Shapes and a-decay of superheavy nuclei

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Structure models for superheavy nuclei

Macro-micro models

The microscopic part (single-particle potential) is adjusted to empirical low-energy single-particle nuclear spectra, and a macroscopic energy formula is constructed separately to reproduce exp. masses.

Self-consistent models based on Energy Density Functionals

- ➡ adjusted to selected bulk nuclear properties, e.g. masses, charge radii, and empirical properties of homogeneous nuclear matter.
- ➡ universal theory framework that can be applied to nuclei over the entire mass table.
- ➡ important for extrapolations to mass regions where only few data are available.

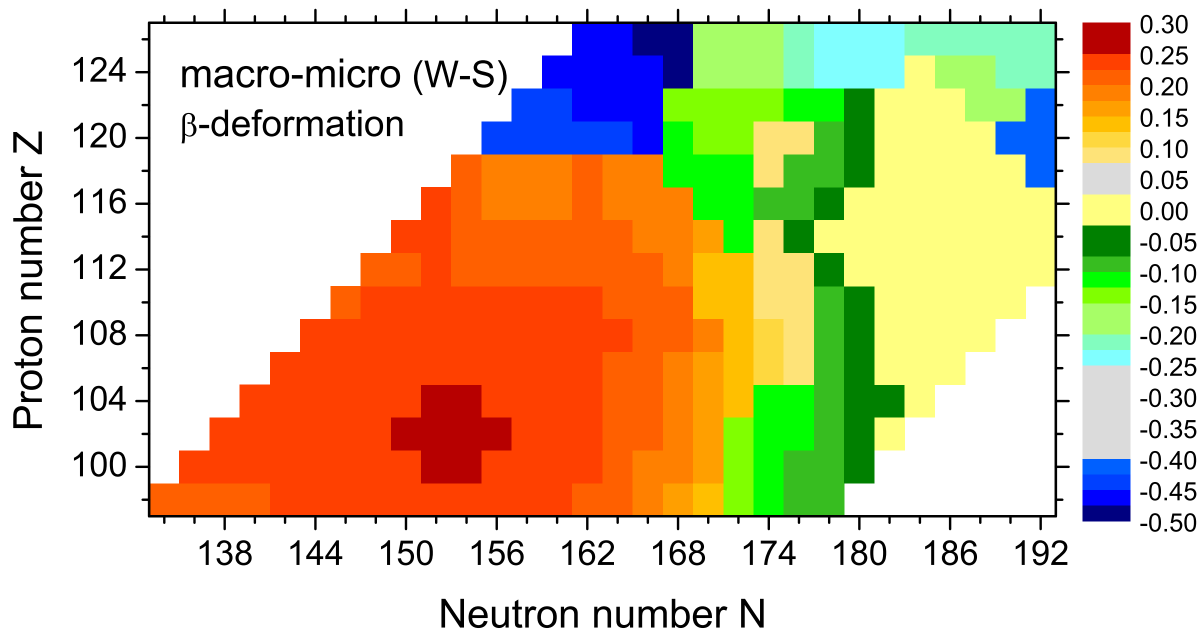
Extrapolation to SHE

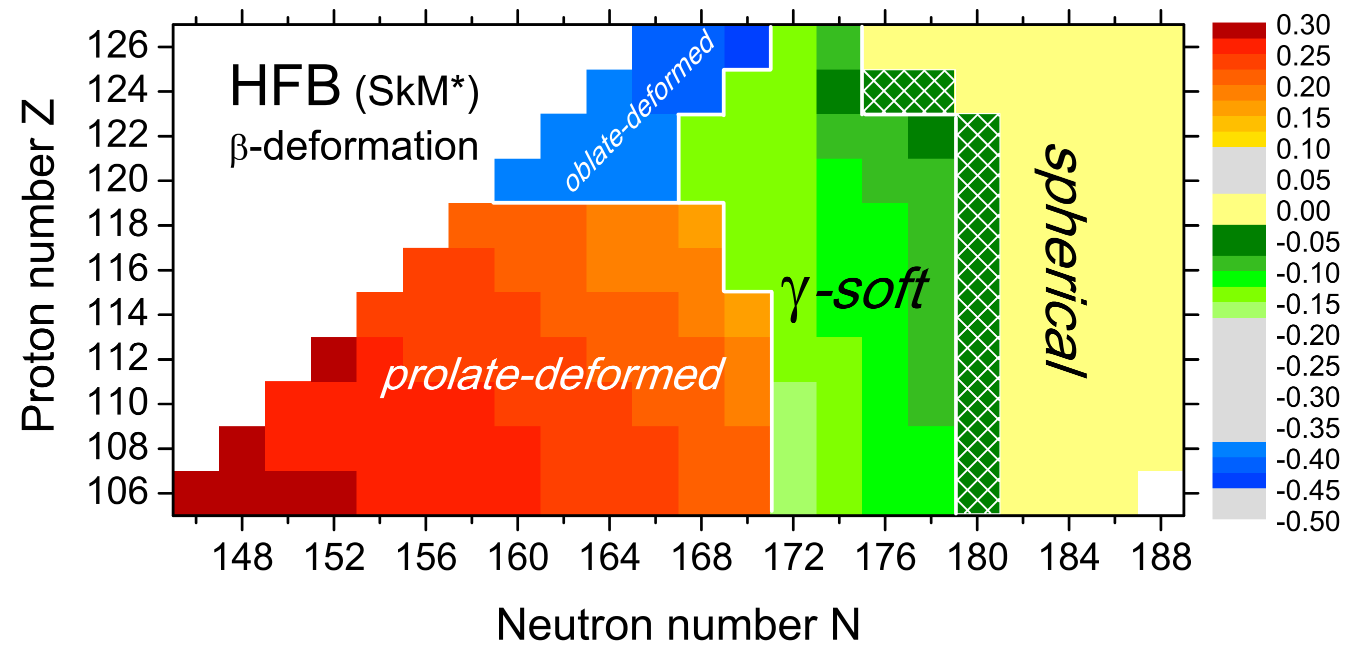
EDFs and the corresponding structure models are applied to a region far from those in which their parameters are determined by data ➠ large uncertainty in model predictions?

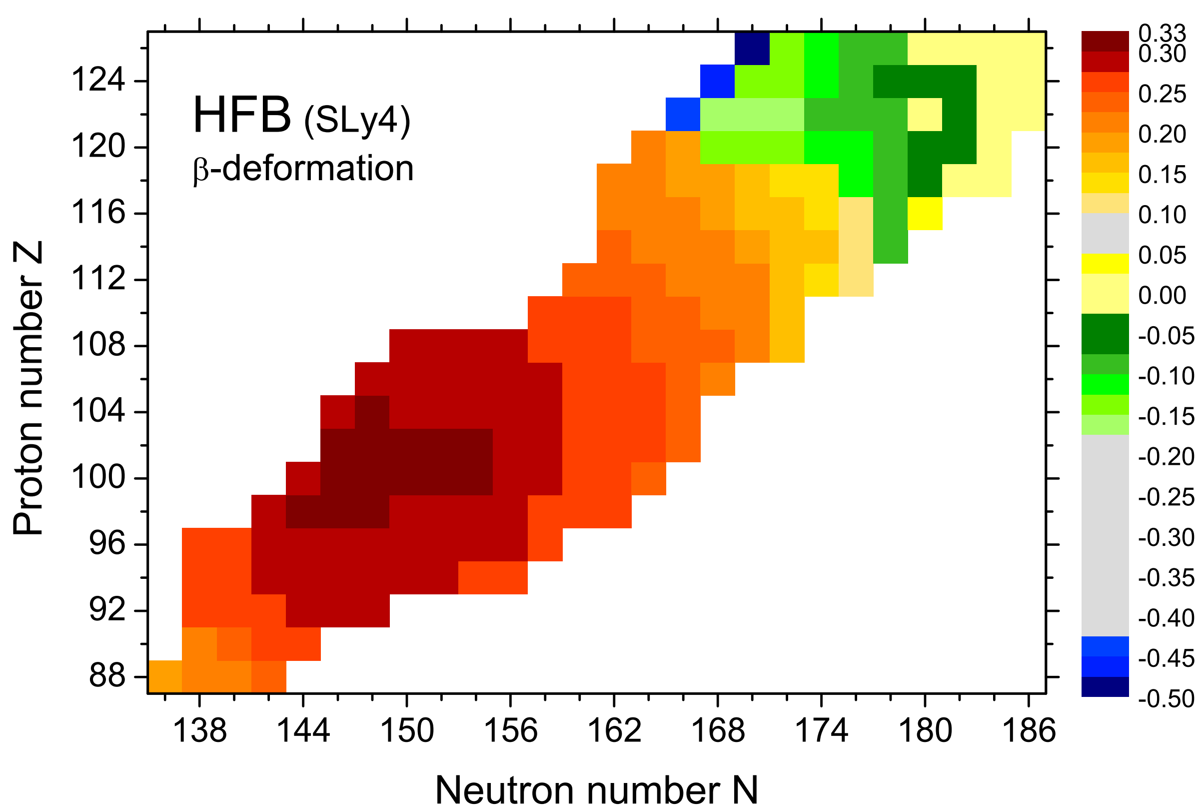
Much higher density of single-particle states close to the Fermi energy ➠ the evolution of deformed shells with nucleon number will have a more pronounced effect on energy gaps, separation energies, Q_α-values, band-heads in odd-A nuclei, K-isomers …

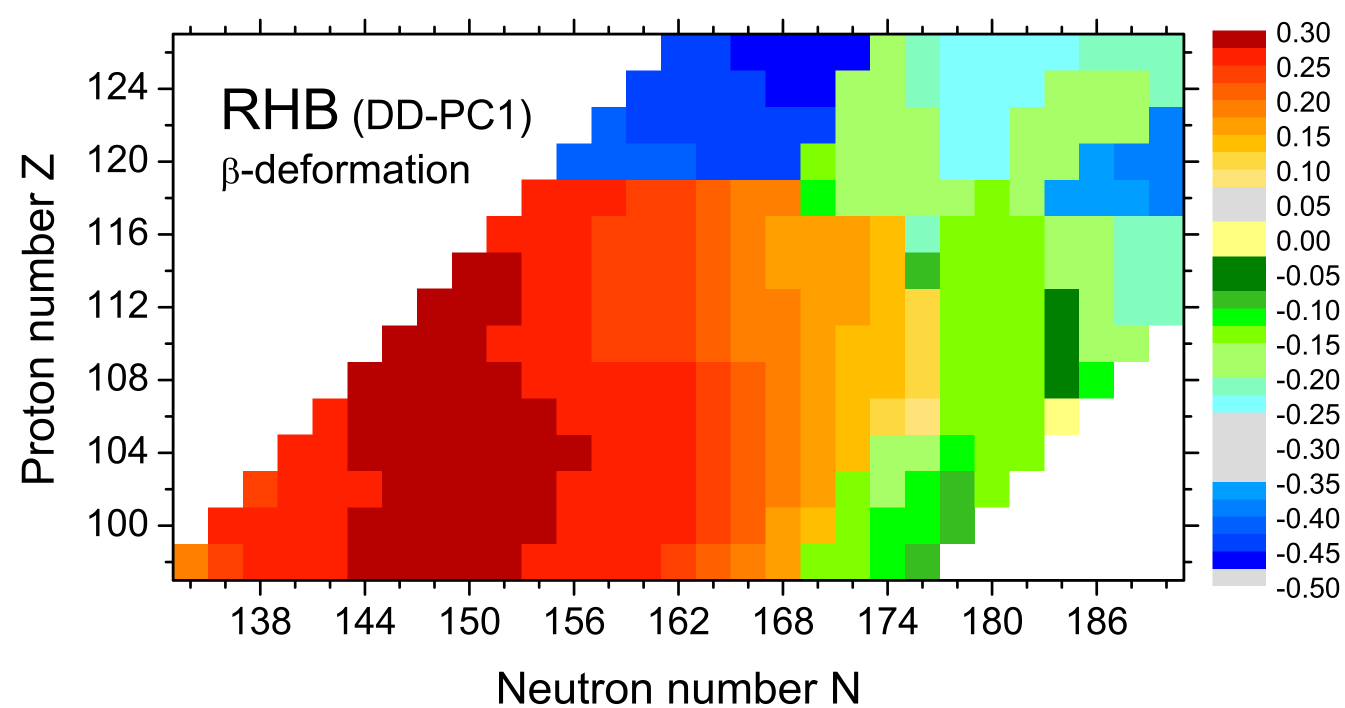
Much stronger competition between the attractive short-range nuclear interaction and the long-range electrostatic repulsion ➠ impact on the Coulomb, surface and isovector energies! Shape transitions! Exotic shapes!

Equilibrium quadrupole deformation parameters β_{20} of even–even SH nuclei









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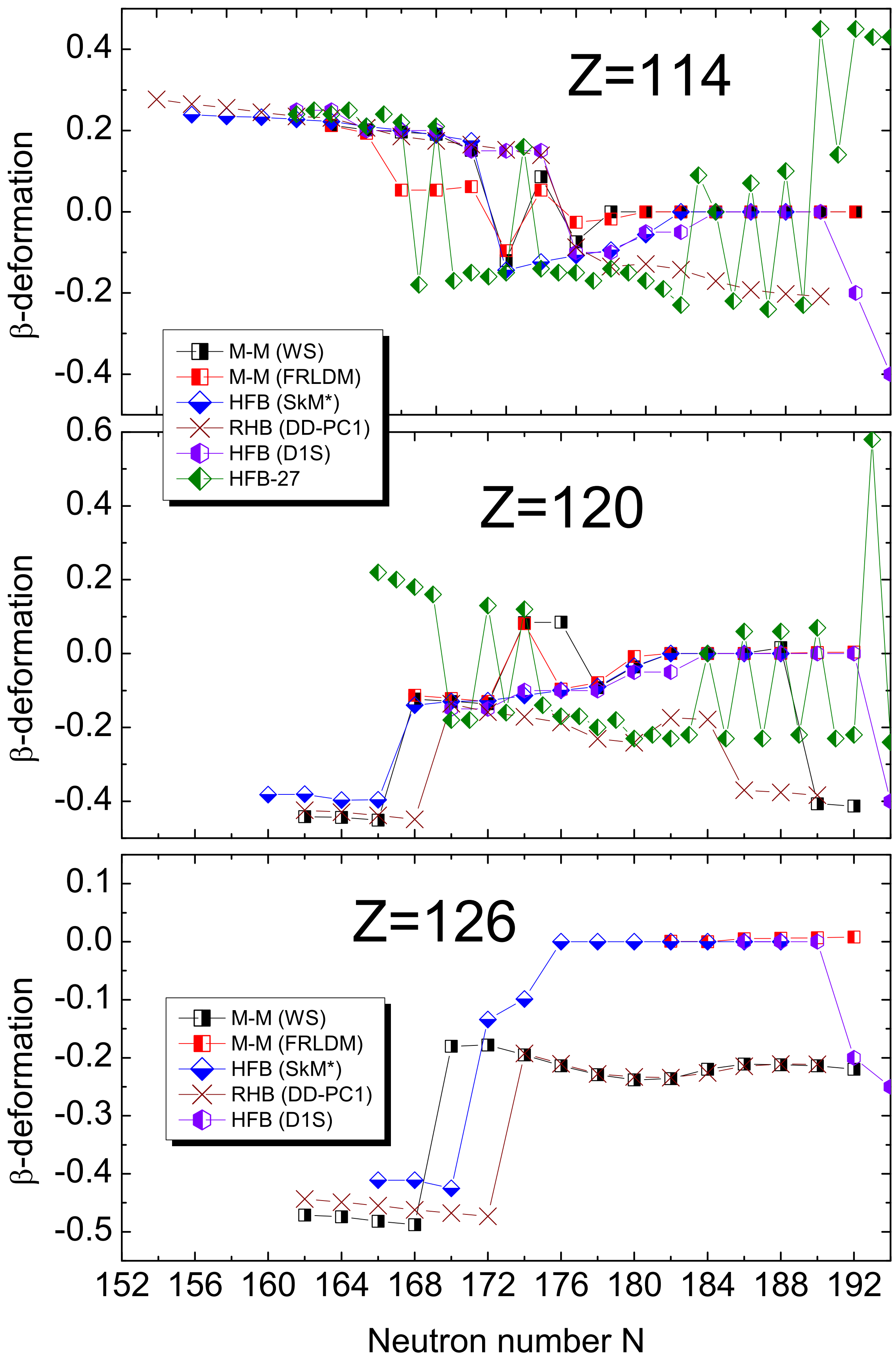
Evolution of shapes



Equilibrium quadrupole deformation parameters β_{20} for the Z = 114, 120 and 126 isotopic chains: macro–micro and mean-field models.

