st order)

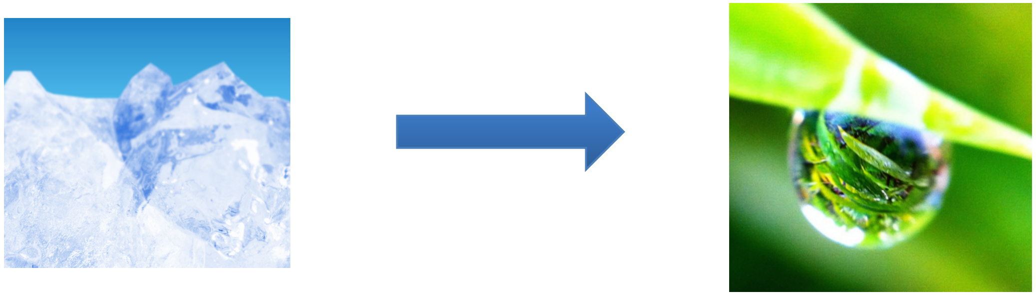
abrupt change

Can this be a “Phase Transition” ?

Phase Transition :

A macroscopic system can change qualitatively from a stable state (*e.g.* ice for H_2O) to another stable state (*e.g.*, water for H_2O) as a function of a certain parameter (*e.g.*, temperature).

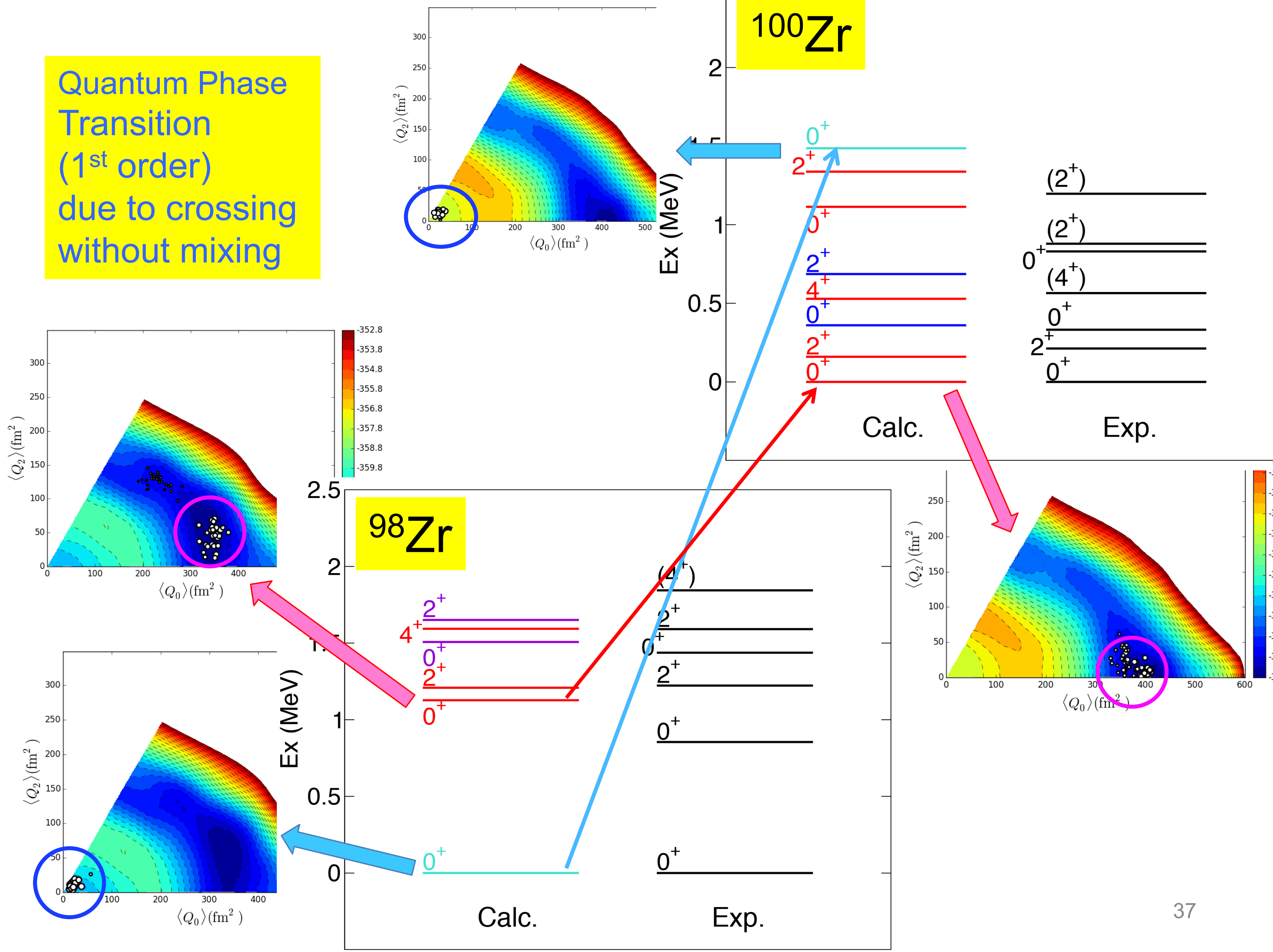
The phase transition implies this kind of phenomena of macroscopic systems consisting of almost infinite number of molecules.

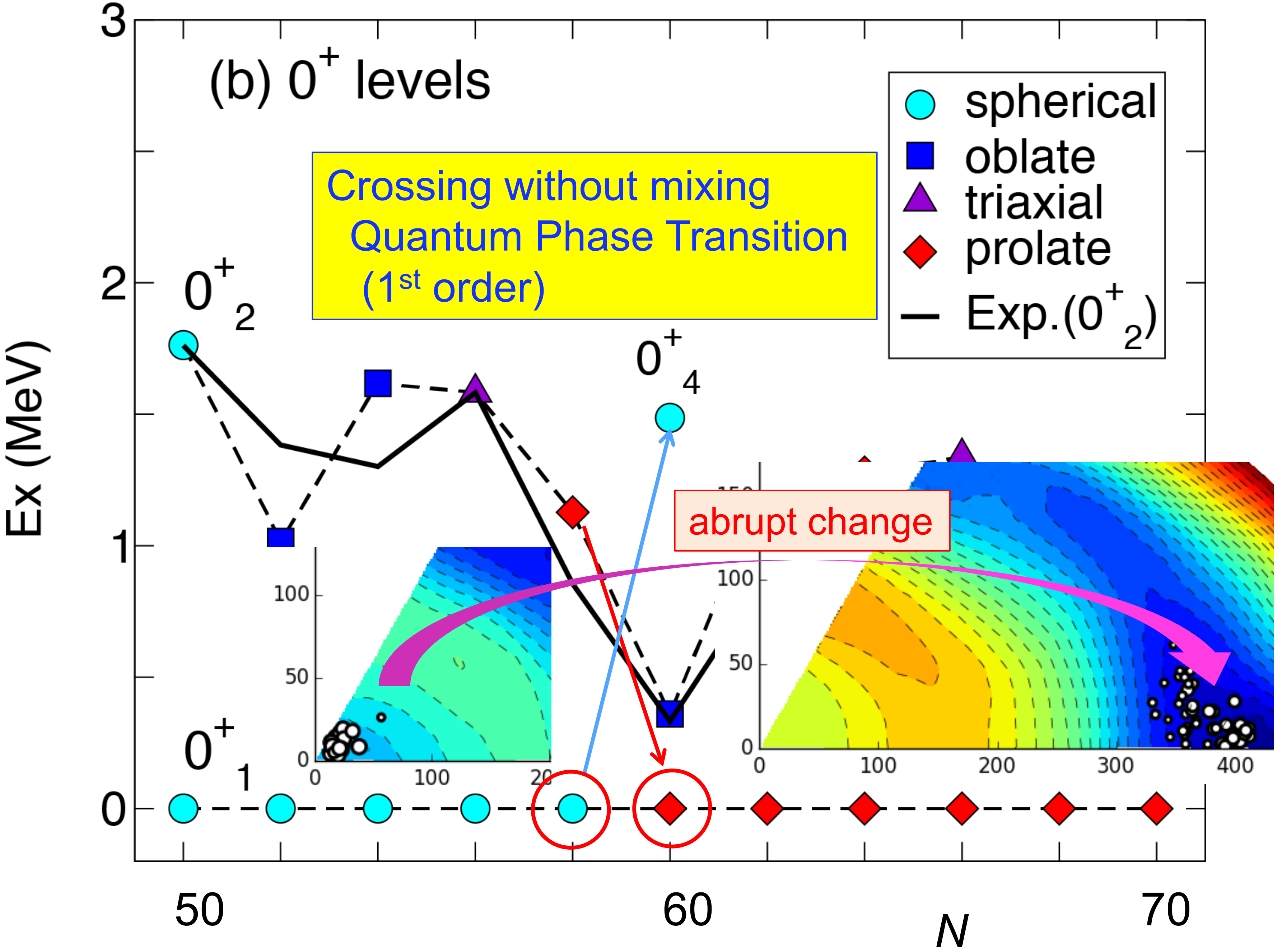


Quantum Phase Transition (QPT)

The concept of the phase transition cannot be applied to microscopic systems as it is. The QPT has been introduced as *an abrupt change* (of order parameter) *in the ground state of a many-body system by varying a physical (i.e., control) parameter at zero temperature. (cf., Wikipedia)*

Quantum Phase Transition (1^{st} order) due to crossing without mixing

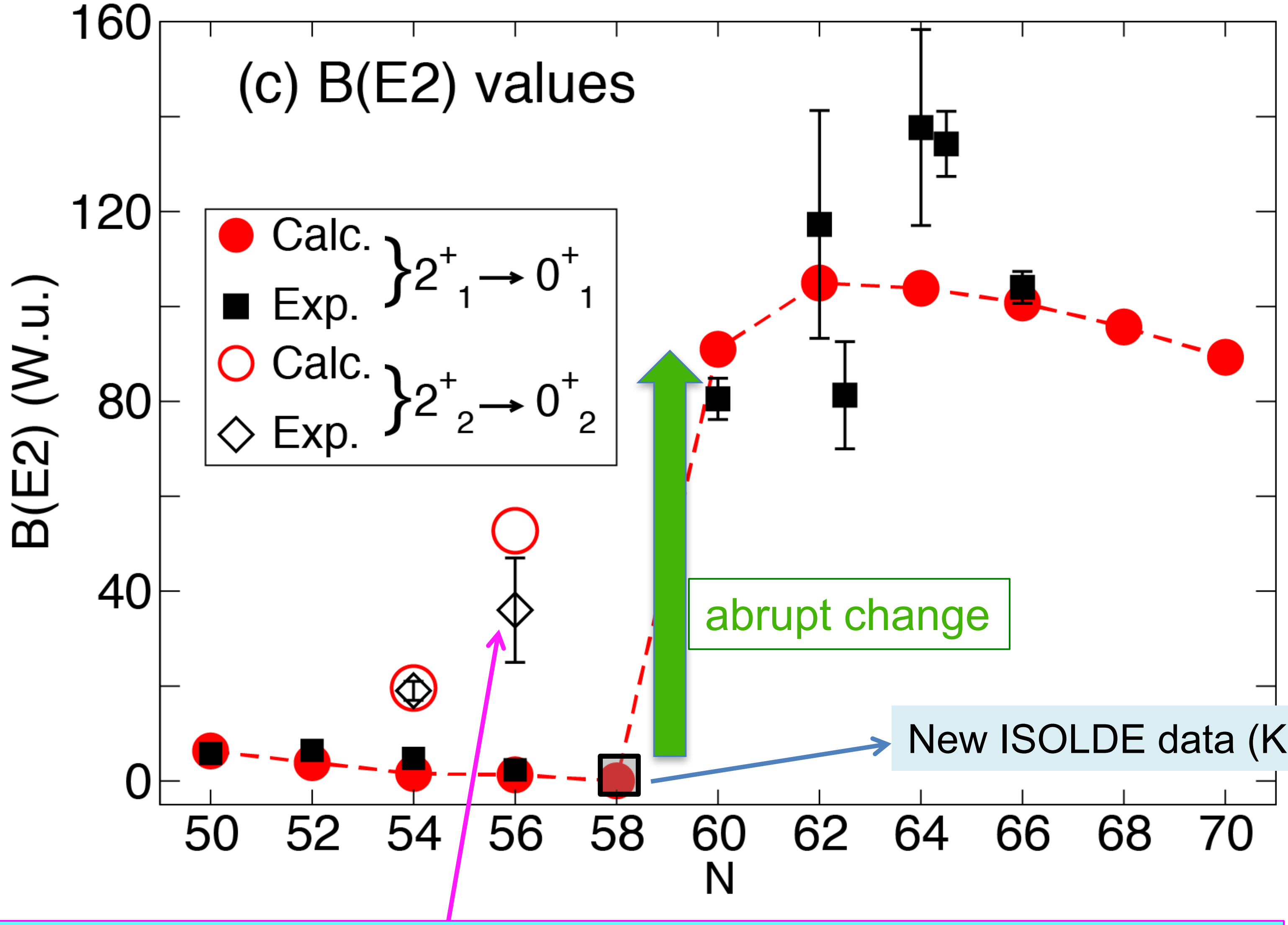




Crossing without mixing
Quantum Phase Transition
(1^{st} order)

abrupt change

B(E2; 2^+ -> 0^+) systematics



abrupt change

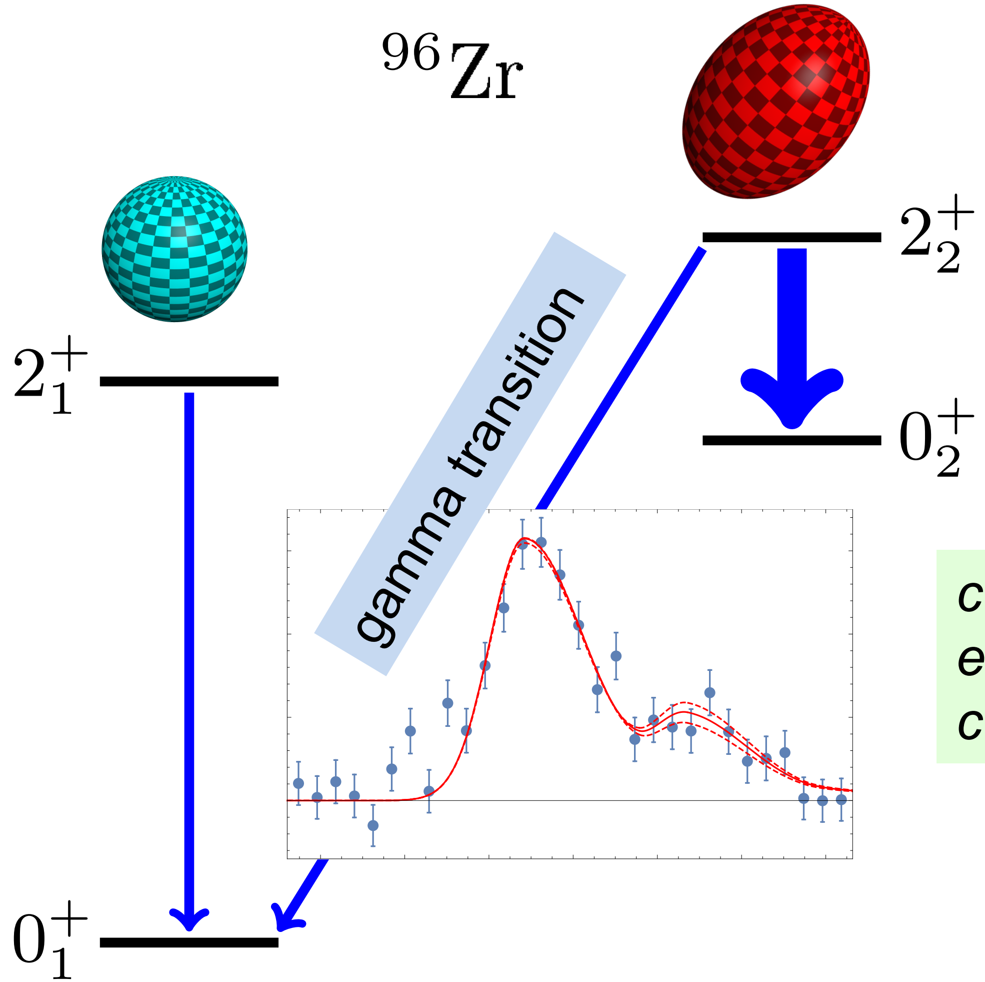
New ISOLDE data (Korten)

New data from Darmstadt, Kremer *et al*. PRL 117, 172503 (2016)

