Exotic hadrons from lattice QCD

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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Dynamics of highly unstable exotic light nuclei and few-body systems
@Saclay, France (Jan. 30 -- Feb. 3, 2017)

Dynamics of strong interaction

★ **underlying theory = QCD (quantum chromodynamics)**

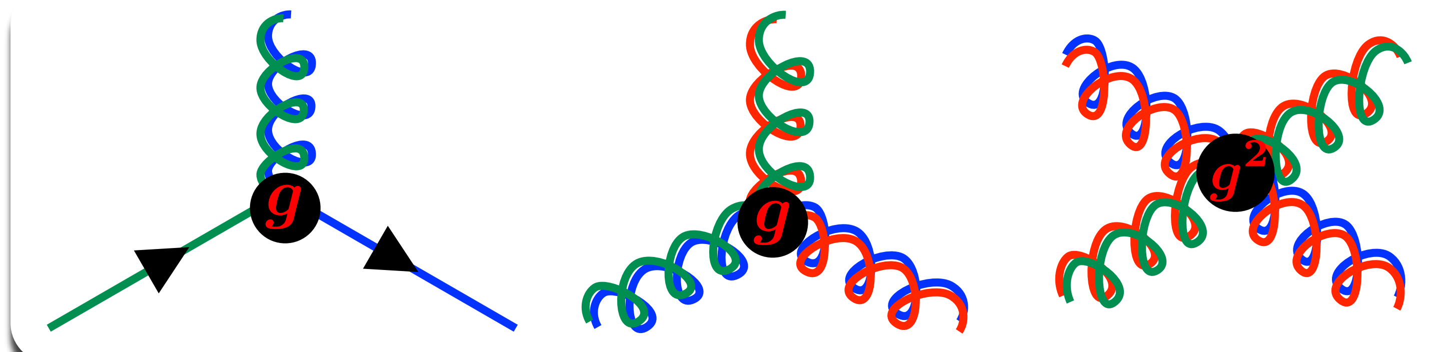
$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \bar{q}\gamma^\mu(i\partial_\mu - g t^a A^a_\mu)q - m\bar{q}q$$

- basic parameters : coupling g + quark masses m (u, d, s, c, b)

✤ **dynamics**

✓ non-Abelian gauge theory

✓ strong coupling $\alpha_s = g^2/4\pi \sim 1$

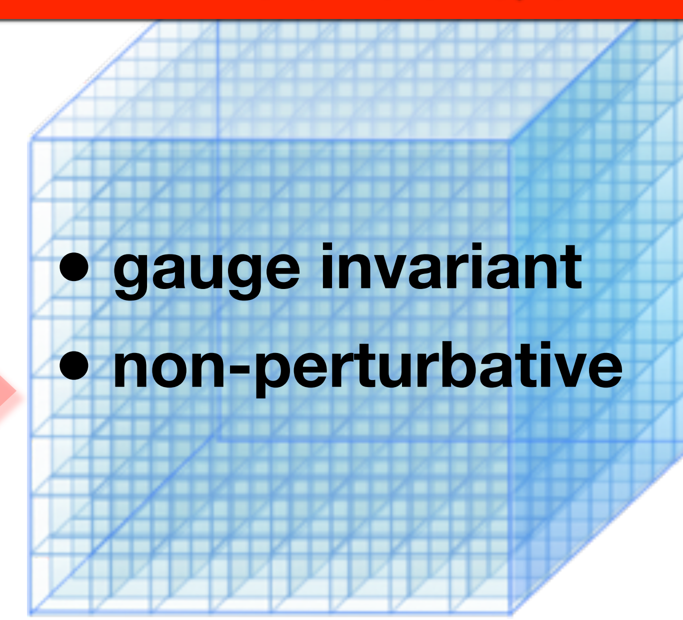


non-Abelian & non-perturbative dynamics

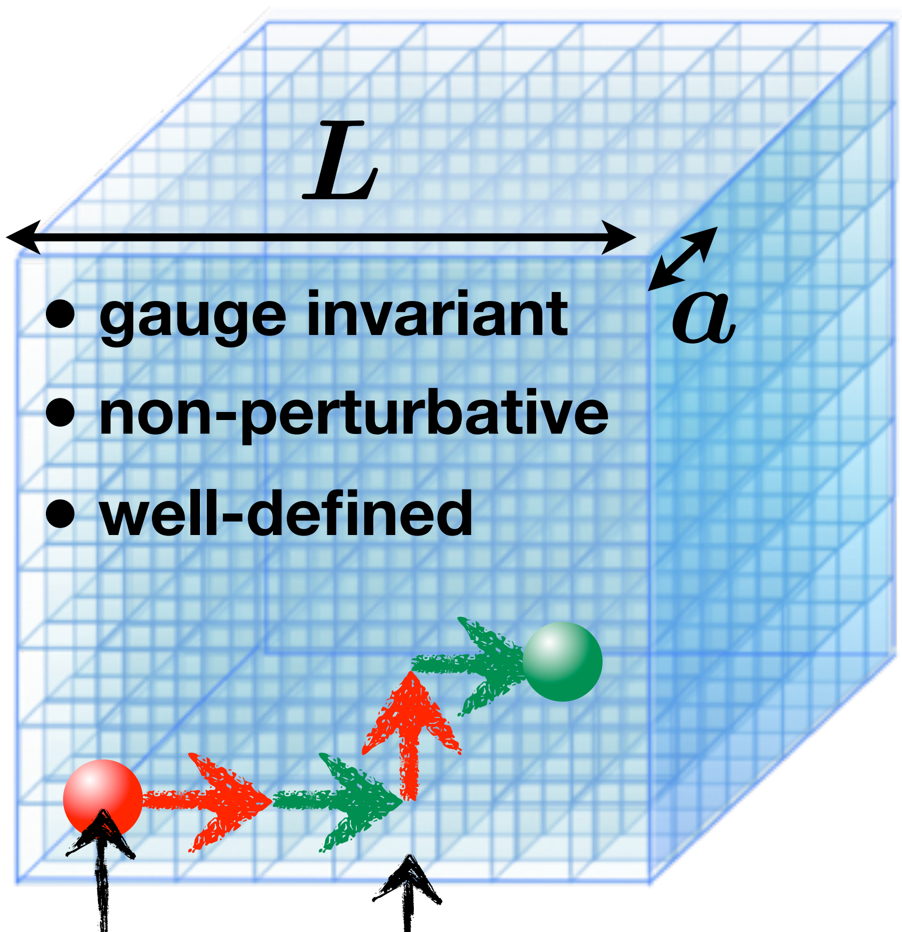
✦ **non-trivial vacuum** in low-energy regime

- color confinement
- mass gap
- condensate, ...

1st principle calculation
= Lattice QCD



Lattice QCD = 1st principle calculation of QCD



- gauge invariant
- non-perturbative
- well-defined

quarks ***q***
on the sites

gluons ***$U_\mu = exp(iaA_\mu)$***
on the links

input parameters

- hopping parameter κ_q --> m_q
- gauge coupling g --> a

- **path integral formulation**

$$Z = \int [dU][d\bar{q}dq]\ \exp\left(-\int d\tau d^3x\,(\mathcal{L}_g + \mathcal{L}_q)\right)$$

$$= \int [dU]\ det\,[D(U, m_q)]\,\exp\left(-\int d\tau d^3x \mathcal{L}_g\right)$$

dynamical quark-antiquark loop

typically $(30\text{-}100)^4 \sim 10^{6\text{-}8}$ sites employed

Monte Carlo Simulations

observables <O>

- ensemble average of O

$$\langle \mathcal{O} \rangle = \langle 0|\mathcal{O}|0\rangle$$

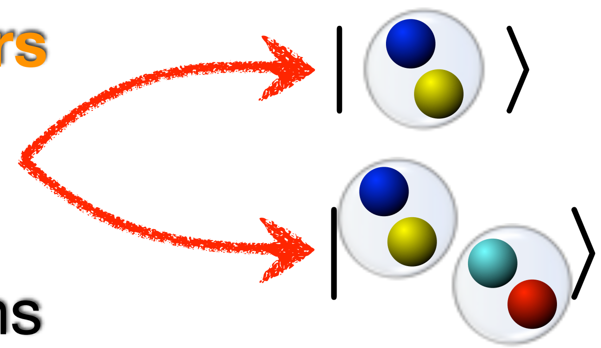
$$= \int [dU] det\,[D(U, m_q)]\, e^{-\int d\tau d^3x \mathcal{L}_g} \mathcal{O}(U, m_q)$$

How to extract observable from lattice QCD

Conventional approach: temporal correlations

1) chose your favorite operators

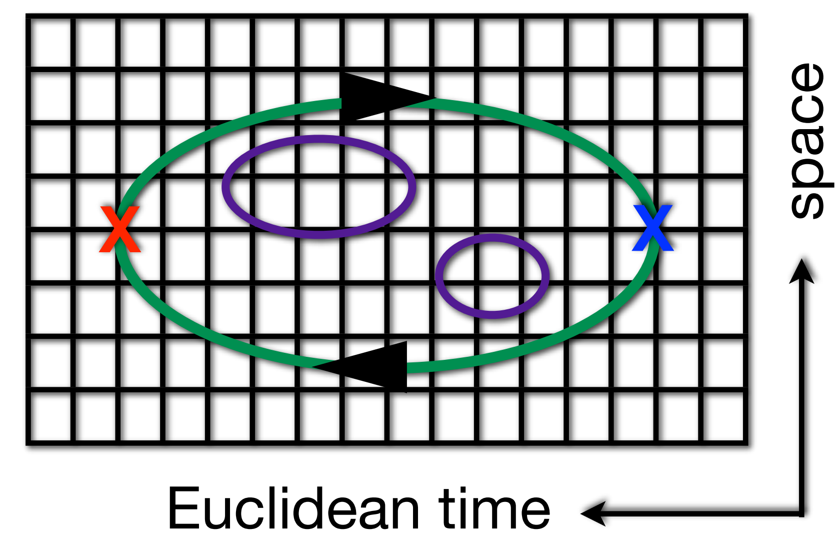
$$\phi(x) = \bar{q}(x)\Gamma q(x)$$

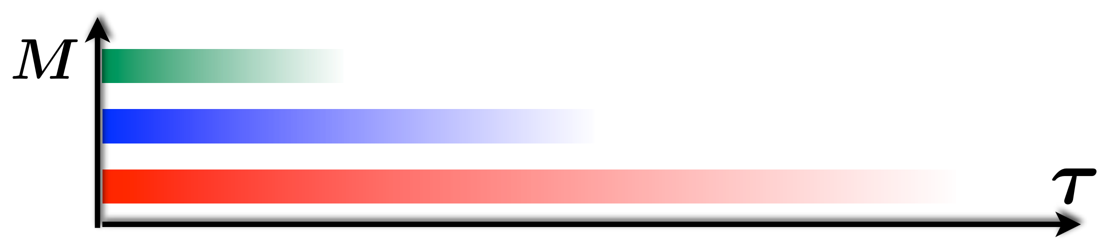


2) measure hadron 2pt functions

$$\langle C(\tau)\rangle = \sum_{\vec{x}} \langle 0|\phi(x)\phi(0)^\dagger|0\rangle$$

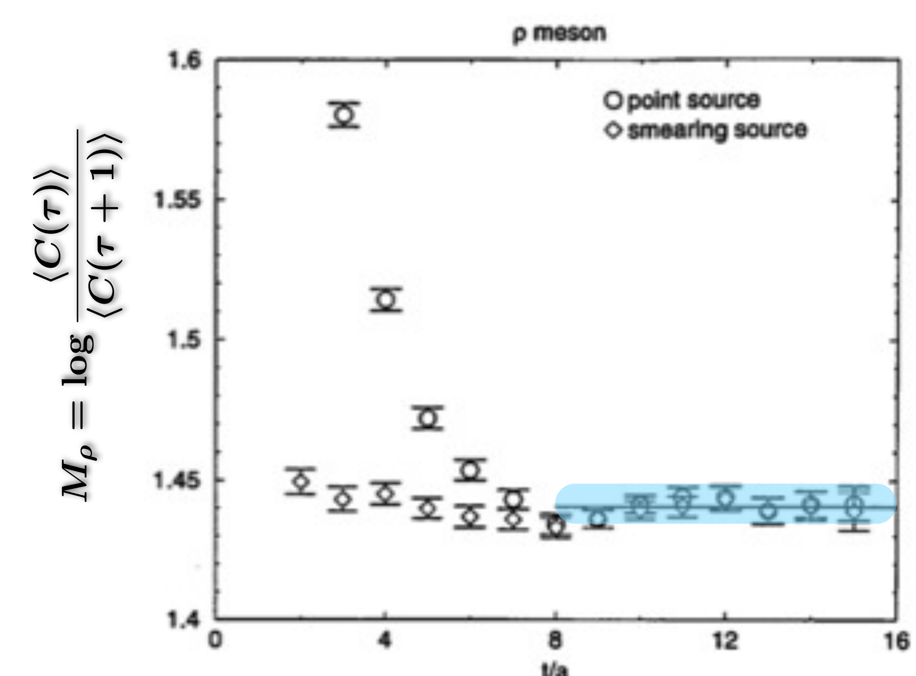
$$= A_1 e^{-M_1\tau} + A_2 e^{-M_2\tau} + A_3 e^{-M_3\tau} + \cdots$$





3) make effective mass plot for hadron masses

$$M = \log \frac{\langle C(\tau)\rangle}{\langle C(\tau+1)\rangle}$$



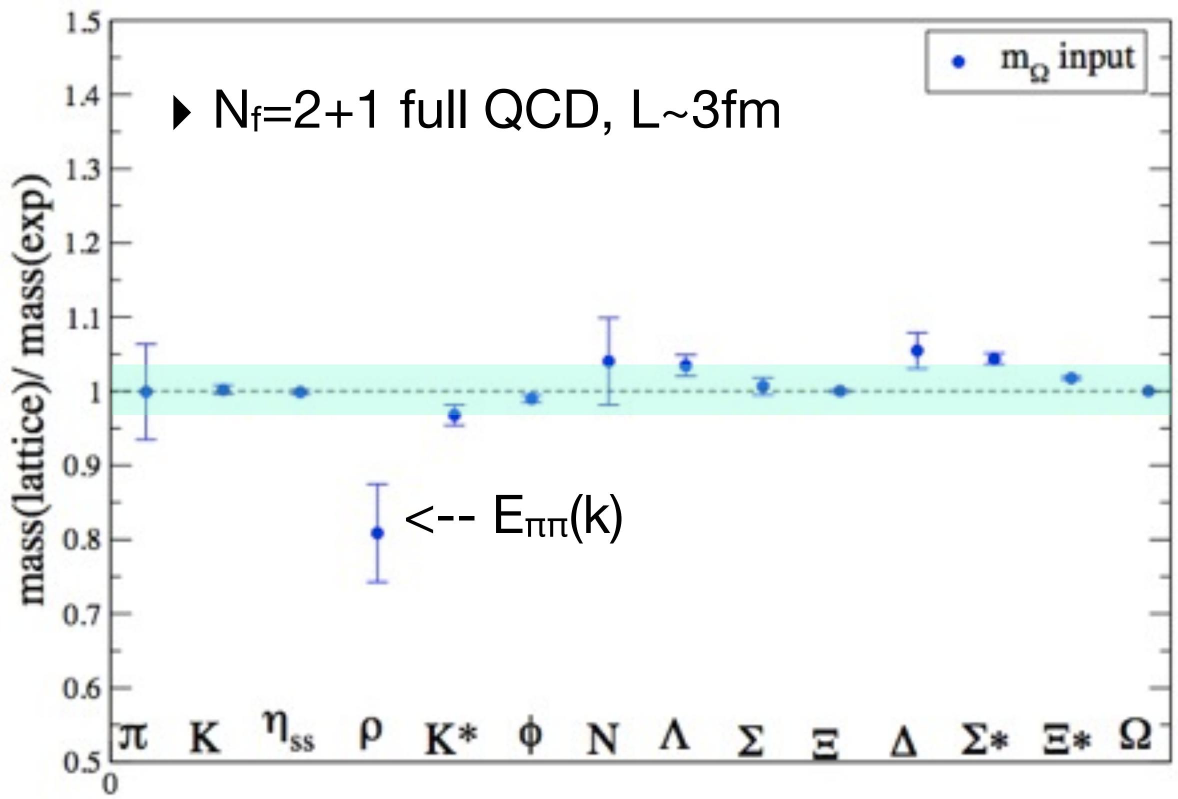
4) obtain masses from plateau (large τ region for ground state)