Importance of collective correlations that arise from restoration of broken symmetries and fluctuations of collective variables!

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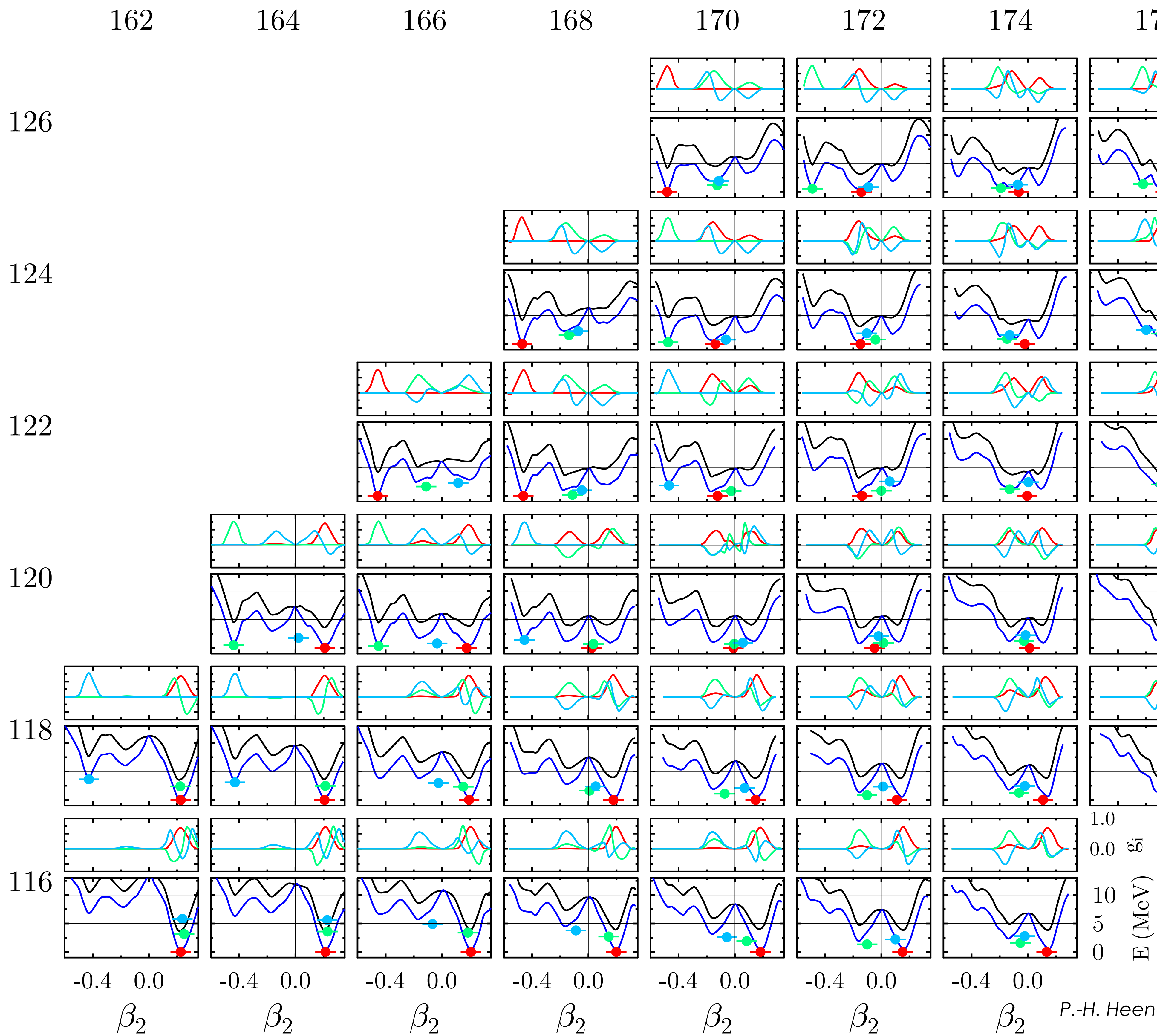
Deformation energy curves (SLy4 EDF): projection on particle numbers only (black), and projection on angular momentum I = 0 (blue). Collective wave function, energy and mean deformation of the three lowest 0^+ states.



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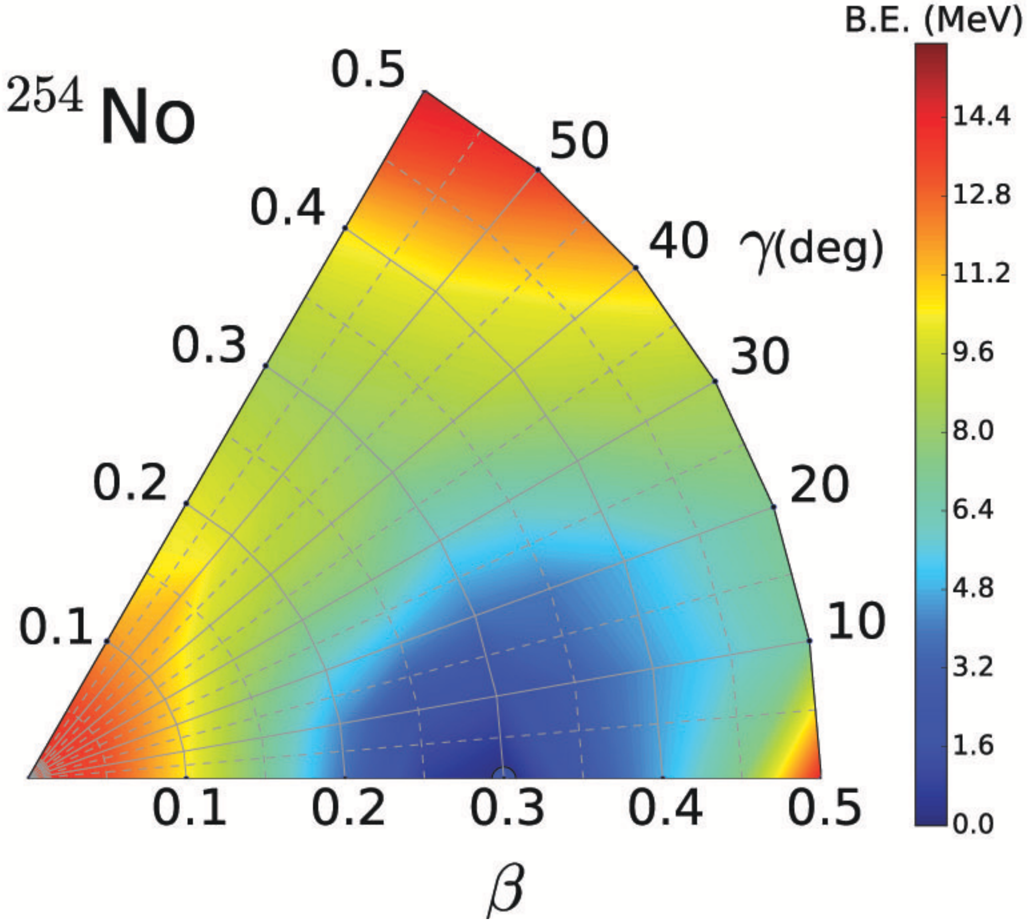
Collective Hamiltonian

Prog. Part. Nucl. Phys. **66**, 519 (2011).
Phys. Rev. C **79**, 034303 (2009).



... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom:

$$H_{\text{coll}} = \mathcal{T}_{\text{vib}}(\beta,\gamma) + \mathcal{T}_{\text{rot}}(\beta,\gamma,\Omega) + \mathcal{V}_{\text{coll}}(\beta,\gamma)$$

$$\mathcal{T}_{\text{vib}} = \frac{1}{2}B_{\beta\beta}\dot{\beta}^2 + \beta B_{\beta\gamma}\dot{\beta}\dot{\gamma} + \frac{1}{2}\beta^2 B_{\gamma\gamma}\dot{\gamma}^2$$

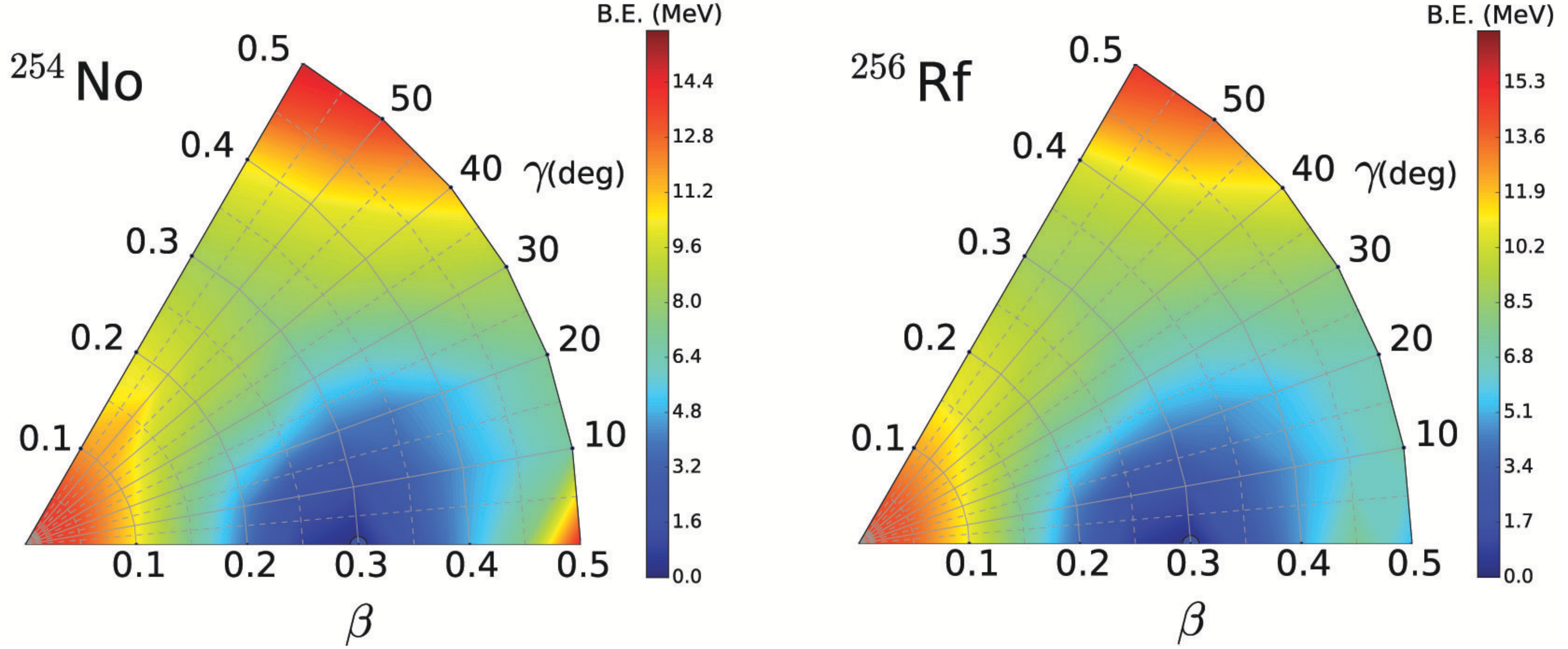
$$\mathcal{T}_{\text{rot}} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{I}_k\omega_k^2$$

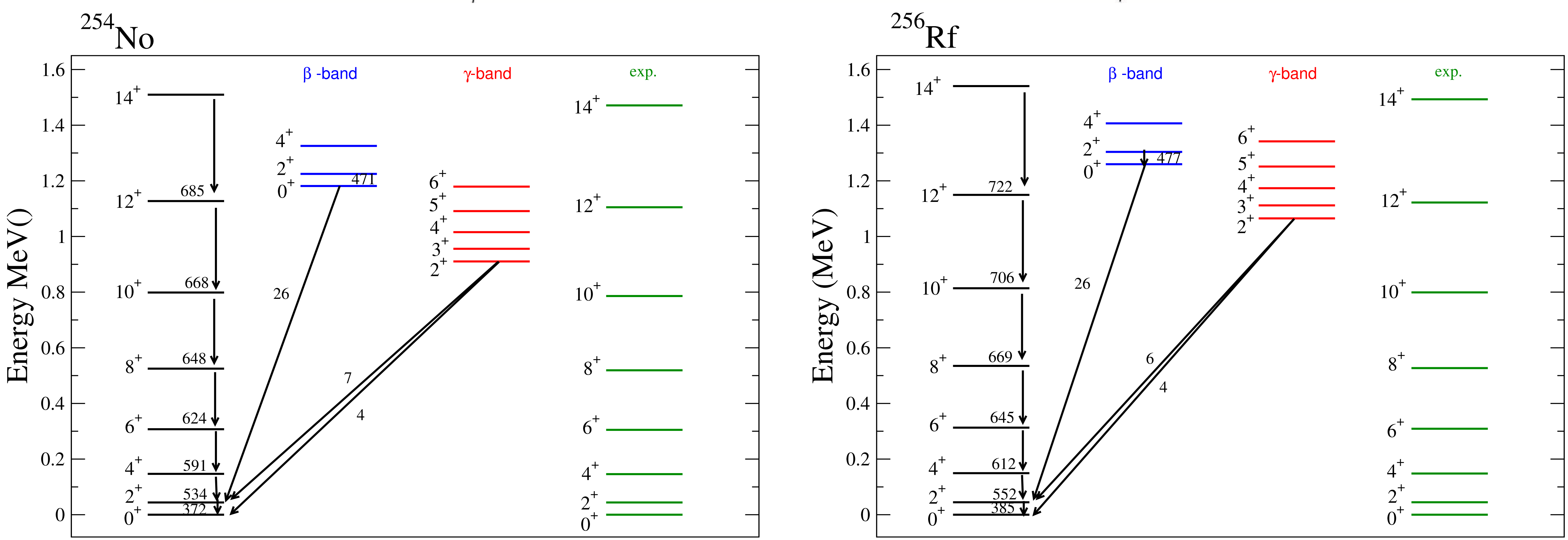
The dynamics of the collective Hamiltonian is determined by: the self-consistent collective potential, the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the three moments of inertia I_k, functions of the intrinsic deformations β and γ.

