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Coexistence of nuclear shapes: mean-field and beyond

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Supported by: Croatian Science Foundation

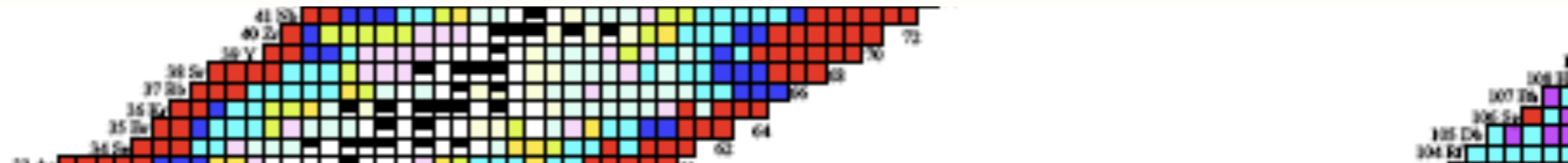


Theoretical framework: energy density functionals

✓ the nuclear many-body problem is effectively mapped onto a one-body problem without explicitly involving internucleon interactions



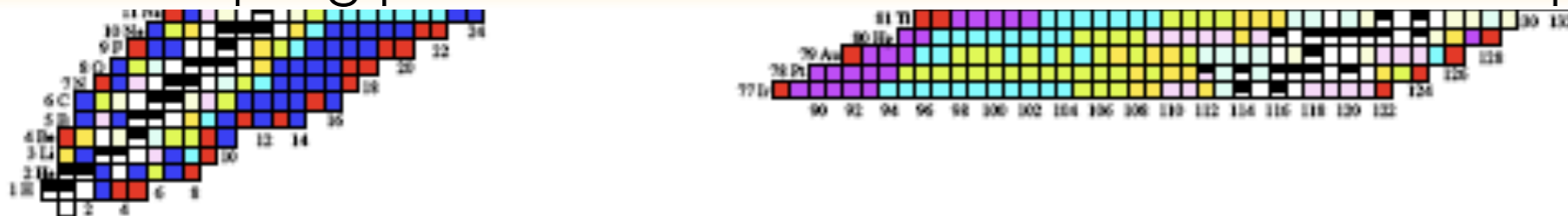
✓ the exact density functional is approximated with powers and gradients of ground state densities and currents



✓ universal density functionals can be extended from relatively light systems to superheavy nuclei and from the valley of stability to the particle drip line

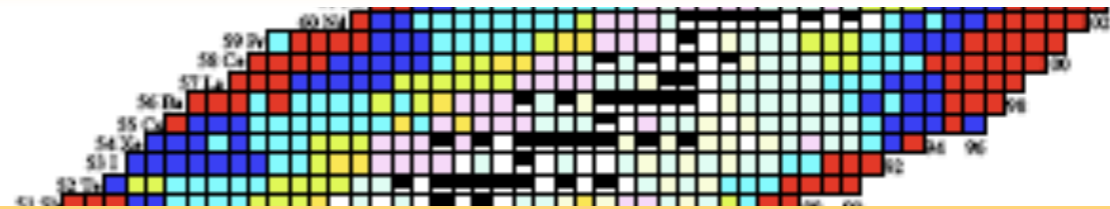


✓ the coupling parameters of the EDF are fine-tuned to empirical data

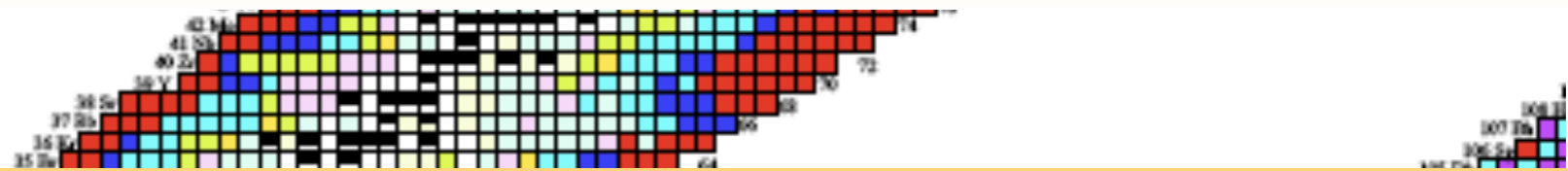


Covariant nuclear energy density functionals

✓ covariant EDFs – built from densities and currents bilinear in the Dirac spinor field of the nucleon



✓ unique parameterization of time-odd components (currents) of the nuclear mean-field



✓ the distinction between scalar and vector self-energies leads to a natural saturation mechanism for nuclear matter

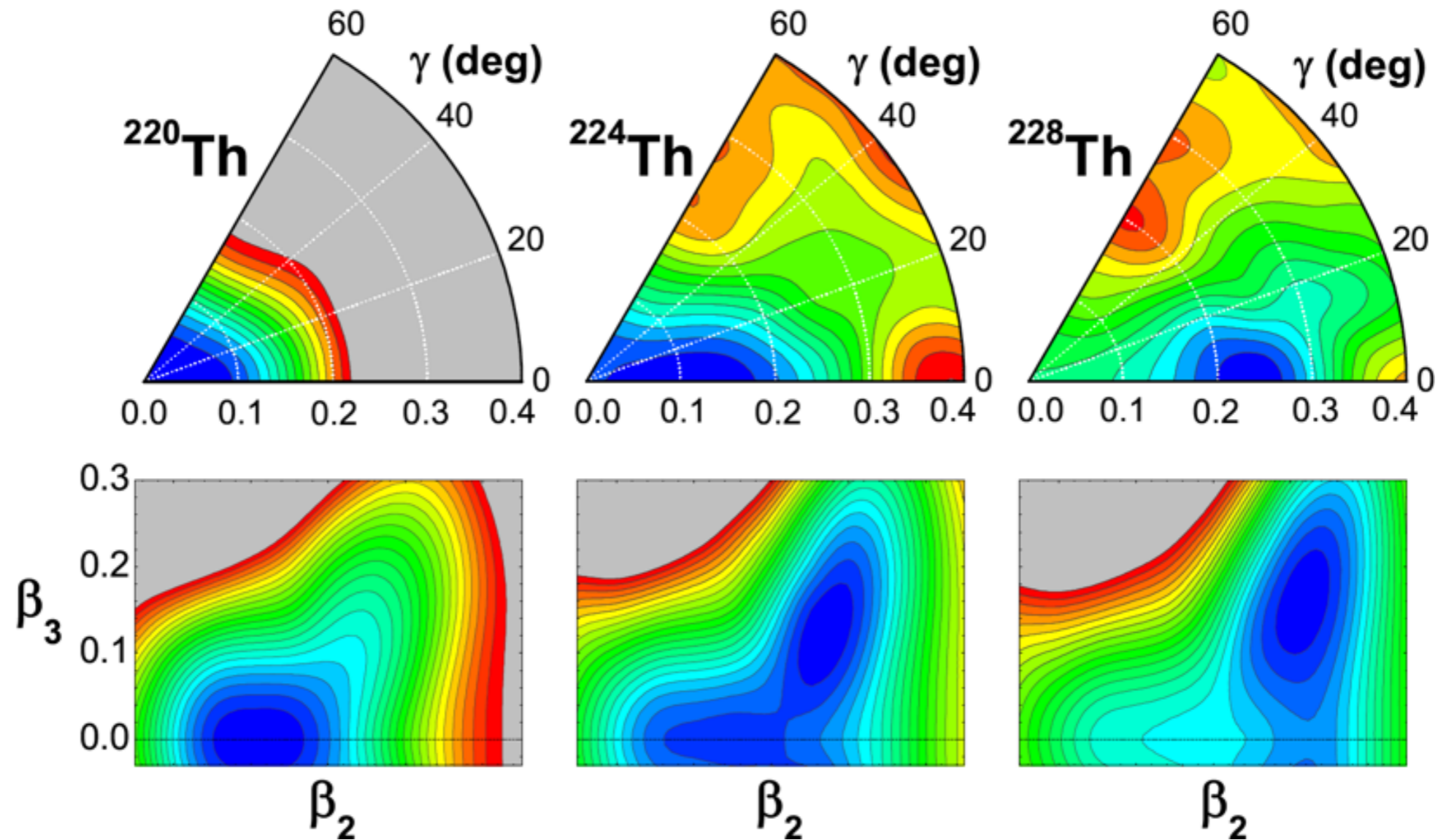


✓ natural inclusion of the spin degree of freedom -> spin-orbit potential with empirical strength



Basic implementation: self-consistent mean-field method

- produces energy surfaces as functions of intrinsic deformation parameters

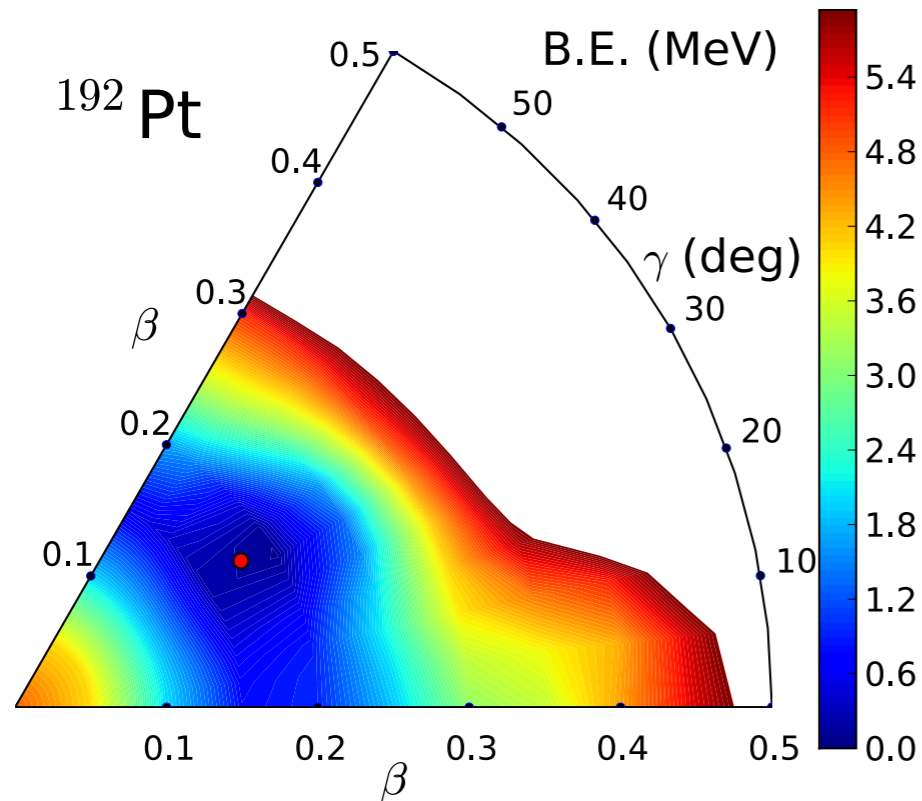


- includes static correlations: deformations and pairing
- does not include collective correlations originating from symmetry restoration and quantum fluctuations around mean-field minima

Beyond mean-field correlations: Collective Hamiltonian

Prog. Part. Nucl. Phys. 66, 519 (2011).
Phys. Rev. C 79, 034303 (2009).

... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom



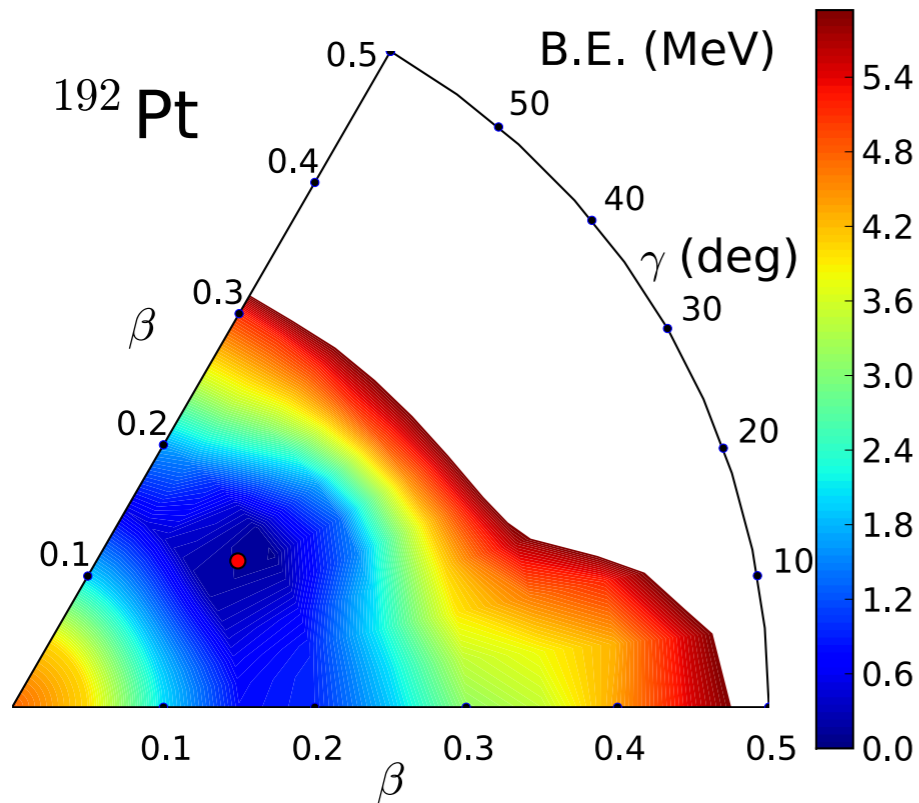
$$H_{\text{coll}} = \mathcal{T}_{\text{vib}}(\beta, \gamma) + \mathcal{T}_{\text{rot}}(\beta, \gamma, \Omega) + \mathcal{V}_{\text{coll}}(\beta, \gamma)$$

$$\mathcal{T}_{\text{vib}} = \frac{1}{2} B_{\beta\beta} \dot{\beta}^2 + \beta B_{\beta\gamma} \dot{\beta} \dot{\gamma} + \frac{1}{2} \beta^2 B_{\gamma\gamma} \dot{\gamma}^2$$

$$\mathcal{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{I}_k \omega_k^2$$

The entire dynamics of the collective Hamiltonian is governed by the seven functions of the intrinsic deformations β and γ : the collective potential, the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the three moments of inertia \mathcal{I}_k .

