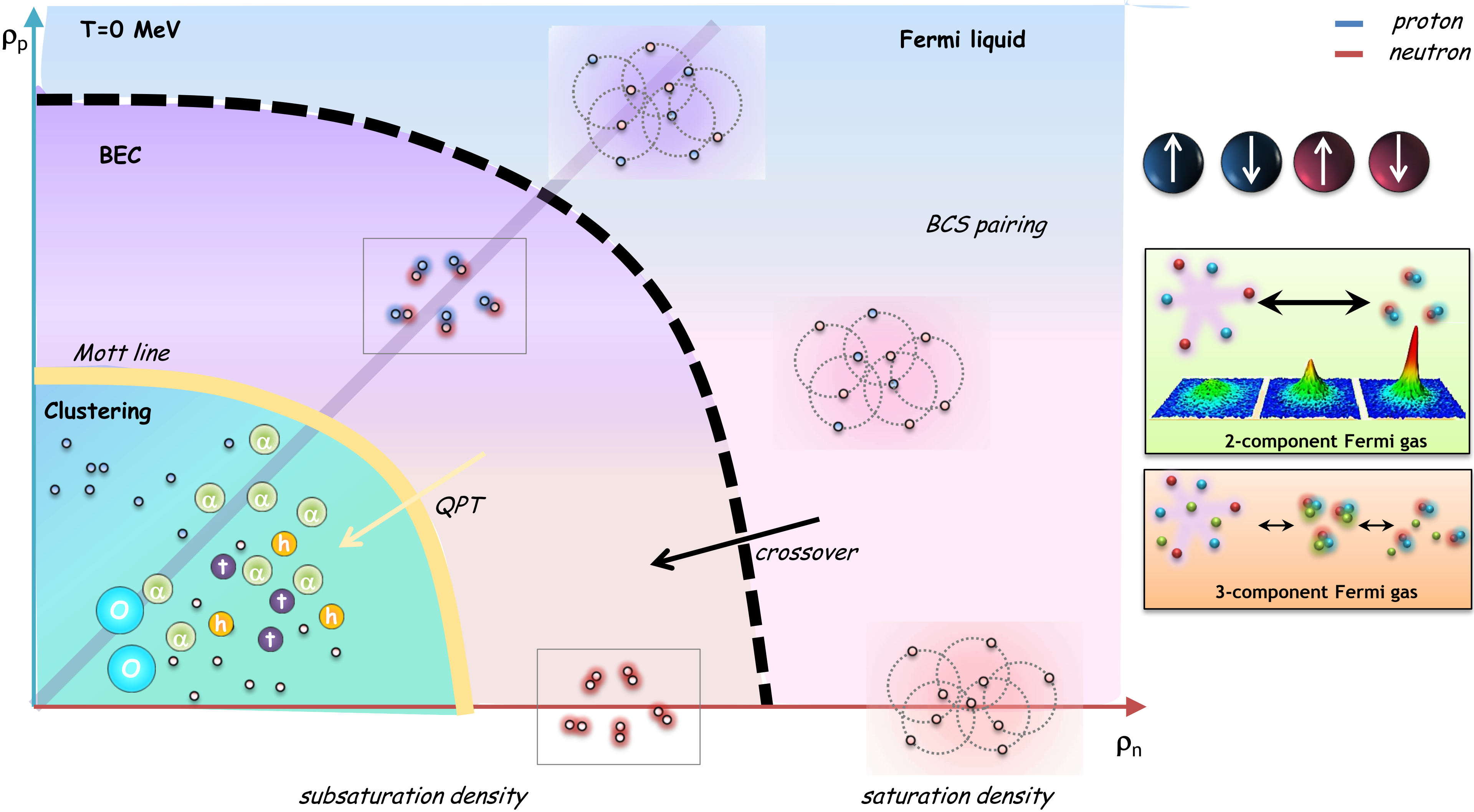
-Nuclear Clustering in the Energy Density Functional Approach-

P. Marevic[1,2], R.D. Lasseri[2], J.-P. Ebran[1], E. Khan[2], T. Niksic[3], D.Vretenar[3]

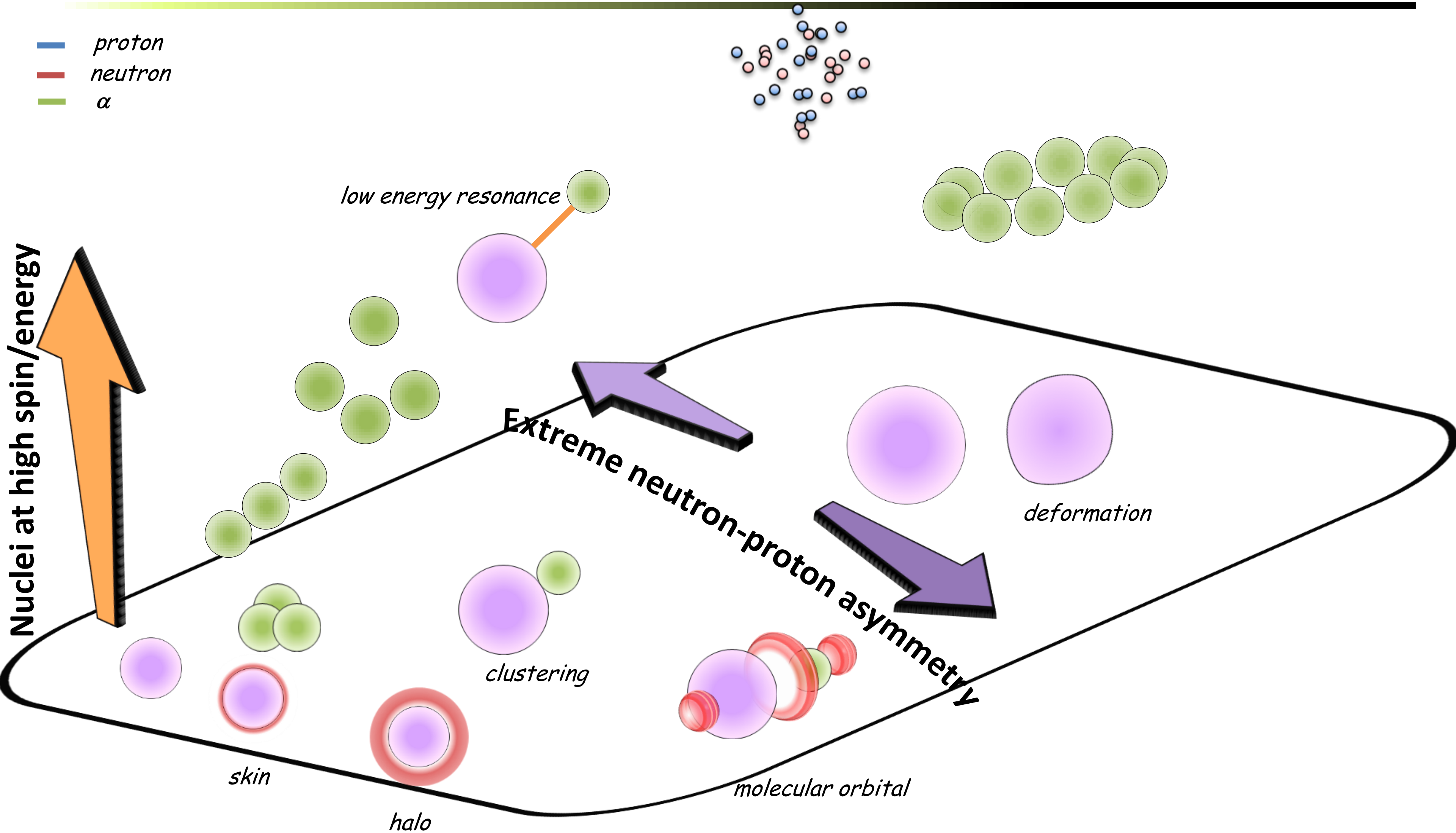
[1] CEA,DAM,DIF
[2] IPNO
[3] University of Zagreb

✪ Nuclear systems as a mixture of 4 types of fermion



➲ These features leave fingerprints in finite nuclei

✪ Emergence of ordered sub-structures inside nuclei



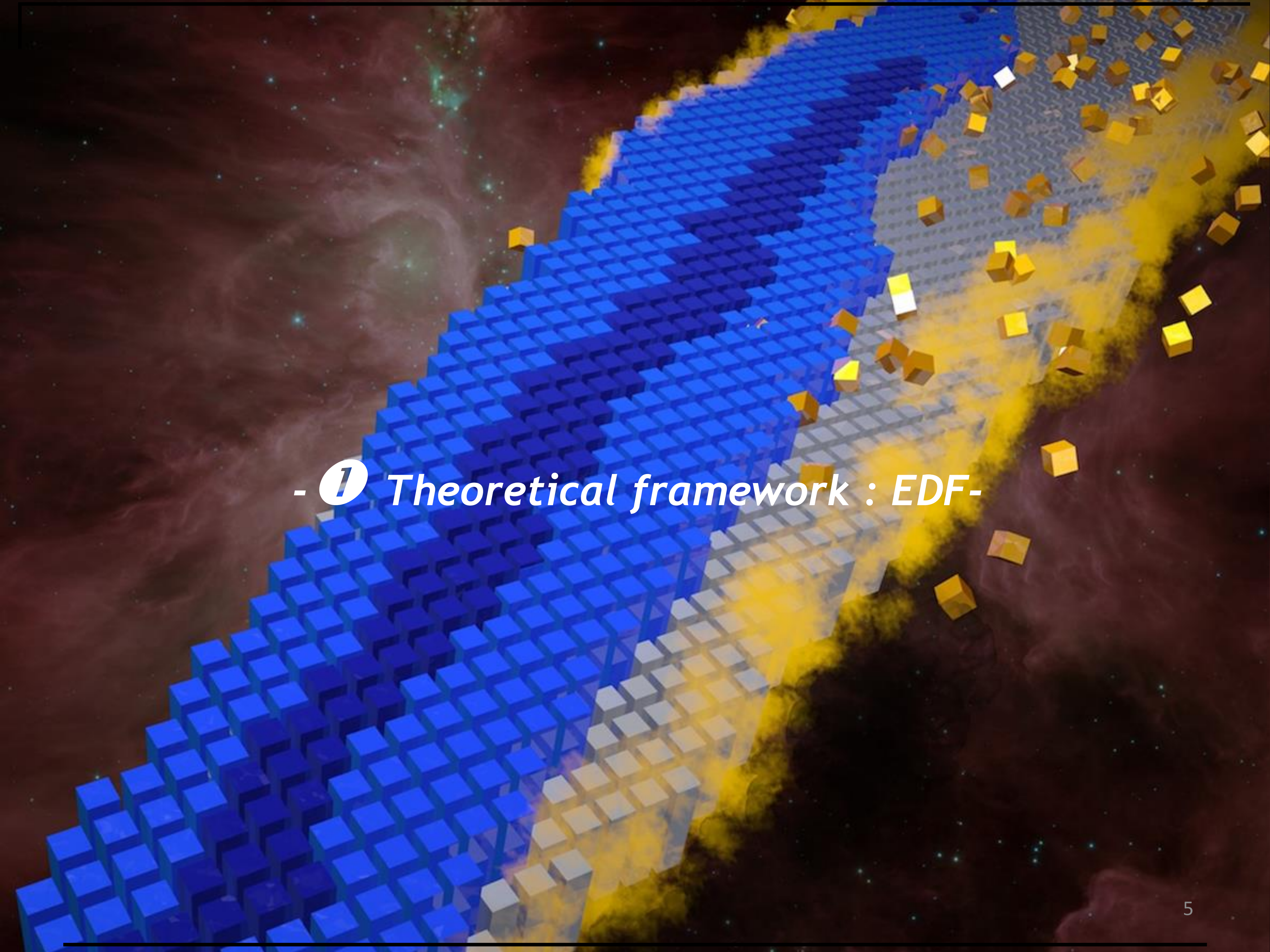
➲ Nuclear EDFs provide a unified and consistent description of these various properties

✪ Outline

❶ Theoretical framework : EDF

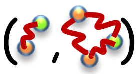
❷ Pair correlations

❸ Nuclear clustering



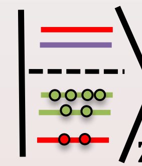
-❶ *Theoretical framework : EDF*-

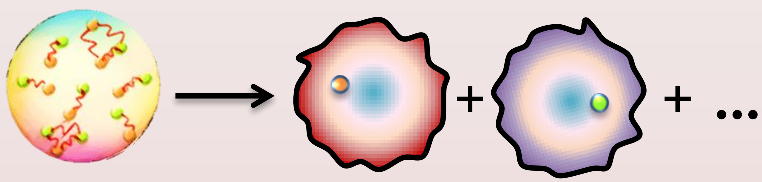
Nuclear many-body problem : strategies

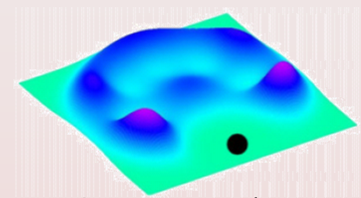
- Challenge : extract from H() the nuclear observables of interest $\Rightarrow$ Many-body problem

- Many-body approaches as implementation of different strategies to apprehend the many-body problem

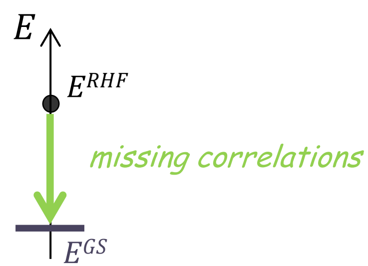
Traditional starting point of a many-body approach : **HF product state**



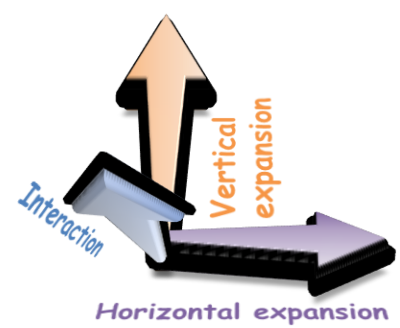




Fermi correlations



- How to incorporate such correlations starting with a HF product state ?



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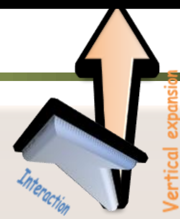


Vertical Philosophy

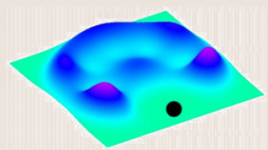
⋮

Excit. 2p2h Z,N,J^π

Excit. 1p1h Z,N,J^π

Reference vacuum Z,N,J^π

- **non collective** excitations
- physics accounted through numerous **rearrangements** of nucleons within the orbitals given by the reference vacuum
- **conservation** of symmetries



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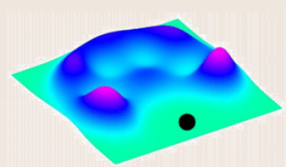
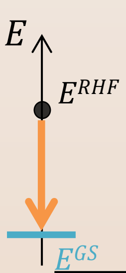
Major contribution

☑ **Dynamical** correlations

- ❖ in all nuclei
- ❖ $|HF\rangle \sim |\,\rangle$





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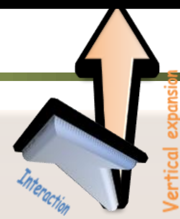

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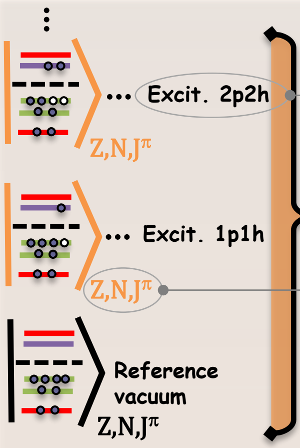

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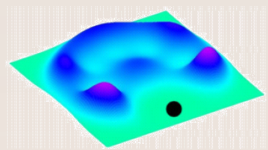

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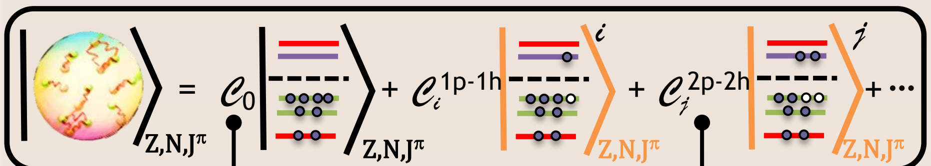

comparable contributions

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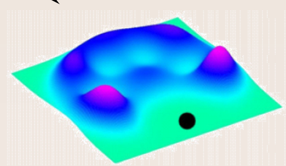
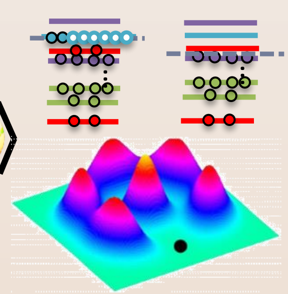
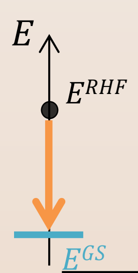
- |HF⟩ ~



Nondynamical correlations

- open shell systems
- |HF⟩≁





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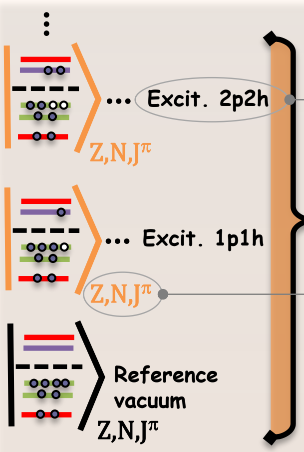
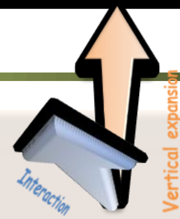
Vertical Philosophy