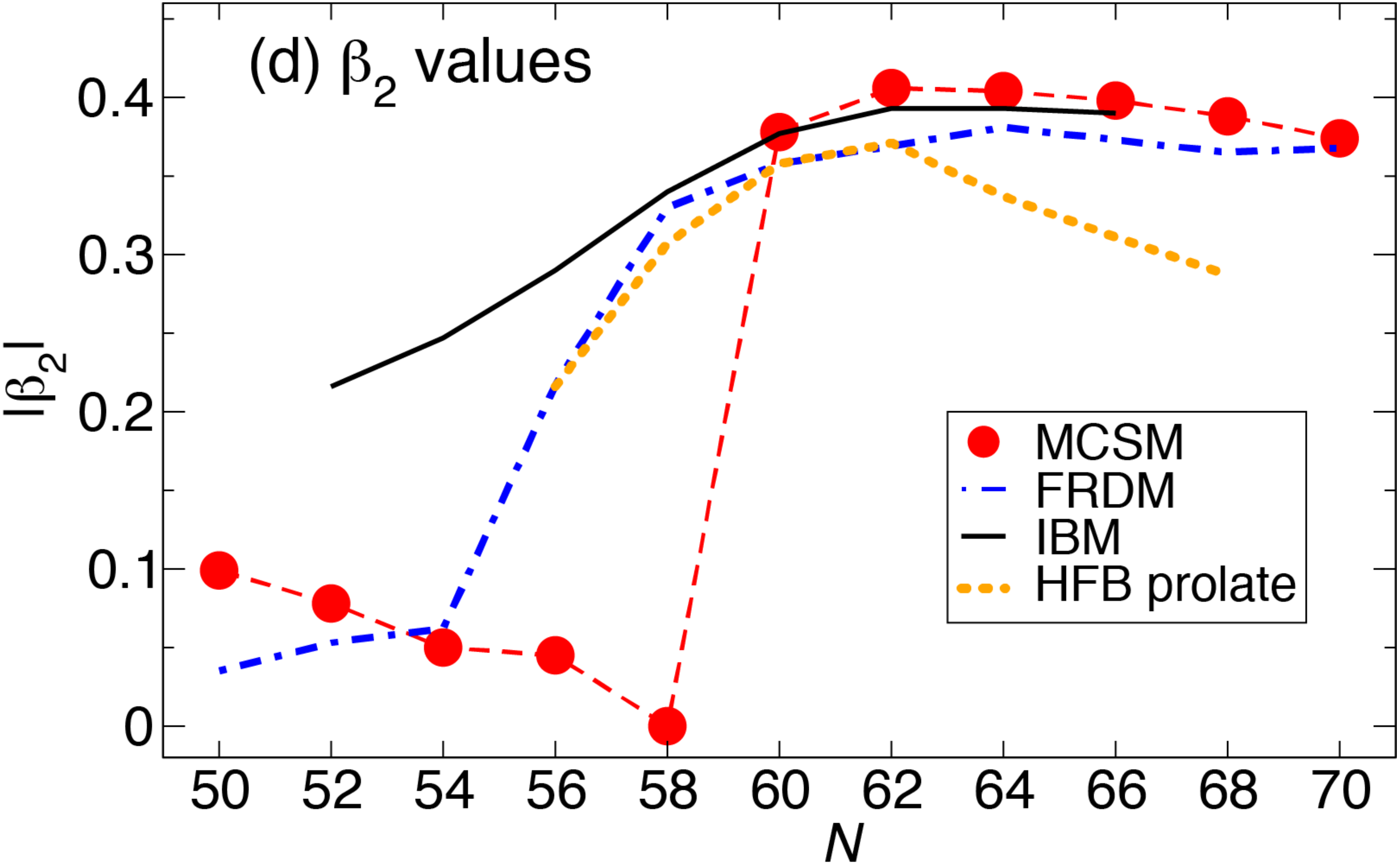
PRL 117, 172503 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

First Measurement of Collectivity of Coexisting Shapes Based on Type II Shell Evolution: The Case of ^{96}Zr

C. Kremer,[1] S. Aslanidou,[1] S. Bassauer,[1] M. Hilcker,[1] A. Krugmann,[1] P. von Neumann-Cosel,[1]
T. Otsuka,[2,3,4,5] N. Pietralla,[1] V. Yu. Ponomarev,[1] N. Shimizu,[3] M. Singer,[1] G. Steinhilber,[1]
T. Togashi,[3] Y. Tsunoda,[3] V. Werner,[1] and M. Zweidinger[1]



co-existence experimentally confirmed



FRDM: S. Moeller et al. At. Data Nucl. Data Tables 59, 185 (1995).

IBM: M. Boyukata et al. J. Phys. G 37, 105102 (2010).

HFB: R. Rodriuez-Guzman et al. Phys. Lett. B 691, 202 (2010).

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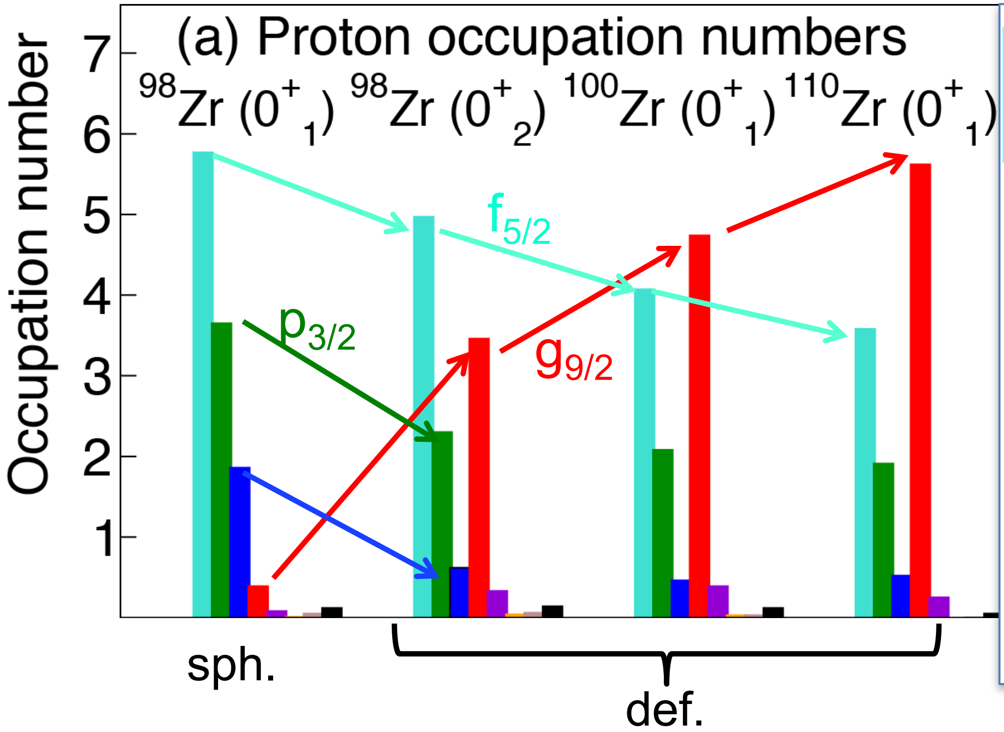
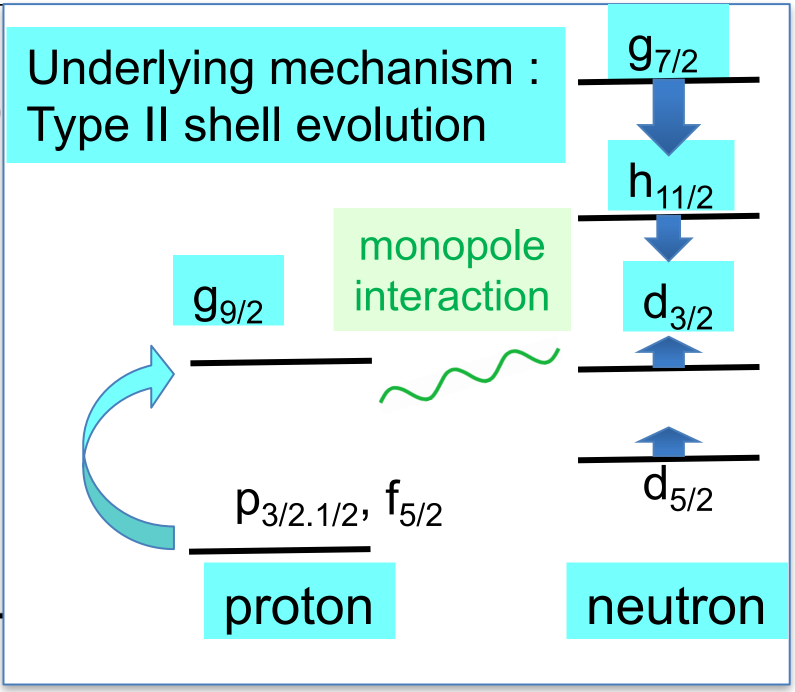
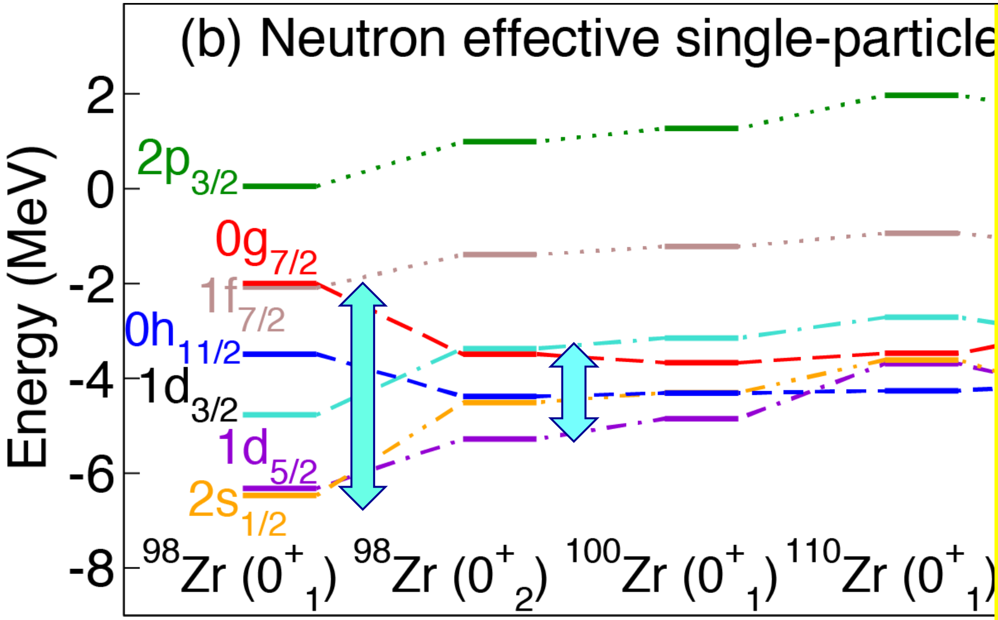

(a) Proton occupation numbers


Underlying mechanism :
Type II shell evolution


(b) Neutron effective single-particle

Relevant neutron single-particle levels get closer as a combined effect of nuclear forces (tensor and central) and particular configurations. The resistance power against deformation is then reduced.

Large difference in ESPEs and configurations ➔ crossing w/o mixing

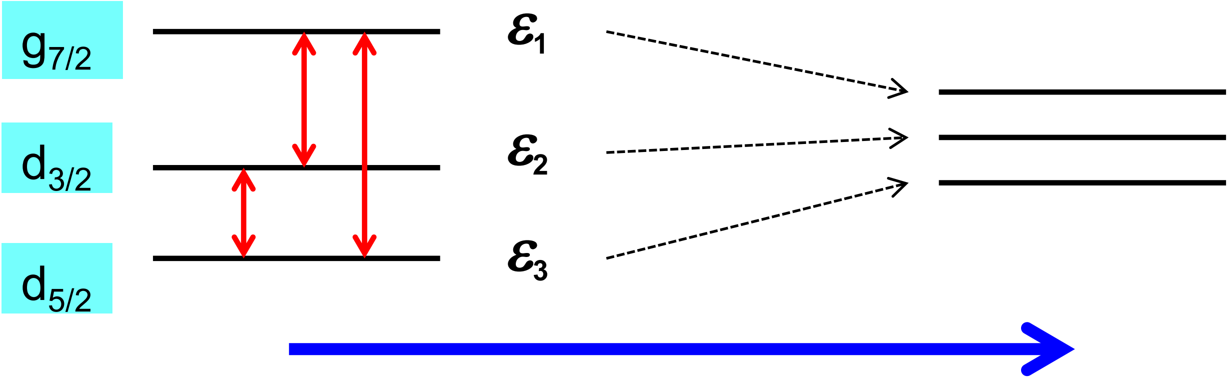
Reminder I : Jahn –Teller effect for nuclear deformation

(Self-consistent) quadrupole deformed field $\propto Y_{2,0}(\theta,\phi)$ mixes the orbits below

$$\Psi\,(J_z{=}1/2) = c_1\,|g_{7/2};\,j_z{=}1/2> + c_2\,|d_{3/2};\,j_z{=}1/2> + c_3\,|d_{5/2};\,j_z{=}1/2>$$

stronger mixing = larger quadrupole deformation

Mixing depends not only on the strength of the $Y_{2,0}(\theta,\phi)$ field, but also the spherical single-particle energies ε_1, ε_2, ε_3, etc.



closer to degeneracy : more mixing

large (or maybe realistic) splitting is certainly an enemy of deformation

Reminder II : Monopole interaction

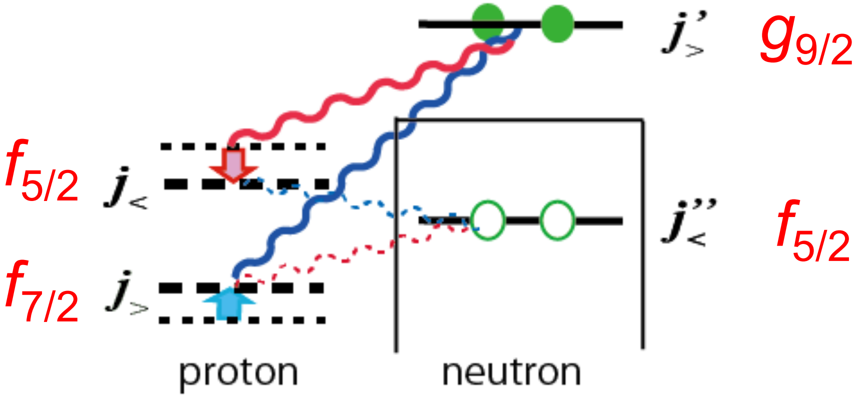
A part of the nucleon-nucleon interaction.

Between a proton in the orbit *j* and a neutron in the orbit *j'* , it is written as

$$v(j, j')\, n^{p}_{j}\, n^{n}_{j'}$$

v(j, j'): monopole matrix element, n^{p}_{j} & $n^{n}_{j'}$: number operators

Ex. Monopole effect from tensor force



1. Proportional to occupation number (linear scaling)
   Its effect can be magnified.

2. Single-particle energies are changed effectively
   Ex: $\Delta\varepsilon^{p}_{j} = v(j, j')\, \Delta n^{n}_{j'}$

3. Also for holes with the opposite sign

4. v(j, j') *not uniform*
   central and tensor forces

Variation of monopole matrix element from a central force : A=70

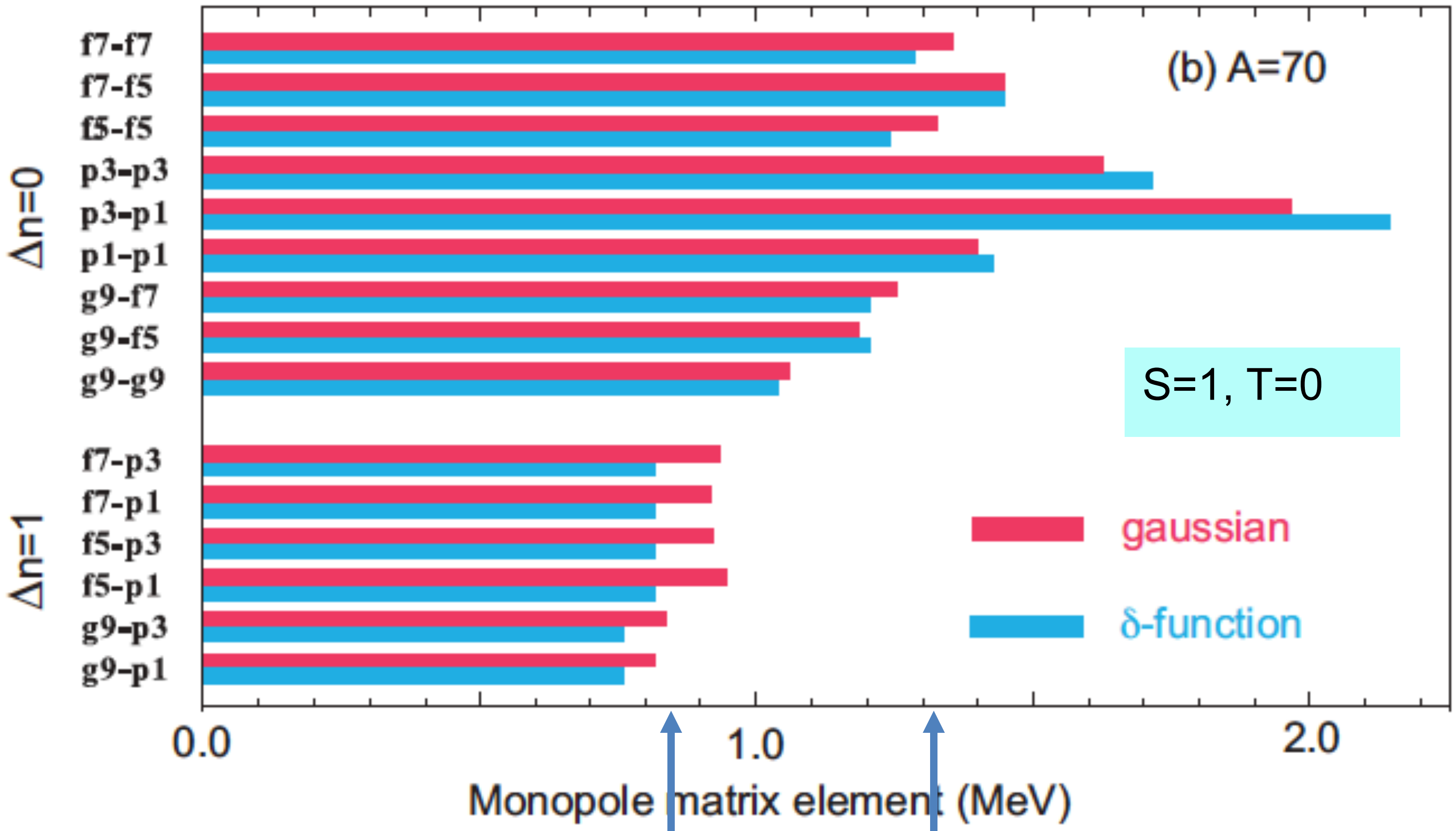


Figure 26 Monopole matrix elements of central gaussian and delta interactions for $(S = 1, T = 0)$ channel. The orbit labeling is abbreviated like g9 for $1g_{9/2}$, *etc.* The orbits are from valence shell for (a) $A = 100$ and (b) $A = 70$.