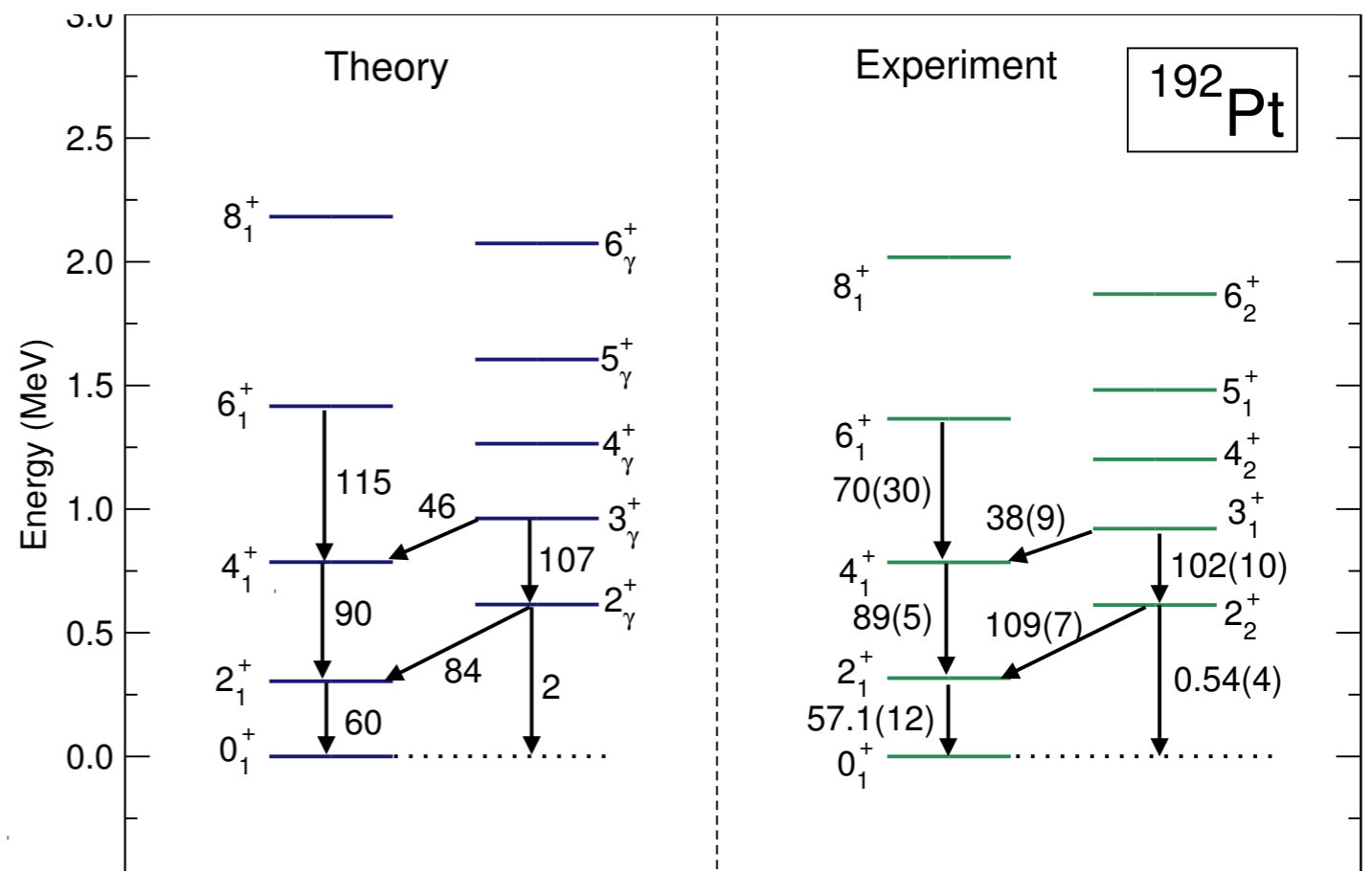
... collective eigenfunction: $\Psi_{\alpha}^{IM}(\beta, \gamma, \Omega) = \sum_{K \in \Delta I} \psi_{\alpha K}^I(\beta, \gamma) \Phi_{MK}^I(\Omega)$



✓ an intuitive interpretation of mean-field results in terms of *intrinsic shapes* and *single-particle states*

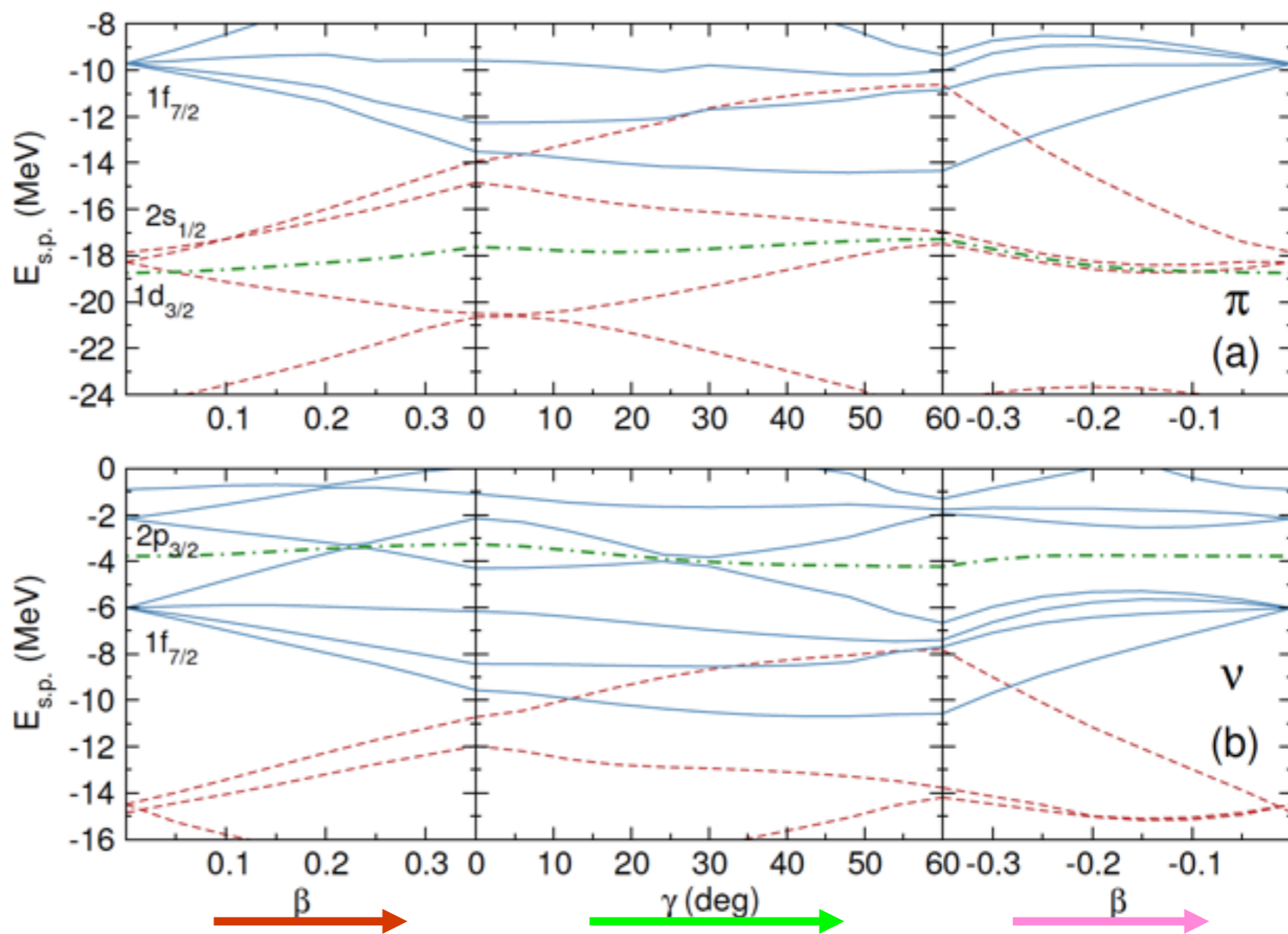
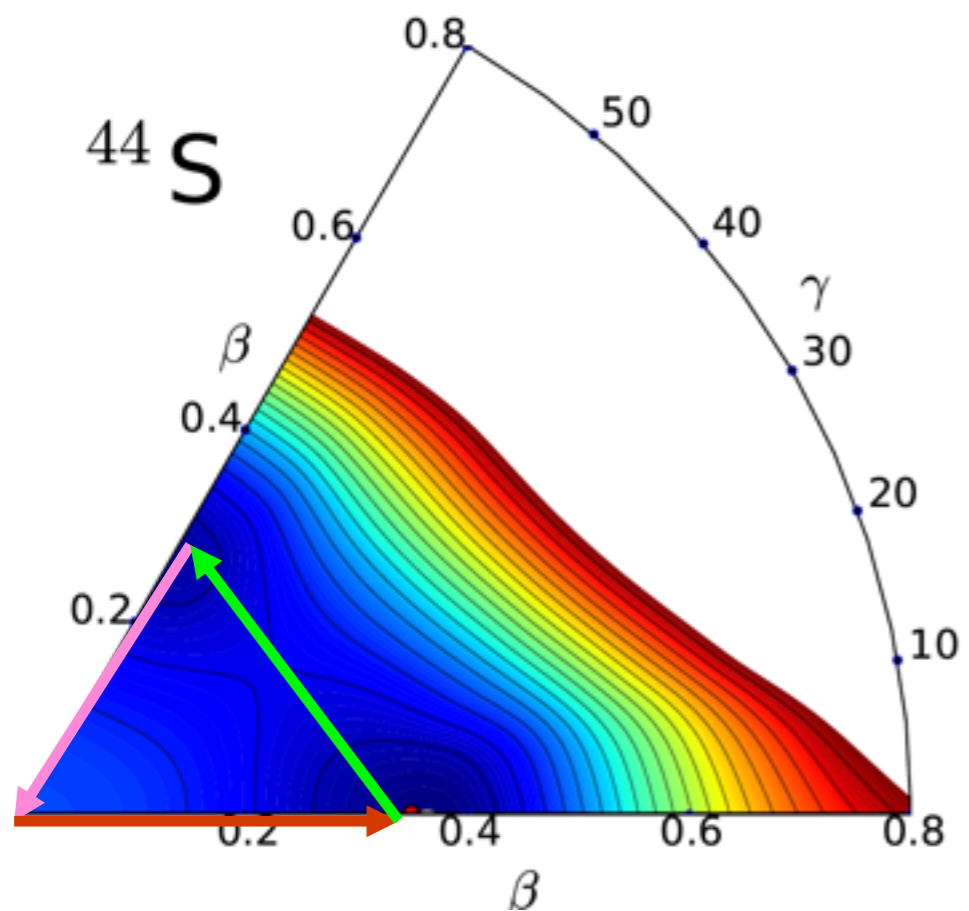
✓ the *full model space* of occupied states can be used; no distinction between core and valence nucleons, *no need for effective charges!*

Prog. Part. Nucl. Phys. 66, 519 (2011).



Coexisting shapes in N=28 isotones

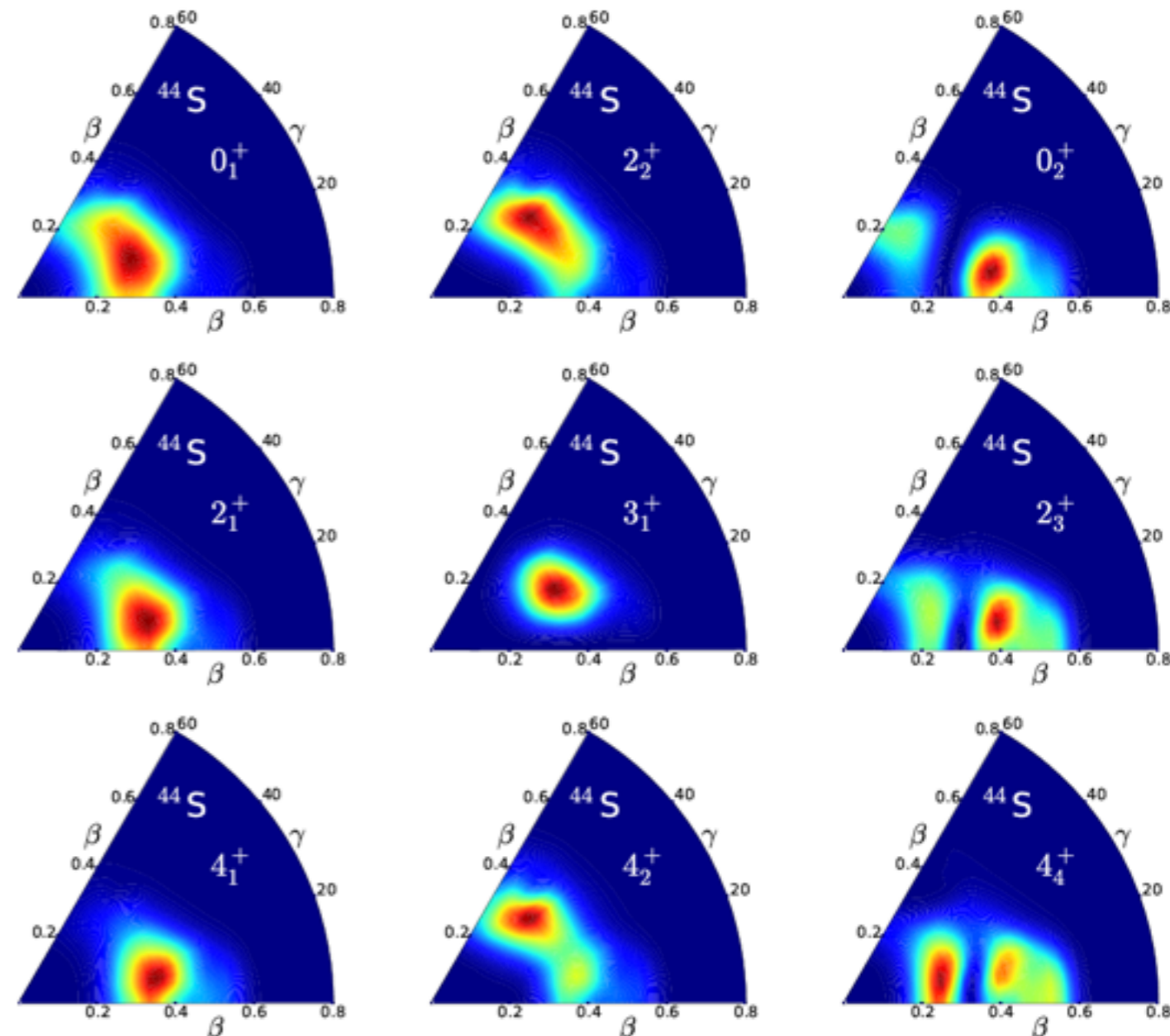
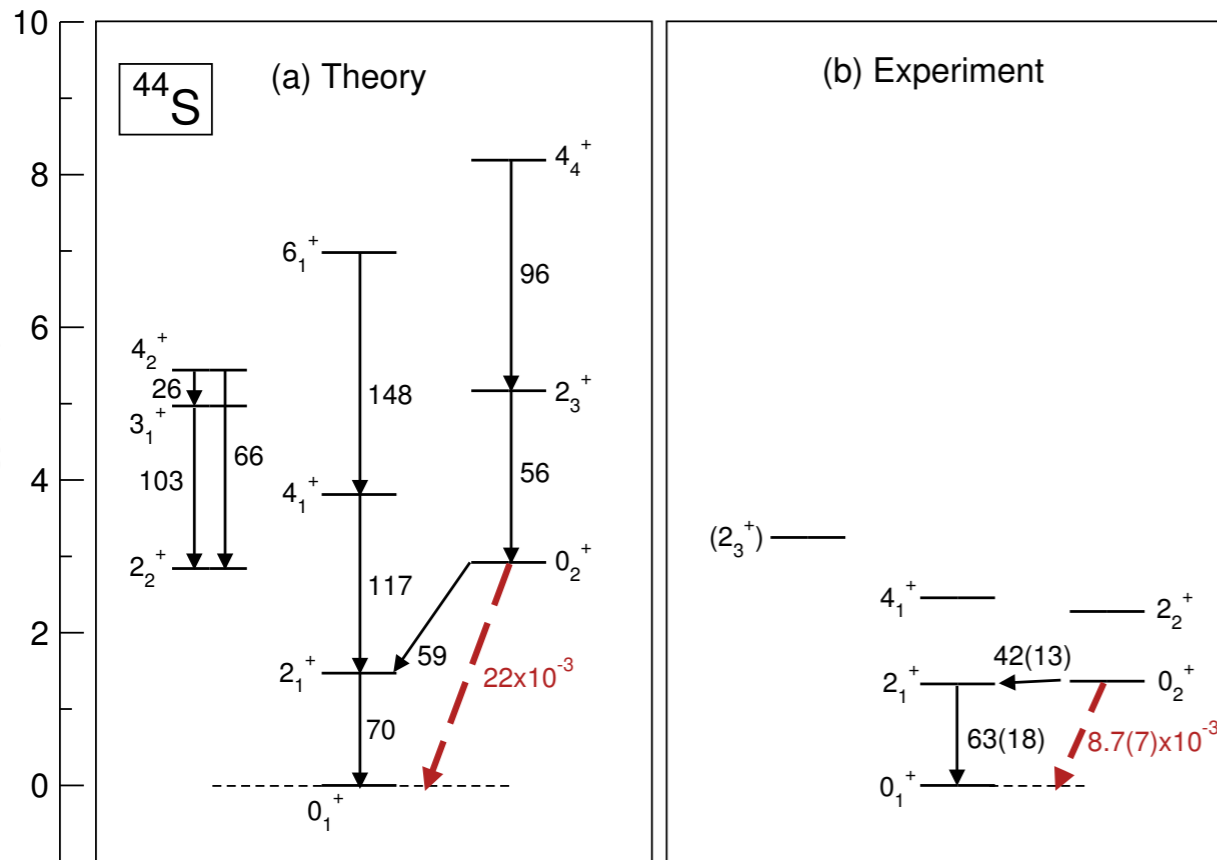
J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005