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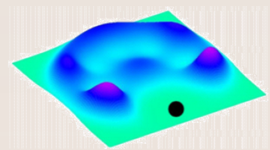

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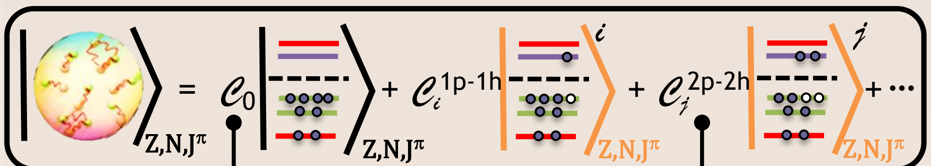

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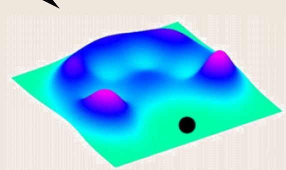

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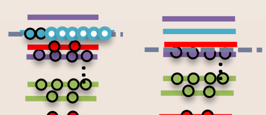
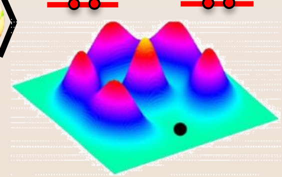
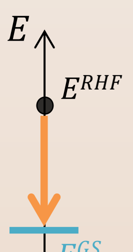
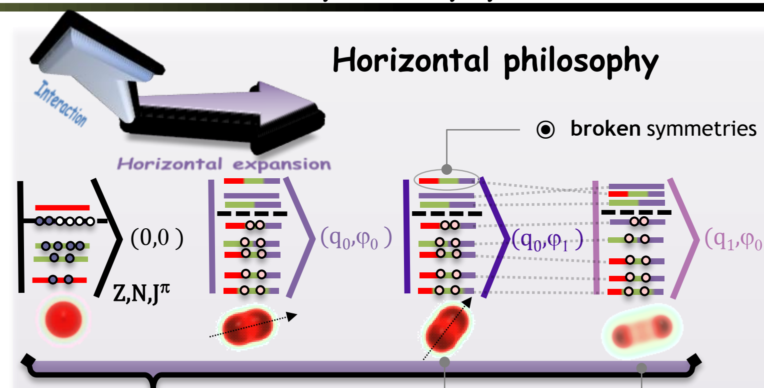






Horizontal philosophy



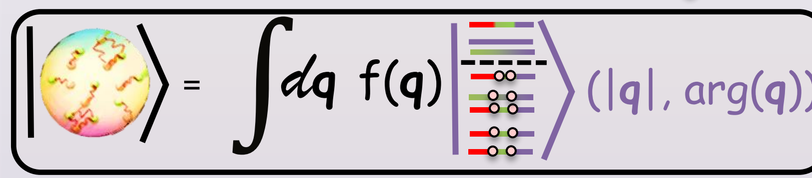
◉ **broken** symmetries

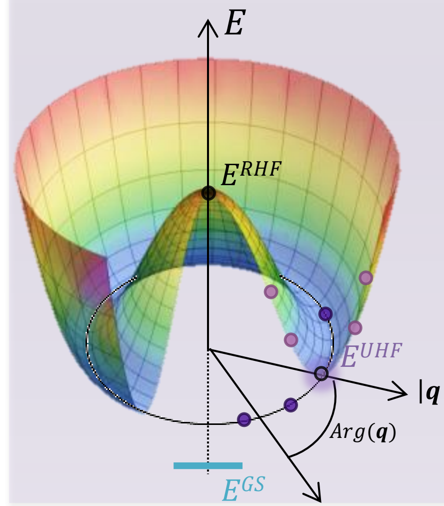
◉ Physics accounted for through

◉ **Collective** excitations

spontaneous symmetry breaking
+ fluctuations of the order parameter

➲ dynamical correlations transferred in H_{eff}(,)





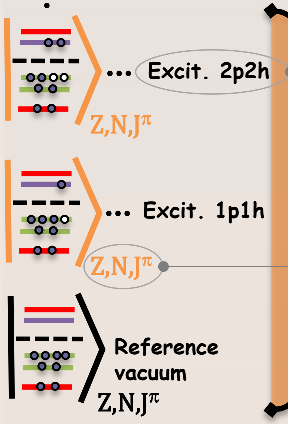
❶ EDF

✪ Nuclear many-body problem : strategies

Interaction — Vertical expansion

Interaction — Horizontal expansion

Vertical Philosophy



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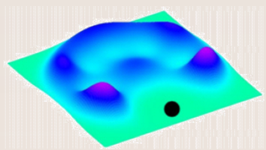

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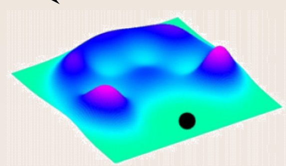

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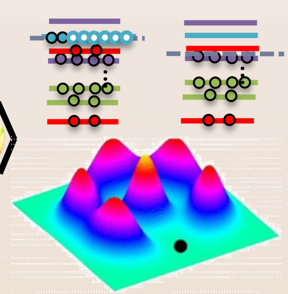

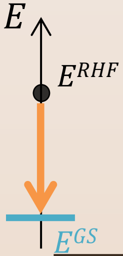

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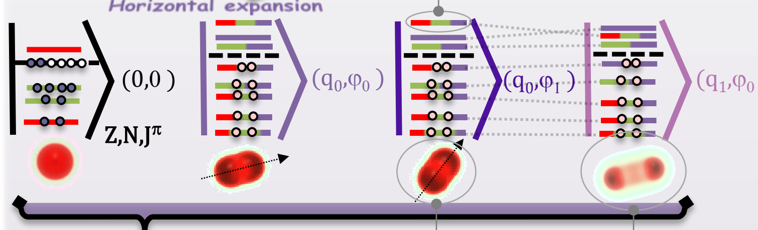

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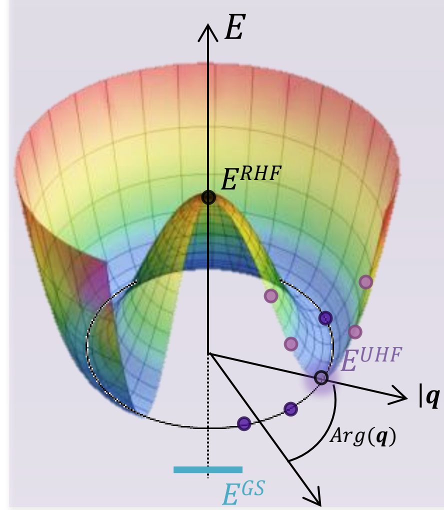

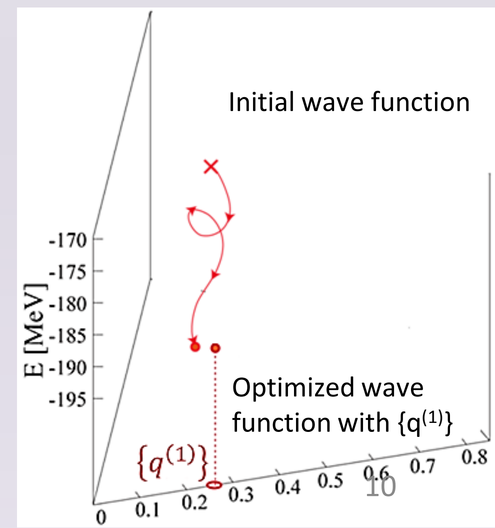

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Vertical expansion

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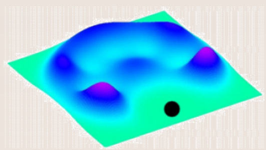

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comparable contributions

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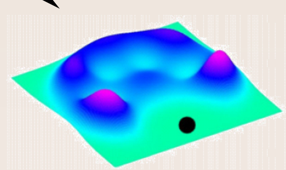

◉ **Nondynamical** correlations

- ❖ open shell systems
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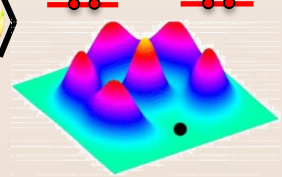

E, E^{RHF}, E^{GS}

Horizontal philosophy

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$(0,0)$ Z,N,J^π (q_0,φ_0) (q_0,φ_1) (q_1,φ_0)

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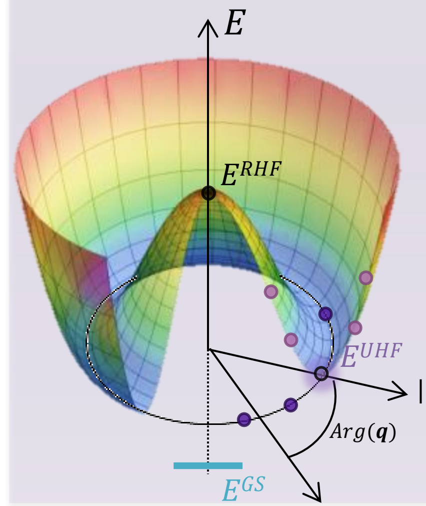
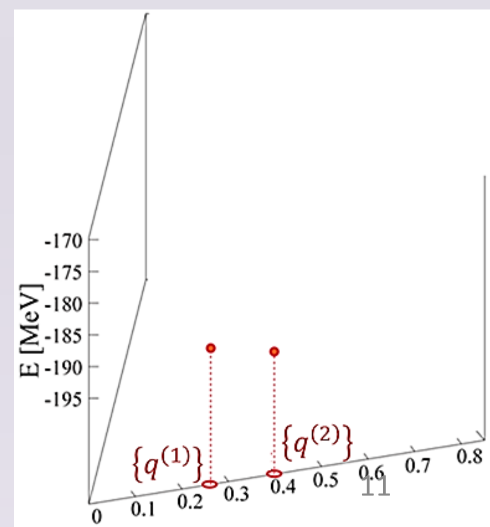
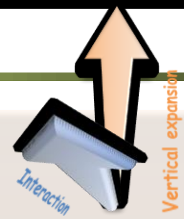
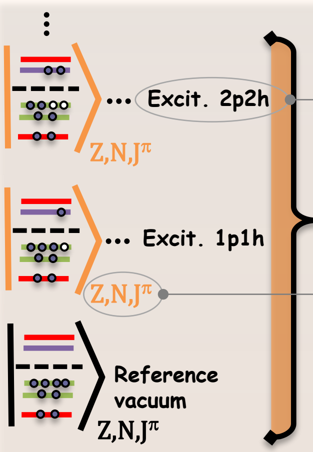




Nuclear many-body problem : strategies

Vertical Philosophy





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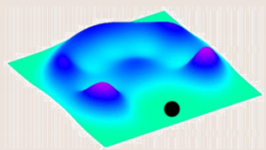

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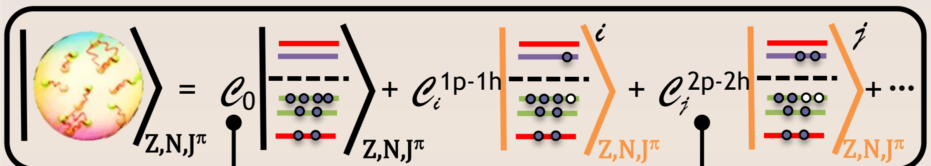

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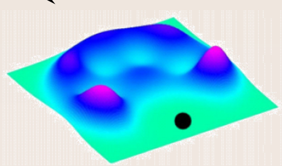

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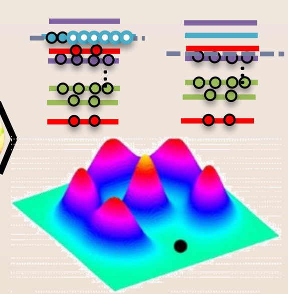
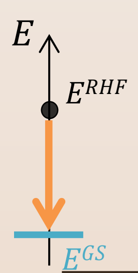
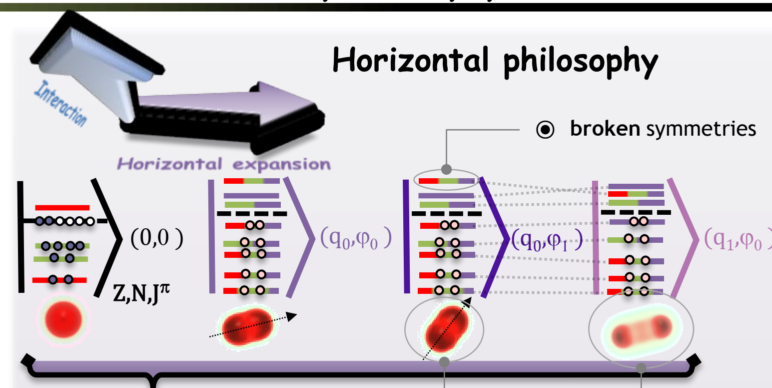




Horizontal philosophy



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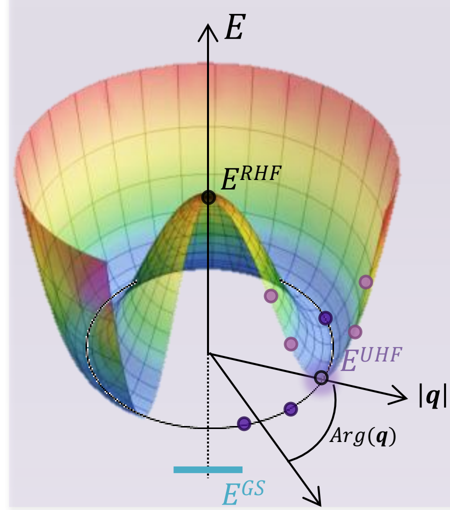

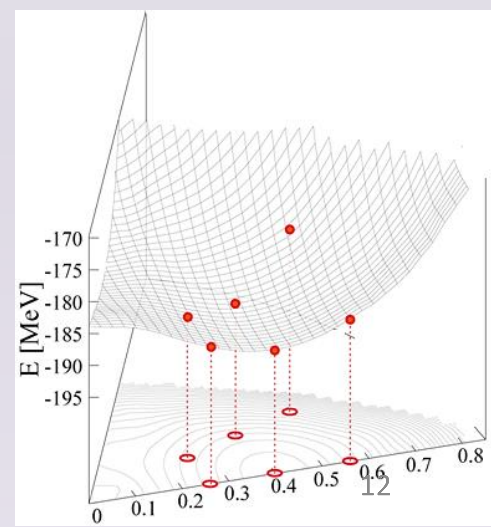

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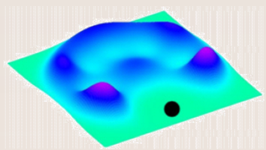

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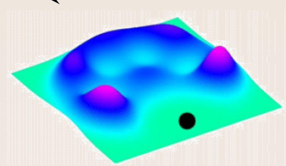

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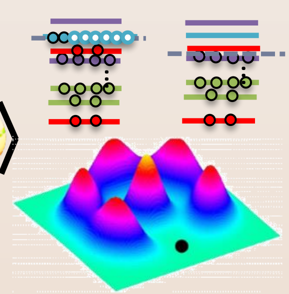

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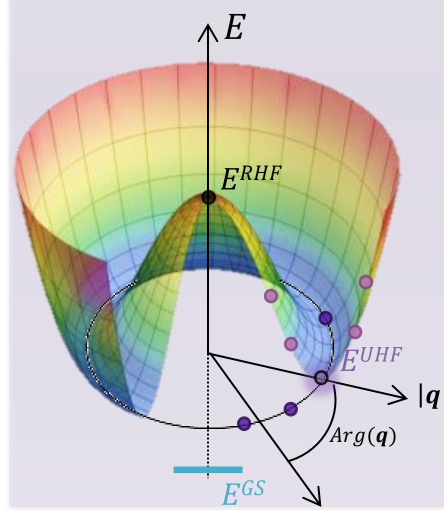
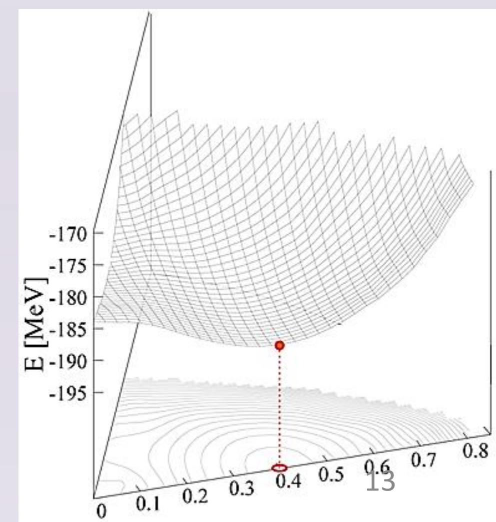
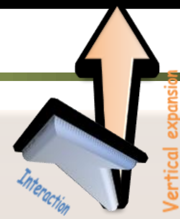
➲ dynamical correlations transferred in $H_{eff}(\ ,\ )$

$$\rangle = \int dq\ f(q) \rangle_{(|q|,\ \arg(q))}$$

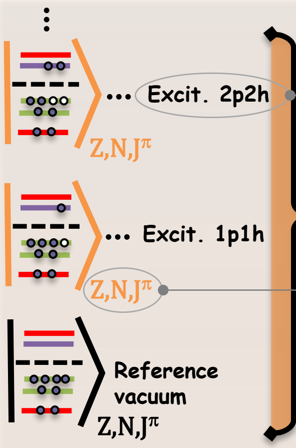




✪ Nuclear many-body problem : strategies



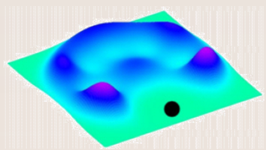
Vertical Philosophy



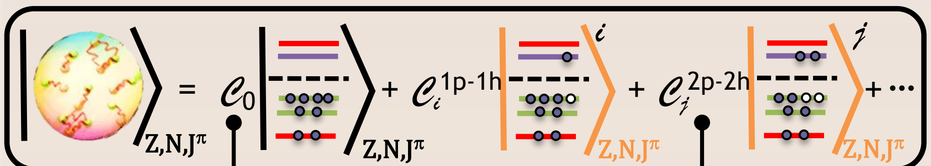
◉ non collective excitations

◉ physics accounted through numerous **rearrangements** of nucleons within the orbitals given by the reference vacuum