mean values ~0.8 MeV ~1.3 MeV

difference ~ 0.5 MeV

variations ~0.1 MeV ~0.3 MeV

Variation of monopole matrix element from tensor force : A=70

variations 0.5 ~ 1 MeV

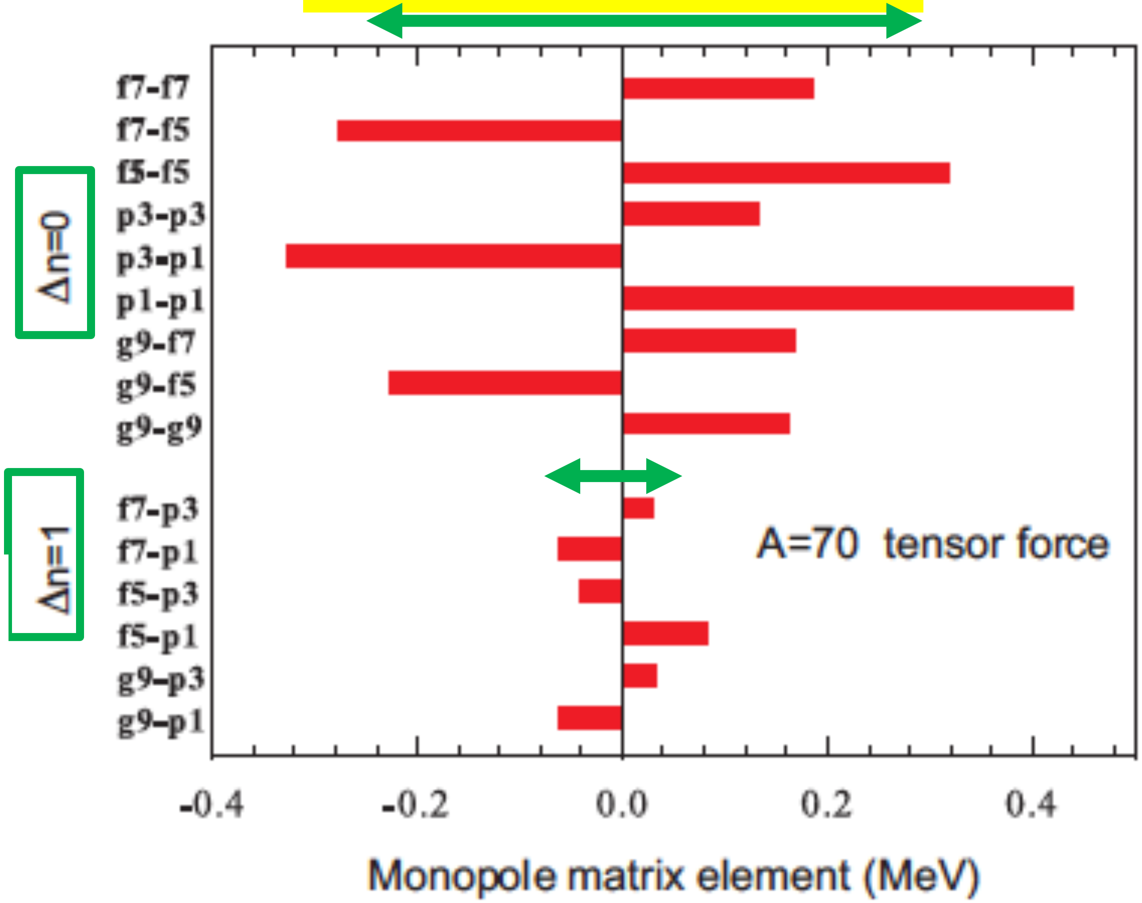
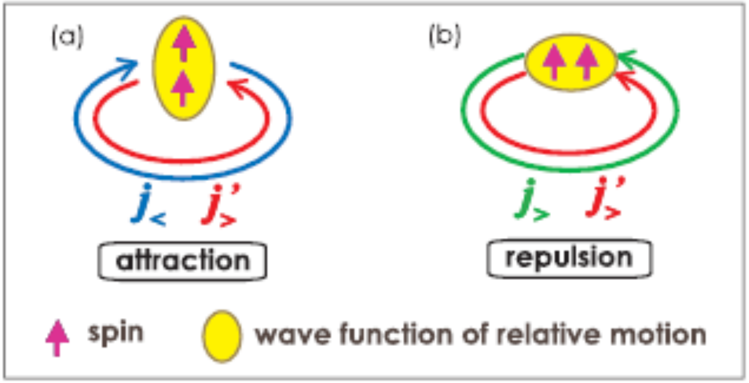
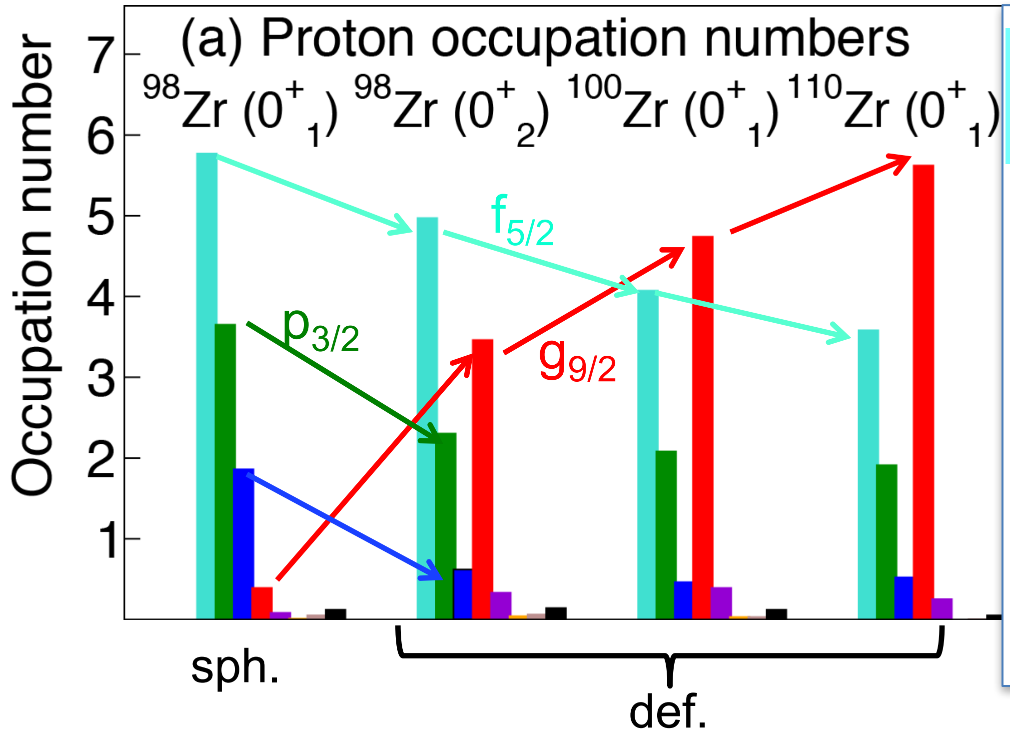


Figure 34 Monopole matrix elements of the tensor force in the T=0 channel. The orbit labeling is abbreviated like f7 for $1f_{7/2}$, *etc.* The orbits are from valence shell for $A = 70$.

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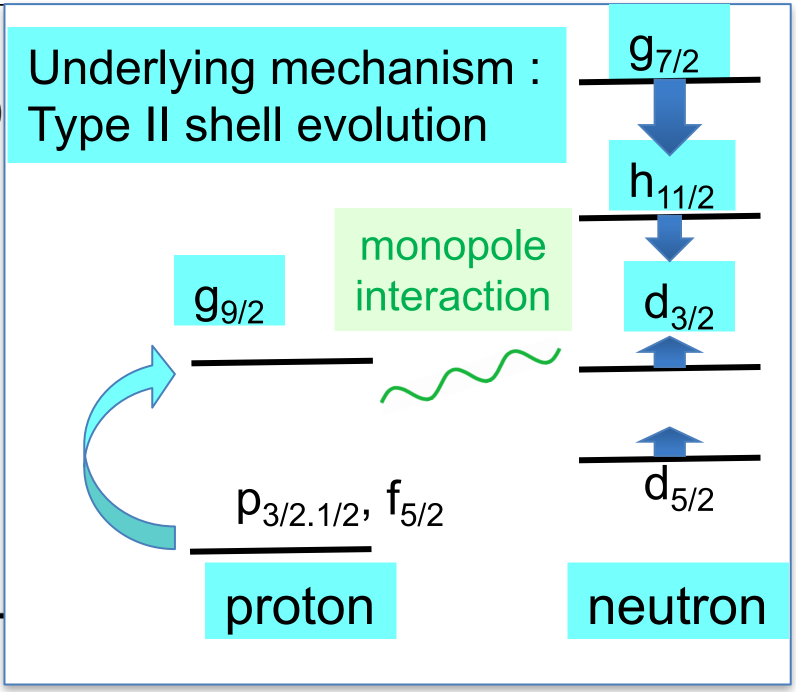
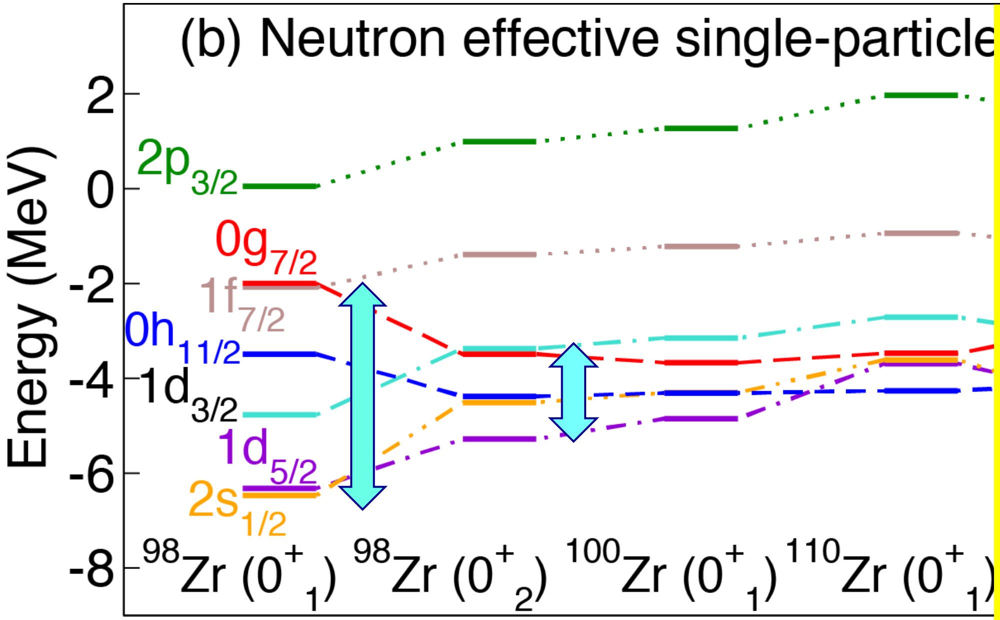

Underlying mechanism : Type II shell evolution


(b) Neutron effective single-particle

Relevant neutron single-particle levels get closer as a combined effect of nuclear forces (tensor and central) and particular configurations.

The resistance power against deformation is then reduced.

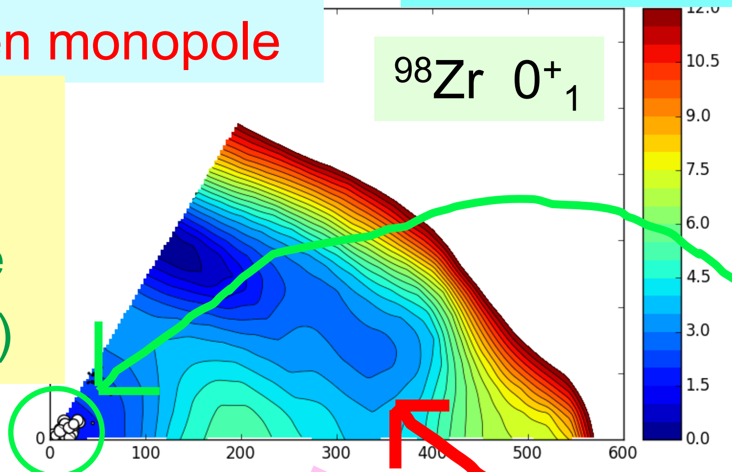
Large difference in ESPEs and configurations ➔ crossing w/o mixing

Anatomy of this effect : ^{98}Zr spherical 0^+_1 and deformed 0^+_2

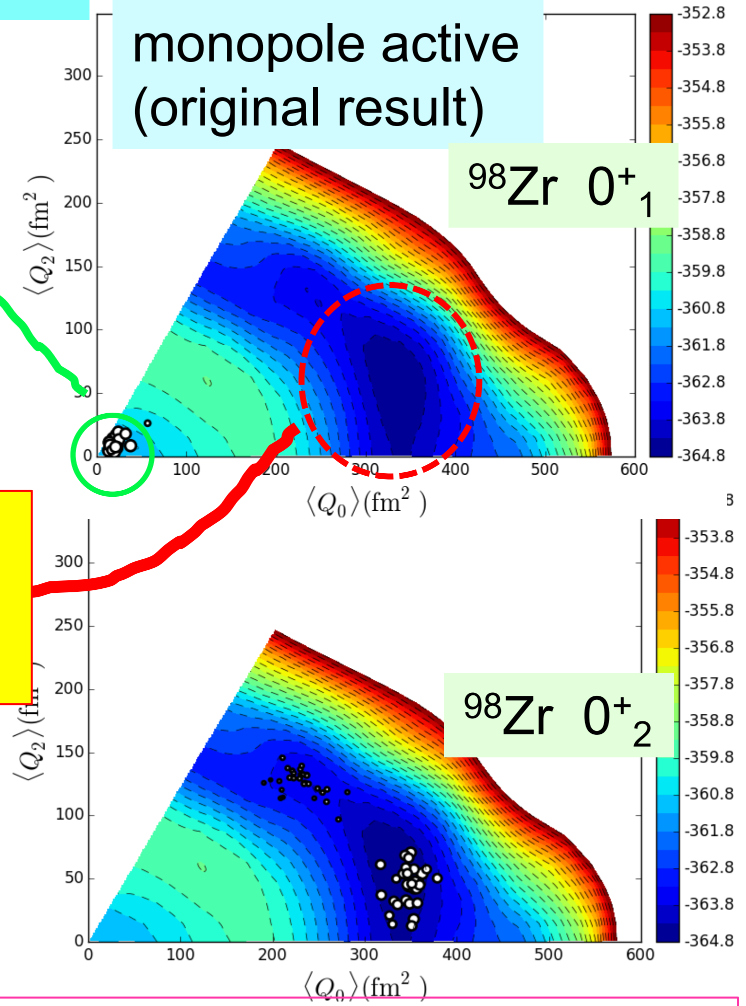
PES with T-plot

Frozen monopole

spherical
ground state
do not change
(overlap~0.98)

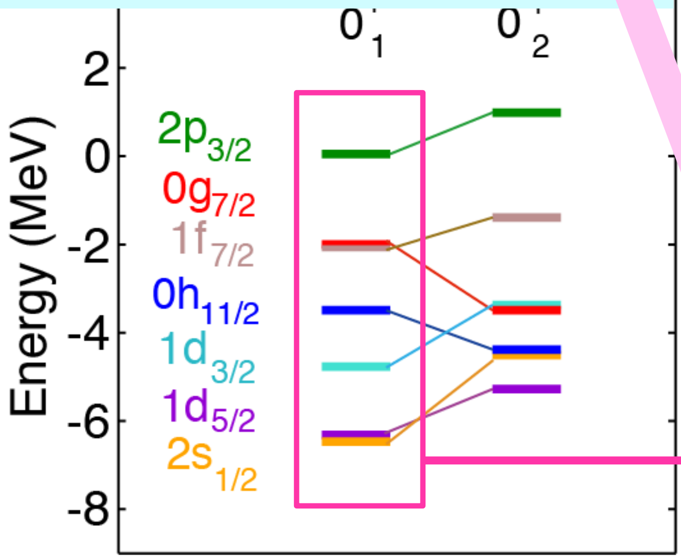


monopole active
(original result)



prolate
minimum
is gone

Effective SPE
- *configuration dependent* -



Use them as constant SPEs
independent of configurations, putting monopole int. aside
➔ Frozen monopole treatment

