Modern nuclear mass models

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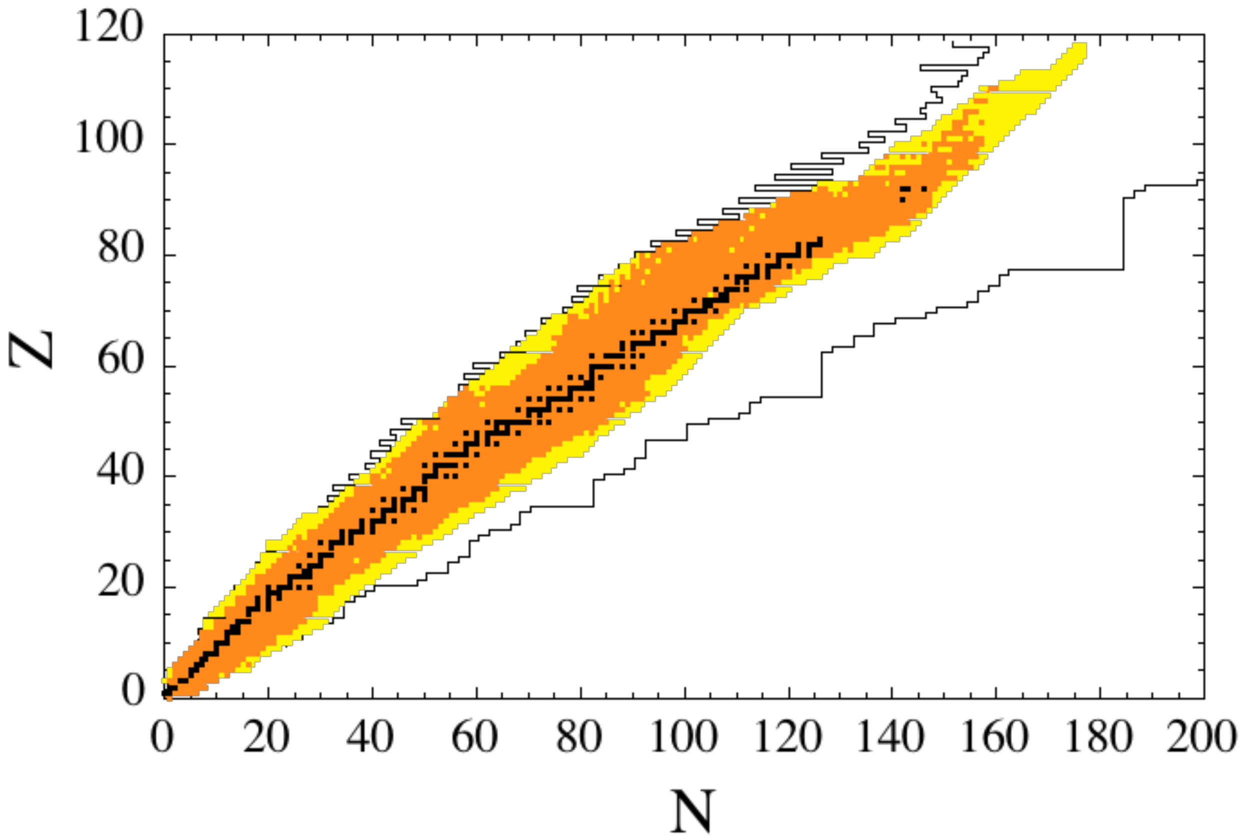
Atomic Mass Evaluation 2012 : 2439 known masses

(“safe” extrapolation to 3353 “recommended” masses)



A wealth of information on nuclear structure; a challenge for theory

Lots of information (sph-def, odd-even, shell, rot-vib, …) hidden in the Mass

A global mass model aims at reproducing known masses with a σ_{rms}~800 keV

(particularly relevant for applications)

Nuclear mass models

Nuclear mass models provide all basic nuclear ingredients:

Mass excess (Q-values), deformation, GS spin and parity

but also

single-particle levels, pairing strength, density distributions, … in the GS
as well as non-equilibrium (e.g fission path) configuration

Building blocks for the prediction of ingredients of relevance in the determination of nuclear reaction cross sections, β-decay rates, … such as

- nuclear level densities
- γ-ray strengths
- optical potentials
- fission probabilities & yields
- etc …

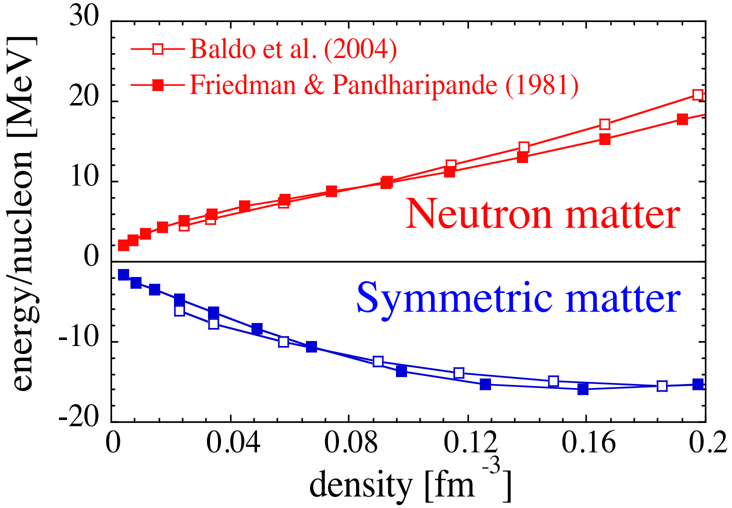
as well as for the nuclear/neutron matter Equation of State **(NEUTRON STARS)**

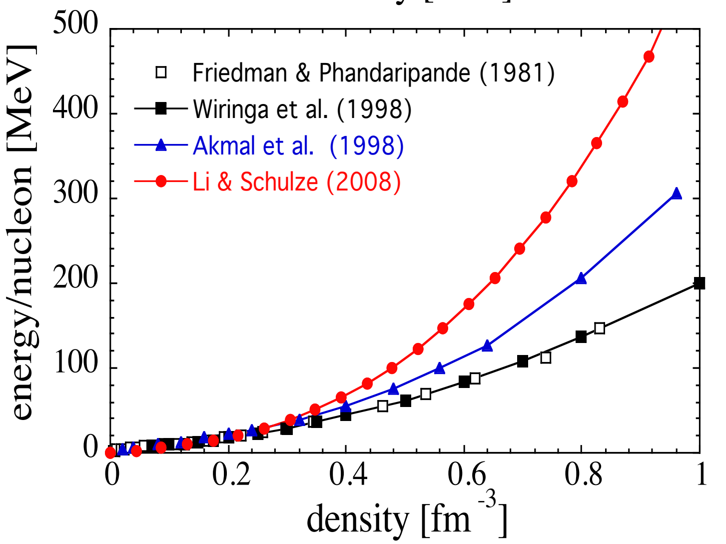
The criteria to qualify a mass model should NOT be restricted to the rms deviation wrt to exp. masses, but also include
- the quality of the underlying physics (sound, coherent, "microscopic", …)
- all the observables of relevance in the specific applications of interest (e.g astro)

Challenge for modern mass models: to reproduce as many observables as possible

- **2353 experimental masses** from AME'2012 → rms ~ 800keV
- 782 exp. charge radii (rms ~ 0.03fm), charge distributions, as well as ~26 n-skins
- Isomers & Fission barriers (scan large deformations)
- Symmetric nuclear matter properties
  - $m^* \sim 0.6$ - 0.8 (BHF, GQR) & $m^*_n(\beta) > m^*_p(\beta)$
  - $K \sim 230$ - 250 MeV (breathing mode)
  - E_{pot} from BHF calc. & in 4 (S,T) channels
  - Landau parameters $F_l(S,T)$
    - stability condition: $F_l^{ST} > -(2l+1)$
    - empirical $g_0 \sim 0$; $g_0' \sim 0.9$-1.2
    - sum rules $S_1 \sim 0$; $S_2 \sim 0$
  - Pairing gap (with/out medium effects)
  - Pressure around 2-3ρ_0 from heavy-ion collisions
- Neutron matter properties
  - $J \sim 29 - 32$MeV
  - E_n/A from realistic BHF-like calculations
  - Pairing gap
  - Stability of neutron matter at all polarizations
- Giant resonances
  - ISGMR, IVGDR, ISGQR
- Additional model-dependent properties
  - Nuclear Level Density (pairing-sensitive)
  - Properties of the lowest 2+ levels (519 e-e nuclei)
  - Moment of inertia in superfluid nuclei (back-bending)

~ model-dependent





Mean Field mass models

$$E = E_{MF} - E_{coll} - E_W - E_{b\infty}$$

E_{MF} : HFB or HF-BCS (or HB) main contribution

E_{coll}: Quadrupole Correlation corrections to restore broken symmetries and include configuration mixing

E_W : *Wigner* correction contributes significantly only for nuclei along the $Z \sim N$ line (and in some cases for light nuclei)

$E_{b\infty}$: Correction for infinite basis

Skyrme-HFB | Gogny-HFB | Relativistic MF

Modern Mean Field mass models

Adjustement of an effective interaction / density functional to all (2353) experimental masses (AME'12)

σ_{rms}(M) = 0.5-0.8 MeV on 2353 (Z ≥ 8) experimental masses

To be compared with

- Droplet-like approaches : e.g FRDM → σ_{rms}(M)~0.65 MeV
- Other Mean-Field predictions :

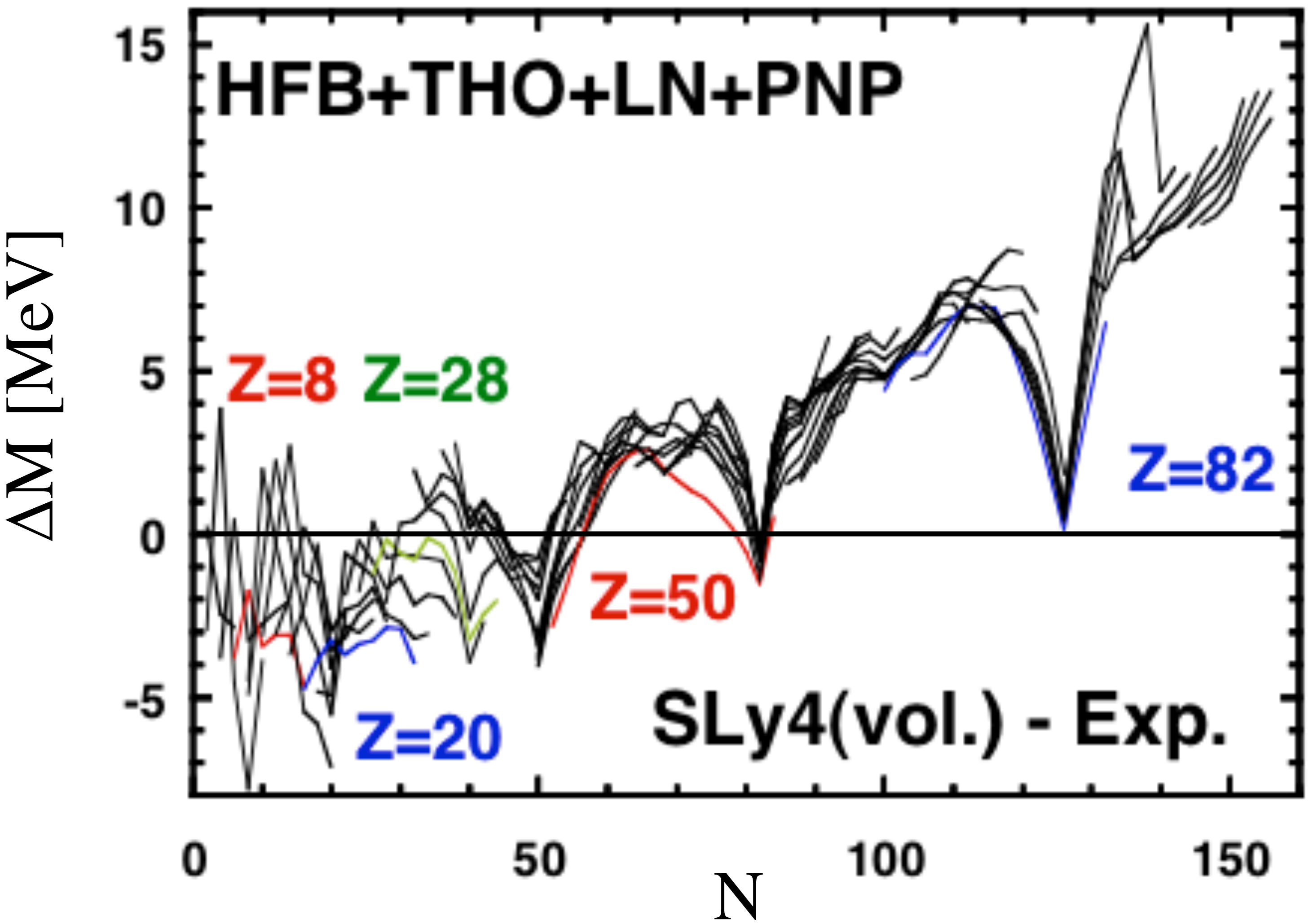
Traditional Skyrme or Gogny forces: rms > 2 MeV

e.g. Oak Ridge "Mass Table" based on HFB with SLy4

rms(M)=5.1MeV on 570 e-e sph+def nuclei

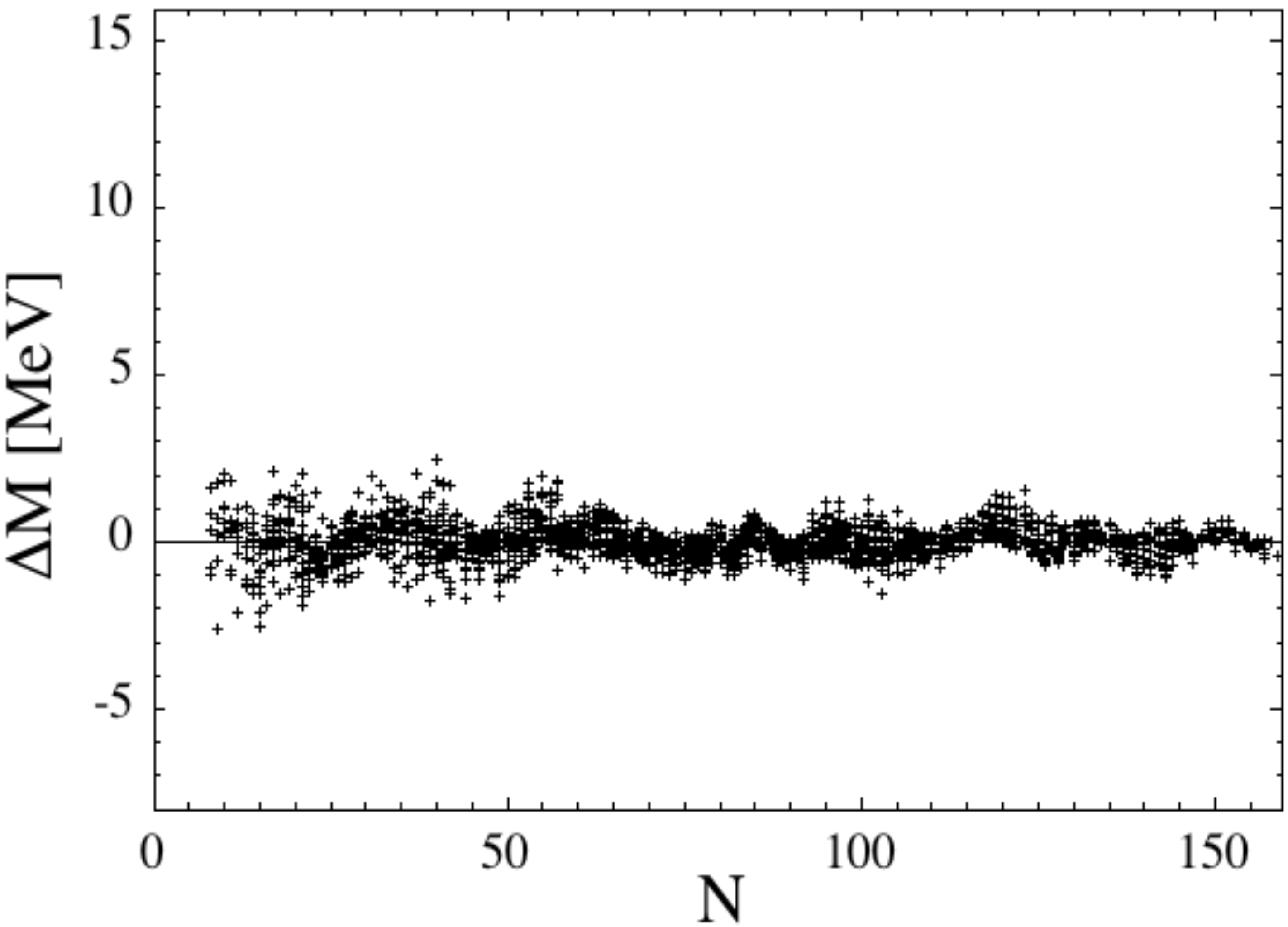
Different fitting protocols for mass models !