◉ **conservation** of symmetries



Fermi correlations

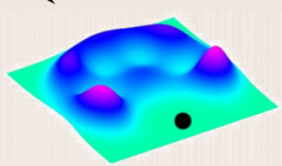


comparable contributions

☑ **Dynamical correlations**

❖ in all nuclei

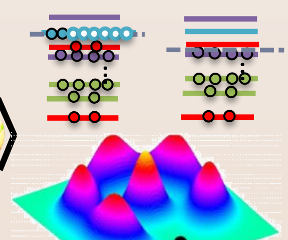
❖ |HF⟩ ~ |Ψ⟩

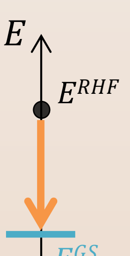


◉ **Nondynamical** correlations

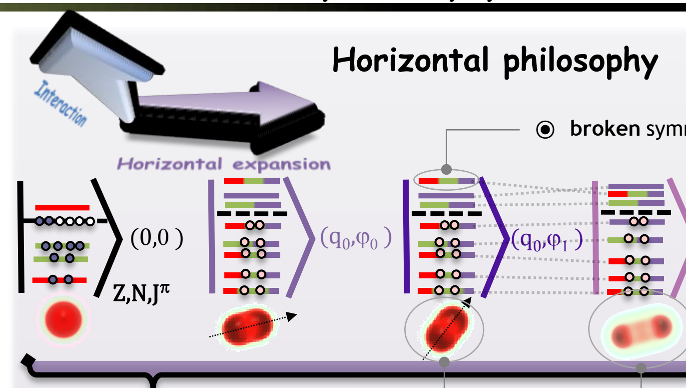
❖ open shell systems

❖ |HF⟩≁ |Ψ⟩





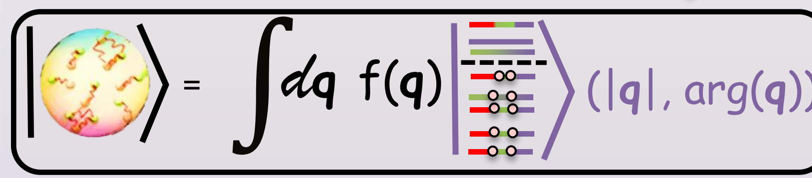
Horizontal philosophy

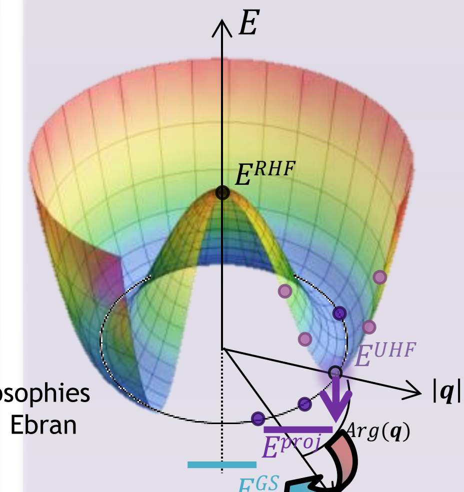


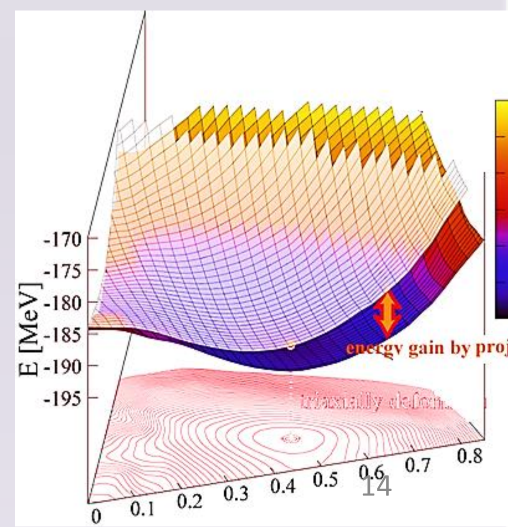
◉ Physics accounted for through spontaneous symmetry breaking

◉ **Collective** excitations + **fluctuations of the order parameter**

➲ dynamical correlations transferred in H_{eff}







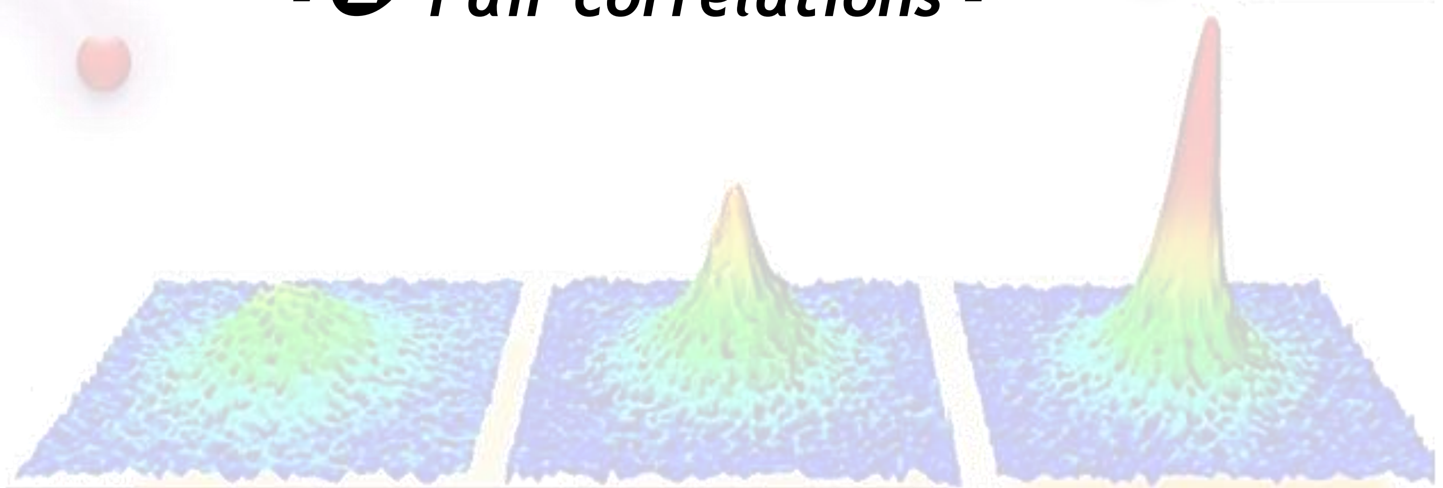
Novel many-body approaches combining both philosophies

P. Arthuis, J. Ripoche, T. Duguet, D. Lacroix, J.-P. Ebran

J.Ripoche et al PRC 95, 014326 (2017)

T. Duguet et al EPJA **51**, 162 (2015)

- ❷ Pair correlations -



✪ *How to describe a pair of nucleons in the medium ?*

Reduced density matrix

◉ All the information of a many-body system is contained in its Nth-order density matrix

$$D_N\left(1,...,N;1',...,N'\right)=\Psi\left(1,...,N\right)\Psi^*\left(1',...,N'\right)$$

◉ Only keep information about p-“cluster” embedded in the medium composed by the other N-p particles :

$$\Gamma_p\left(1,...,p;1',...,p'\right)=\begin{pmatrix} N \\ p \end{pmatrix}\int d(p+1)...dN\,\Psi\left(1,...,N\right)\Psi^*\left(1',...,p',p+1,...,N\right)$$

◉ 2-RDM : eigenfunctions provide an in-medium pair wave function

Form of the A-body wave function

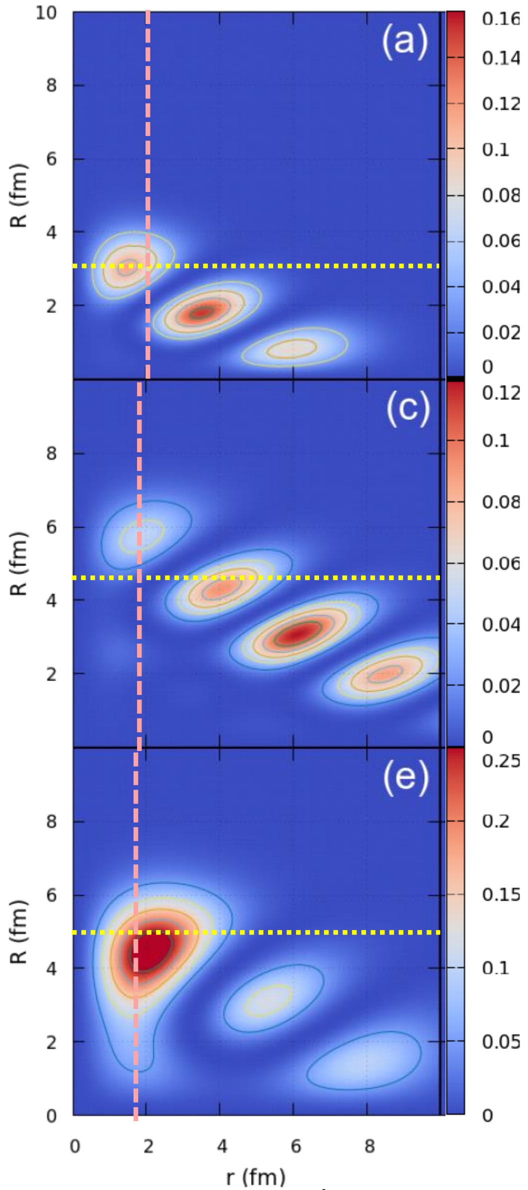
◉ BCS assumption: all the pairs of fermions occupy the same pair wave function :

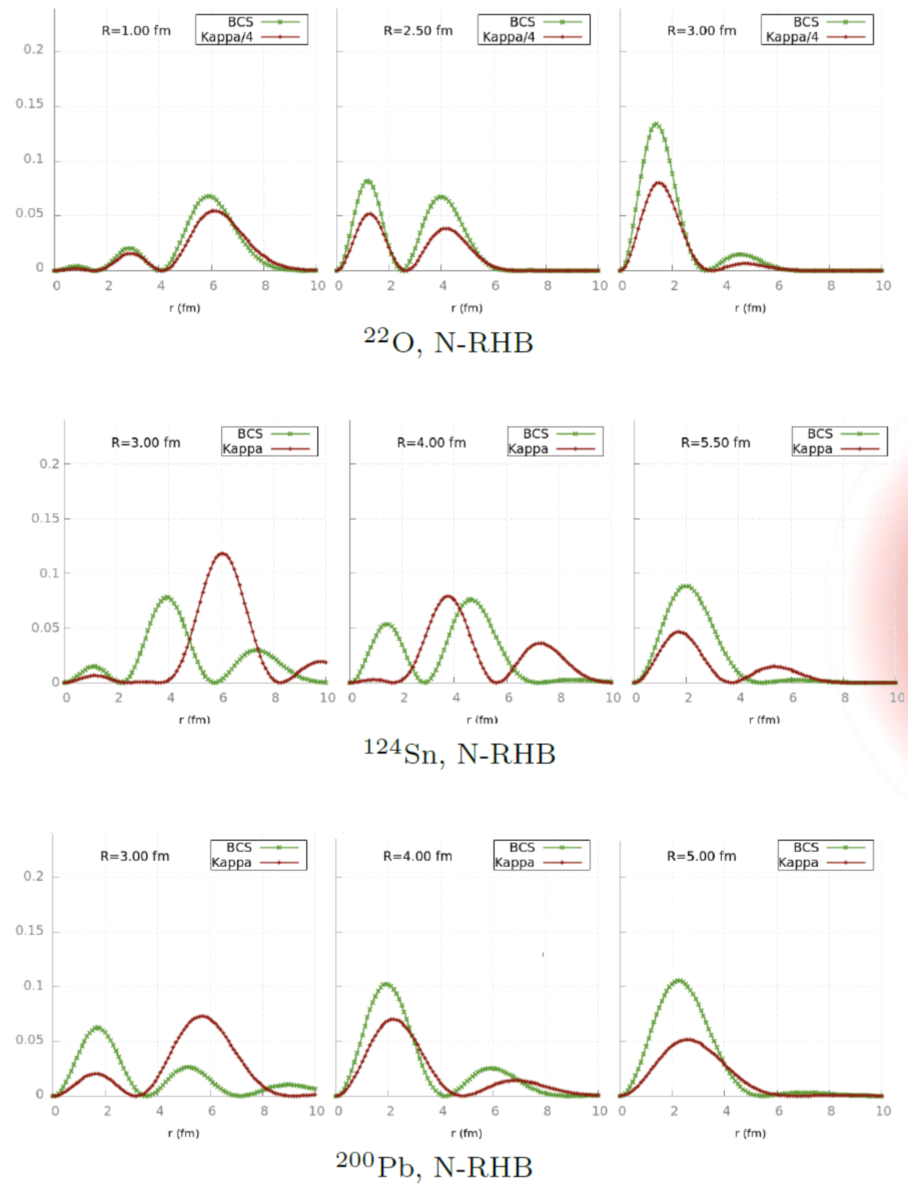
$$\Psi_N=\Psi(\boldsymbol{r}_1\sigma_1,\boldsymbol{r}_2\sigma_2...\boldsymbol{r}_N\sigma_N)=\mathcal{A}[\phi(\boldsymbol{r}_1\sigma_1;\boldsymbol{r}_2\sigma_2)\phi(\boldsymbol{r}_3\sigma_3;\boldsymbol{r}_4\sigma_4)\cdots\phi(\boldsymbol{r}_{N-1}\sigma_{N-1};\boldsymbol{r}_N\sigma_N)]$$