Type II shell evolution is a simplest and visible case of

Quantum Self Organization

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$

resistance power ← pairing force

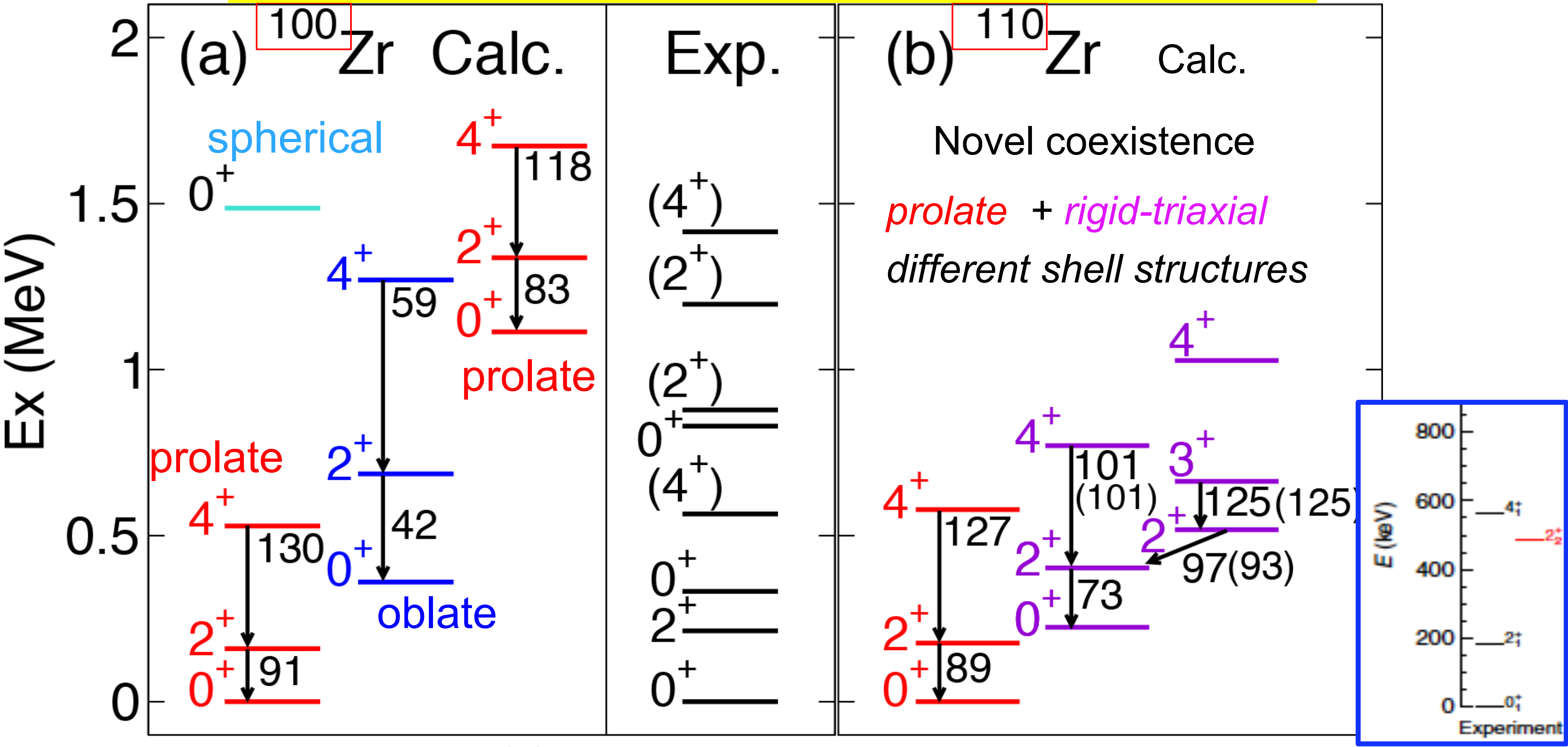
↖ single-particle energies

Atomic nuclei can “organize” their single-particle energies by taking particular configurations of protons and neutrons optimized for each eigenstate, thanks to orbit-dependences of monopole components of nuclear forces (*e.g.*, tensor force).

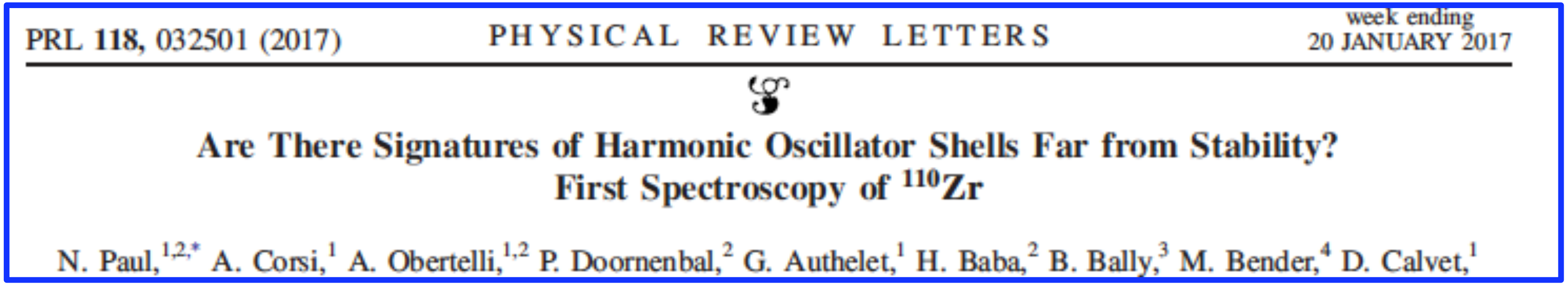
→ an enhancement of Jahn-Teller effect.

Single-particle levels and the number of particles
determines
the shapes

Prolate – rigid-triaxial shape coexistence



( ) : Rigid-triaxial rotor with gamma=28 degrees normalized at 2^+_2 -> 0^+_2

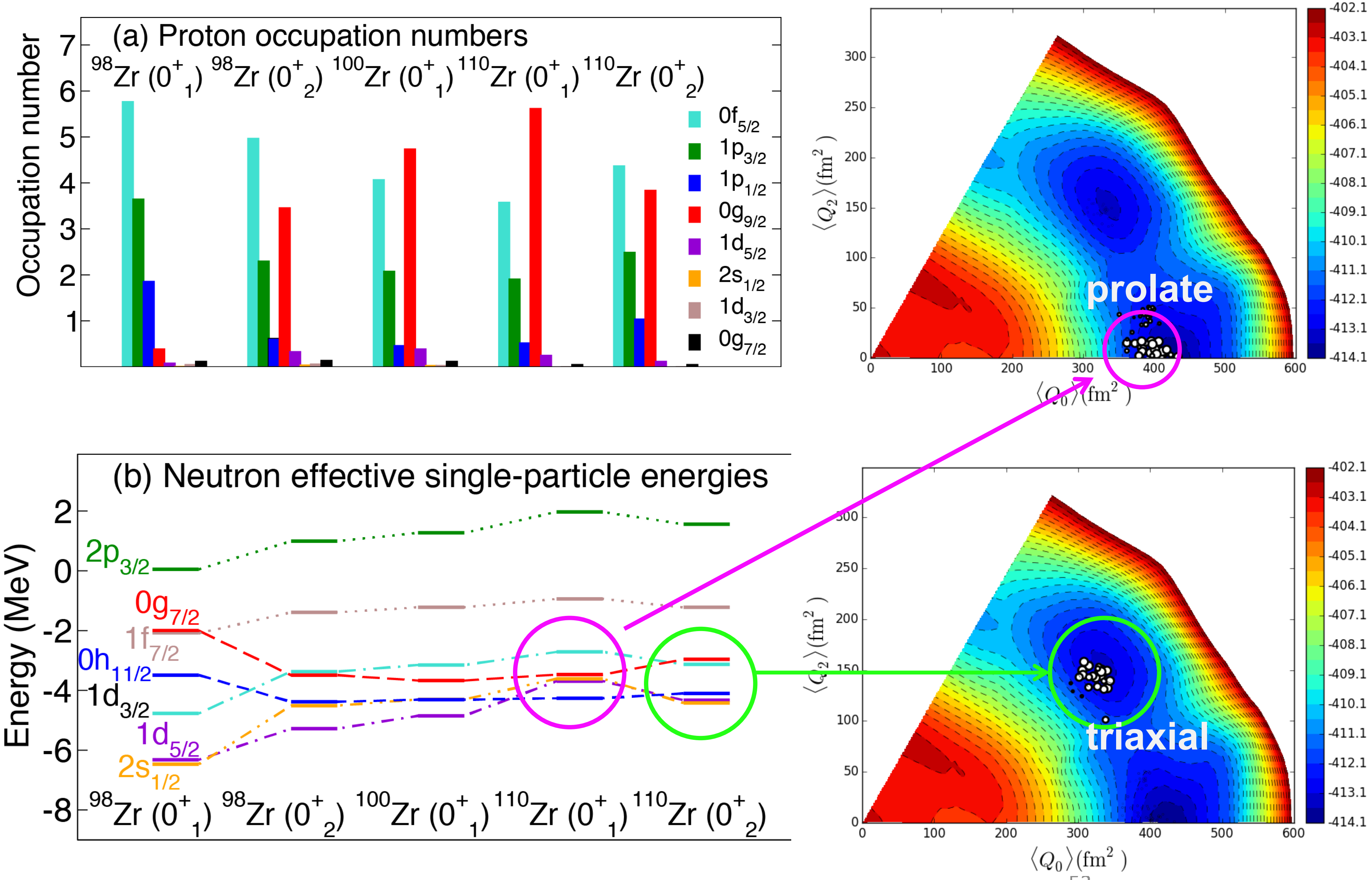


PRL 118, 032501 (2017) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2017

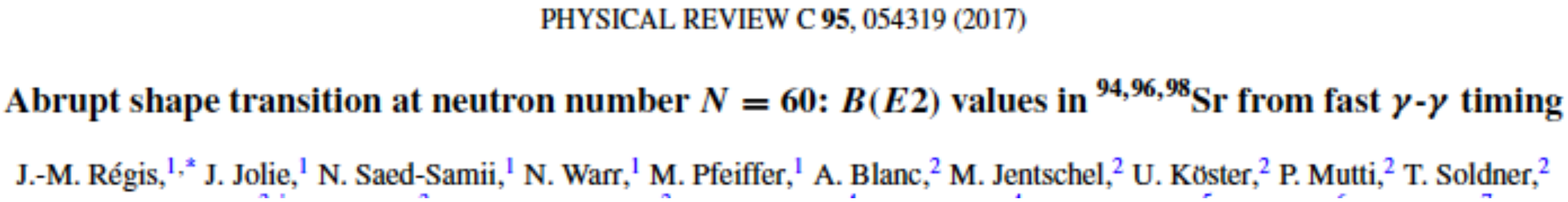
Are There Signatures of Harmonic Oscillator Shells Far from Stability? First Spectroscopy of ^{110}Zr

N. Paul,[1,2,*] A. Corsi,[1] A. Obertelli,[1,2] P. Doornenbal,[2] G. Authelet,[1] H. Baba,[2] B. Bally,[3] M. Bender,[4] D. Calvet,[1]

different shell structures ~ like “different nuclei”



Sr & Kr isotopes within the scope but not yet done well



PHYSICAL REVIEW C **95**, 054319 (2017)

Abrupt shape transition at neutron number $N = 60$: $B(E2)$ values in ^{94,96,98}Sr from fast γ-γ timing

J.-M. Régis,[1,*] J. Jolie,[1] N. Saed-Samii,[1] N. Warr,[1] M. Pfeiffer,[1] A. Blanc,[2] M. Jentschel,[2] U. Köster,[2] P. Mutti,[2] T. Soldner,[2]

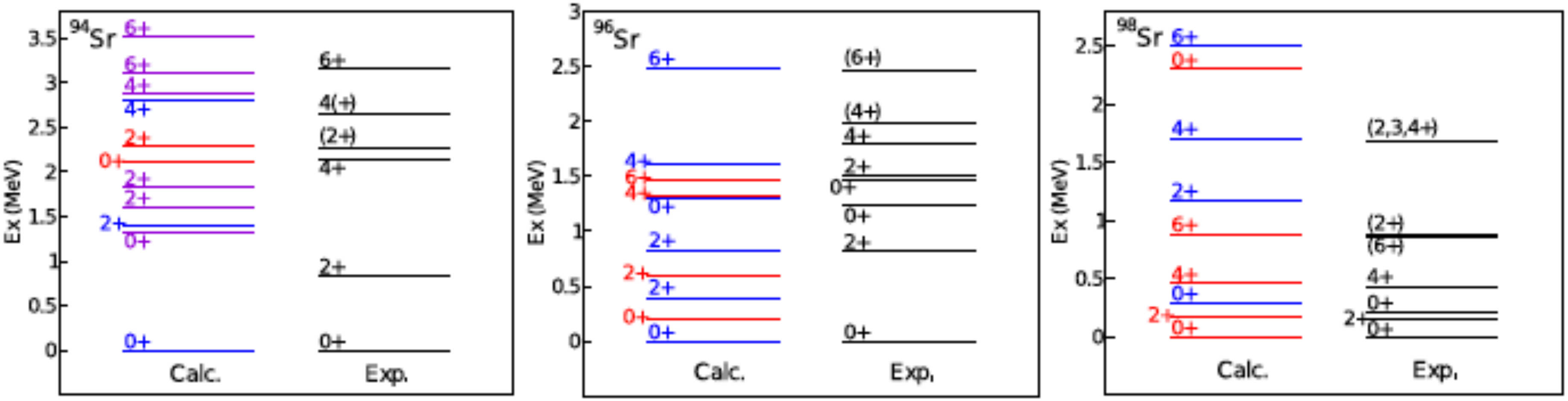


FIG. 7. Comparison between level schemes from MCSM calculations and experimental values for the lowest two excited $I^\pi = 0^+, 2^+, 4^+, 6^+$ states in ^{94,96,98}Sr. Oblate (prolate) deformed states are given in blue (red) and triaxial ones in purple. The experimental data are taken from Refs. [21,26].

Outline

I Introduction

II Presently used numerical methodology of many-body problems

III First application of MCSM to shape coexistence: Ni isotopes

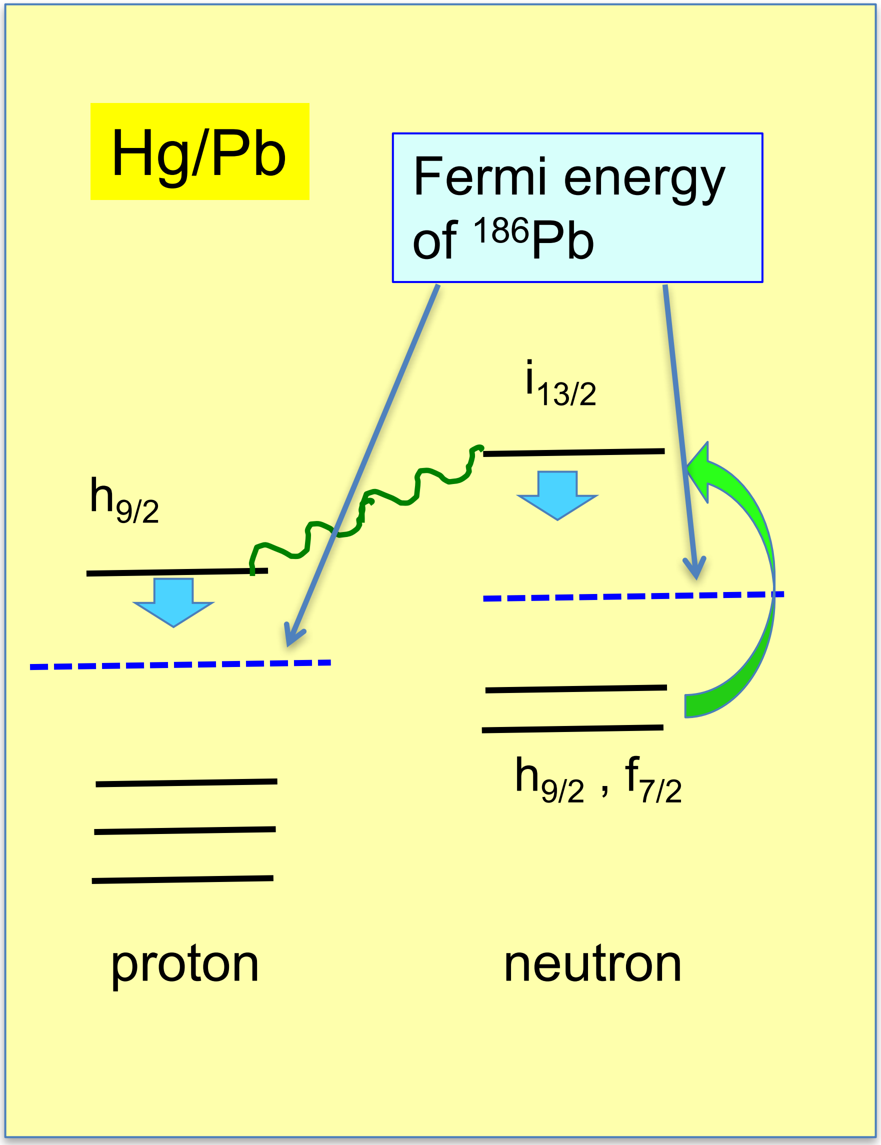
IV An example from Quantum Phase Transition in Zr isotopes