Coexisting shapes in N=28 isotones

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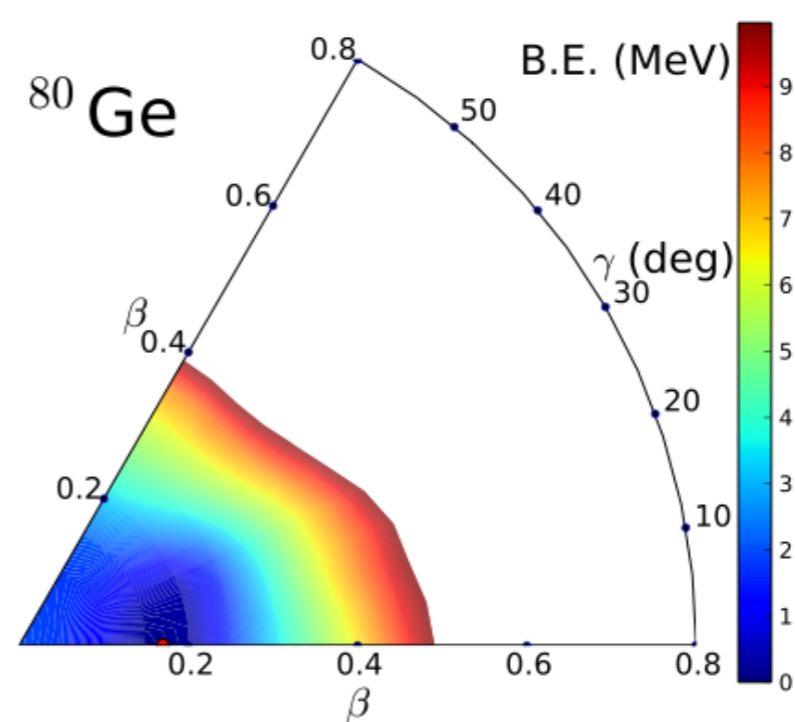
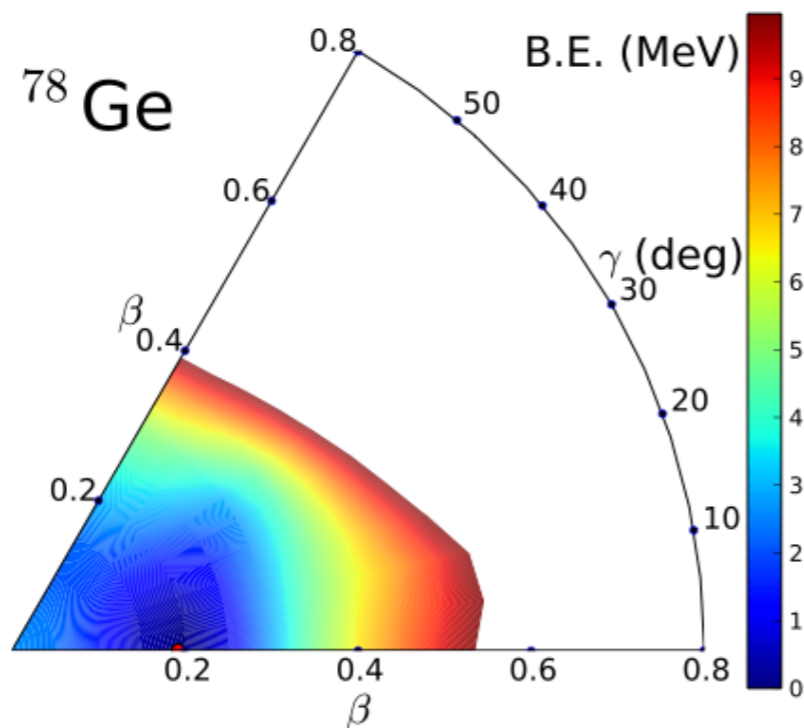
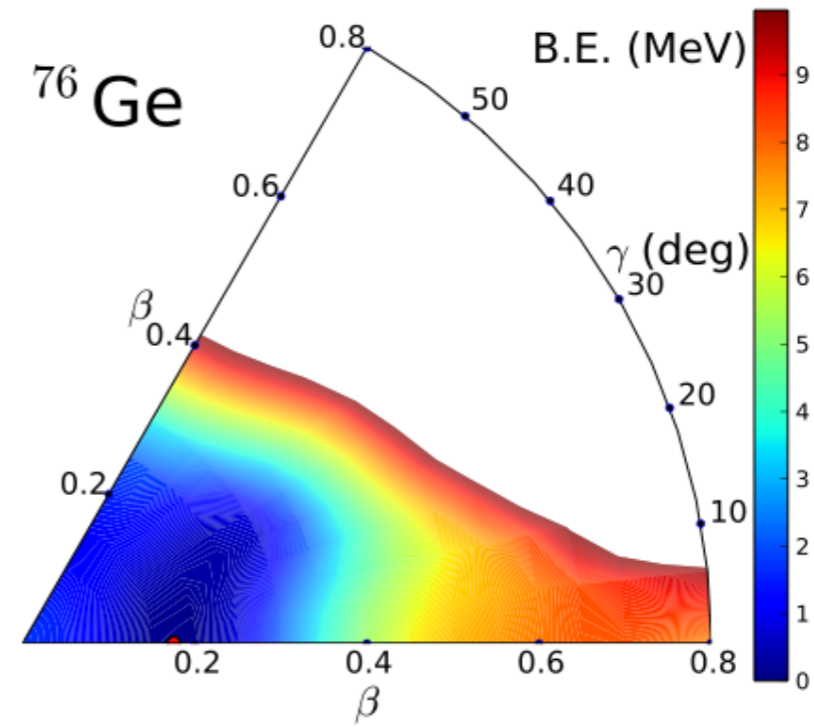
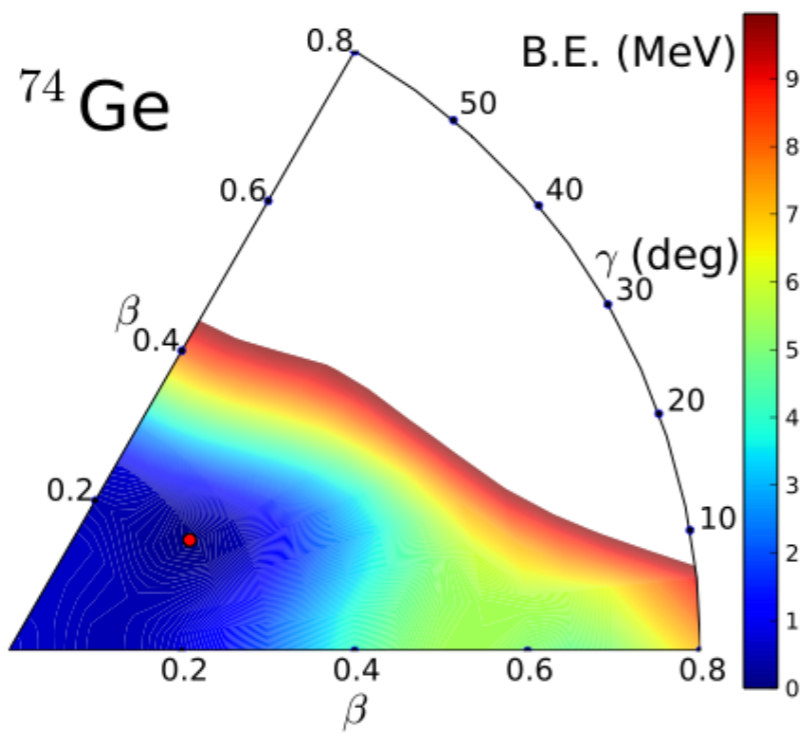
Probability distributions in the β - γ plane

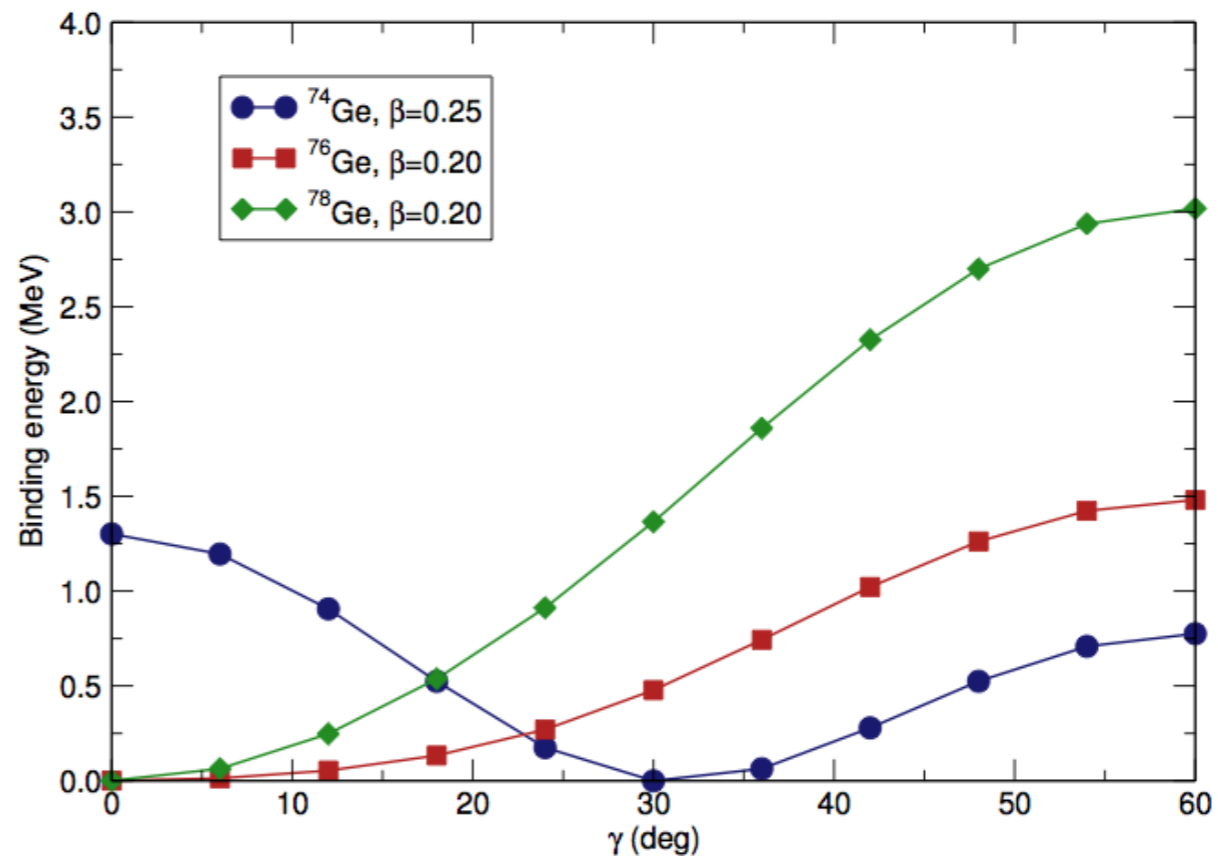
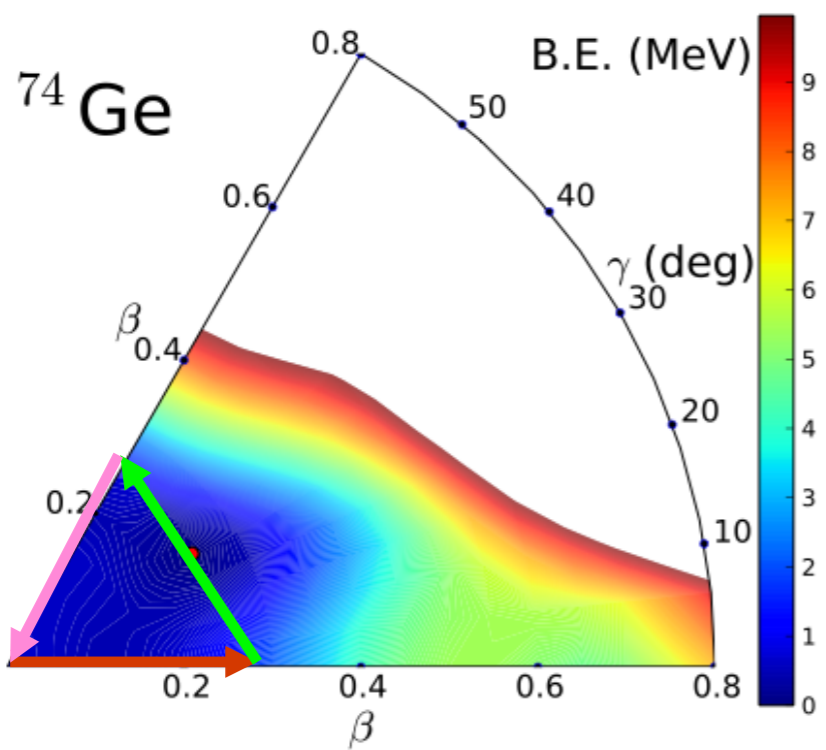
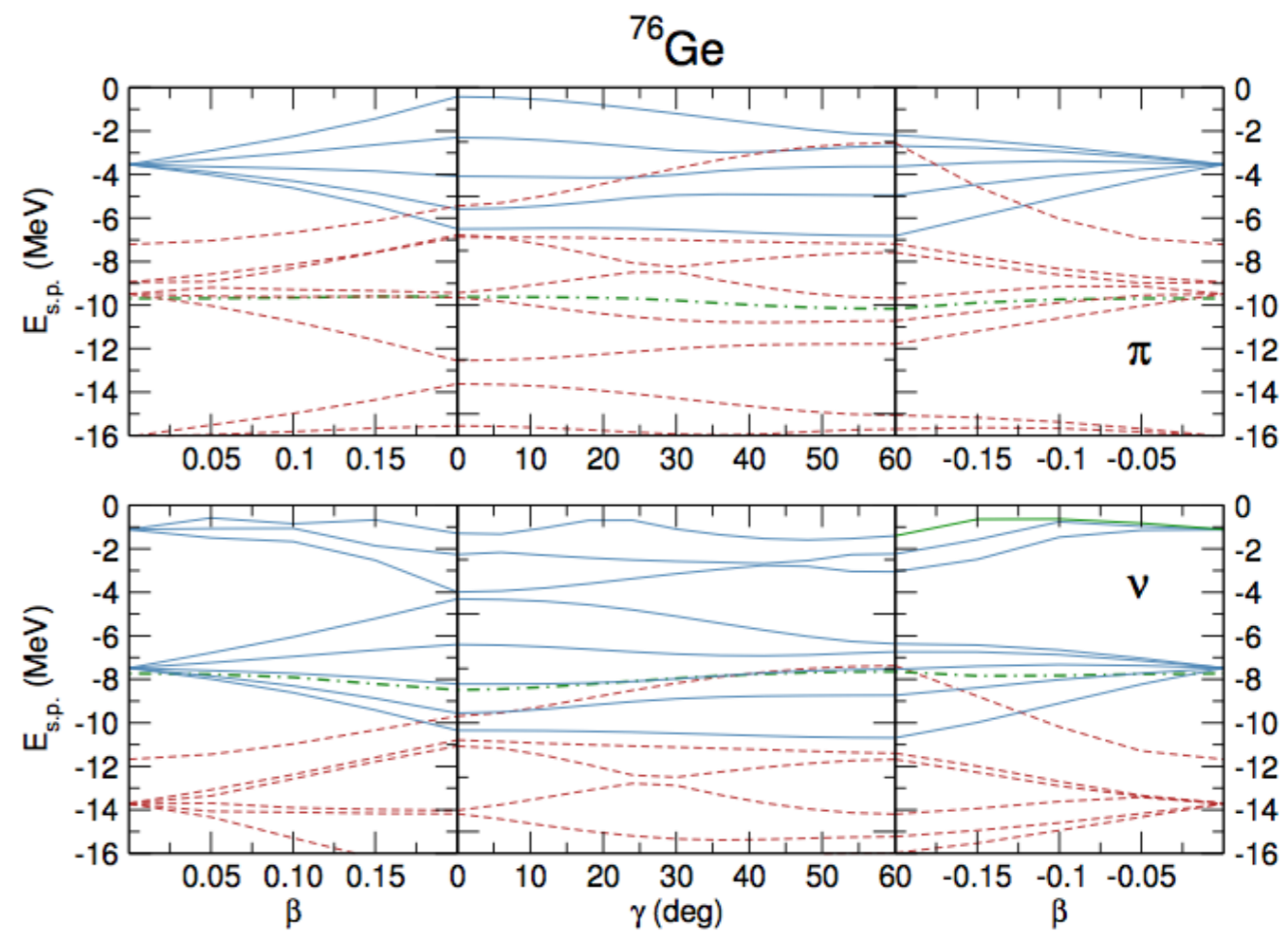
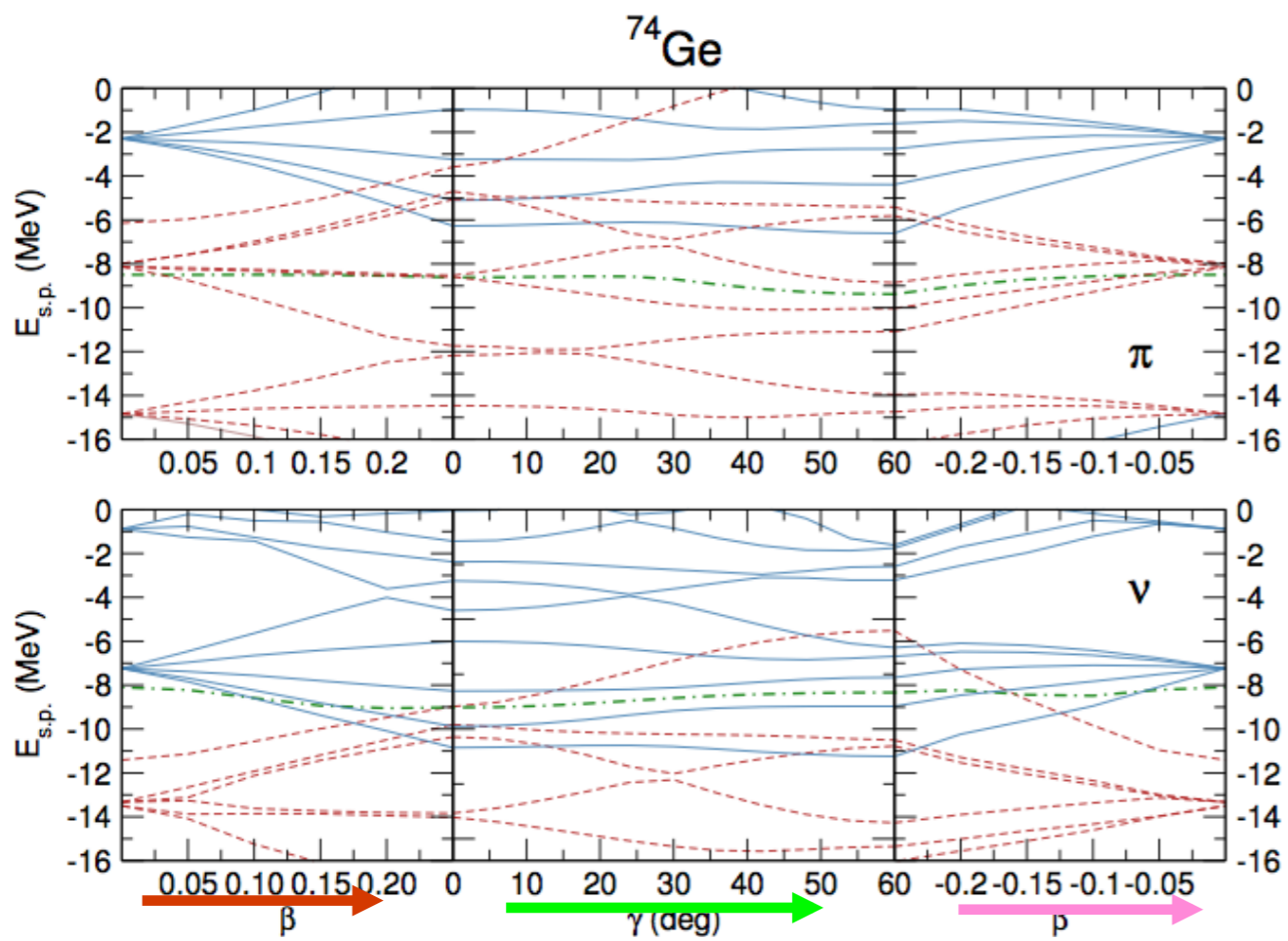