M(SLy4) – M(exp)



Dobaczewski et al., 2004

M(BSk27) – M(exp)



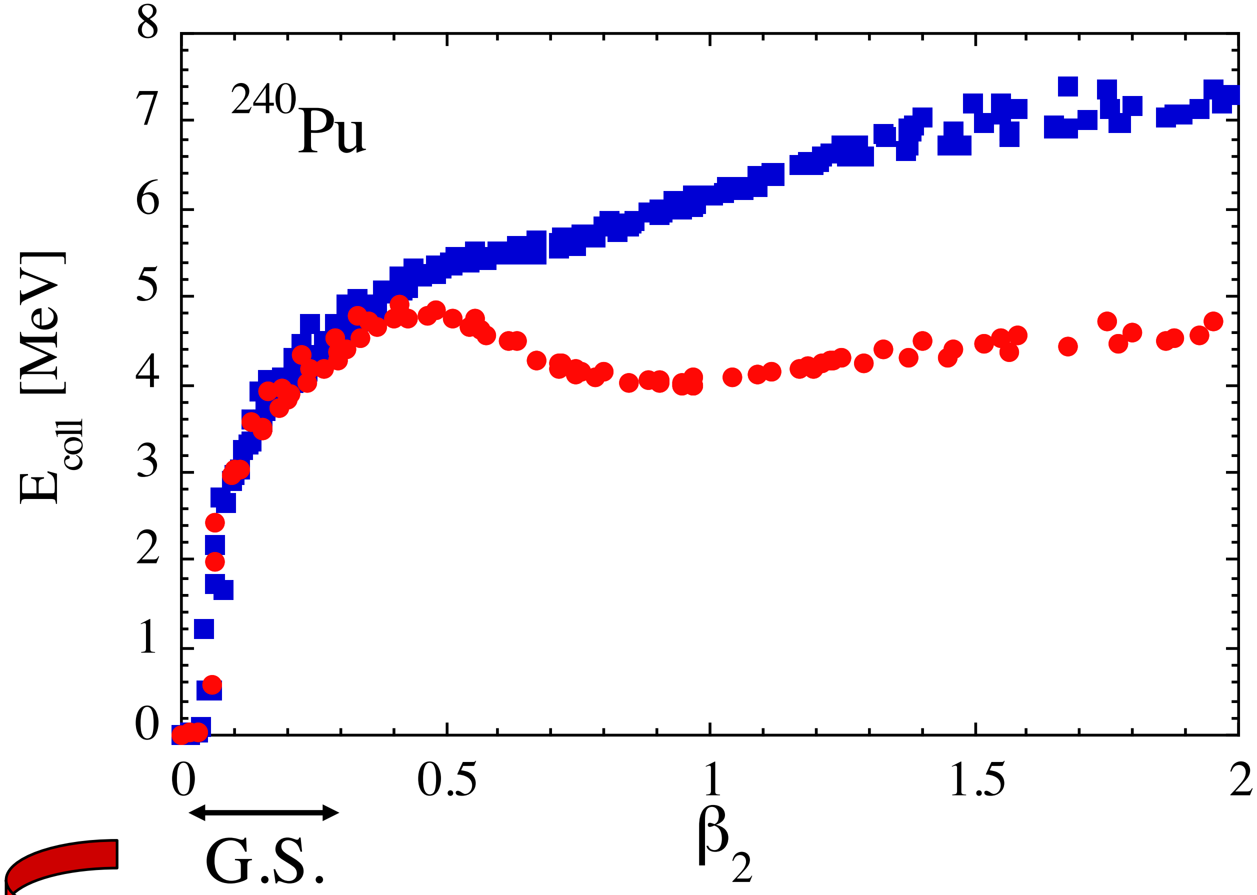
Skyrme-HFB model: a weapon of mass production

The long road in the HFB mass model development		σ_{rms} (2353 AME'12)	
HFB-1–2 :	Possible to fit all 2149 exp masses Z≥8	663 keV	
HFB-3:	Volume versus surface pairing	650 keV	
HFB-4–5:	Nuclear matter EoS: m^*=0.92	670 keV	
HFB-6–7:	Nuclear matter EoS: m^*=0.80	670 keV	
HFB-8:	Introduction of number projection	673 keV	
HFB-9:	Neutron matter EoS - J=30 MeV	757 keV	
HFB-10–13:	Low pairing & NLD	724 keV	~
HFB-14:	Collective correction and Fission B_f	734 keV	~
HFB-15:	Including Coulomb Correlations	658 keV	
HFB-16:	with Neutron Matter pairing	628 keV	
HFB-17:	with Neutron & Nuclear Matter pairing	569 keV	
HFB-18–21:	Non-Std Skyrme (t_4-t_5 terms) - Fully stable	572 keV	
HFB-22–26:	New AME'12 masses, J=30-32MeV, EoS	567 keV	
HFB-27:	Standard Skyrme	**500 keV**	
HFB-28–29:	Sentivity to Spin Orbit terms	520 keV	
HFB-30–32:	Self-energy effects in pairing, J=30-32MeV	560 keV	

Correction for quadrupole correlations

!! of particular relevance at large deformation --> Fission calculations !!

- a perturbative *cranking* correction for rotational correlations
- a *phenomenological* correction for "vibrational" correlations



rotational

$$E_{coll} = b\, E_{rot}^{crank} \tanh(c\beta_2)$$

rotational + vibrational

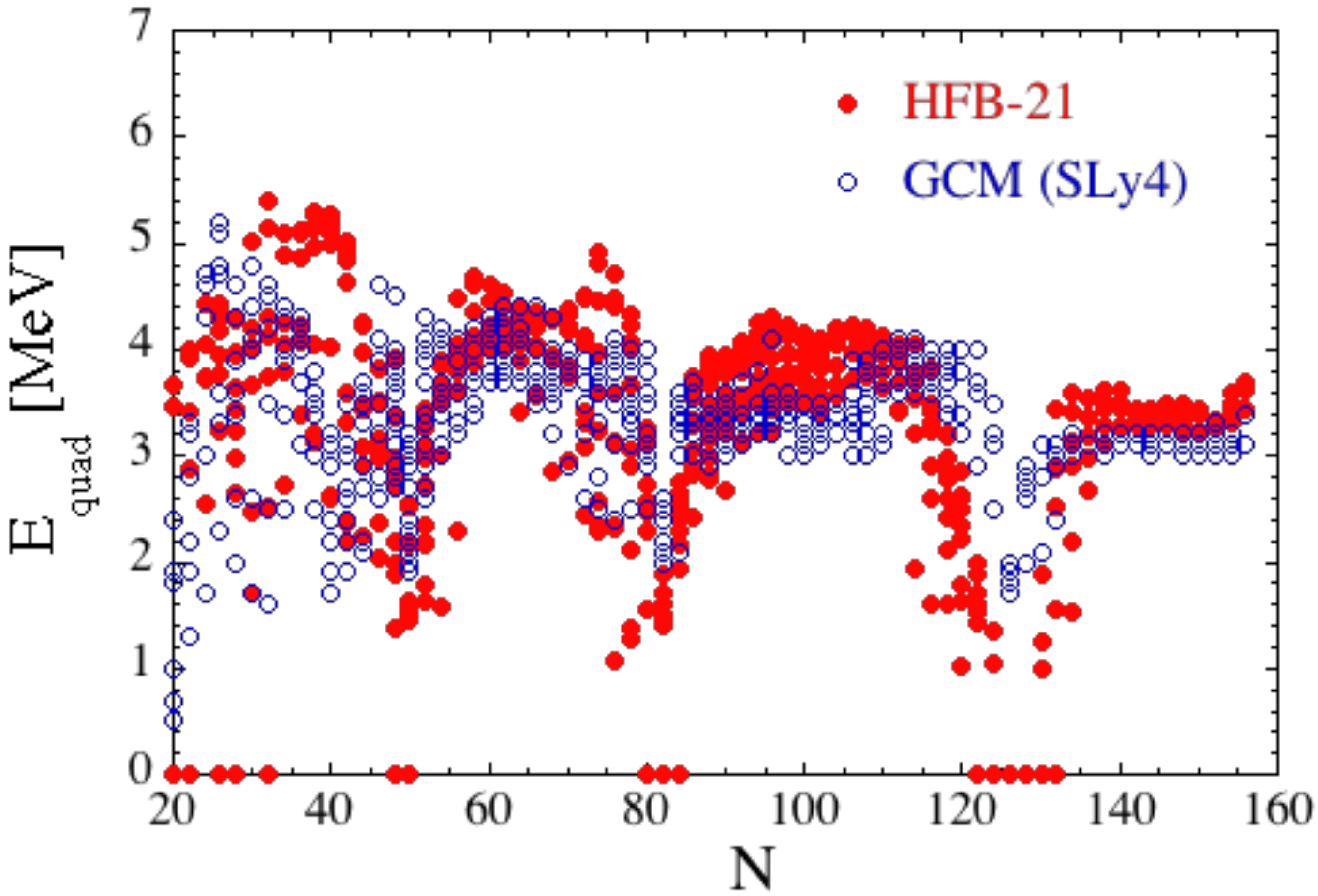
$$E_{coll} = b\, E_{rot}^{crank} \tanh(c\beta_2) + d\, E_{rot}^{crank} \exp\left[-l(\beta_2 - \beta_2^0)^2\right]$$

(still unsatisfactory description)

Small impact on GS energy, but significant on Isomers and fission barriers

Quadrupole corrections to the binding energy

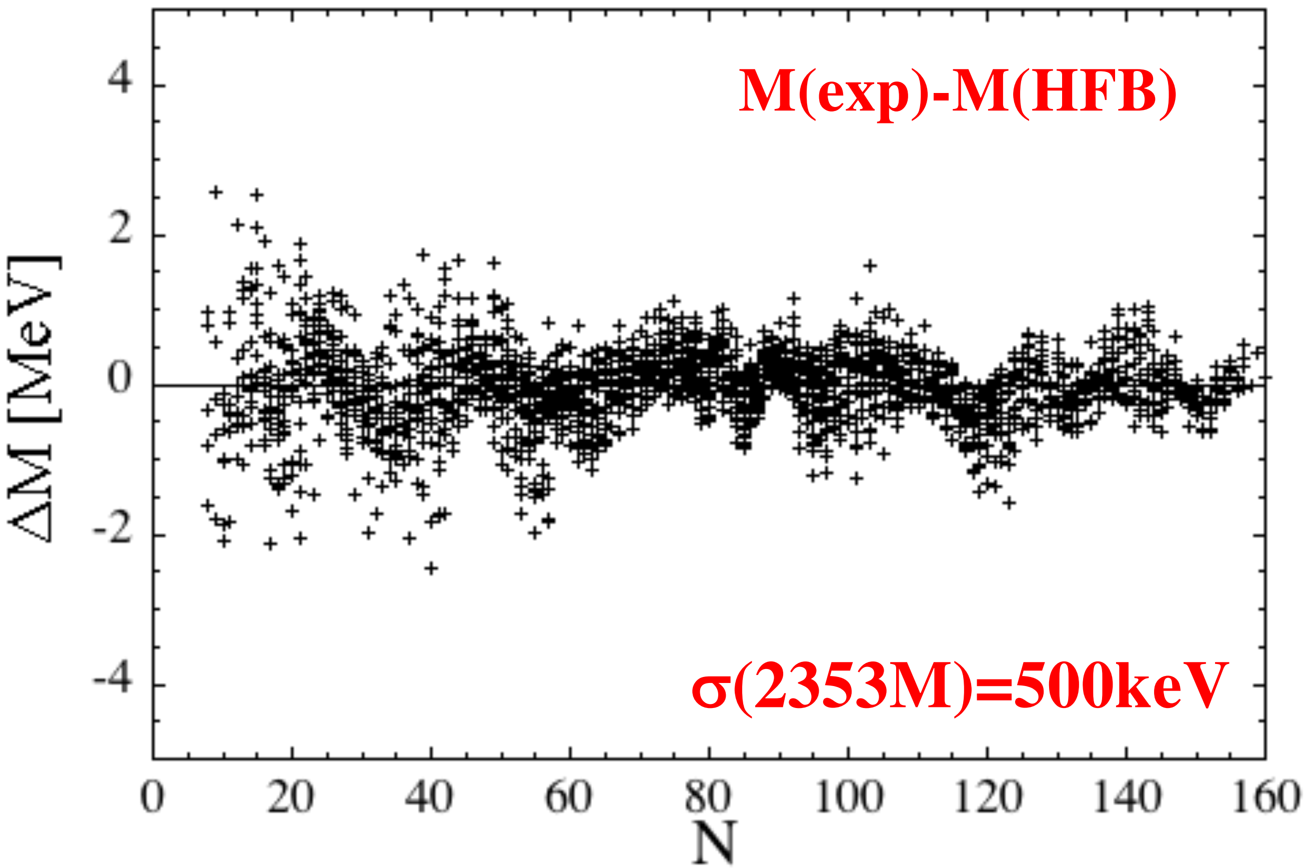
Comparison with the GCM (SLy4) calculation of Bender (2004)