V Basic mechanism

VI Shape coexistence and/or critical phenomena in Hg/Pb isotopes

VII Remarks

Moving to heavier nuclei



Hg/Pb

Fermi energy of ^{186}Pb

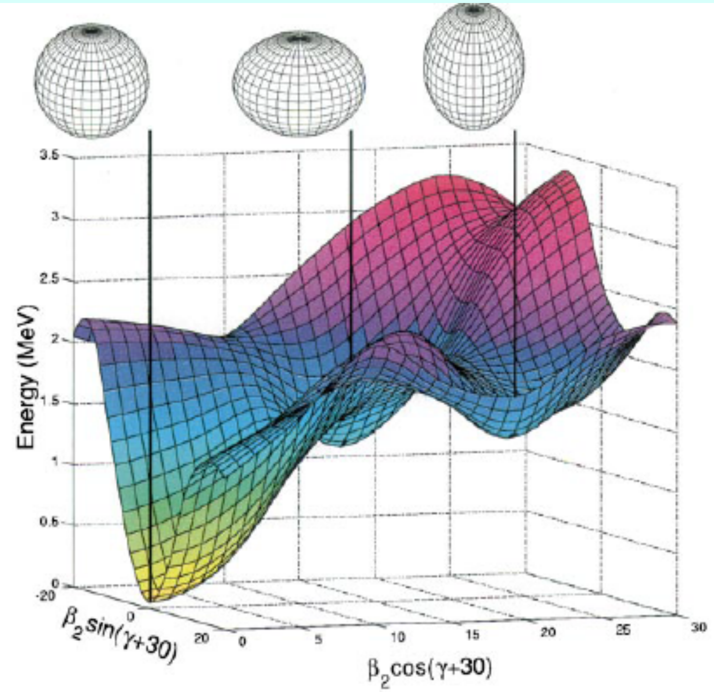
$i_{13/2}$

$h_{9/2}$

$h_{9/2}$, $f_{7/2}$

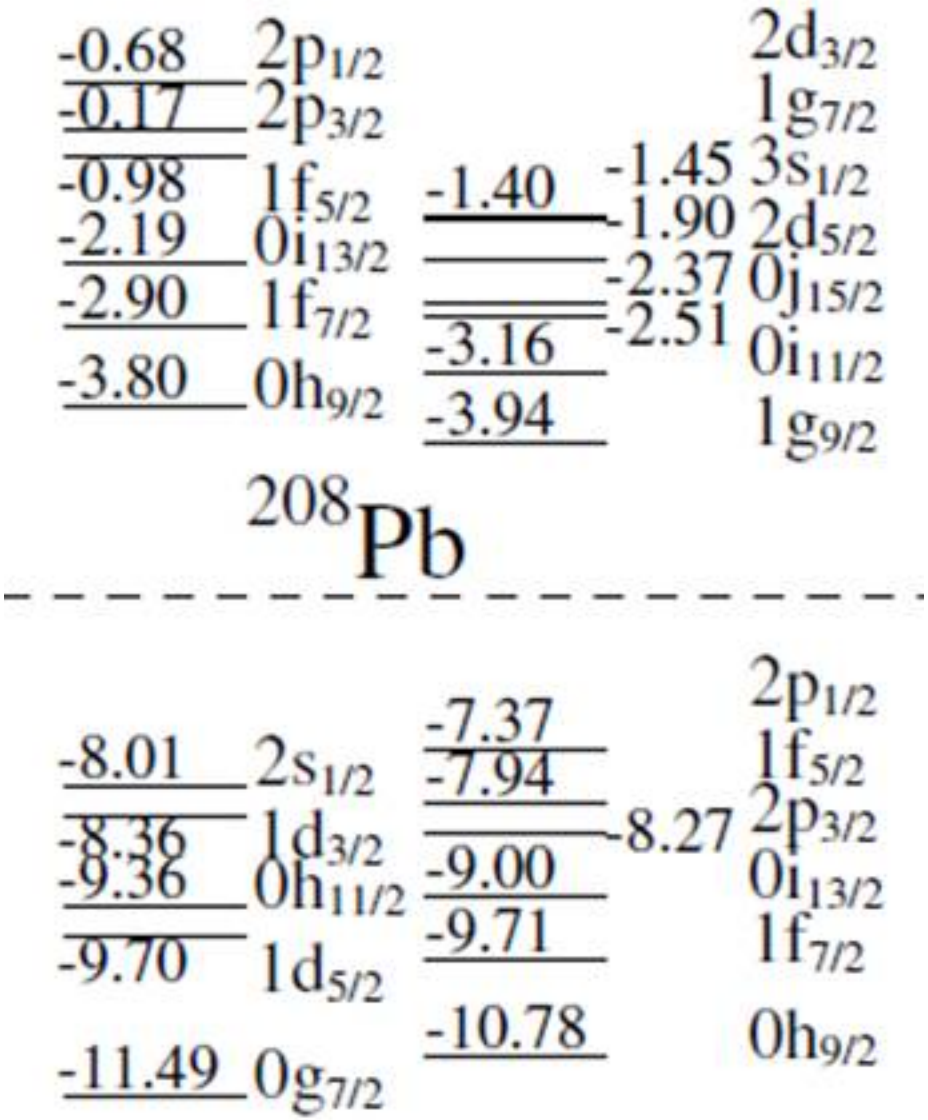
proton

neutron



^{186}Pb Andreyev *et al.*, Nature **405**, 430 (2000)

MCSM calculation setup



All these orbits are included in the MCSM calculation.

30 protons activated

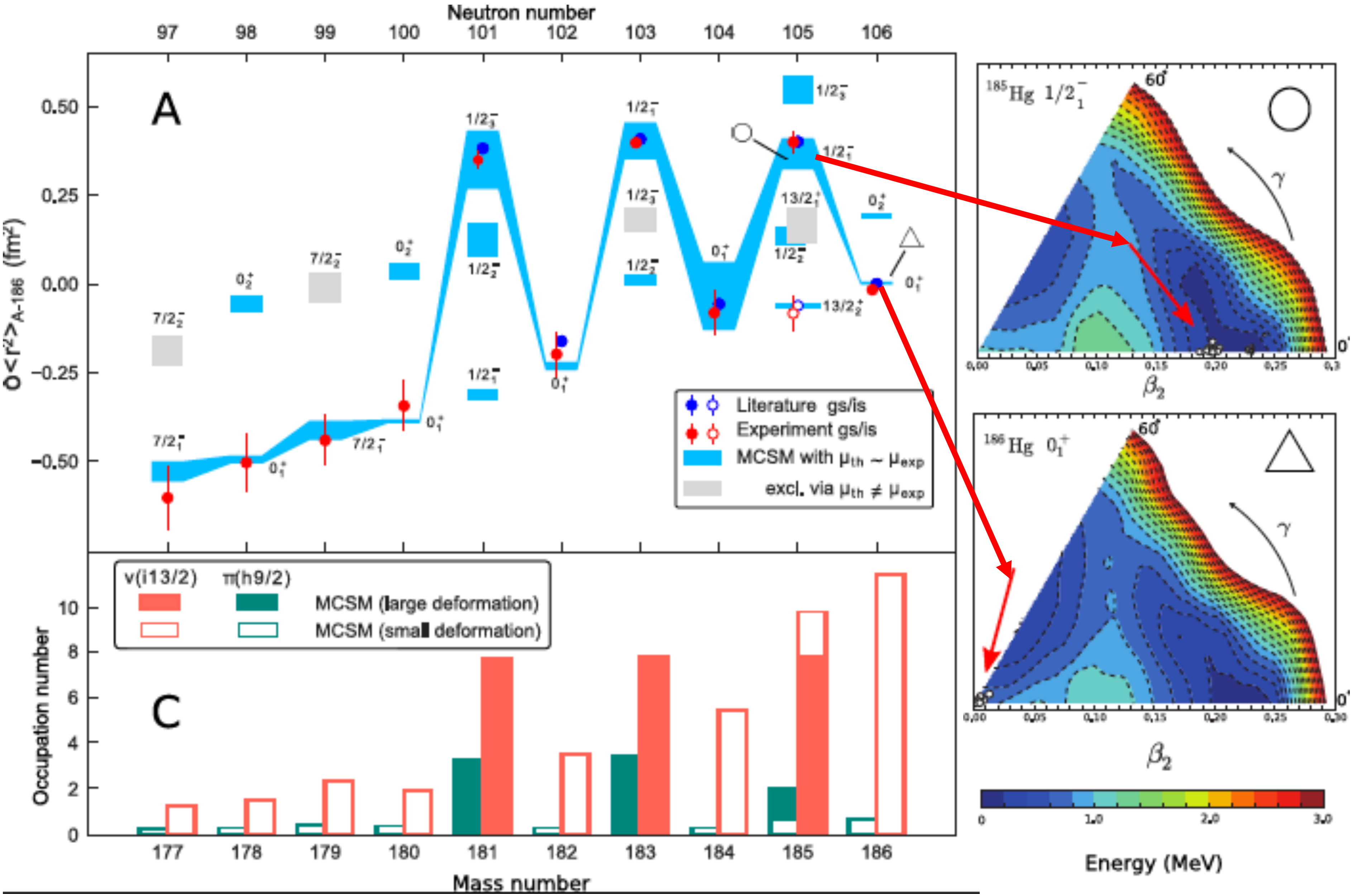
15 (^{177}Hg) – 24 (^{188}Hg) neutrons activated

max dimension in conventional diagonalization ~ 2 x 10 ^{42}

(feasible in 60 years)

p-p, n-n effective interactions are taken from B.A. Brown, PRL 85, 5300 (2000)

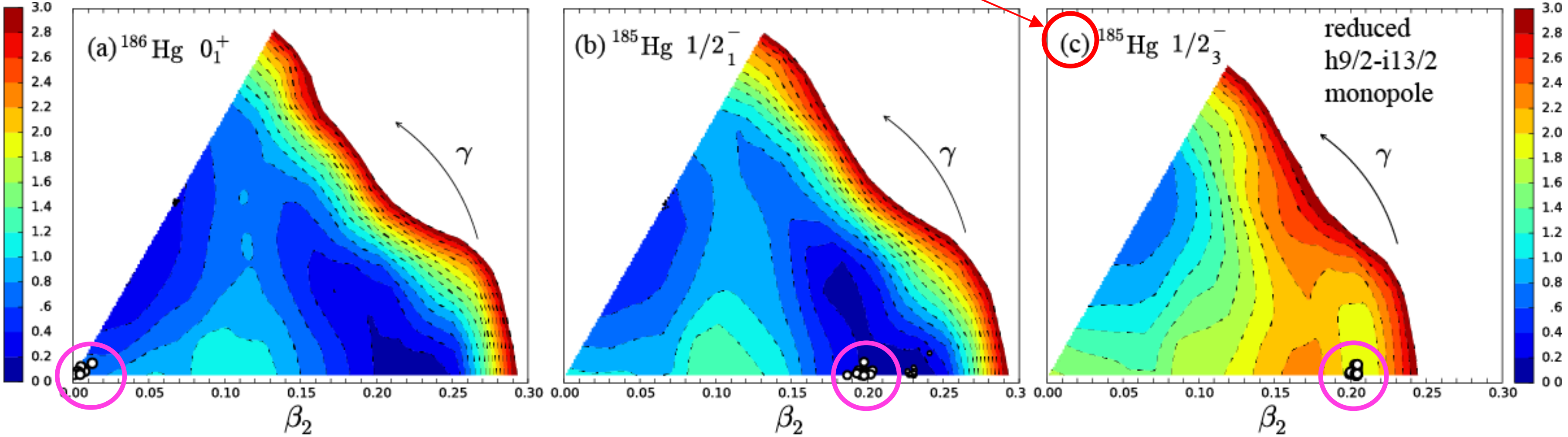
p-n effective interaction is taken from T. Otsuka, PRL 104, 012501 (2010) (VMU: central + tensor) + 2body LS from M3Y


Neutron number
97
98
99
100
101
102
103
104
105
106
A
δ<r²>A-186 (fm²)
0.50
0.25
0.00
−0.25
−0.50
7/2₂⁻
7/2₁⁻
0₂⁺
0₁⁺
7/2₂⁻
7/2₁⁻
0₂⁺
0₁⁺
1/2₃⁻
1/2₂⁻
1/2₁⁻
0₁⁺
1/2₁⁻
1/2₃⁻
1/2₂⁻
0₁⁺
1/2₃⁻
1/2₁⁻
13/2₁⁺
1/2₂⁻
13/2₂⁺
0₂⁺
0₁⁺
Literature gs/is
Experiment gs/is
MCSM with μth ∼ μexp
excl. via μth ≠ μexp
C
Occupation number
ν(i13/2)
π(h9/2)
MCSM (large deformation)
MCSM (small deformation)
0
2
4
6
8
10
177
178
179
180
181
182
183
184
185
186
Mass number
¹⁸⁵Hg 1/2₁⁻
60°
γ
β₂
0.00
0.05
0.10
0.15
0.20
0.25
0.3
0°
¹⁸⁶Hg 0₁⁺
60°
γ
β₂
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0°
0
1.0
2.0
3.0
Energy (MeV)

Single-particle structure is self-organized in prolate states:

proton $h_{9/2}$ – neutron $i_{13/2}$ monopole interaction is particularly strong due to the central + tensor force.

If we weaken it to an averaged magnitude of monopole interactions between other orbits, we get (c)



^{186}Hg 0^+_1

^{185}Hg $1/2^-_1$

^{185}Hg $1/2^-_3$

Note : oblate region is not affected so much

Remarks

Naïve Fermi liquid picture (a la Landau) is revised, as *atomic nuclei are not necessarily like simple solid vases containing almost free nucleons*.

Nuclear forces are rich enough to optimize single-particle energies for each eigenstate (especially in the cases of collective-mode states), as referred to as quantum self-organization.

The quantum self-organization produces sizable effects with
(i) two quantum fluids (protons and neutrons),
(ii) two major forces : *e.g.*, quadrupole interaction to drive collective mode
monopole interaction to control resistance

Thus, non-specific forces, e.g. monopole interaction, work coherently so that single-particle states are not always enemies but can be friends of collective modes.

Interesting topics may include

- prolate shape is more favored (reason for prolate > oblate ?)
- Majorana force in IBM may be explained for the first time
- more important for heavier nuclei → stability of superheavy elements
- time dependent version … intriguing project

Remarks - *continued*

Type II shell evolution is a simpler case of the quantum self-organization, involving closed-shell structure (~shape coexistence).
Quantum phase transition, shape coexistence, shape transition (e.g., Sm), superdeformation, fission, … seem to be relevant.

heavier nuclei : more particles and more orbits => more important

From known examples,

Ni (+Co ..) shape coexistence with lowest Ex~1 MeV

Zr (+Sr ..) quantum phase transition an N=60

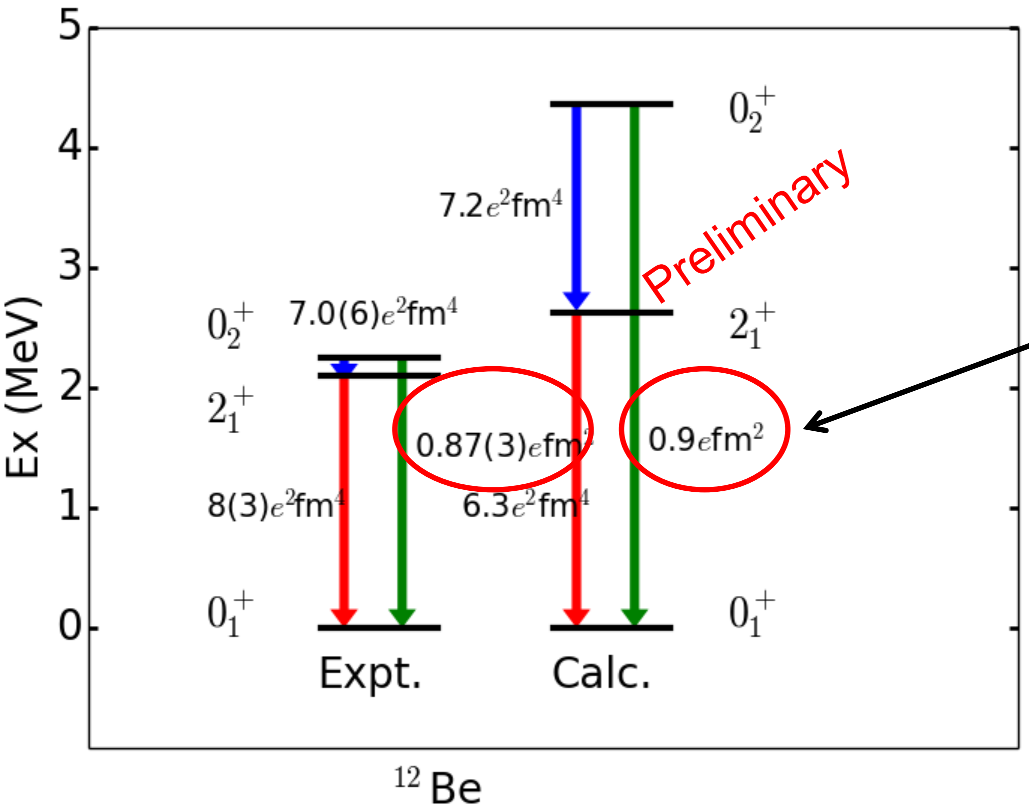
Hg (+Pb ..) even-even isotopes : shape coexistence

even-odd : alternation between spherical/weak oblate and
prolate → degeneracy of phases (critical phenomenon)
even or odd of neutron number controls

E2 & E0 Transition strength of ^{12}Be

Current status

E0 is difficult for the shell model. But could ab initio shell model solve this ?