	$K = 0$	$K = 2$	Q_{spec}
2_1^+	89%	11%	-10.8
2_2^+	21%	79%	8.2
2_3^+	78%	22%	-7.3

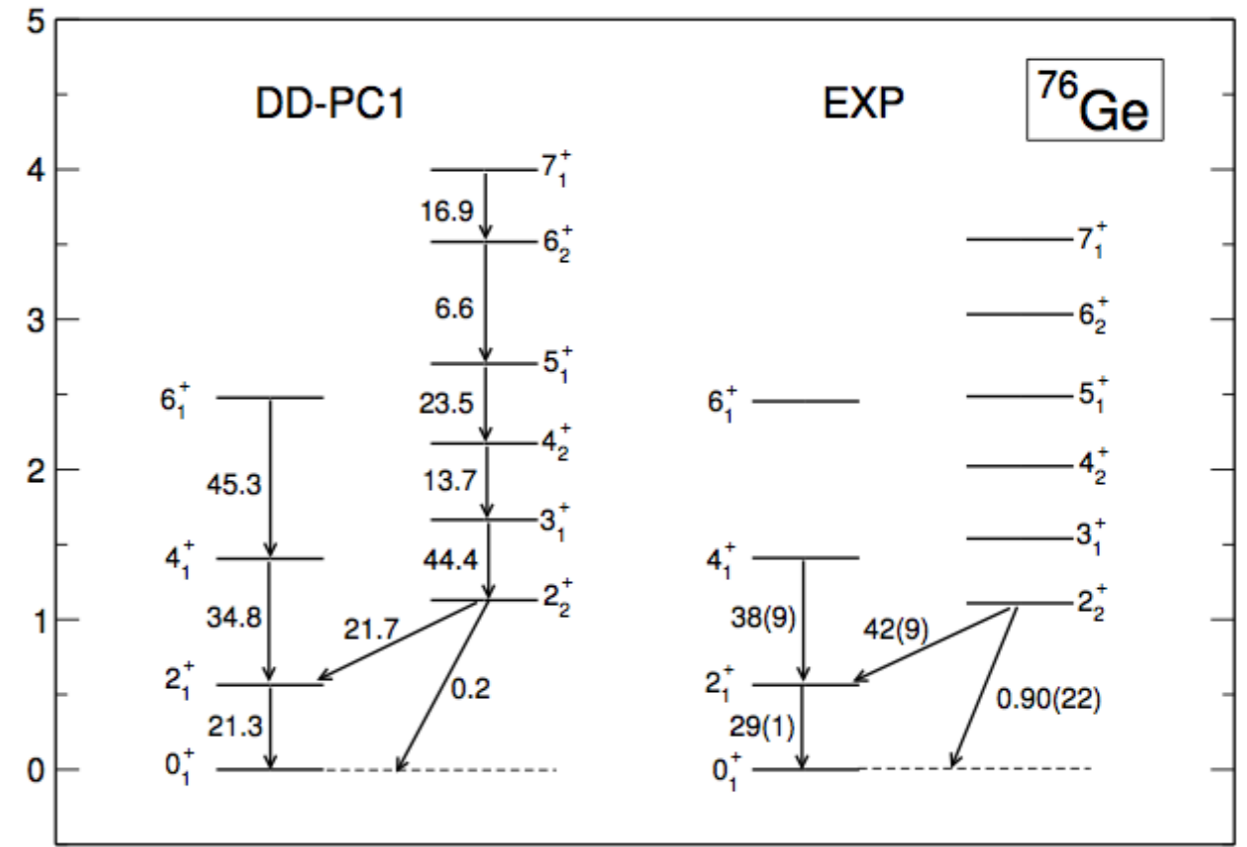
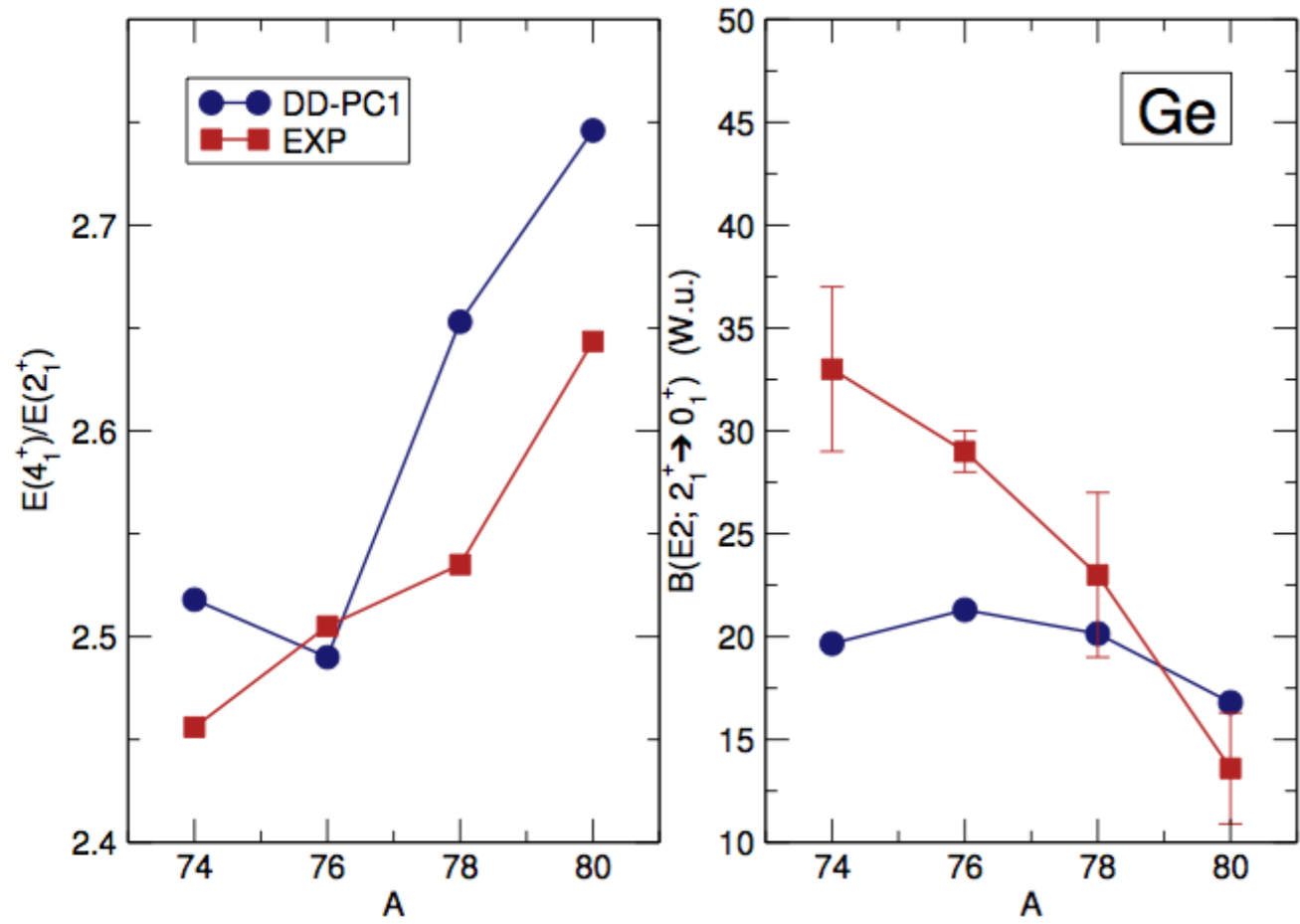
Shape evolution and triaxiality in germanium isotopes

Phys. Rev. C 89, 044325 (2014).



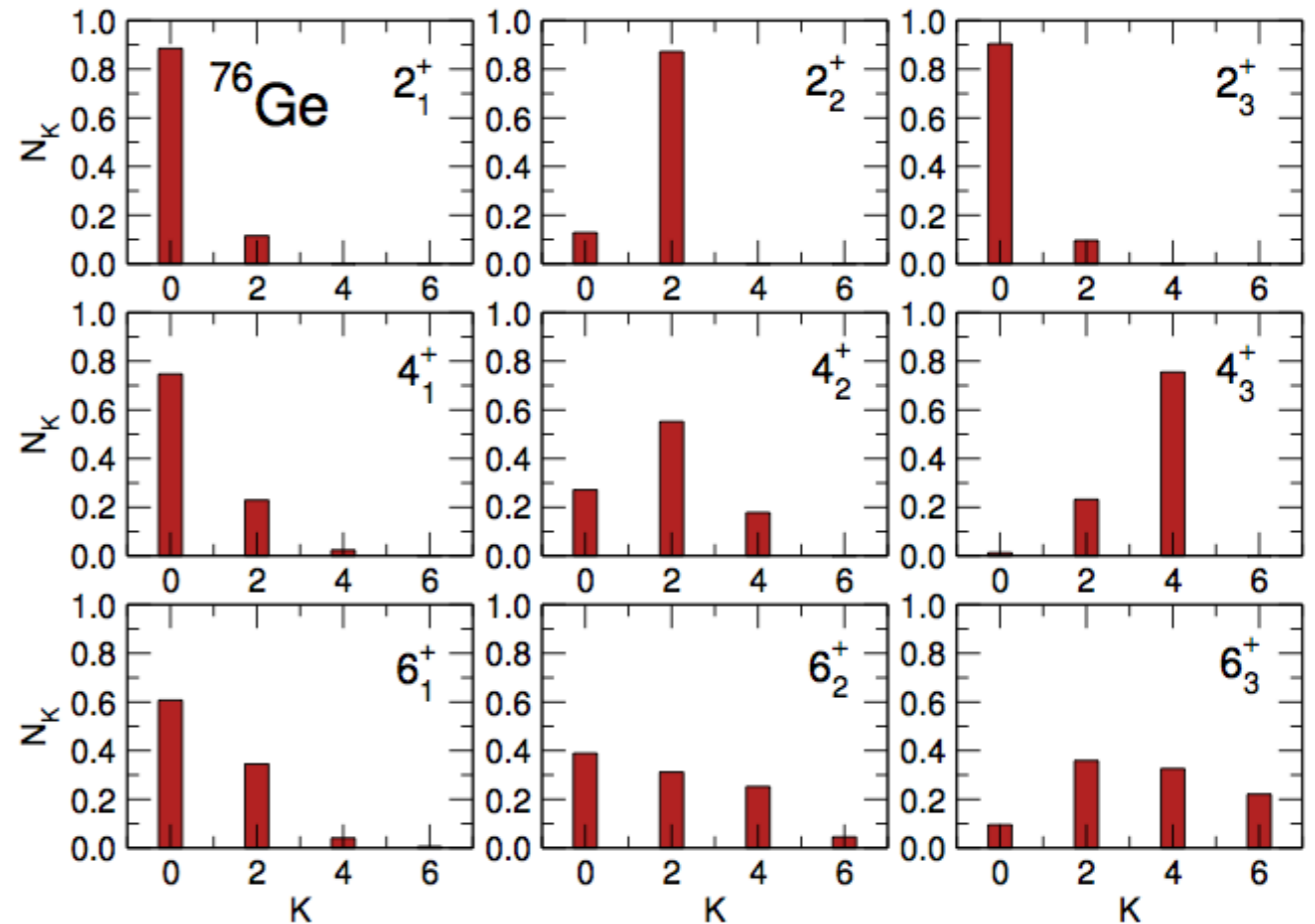


Quadrupole collective Hamiltonian based on the functional DD-PCI



Distribution of K components (projection of the angular momentum on the body-fixed symmetry axis) in the collective wave functions of the nucleus ^{76}Ge .

$$N_K = 6 \int_0^{\pi/3} \int_0^\infty |\psi_{\alpha,K}(\beta, \gamma)|^2 \beta^4 |\sin 3\gamma| d\beta d\gamma.$$