606 e-e nuclei with 8 ≤ Z ≤ 108

Comparison between HFB-27 and experimental masses

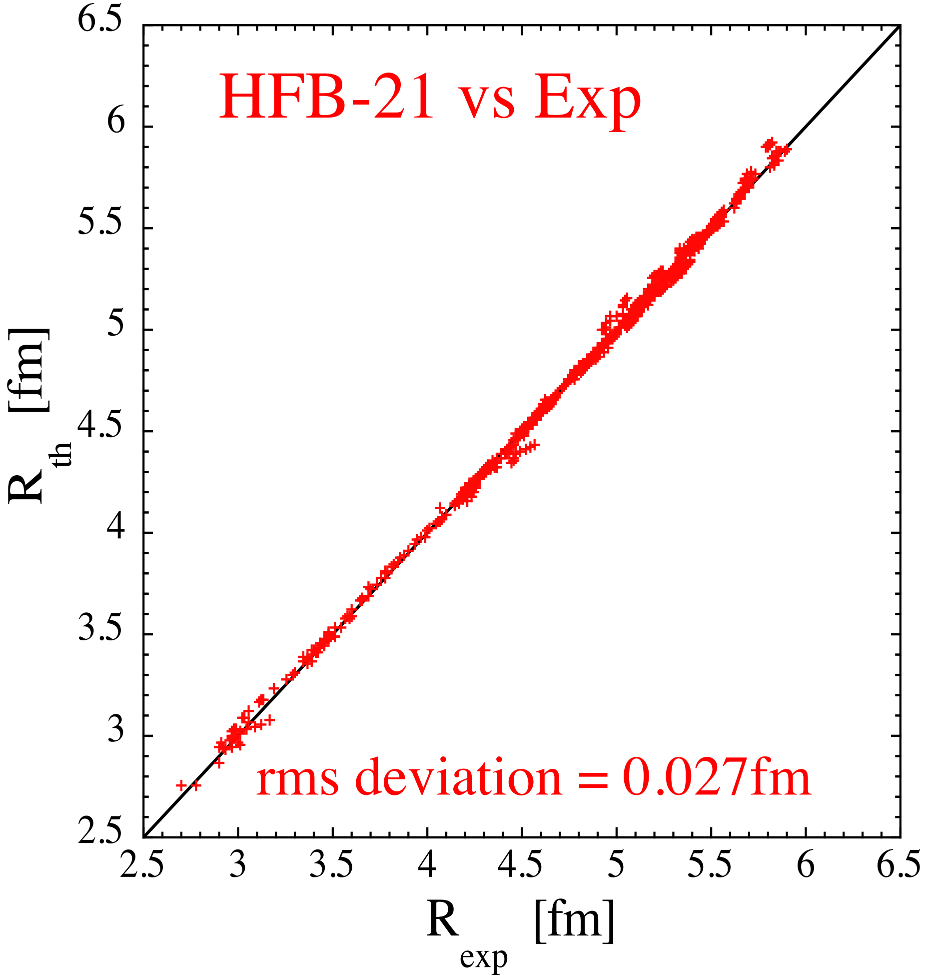
AME'12



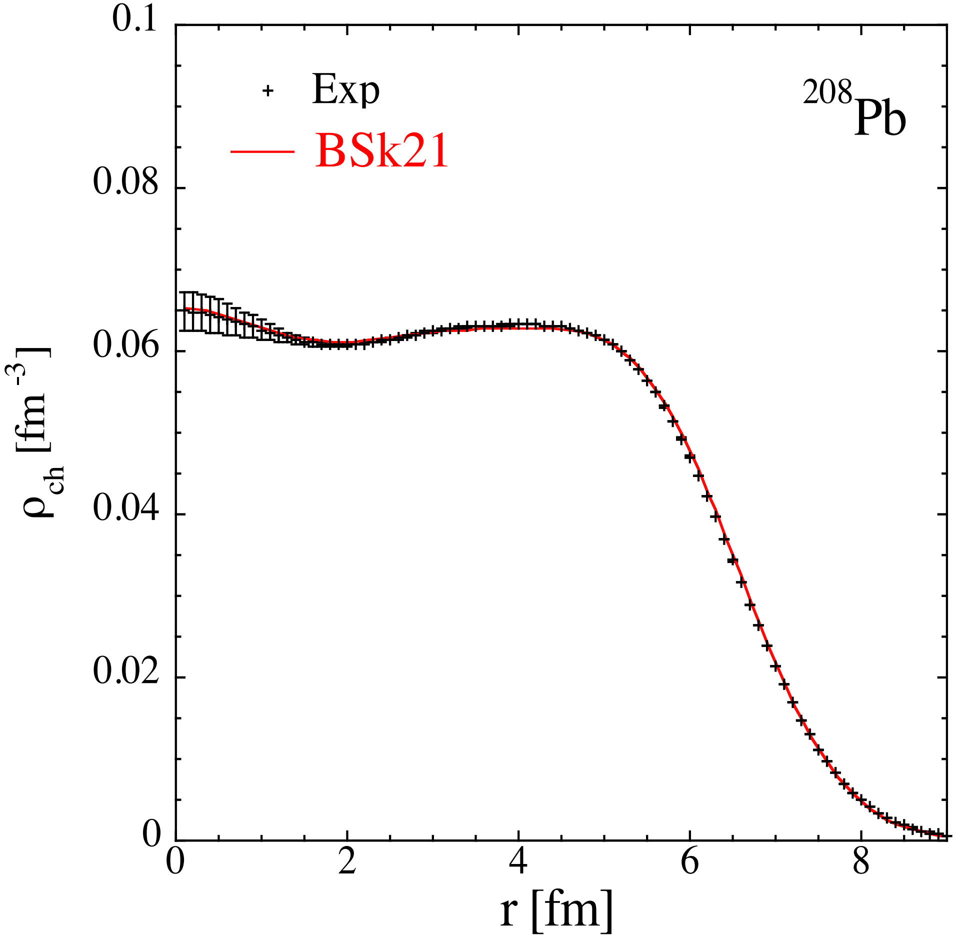
	σ(HFB27)	σ(HFB24)	σ(FRDM)
2353 M (AME 2012):	512 keV	549 keV	654 keV
2353 M (AME 2012): model error	500 keV	542 keV	648 keV
257 M from AME'12 with S_n<5MeV:	645 keV	702 keV	857 keV
128 M (28≤Z≤46, n-rich) at JYFLTRAP (2012):	508 keV	546 keV	698 keV

Some examples for nuclear structure properties of interest for applications

Charge radii for 782 nuclei

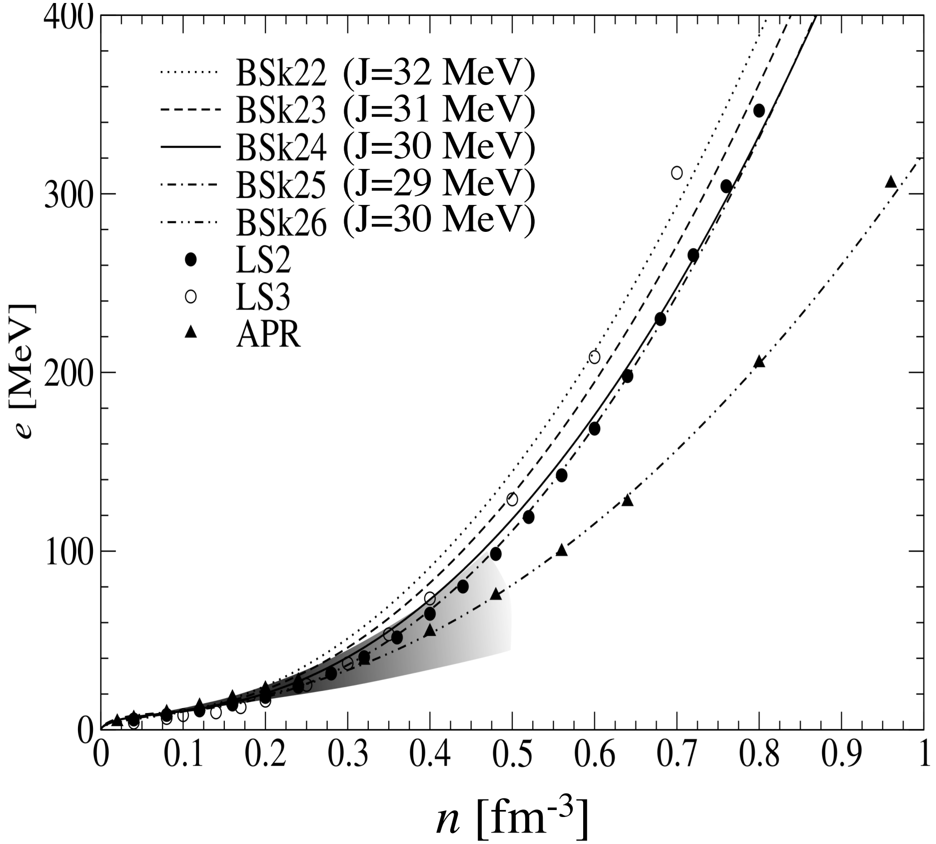


Charge distribution of ^{208}Pb

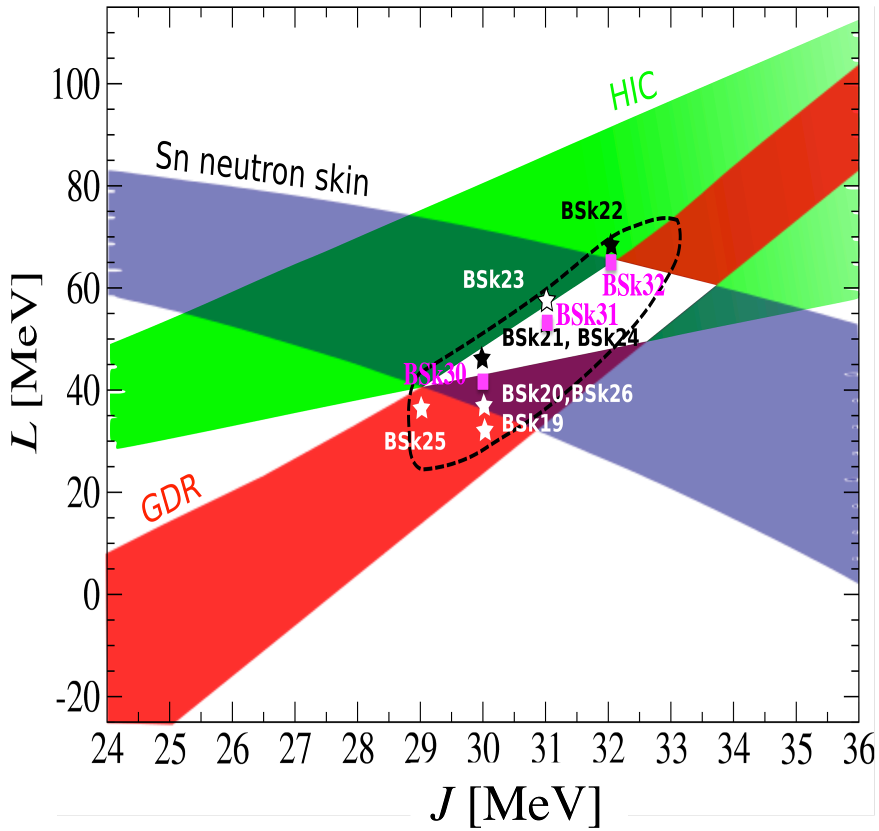


Nuclear matter properties & constraints from "realistic calculations"

Energy per nucleon in neutron matter



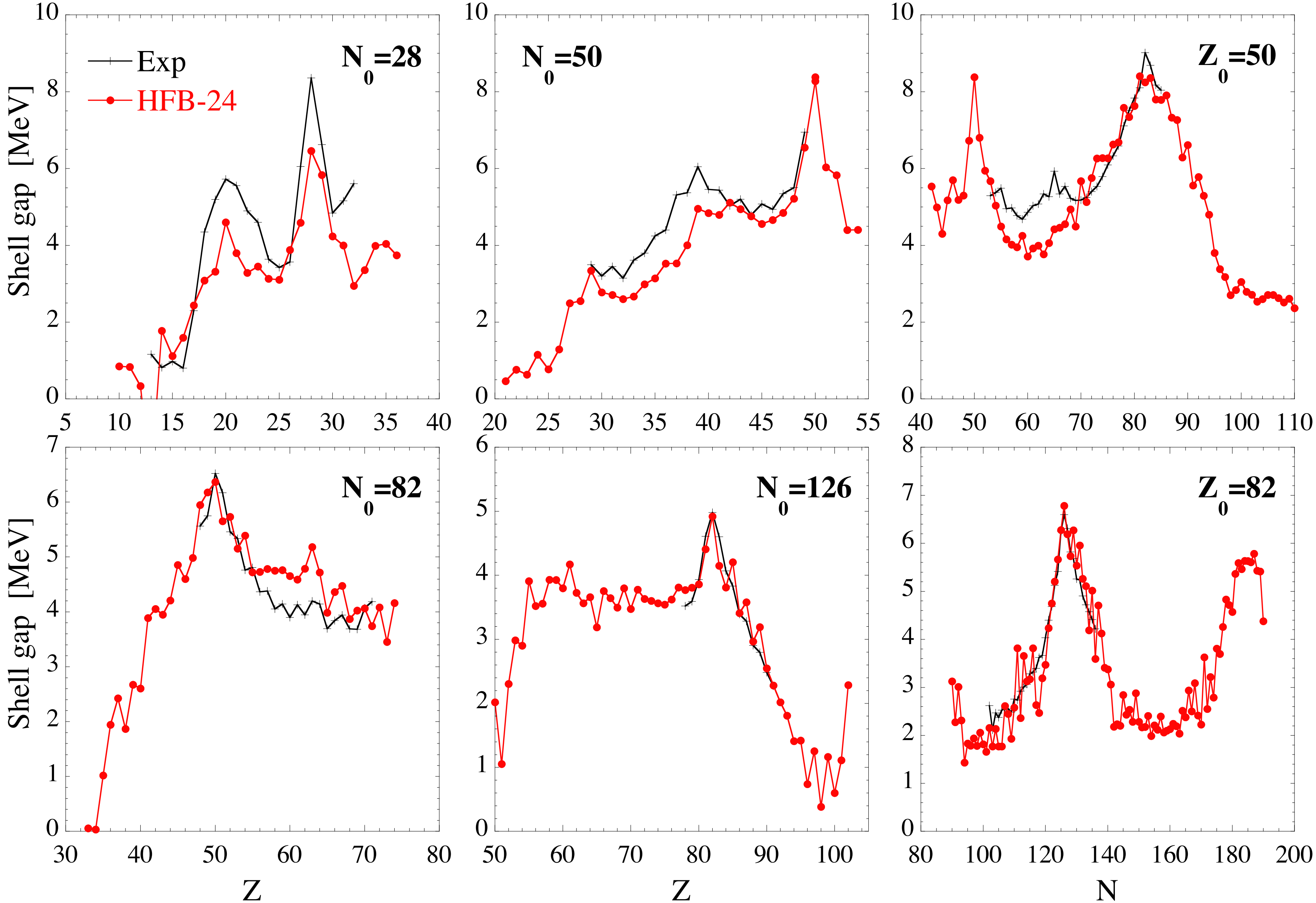
Experimental constraints on *J* / *L*



- Stable neutron matter at all polarisations (no ferromagnetic instability)
- Maximum NS mass : M_{max}> 2.0 M_o for HFB-20–26; 28-32
  as constrained by NS observation

Shell gaps obtained with HFB-24 mass model

$$\Delta_n(N_0, Z) = S_{2n}(N_0, Z) - S_{2n}(N_0 + 2, Z)$$



A new generation of mass models

Gogny-HFB mass table
beyond mean field !