excited states are extracted using diagonalization of correlation functions

Single hadron spectroscopy from LQCD

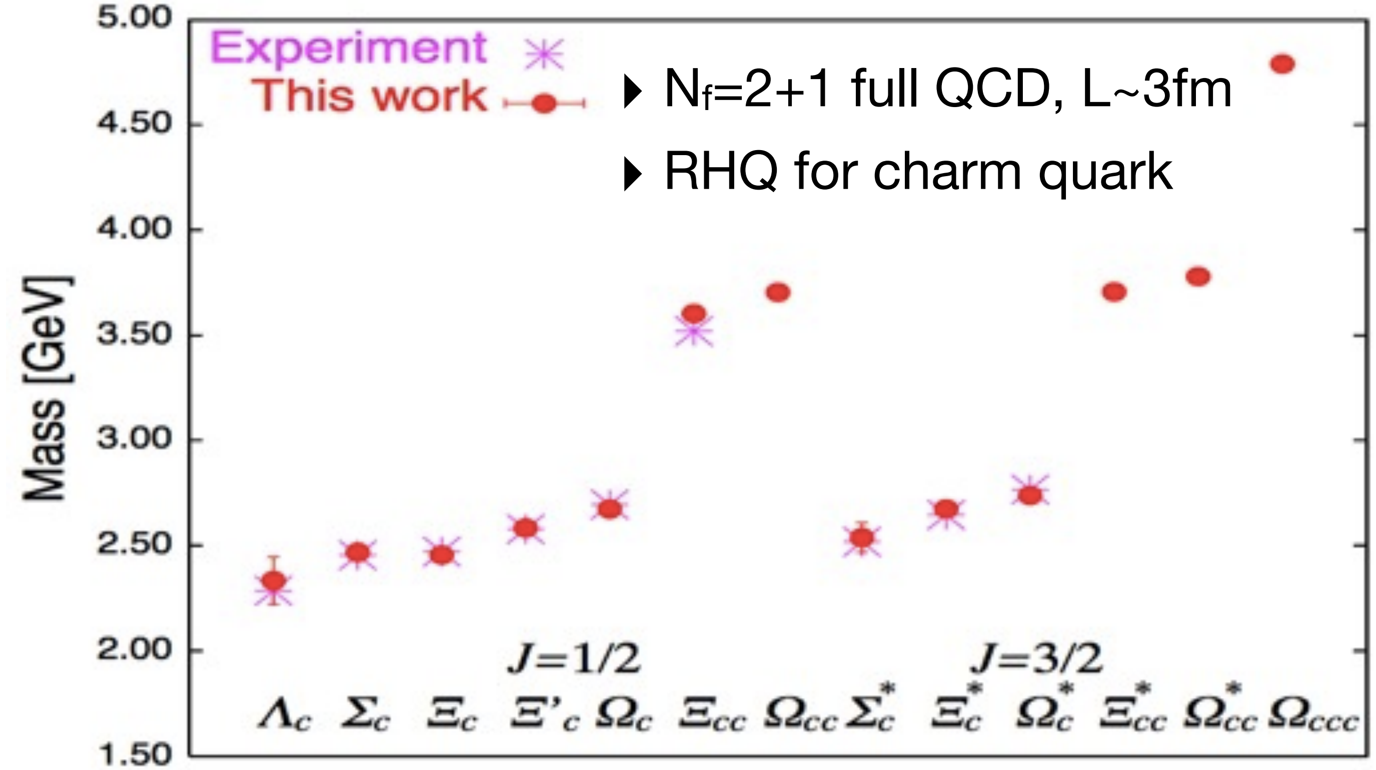
★ **Low-lying hadrons on physical point (physical m_q)**

light-quark sector

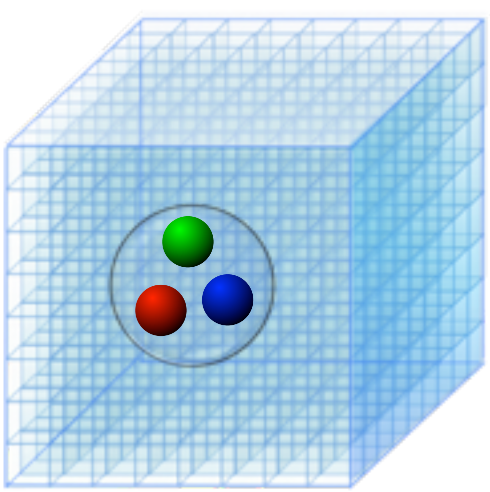


Aoki et al. (PACS-CS), PRD81 (2010).

charm baryons



Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).



a few % accuracy already achieved for single hadrons

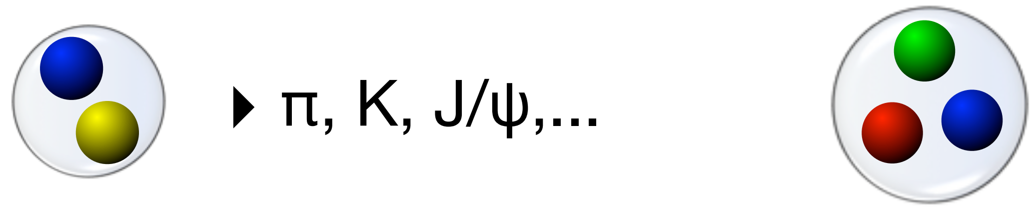
LQCD now can predict undiscovered charm hadrons (Ξ^*_{cc}, Ω_{ccc},...)

➡ **Next challenge in spectroscopy : hadron resonances**

Hadrons (baryons & mesons)

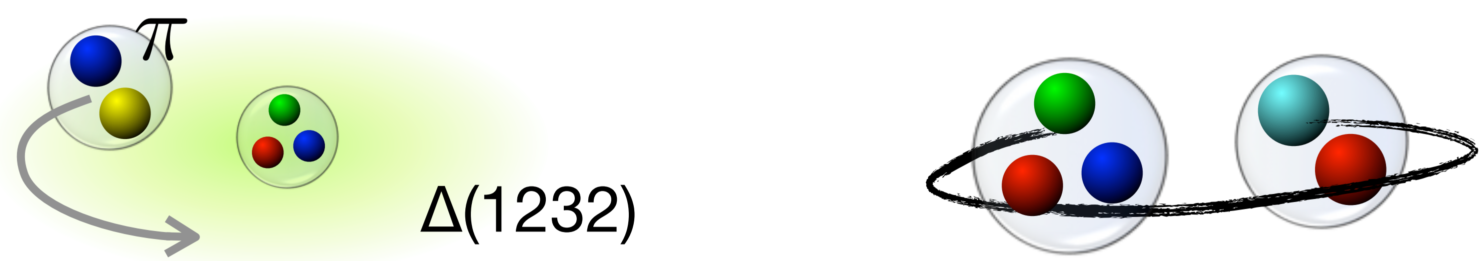
- Low-lying hadrons are established as simple $q\bar{q}$ & qqq

  ★ wave function renormalization in QCD, quark model



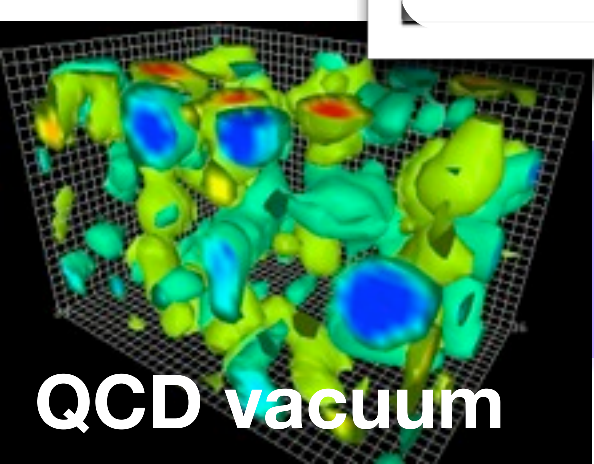
‣ π, K, J/ψ,...   ‣ N, Λ, Σ, Ξ, ...

- Several excited states are considered to be...



Δ(1232)   Λ(1405)

- “Exotic” hadrons are not established yet.

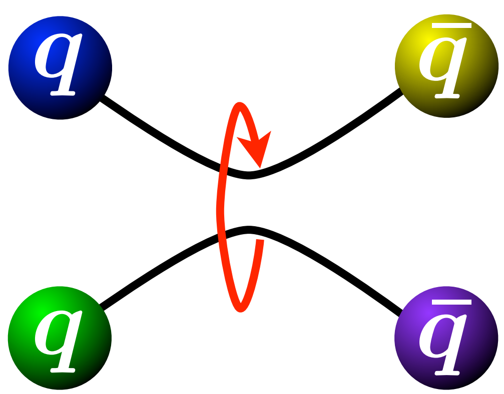


Miserable convergence of Fock space expansion...

$$|B=1, J^P\rangle = |3q\rangle + |\text{meson} + 3q\rangle + |\text{penta-quark}\rangle + \cdots$$

Why exotic hadrons?

★ Exotic hadrons are “colorful” --> hint for color confinement



$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

➡ Color non-singlet $(qq)_6$, $(qq)_8$ configurations are allowed only in multi-quarks

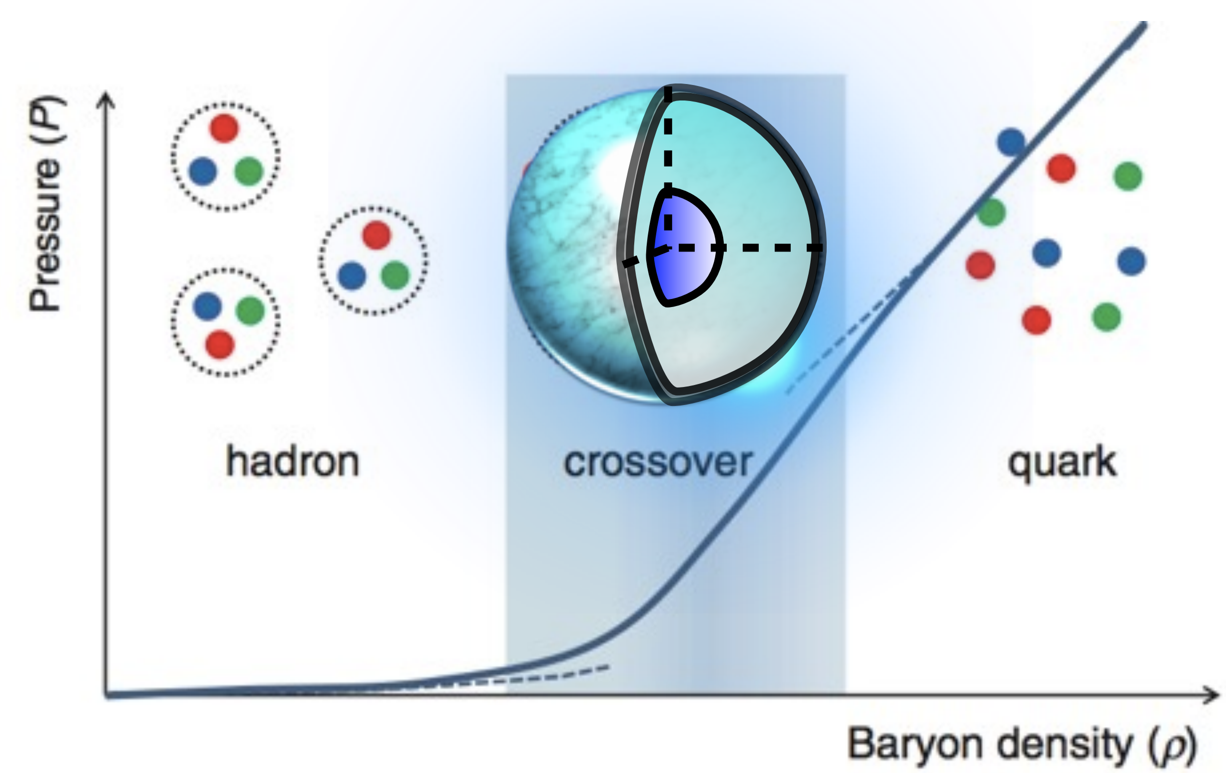
★ Color non-singlet interactions play important role for dense matter through condensates

➡ hadron phase (confinement)

$$\langle q\bar{q} \rangle \neq 0$$

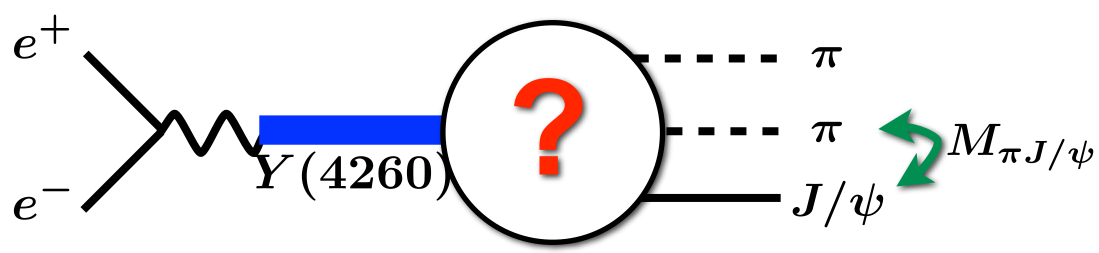
➡ quark phase (deconfinement)

$$\langle qq \rangle_\rho \neq 0$$

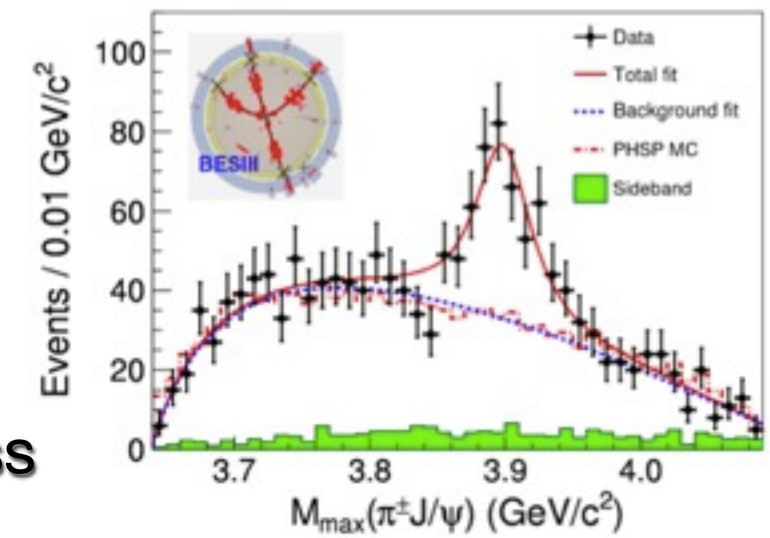


Where exotic hadrons expected?

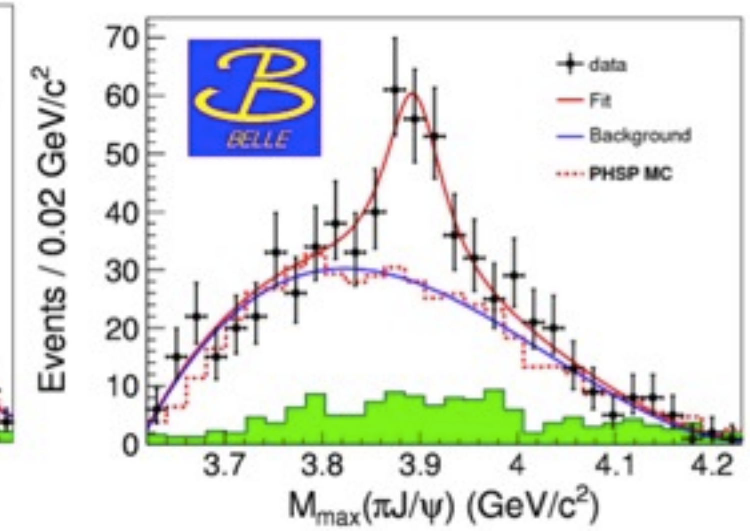
- Tetra-quark candidate $Z_c(3900)$



BESIII Coll., PRL110 (2013).