... collective eigenfunction:

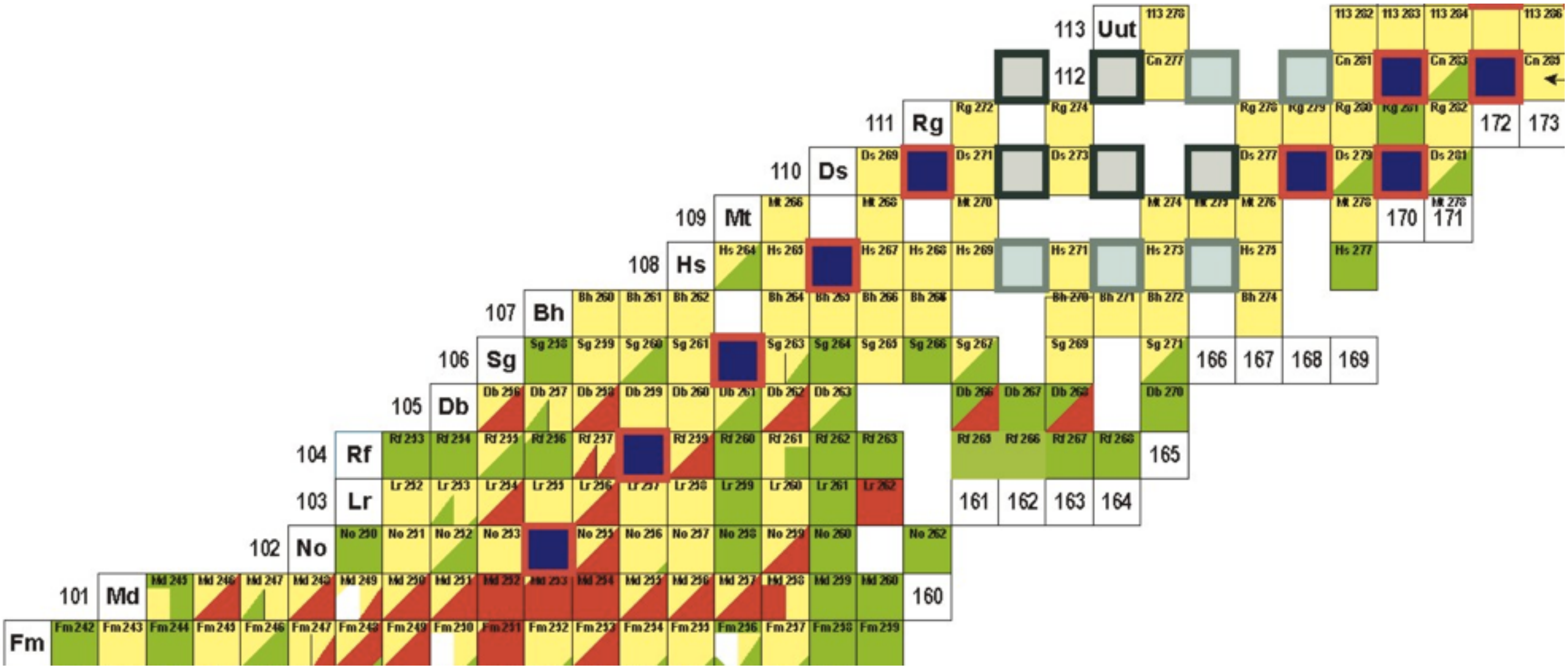
$$\Psi_{\alpha}^{IM}(\beta,\gamma,\Omega) = \sum_{K\in\Delta I}\psi_{\alpha K}^{I}(\beta,\gamma)\Phi_{MK}^{I}(\Omega)$$

Self-consistent RHB triaxial energy maps of ^{254}No and ^{256}Rf isotopes in the β–γ plane (0 ≤ γ ≤ 60°). DD-PC1 energy density functional and a separable pairing force of finite range.