(M. Girod, S. Hilaire, S. Péru: Bruyères-le-Châtel, France)

The total binding energy is estimated from

$$E_{tot} = E_{HFB} - E_{Quad} - E_{b\infty}$$

• E_{HFB}: deformed HFB binding energy obtained with a *finite-range* standard Gogny-type force

$$V(1,2) = \sum_{j=1,2} e^{-\frac{(\vec{r}_1-\vec{r}_2)^2}{\mu_j^2}} (W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau)$$
$$+ t_0 (1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \left[\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\right]^\alpha$$
$$+ i W_{LS} \overleftarrow{\nabla}_{12} \delta(\vec{r}_1 - \vec{r}_2) \times \overrightarrow{\nabla}_{12} . (\vec{\sigma}_1 + \vec{\sigma}_2) .$$

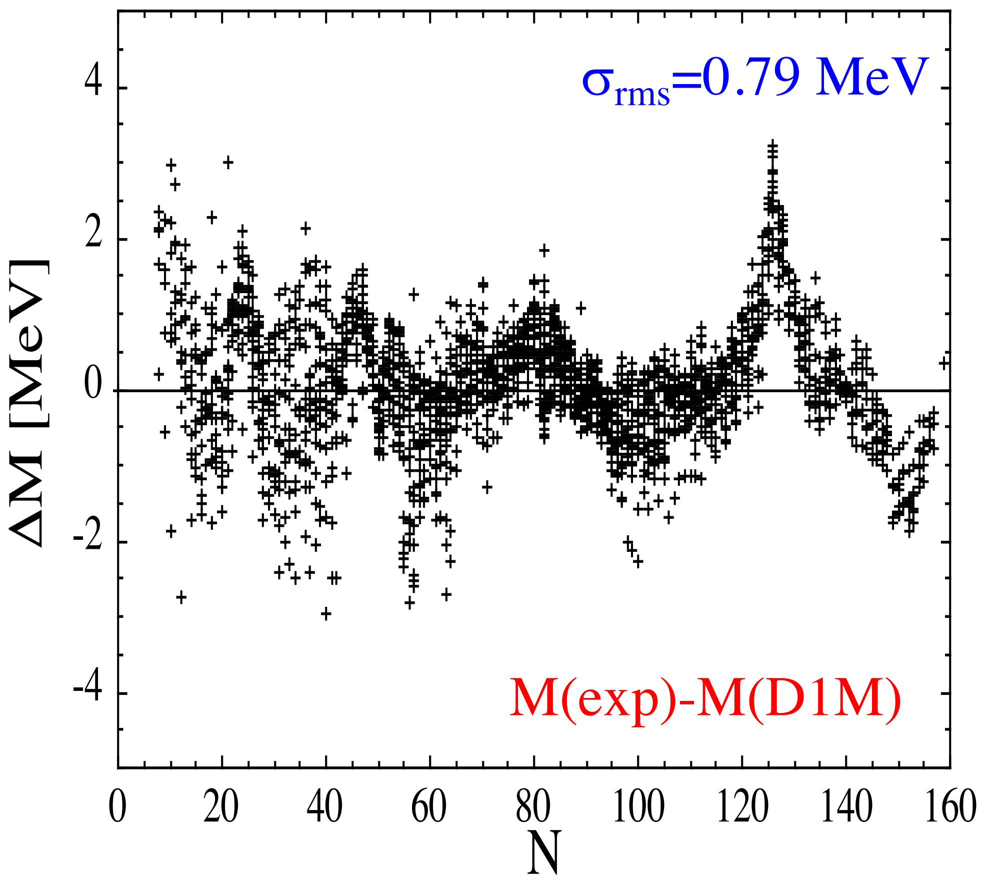
• E_{Quad} : quadrupolar correction energy determined with the *same* Gogny force (no "double counting") in the framework of the GCM+GOA model for the five collective quadrupole coordinates, i.e. rotation, quadrupole vibration and coupling between these collective modes (axial and triaxial quadrupole deformations included)

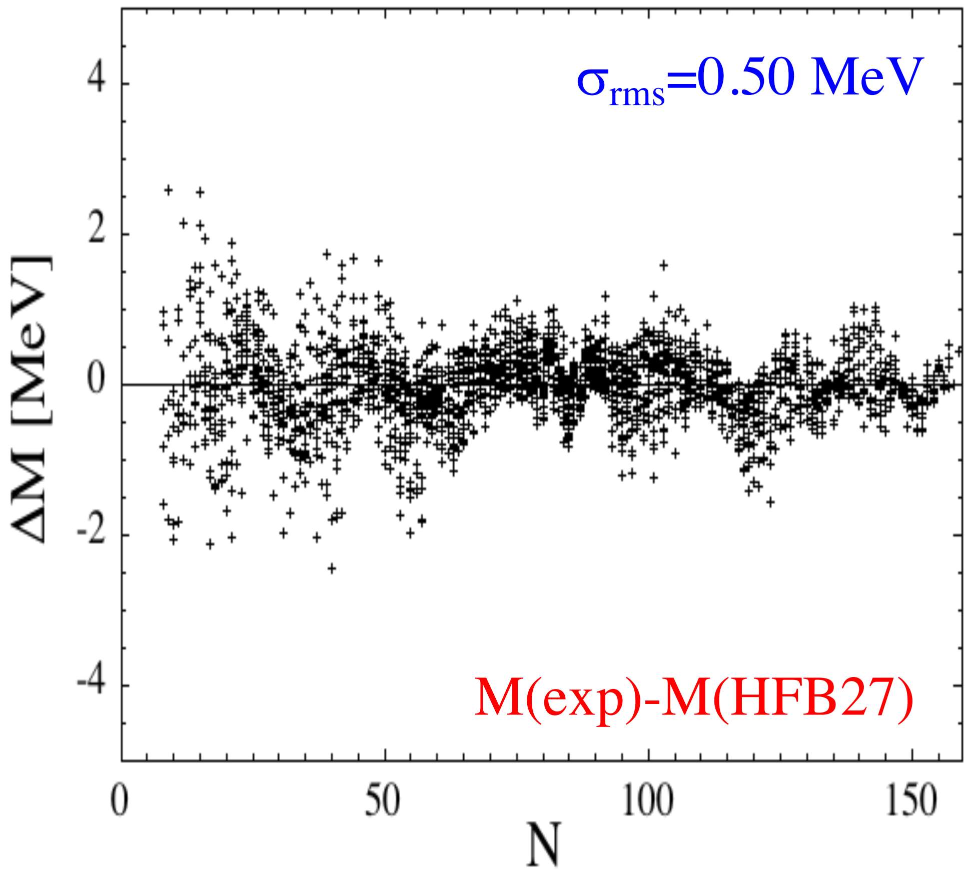
Girod, Berger, Libert, Delaroche

Gogny-HFB mass formula (D1M force)

2353 Masses: σ_{rms}=0.79 MeV with coherent E_{Quad} & E_{HFB} !

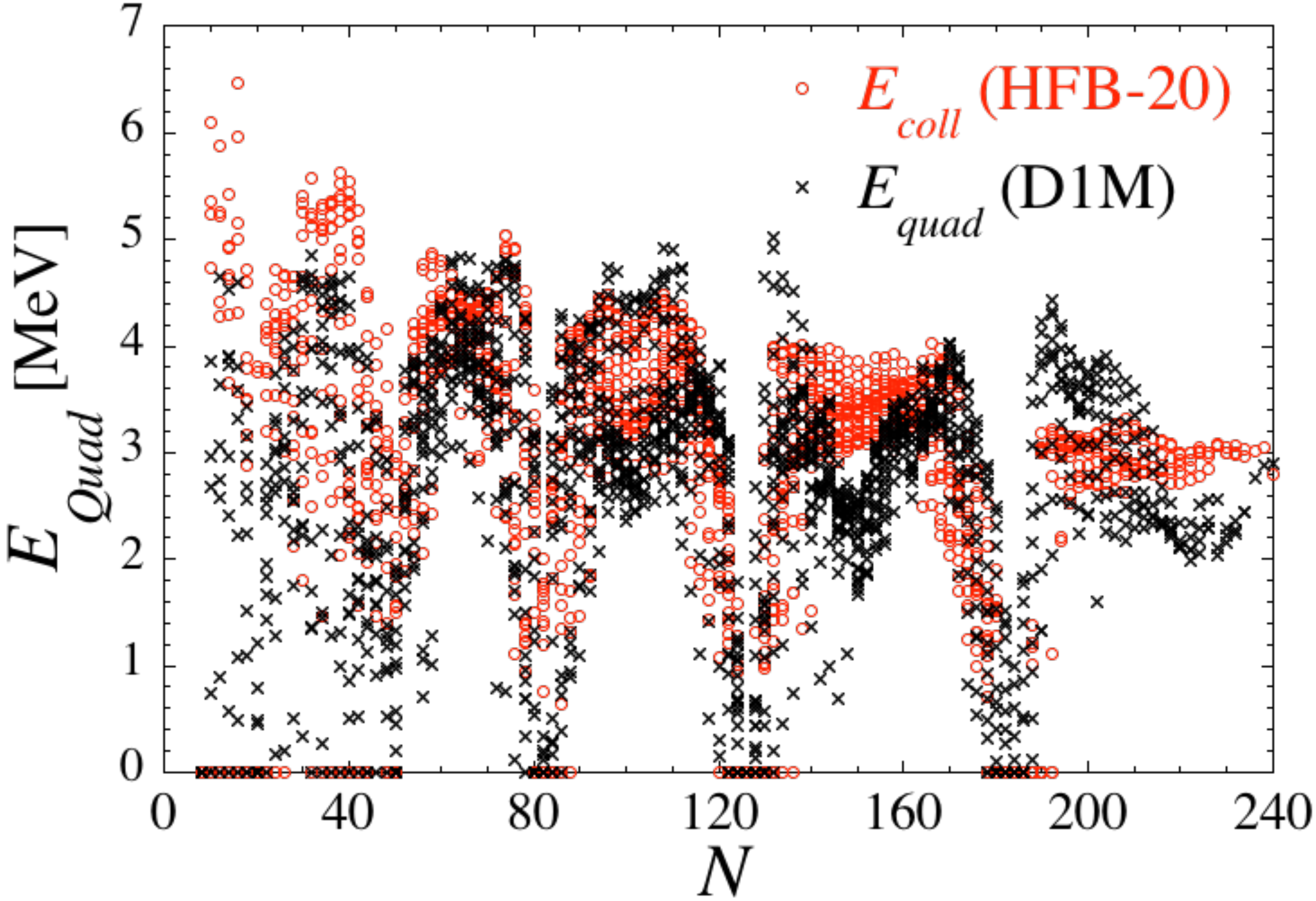
707 Radii: σ_{rms}=0.031 fm (with Q corrections)



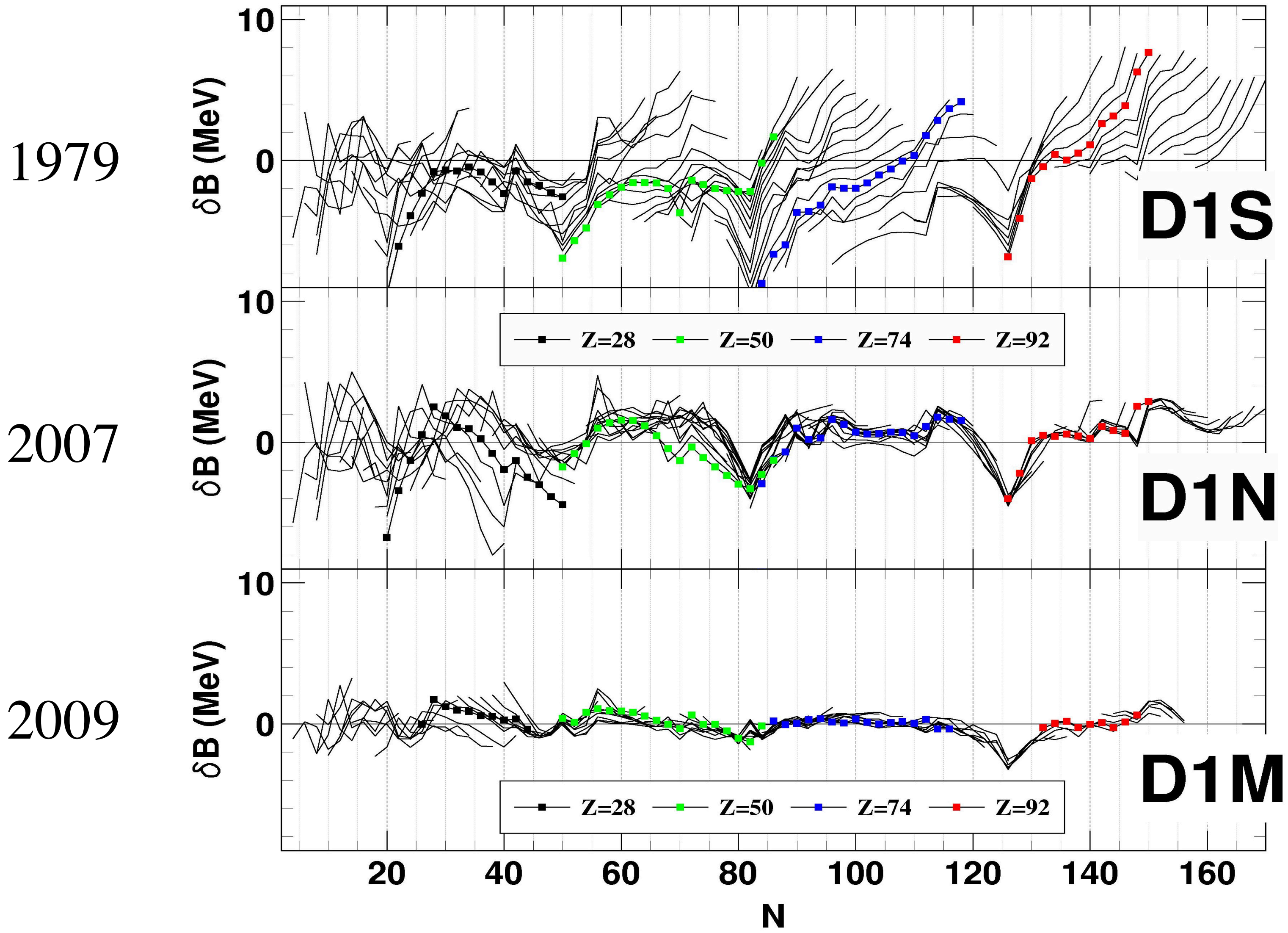


--> It is possible to adjust a Gogny force to reproduce all experimental masses “accurately”

Quadrupole corrections to the binding energy



$\delta B = M(\text{th}) - M(\text{exp})$



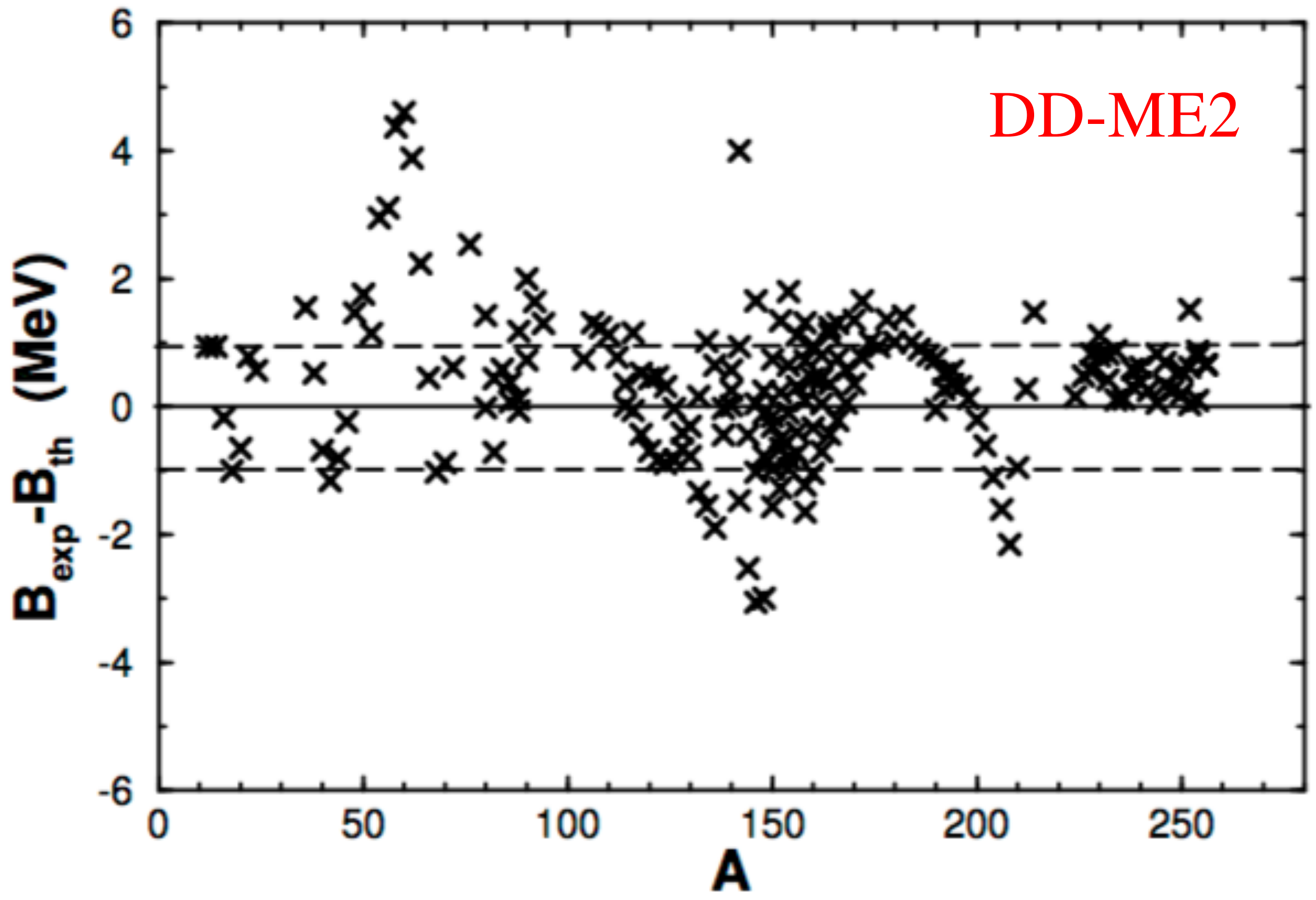
So far, D1M reproduces other observables with the same quality as D1S

What about
Relativistic Mean Field
mass predictions ?