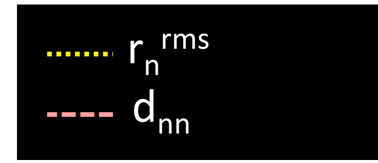
❷ Pair correlation

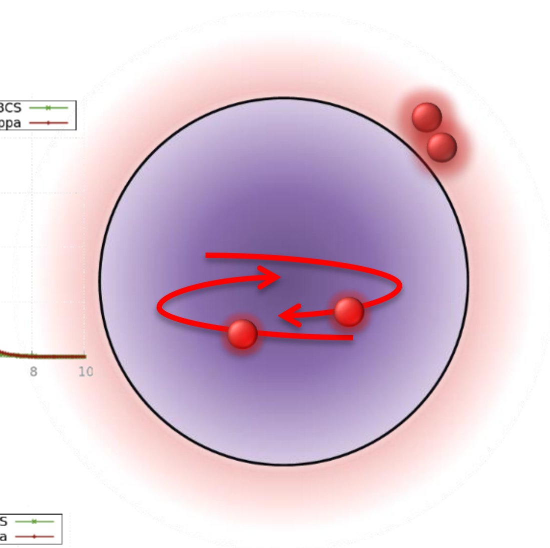
✪ 2-neutron distribution properties

Covariant HFB with projection on good particle number prior to variation





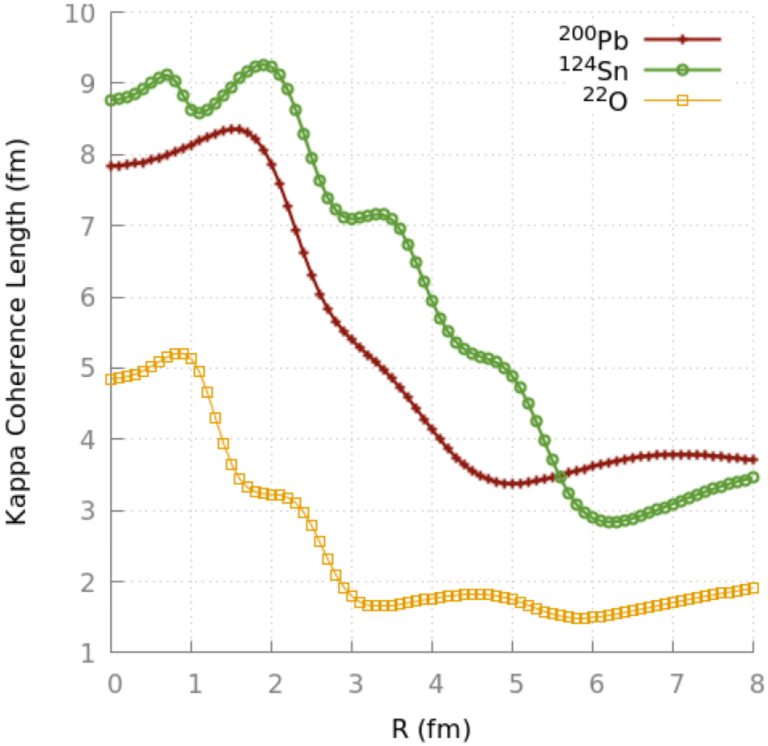


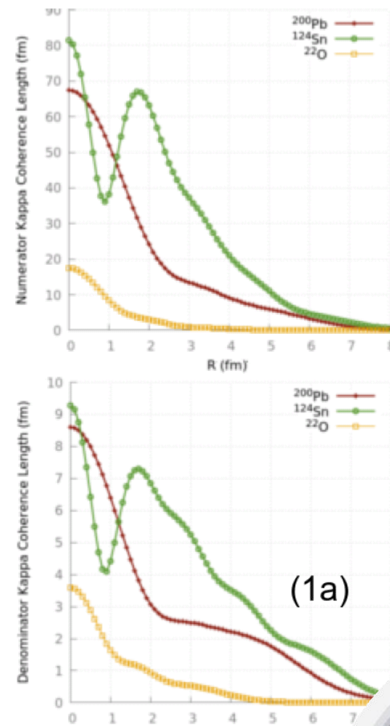


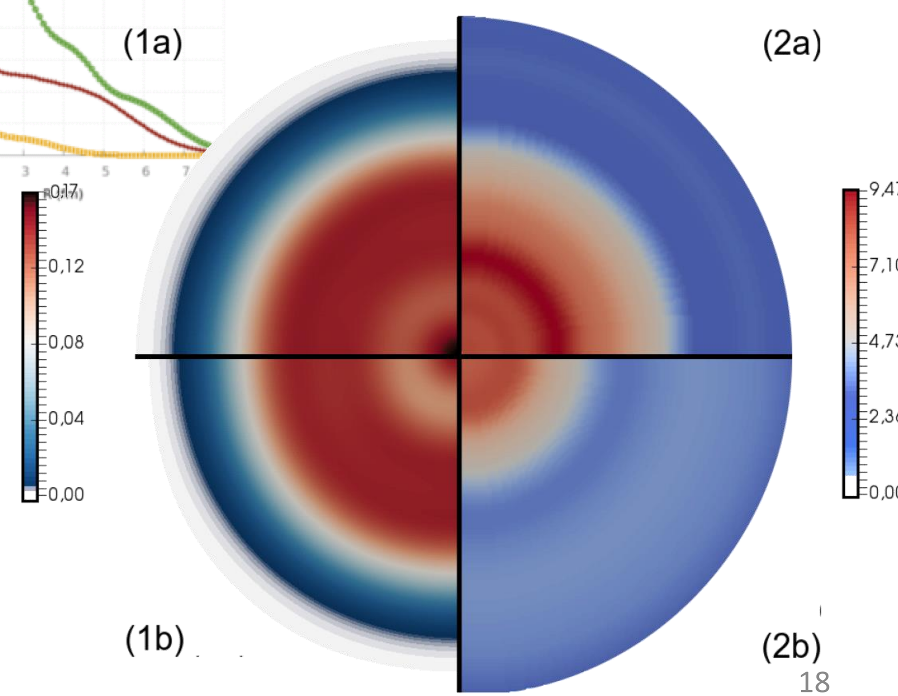
R.D. Lasseri et al, in prep

❷ Pair correlation

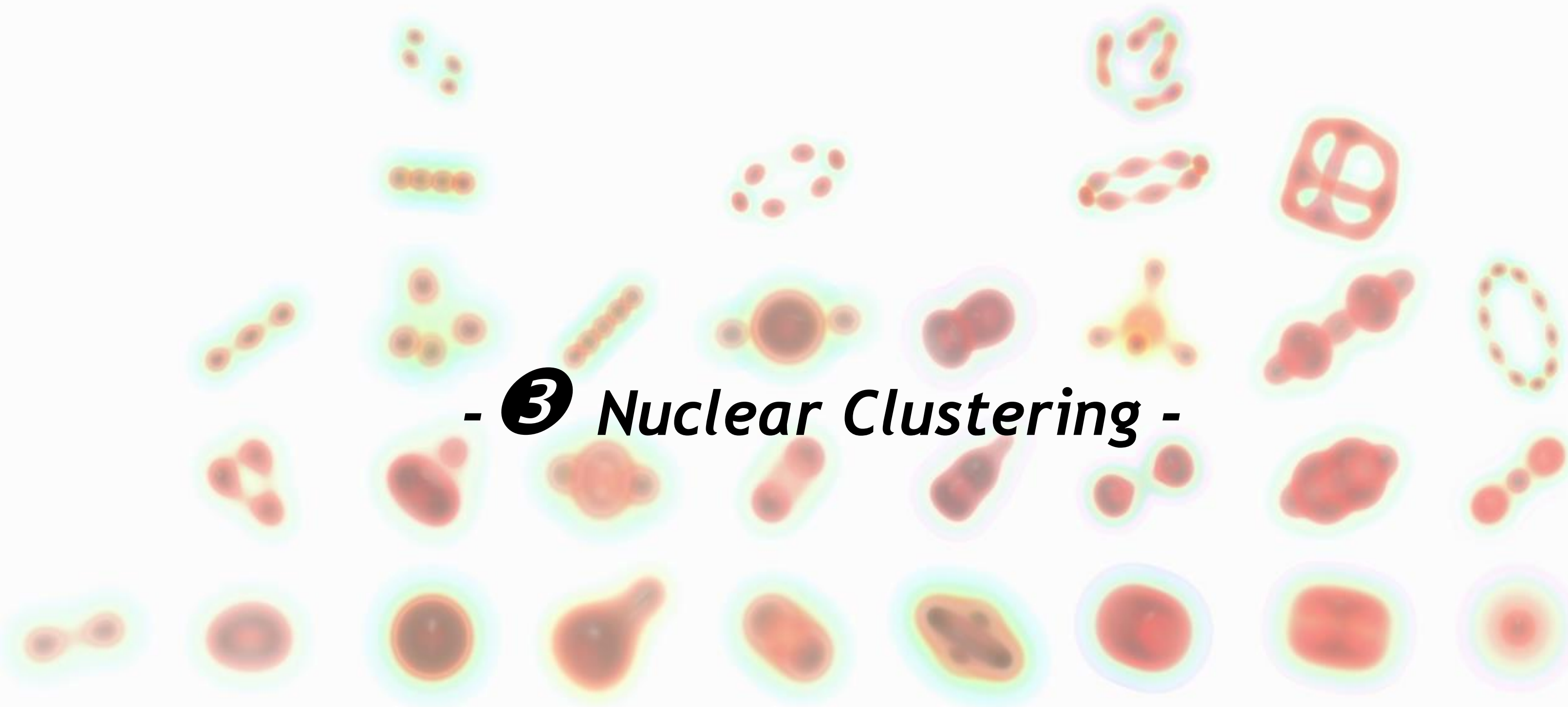
✪ 2-neutron distribution properties







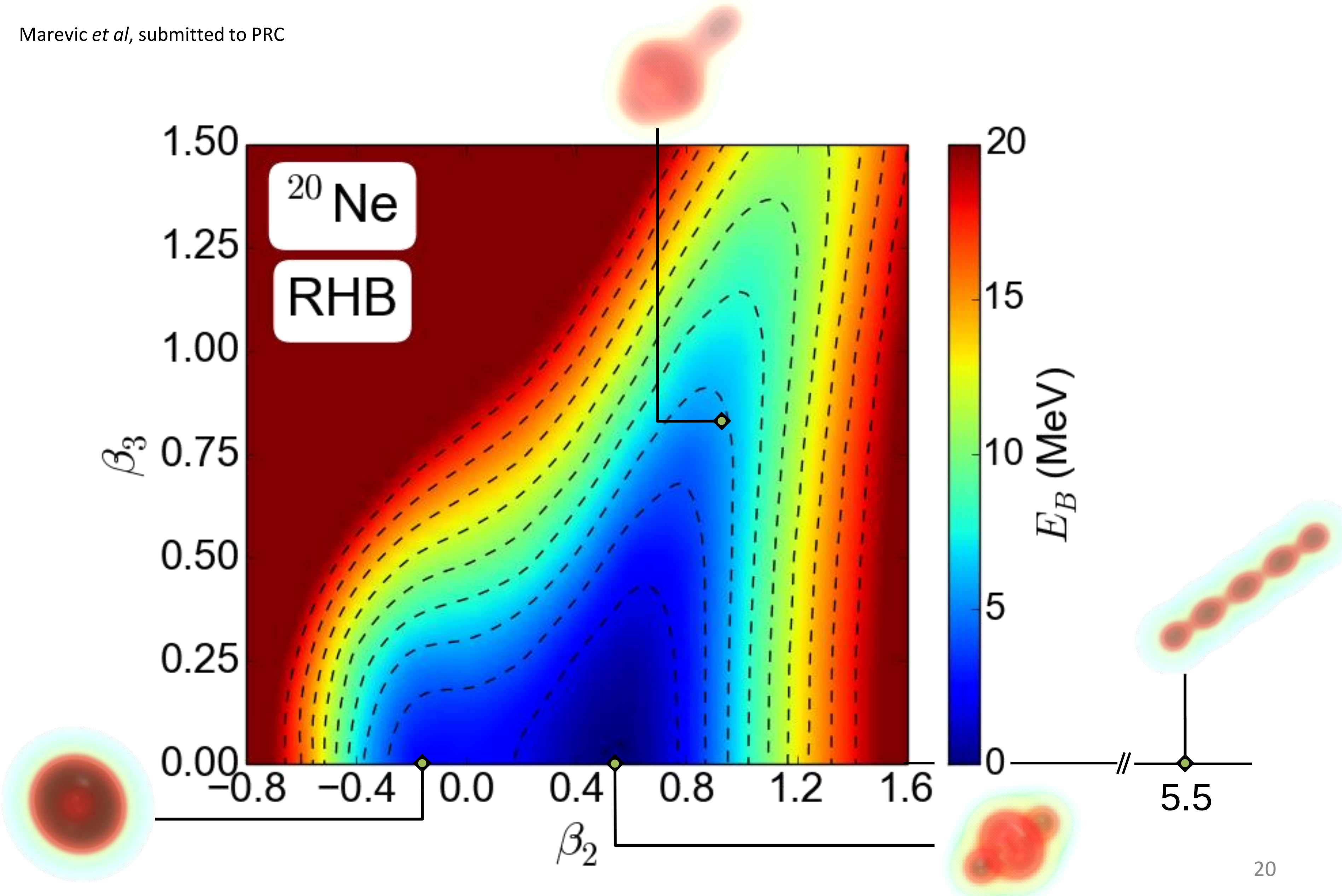
R.D. Lasseri et al, in prep



- ❸ Nuclear Clustering -

✪ First indicator: mass density at the MF level

Marevic *et al*, submitted to PRC



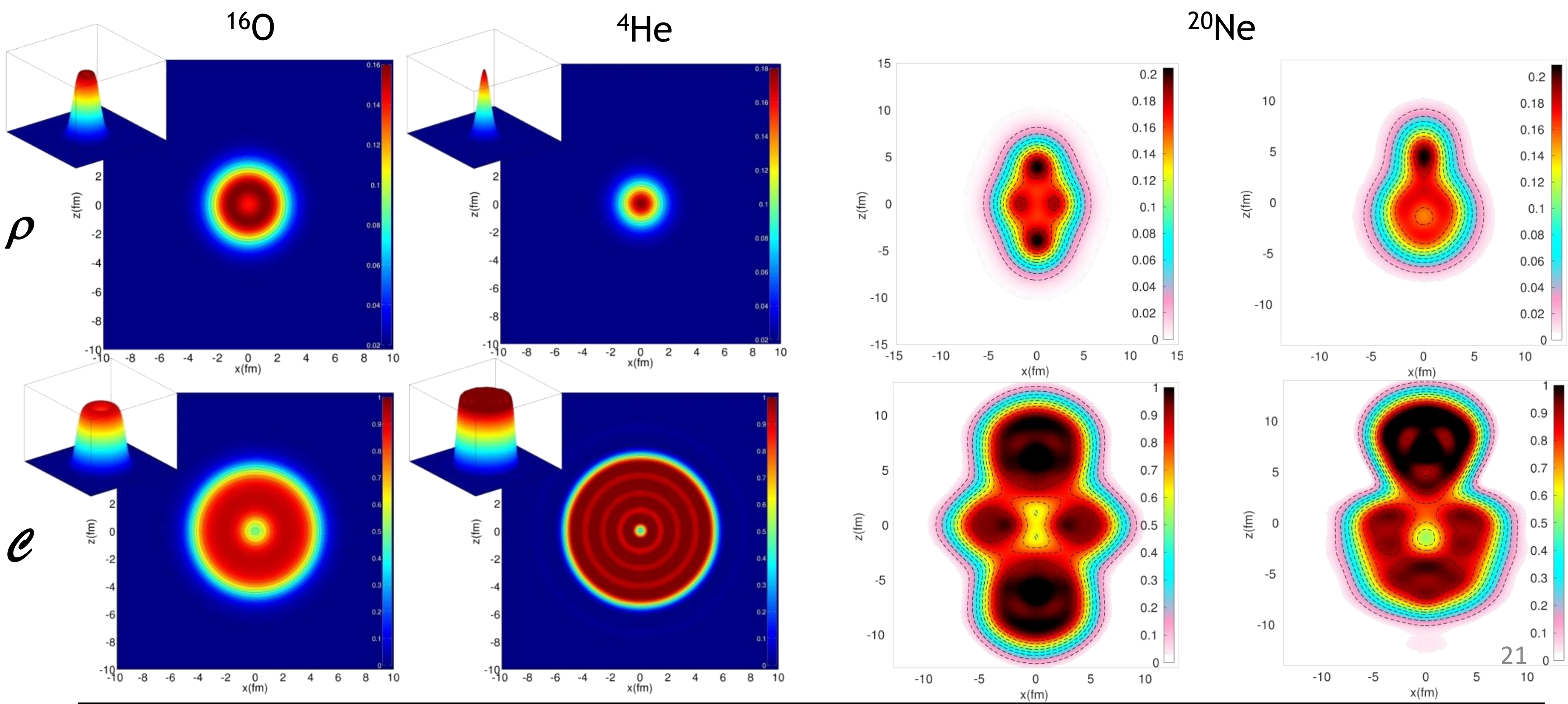
✪ *Localization measure*

➲ Conditional probability $R_{q\sigma}(\mathbf{r},\mathbf{r}') = \rho_{q\sigma}(\mathbf{r}') - \dfrac{|\rho_{q\sigma\sigma}(\mathbf{r},\mathbf{r}')|^2}{\rho_{q\sigma}(\mathbf{r})}$

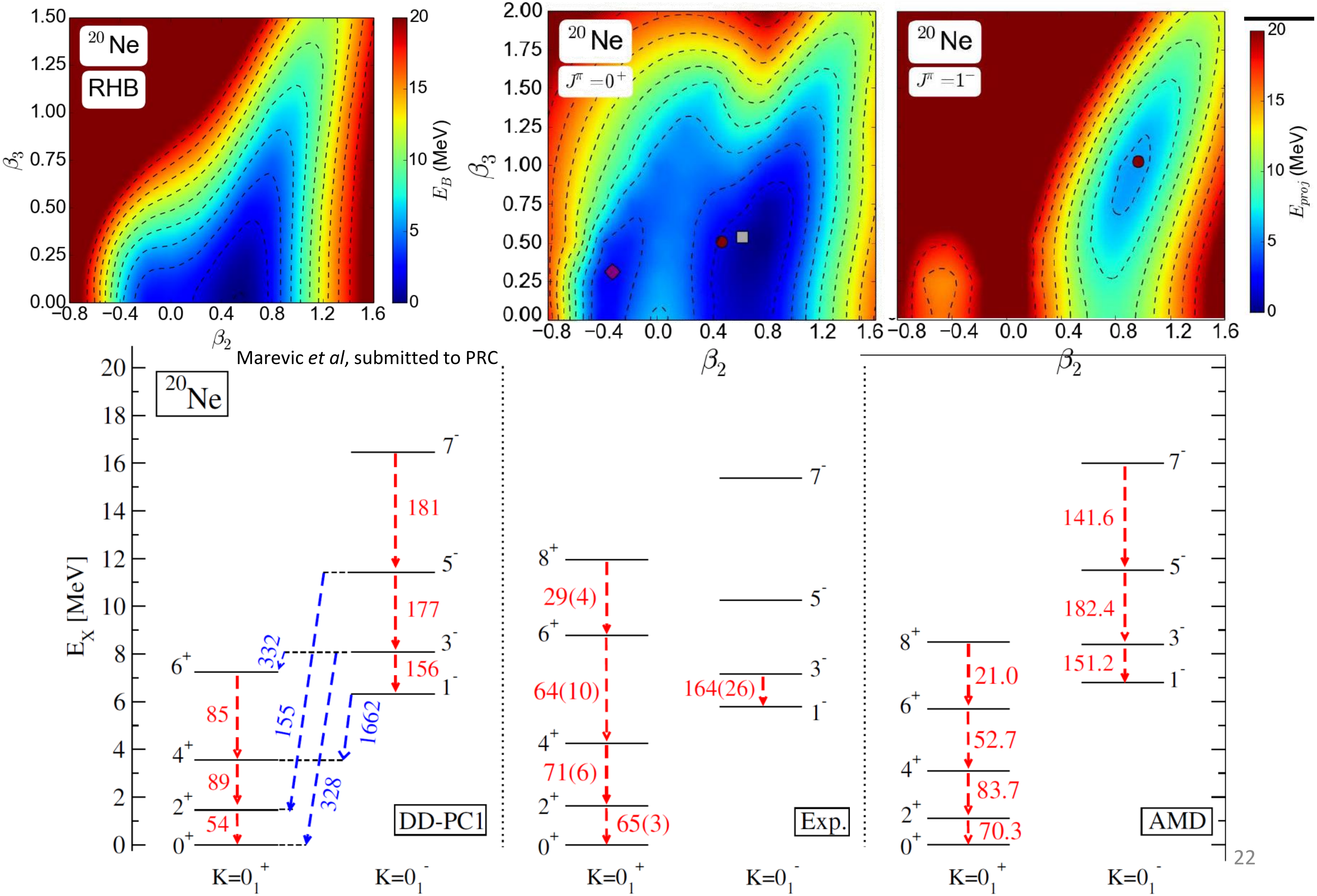
Reinhard *et al*, Phys. Rev. C 83, 034312 (2011)

$$\mathcal{C}_{q\sigma}(\mathbf{r}) = \left[1 + \left(\frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}[\nabla\rho_{q\sigma}]^2 - \mathbf{j}^2_{q\sigma}}{\rho_{q\sigma}\tau^{\mathrm{TF}}_{q\sigma}}\right)^2\right]^{-1}$$