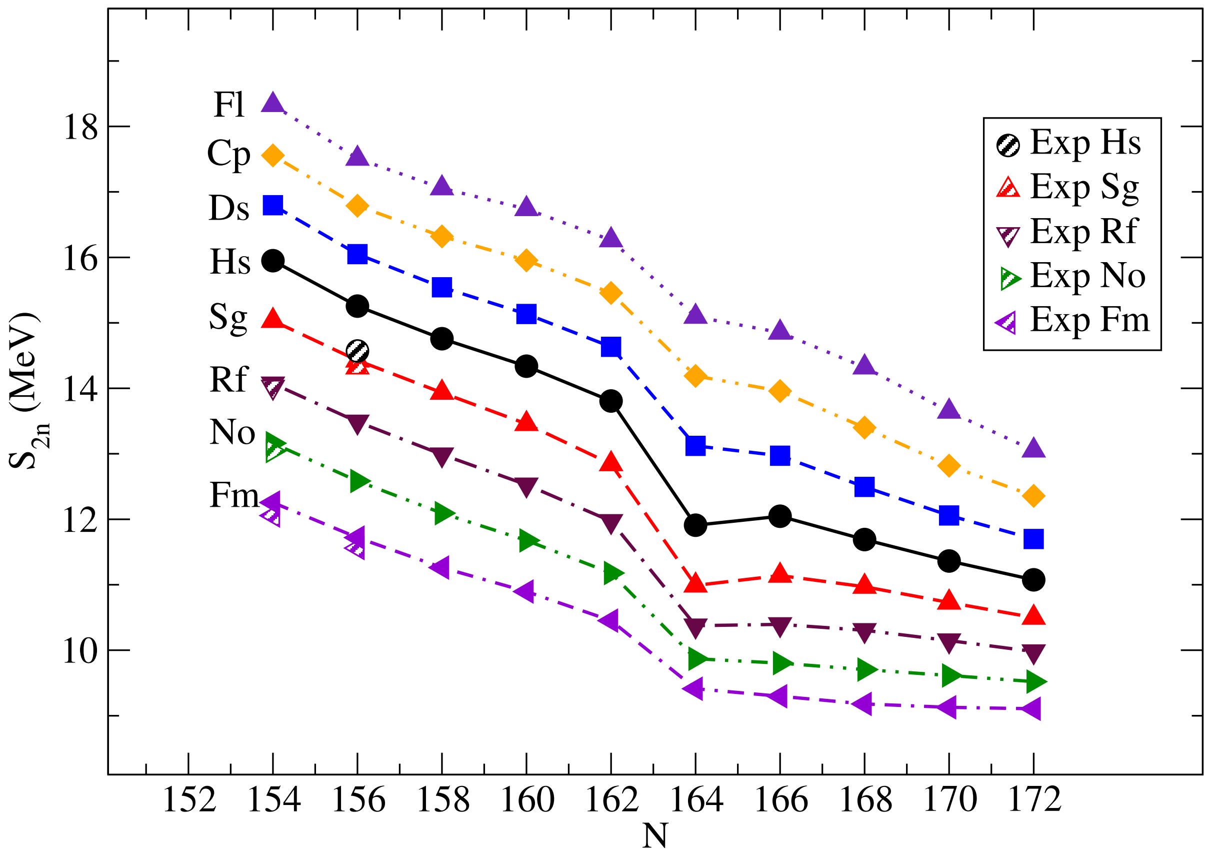




Transactinides

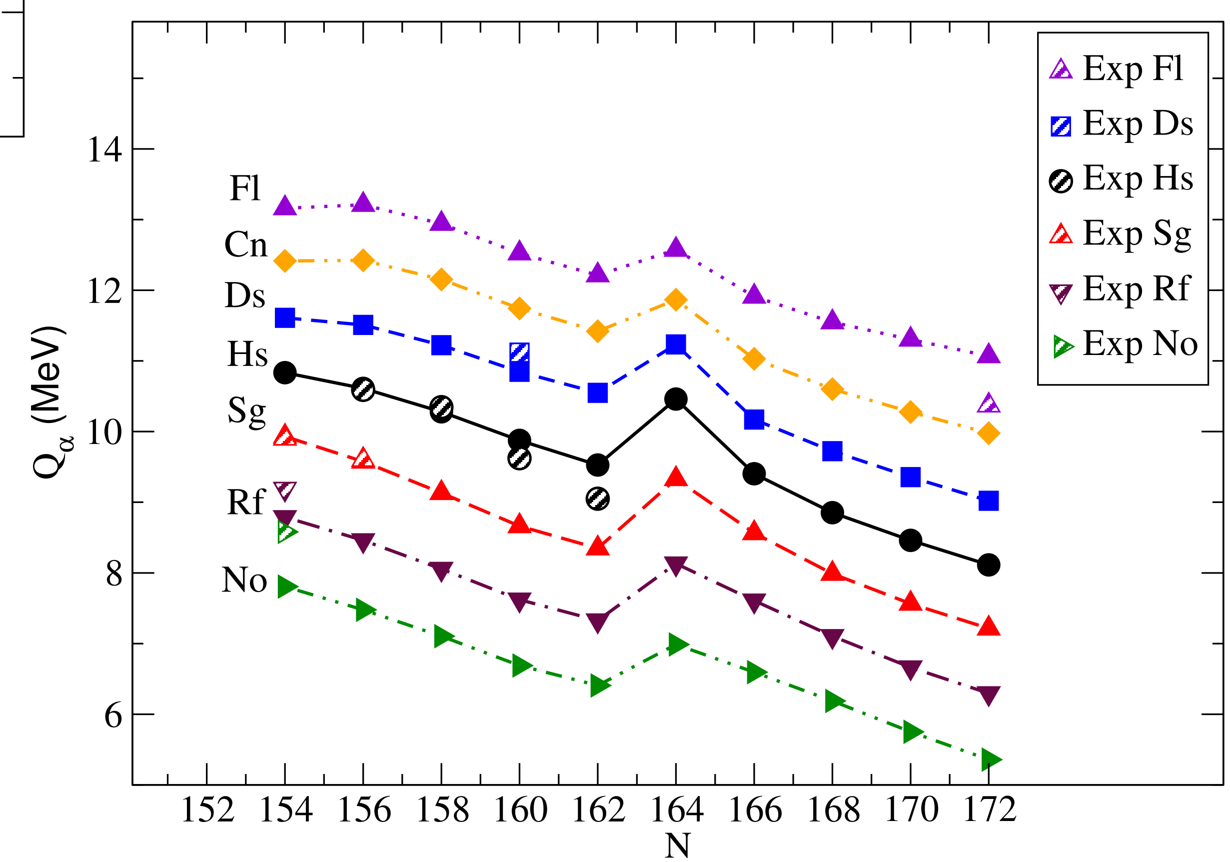


Transactinides



2n separation energies

Q_α values



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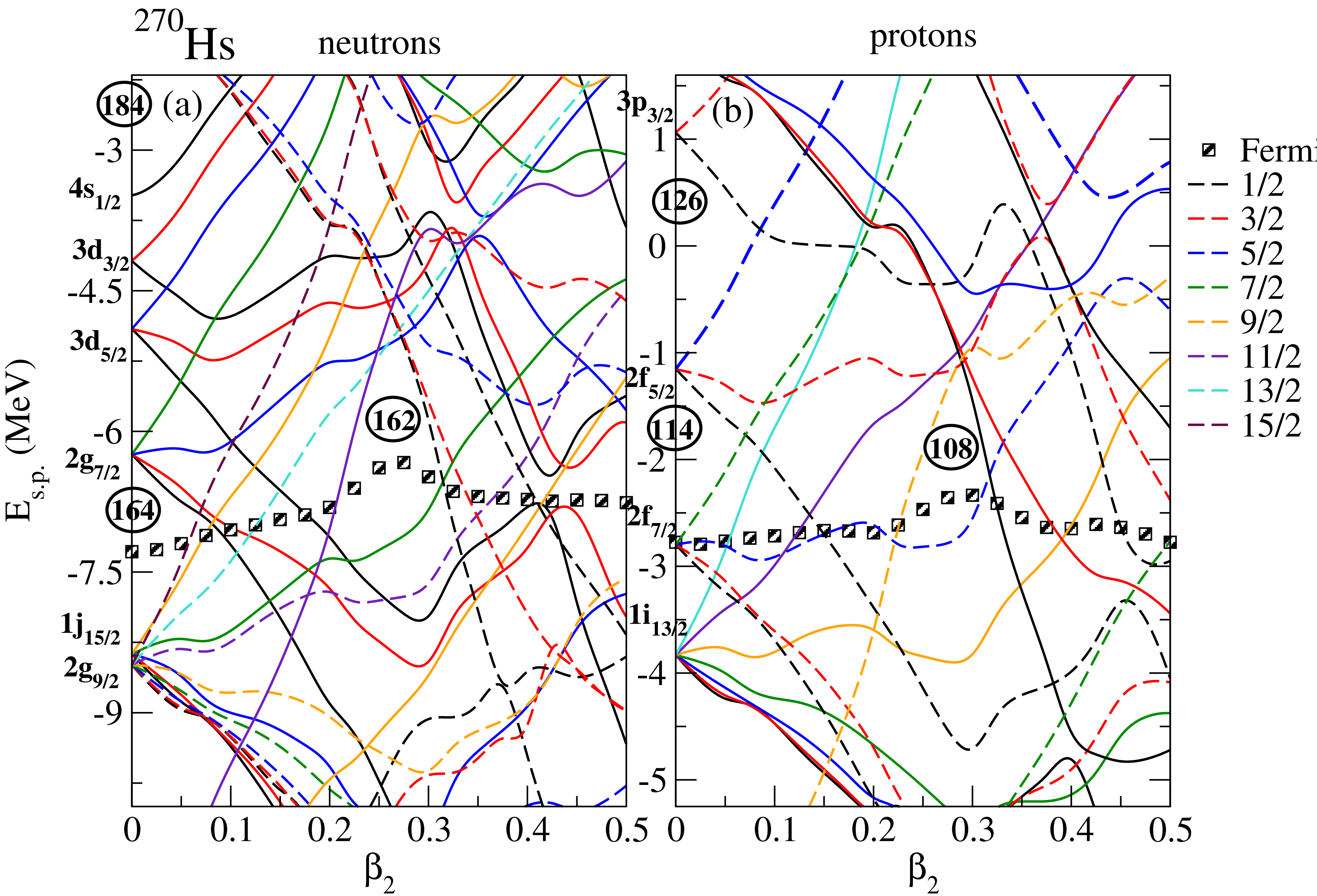
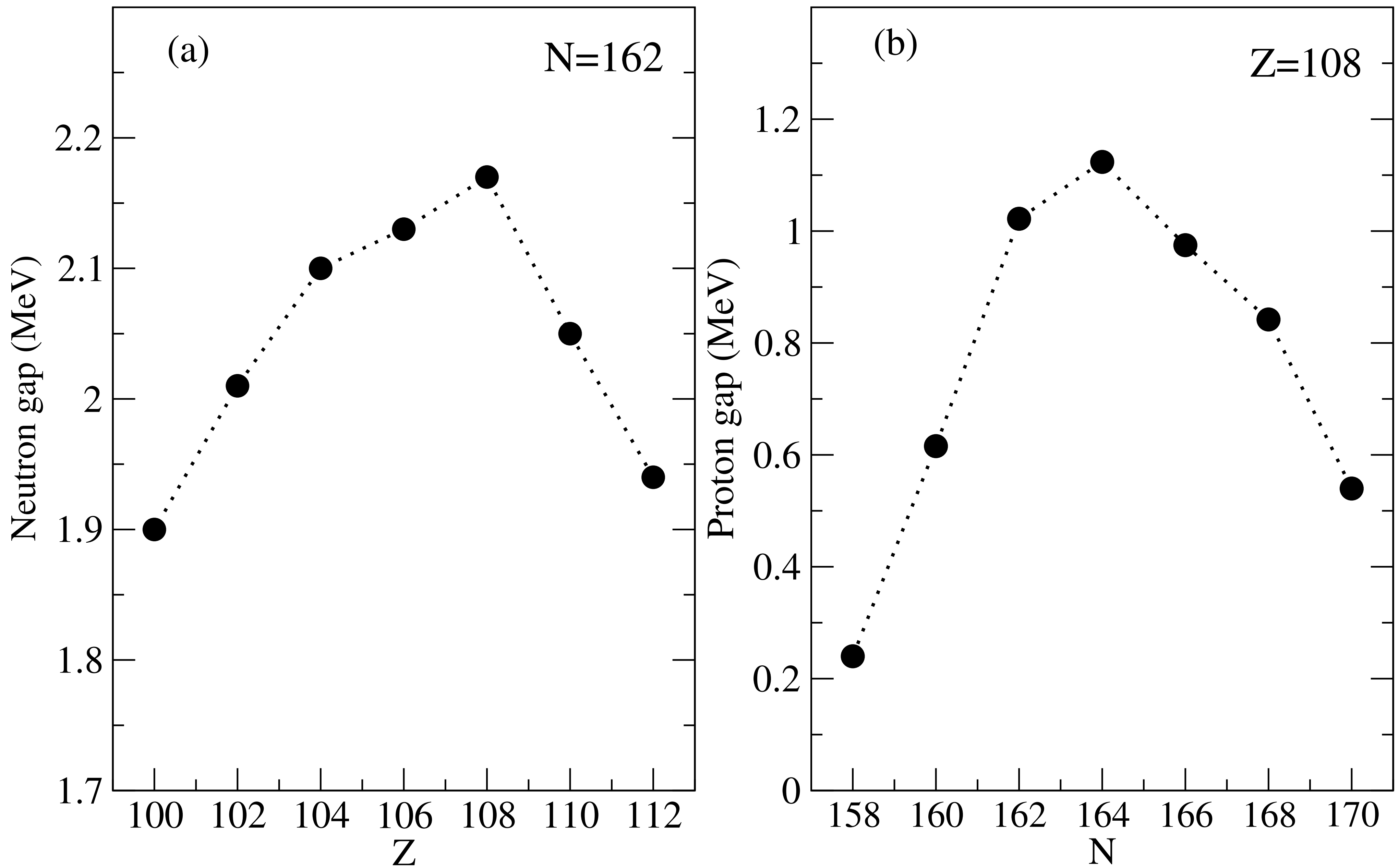
Energy gaps are small! Shape stabilization depends on how fast the shell structures vary with deformation!

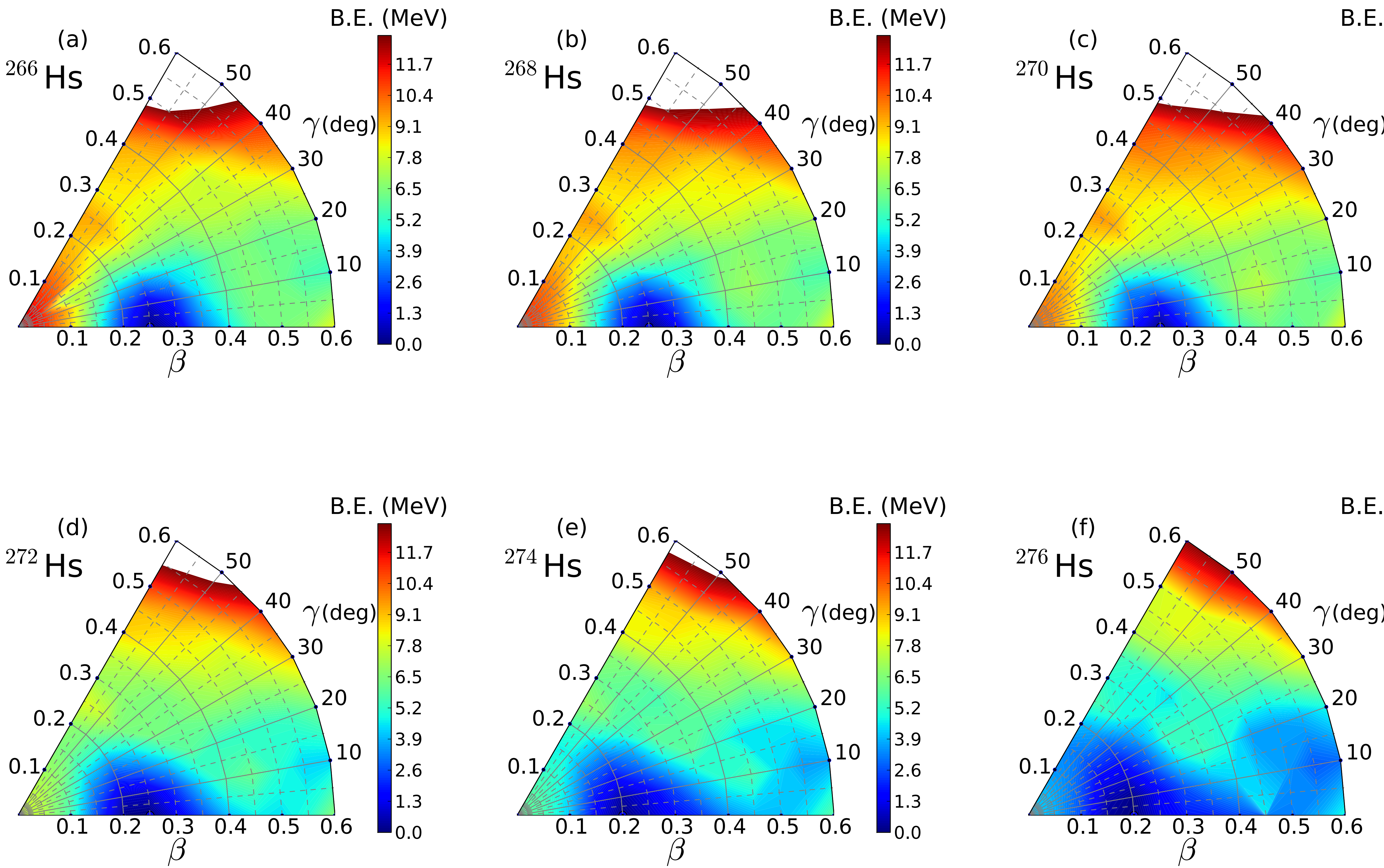


Neutron and proton shell gaps

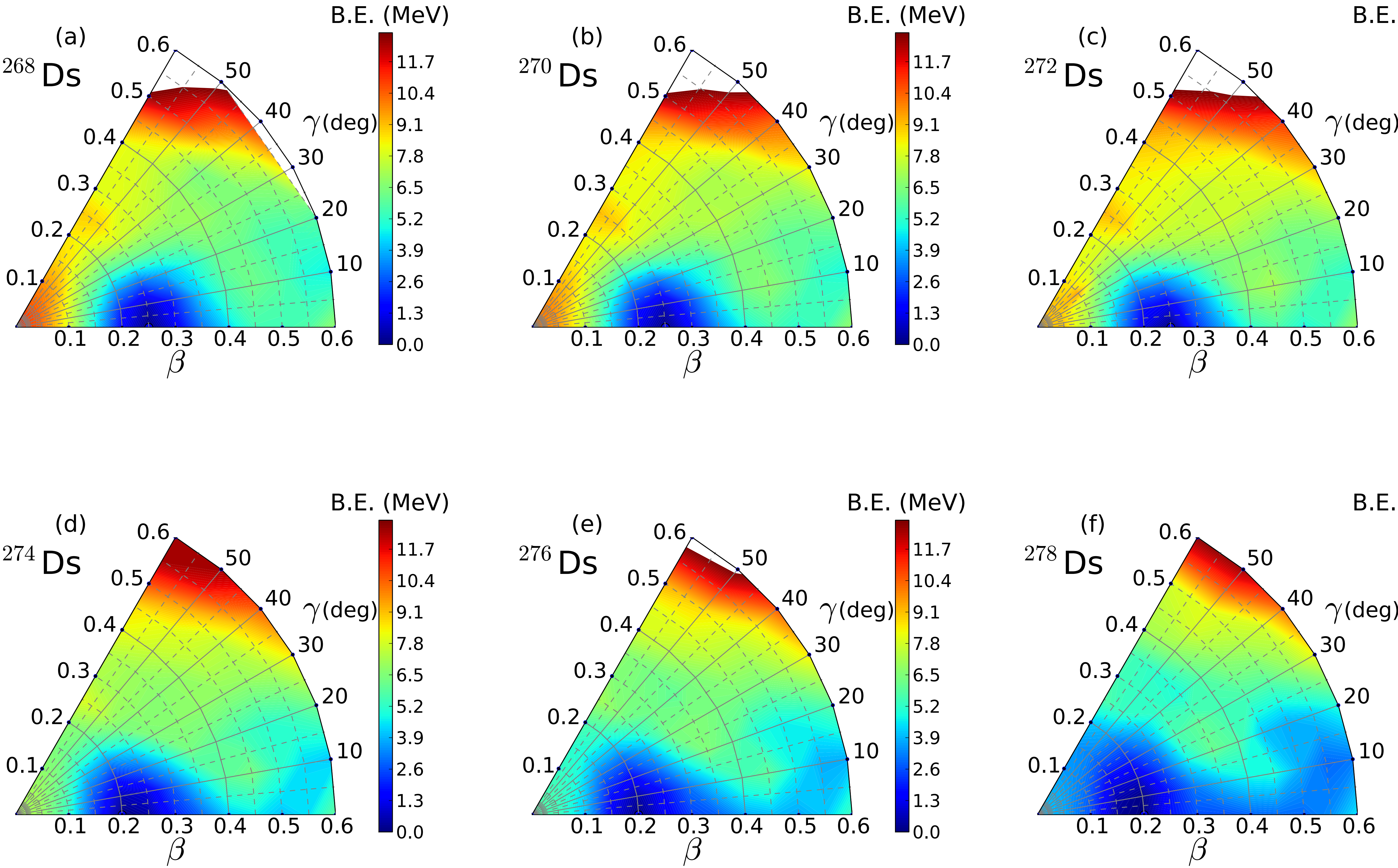


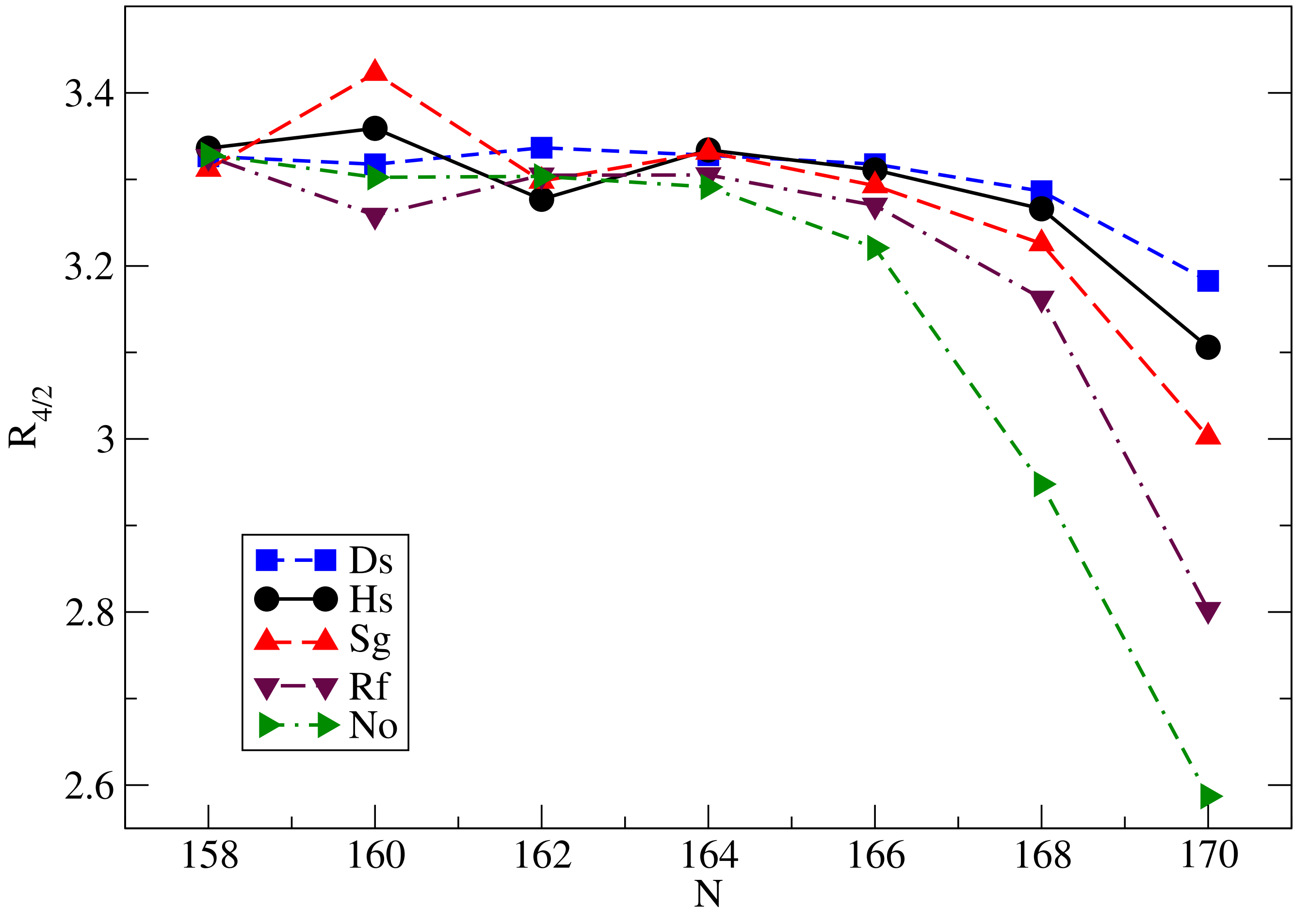
^{270}Hs ➠ deformed “doubly magic” nucleus