RHB interactions for mass estimates

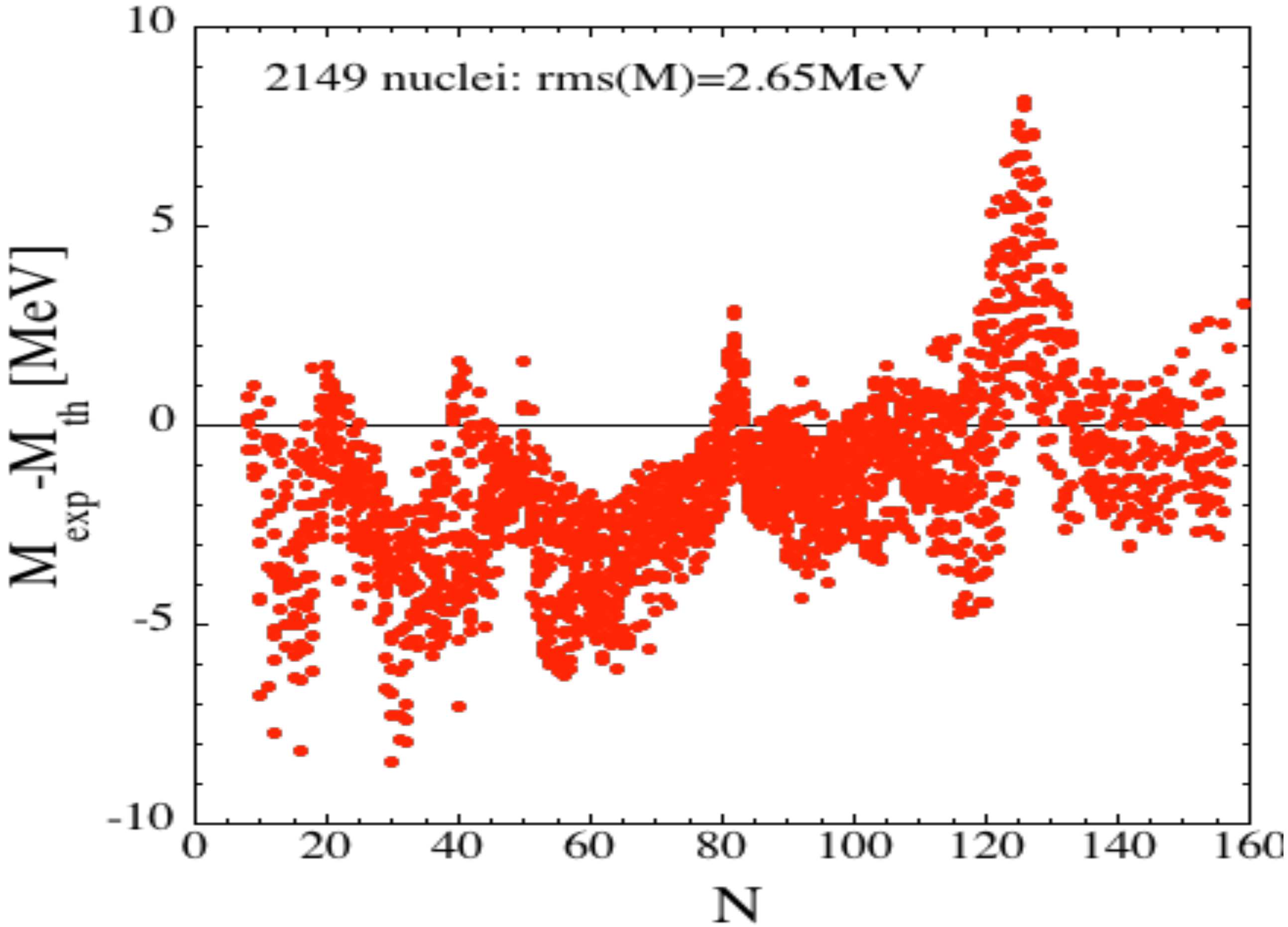
PC-PK1 : σ(2149 nuc - AME03) ~ 1.41MeV (Zhao et al. 2010)

DD-ME2 + Gogny D1S pairing : σ(200 nuc) ~ 0.9MeV (Vretenar et al. 2005)



DD-ME2 masses in comparison with experimental data

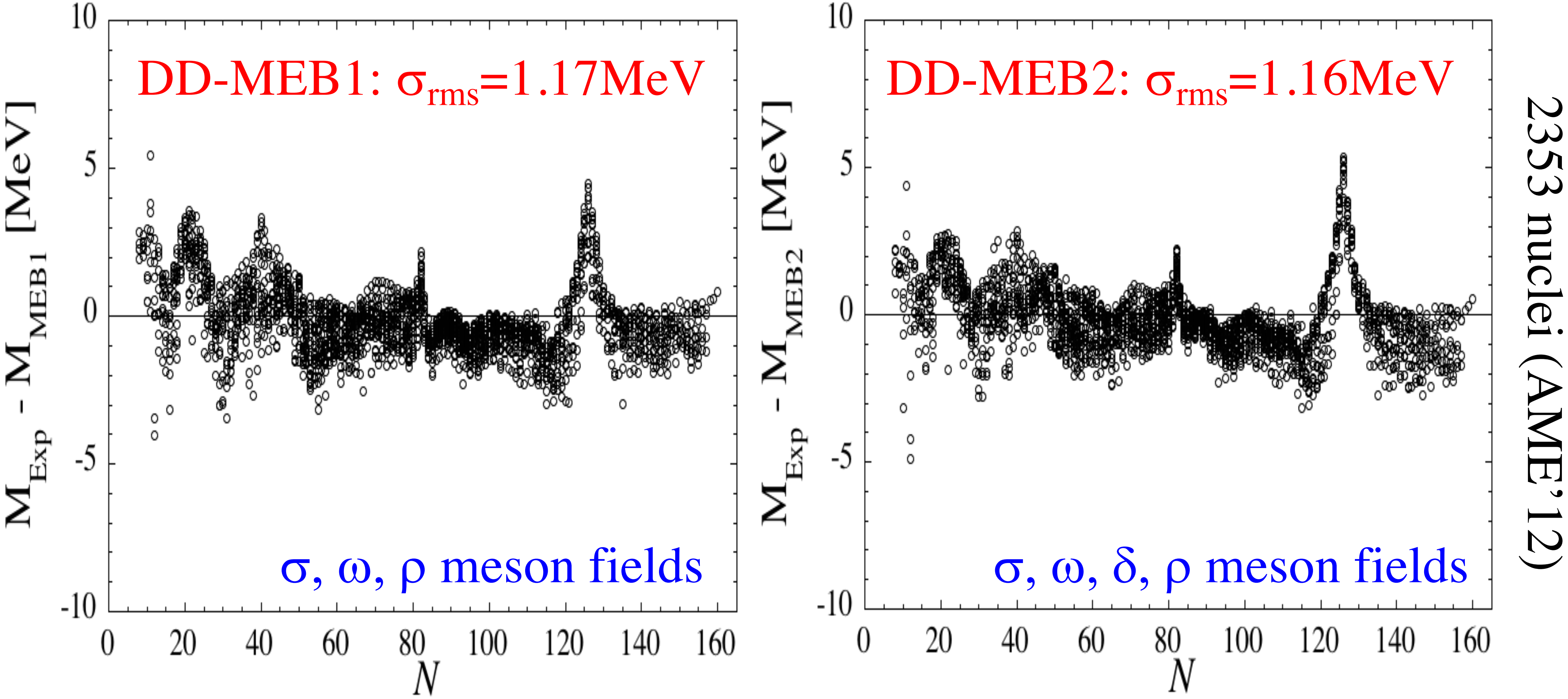
DD-ME2 + Gogny D1S pairing (Vretenar et al. 2005)



NEW effort within the Relativistic Mean Field model

Mass models based on the covariant density functional theory with finite-range density-dependent meson-nucleon couplings.

$$E_{tot} = E_{RMF} - E_{coll} \quad \text{with} \quad E_{coll} = b\, E_{rot}^{crank} \tanh(c\beta_2)$$

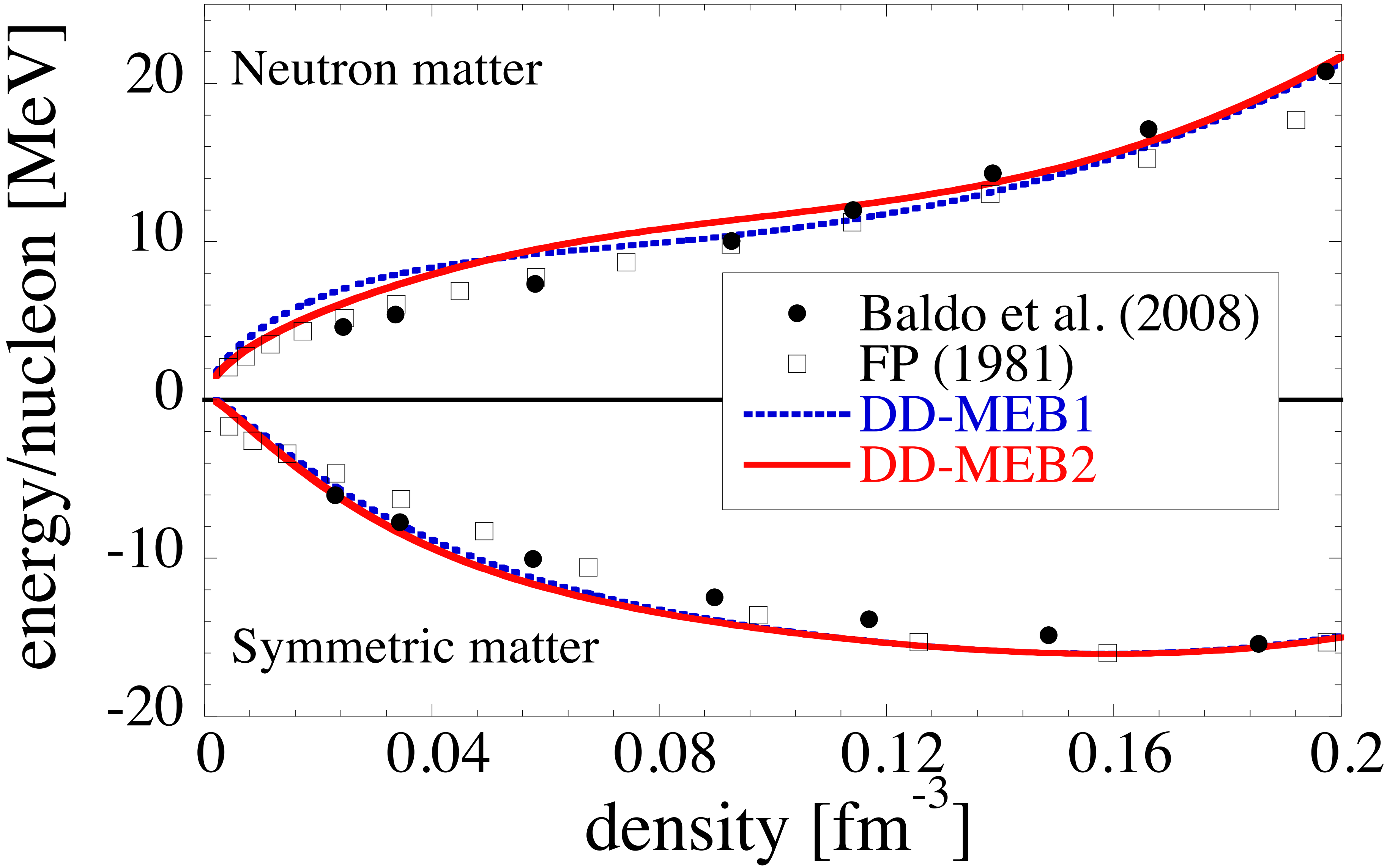


→ Still room for improvement !

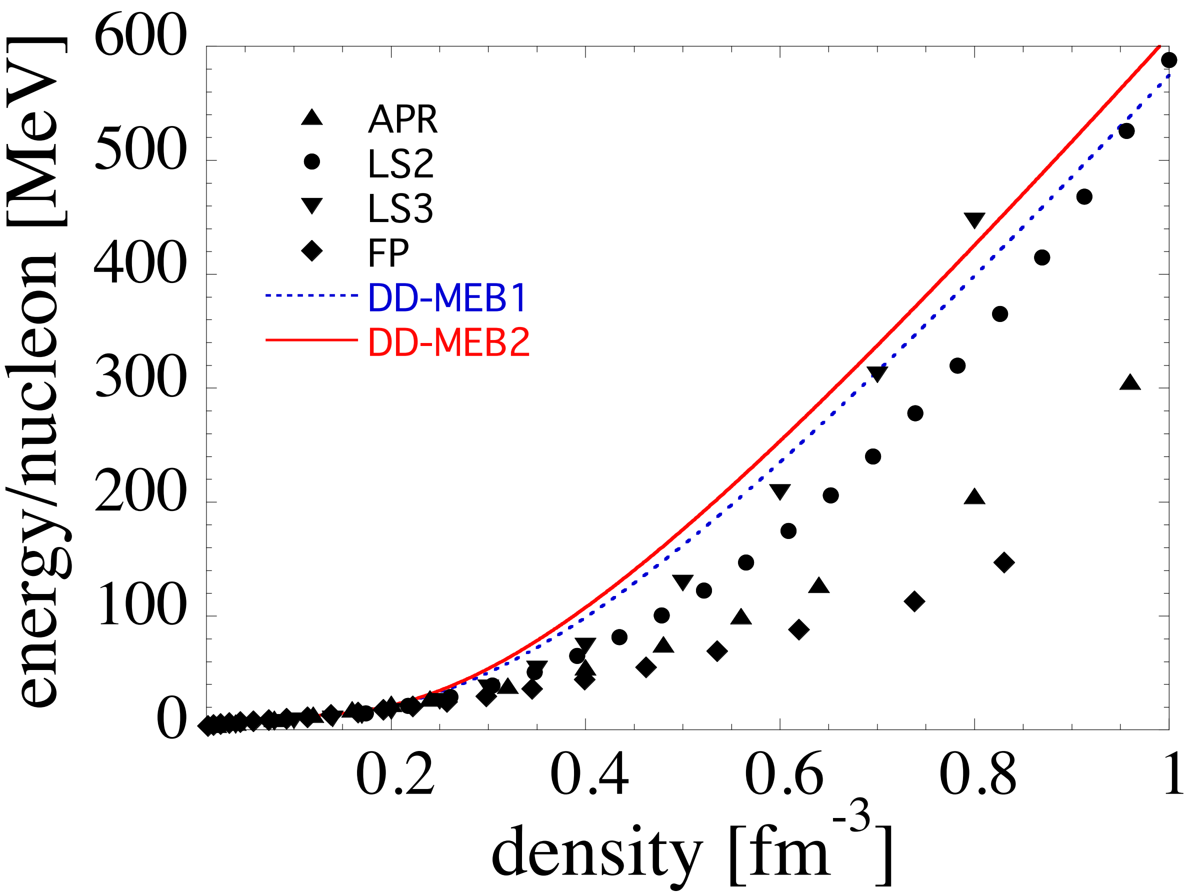
D. Peña-Arteaga et al. (2016)