$B(E0; 0^+_2 \rightarrow 0^+_1)$

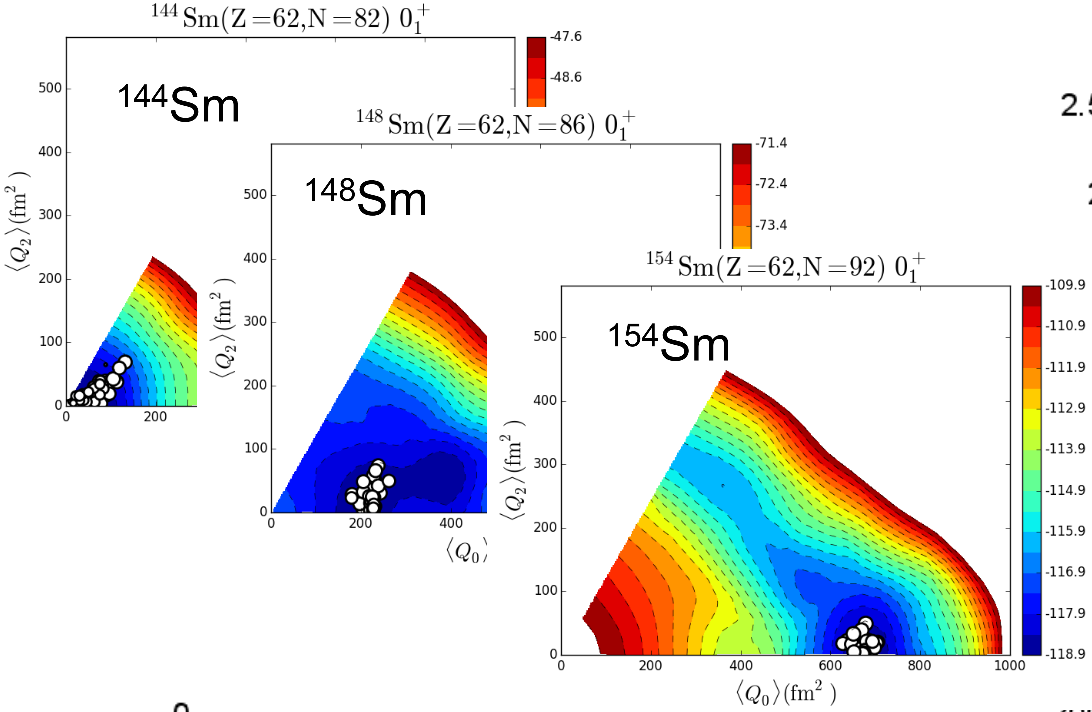
Expt.:

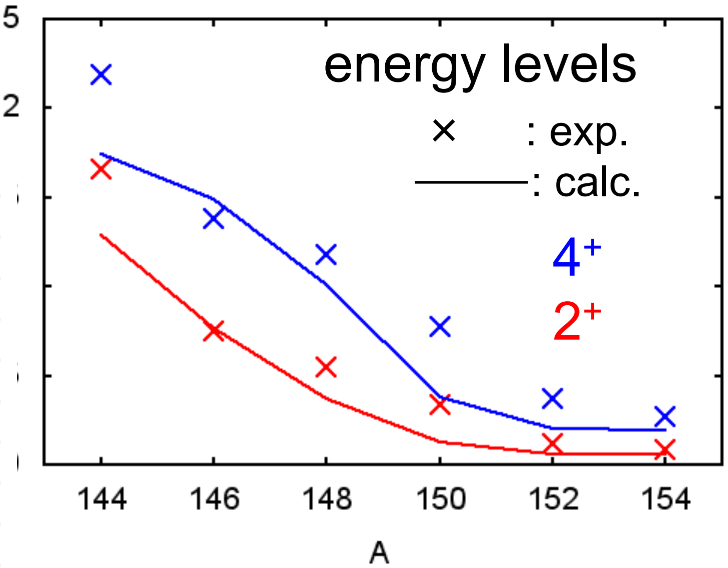
S. Shimoura, et al., Phys. Lett. B 654 87 (2007)

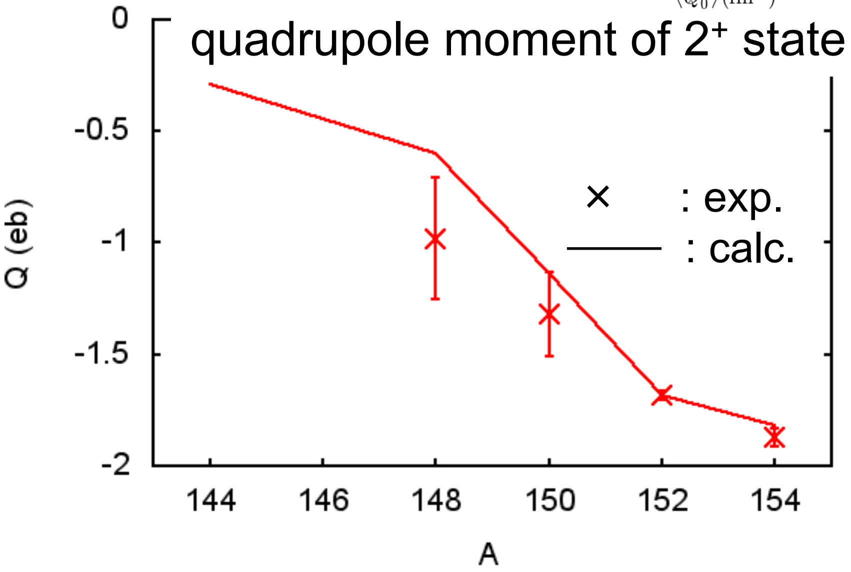
N. Imai, et al., Phys. Lett. B 673 179 (2009)

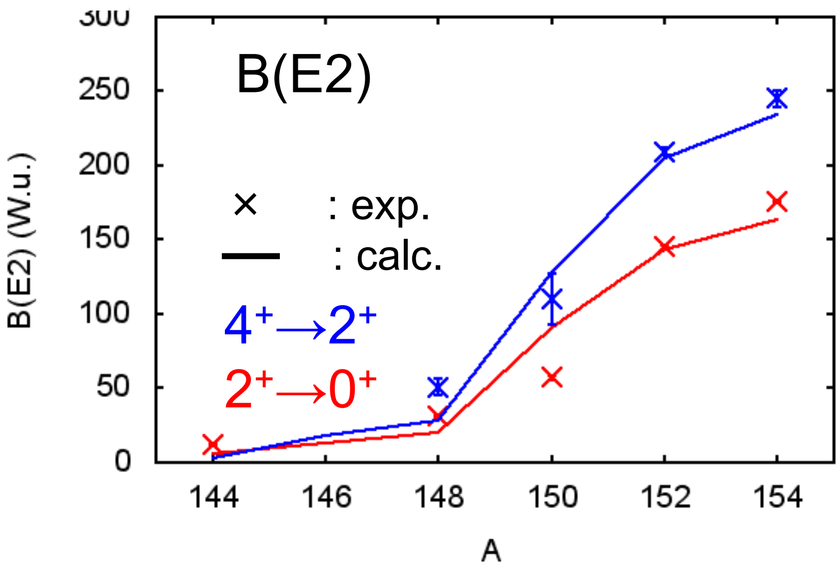
Thank you

Shape evolution in Sm isotopes (very preliminary)





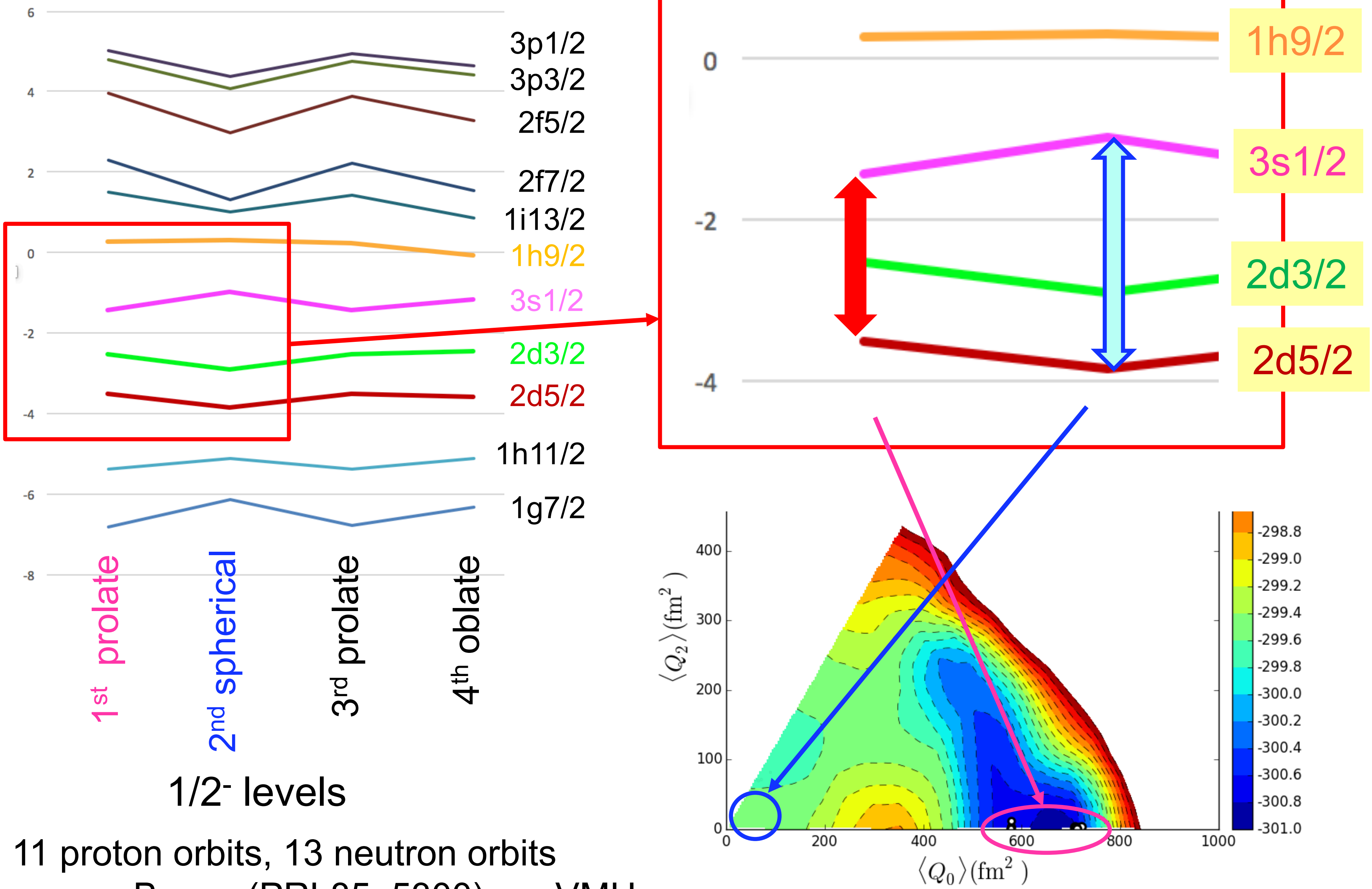




Shape coexistence in Hg/Pb region

Very Preliminary

^{181}Hg proton ESPE (MeV)



1/2$^-$ levels

11 proton orbits, 13 neutron orbits

nn, pp Brown (PRL85, 5300), pn VMU

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$

resistance power ← pairing force

↖ single-particle energies

Analogy to electric current,

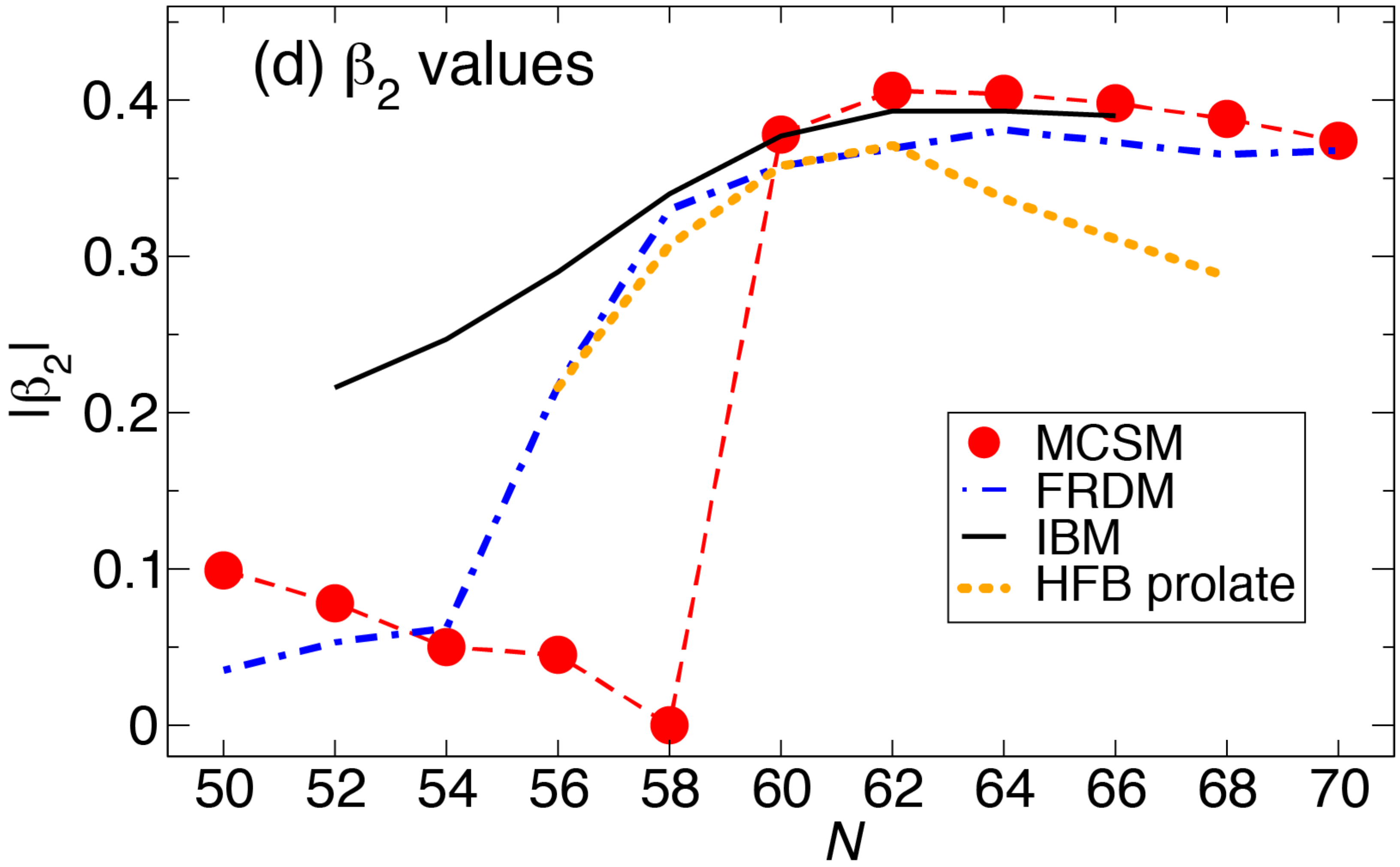
$$\text{current} = \frac{\text{voltage}}{\text{resistance}}$$

Additional remark:

The atomic nucleus can optimize its single-particle properties
for actual mode/shape (or any final form of the structure),
by choosing favorable configurations.

This aspect of the quantum self-organization may be
(one of) the missing correlations Nakatsukasa-san mentioned
this morning.