Triaxial deformation energy maps

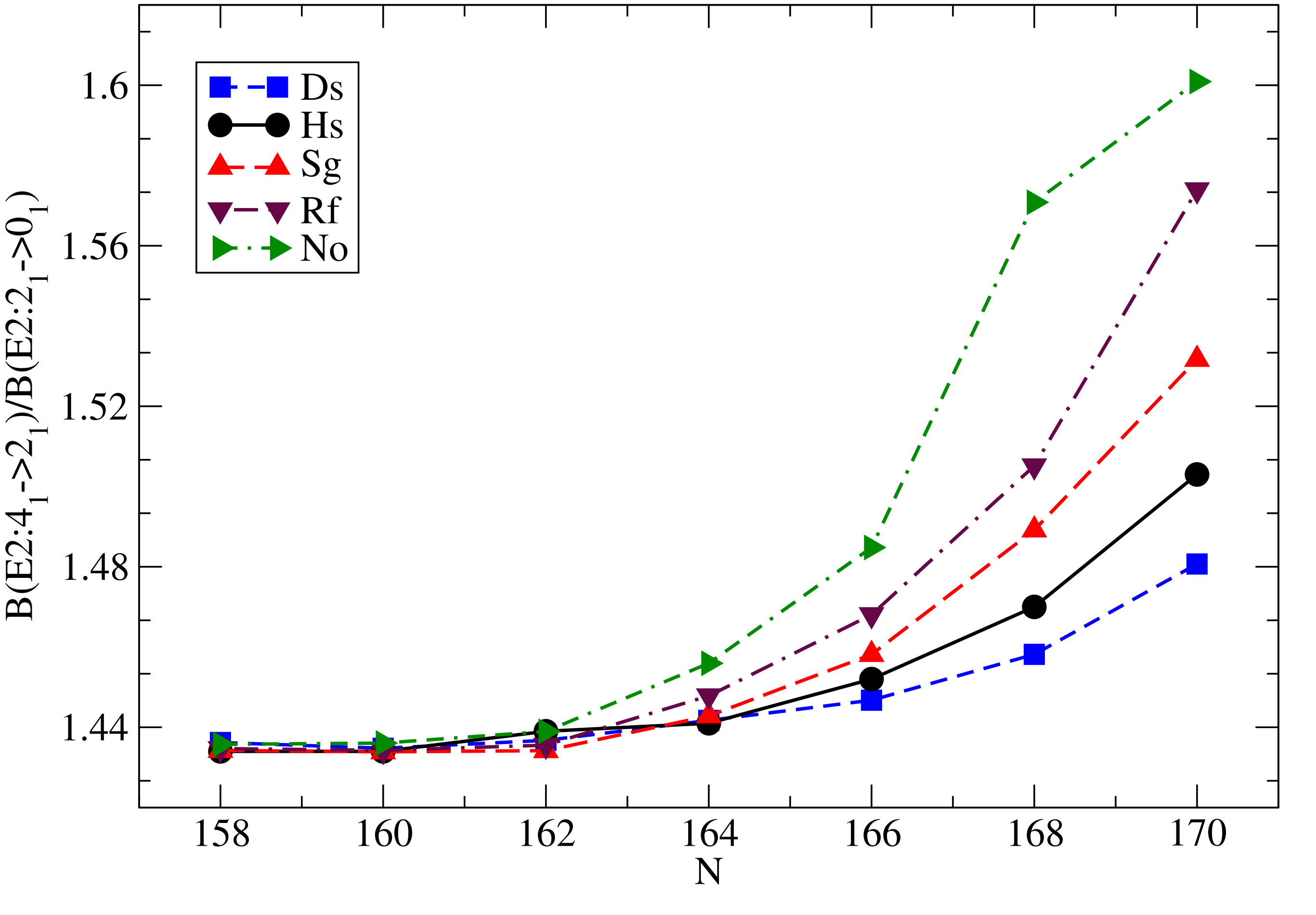


Triaxial deformation energy maps



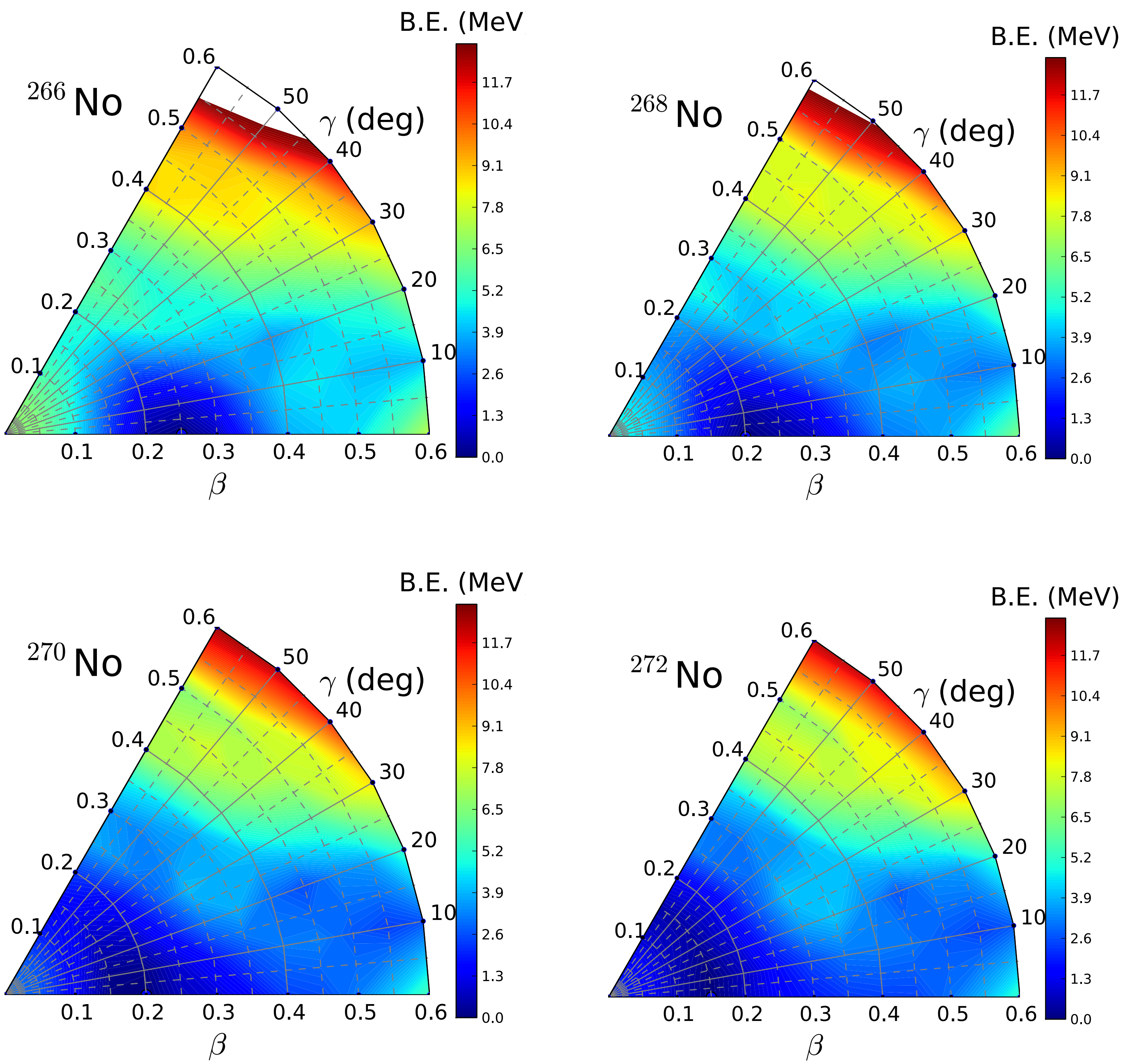


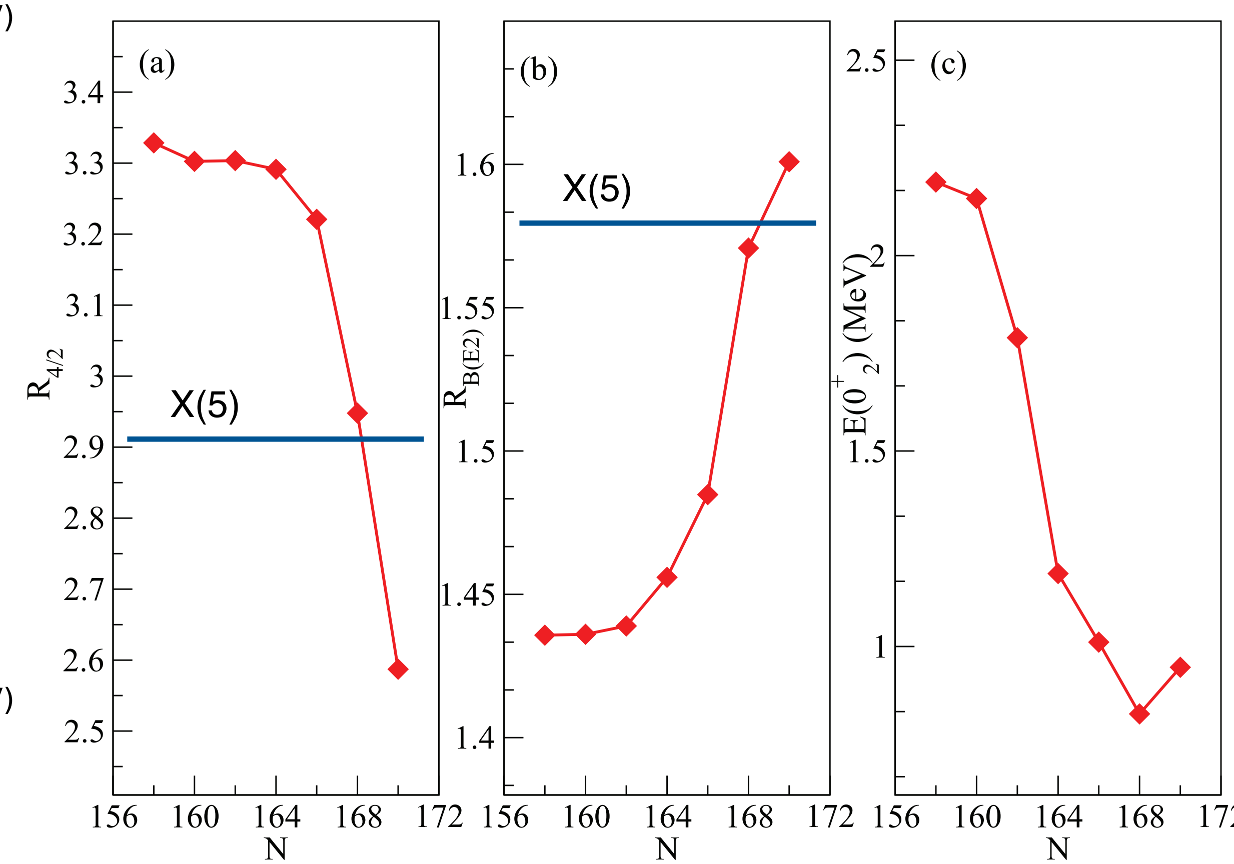
The ratio R4/2 of excitation energies of the yrast states 4^+_1 and 2^+_1 as a function of the neutron number.



The ratio $B(E2; 4^+_1 \rightarrow 2^+_1) / B(E2; 2^+_1 \rightarrow 0^+_1)$ as a function of the neutron number.

Shape-phase transitions and critical-point phenomena in the region of superheavy nuclei