NEW effort within the Relativistic Mean Field model

Constraints on nuclear matter Equation of State

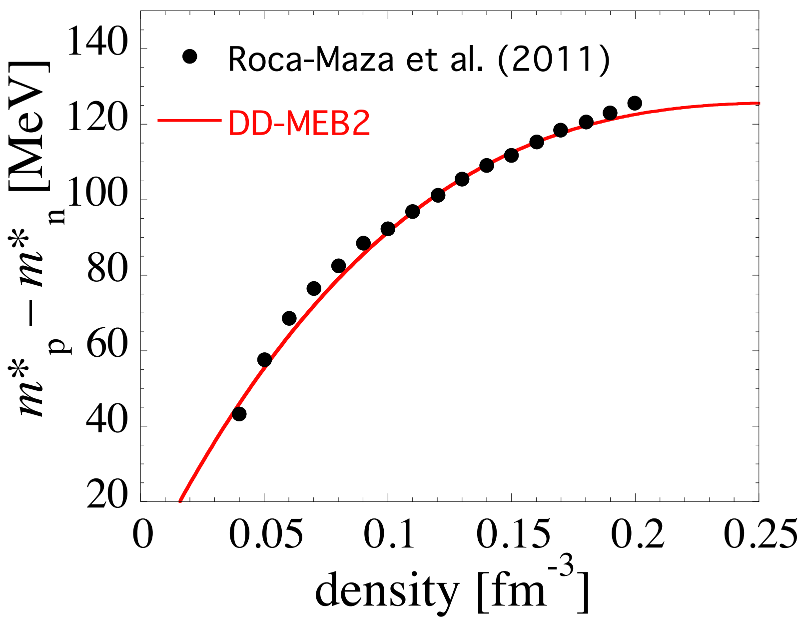


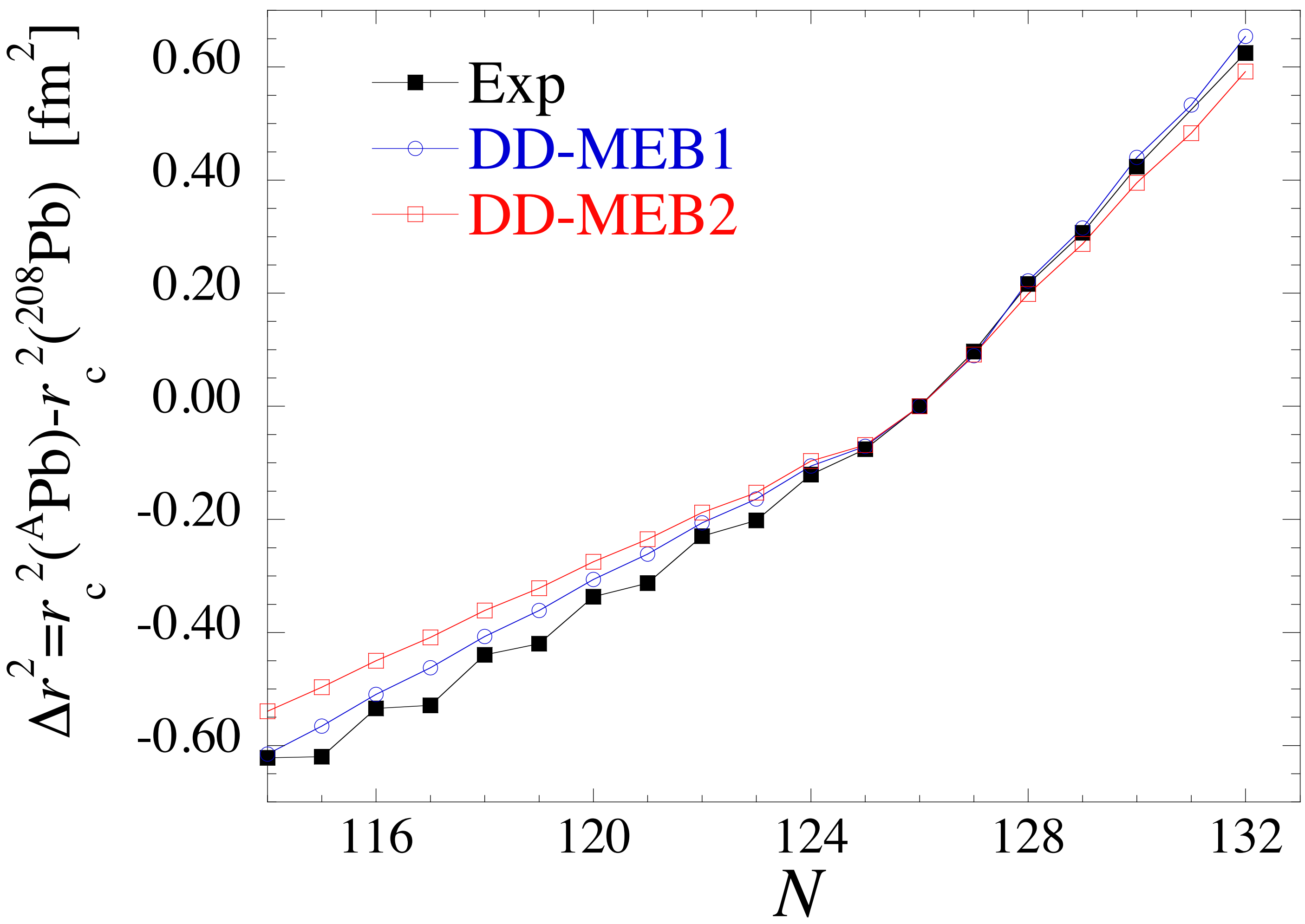
Equation of State in Neutron Matter at high density

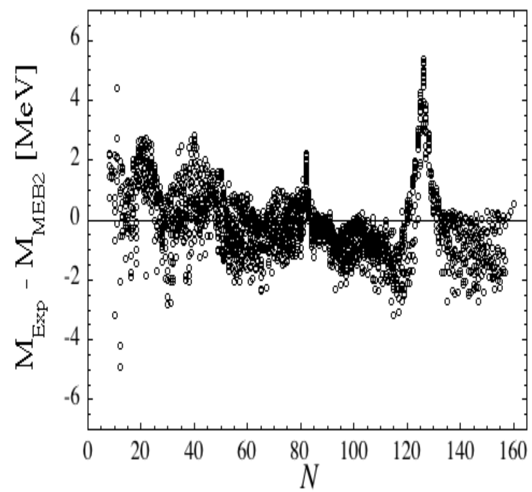


Maximum NS mass : M_{max}~ 2.6 M_o

p-n effective mass splitting in Neutron Matter compared with Dirac-BHF calculations




Exp
DD-MEB1
DD-MEB2
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
116
120
124
128
132
N
$\Delta r^2 = r_c^2(^A\text{Pb}) - r_c^2(^{208}\text{Pb})$ [fm^2]

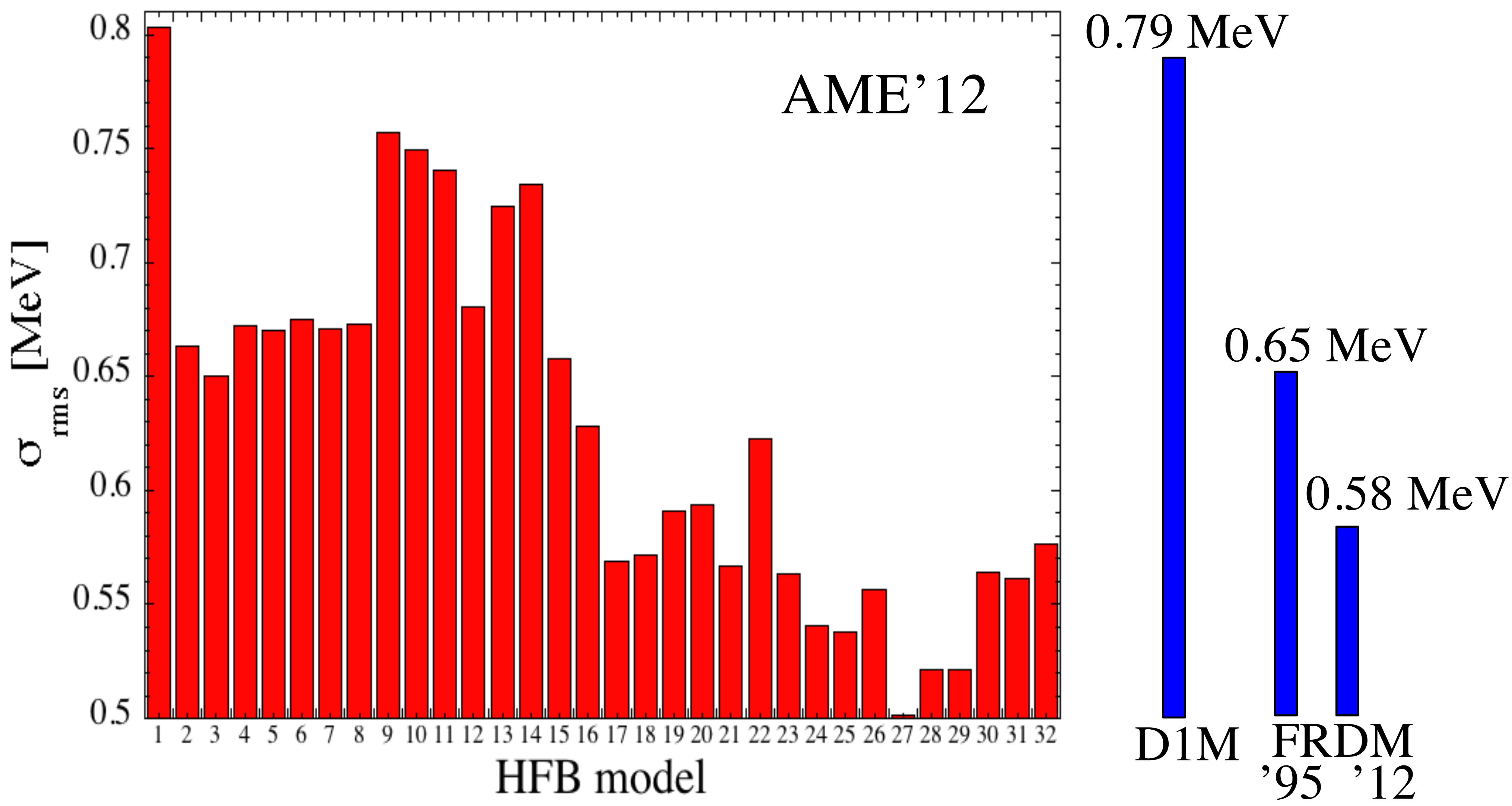

6
4
2
0
-2
-4
-6
0
20
40
60
80
100
120
140
160
N
$M_{Exp} - M_{MEB2}$ [MeV]

Main mass models used for applications

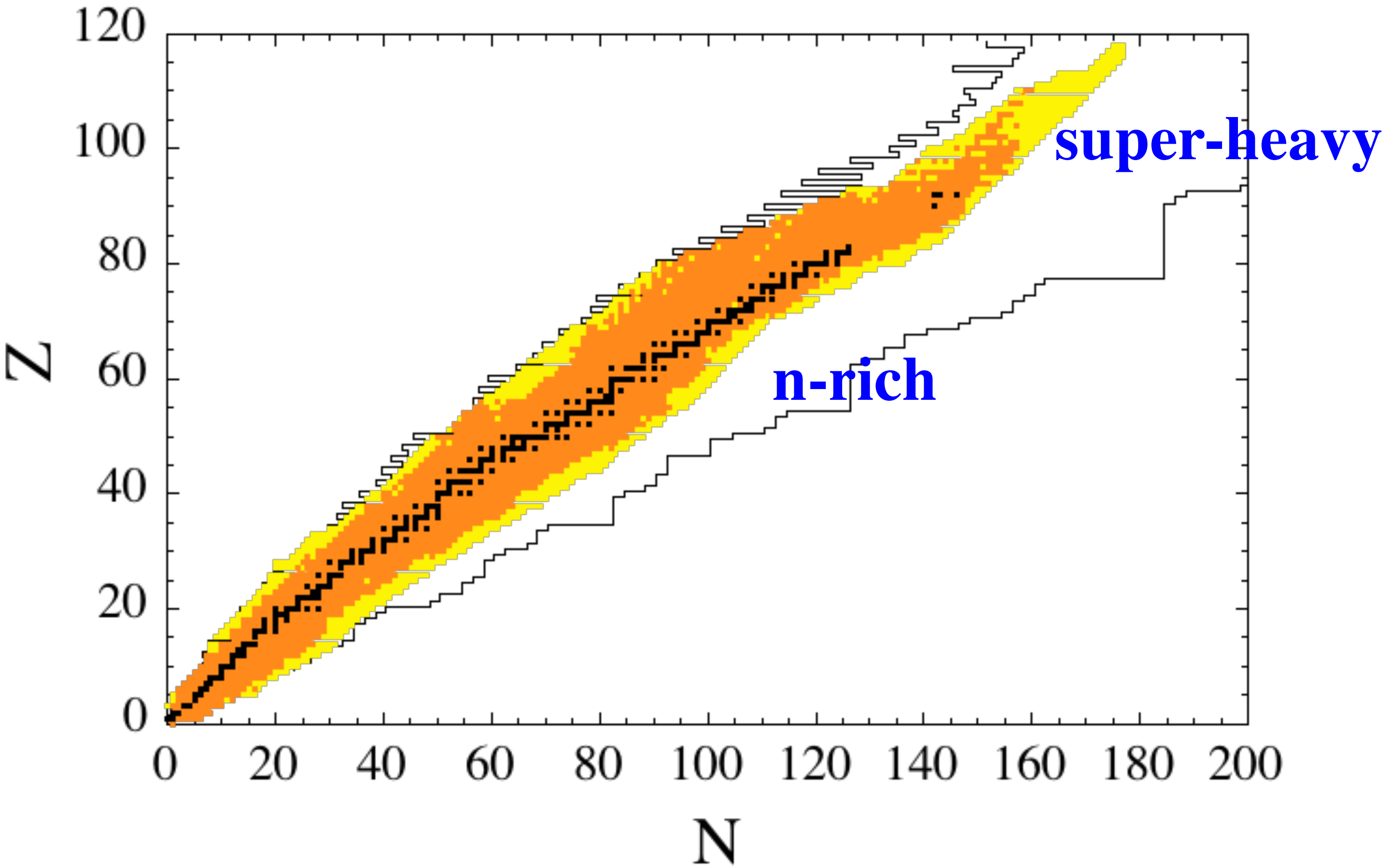
32 Skyrme HFB mass models with $0.5 < \sigma_{rms} < 0.8$ MeV