- 0.5 : *signals a nearly homogeneous Fermi gas*
- 1 : *localized α-like state (in N=Z systems)*

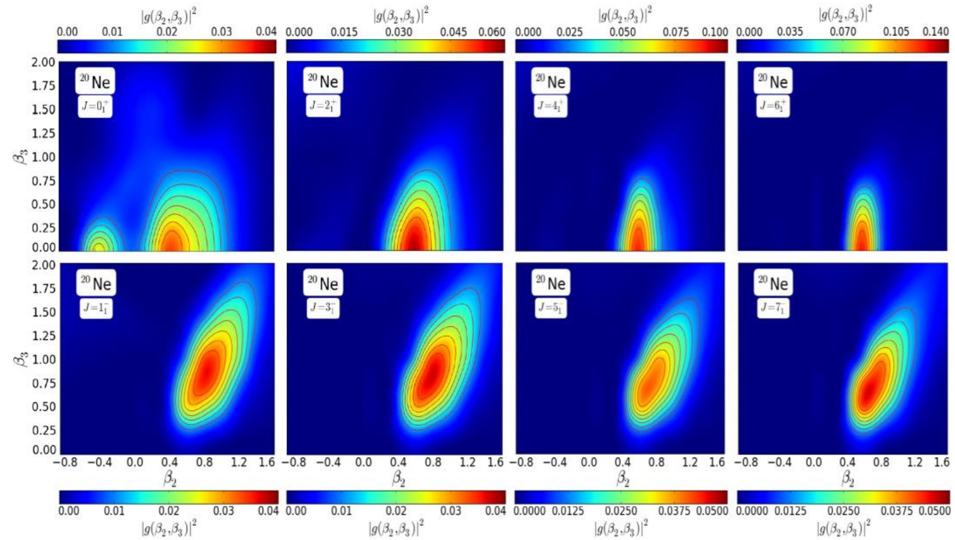


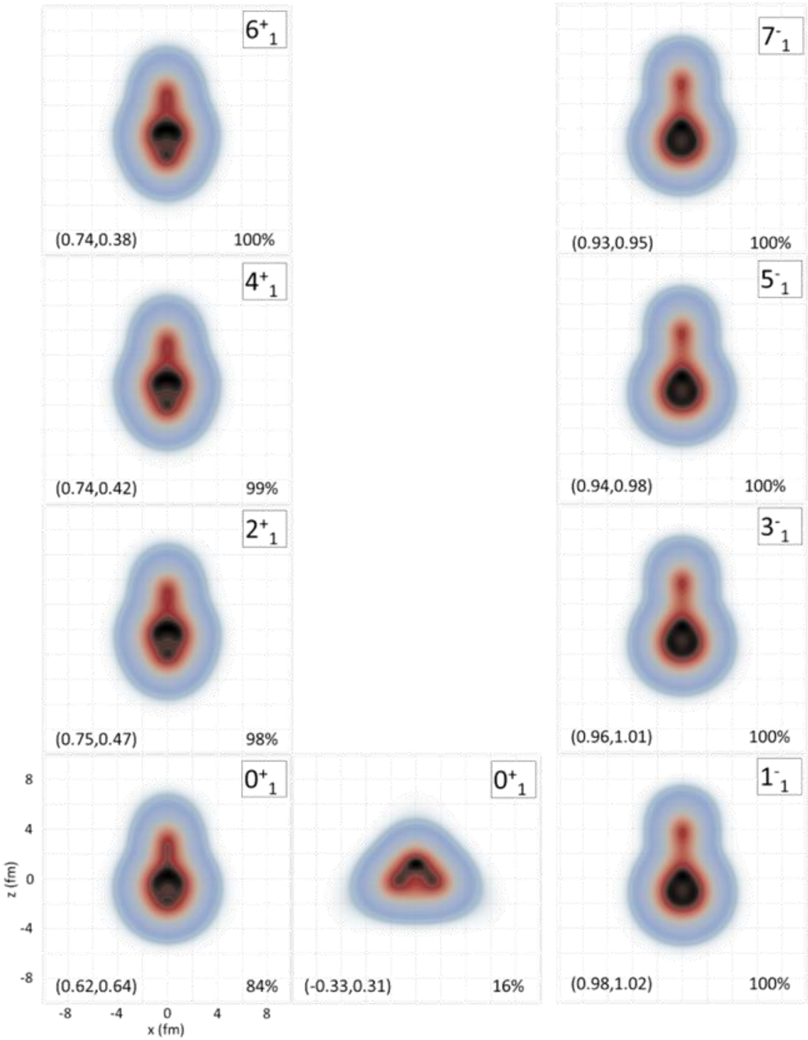
✪ Beyond MF level

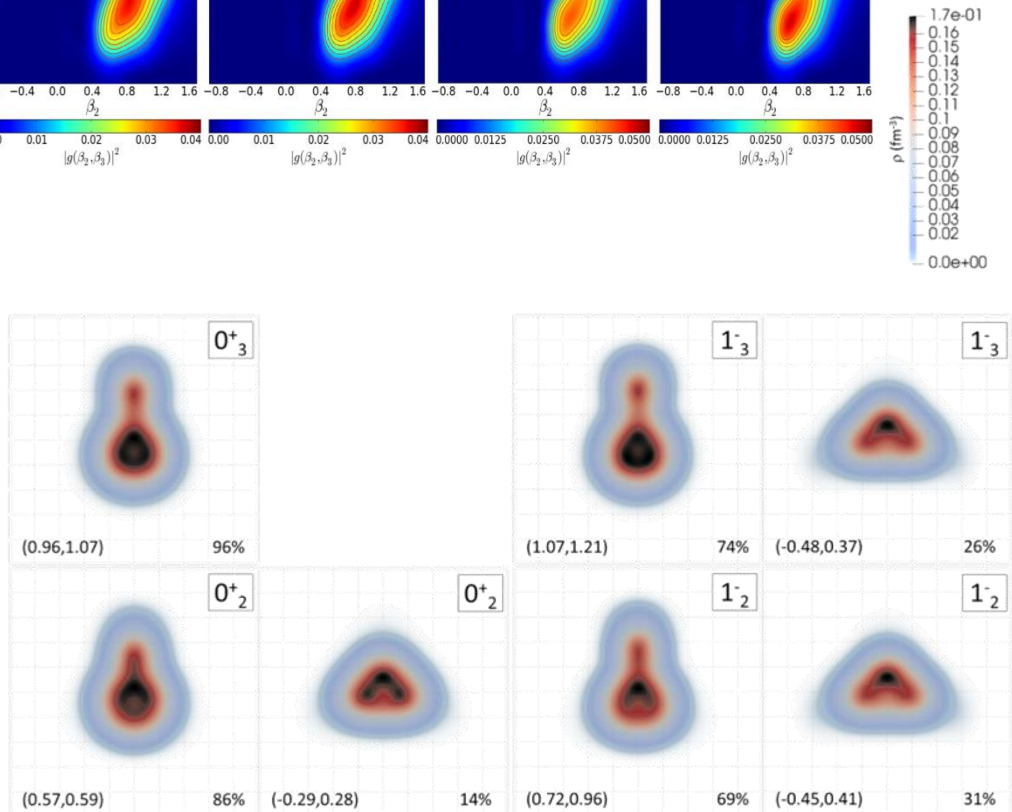


Marevic *et al*, submitted to PRC

✪ Beyond MF level

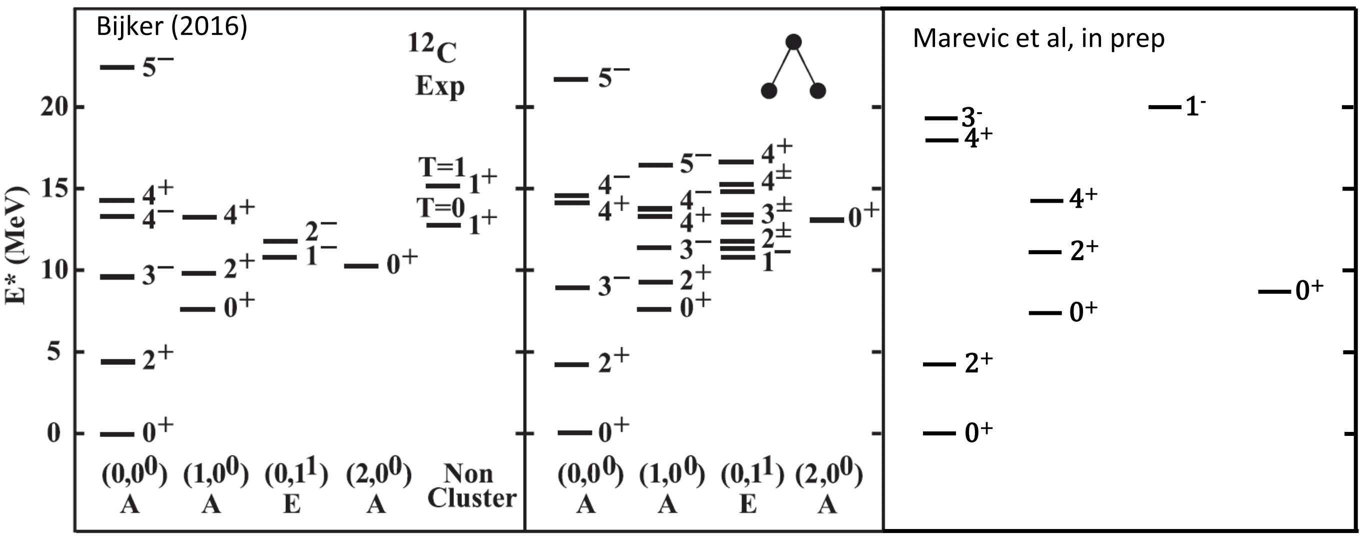


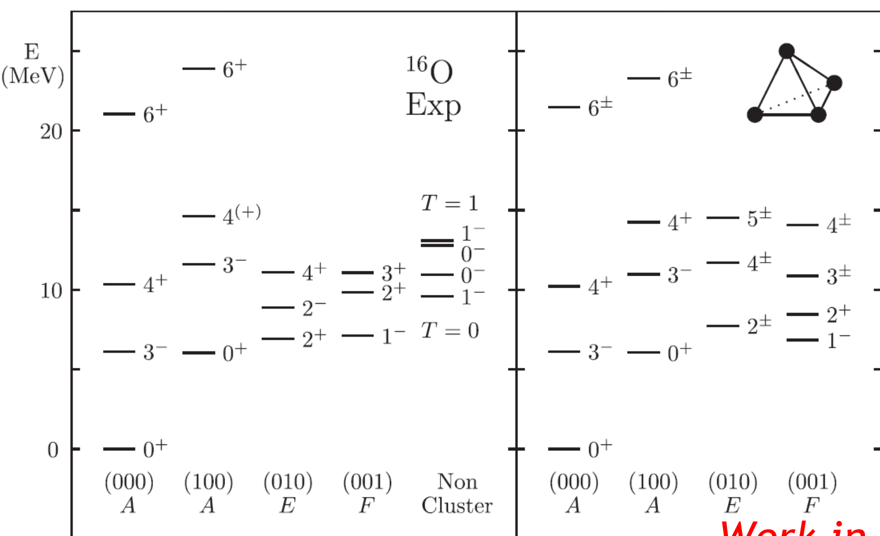




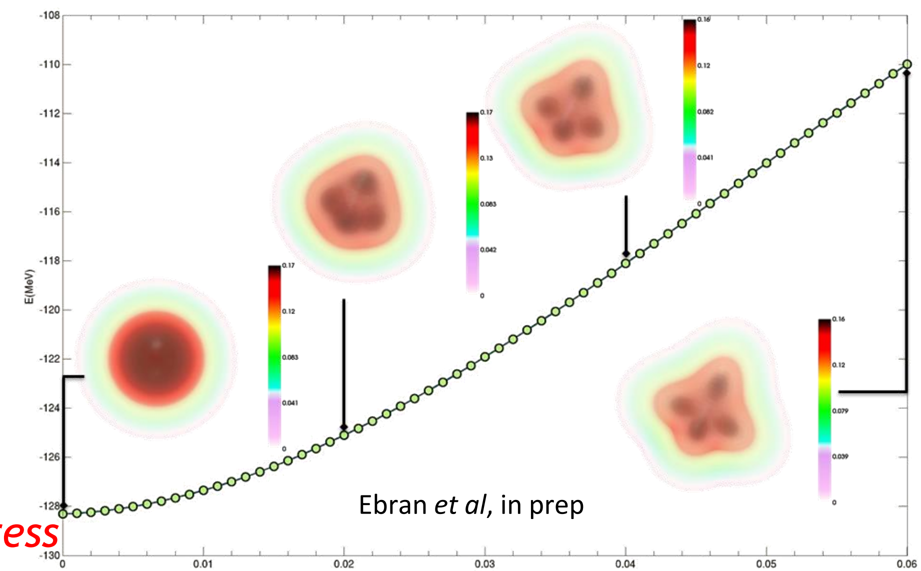
Marevic *et al*, submitted to PRC

✪ rotation-vibration bands





Work in progress



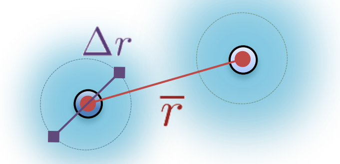
✪ Guidance : the localization parameter

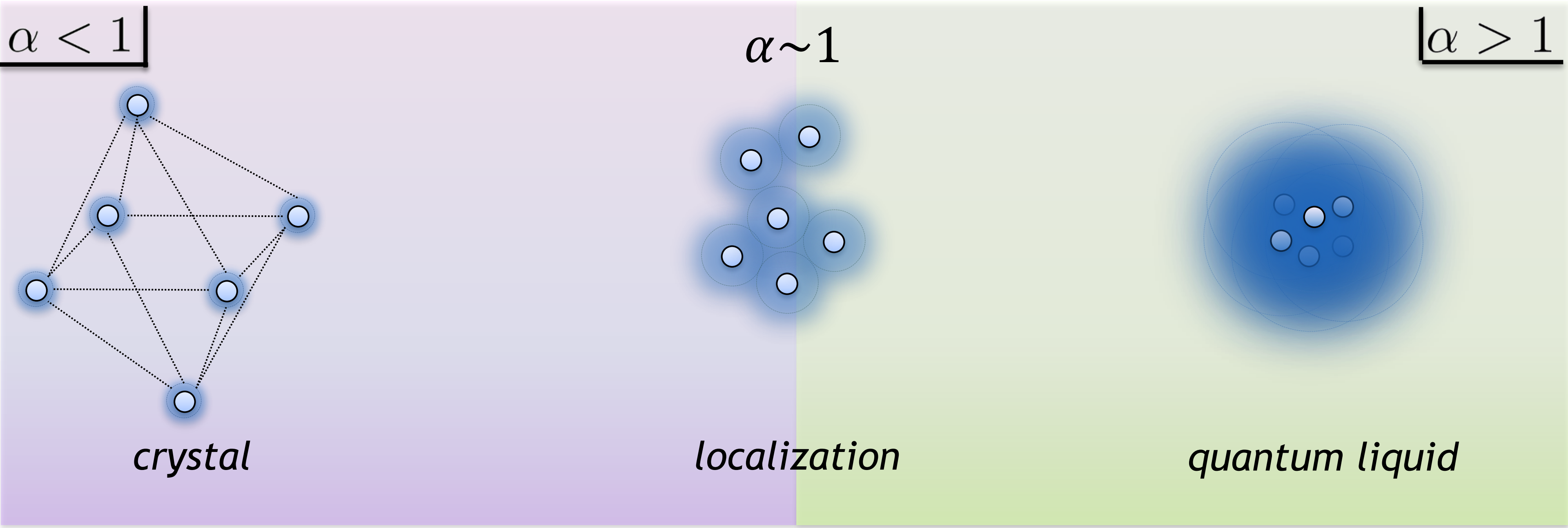
$$\alpha = \frac{\Delta r}{\overline{r}} = f\left(\frac{E_{kin}}{E_{pot}}\right) \propto \left[\frac{A^{1/3}}{\overline{r} V_0^{1/2}}\right]^{\frac{1}{2}}$$