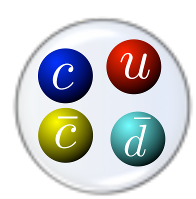
Belle Coll., PRL110 (2013).



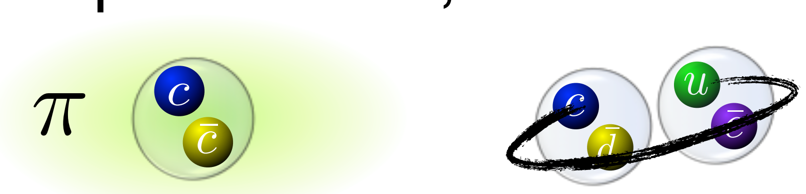
- peak observed in $\pi^{\pm}J/\psi$ invariant mass
- tetra-quark candidate ($c^{bar}c$ $u^{bar}d$)
- $J^P=1^+$, M ~ 3900, Γ ~ 60 MeV (Breit-Wigner fit) --> just above open charm $D^{bar}D^*$ threshold

★ Structure of $Z_c(3900)$ by models

- tetra-quark? Maiani et al. (2013).



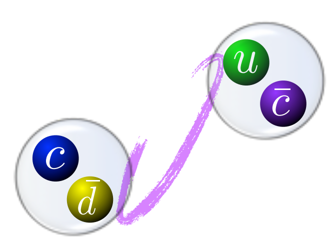
- J/ψ + π cloud, $D^{bar}D^*$ molecule?



Voloshin (2008),
Nieves et al. (2011),
+ many others

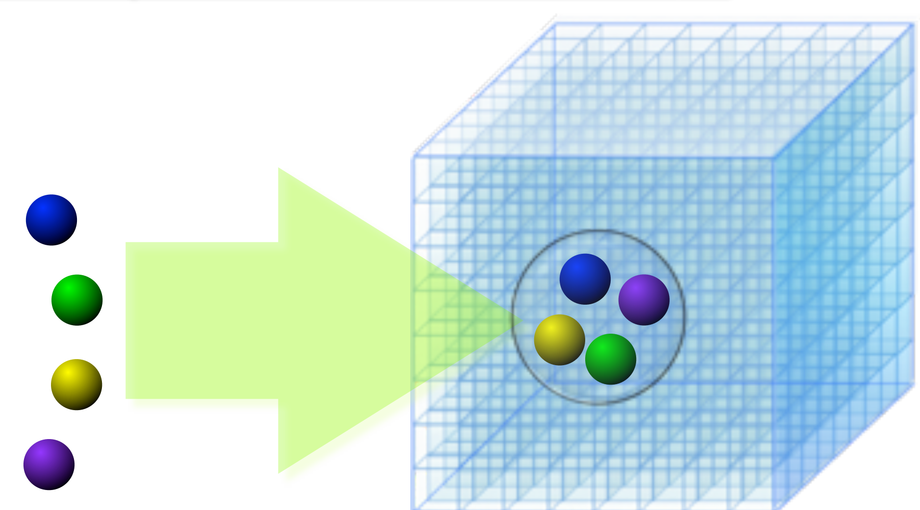
- cusp at $D^{bar}D^*$ threshold

Chen et al. (2013), Swanson (2015).



➡ **poor information on interaction**

★ $Z_c(3900)$ from lattice QCD



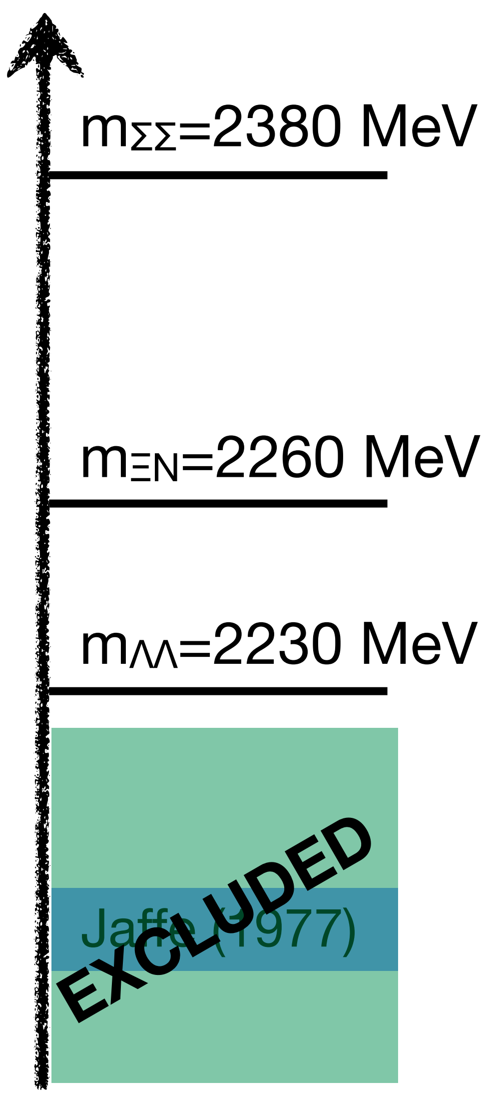
Where exotic hadrons expected?

★ H-dibaryon (flavor singlet uuddss configuration)

- no Pauli blocking in flavor SU(3) singlet
- attractive color magnetic int. (quark model)

➡ **“Perhaps a Stable Dihyperon”**

by R. Jaffe (1977)



★ Constraint from NAGARA event (2001)

$$\Xi^- + {}^{12}C \to {}_{\Lambda\Lambda}^{\ 6}He + {}^{4}He + t$$

Observation of double-Λ hypernuclear excludes possibility of deeply bound H-dibaryon

★ fate of H-dibaryon from lattice QCD

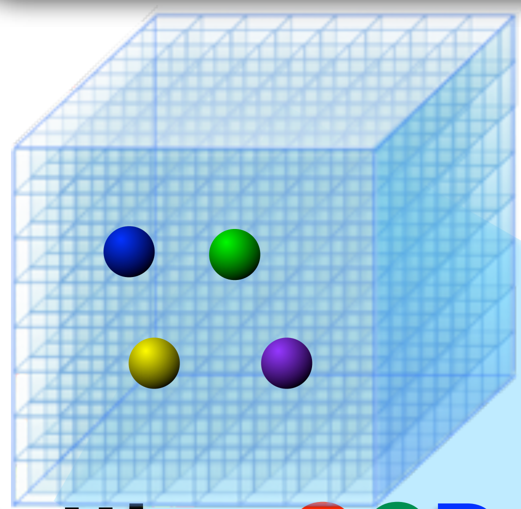
- Is H-dibaryon shallow bound state, resonance, or ...?
- What is structure of H-dibaryon?

  (hexa-quark? molecule?)

Contents

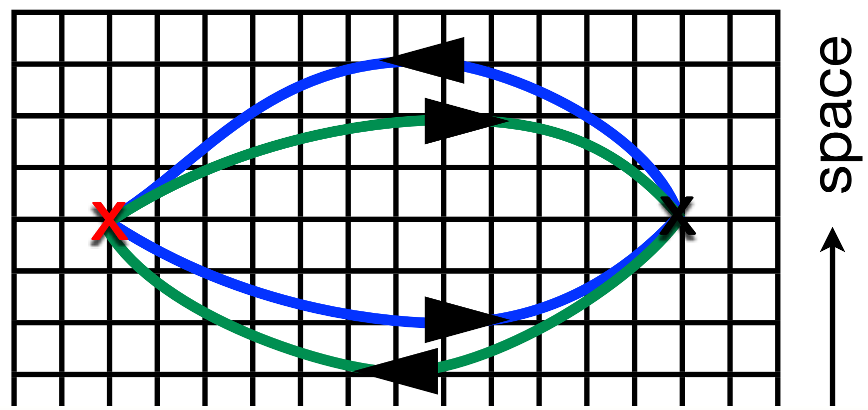
- ✓ Introduction to exotic hadrons
- ✓ How to study exotic hadrons on the lattice?
- ✓ HAL QCD method
- ✓ Nature of exotic candidates from lattice QCD
  - tetra-quark candidate : $Z_c(3900)$
  - hexa-quark candidate : H-dibaryon
- ✓ Summary

How to study exotics on the lattice?



lattice QCD

Lattice QCD spectroscopy

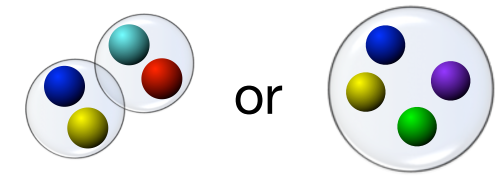


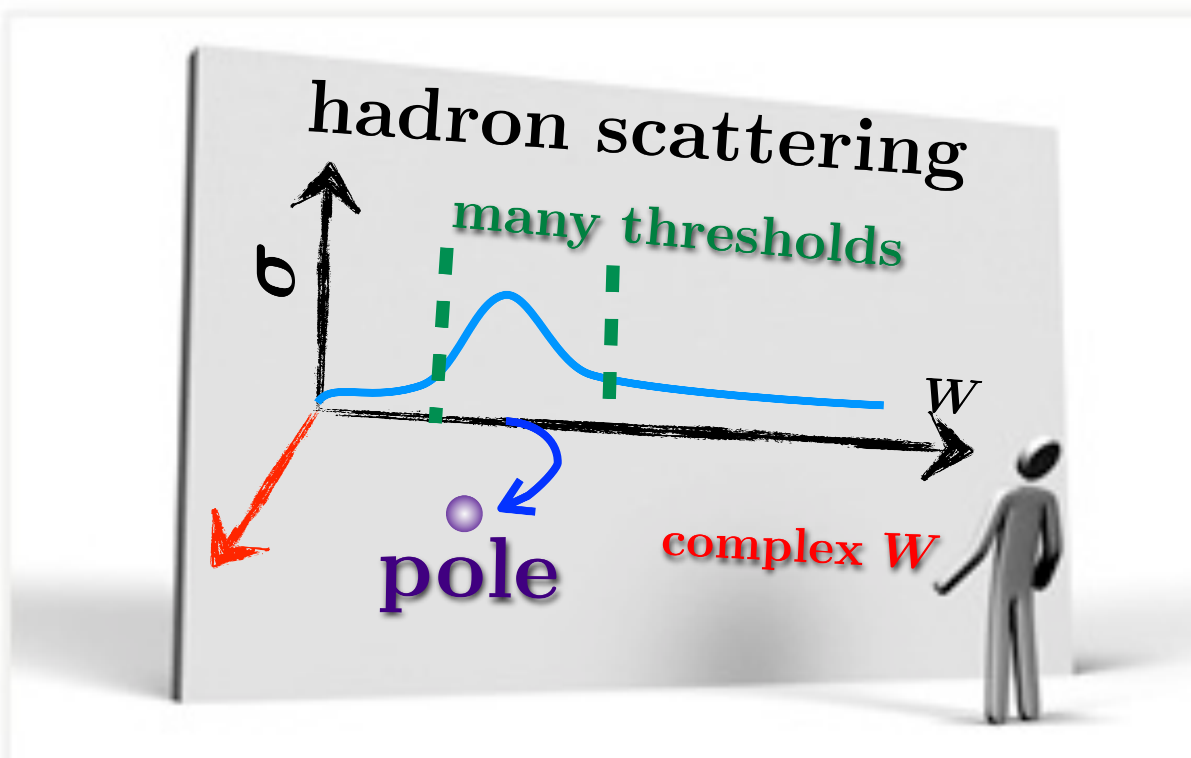
e.g., 4-quark operator

$$\Phi(x) = \bar{q}(x)\bar{q}(x)q(x)q(x)$$

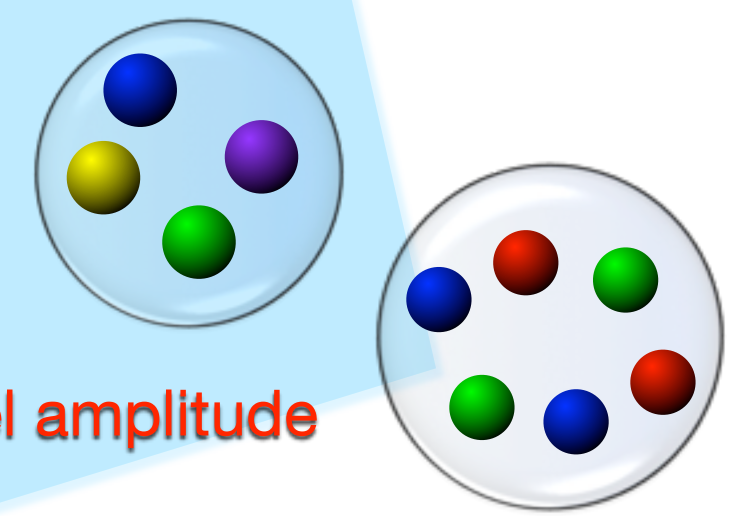
$$= A_1 e^{-W_1 \tau} + A_2 e^{-W_2 \tau} + \cdots$$

(W_1, W_2, ... are QCD eigen-energies)





Exotic hadrons



- ★ Resonance does not correspond to QCD eigen-state
- ★ Resonance energy is determined by pole of coupled-channel amplitude
- ★ How can we determine pole position on the lattice?

How to study resonance from lattice QCD?

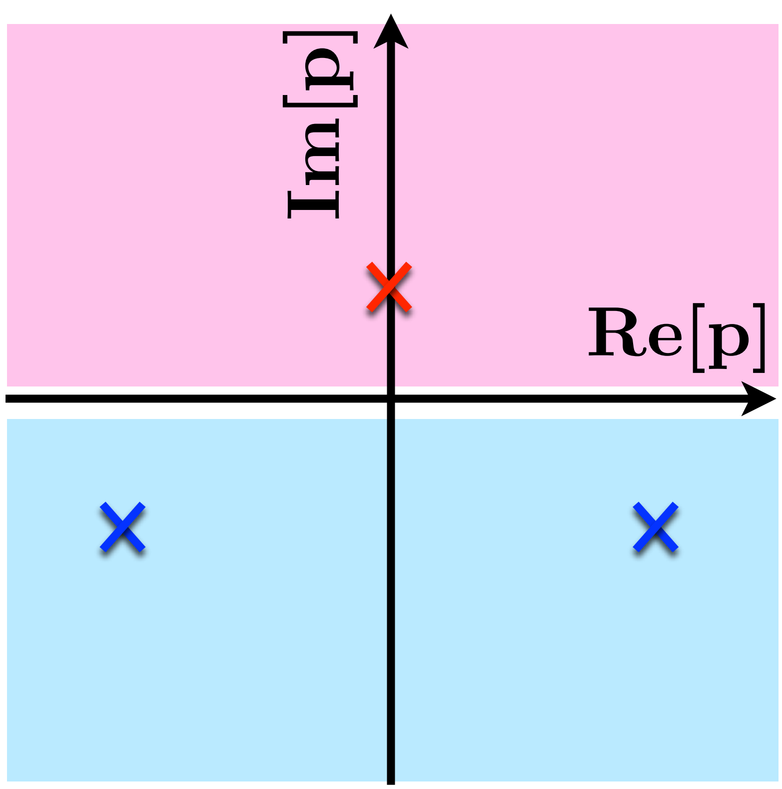
T-matrix in formal scattering theory (N/D method)

$$T^{-1}(\sqrt{s}) = V^{-1} + \frac{1}{2\pi}\int_{s_+}^{\infty} ds' \frac{\rho(s')}{s'-s}$$

Interaction part is not determined within scattering theory

➡ **interactions faithful to S-matrix in QCD**

Analyticity of T-matrix is uniquely determined



Bound states (physical sheet, 1st)