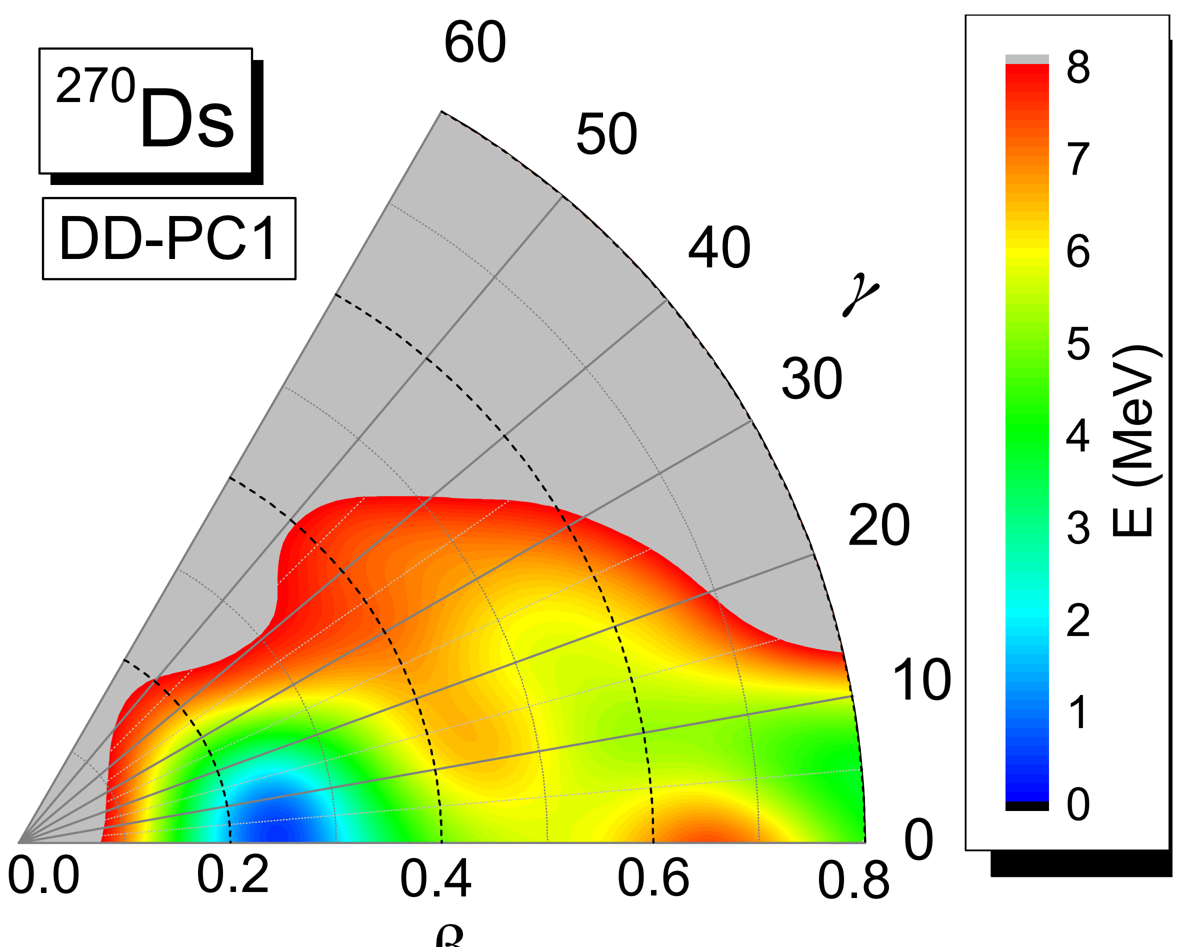


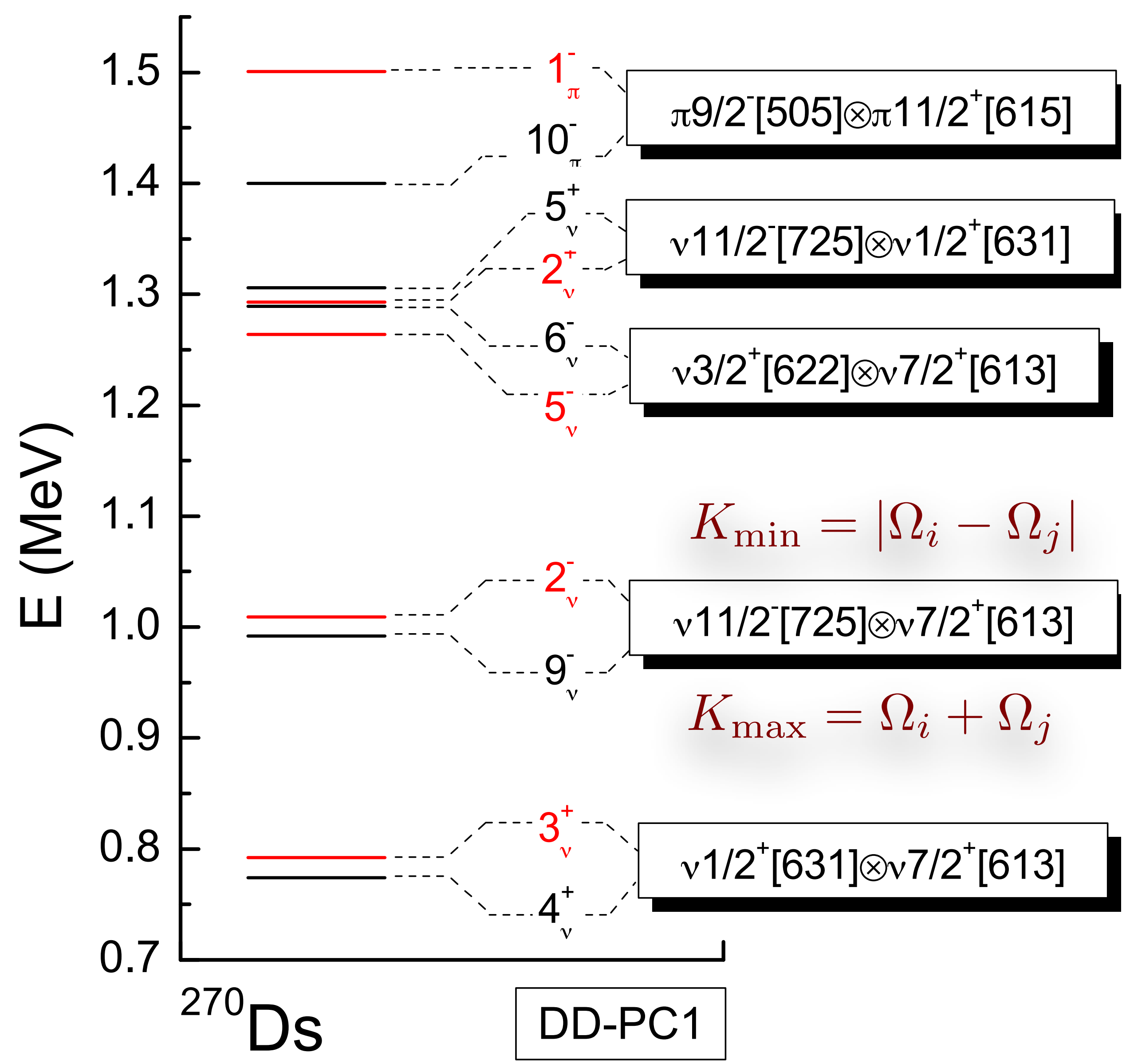
The ratio R4/2 of excitation energies of the levels $4^+{}_1$ and $2^+{}_1$ (a), the ratio of reduced transition probabilities R = B(E2; $4^+{}_1 \rightarrow 2^+{}_1$)/ B(E2; $2^+{}_1 \rightarrow 0^+{}_1$) (b), and the excitation energy of the level $0^+{}_2$ (c), in No isotopes as functions of the number of neutrons.

Two-quasiparticle isomers

Axially deformed nuclei ➠ two-quasiparticle K-isomers

K-forbidden transitions ➠ information on the single-nucleon states, pairing gaps, and residual interactions.





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High-excitation energy of K-isomers ➠ evidence for an axially deformed shell-closure at N=162

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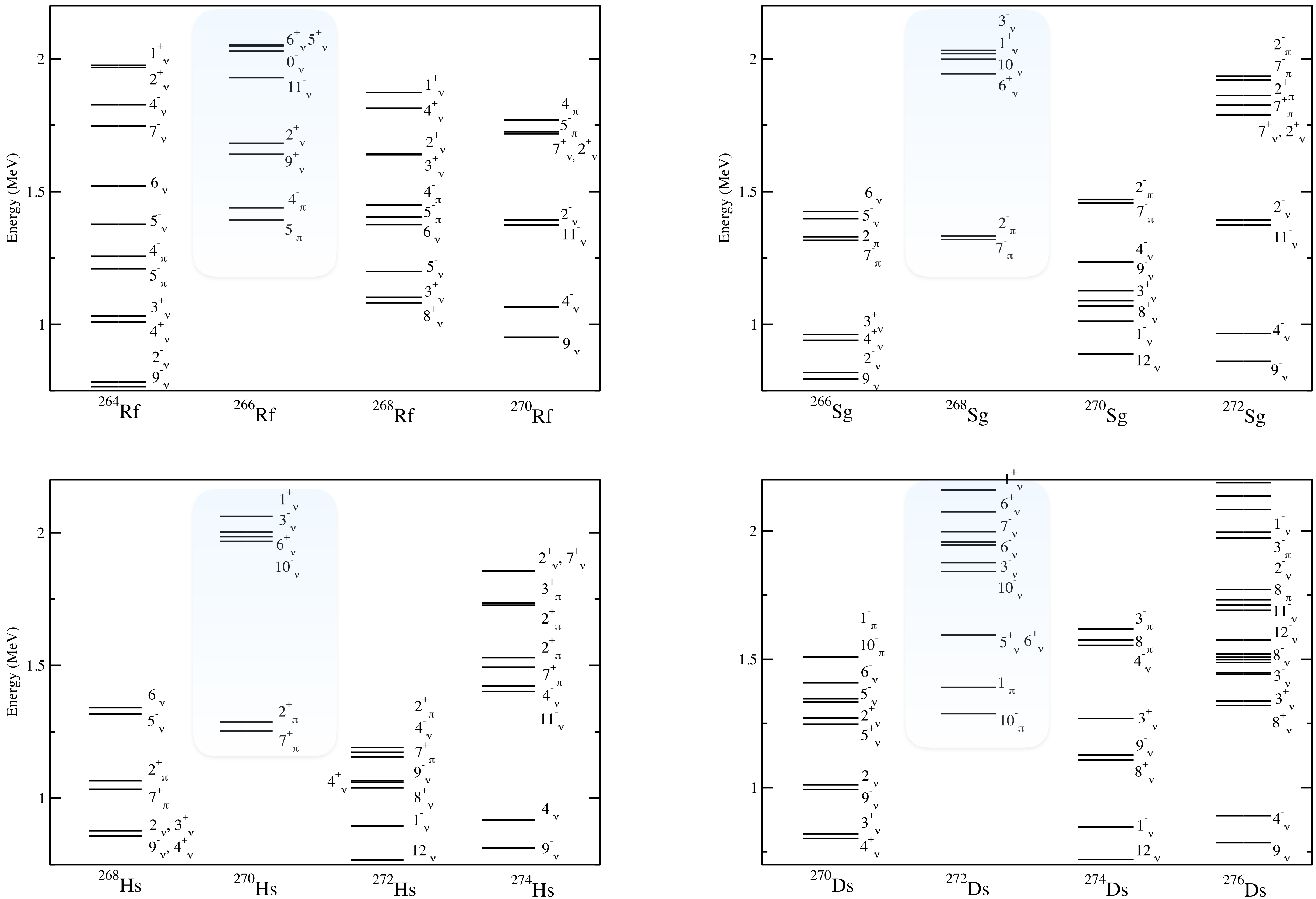
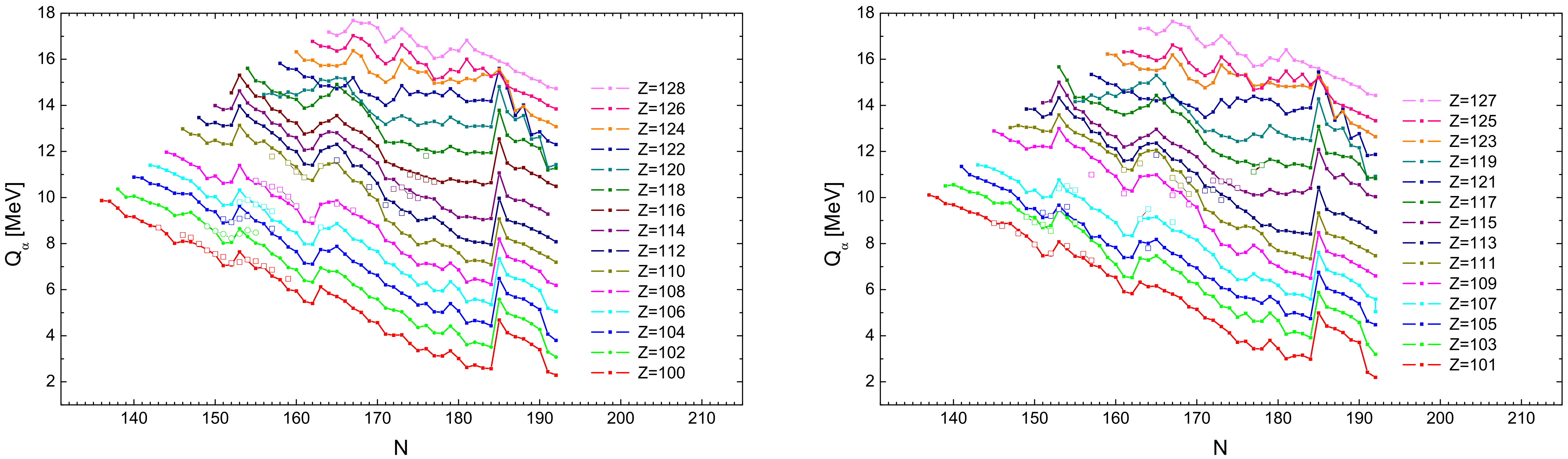


α-decay

…principal decay channel of the heaviest nuclei:

$$Q_\alpha(Z,N) = M(Z,N) - M(Z-2,N-2) - M(2,2)$$
$$= B(Z-2,N-2) - B(Z,N) + B(2,2),$$

Macro–micro Woods–Saxon (WS)



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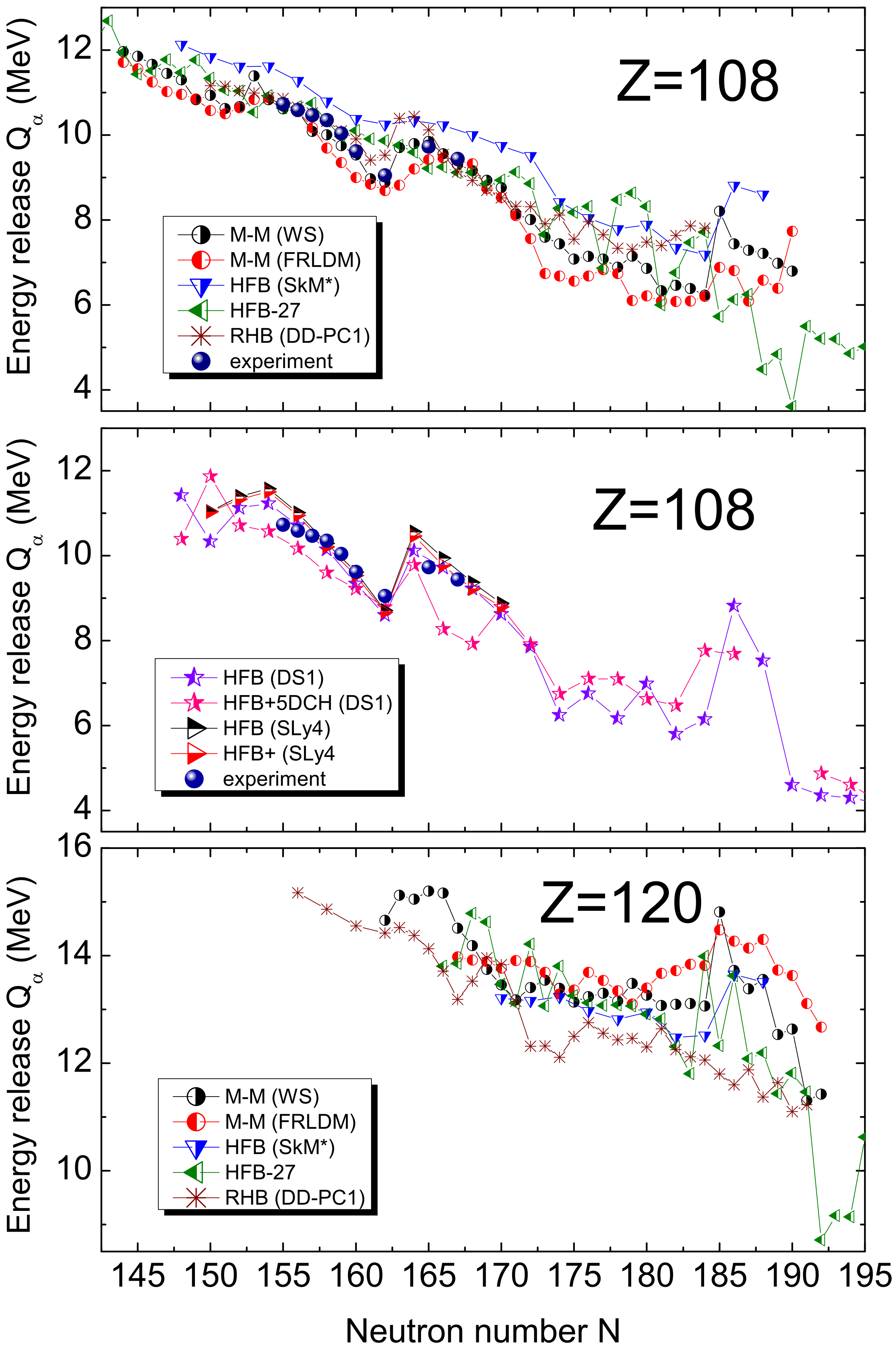