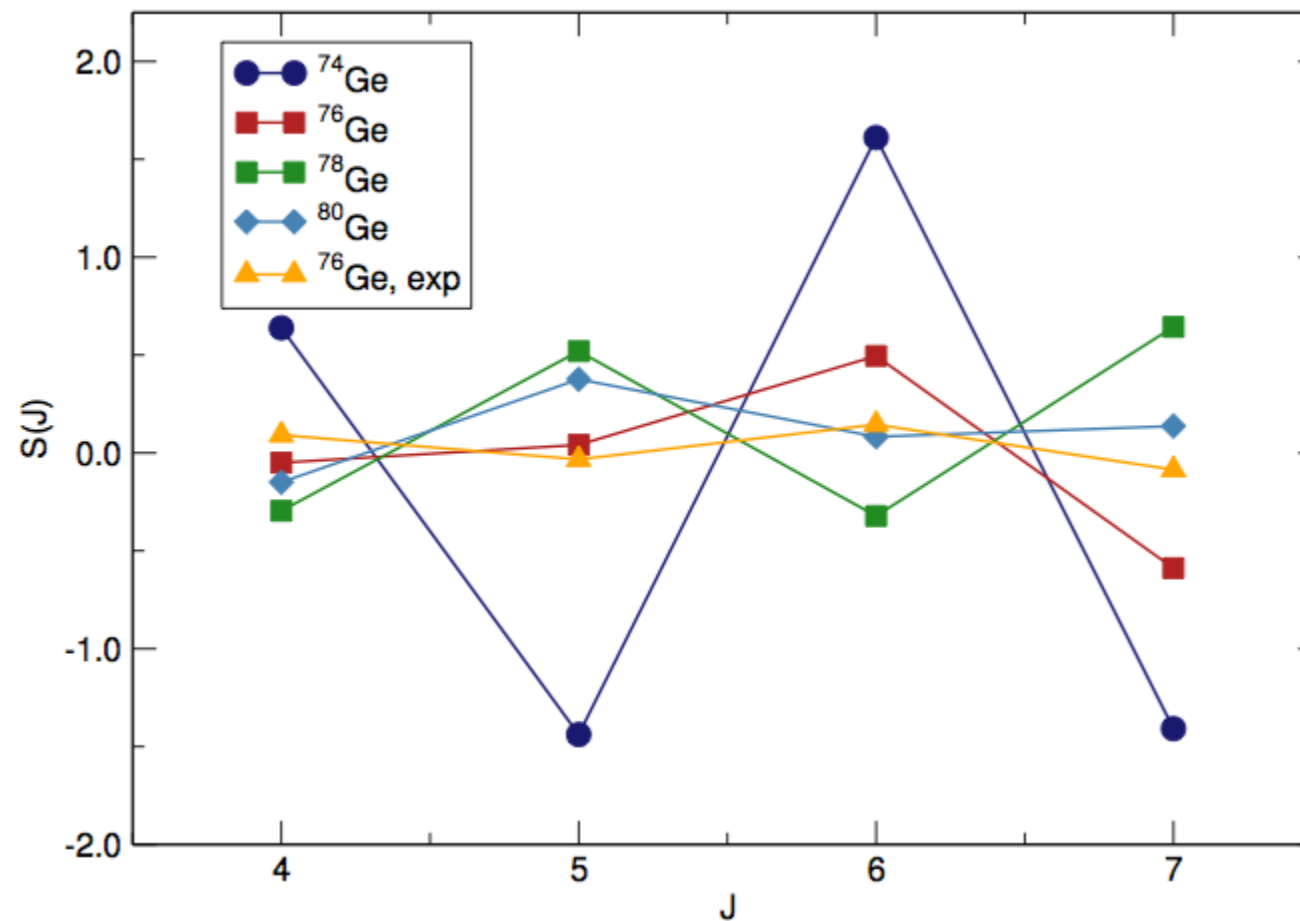
→ more K-mixing for states with higher angular momenta.

The level of K-mixing is reflected in the staggering in energy between odd- and even-spin states in the γ band:

$$S(J) = \frac{E[J_{\gamma}^{+}] - 2E[(J-1)_{\gamma}^{+}] + E[(J-2)_{\gamma}^{+}]}{E[2_{1}^{+}]}$$

Deformed γ -soft potential $\Rightarrow S(J)$ oscillates between negative values for even-spin states and positive values for odd-spin states.

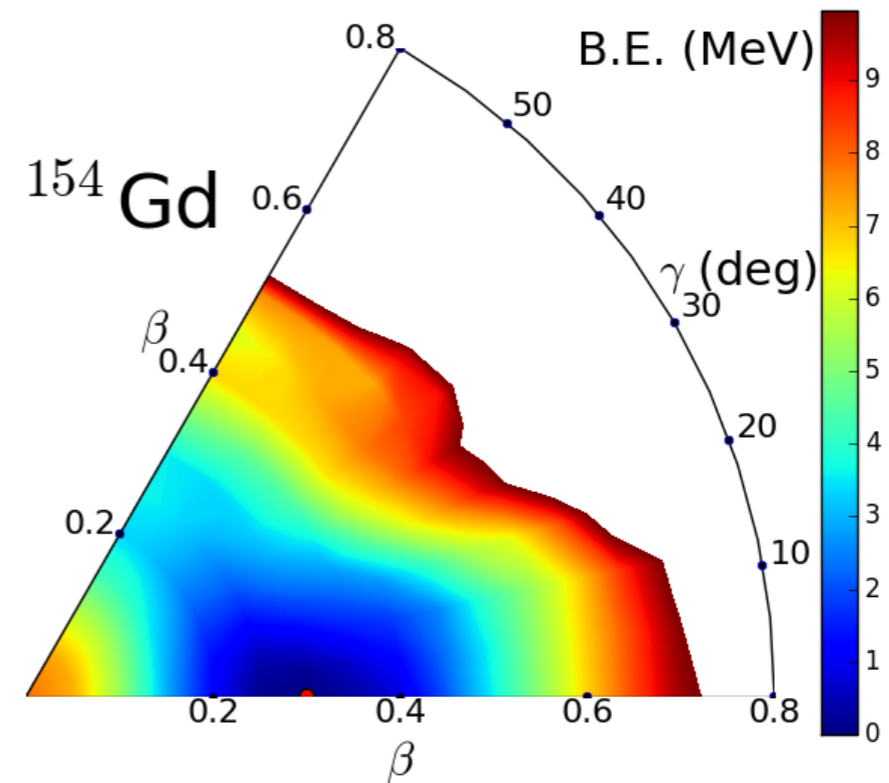
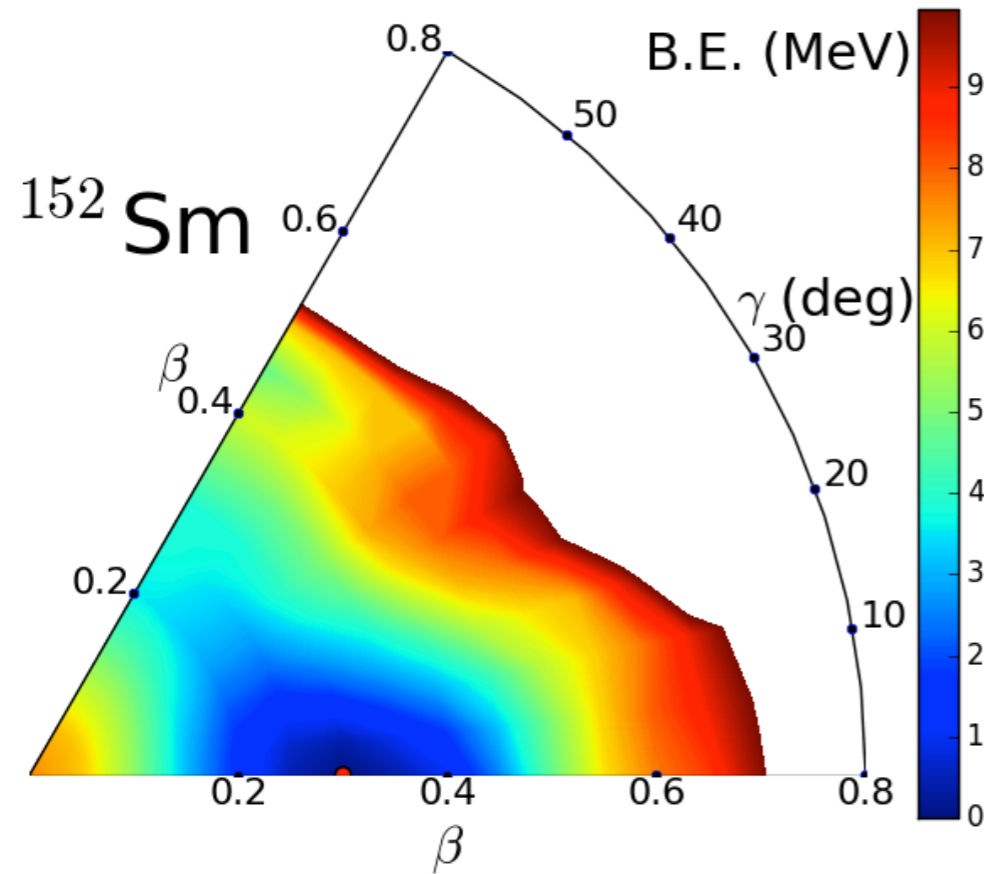
γ -rigid triaxial potential $\Rightarrow S(J)$ oscillates between positive values for even-spin states and negative values for odd-spin states.



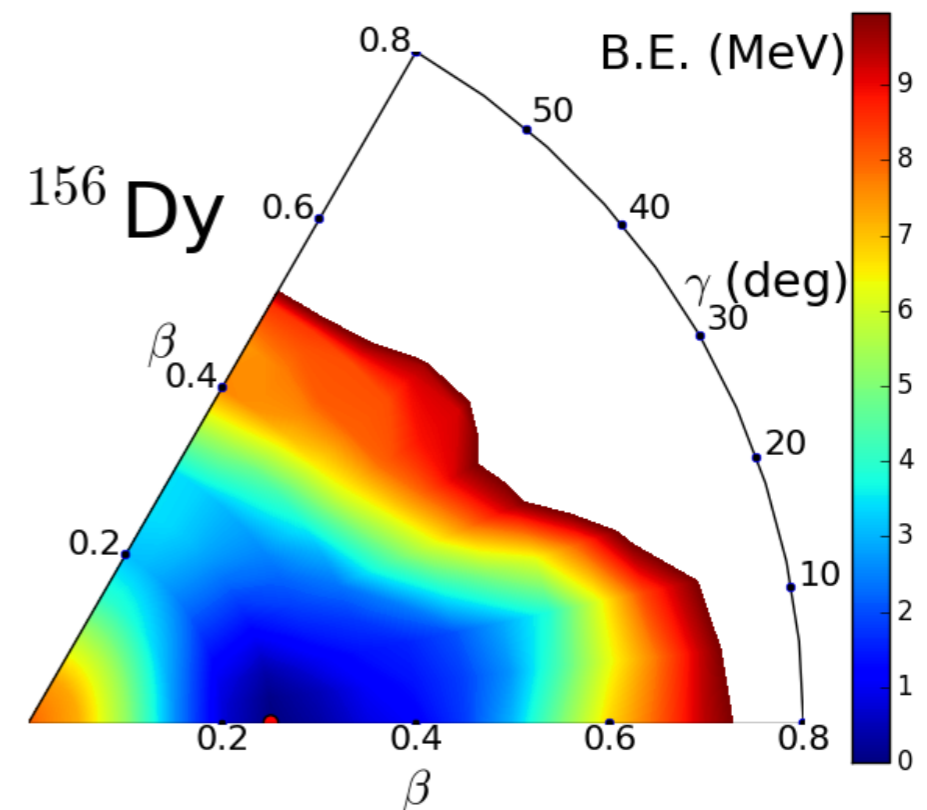
The mean-field potential of ^{76}Ge is γ soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but they are not strong enough to stabilize a $\gamma \approx 30^\circ$ triaxial shape.

Lowest 0^+ excitations in N=90 rare-earth nuclei

J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005

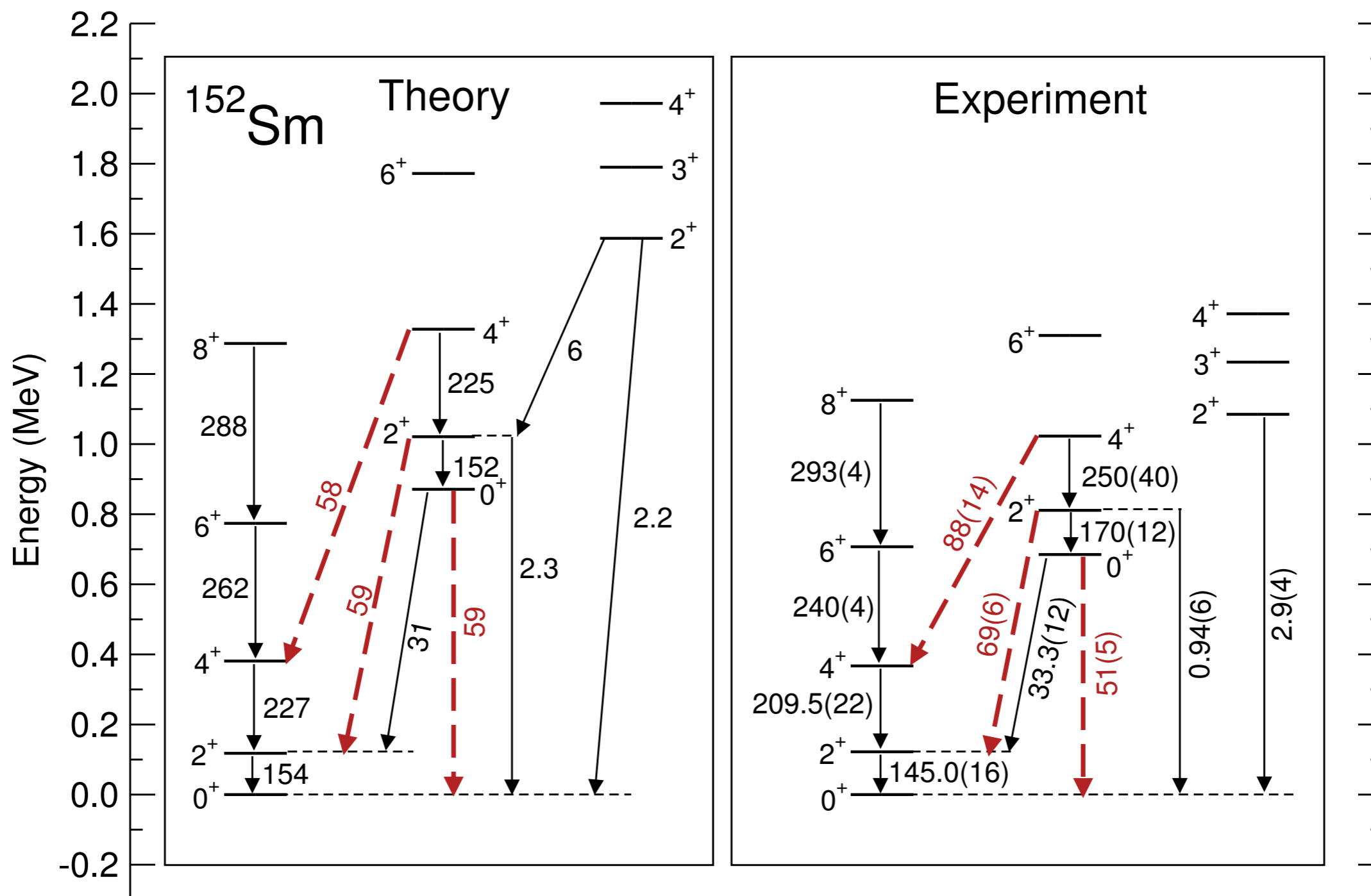


PES (β - γ plane) in N=90 rare-earth nuclei calculated with the DD-PC1 EDF.