1 Gogny HFB mass model with $\sigma_{rms} = 0.79$ MeV

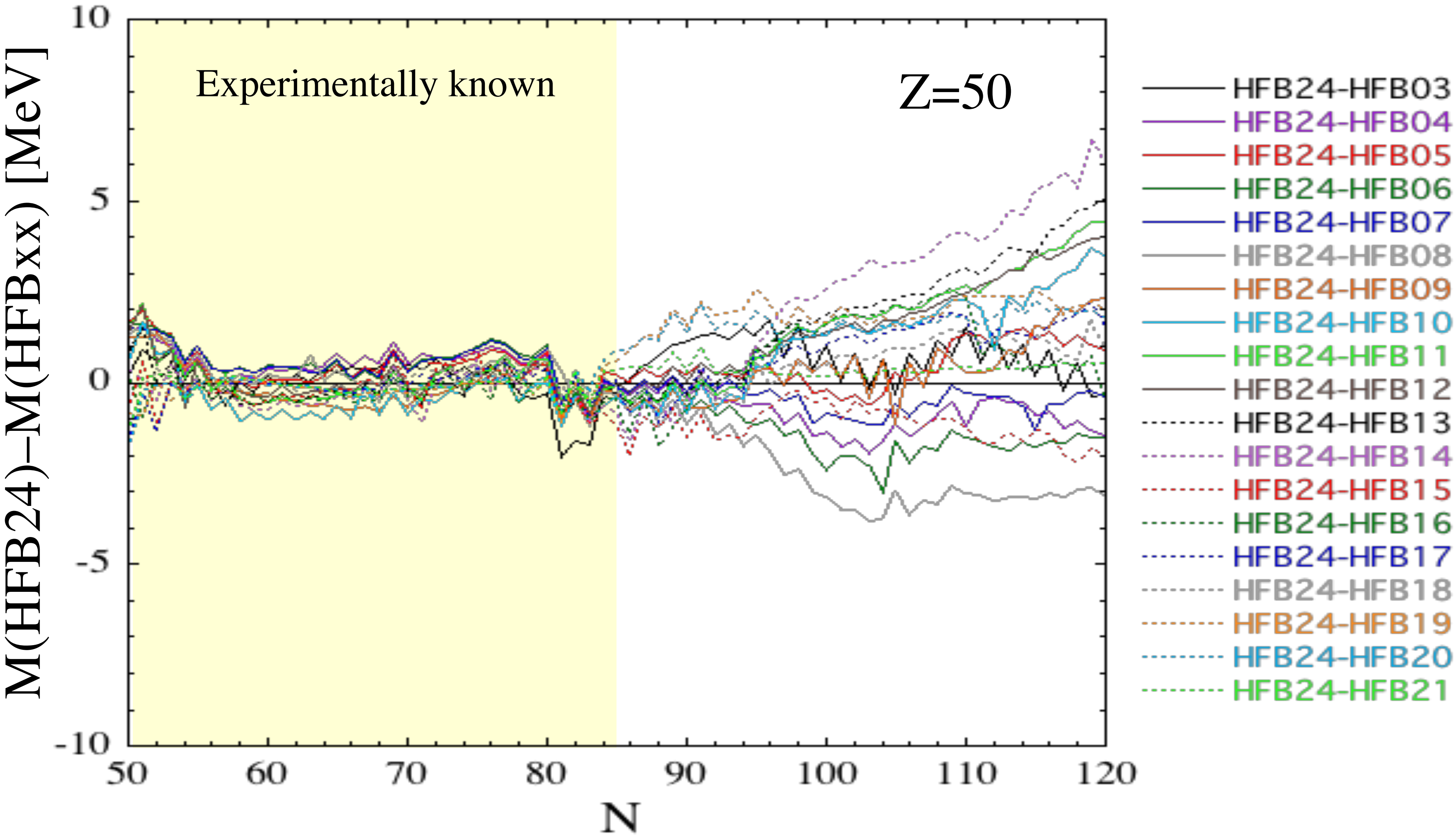
FRDM'12 & 95 mass models with $0.58 < \sigma_{rms} < 0.65$ MeV



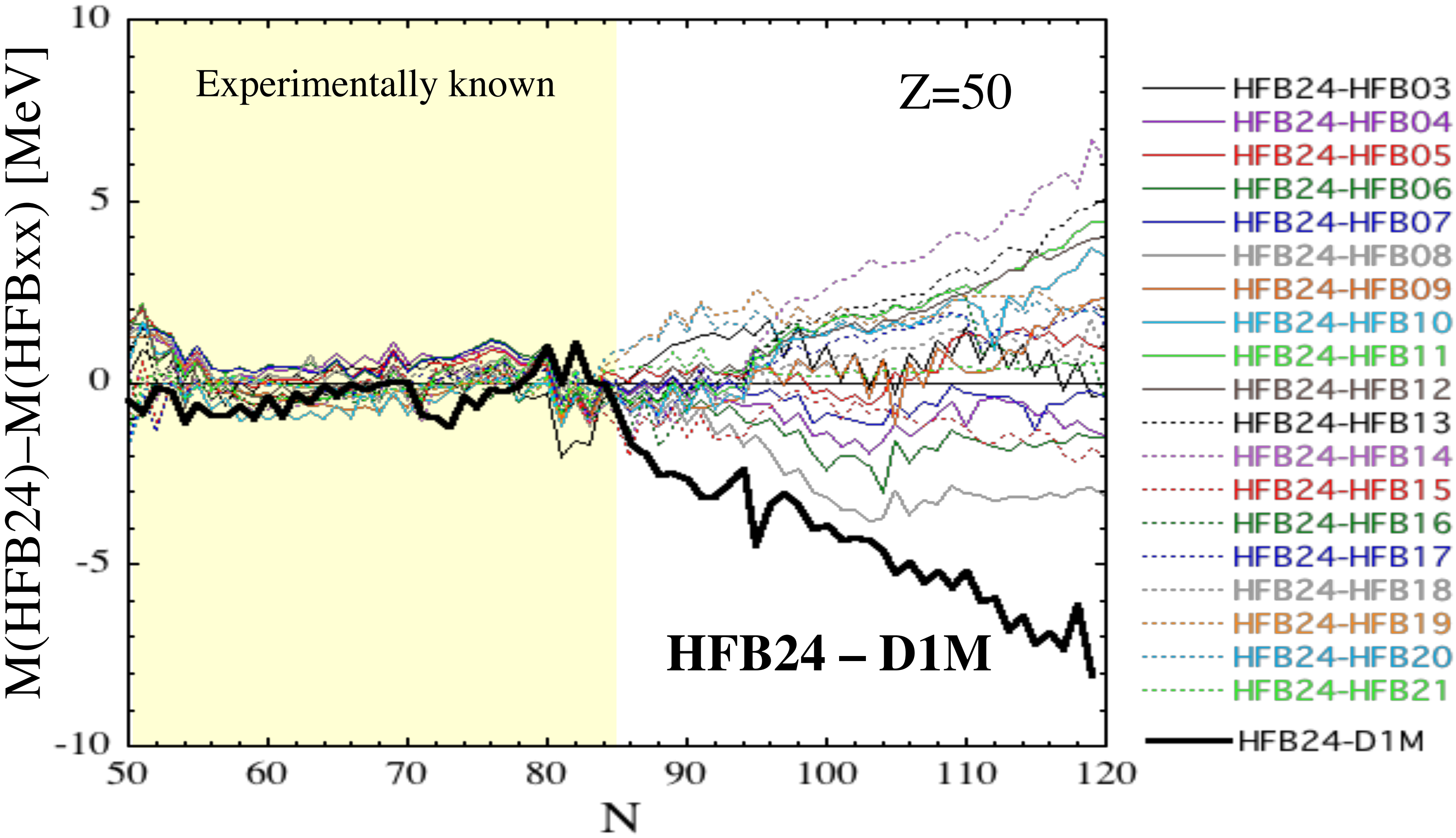
Extrapolation towards experimentally unknown nuclei



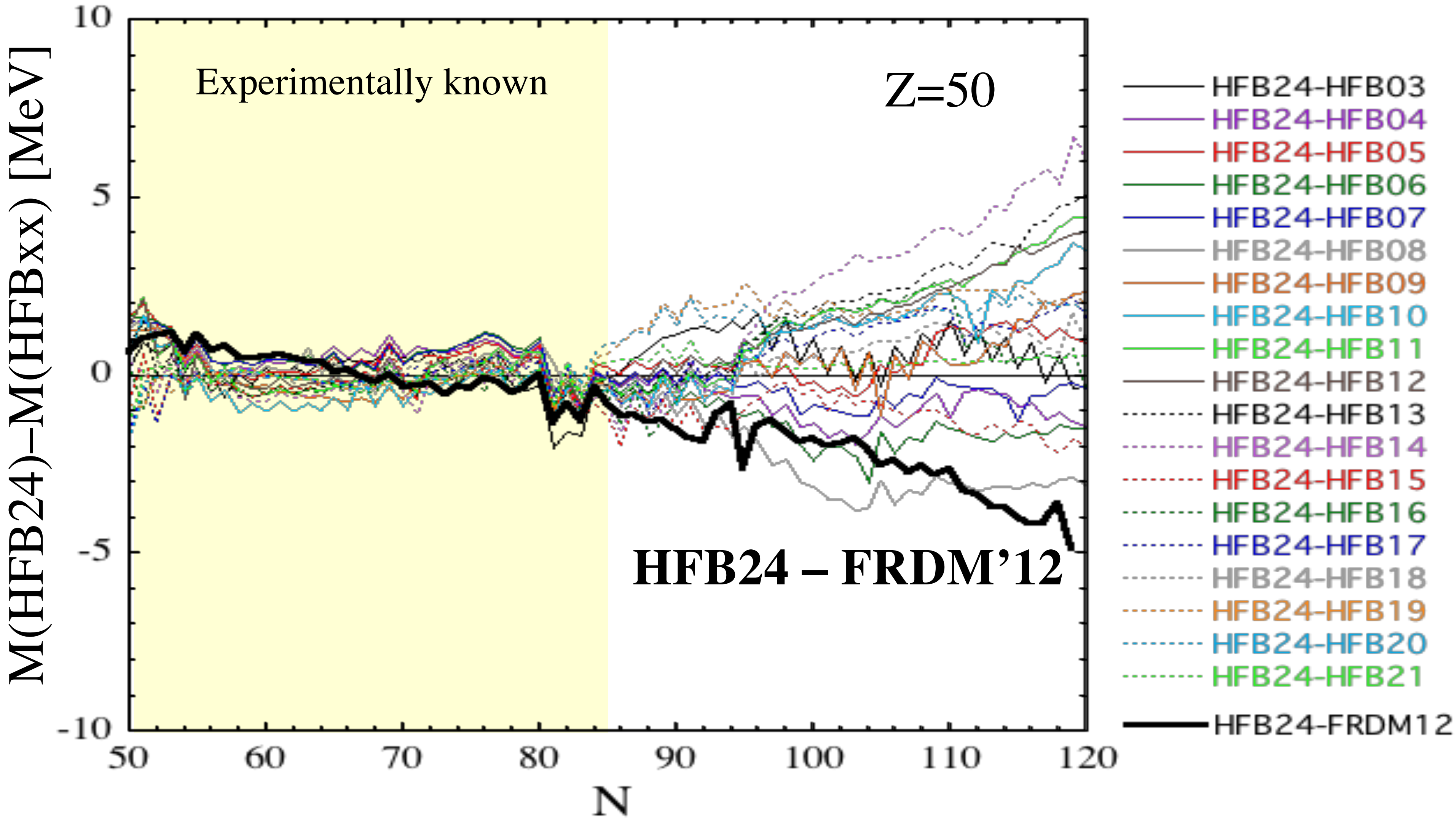
Uncertainties of mass extrapolation in HFB mass models



Uncertainties of mass extrapolation in HFB mass models



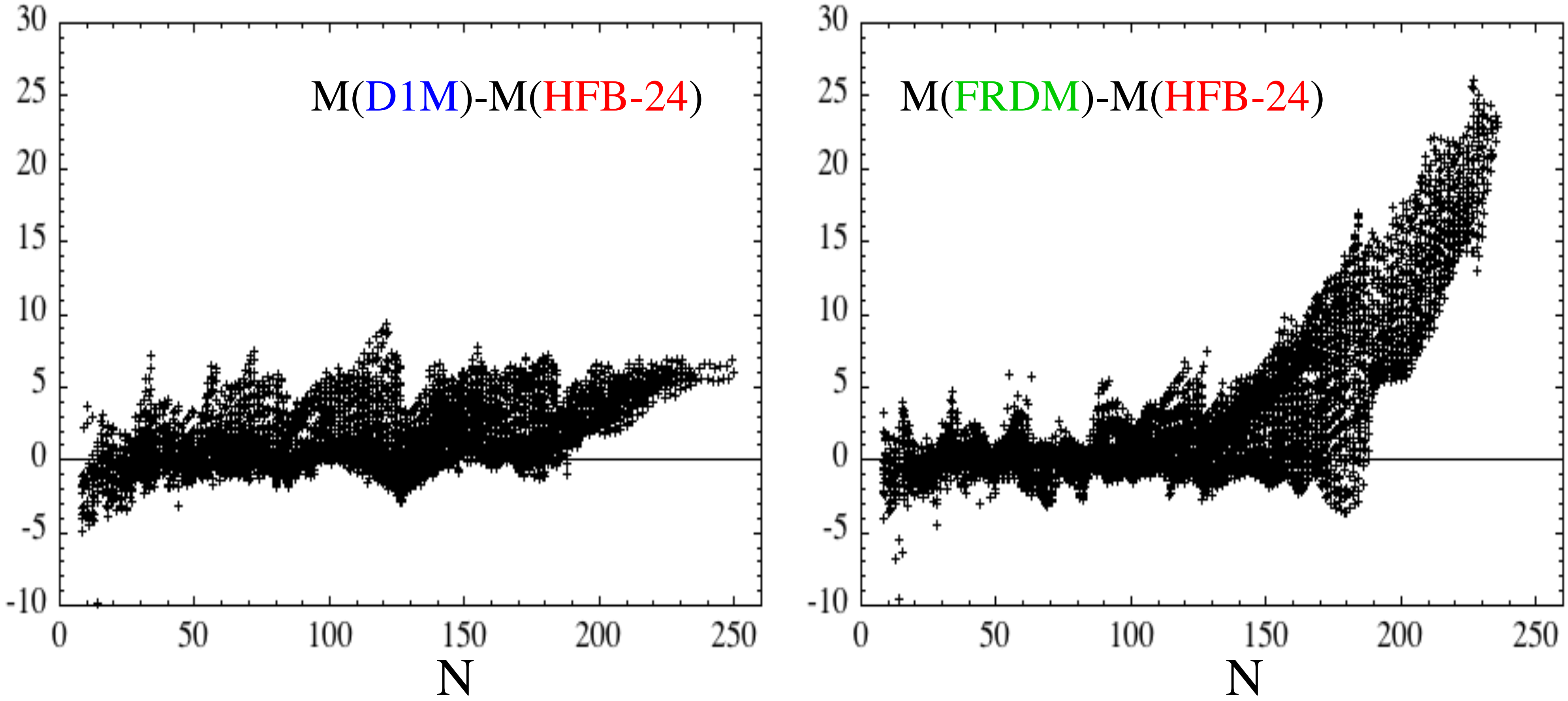
Uncertainties of mass extrapolation in HFB mass models



Comparison between Skyrme-HFB, Gogny-HFB and FRDM

HFB-24: Skyrme HFB mass model	σ(2353 exp masses)=549keV
HFB-D1M: Gogny HFB mass model	σ(2353 exp masses)=789keV
FRDM: Finite Range Droplet mass model	σ(2353 exp masses)=654keV