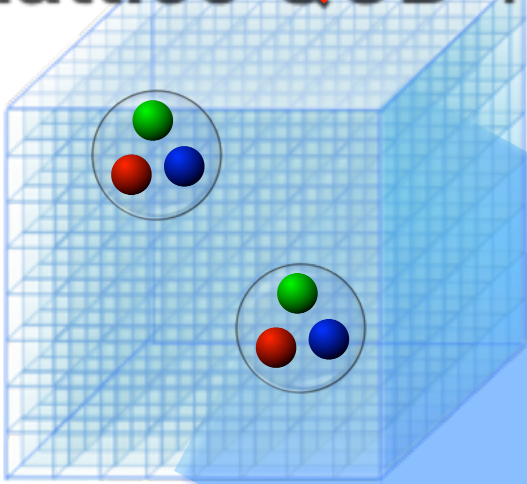
- binding energy --> T-matrix pole position
- coupling --> residue of pole

Resonance/virtual states (unphysical sheet, 2nd)

- Analytic continuation of T-matrix
- resonance energy --> T-matrix pole position
- coupling --> complex residue of pole

HAL QCD strategy (interactions faithful to S-matrix)

lattice QCD + scattering theory

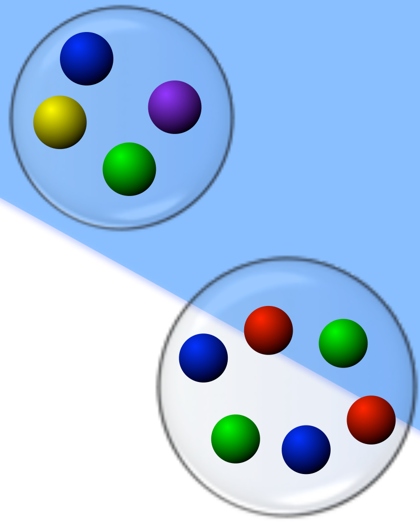


K-computer

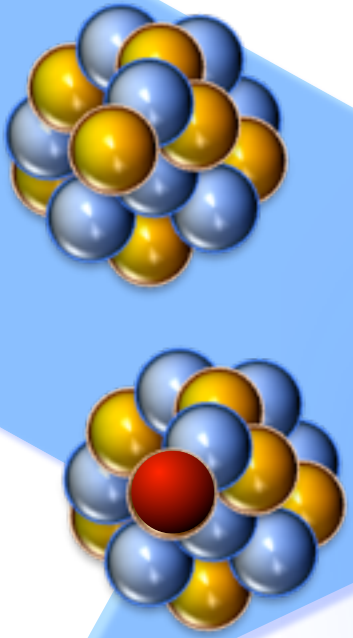
hadron interactions

- resonance pole & residue
- ab initio many-body calc.
- hadronic EoS

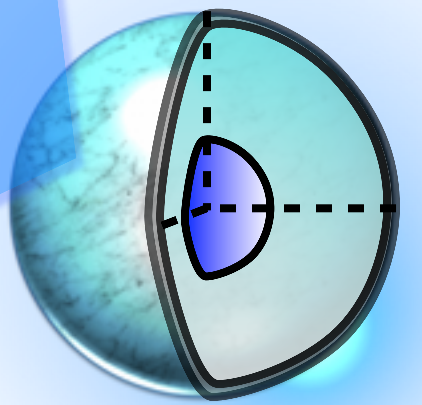
hadron resonances

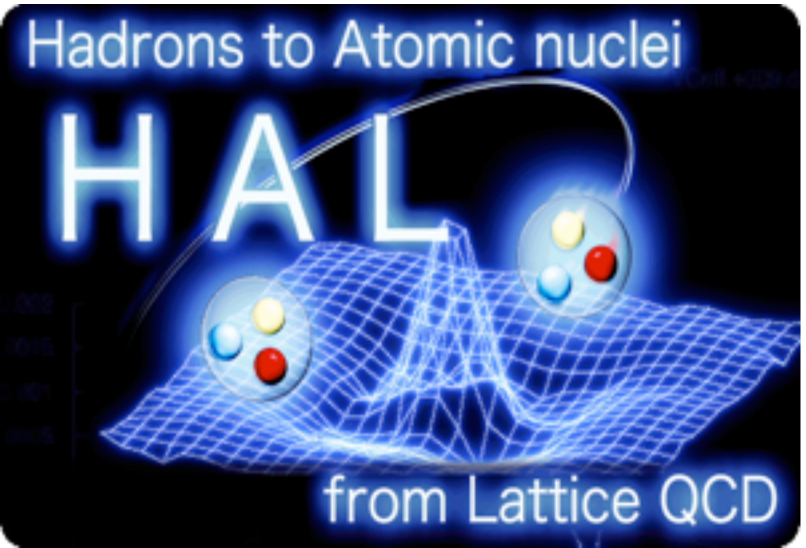


nuclei



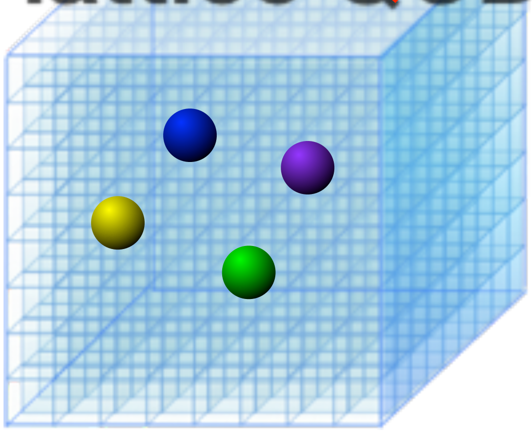
neutron stars





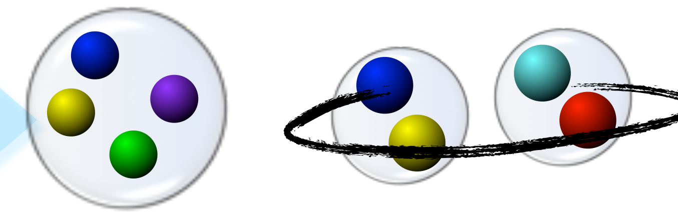
Hadron scattering in LQCD

lattice QCD



hadron interaction
faithful to S-matrix

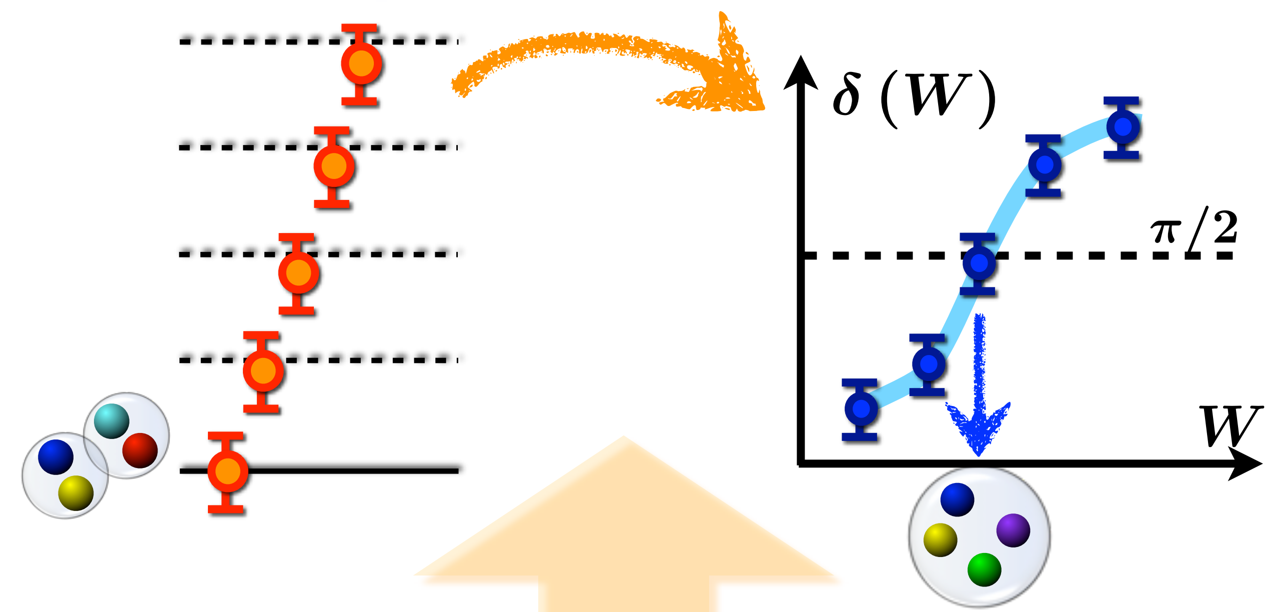
✓ **hadron resonances**

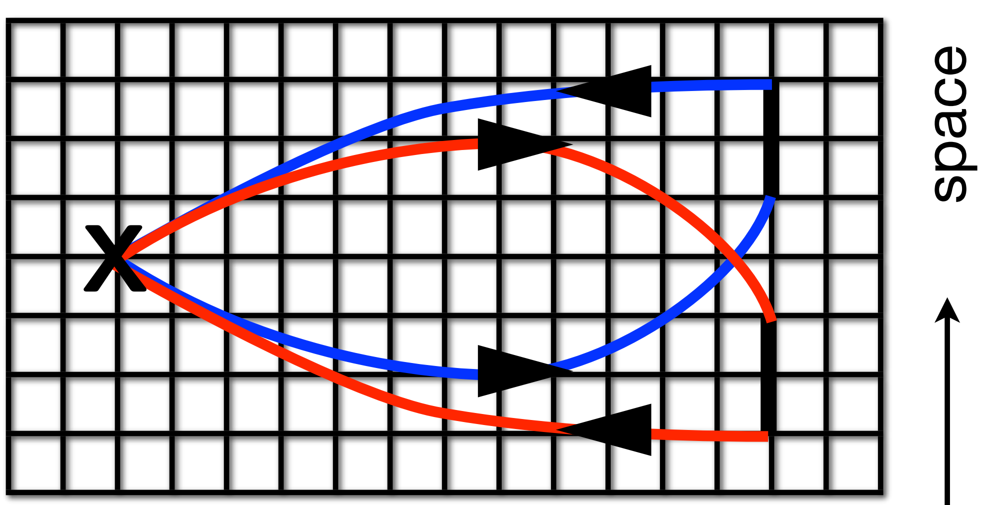


★ single channel scattering

$$\langle 0|\Phi(\tau)\Phi^\dagger(0)|0\rangle = \sum_n A_n e^{-W_n \tau}$$

$$\left(W(k_n) = \sqrt{m_1^2 + k_n^2} + \sqrt{m_2^2 + k_n^2} \right)$$





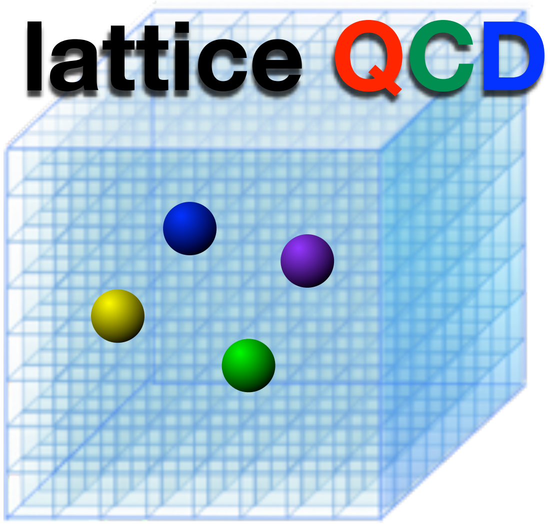
❖ Lüscher's formula   Lüscher, NPB354 (1991).

‣ **finite V spectrum --> phase shift δ(W_n)**

$$k_n \cot\delta(k_n) = \frac{4\pi}{L^3} \sum_{m\in\mathbb{Z}^3} \frac{1}{\vec{p}_m^{\,2} - k_n^2}$$

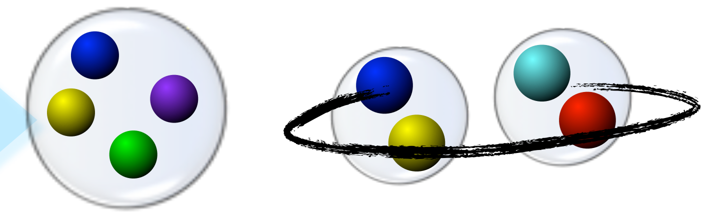
Problem in coupled-channel scattering

lattice QCD



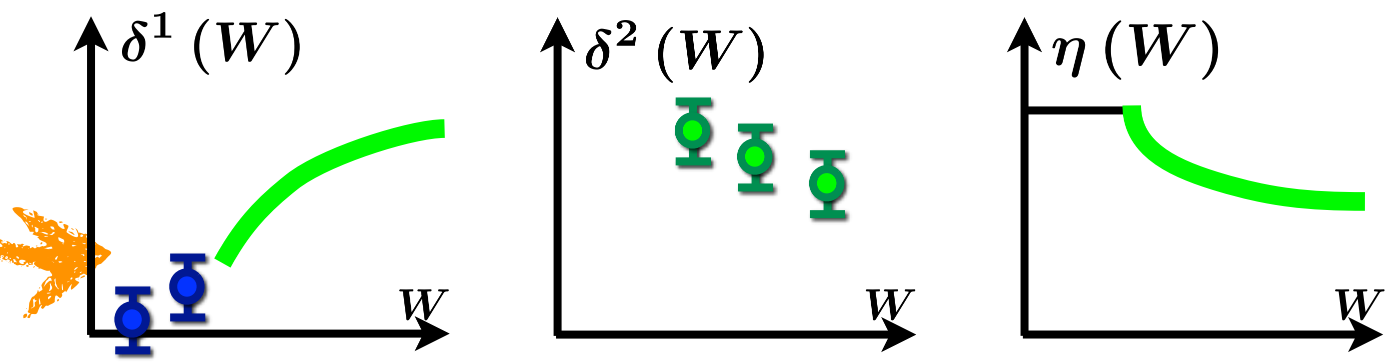
hadron interaction
faithful to S-matrix

✓ **hadron resonances**



★ coupled-channel scattering

$$\langle 0|\Phi(\tau)\Phi^\dagger(0)|0\rangle = \sum_n A_n e^{-W_n \tau}$$



➡ coupled-channel Lüscher's formula

elastic region: W --> δ(W)

inelastic region: W --> $\delta^1(W)$, $\delta^2(W)$, $\eta(W)$ **--> find W(L_1)=W(L_2)=W(L_3)**

➡ assumptions about interactions or K-matrices necessary...

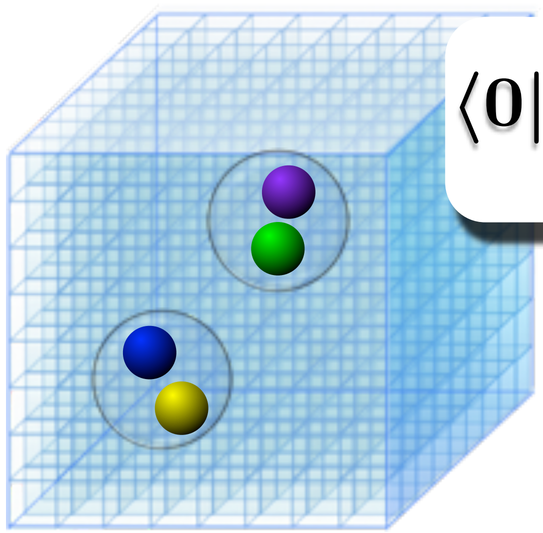
★ indicate more information mandatory to solve coupled-channel scatterings

➡ What can we measure in addition to temporal correlations?