◉ particle number

◉ depth of the confining potential

◉ average inter-particle distance ⇒ density



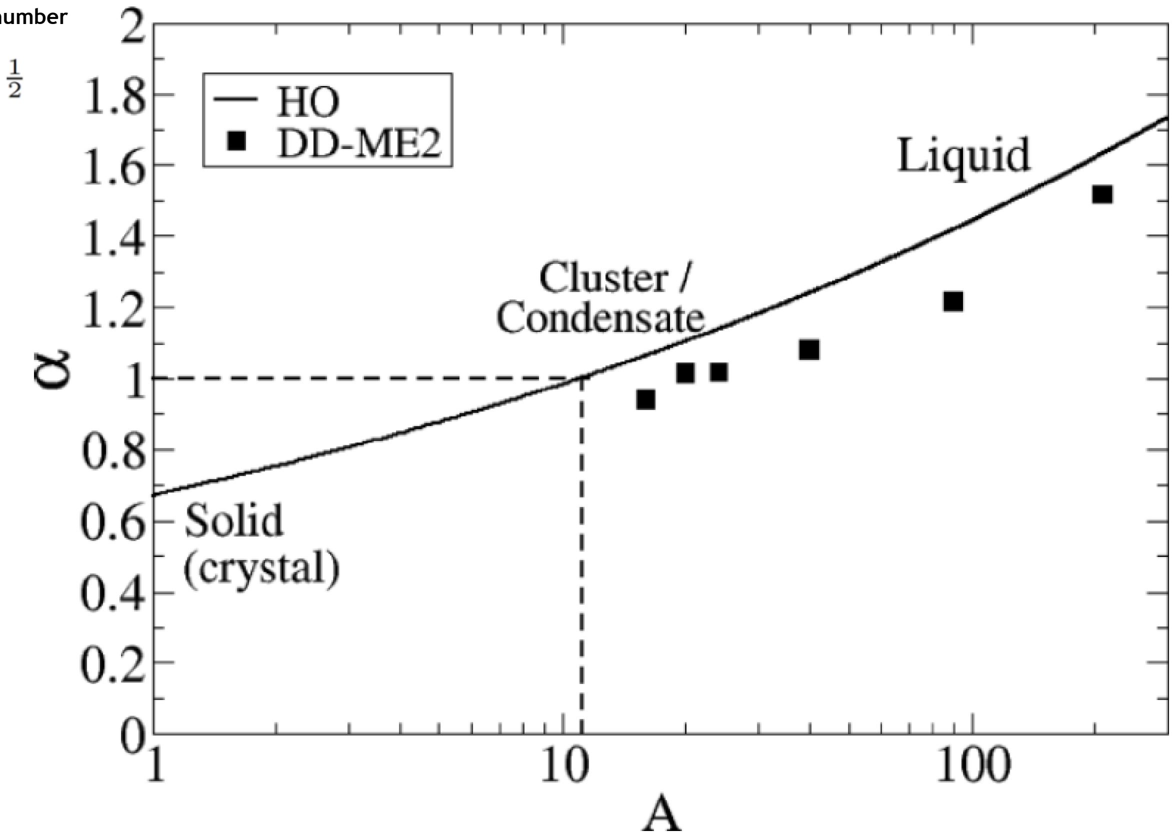


Ebran, Khan, Niksic & Vretenar, Nature 487, 341-344 (2012) - PRC 87, 044307 (2013)

✪ Influence of the particle number

particle number

$$\alpha = \frac{\Delta r}{\overline{r}} = f\left(\frac{E_{kin}}{E_{pot}}\right) \propto \left[\frac{A^{1/3}}{\overline{r} V_0^{1/2}}\right]^{\frac{1}{2}}$$



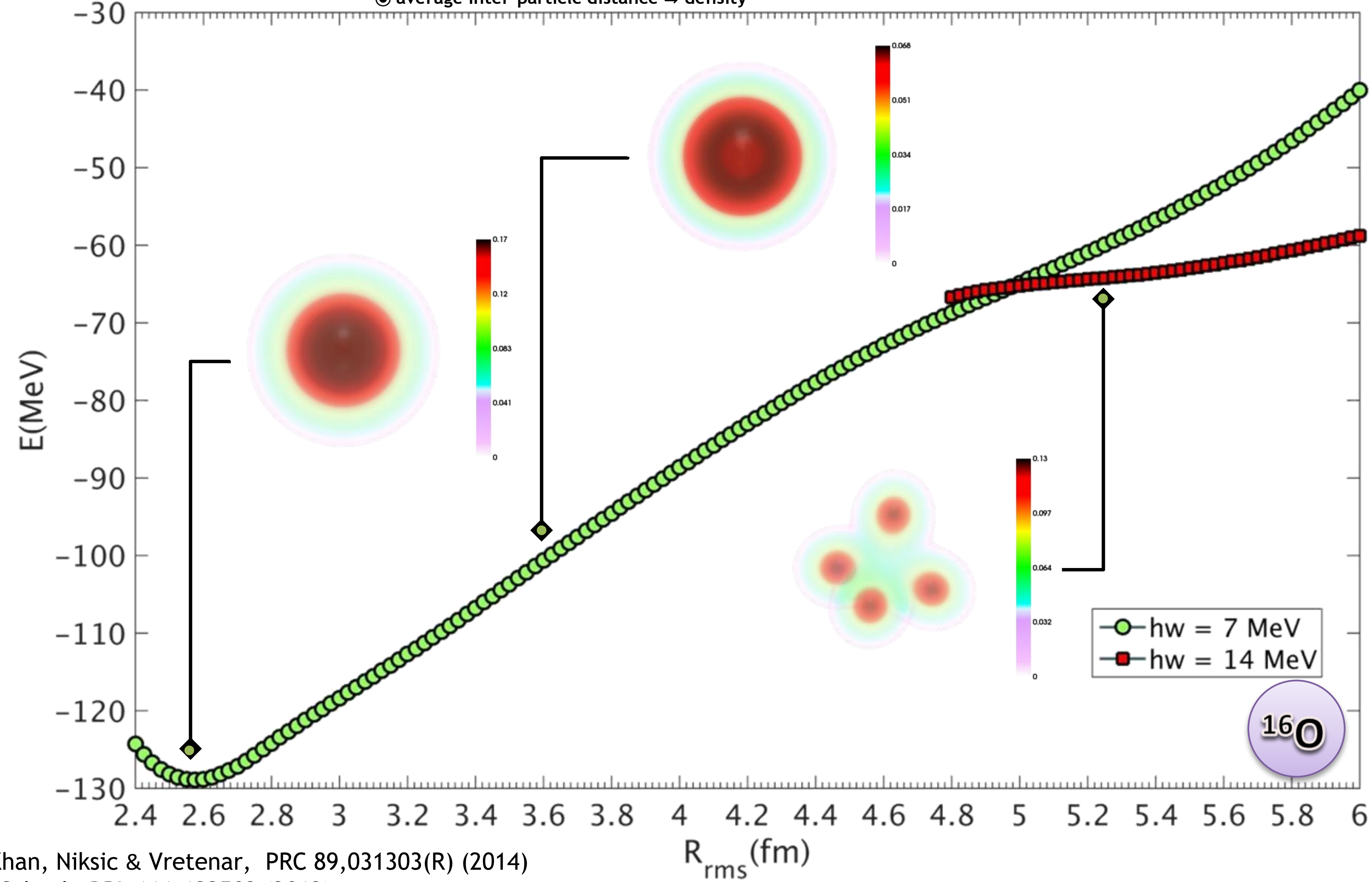
➲ Clustering is more likely to be found in light systems

Ebran, Khan, Niksic & Vretenar, PRC 87, 044307 (2013) - PRC 89, 031303(R) (2014)

✪ Influence of the density

$$\alpha = \frac{\Delta r}{\overline{r}} = f\left(\frac{E_{kin}}{E_{pot}}\right) \propto \left[\frac{A^{1/3}}{\overline{r} V_0^{1/2}}\right]^{\frac{1}{2}}$$

◉ average inter-particle distance ⇒ density



Ebran, Khan, Niksic & Vretenar, PRC 89,031303(R) (2014)
Girod & Schuck, PRL 111,132503 (2013)

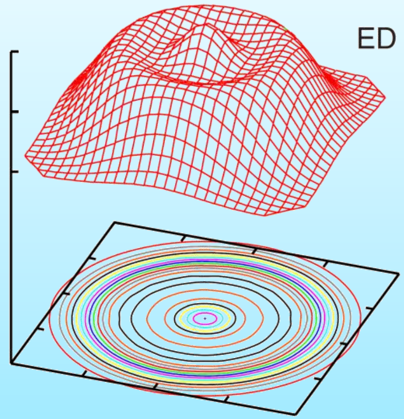
✪ Influence of the confining potential

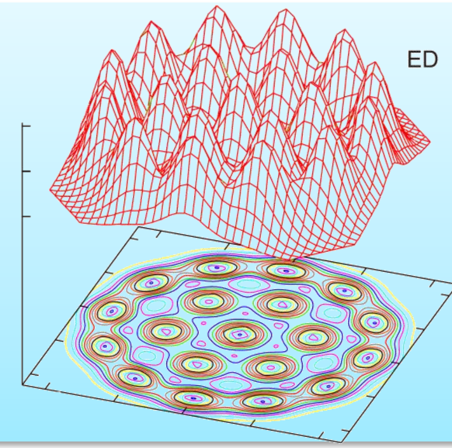
$$\alpha = \frac{\Delta r}{\overline{r}} = f\left(\frac{E_{kin}}{E_{pot}}\right) \propto \left[\frac{A^{1/3}}{\overline{r}V_0^{1/2}}\right]^{\frac{1}{2}}$$

◉ depth of the confining potential

Deeper confining potential

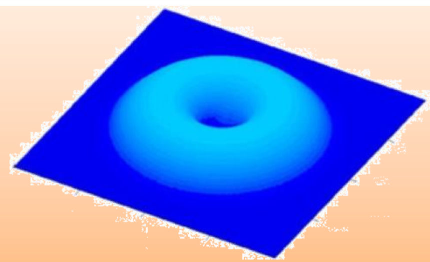
➲ Electrons in quantum dots

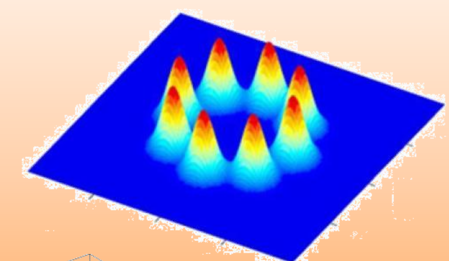




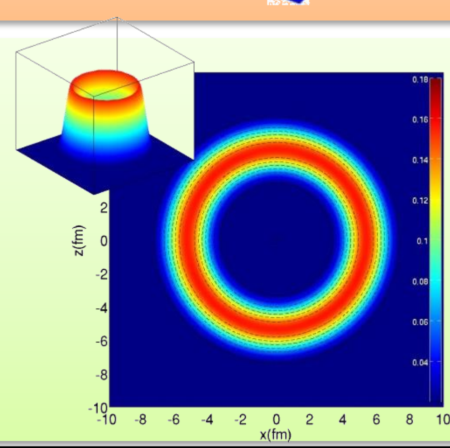
Yannouleas and Landman, Rep.Prog.Phys. 70, 2067 (2007)