Lowest 0^+ excitations in $N=90$ rare-earth nuclei

J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005

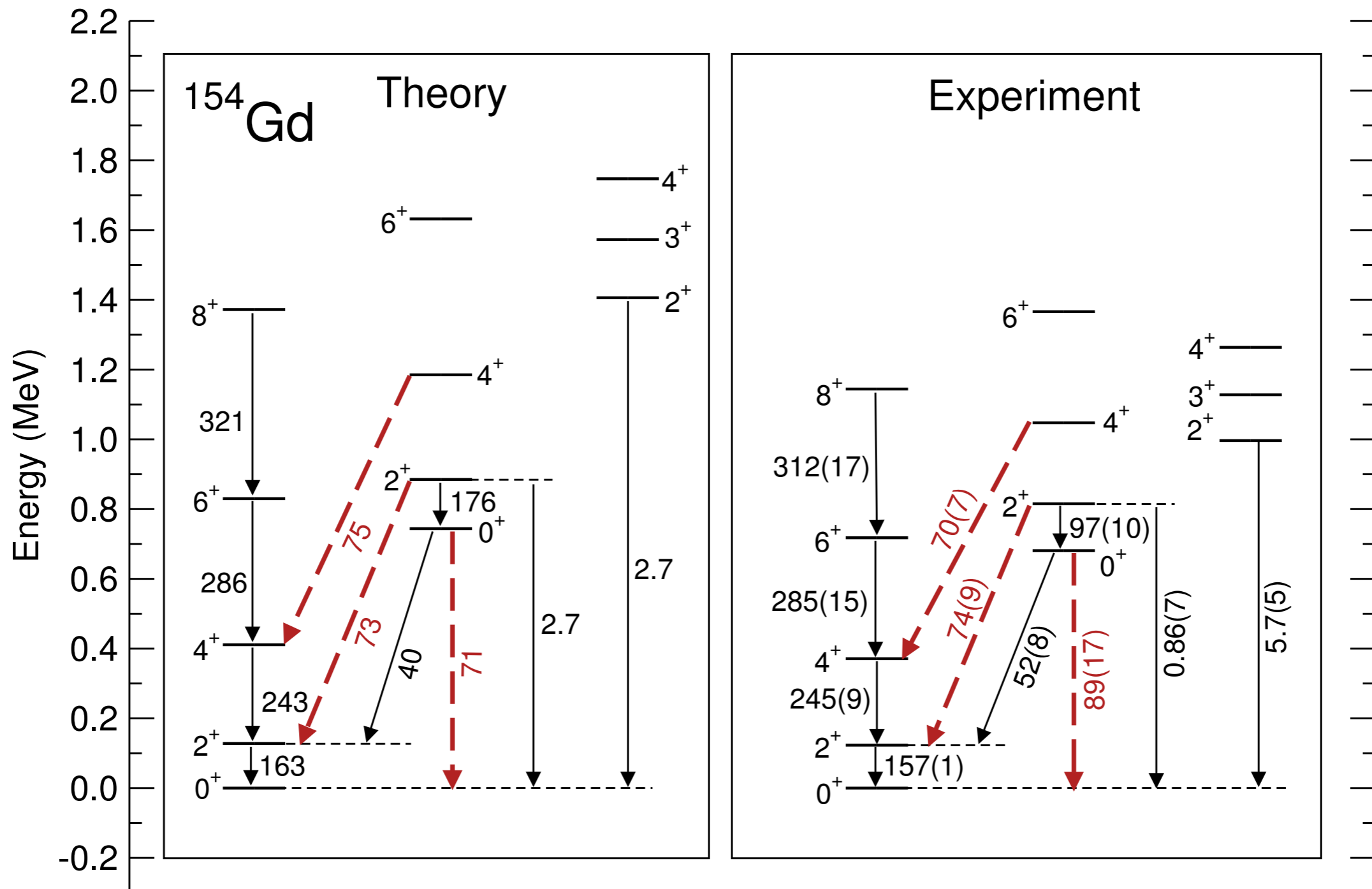


Solid black lines: $B(E2)$ values in Weisskopf units.

Dashed red lines: $E0$ transitions with the corresponding $Q^2(E0) \times 10^3$ values

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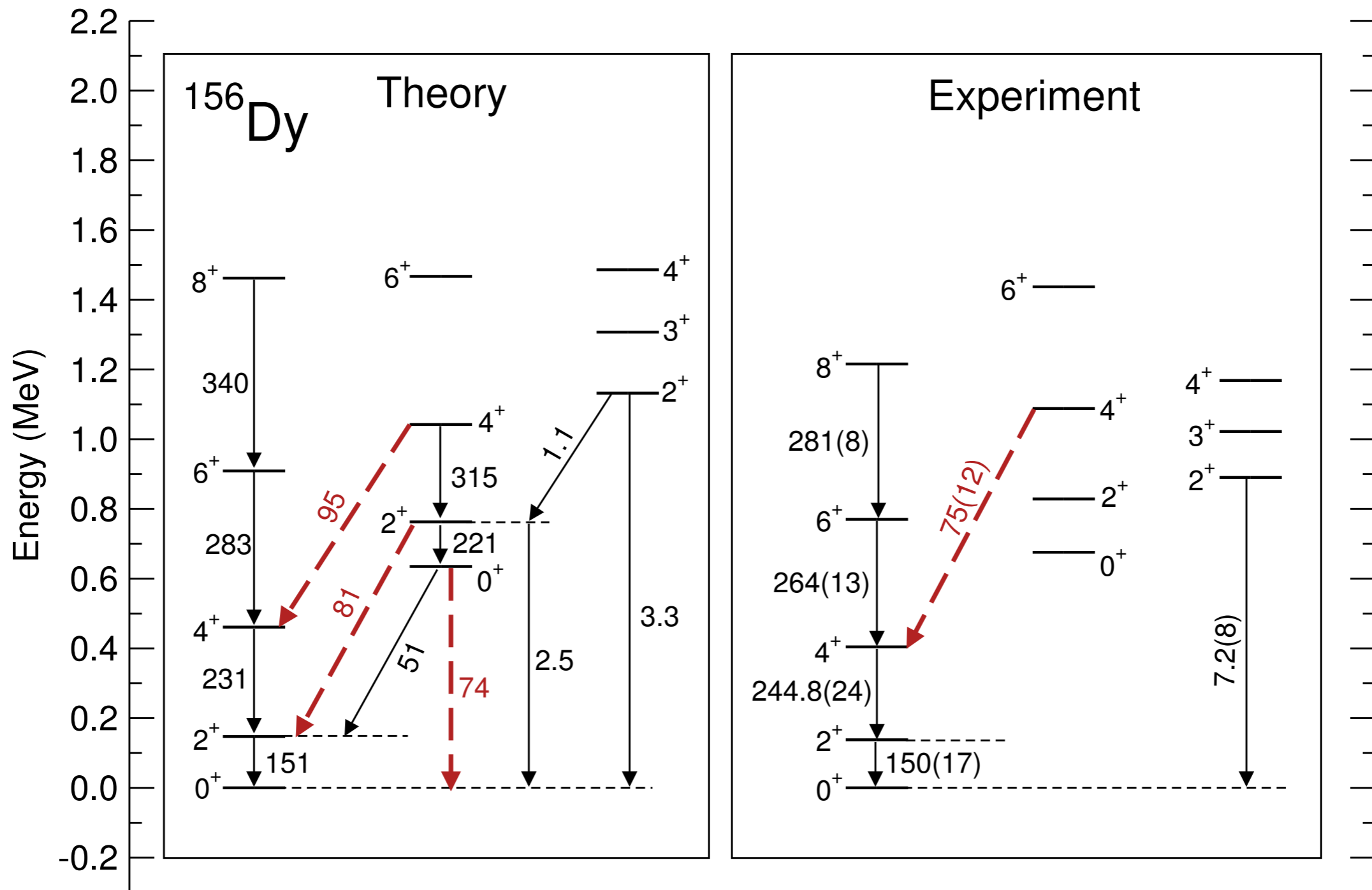


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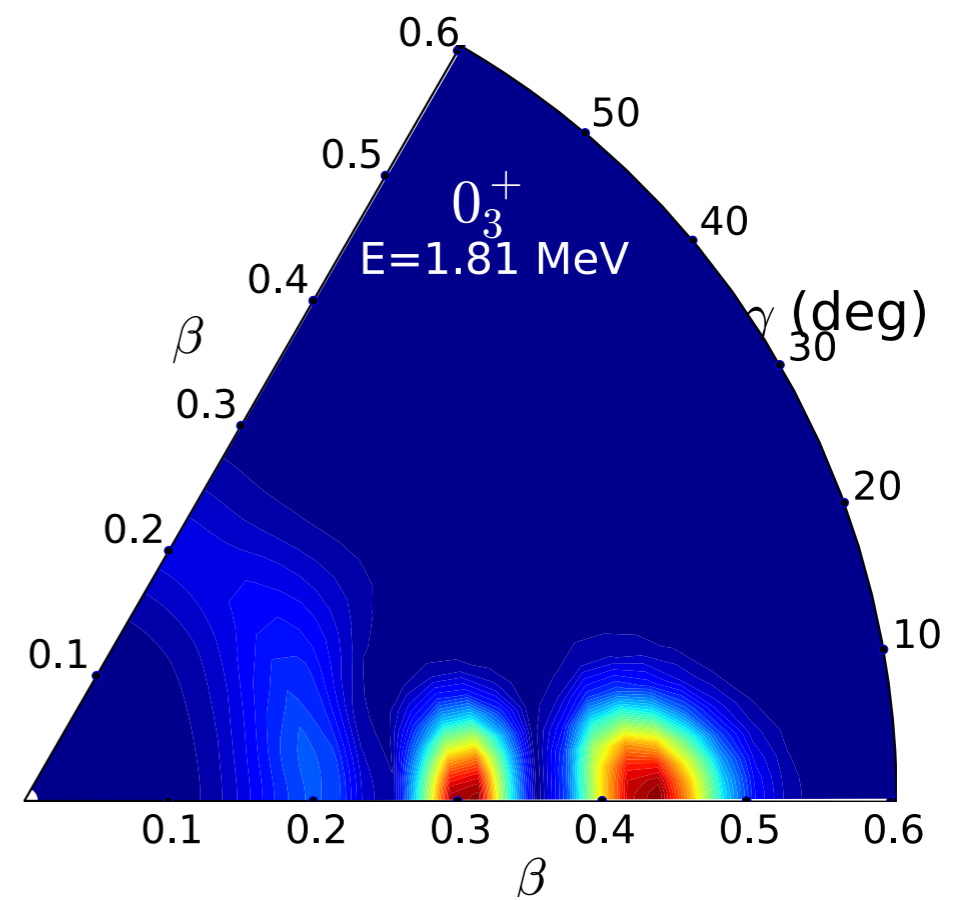
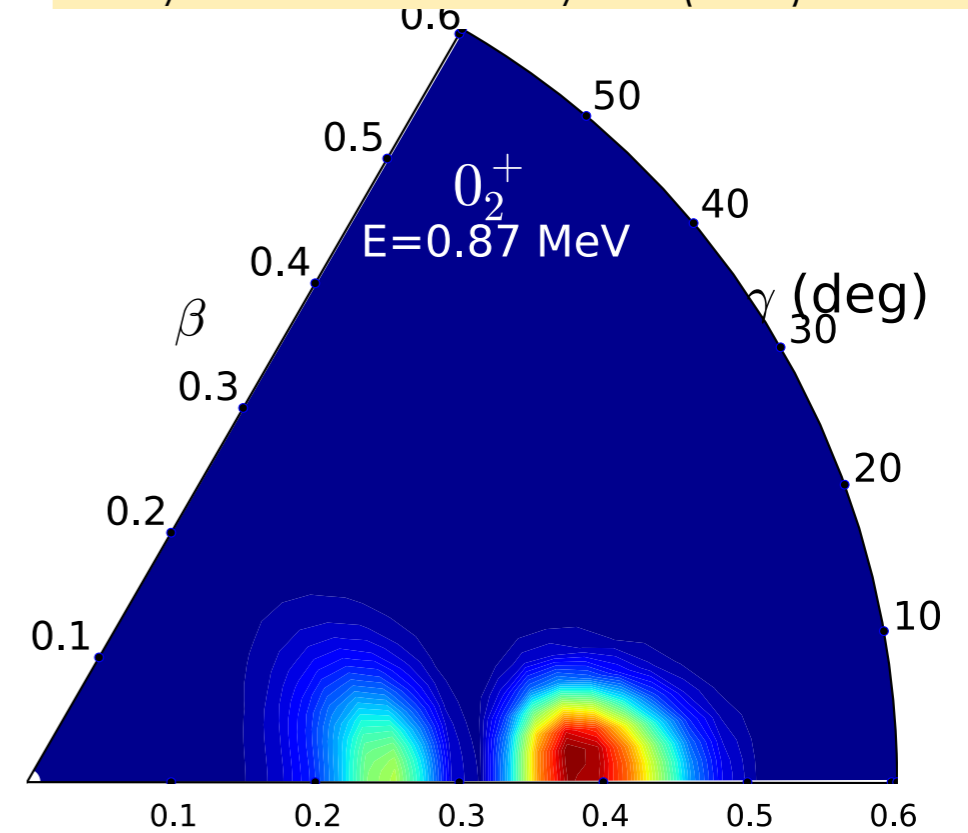
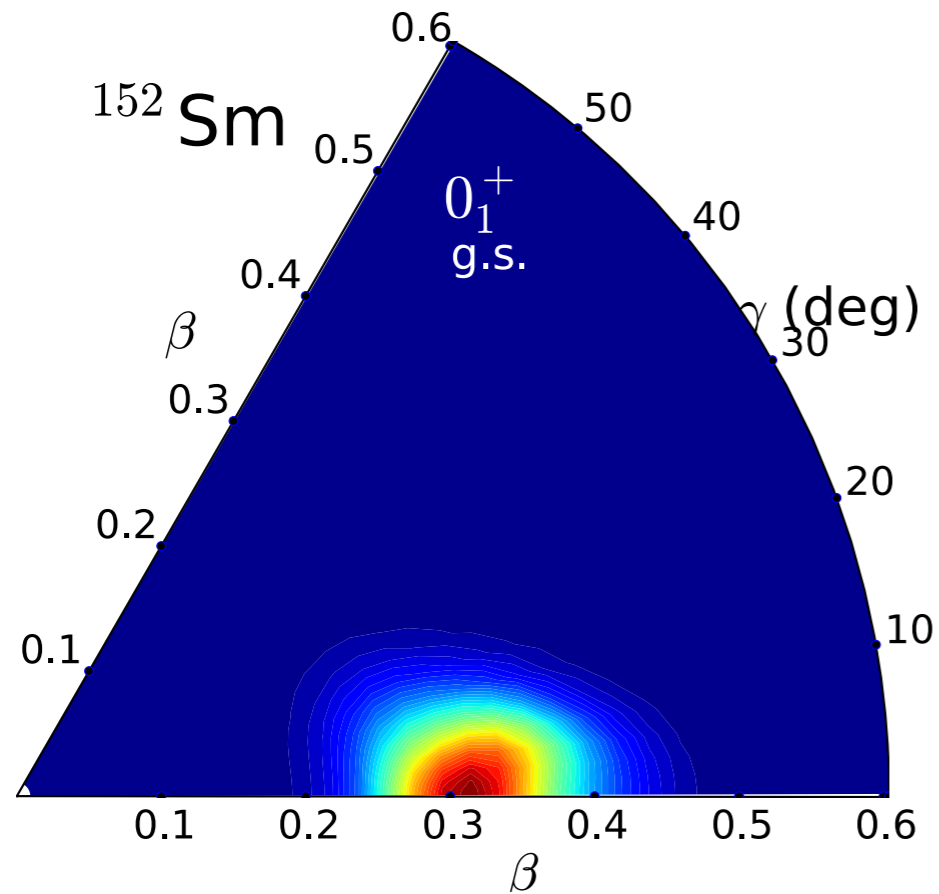