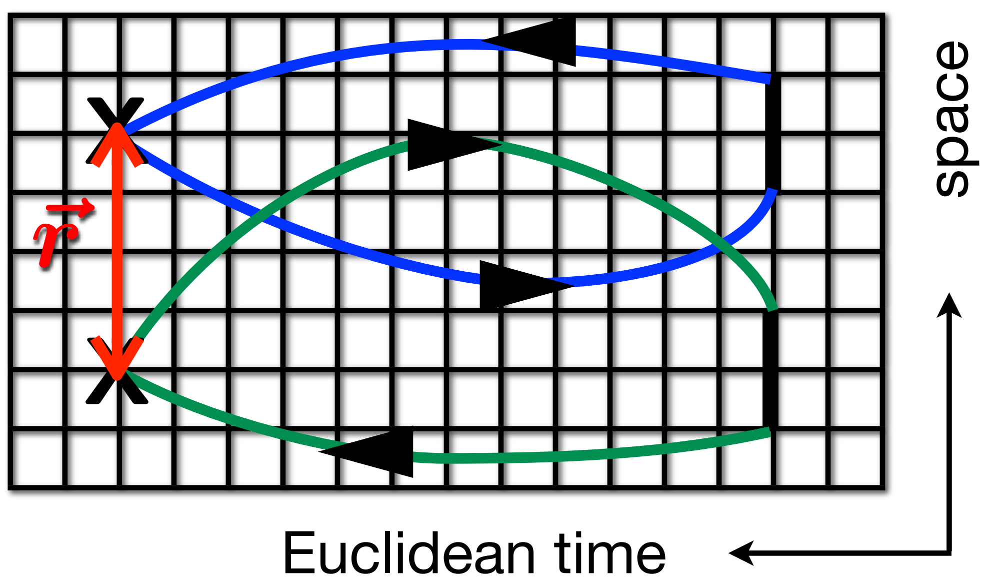
HAL QCD approach "potential" as representation of S-matrix

✦ HAL QCD approach: extract energy-independent interaction kernel

➡ measure spatial correlation as well as temporal correlation



$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1 Z_2}\sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al,(HAL QCD), PLB712, 437(2012).

★ Nambu-Bethe-Salpeter wave functions: $\psi_n(r)$

- NBS wave functions outside interactions --> free Klein-Gordon equation

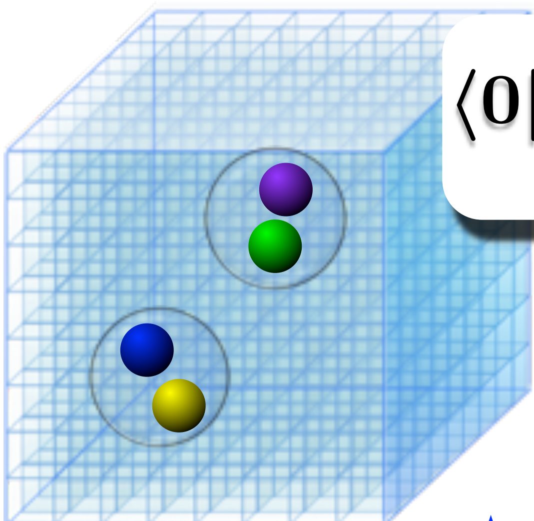
$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

➡ S-matrix

HAL QCD approach "potential" as representation of S-matrix

✦ HAL QCD approach: extract energy-independent interaction kernel

➡ measure spatial correlation as well as temporal correlation

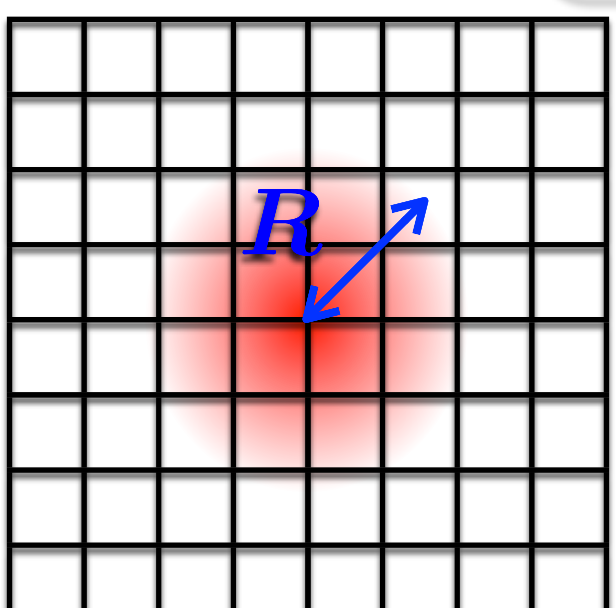


$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1 Z_2}\sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al,(HAL QCD), PLB712, 437(2012).

★ NBS wave functions inside interactions: half-offshell T-matrix

$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r},\vec{r}')\psi_n(\vec{r}')$$

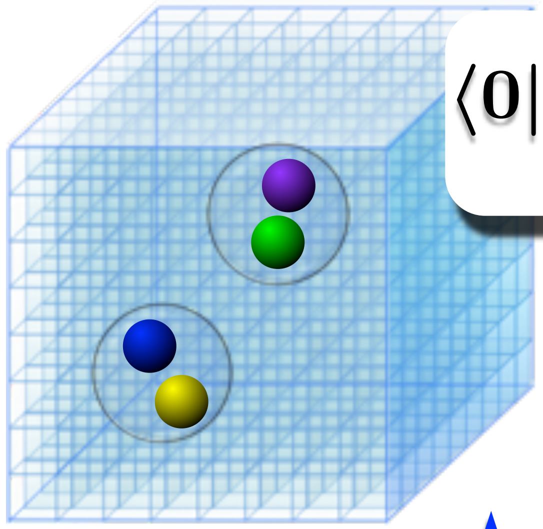


- U(r,r') is faithful to S-matrix in elastic region
- U(r,r') is energy-independent (until new threshold opens)
- U(r,r') contains all 2PI contributions
- U(r,r') is not an observable (applied to ab initio calc.)

Coupled-channel HAL QCD approach

✦ HAL QCD approach: extract energy-independent interaction kernel

➡ measure spatial correlation as well as temporal correlation



$$\langle 0|\phi_1^a(\vec{x}+\vec{r},\tau)\phi_2^a(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1^a Z_2^a}\sum_n A_n \psi_n^a(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al,(HAL QCD), PLB712, 437(2012).

★ channel wave functions defined in asymptotic region: $\psi^a_n(r)$

$$\left(\nabla^2 + (\vec{k}_n^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

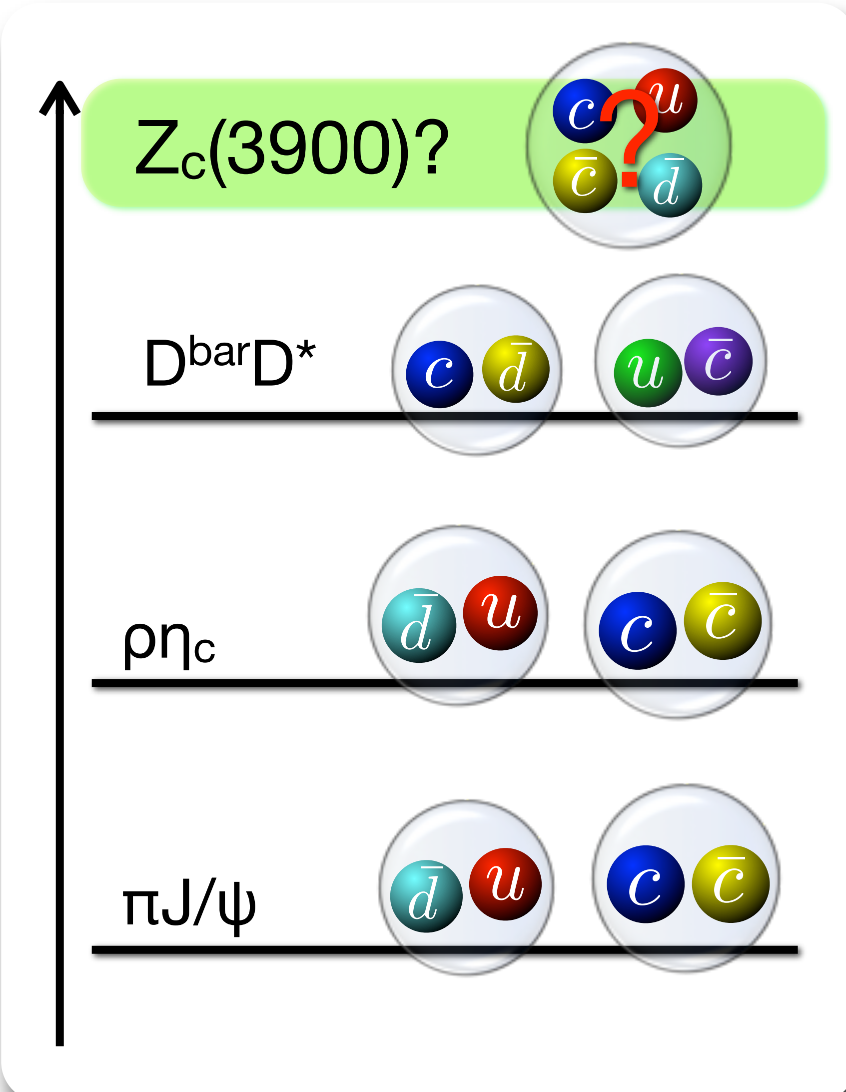
★coupled-channel potential $U^{ab}(r,r')$:

- $U^{ab}(r,r')$ is faithful to S-matrix in both elastic and inelastic regions
- $U^{ab}(r,r')$ is energy-independent (until new threshold opens)
- $U^{ab}(r,r')$ contains all 2PI contributions

Full details, Aoki et al. [HAL QCD Coll.], PTEP 2012, 01A105 (2012); Proc. Jpn. Acad., Ser. B, 87 (2011).

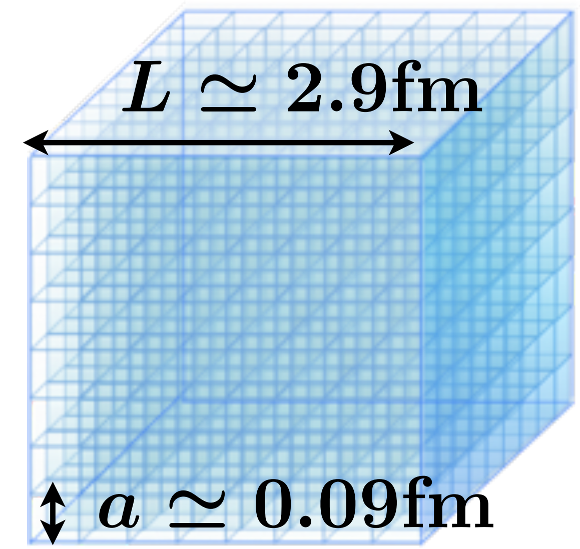
$Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$: $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel

Y. Ikeda et al., [HAL QCD], PRL117, 242001 (2016).



❖ N_f=2+1 full QCD

- Iwasaki gauge
- clover Wilson quark
- 32^3 x 64 lattice



❖ Relativistic Heavy Quark (charm)

- remove leading cutoff errors $O((m_c\, a)^n)$, $O(\Lambda_{QCD}\, a)$, ...

➡ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

light meson mass (MeV)

m_π= 411(1), 572(1), 701(1)

m_ρ= 896(8), 1000(5), 1097(4)

charmed meson mass (MeV)

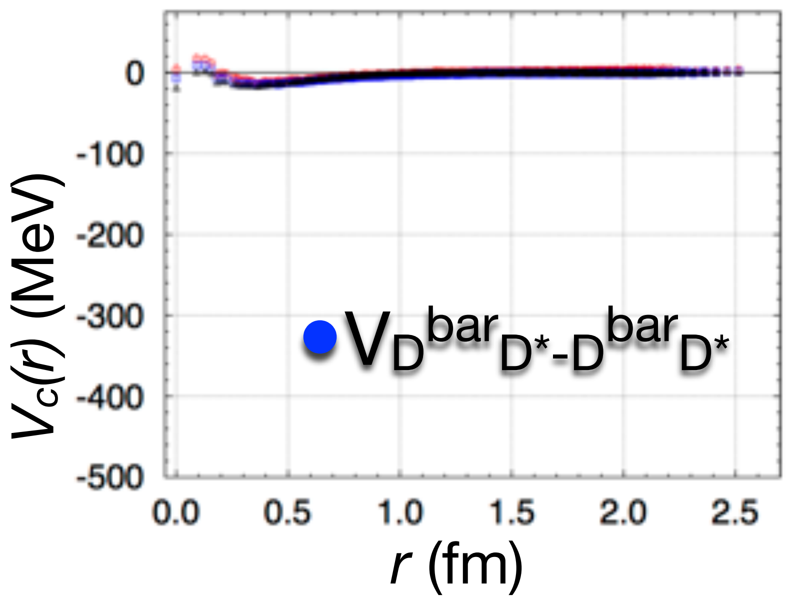
$m_{\eta c}$= 2988(1), 3005(1), 3024(1)

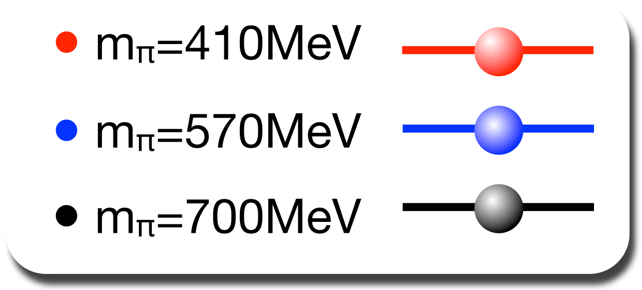
$m_{J/\psi}$= 3097(1), 3118(1), 3143(1)

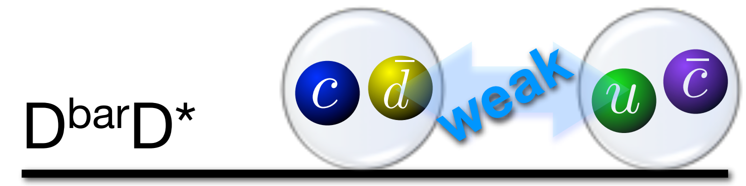
m_D= 1903(1), 1947(1), 2000(1)

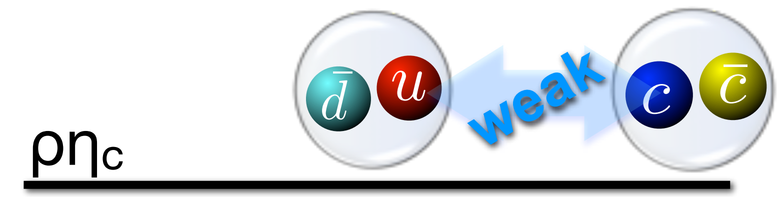
m_{D^*}= 2056(3), 2101(2), 2159(2)

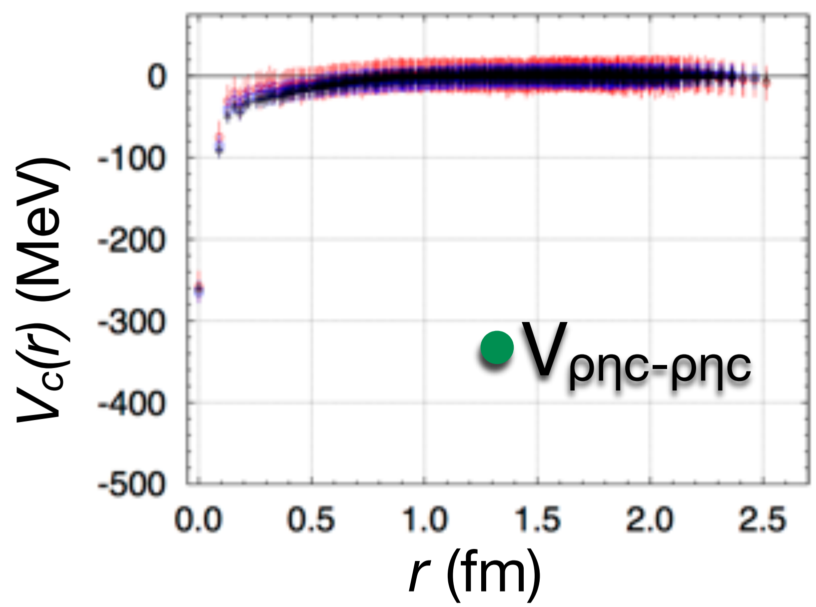
Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