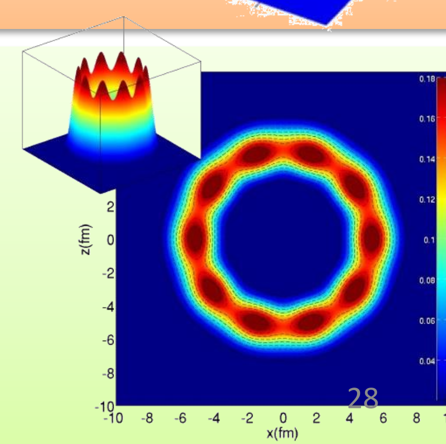
➲ Neutral bosons in rotating trap



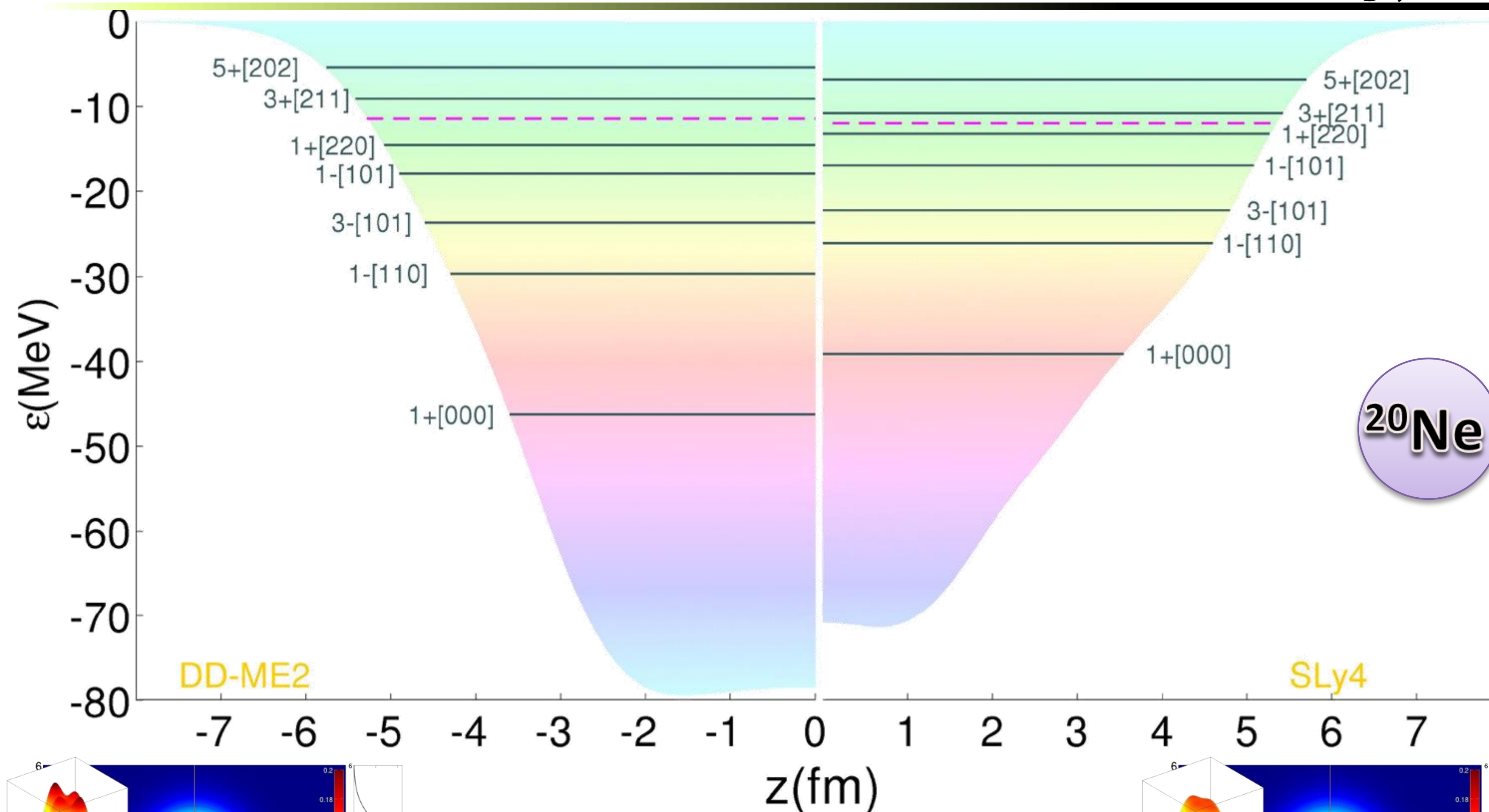


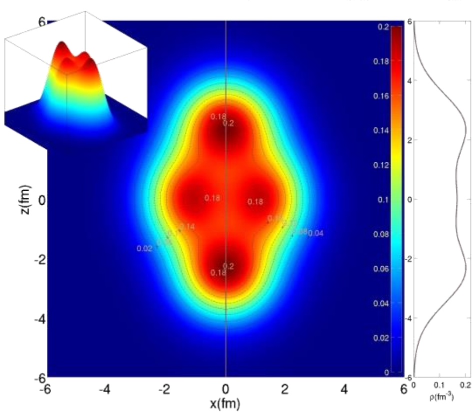
➲ Nucleons in ^{40}Ca

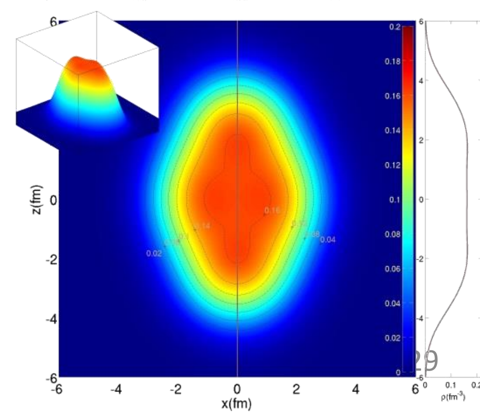




✪ Influence of the confining potential





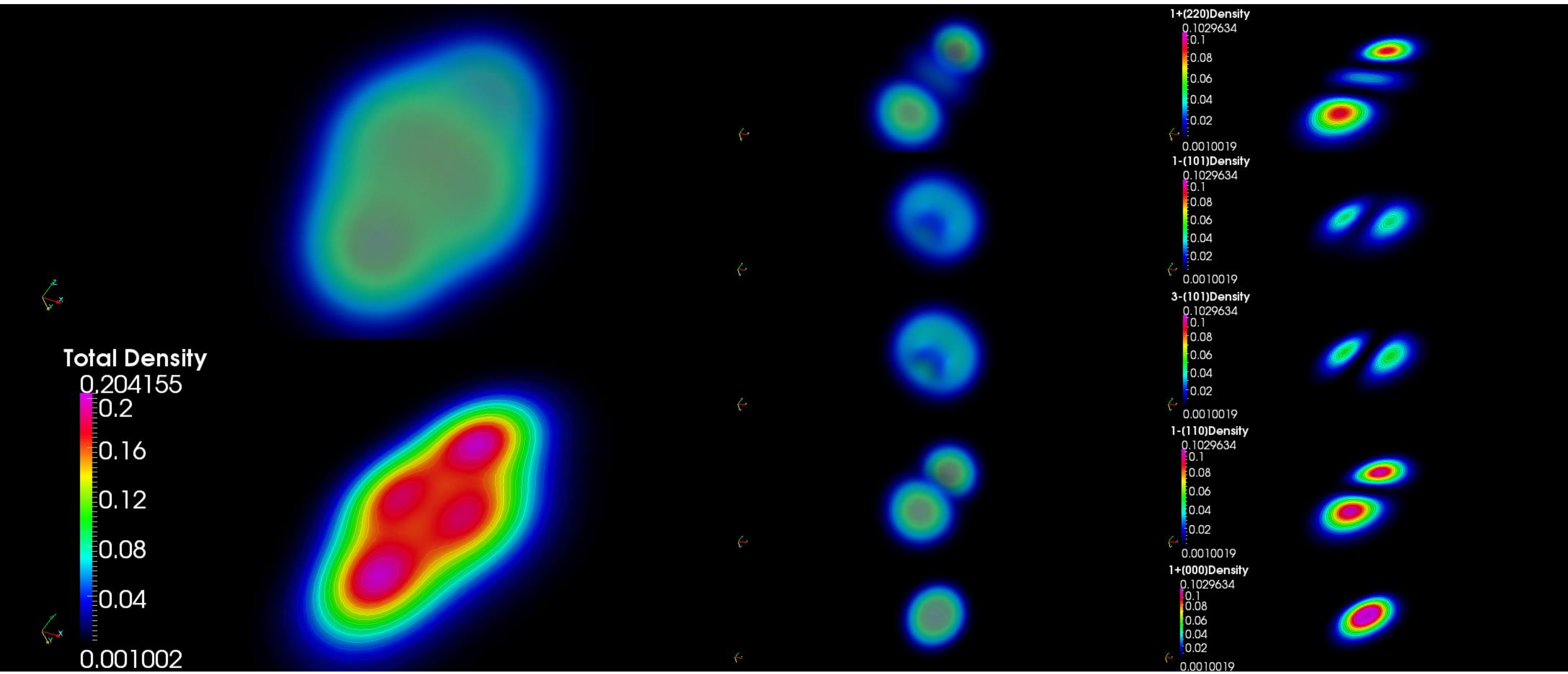


✪ *Influence of the confining potential*

$$\alpha = \frac{\Delta r}{\bar{r}} = f\left(\frac{E_{kin}}{E_{pot}}\right) \propto \left[\frac{A^{1/3}}{\bar{r} V_0^{1/2}}\right]^{\frac{1}{2}}$$

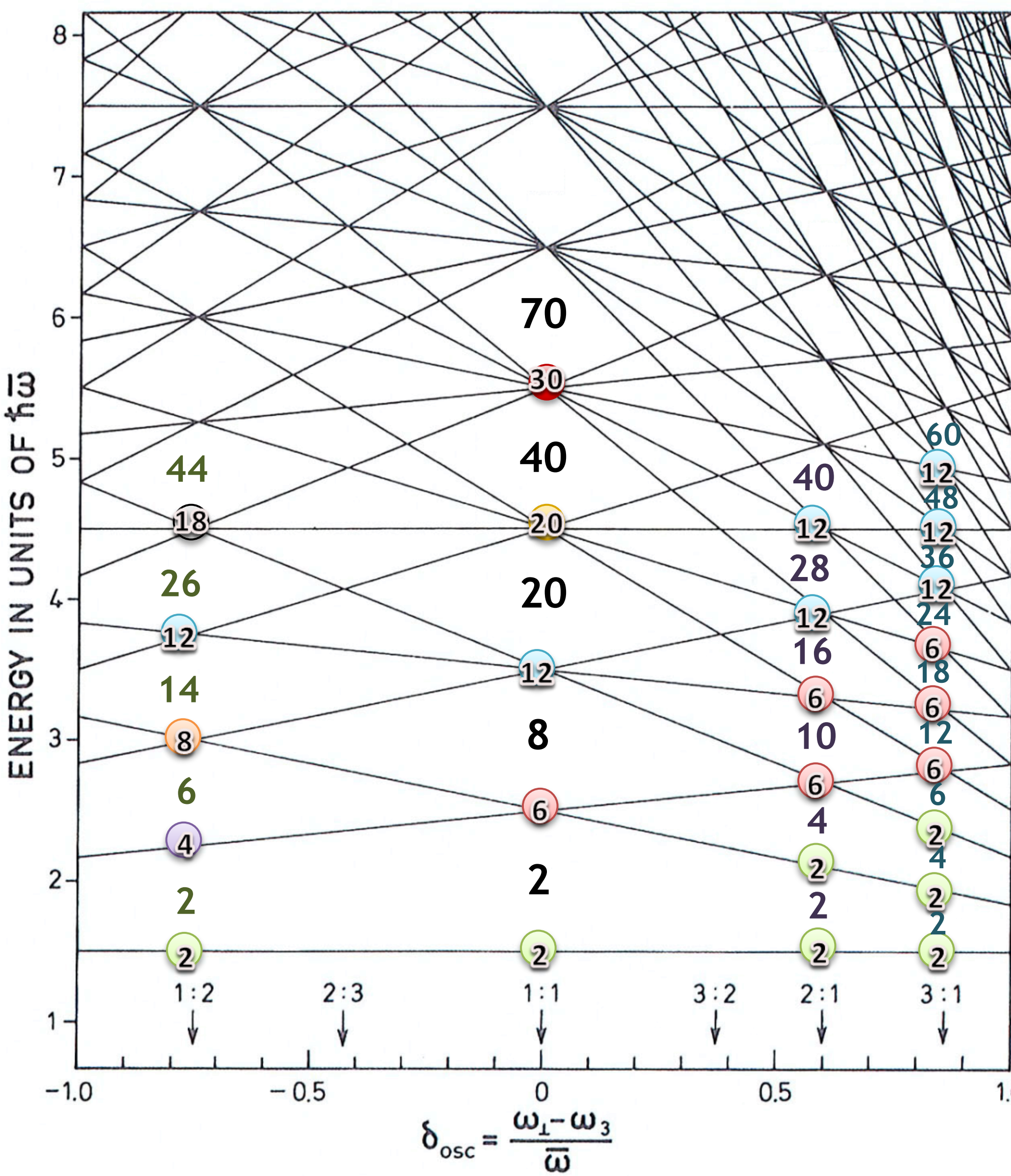
^{20}Ne

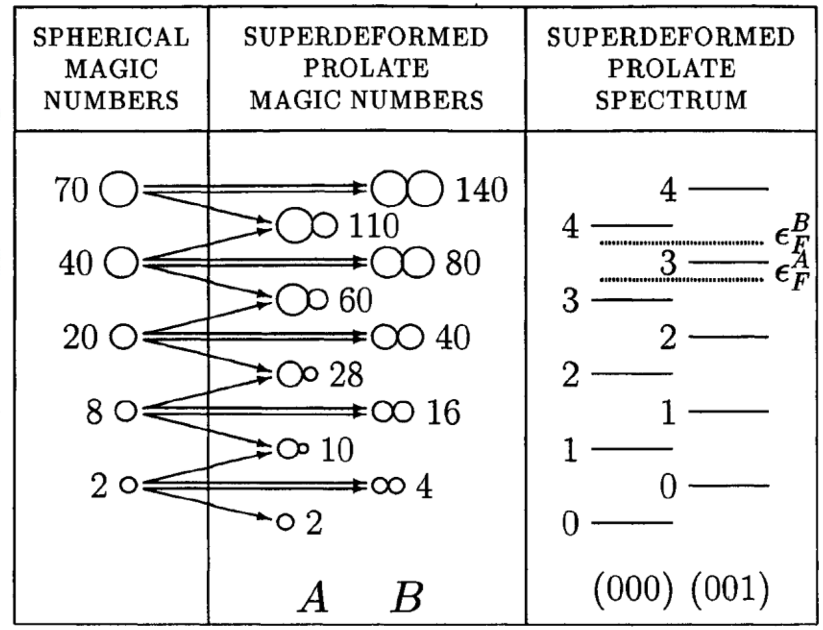
◉ **depth of the confining potential**

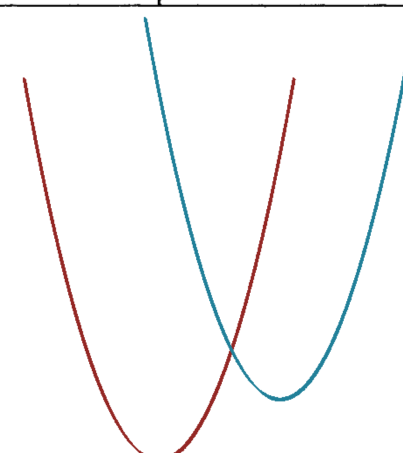


✪ Influence of the confining potential

Harmonic oscillator case





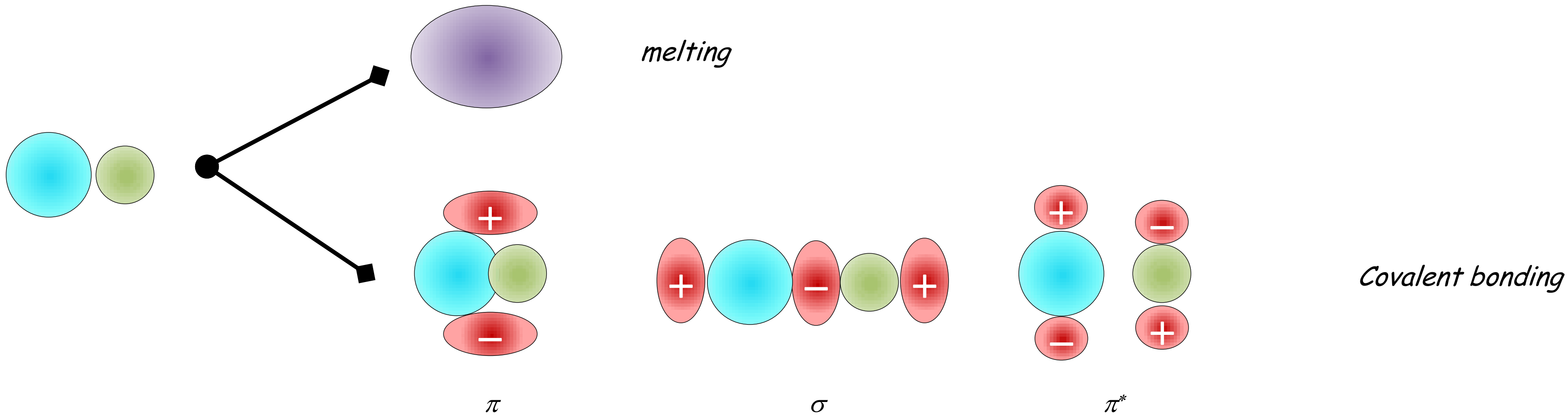


Nazarewicz *et. al.*, AIP Conf. Proc. 259, 30 (1992)