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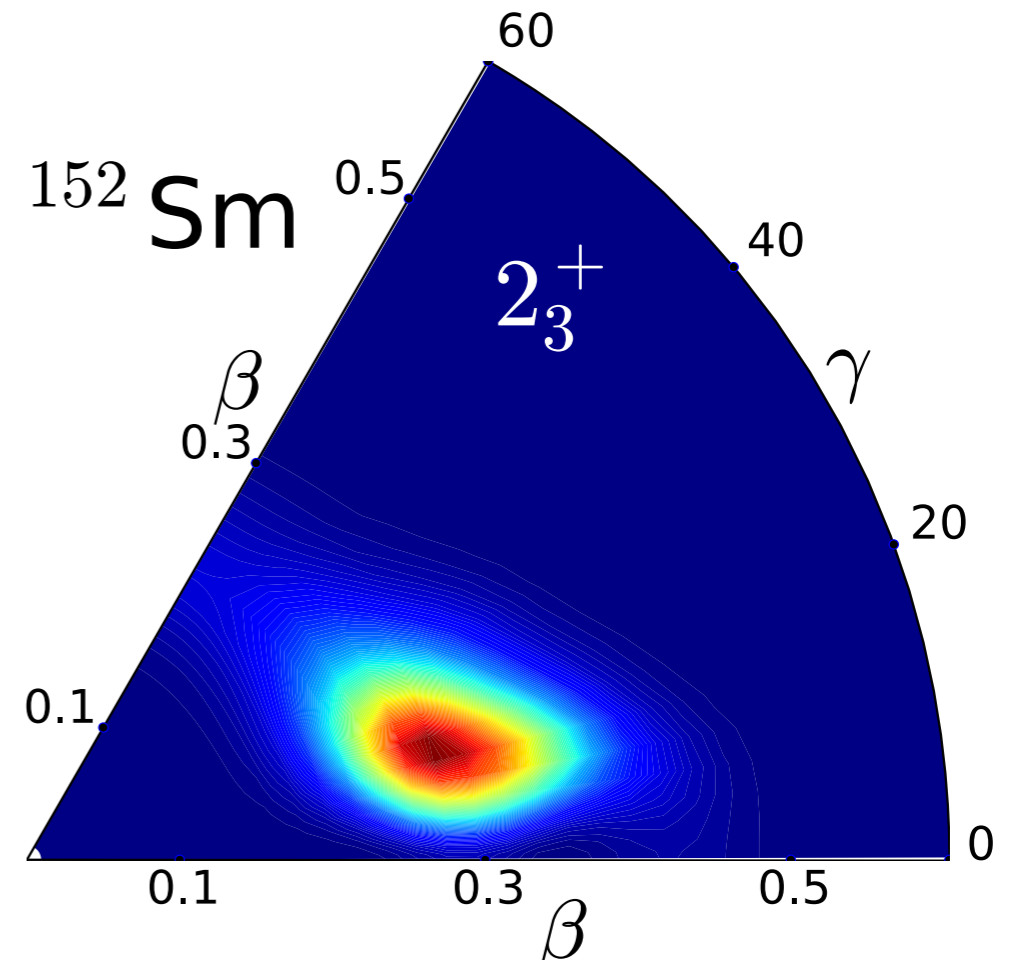


Probability distributions in the β - γ plane for the lowest collective 0^+ states of the ^{152}Sm isotope.

Lowest 0^+ excitations in $N=90$ rare-earth nuclei

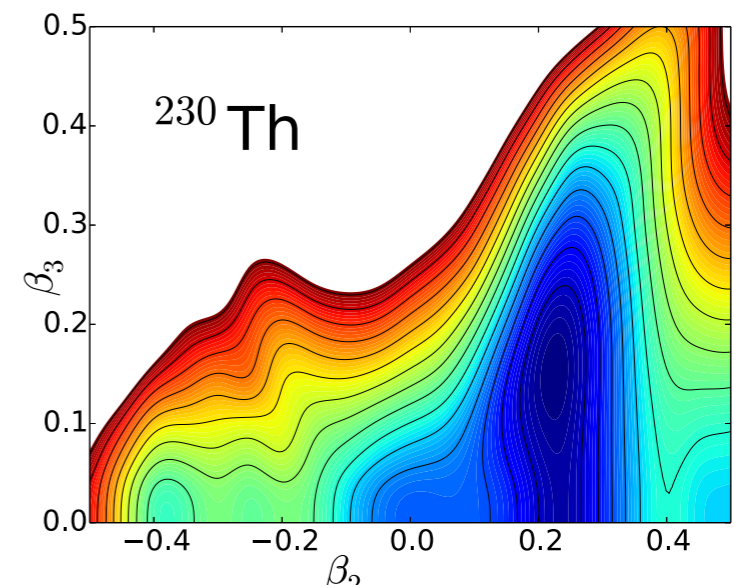
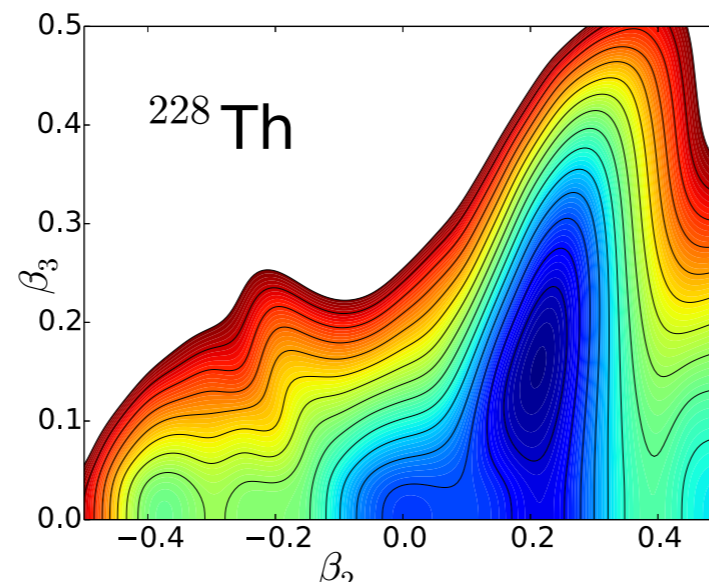
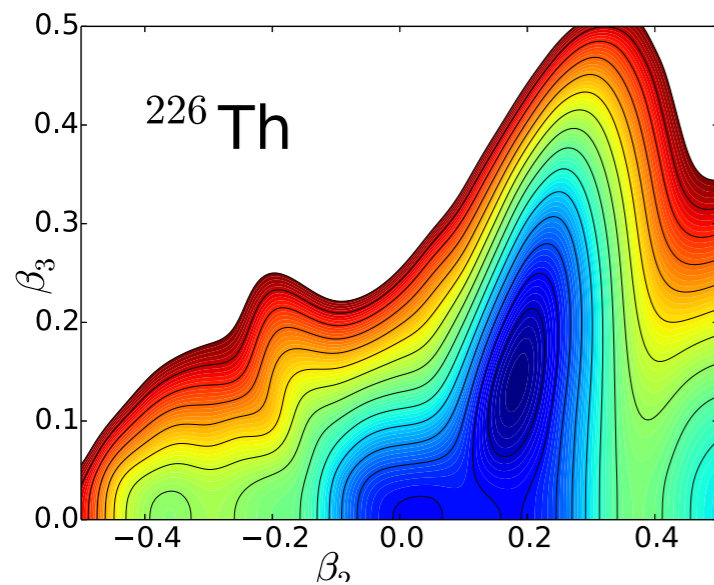
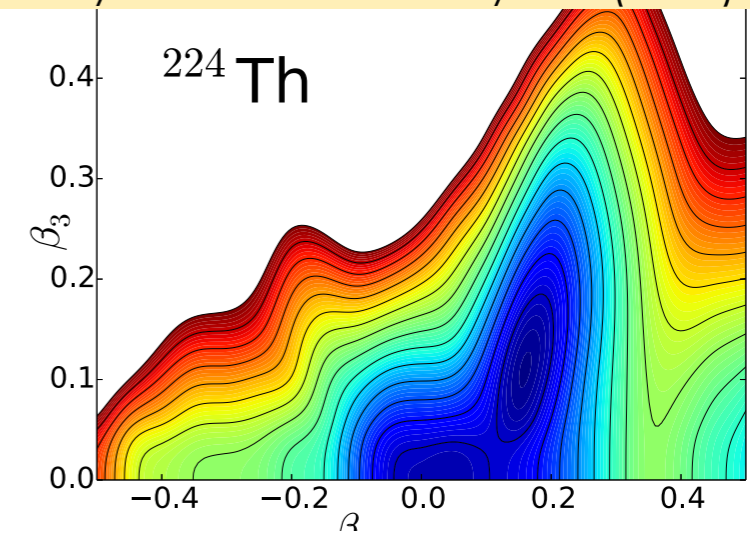
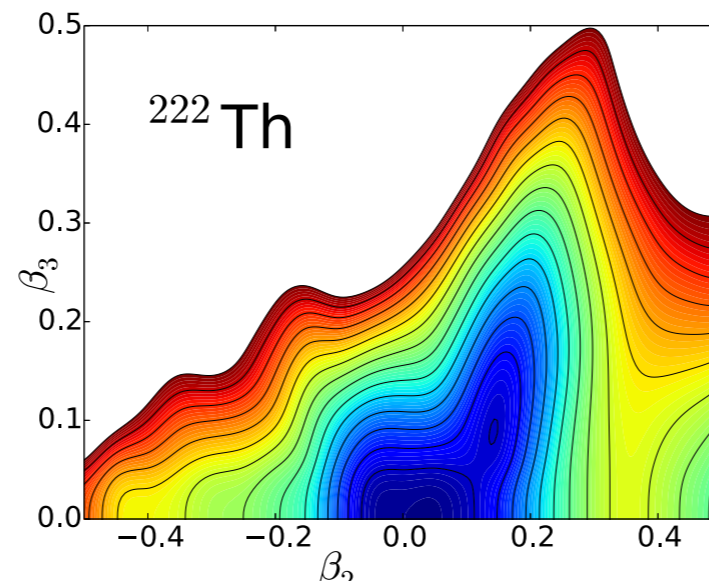
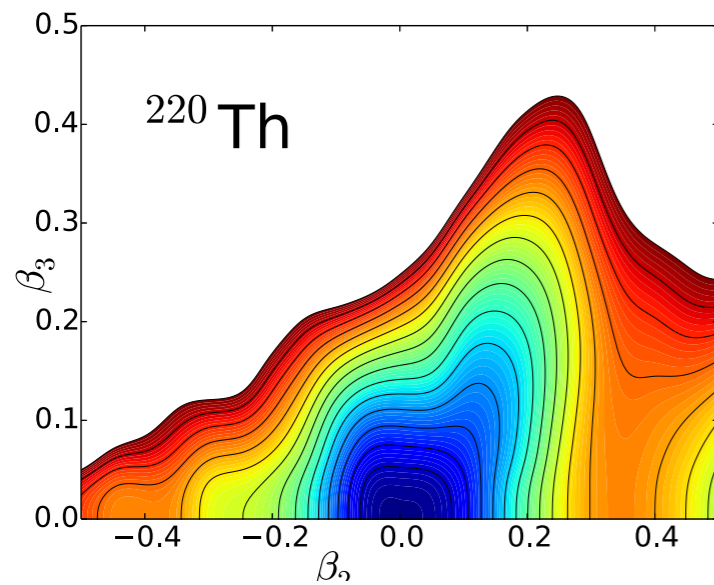
J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005

Probability distributions in the β - γ plane for the band-head of the $K=2$ γ -band of the ^{152}Sm isotope.



Quadrupole and octupole shape transition in thorium

J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005



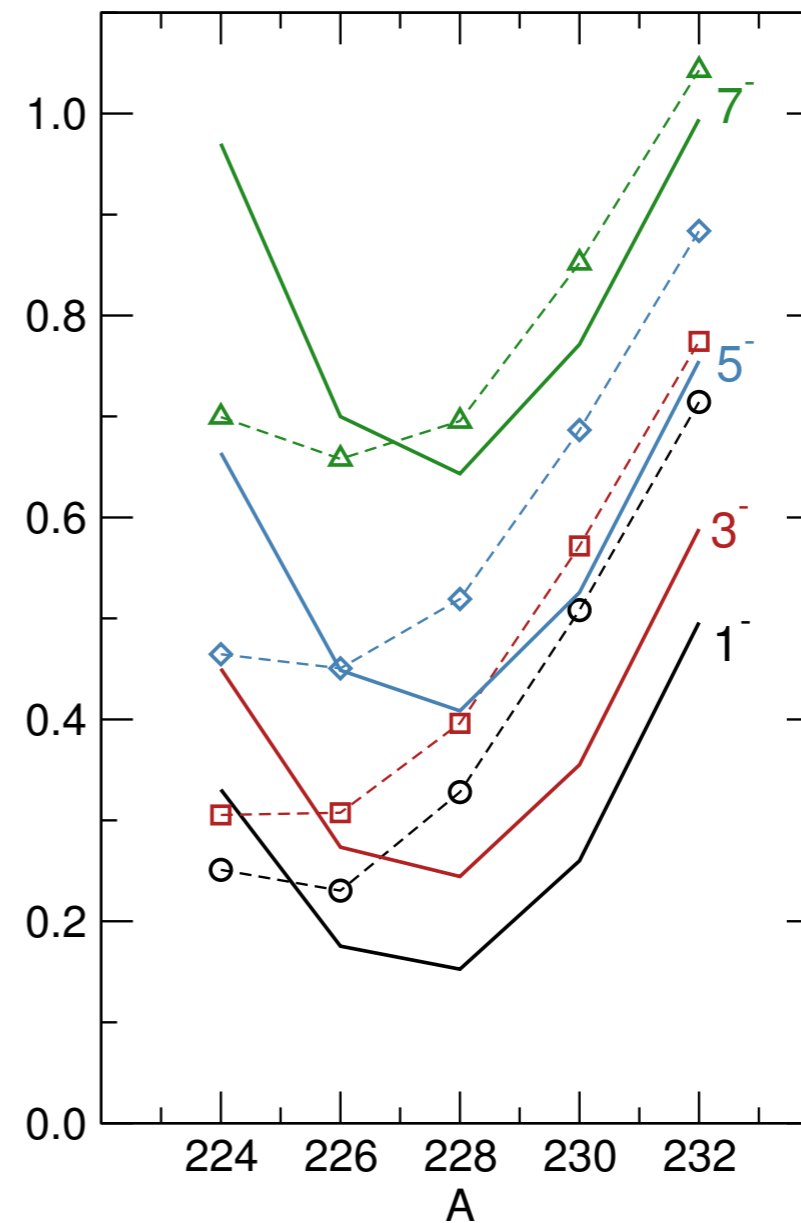
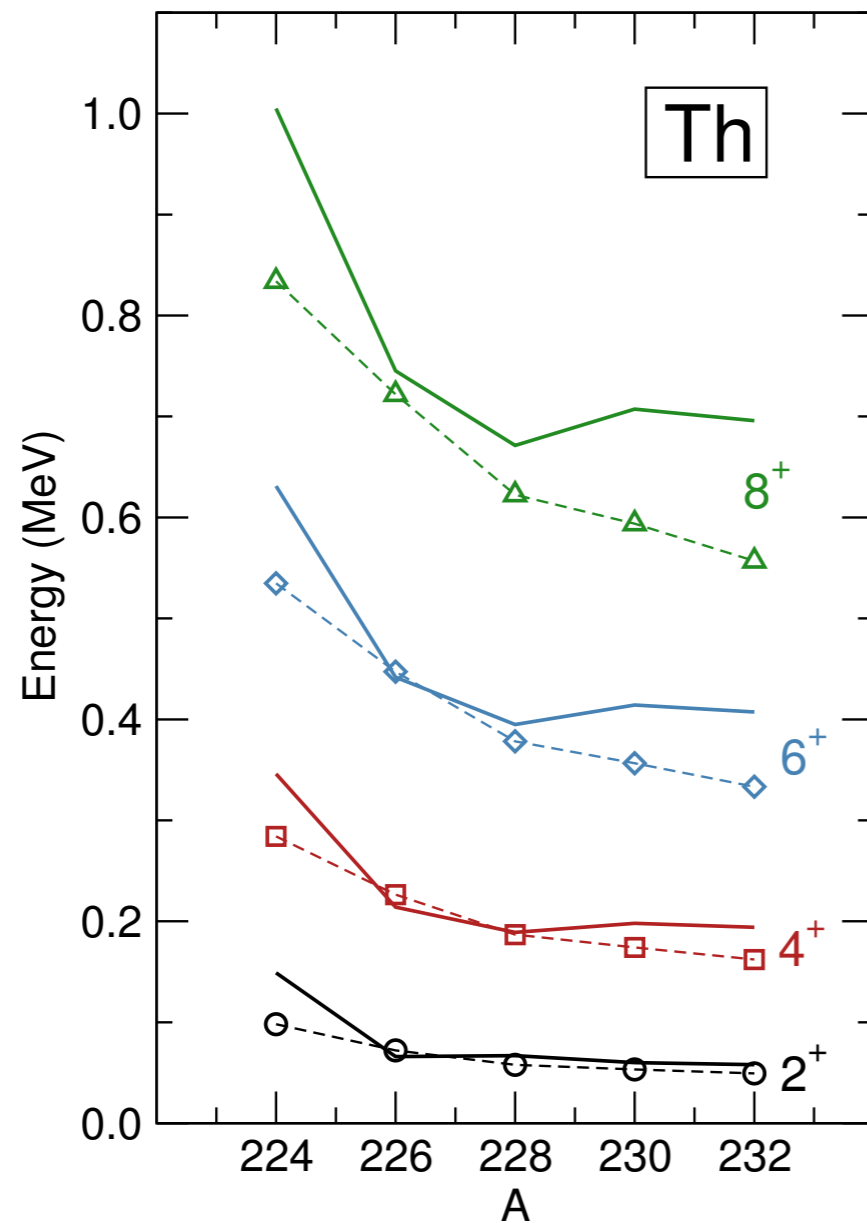
...quadrupole-octupole collective Hamiltonian:

$$H_{coll} = -\frac{\hbar^2}{2\sqrt{w\mathcal{I}}} \left[\frac{\partial}{\partial\beta_2} \sqrt{\frac{\mathcal{I}}{w}} B_{33} \frac{\partial}{\partial\beta_2} - \frac{\partial}{\partial\beta_2} \sqrt{\frac{\mathcal{I}}{w}} B_{23} \frac{\partial}{\partial\beta_3} - \frac{\partial}{\partial\beta_3} \sqrt{\frac{\mathcal{I}}{w}} B_{23} \frac{\partial}{\partial\beta_2} + \frac{\partial}{\partial\beta_3} \sqrt{\frac{\mathcal{I}}{w}} B_{22} \frac{\partial}{\partial\beta_3} \right] + \frac{\hat{j}^2}{2\mathcal{I}} + V(\beta_2, \beta_3)$$

Quadrupole and octupole shape transition in thorium

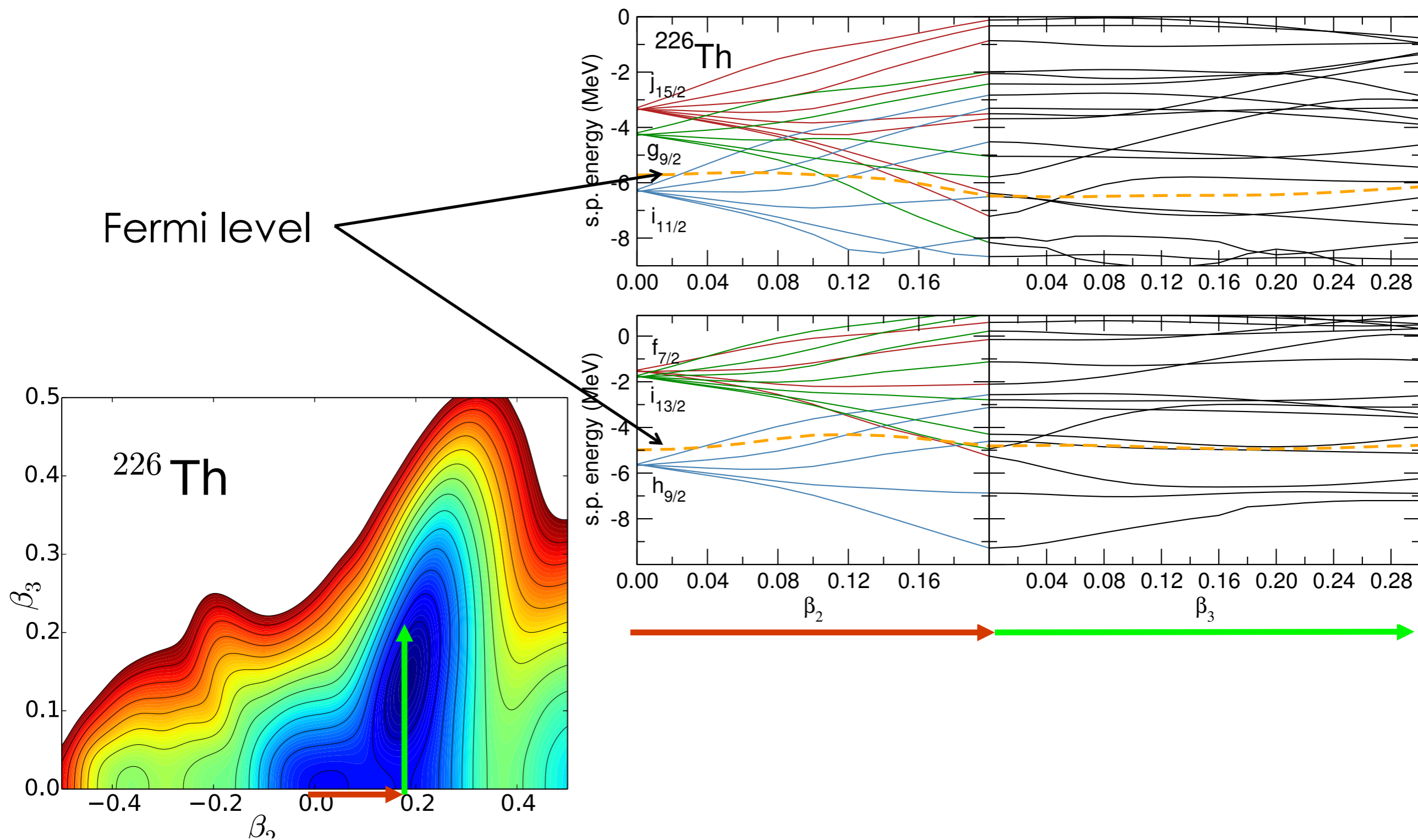
J. Phys. G: Nucl. Part. Phys. 43 (2016) 024005

... systematics of energy spectra of the positive-parity ground-state band ($K^\pi = 0^+$) and the lowest negative-parity ($K^\pi = 0^-$) sequences in $^{224-232}\text{Th}$.



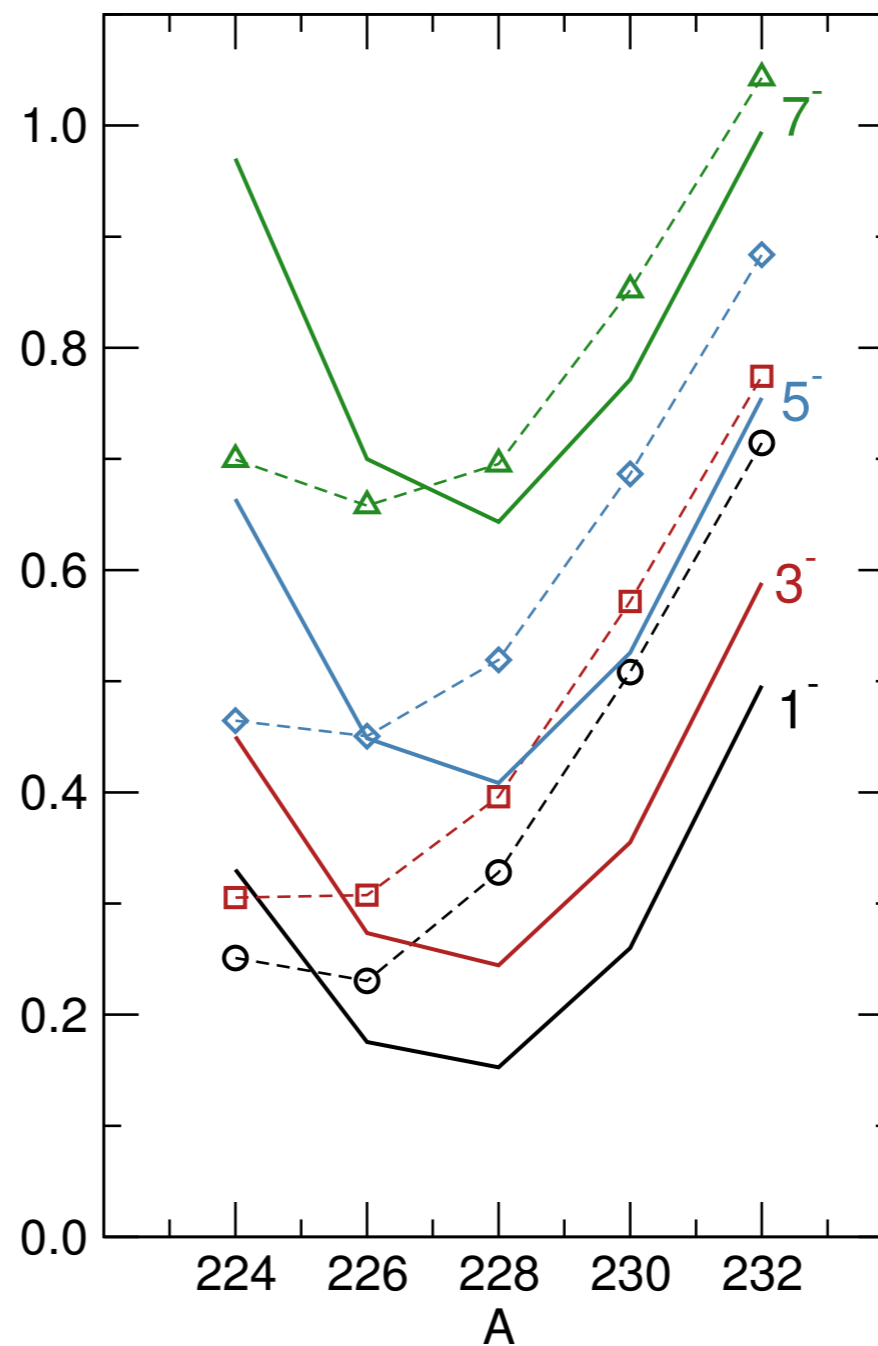
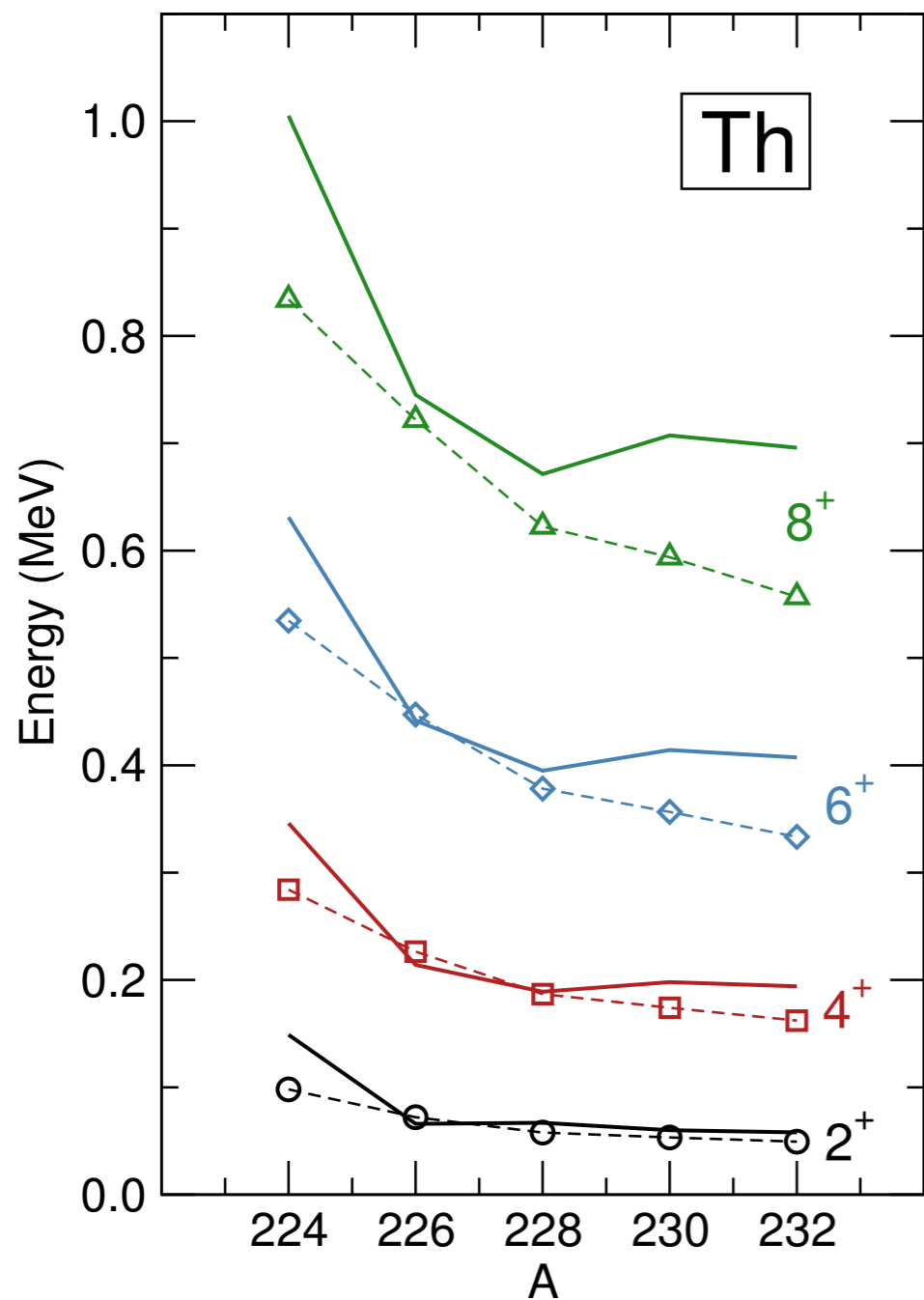
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Quadrupole and octupole shape transition in thorium

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Solid lines – theory
Dashed lines – exp.

Some other recent applications...

✓ level of accuracy (rms deviation of experimental masses) of covariant EDFs is still below the state-of-the-art non-relativistic HFB mass models: Should additional terms be included in the EDF? (price to pay: increased model complexity)

J. Phys. G. 42, 034008 (2015)

✓ quantification of theoretical uncertainties within the EDF framework

Phys. Rev. C 95, 054304 (2017)

Phys. Rev. C 94, 024333 (2016)

✓ is it possible to systematically reduce the number of parameters defining the EDF? → manifold boundary approximation method

✓ Extrapolation to superheavy elements (energy gaps; separation energies; Q_α -values; K-isomers...)

Nucl. Phys. A 944, 415 (2015)

Phys. Rev. C 91, 034324 (2015)

Phys. Rev. C 89, 024312 (2014)

✓ Description of fission dynamics (fission barriers, paths and lifetimes; induced fission dynamics)

Phys. Rev. C 96, 024319 (2017)

Phys. Rev. C 93, 044315 (2016)

Phys. Rev. C 89, 064315 (2015)

Summary

- ✓ NEDFs provide an economic, global and accurate microscopic approach to nuclear structure that can be extended from relatively light systems to superheavy nuclei, and from the valley of β -stability to the particle drip-lines.
- ✓ NEDF-based structure models that take into account collective correlations → microscopic description of low-energy observables: excitation spectra, transition rates, changes in masses, isotope and isomer shifts, related to shell evolution with nuclear deformation, angular momentum, and number of nucleons.

Collaborators

✓ **UniZg**: Dario Vretenar, Kosuke Nomura (University of Tsukuba), Jie Zhao (China Academy of Engineering Physics, Chengdu), Vaia Prassa (Thessaloniki)

✓ **PKU, CAS, Southwest University**: Jie Mong, Shan-Gui Zhou, Bing-Nan Lu, Zhipan Li, Jiangming Yao