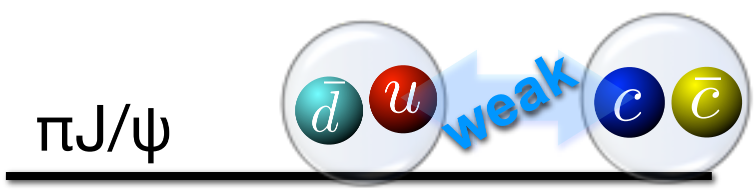




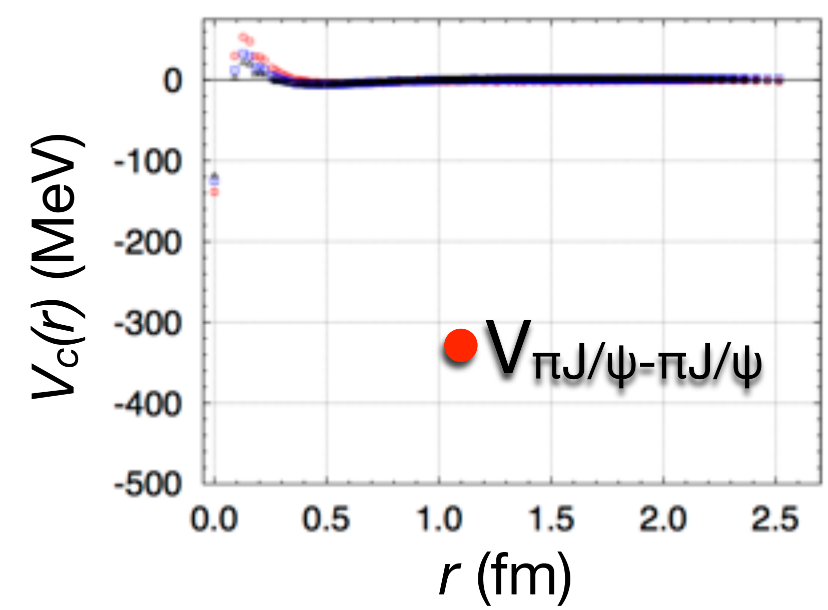


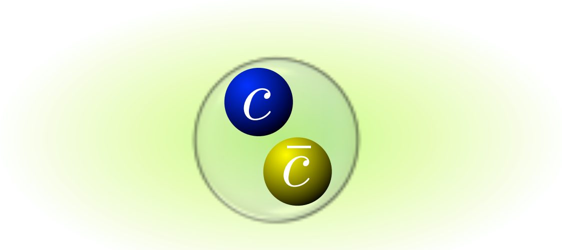


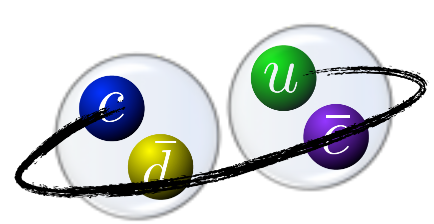




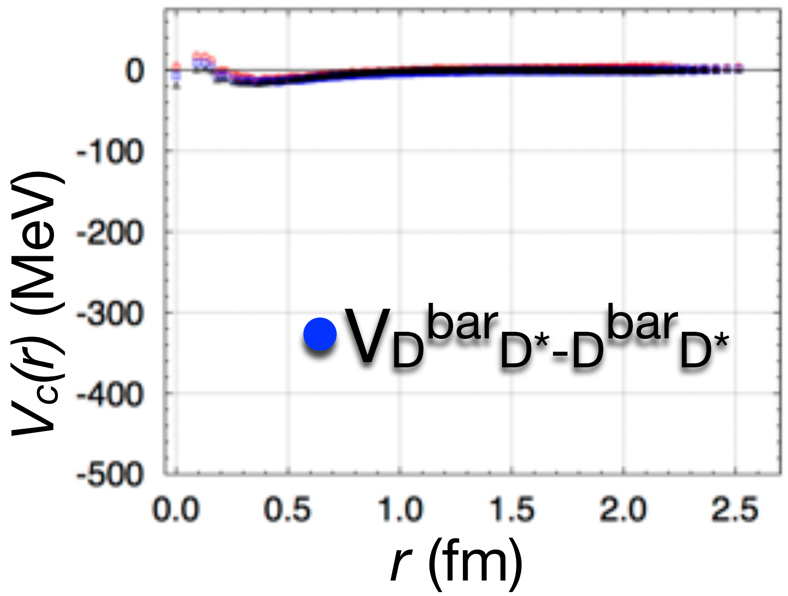
- All diagonal potentials are weak
  - ➡ no bound/resonant $\pi J/\psi$, $D^{bar}D^*$

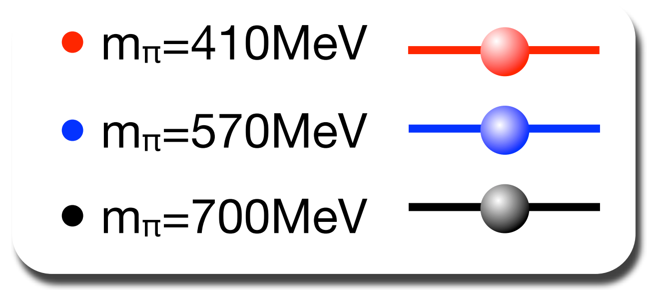


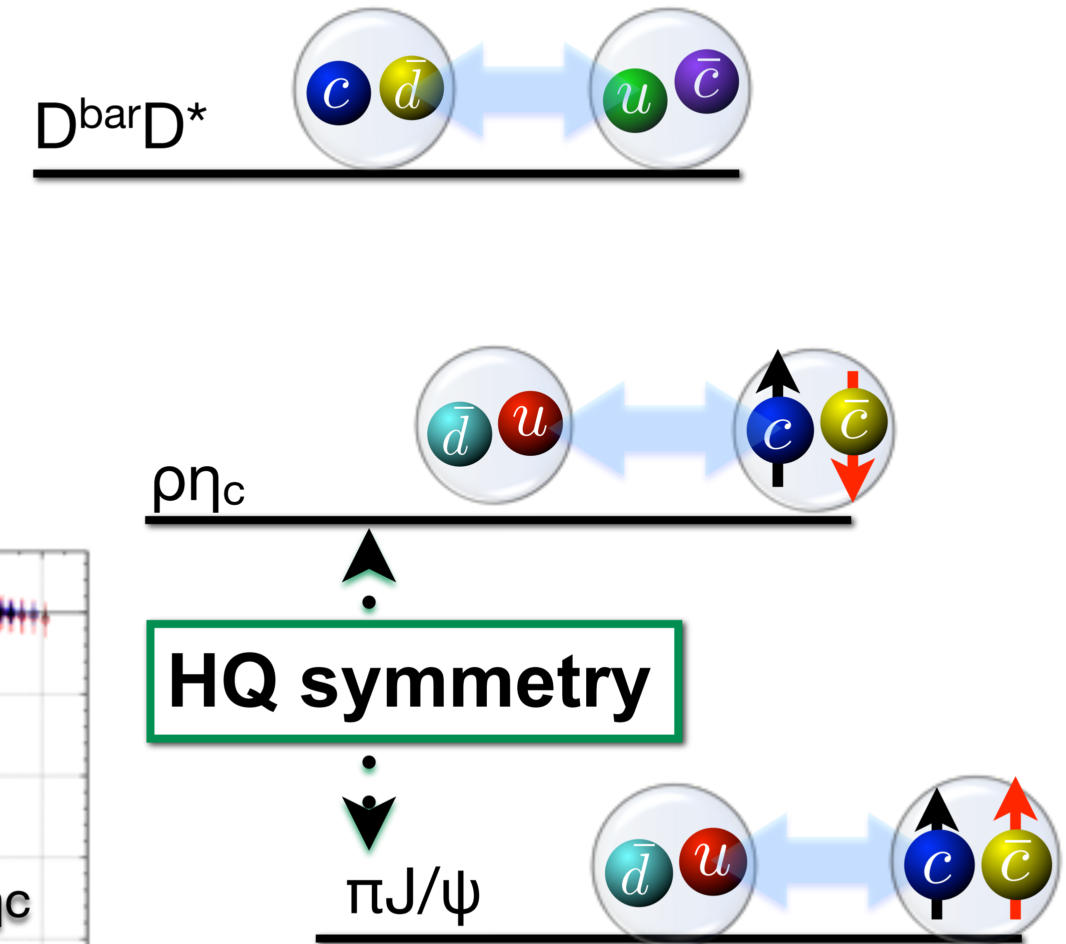


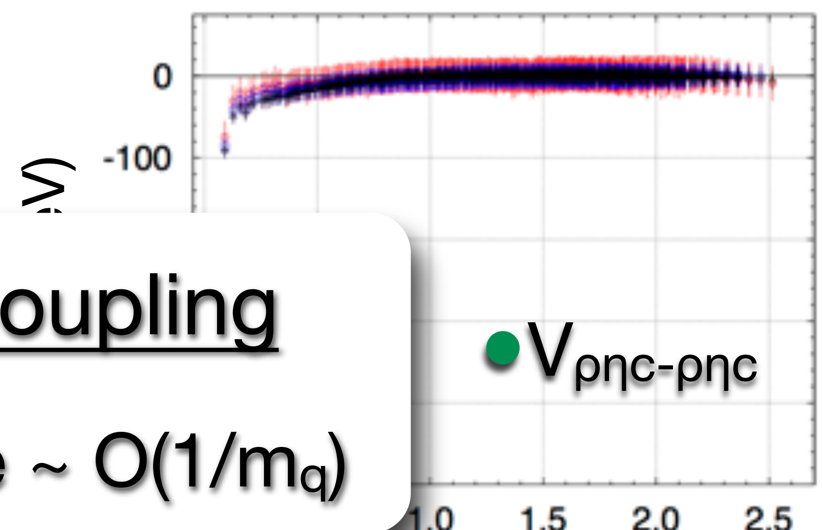


Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)



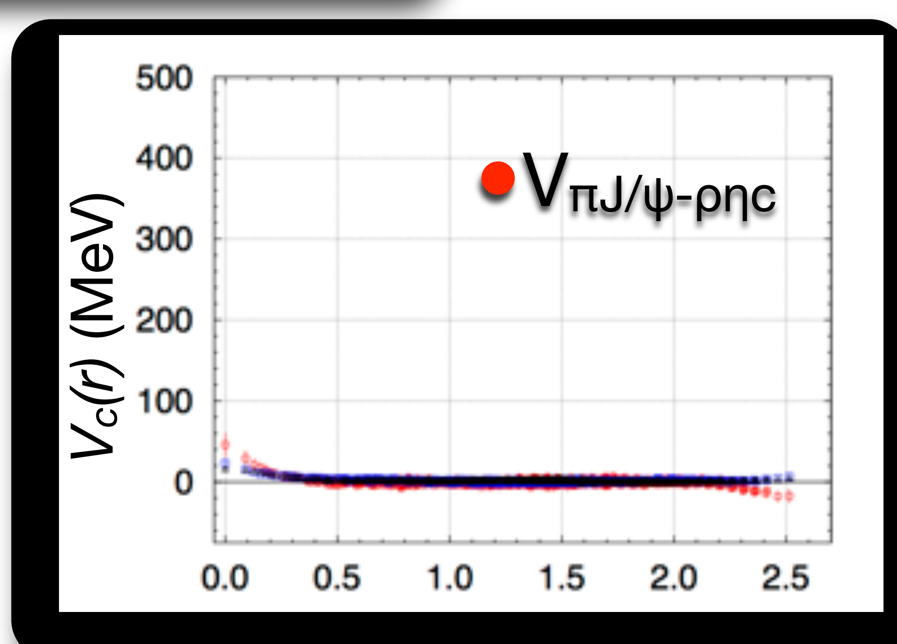


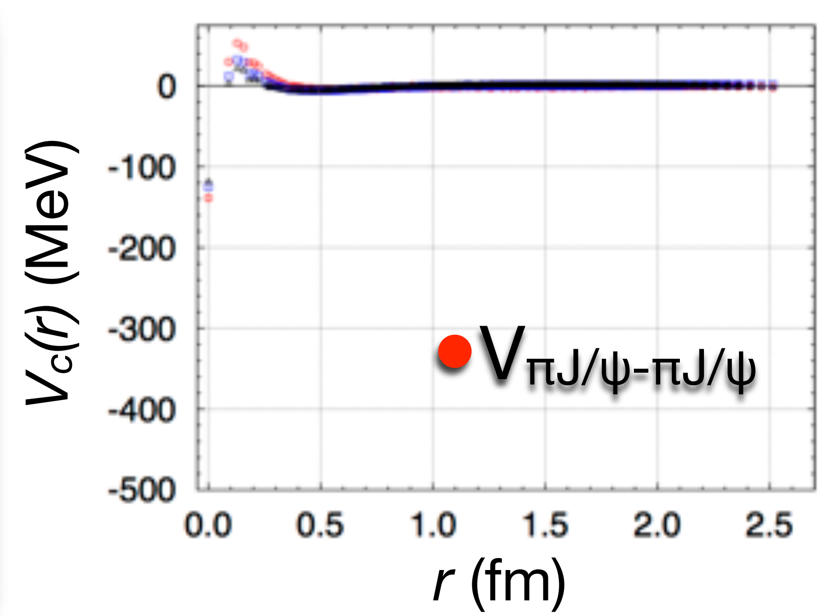




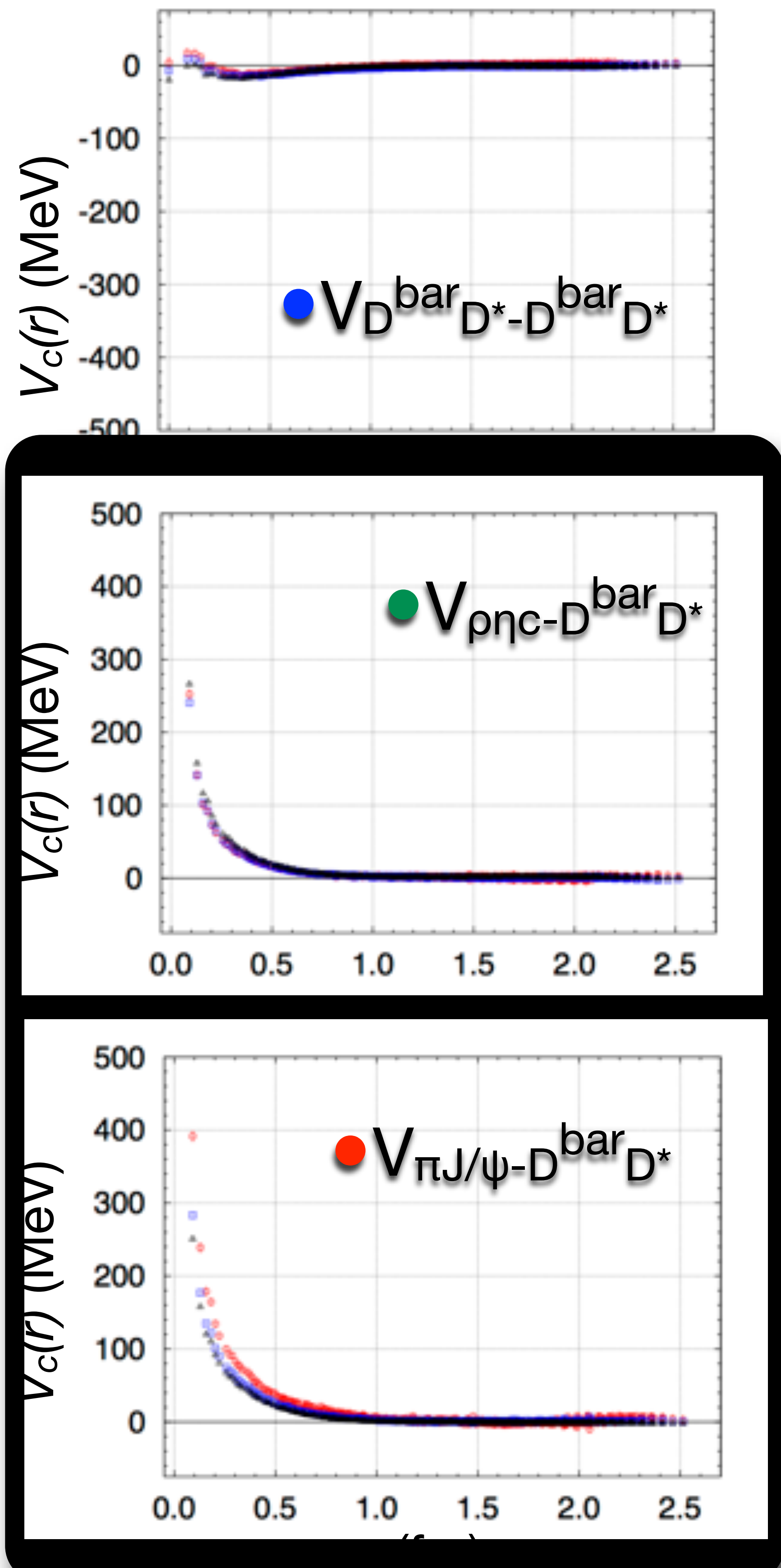
- Weak $\pi J/\psi$-$\rho\eta_c$ coupling