Deformation parameter β_2 varies as the neutron number *N*

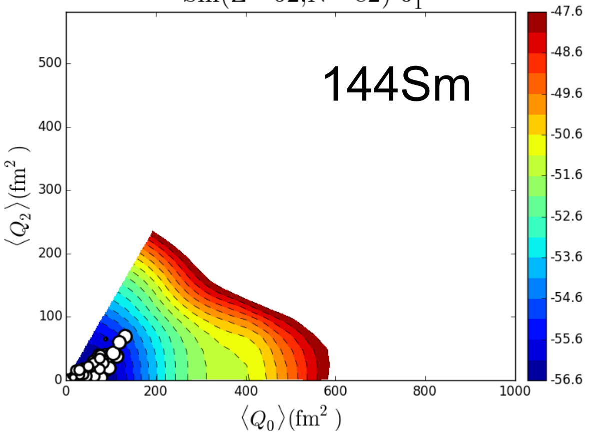


Neutron number

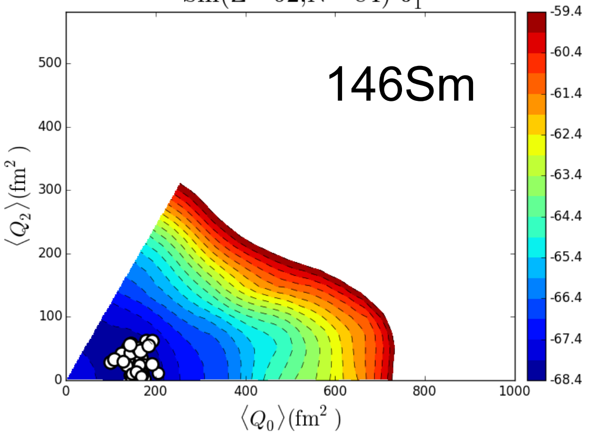
Sm同位体(Z=62)のT-plot

- ^{144}Smから^{154}Smにかけて変形度が増加(^{154}Smでβ～0.3)
- ^{150}Smから配位が変わり、$\pi 0h_{11/2}$, $\nu 0g_{9/2}$, $\nu 0i_{13/2}$などに励起する

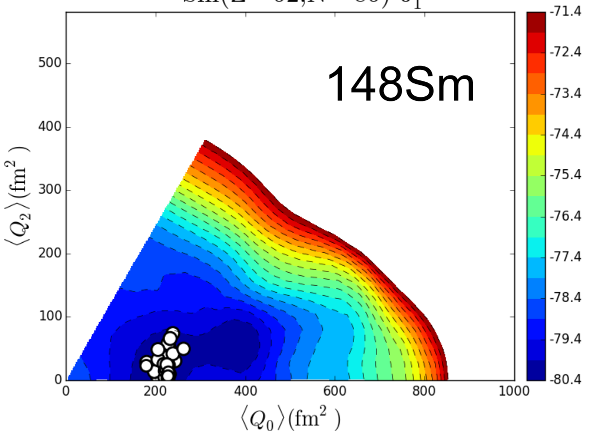
$^{144}\mathrm{Sm}(Z=62, N=82)\ 0_1^+$



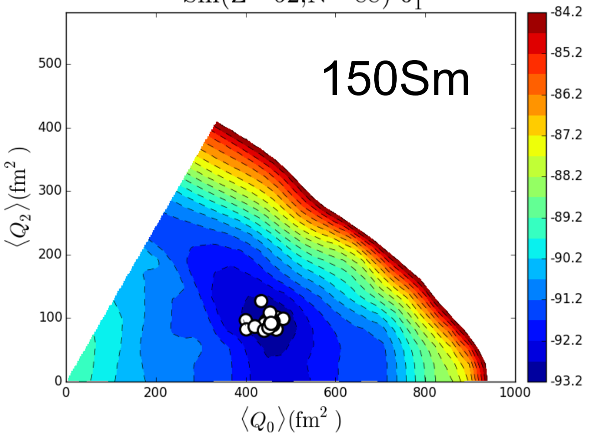
$^{146}\mathrm{Sm}(Z=62, N=84)\ 0_1^+$



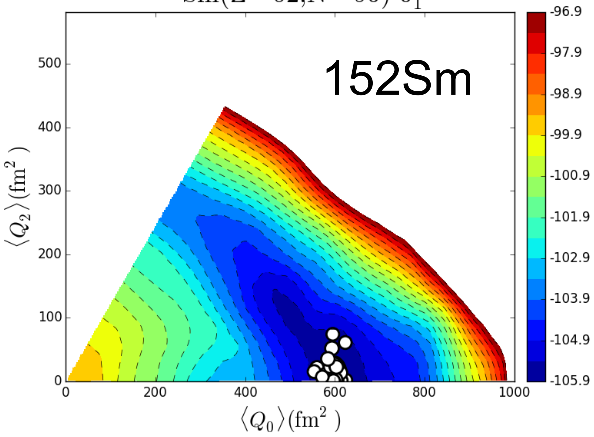
$^{148}\mathrm{Sm}(Z=62, N=86)\ 0_1^+$



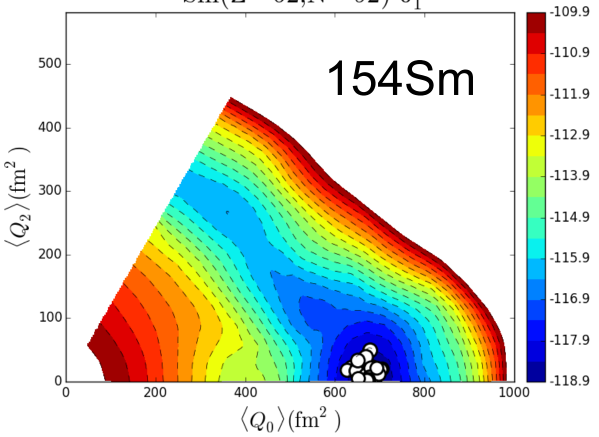
$^{150}\mathrm{Sm}(Z=62, N=88)\ 0_1^+$



$^{152}\mathrm{Sm}(Z=62, N=90)\ 0_1^+$



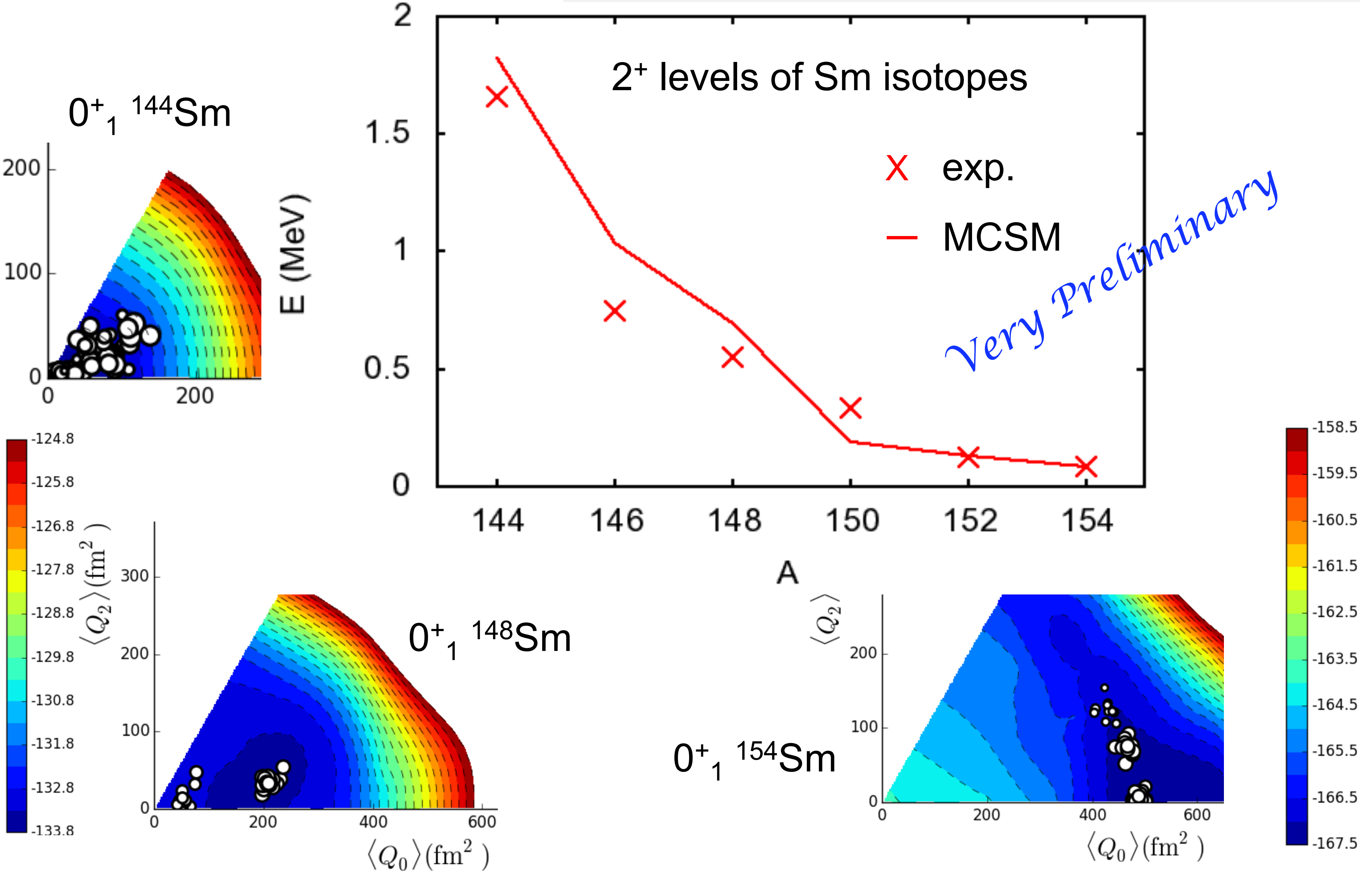
$^{154}\mathrm{Sm}(Z=62, N=92)\ 0_1^+$



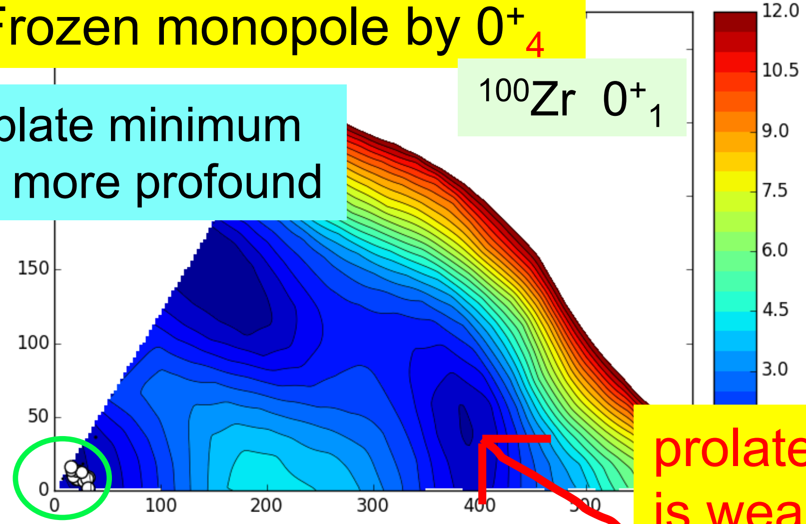
Sm isotopes

proton 8 orbits
neutron 10 orbits

Interaction VMU (gaussian central + π+ρ tensor)



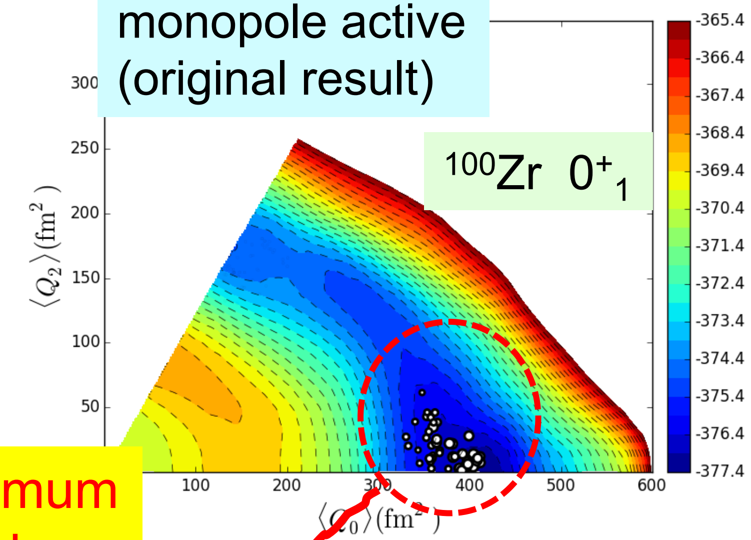
^{100}Zr prolate 0^+_1 and spherical 0^+_4 (T-plot)



Frozen monopole by 0^+_4

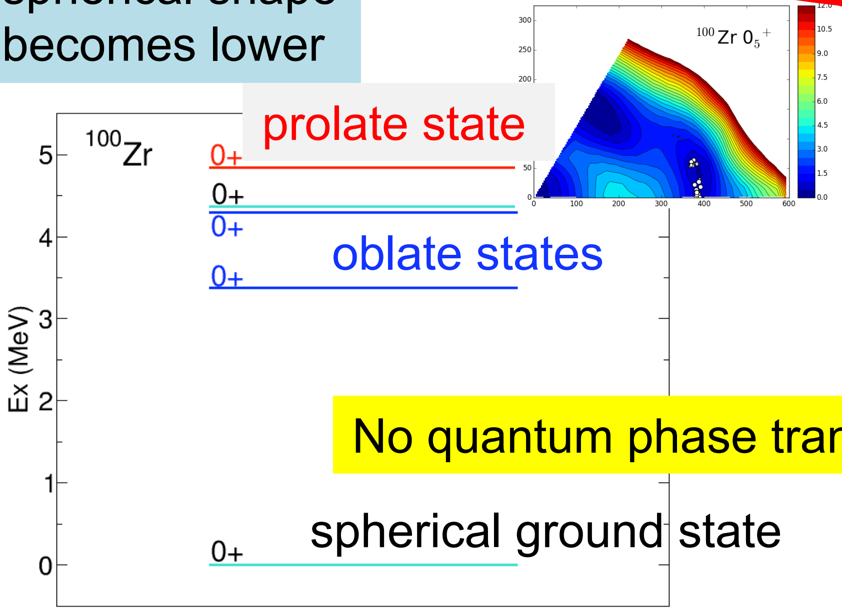
oblate minimum is more profound

prolate minimum is weakened.



monopole active
(original result)