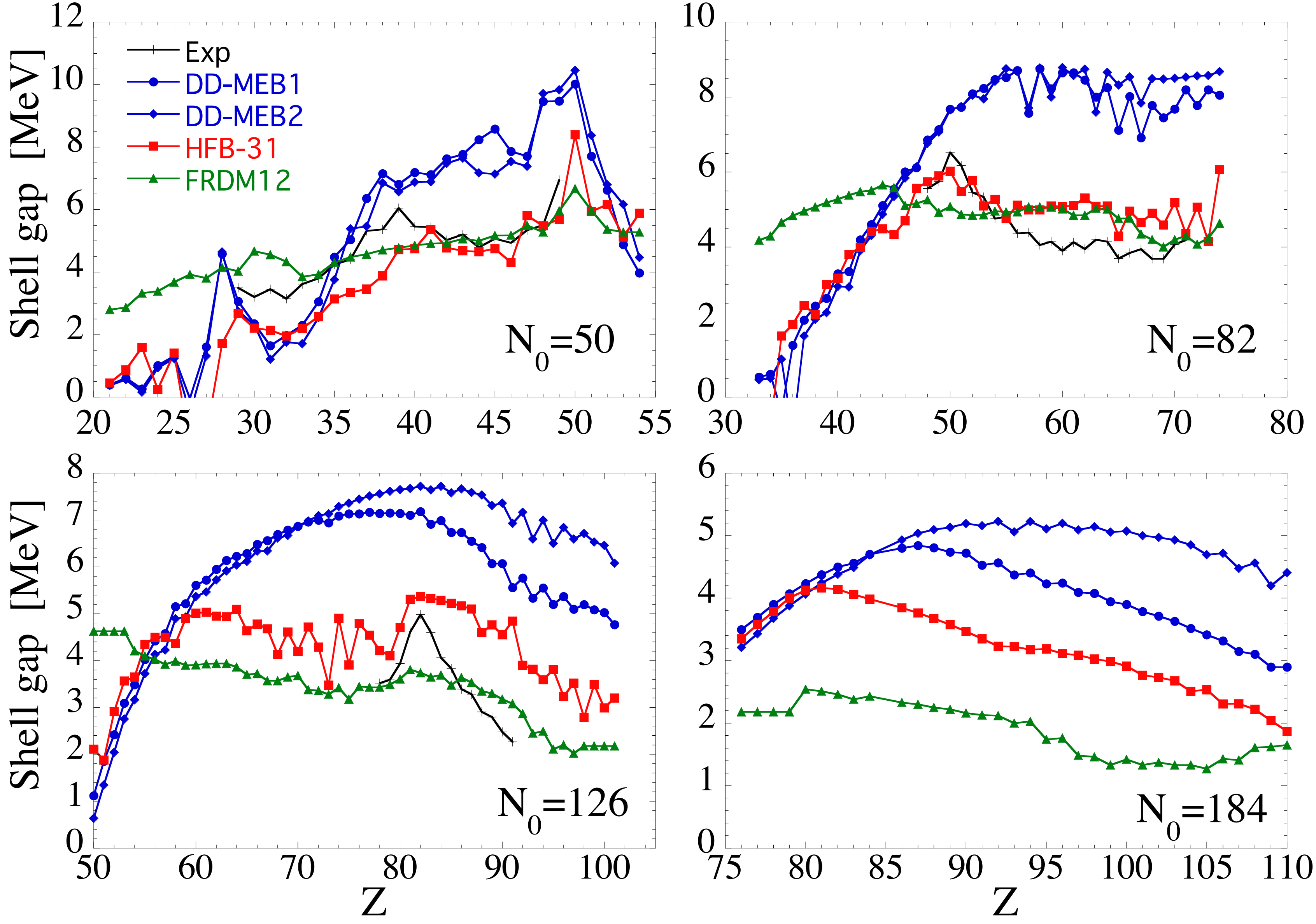
~ 8500 nuclei with $8 \leq Z \leq 110$



Different trends due to different INM, shell & correlation energies

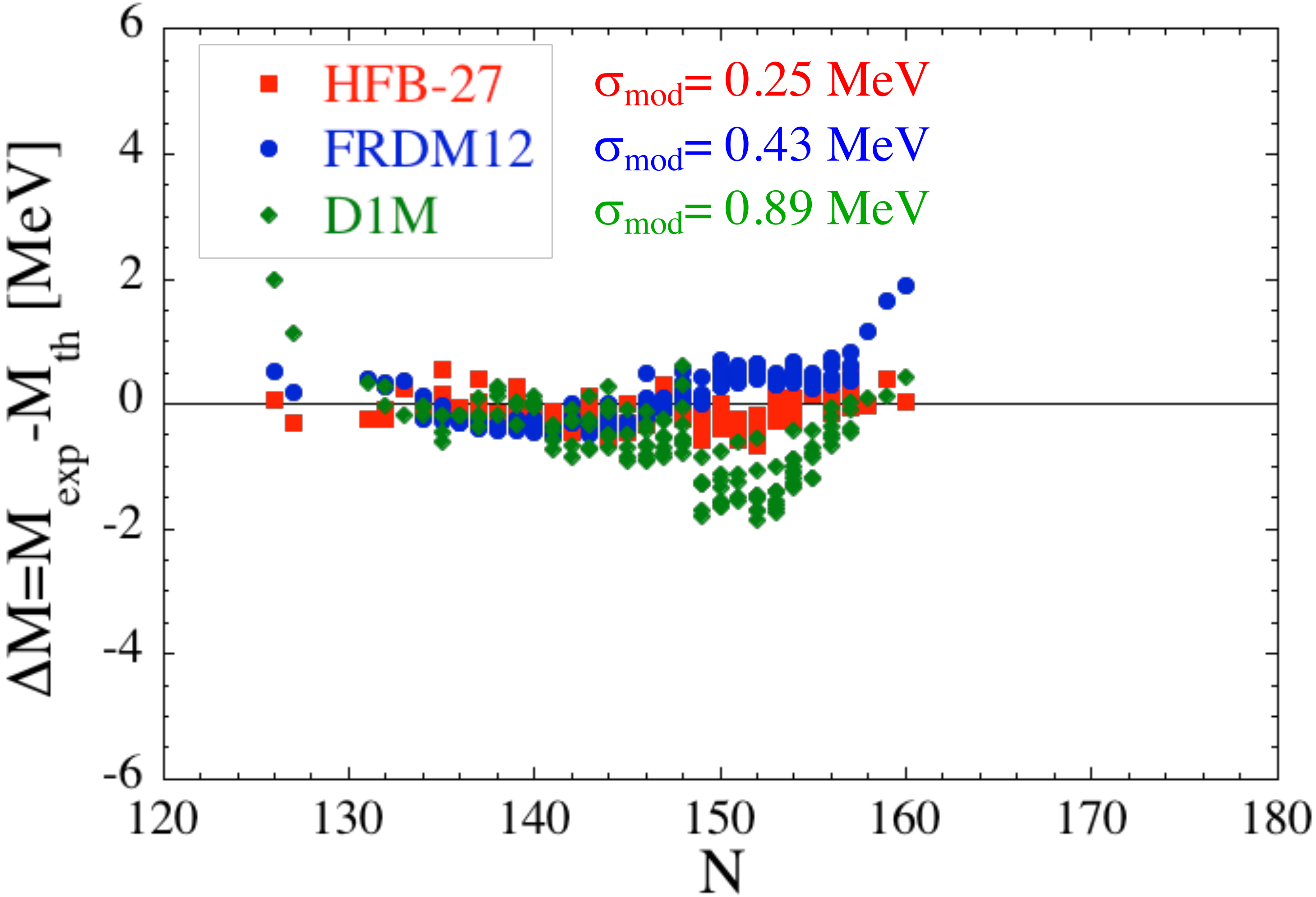
NEW effort within the Relativistic Mean Field model

Relatively strong shell effects $\Delta_n(N_0,Z) = S_{2n}(N_0,Z) - S_{2n}(N_0+2,Z)$



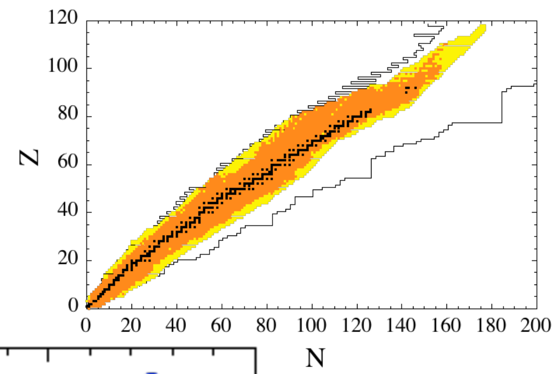
TOWARDS SUPER-HEAVY NUCLEI

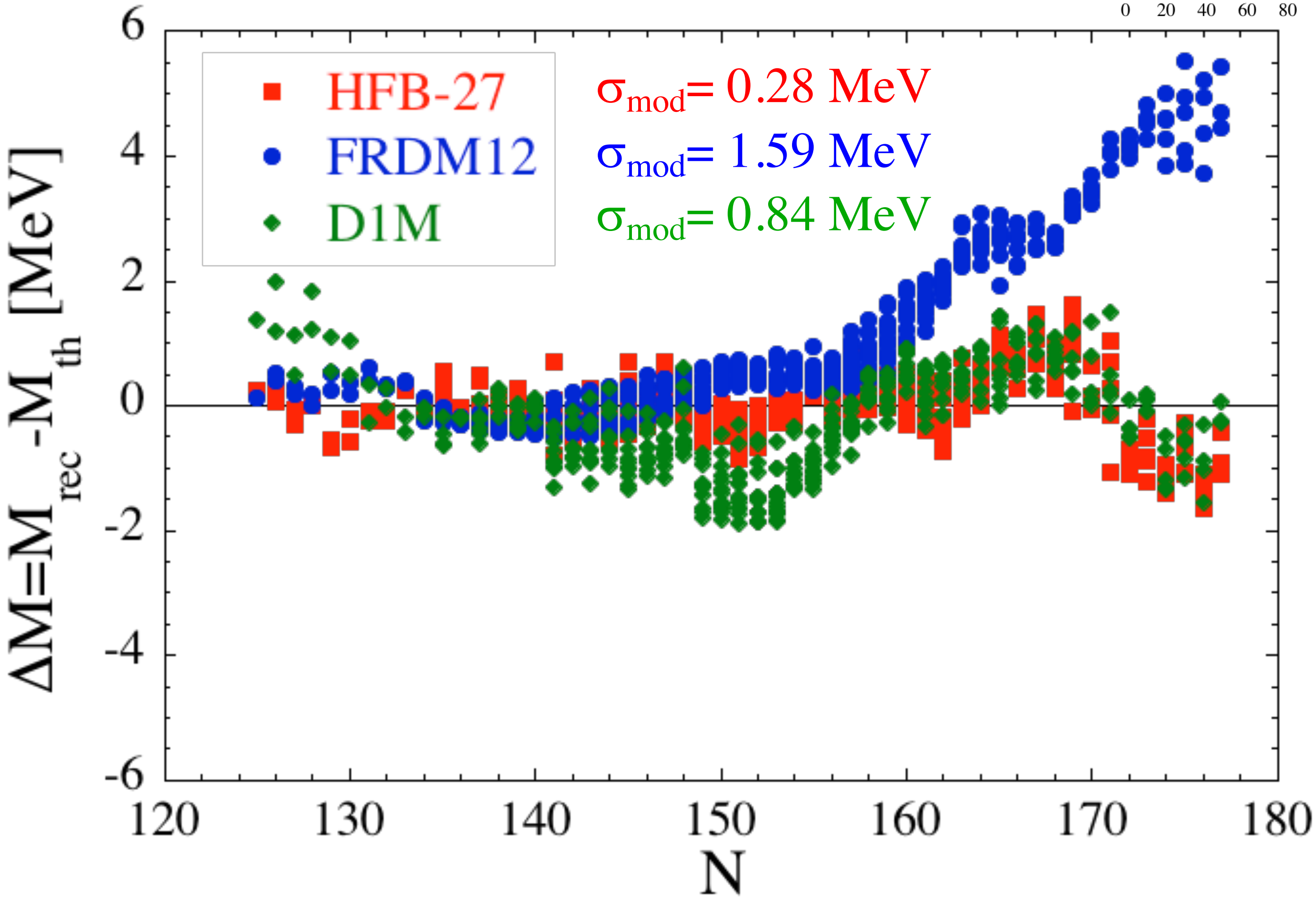
146 nuclei with $92 \leq Z$ (≤ 110) in the AME'12



410 nuclei with 92 ≤ Z ≤ 118

Exp + Estimated by AME'12 systematics

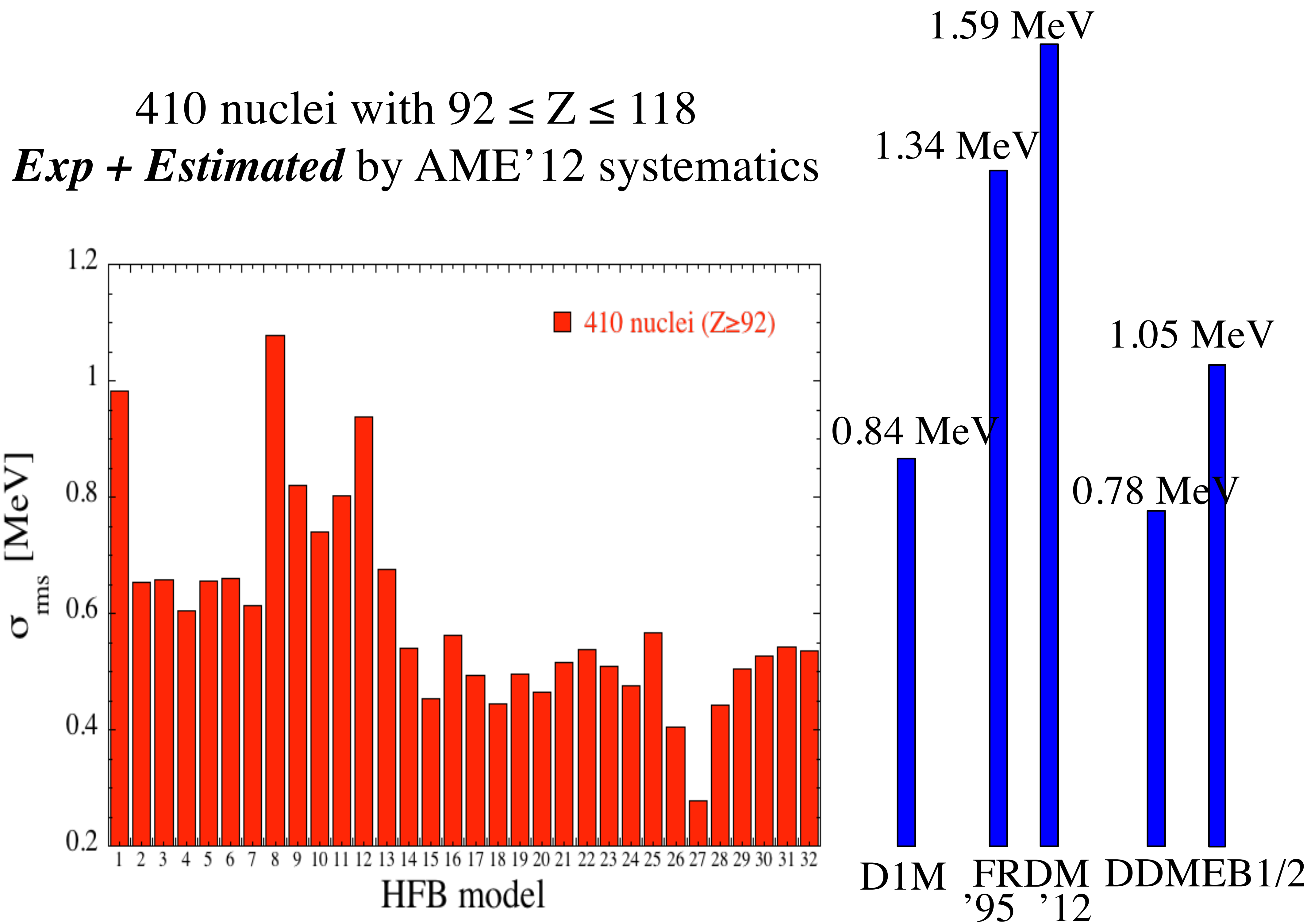




… towards N=184 shell closure !?!

410 nuclei with 92 ≤ Z ≤ 118

Exp + Estimated by AME’12 systematics



Future challenges for modern mass models

1. **To include the state-of-the-art theoretical framework**
   - To include explicitely correlations (quadrupole, octupole, …)
     → GCM
   - To include relevant degrees of freedom for deformation (triaxility, l-r symmetry, …)
   - To include "proper" description for odd nuclei, N~Z nuclei
   - To include "extended" interactions (tensor, D2-type, …)
2. **To reproduce as many "observables" as possible ("exp." & "realistic")**
   - Experimental masses (rms < 0.8 MeV)
   - Radii, density distributions, and neutron skins
   - Fission and isomers
   - Infinite nuclear matter properties (Symmetric, Neutron matter)
   - Giant resonances
   - Spectroscopy
   - Neutron Star maximum mass
   - Etc…
3. **To consider different frameworks**
   - Relativistic, non-relativistic
   - Skyrme-type, Gogny-type (D1 & D2 interactions), DDME, PC, …
   - Non-empirical, Shell Model, etc…

Conclusions

Experimental masses of more than 2300 nuclei provide a wealth of information that can help us to further constrain theoretical models and shed light on microscopic physics

The future challenge lies in a unified description of masses and all other nuclear properties, such as deformations, densities, quadrupole moments, spins, nuclear and neutron matter properties, but also Level Densities, Fission, GR…

A new generation of mass models beyond mean field is emerging
A mass model within the relativistic mean field still need to be built

More experimental data & theoretical works are needed