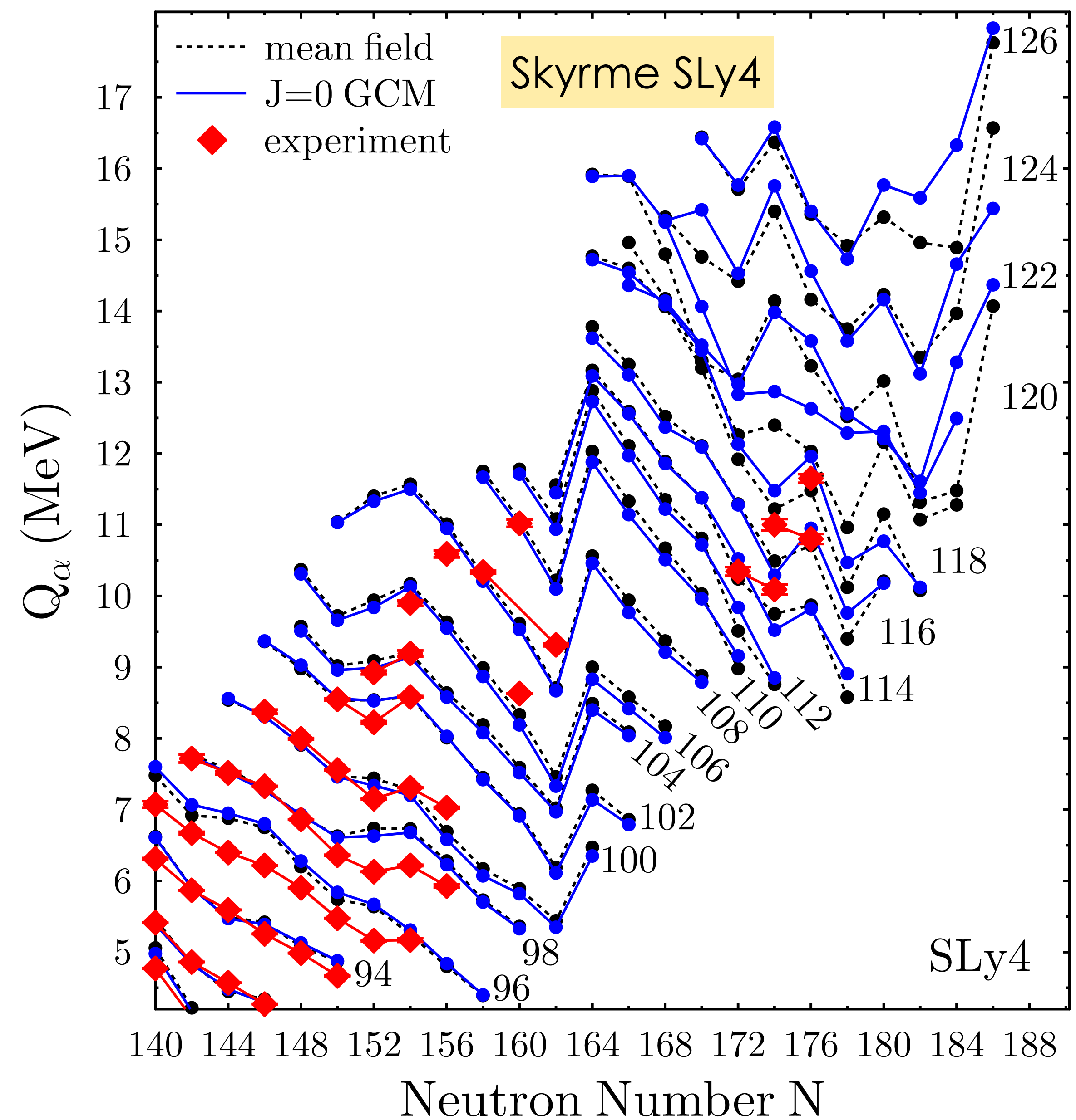

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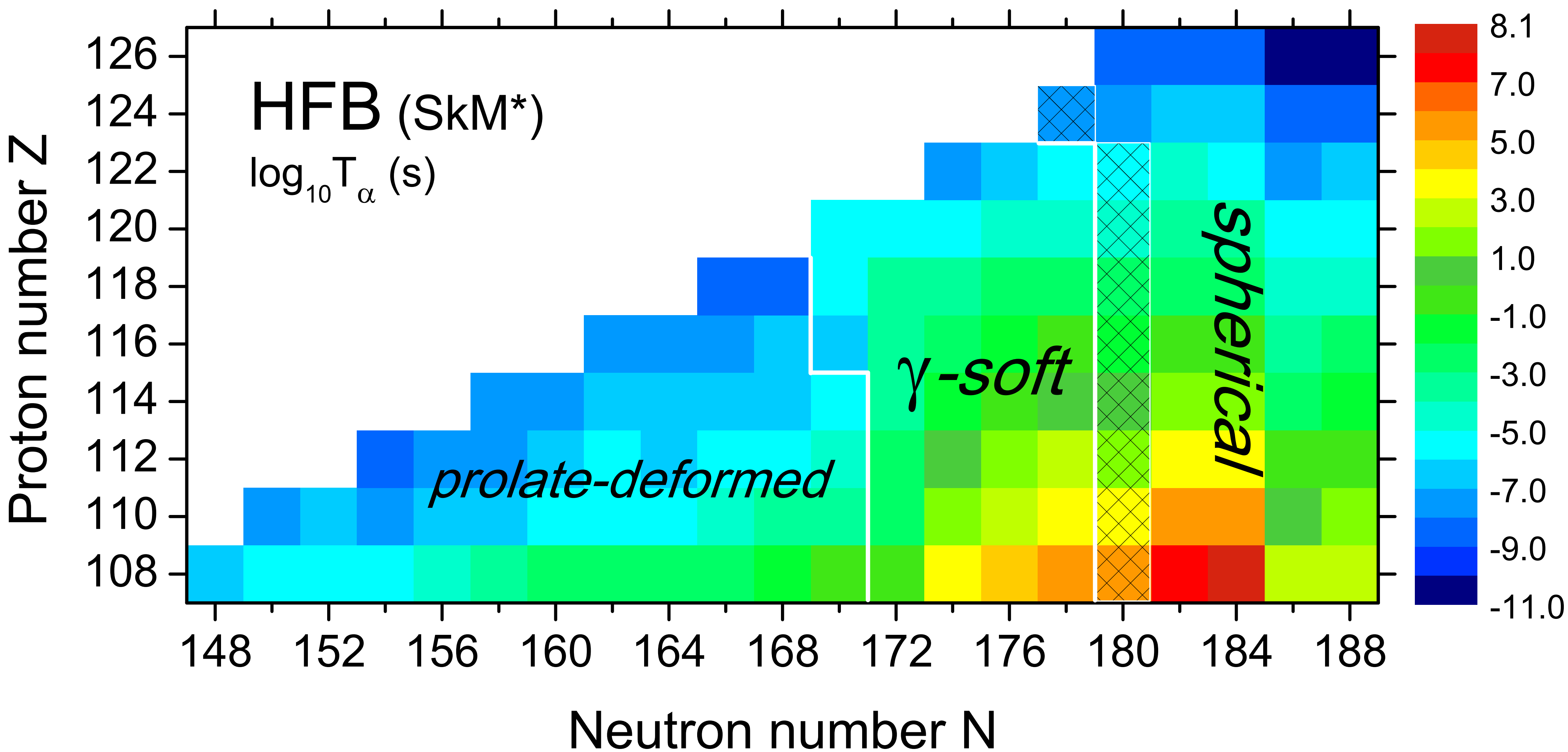
$\log_{10}$ Tα values calculated for even–even SH nuclei from the HFB SkM* Qα values

$$\log_{10} T_{\alpha}^{\mathrm{th}}(Z, N) = aZ\,[Q_{\alpha}(Z, N)]^{-1/2} + bZ + c,$$

$$a = 1.5372, \quad b = -0.1607, \quad c = -36.573$$



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Theoretical predictions for the nucleus $^{296}118$

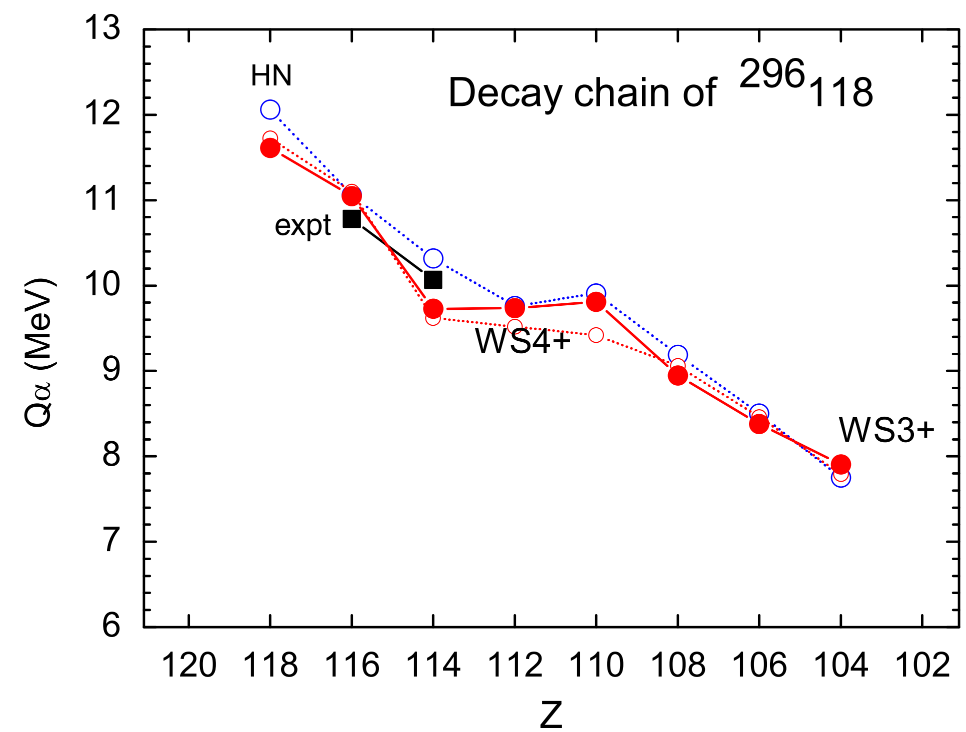
A. SOBICZEWSKI
PHYSICAL REVIEW C **94**, 051302(R) (2016)

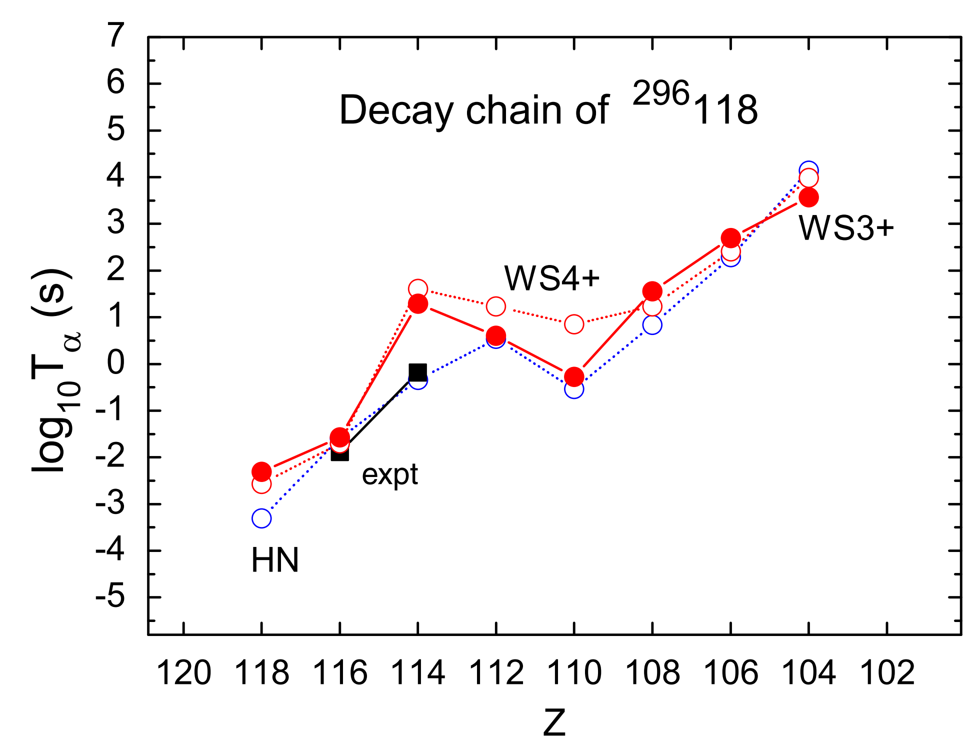
TABLE I. Rms (in keV) of the discrepancies between measured and calculated masses. The latter are obtained with the use of the indicated models for the regions of global ($Z, N \geqslant 8$), heavy ($Z \geqslant 82$, $N \geqslant 126$) and very heavy ($Z \geqslant 100$) nuclei. The year of publication of each model, as well as the number of nuclei with measured masses in each region, N_{nucl}, are also specified.

Model Year	FRDM 1995	DZ 1995	INM 2012	WS3+ 2010	WS4+ 2014	HN 2001	N_{nucl}
$Z, N \geqslant 8$	654	394	362	248	170		2353
$Z \geqslant 82$, $N \geqslant 126$	484	398	258	136	115	355	312
$Z \geqslant 100$	676	828	471	126	130	118	36

TABLE III. Calculated and measured values of the α-decay energies Q_α (in MeV), α-decay and spontaneous-fission half-lives, T_α and T_{sf}, for the decay chain of the nucleus $^{296}118$. Some quantities derived from them are also given (see text).

Nucleus	$^{296}118$	^{292}Lv	^{288}Fl	Avg.
Q_α(WS3+)	11.62	11.05	9.73	
Q_α(WS4+)	11.73	11.10	9.62	
Q_α(HN)	12.06	11.06	10.32	
Q_α(expt)		10.78	10.07	
δQ_α(WS3+)		0.27	−0.34	0.30
δQ_α(WS4+)		0.32	−0.45	0.38
δQ_α(HN)		0.28	0.25	0.26
T_α(WS3+)	4.8 ms	27 ms	19 s	
T_α(WS4+)	2.7 ms	20 ms	41 s	
T_α(HN)	0.50 ms	25 ms	0.45 s	16
f(WS3+)		2.1	29	32
f(WS4+)		1.5	62	1.7
f(HN)		1.9	1.5	
T_α^{expt}		13 ms	0.66 s	
T_{sf}^{th}	1.3×10^4 s	1.4×10^5 s	2.1×10^3 s	
T_{sf}^{expt}			0.30 s	





The half-life T_α of the nucleus $^{296}118$ is predicted to be larger than needed (around 1 μs) for its observation.