spherical shape becomes lower

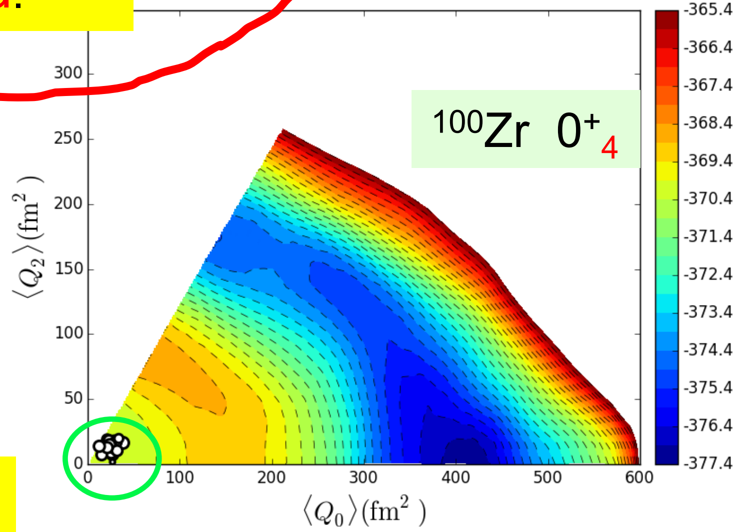


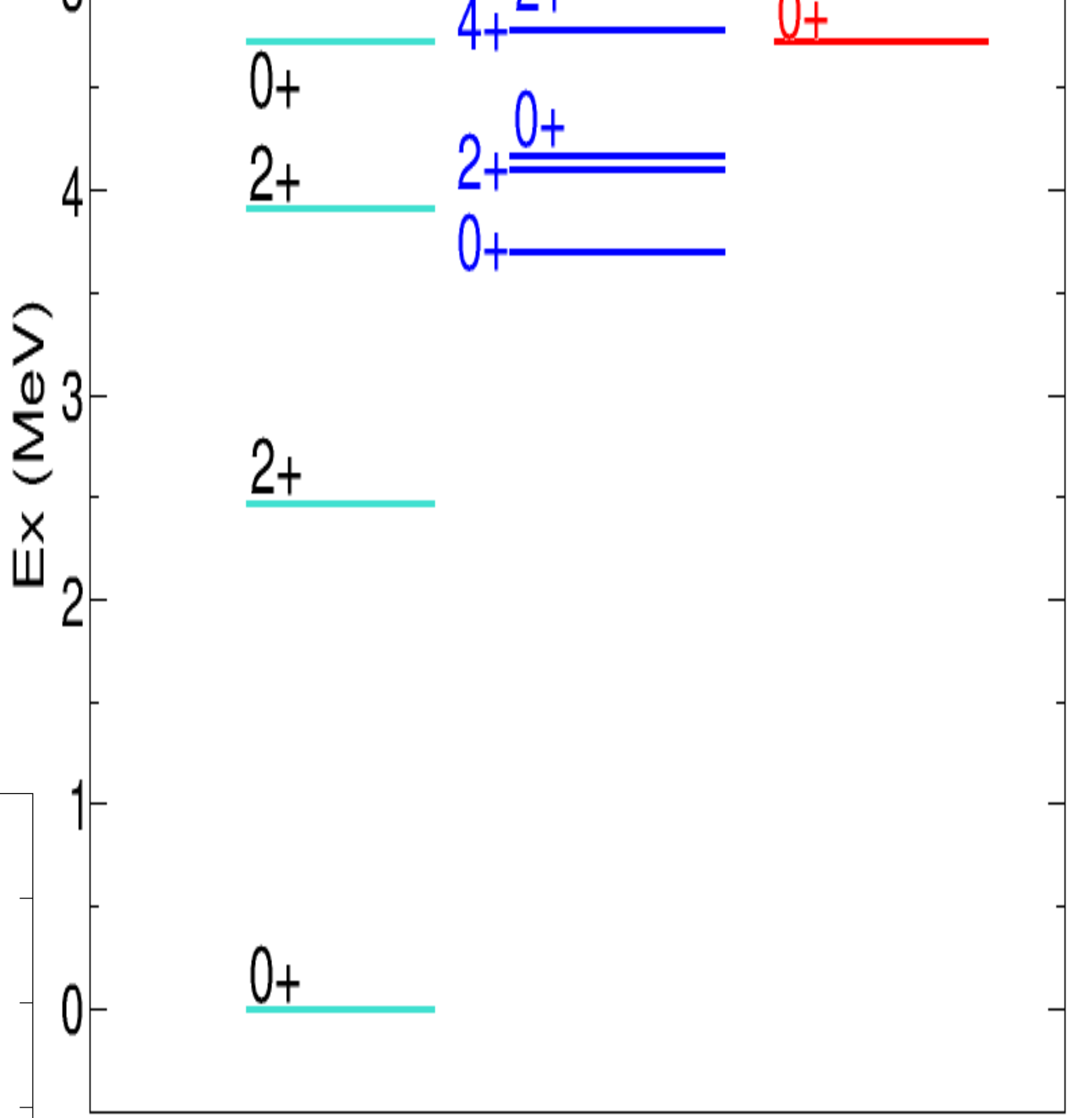
prolate state

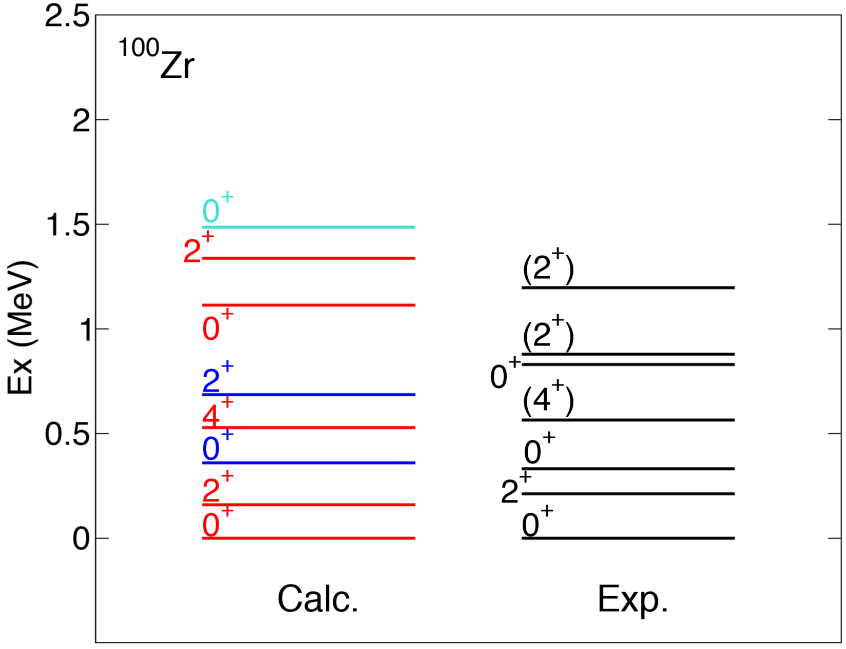
oblate states

No quantum phase transition

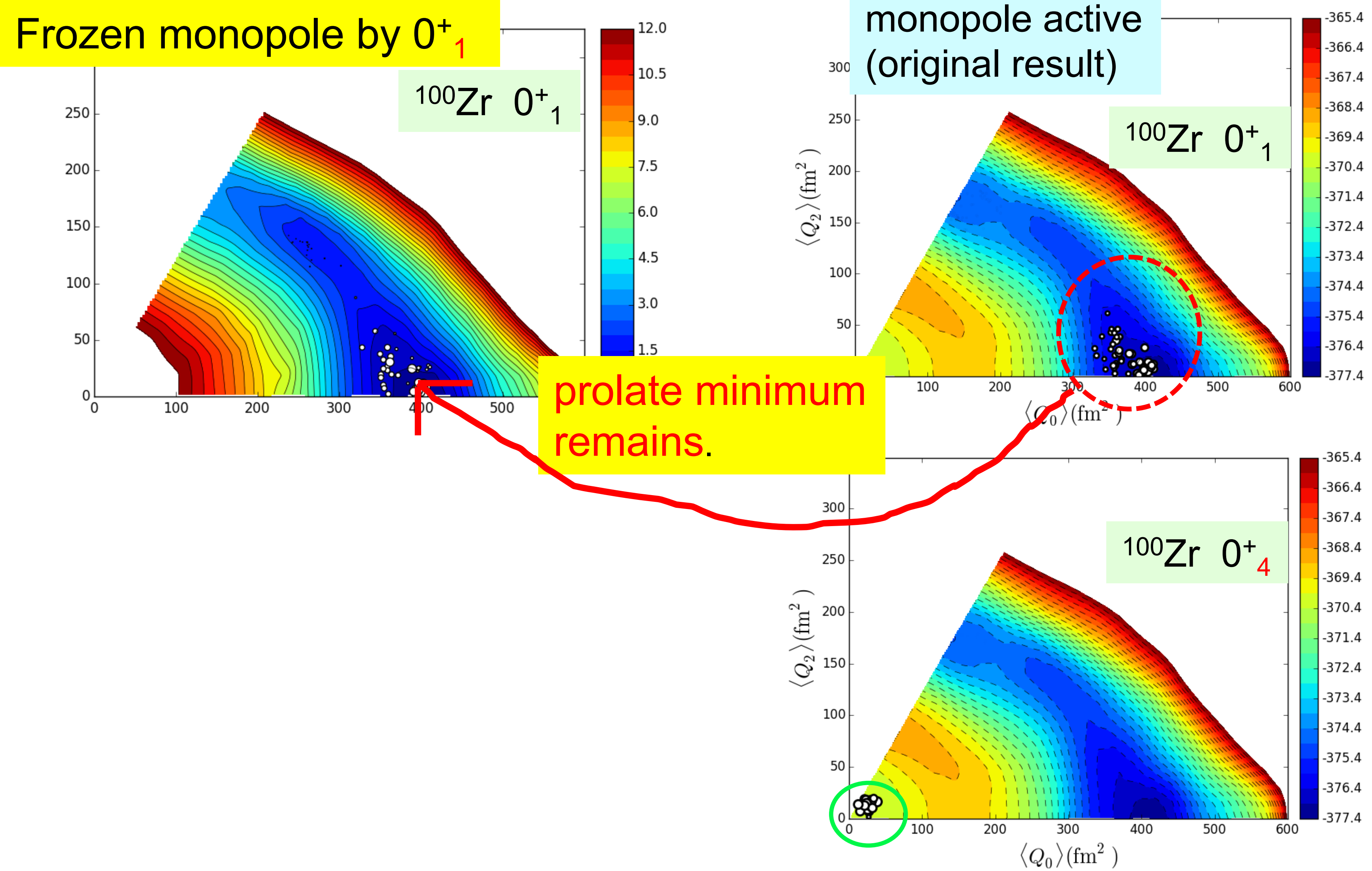
spherical ground state




0+
4+
2+
0+
0+
2+
2+
0+
2+
0+
Ex (MeV)
0
1
2
3
4


100Zr
2.5
2
1.5
1
0.5
0
Ex (MeV)
0+
2+
0+
2+
4+
0+
2+
0+
(2+)
(2+)
0+
(4+)
0+
2+
0+
Calc.
Exp.

^{100}Zr prolate 0^+_1 by frozen S.P.E. (T-plot)

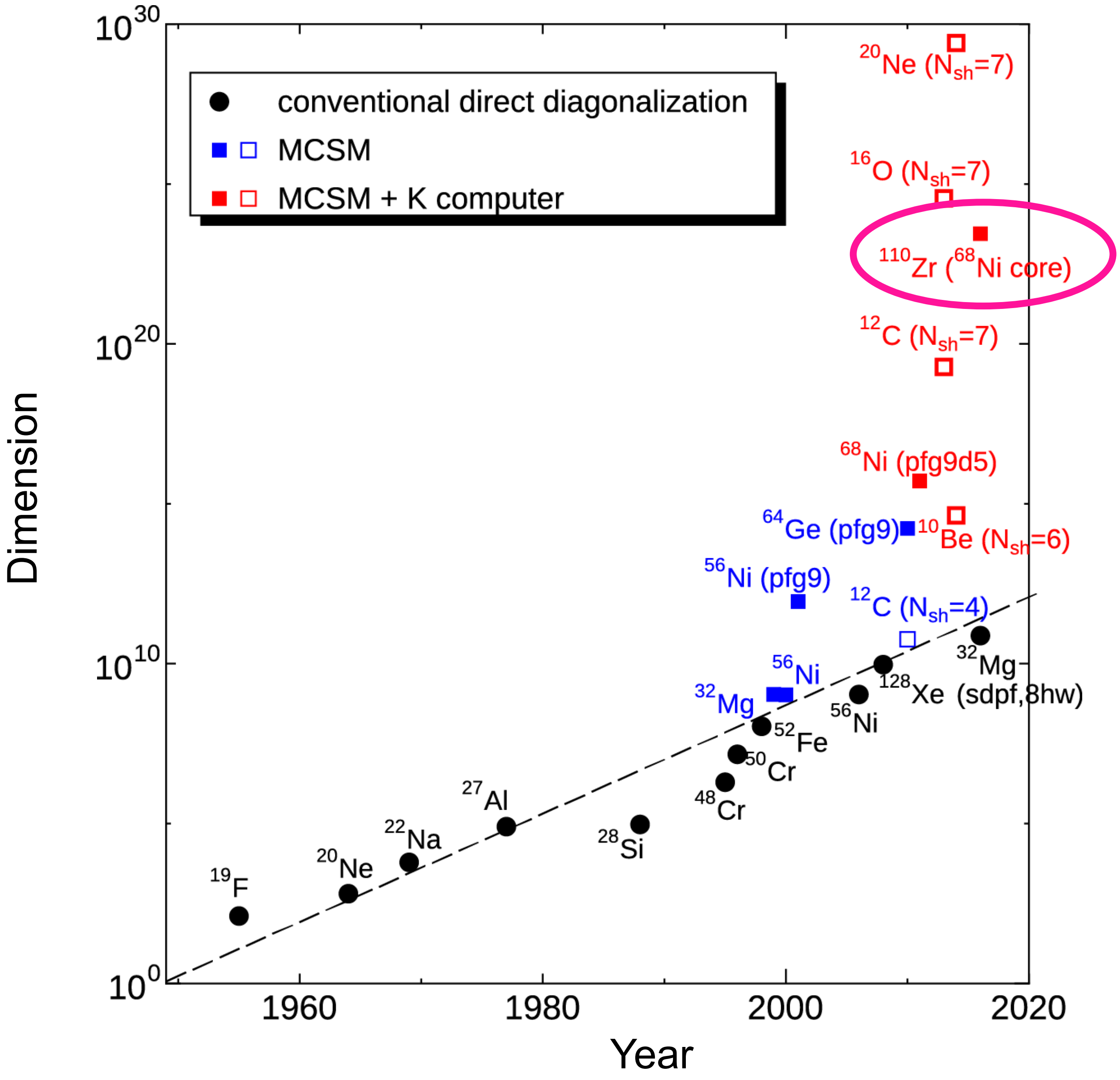


Frozen monopole by 0^+_1

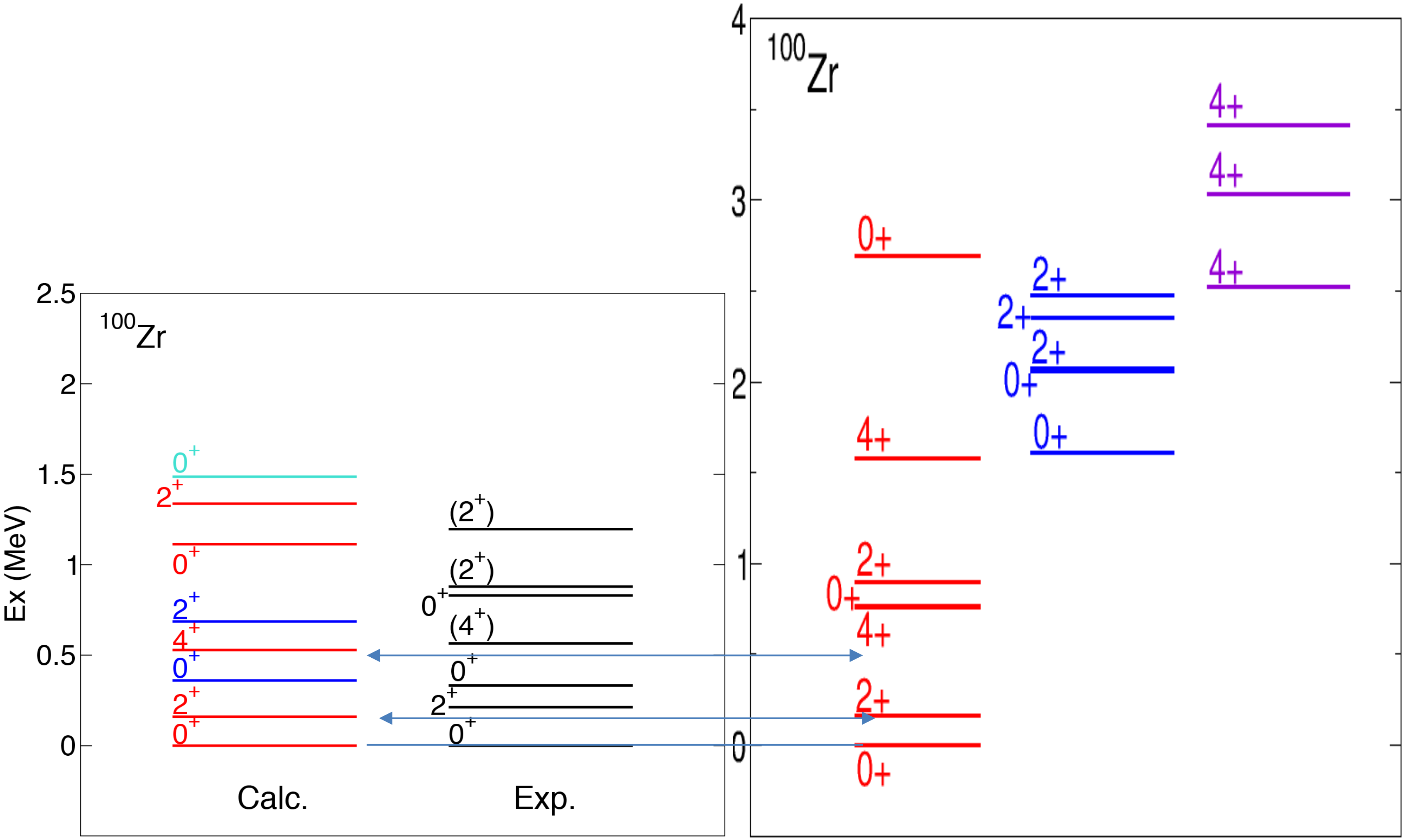
monopole active (original result)

prolate minimum remains.

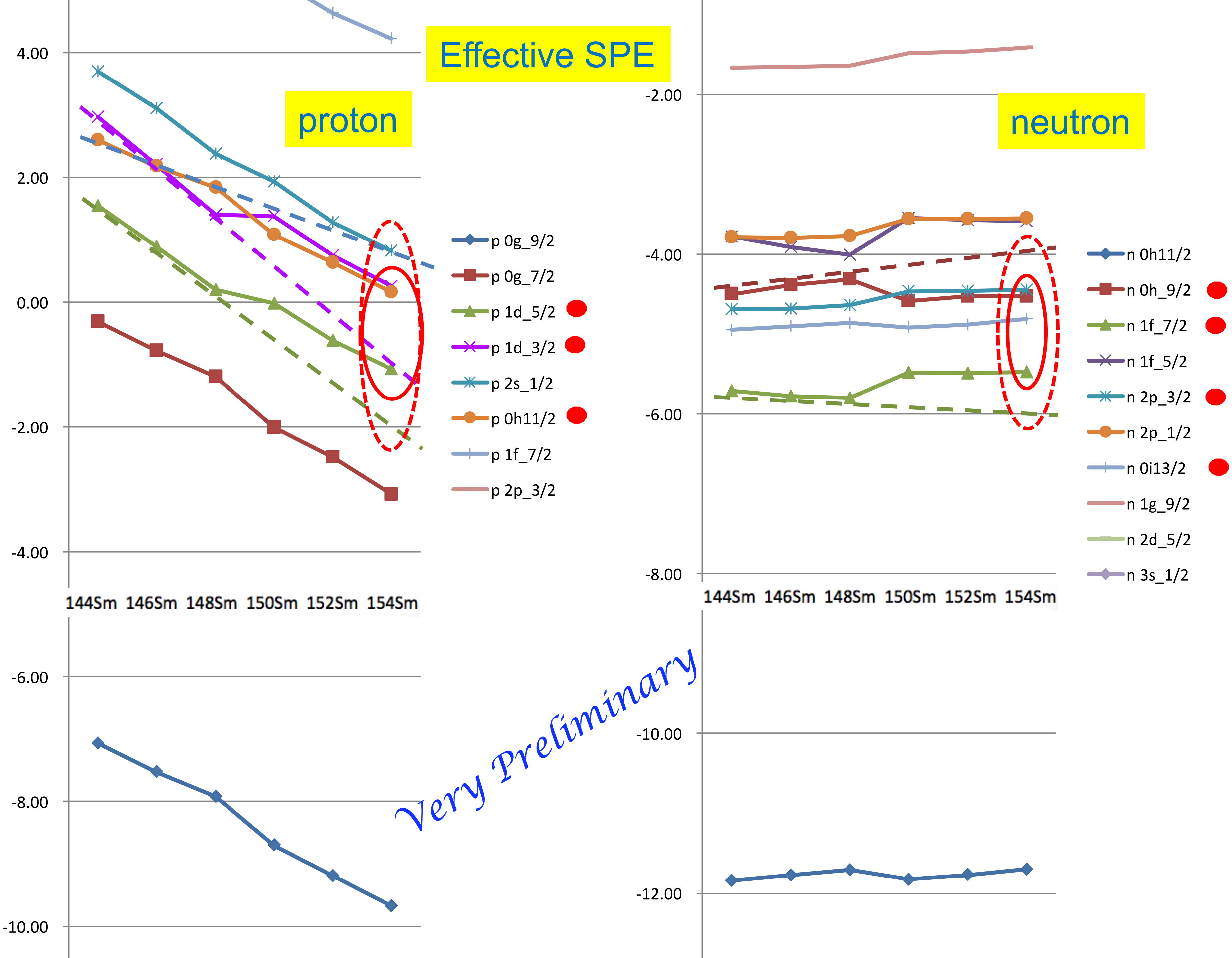
Development of shell-model calculation



~ Avogadro number


^{100}Zr
Ex (MeV)
2.5
2
1.5
1
0.5
0
0^+
2^+
0^+
2^+
4^+
0^+
2^+
0^+
(2^+)
(2^+)
0^+
(4^+)
0^+
2^+
0^+
Calc.
Exp.
^{100}Zr
4
3
2
1
0
0+
4+
2+
0+
4+
2+
0+
2+
2+
2+
0+
0+
4+
4+
4+

Effective SPE



Very Preliminary