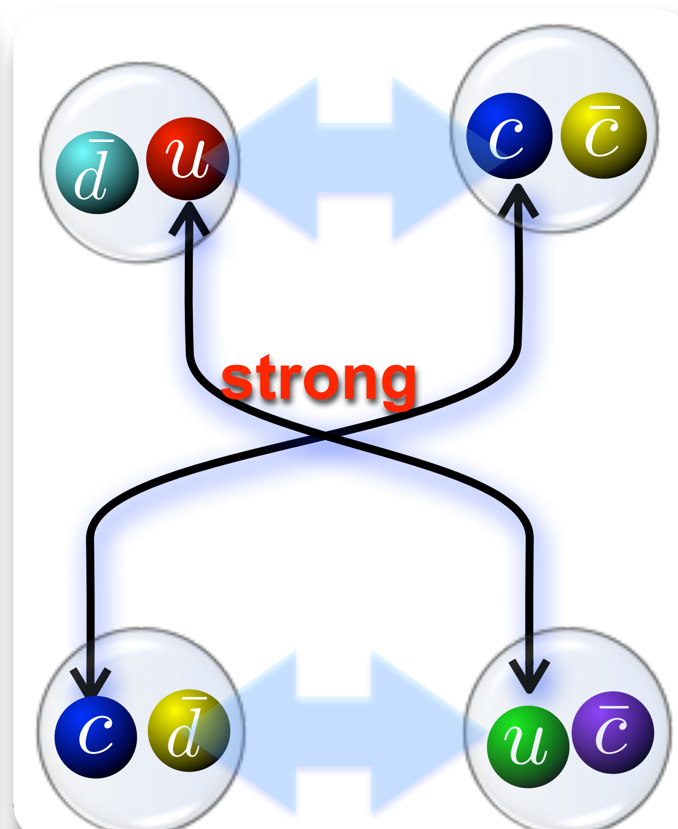
➡ spin-flip amplitude ~ $O(1/m_q)$

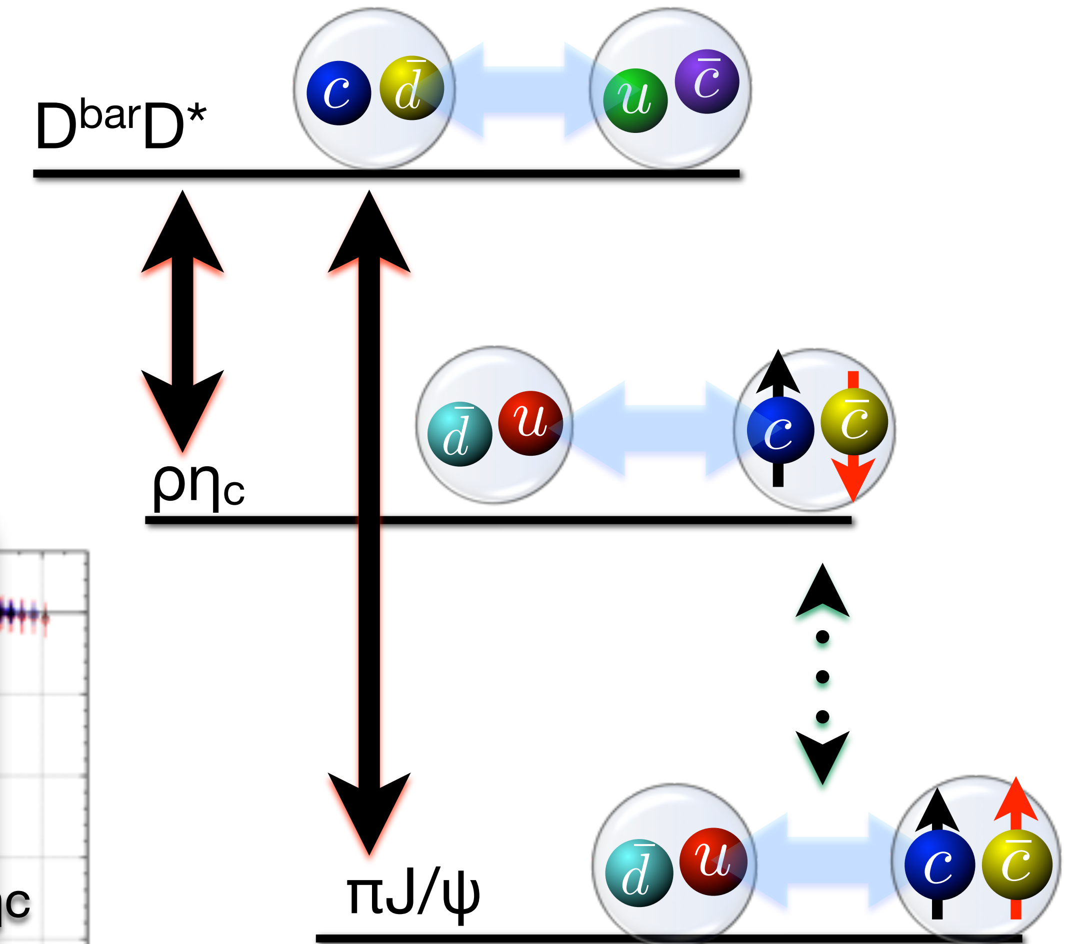




Potential matrix (πJ/ψ - ρη$_c$ - D^{bar}D*)



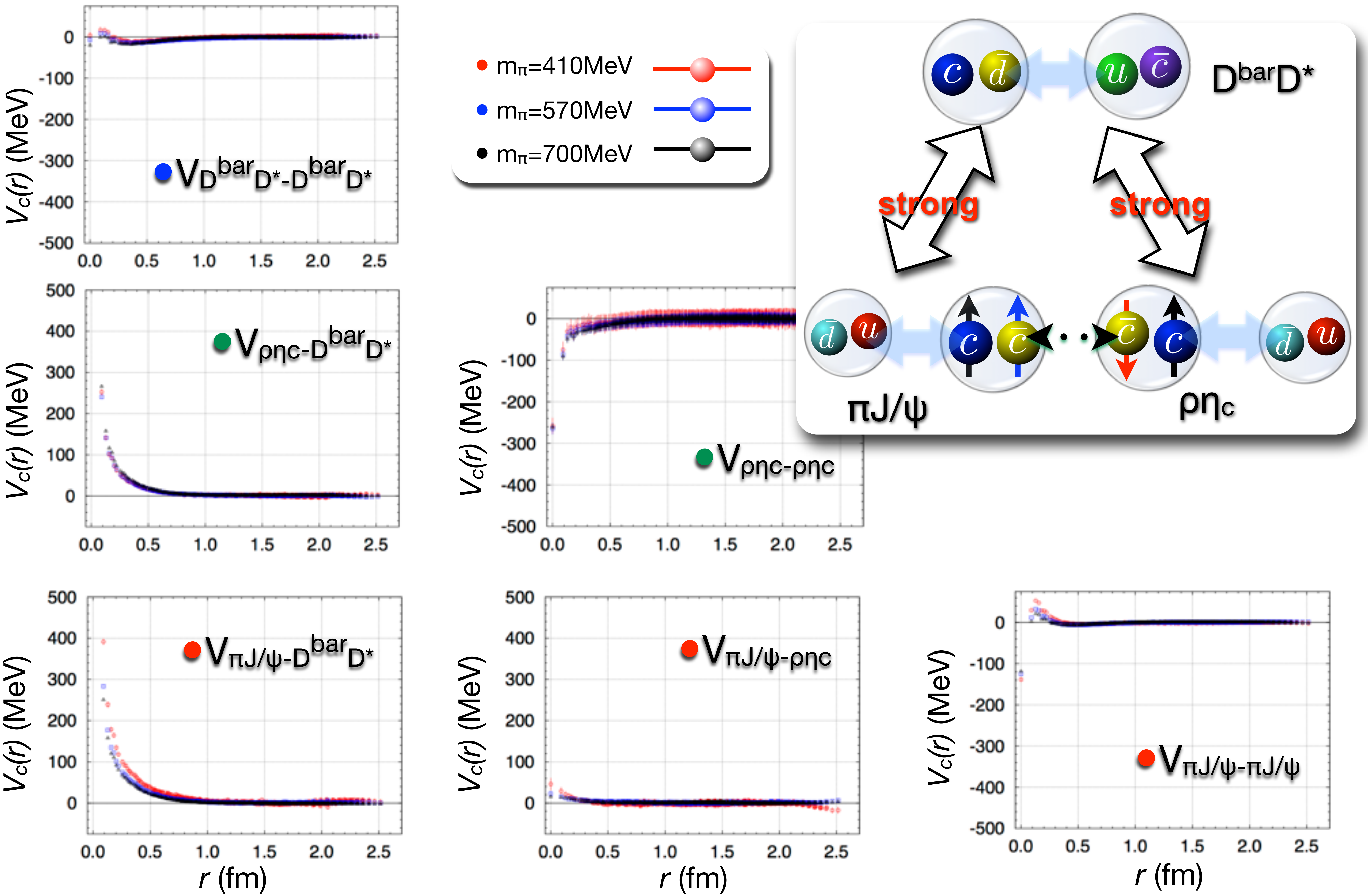




- **Strong off-diagonal D^{bar}D* potentials**

✓ strong charm-quark-exchange interactions

Potential matrix (πJ/ψ - ρηc - DbarD*)



Two-body scattering: structure of $Z_c(3900)$

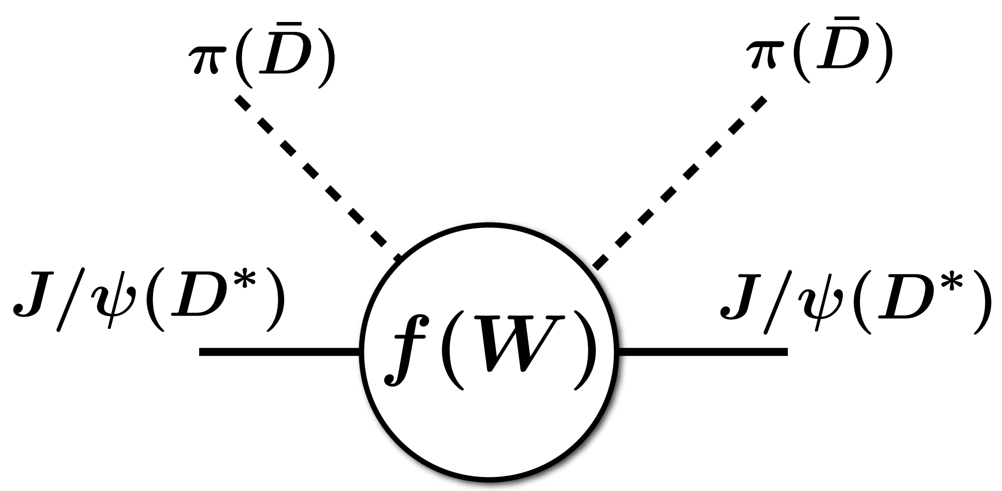
- Two-body s-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel scattering

➡ most ideal reaction to study structure of $Z_c(3900)$

1. invariant mass spectrum

number of scattering particles

$$N_{sc} \propto (\text{flax}) \cdot \sigma(W) \propto \mathrm{Im} f(W)$$



2. pole of amplitude

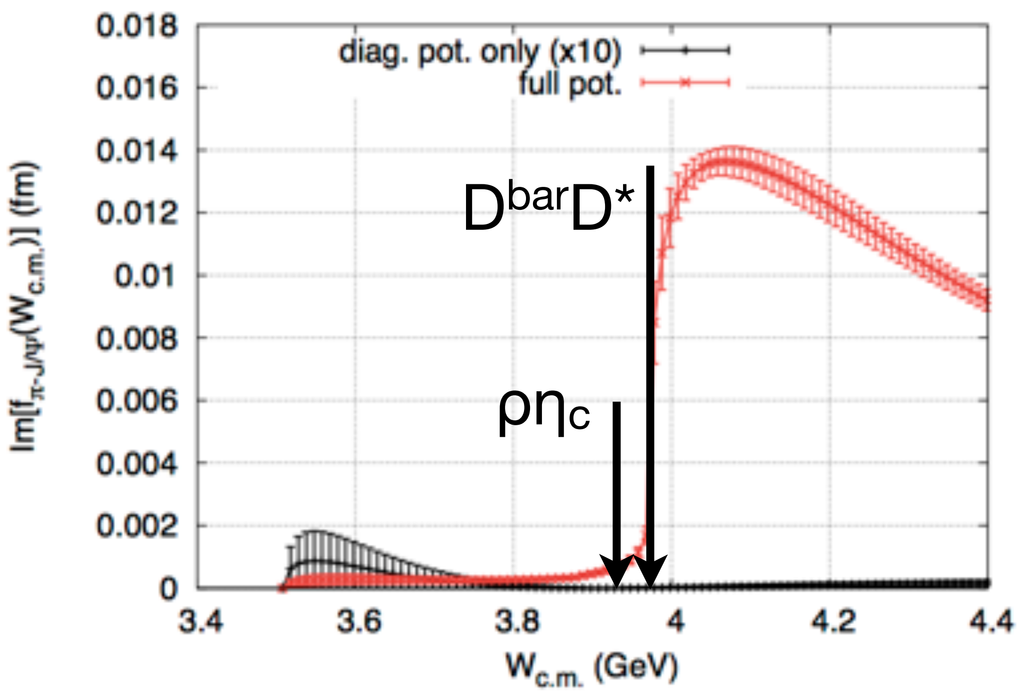
analytic continuation of amplitude onto complex energy plane

- results with mπ=410MeV are shown (weak quark mass dependence)

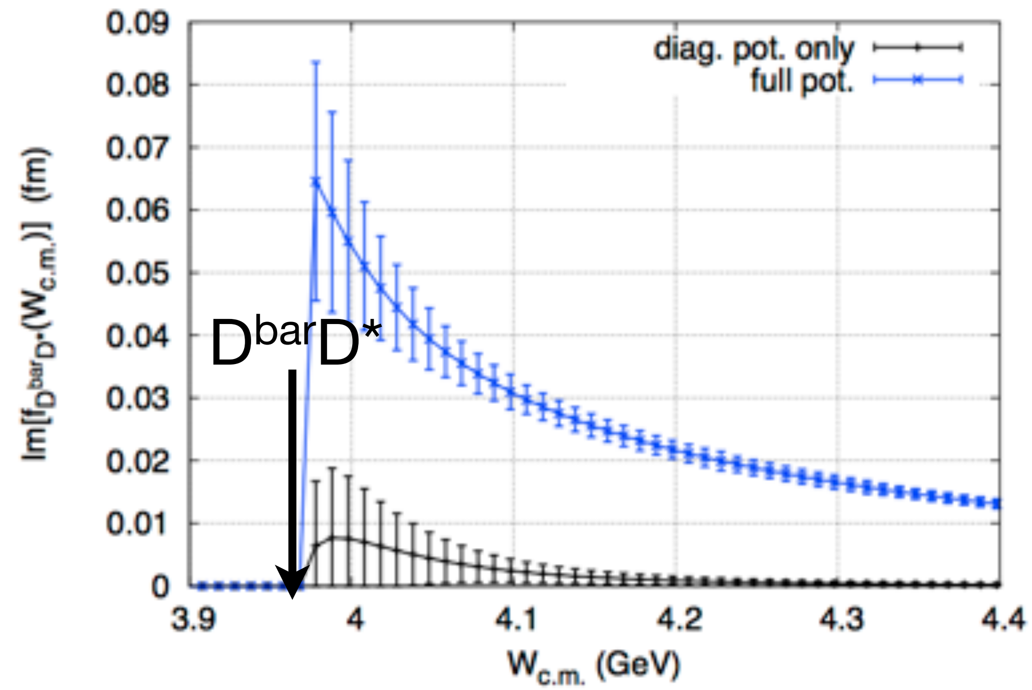
Invariant mass spectra of $\pi J/\psi$ & $D^{bar}D^*$