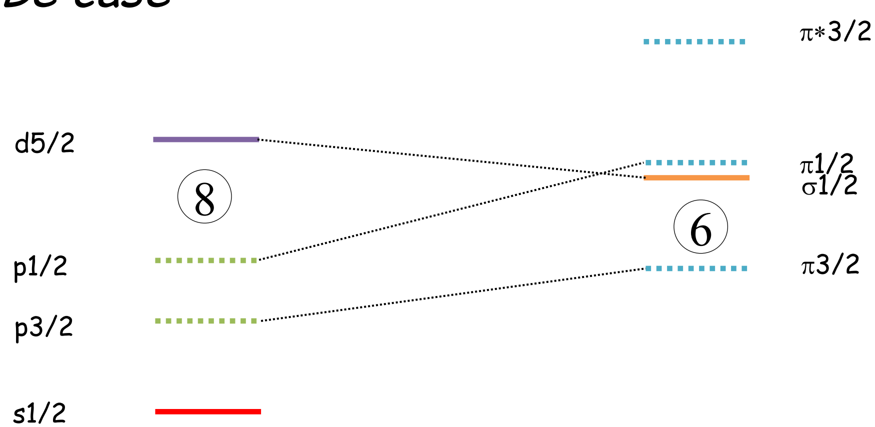
✪ Influence of the neutron excess

2-center cluster in a N=Z system

Neutron rich isotope

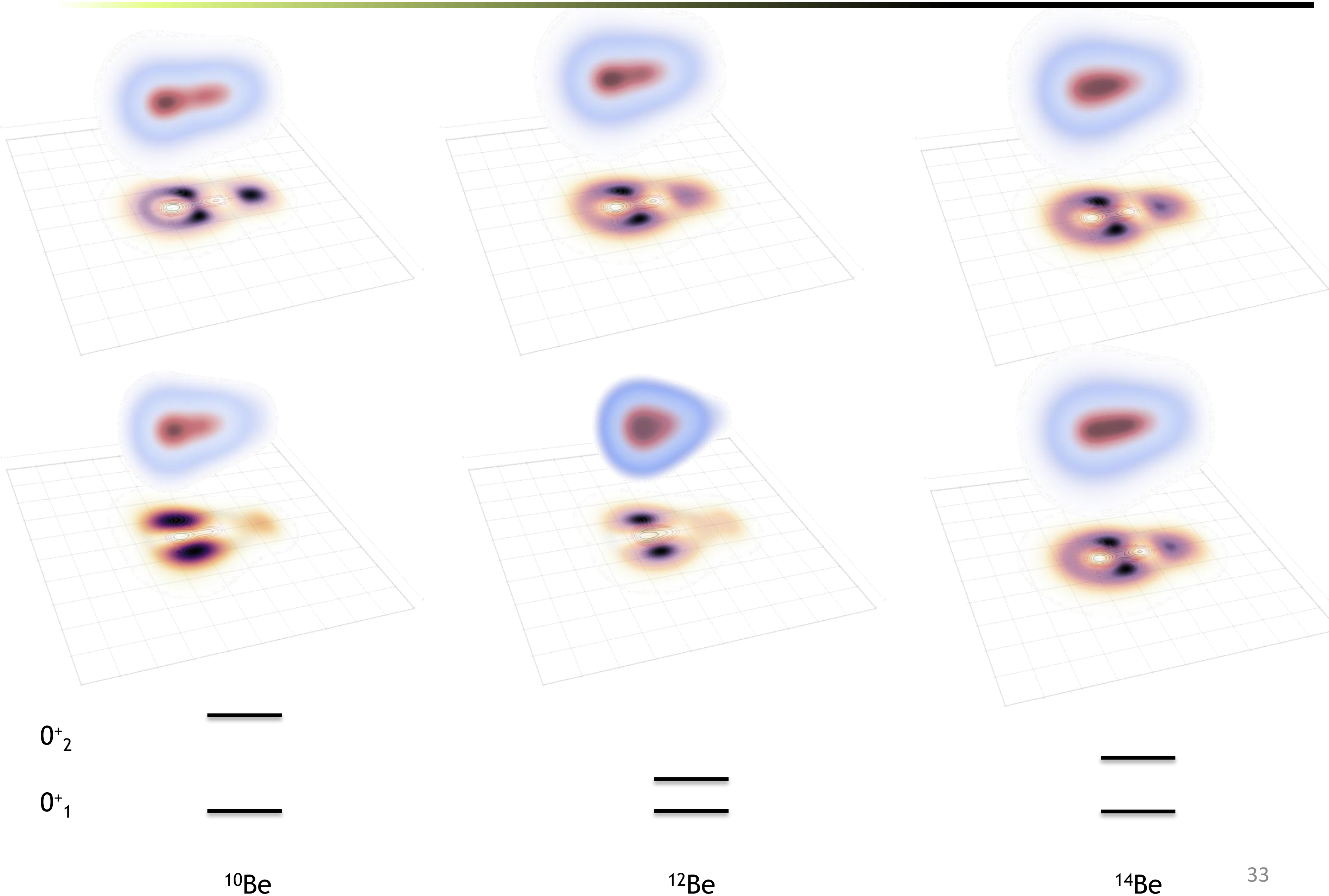


Be case

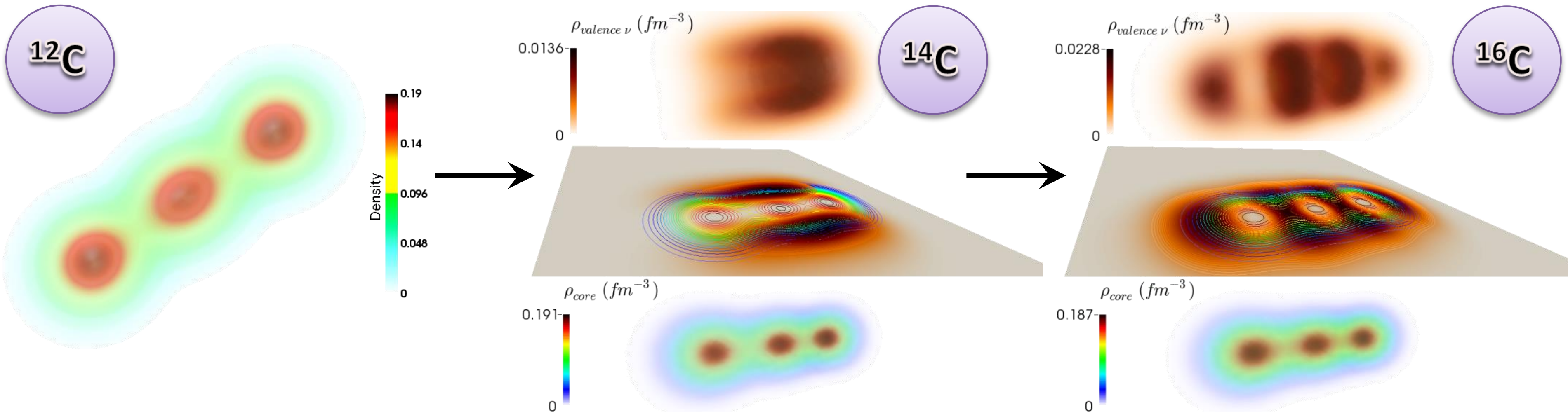


N=8 magic number breaking

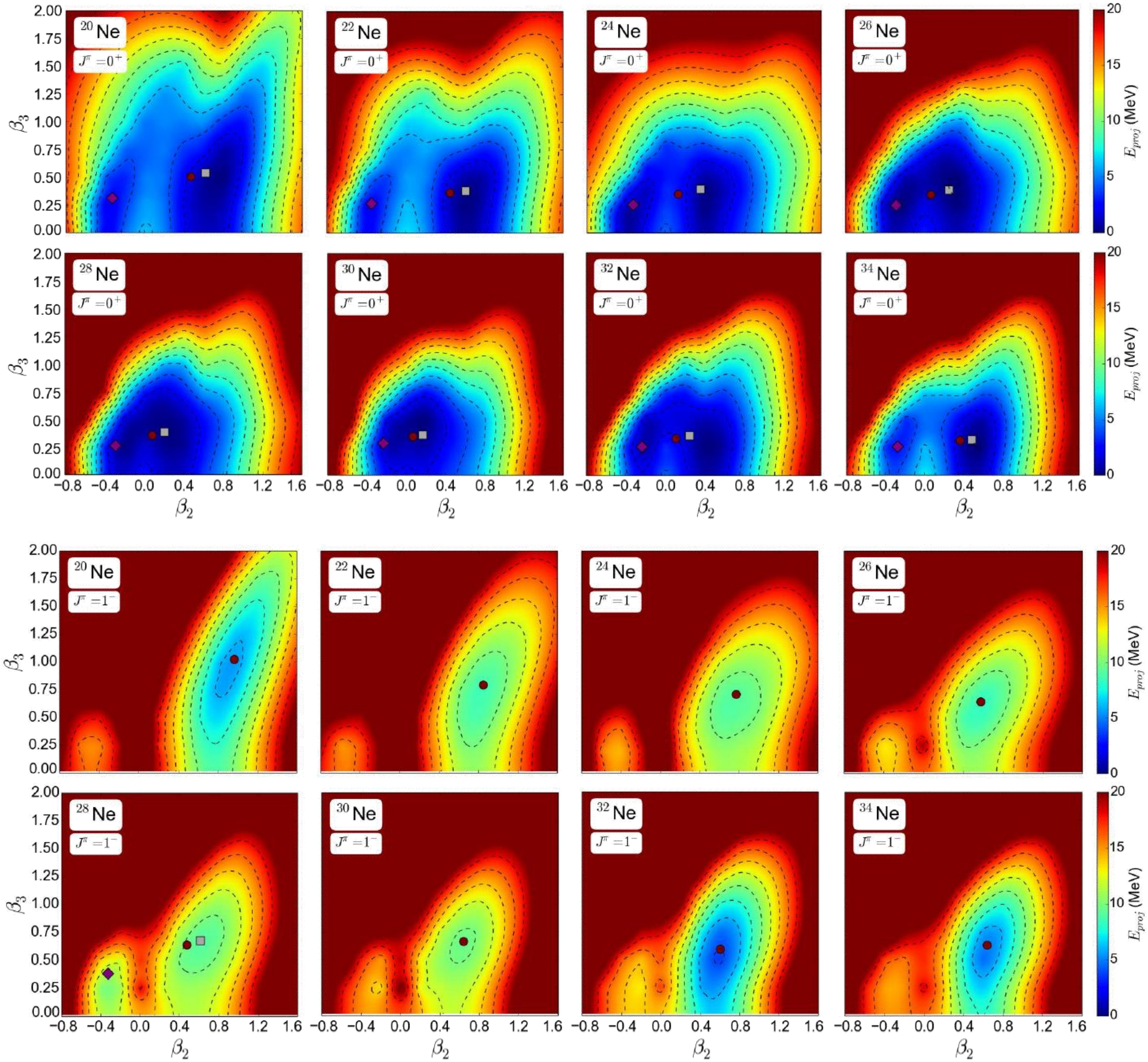
✪ Influence of the neutron excess



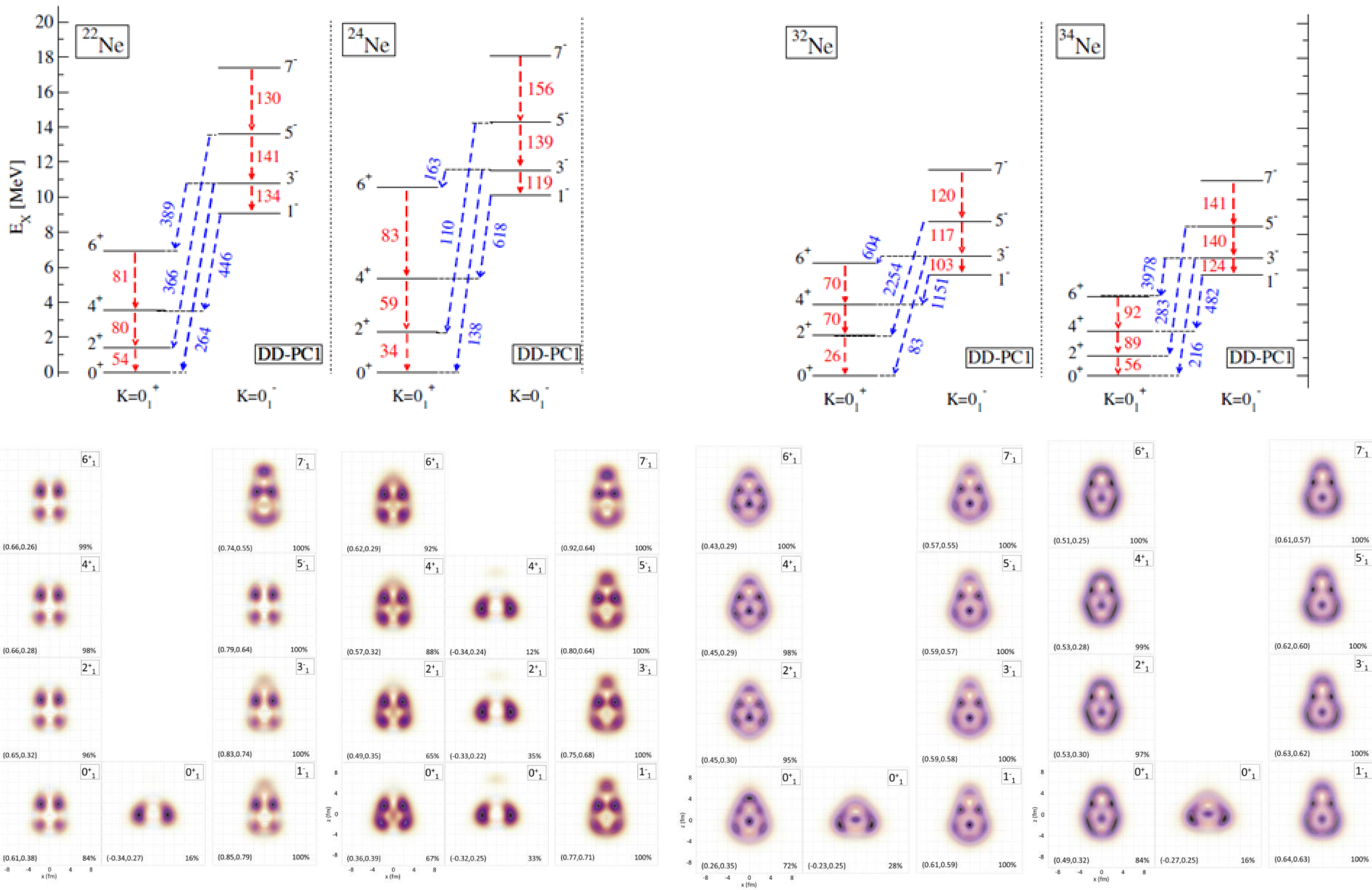
✪ Influence of the neutron excess



✪ Influence of the neutron excess



✪ Influence of the neutron excess



Conclusion & Perspectives

- ➲ Nuclear EDFs frame the various nuclear properties in a unified and consistent way

- ➲ Key feature : breaking/restoration of symmetries to efficiently account for nondynamical correlations

- ➲ Di-neutron like configuration at the surface of superfluid nuclei

- ➲ Clustering in ground and excited states of nuclei : impact of the average density and the depth of the confining potential

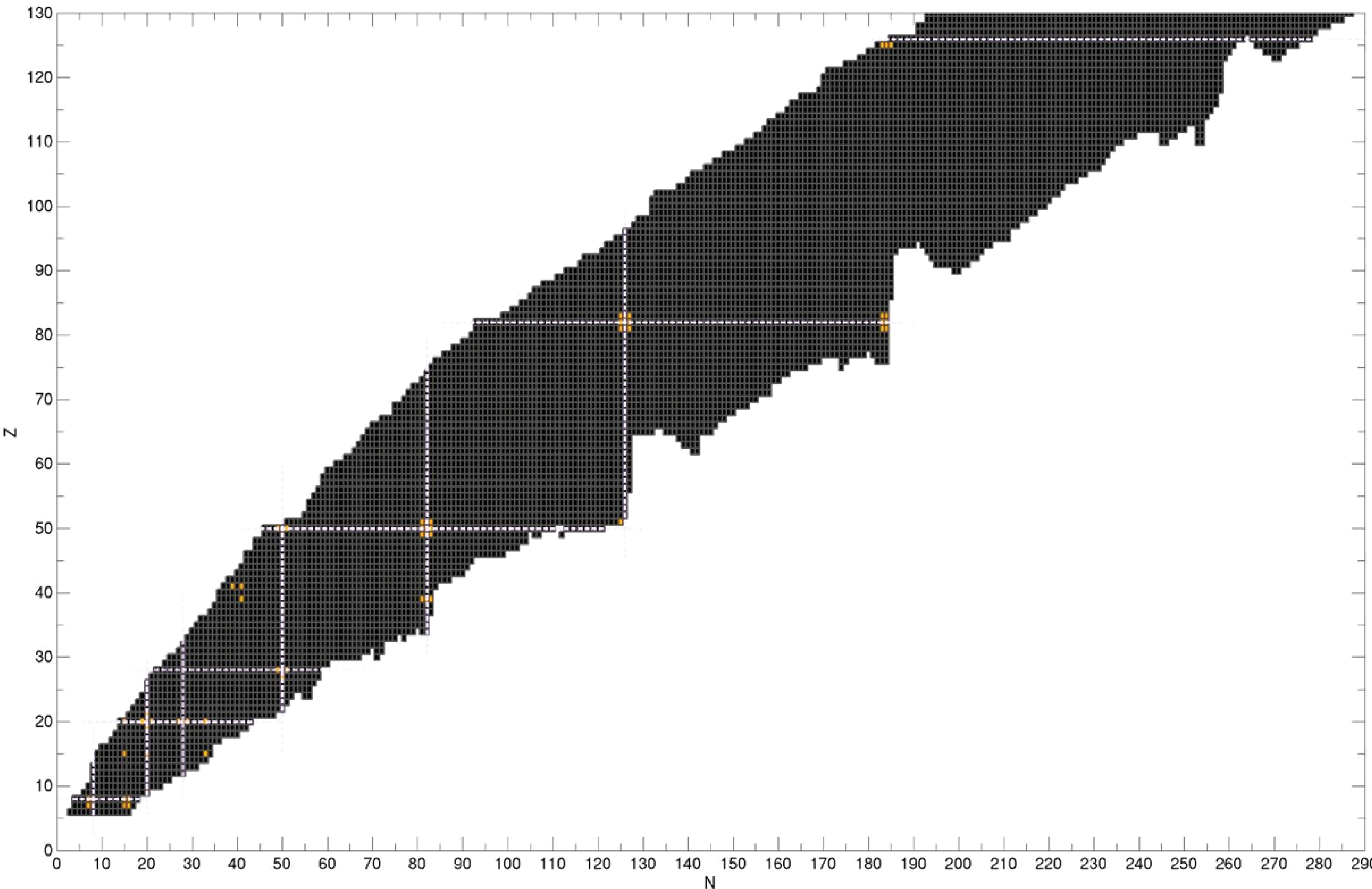
- ➲ Covalent bonding in neutron rich systems

- ➲ Particle number projection, conditional probability distribution, form factor, quarteting … under development

Thank you for your attention

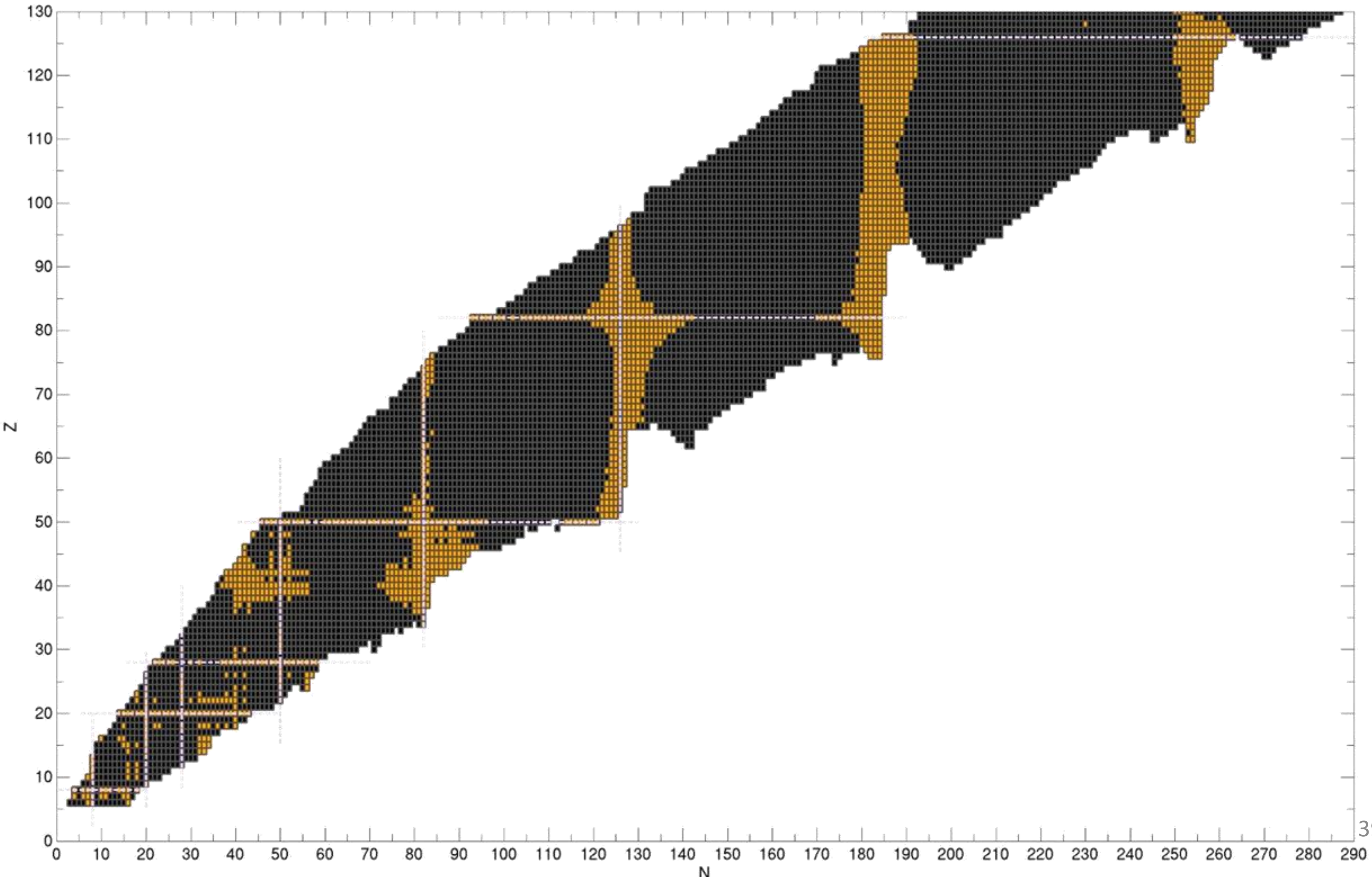
✪ Nuclear many-body problem : strategies

➲ Good description of the ground state of ~50 nuclei



✪ Nuclear many-body problem : strategies

➲ Breaking ***U(1)*** : good description of the ground state of ~300 nuclei



✪ Nuclear many-body problem : strategies

➲ Breaking ***U(1)*** and ***O(3)*** : good description of the ground state of all nuclei