Spontaneous fission

… penetration probability: $$P = \frac{1}{1 + \exp[2S(L)]} \qquad T_{1/2} = \ln 2/(nP)$$

⇒ fission action integral: $$S(L) = \int_{s_{\text{in}}}^{s_{\text{out}}} \frac{1}{\hbar}\sqrt{2\mathcal{M}_{\text{eff}}(s)[V_{\text{eff}}(s) - E_0]}\, ds$$

The effective inertia and collective potential calculated in a SCMF approach based on EDFs.

colective coordinates

$$\mathcal{M}_{\text{eff}}(s) = \sum_{ij} \mathcal{M}_{ij} \frac{dq_i}{ds}\frac{dq_j}{ds}$$

The inertia tensor is computed using the ATDHFB method in the nonperturbative cranking approximation:

$$\mathcal{M}^C_{ij} = \frac{\hbar^2}{2\dot{q}_i\dot{q}_j} \sum_{\alpha\beta} \frac{F^{i*}_{\alpha\beta}F^{j}_{\alpha\beta} + F^{i}_{\alpha\beta}F^{j*}_{\alpha\beta}}{E_\alpha + E_\beta}$$

$$\frac{F^i}{\dot{q}_i} = U^\dagger \frac{\partial\rho}{\partial q_i} V^* + U^\dagger \frac{\partial\kappa}{\partial q_i} U^* - V^\dagger \frac{\partial\rho^*}{\partial q_i} U^* - V^\dagger \frac{\partial\kappa^*}{\partial q_i} V^*$$

Asymmetric fission of ^{250}Fm

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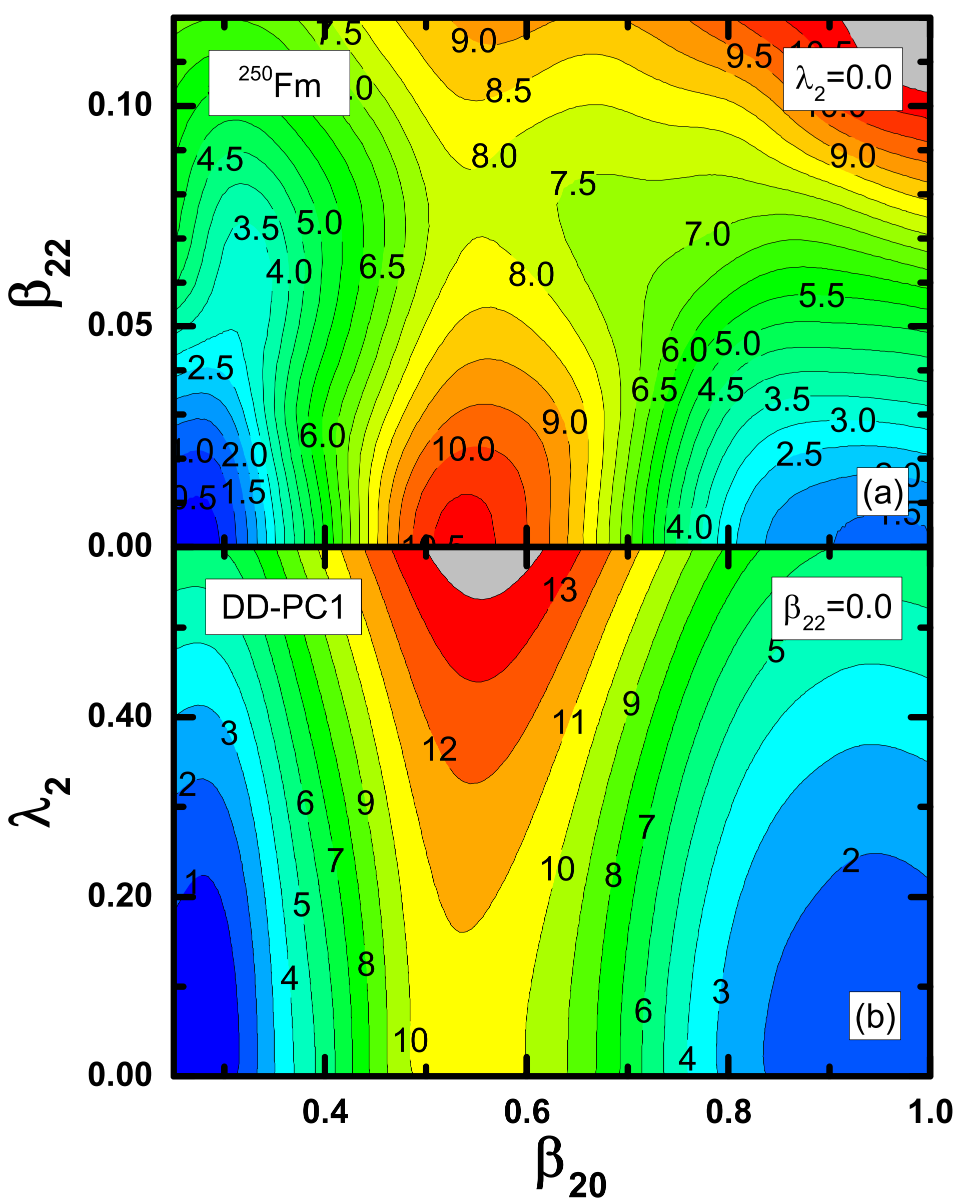
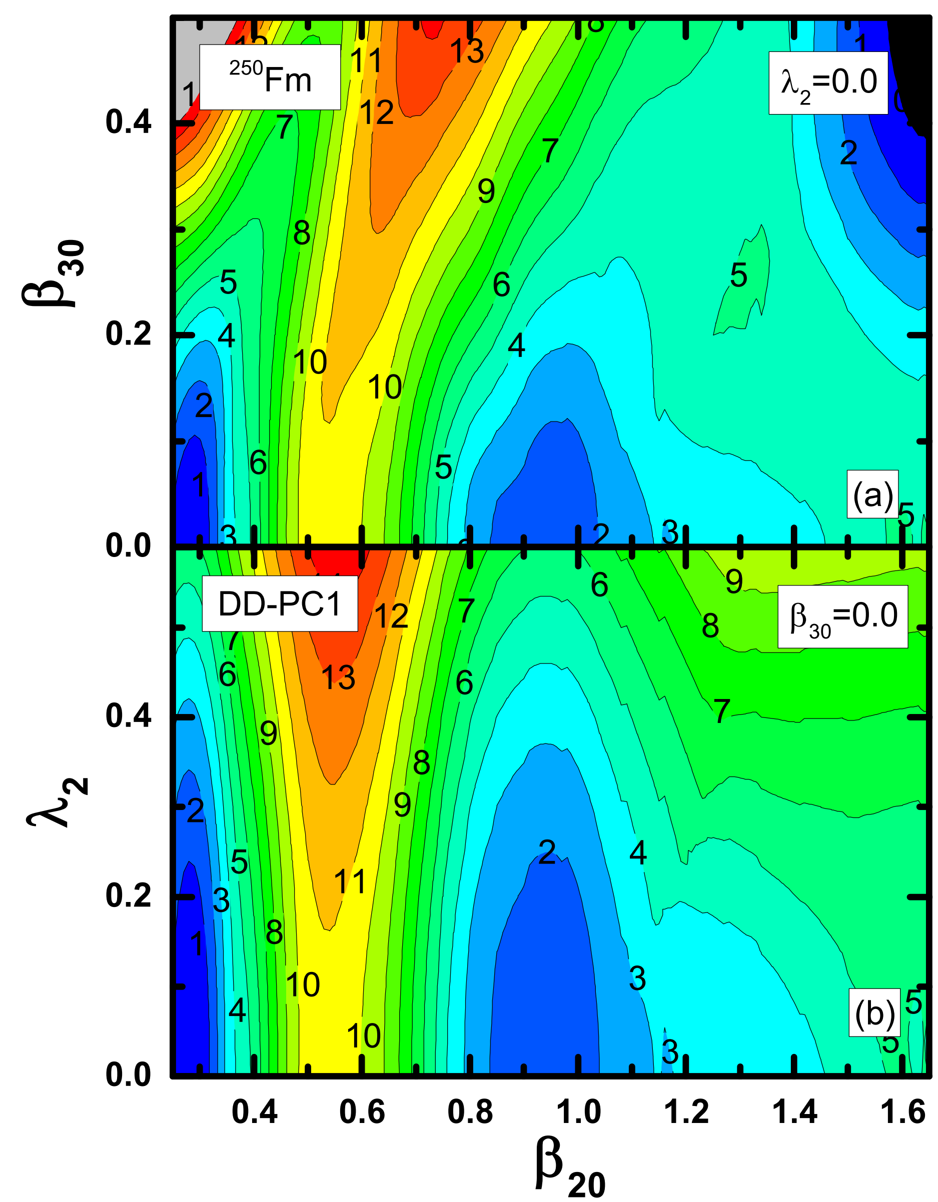
… the Routhian: $$E' = E_{\mathrm{RMF}} + \sum_{\lambda\mu} \frac{1}{2} C_{\lambda\mu} Q_{\lambda\mu} + \lambda_2 \Delta \hat{N}^2$$

E_{RMF} → total SCMF energy including static pairing correlations

⇒ 3-dim. collective spaces of shape and pairing coordinates (β_{20}, β_{22}, λ_2) and (β_{20}, β_{30}, λ_2)

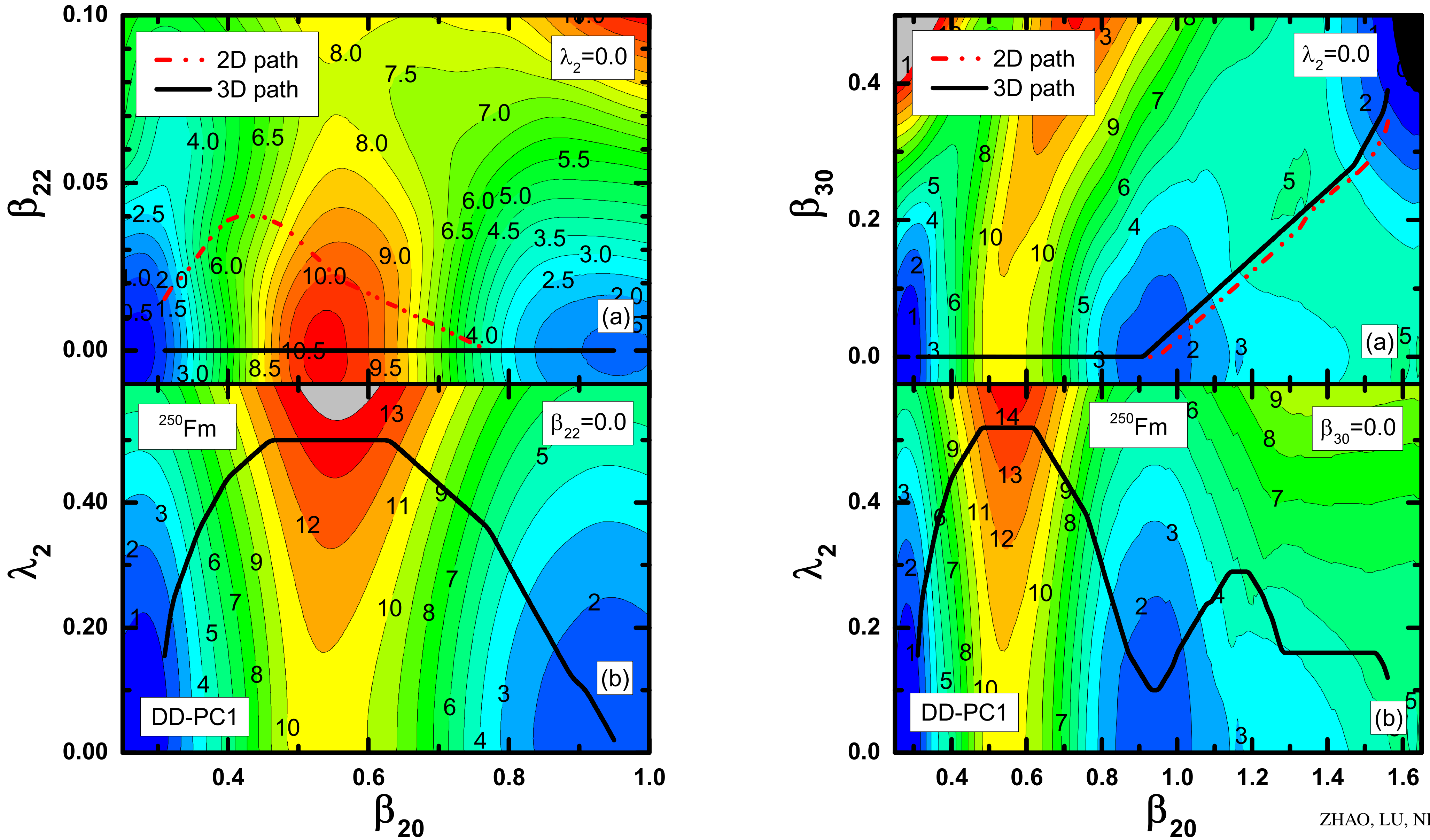




Dynamical coupling between shape and pairing degrees of freedom

The effective inertia and collective potential depend on the strength of pairing correlations:

$$\mathcal{M} \sim \Delta^{-2} \qquad V \sim (\Delta - \Delta_0)^2$$



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To reduce the collective inertia, the fissioning nucleus favors an increase in pairing over the static self-consistent solution, at the expense of a larger potential energy. Because of pairing fluctuations, the corresponding fission action integral is reduced and, consequently, the half-life is orders of magnitude shorter than in the case without the dynamic pairing degree of freedom.

Action integrals and SF half-lives of ^{264}Fm and ^{250}Fm

Nucleus	Path	$S(L)$	$\log_{10}(T_{1/2}/\mathrm{yr})$
^{264}Fm	2D	19.58	-11.03
	3D	14.15	-15.75
^{250}Fm	2D	32.09	-0.16
	3D	22.33	-8.64

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The predicted SF path strongly depends on the choice of the collective inertia!

➠calculation of the full ATDHFB inertia tensor!

➠dynamical effects caused by the competition between triaxial and reflection asymmetric degrees of freedom, and pairing correlations.

Nuclear Energy Density Functionals

✔ unified microscopic description of the structure of stable and nuclei far from stability, and extrapolations toward the region of superheavy nuclei.

✔ when extended to take into account collective correlations, EDFs describe deformations, shape-coexistence and shape transition phenomena associated with shell evolution. Separation energies, Q_α-values, excitation energies of band-heads in odd-A nuclei, excitation energies of high-K isomers, and rotational spectra can be directly compared to data.

✔ Time-dependent NDFT ➠ large amplitude collective motion, spontaneous fission dynamics