★ **2-body scattering (ideal setting to understand $Z_c(3900)$ structure)**

● $\pi J/\psi$ invariant mass



● $D^{bar}D^*$ invariant mass

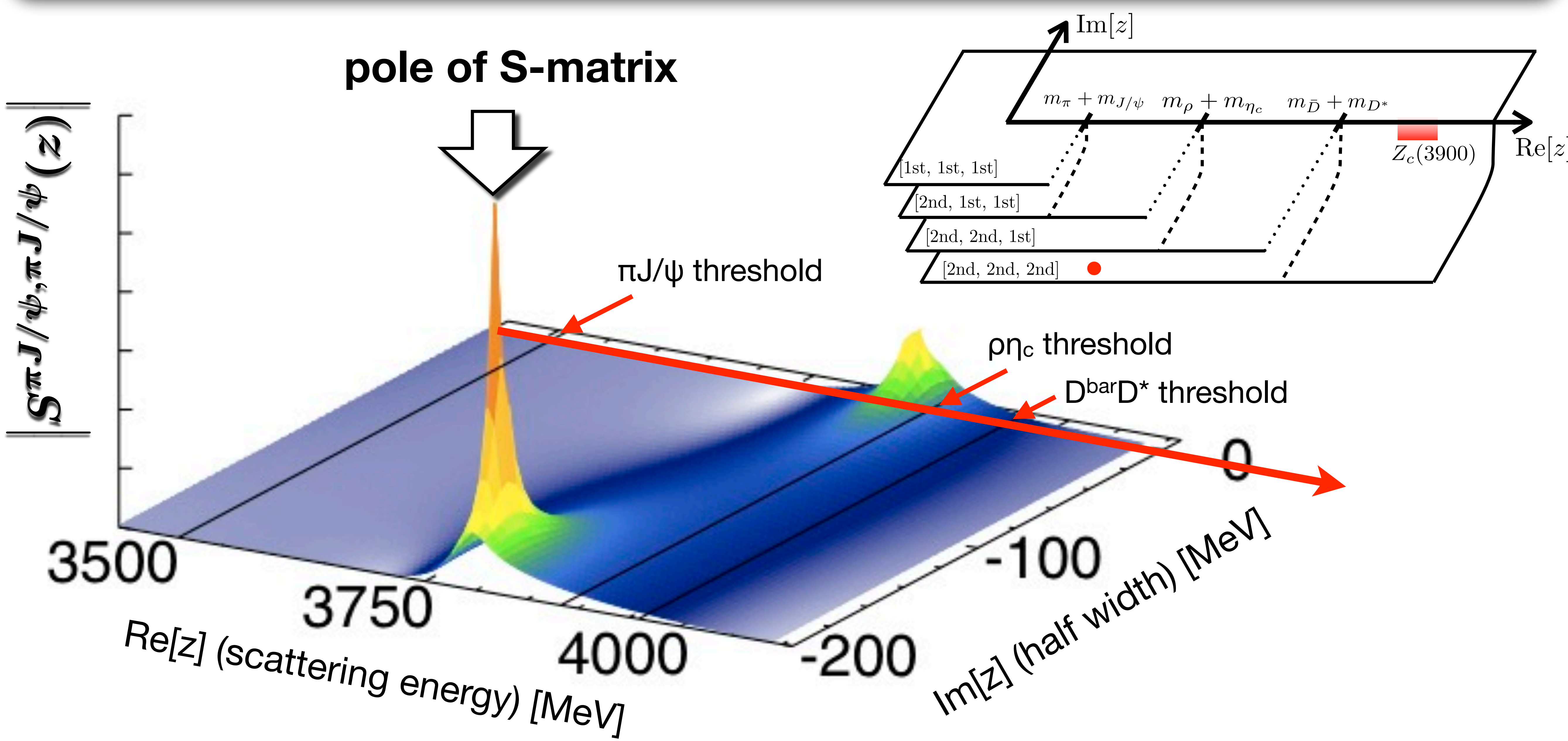


✓ Enhancement near $D^{bar}D^*$ threshold due to strong $V^{\pi J/\psi,\, DbarD^*}$

- peak in $\pi J/\psi$ (not Breit-Wigner line shape)
- threshold enhancement in $D^{bar}D^*$

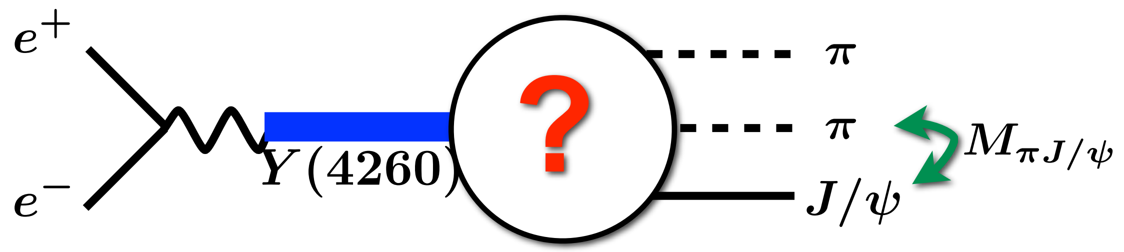
✓ **Is $Z_c(3900)$ a conventional resonance? --> pole position**

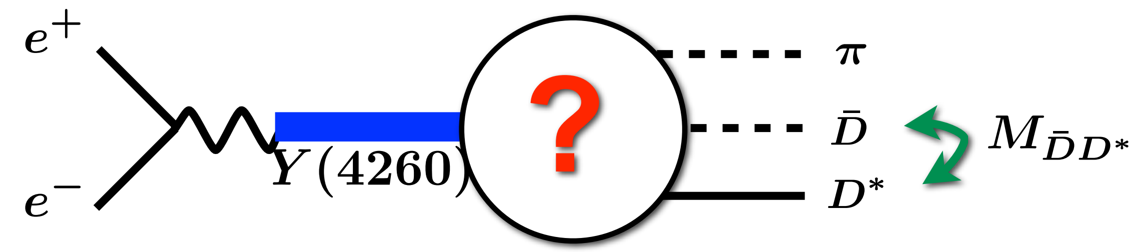
Complex pole position ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)

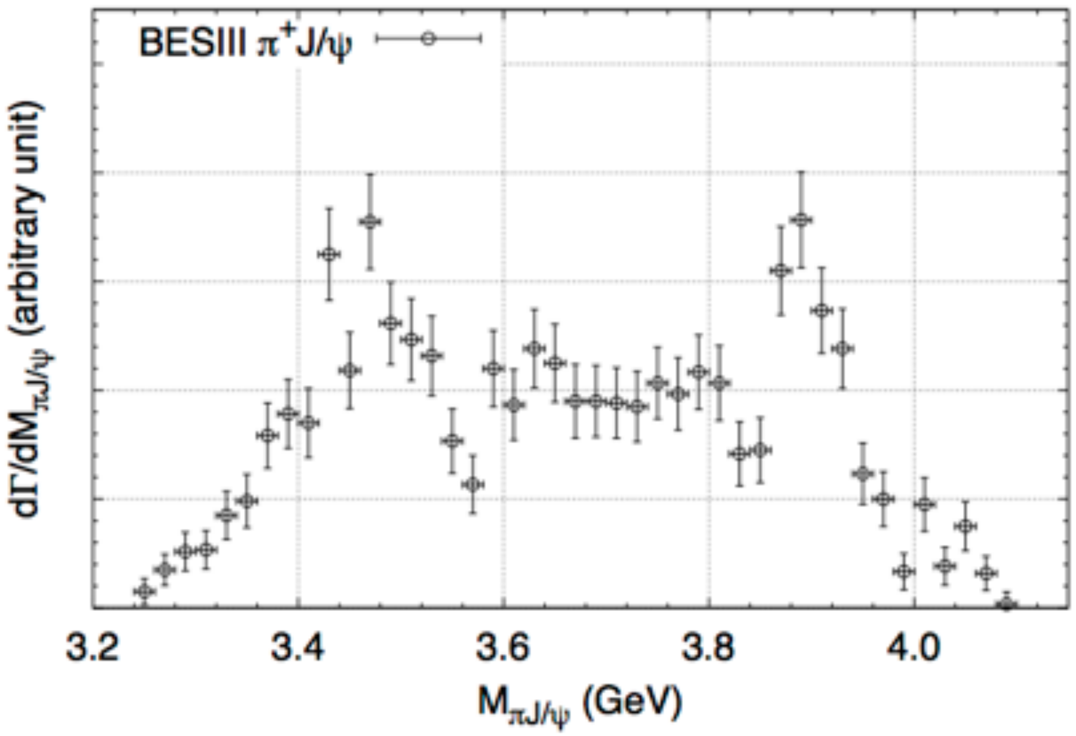


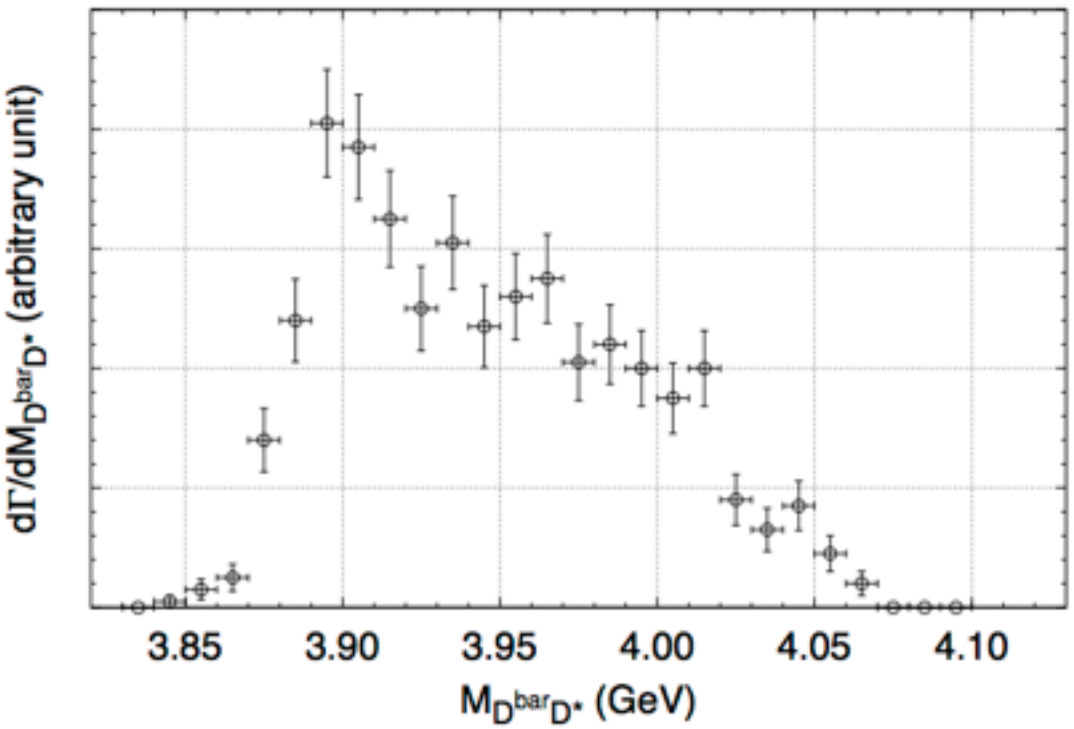
- "Virtual" pole on [2nd, 2nd, 2nd] sheet is found (far below $D^{bar}D^*$ threshold)
- No pole on other relevant sheets to $Z_c(3900)$
- $Z_c(3900)$ is not a conventional resonance but **threshold cusp induced by strong $\pi J/\psi$ - $D^{bar}D^*$ coupling**

Comparison with expt. data : $Z_c(3900)$ production via Y(4260) decay









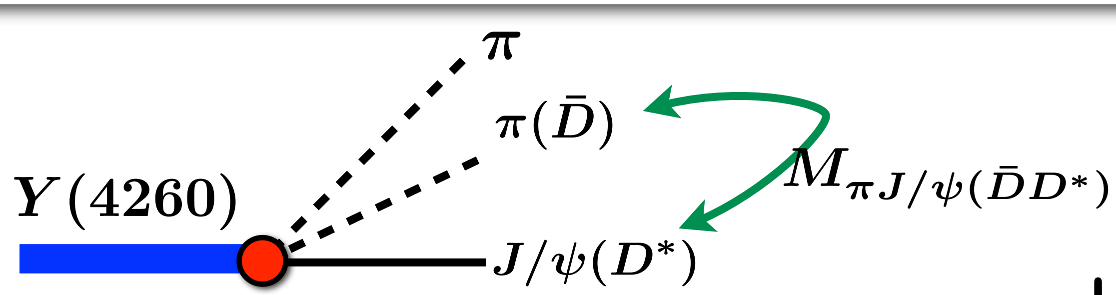
BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).

BESIII Coll., PRL112, 022001, (2014).
B. Wang (BESIII Coll.), MENU2016 talk

✓ check whether expt. data of Y(4260) decay can be reproduced with HAL QCD coupled-channel potential (m_π=410 MeV)

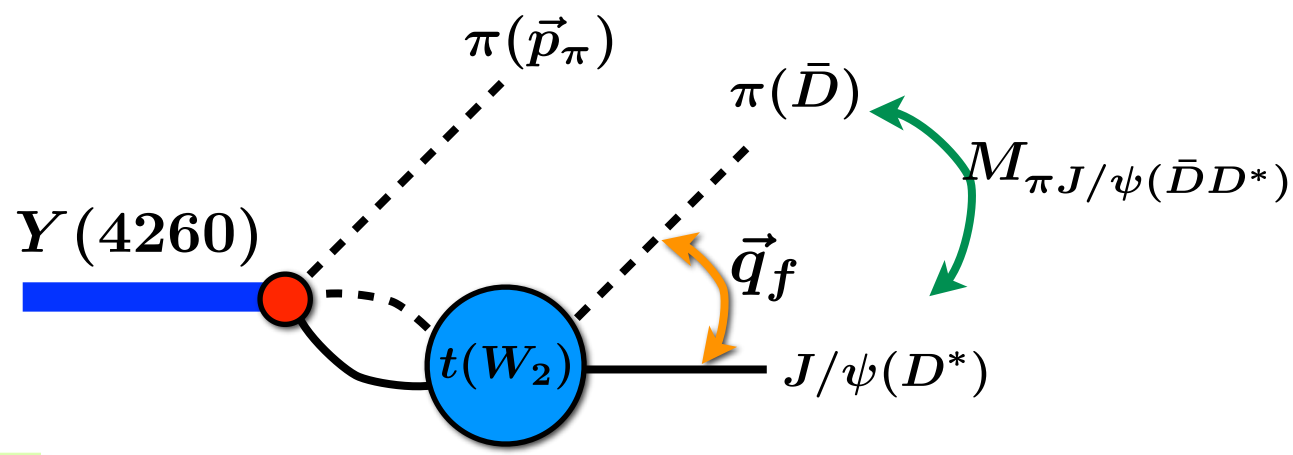
Three-body decay of Y(4260)

$$d\Gamma_f \propto (2\pi)^4 \delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f)) d^3p_\pi d^3q_f |T_f(\vec{p}_\pi, \vec{q}_f; W_3)|^2$$



➡ constant primary vertex is assumed (2 parameters)

$+$



★ input $V^{LQCD}(r)$ --> *t*-matrix

➡ No parameter

✓ Three-body amplitudes

$$T_f(\vec{p}_\pi, \vec{q}_f; W_3) = \sum_{n=\pi\pi J/\psi, \pi\bar{D}D^*} C_n^{Y(4260)} \left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q}', \vec{q}_f, \vec{p}_\pi; W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}', \vec{p}_\pi) + i\epsilon} \right]$$

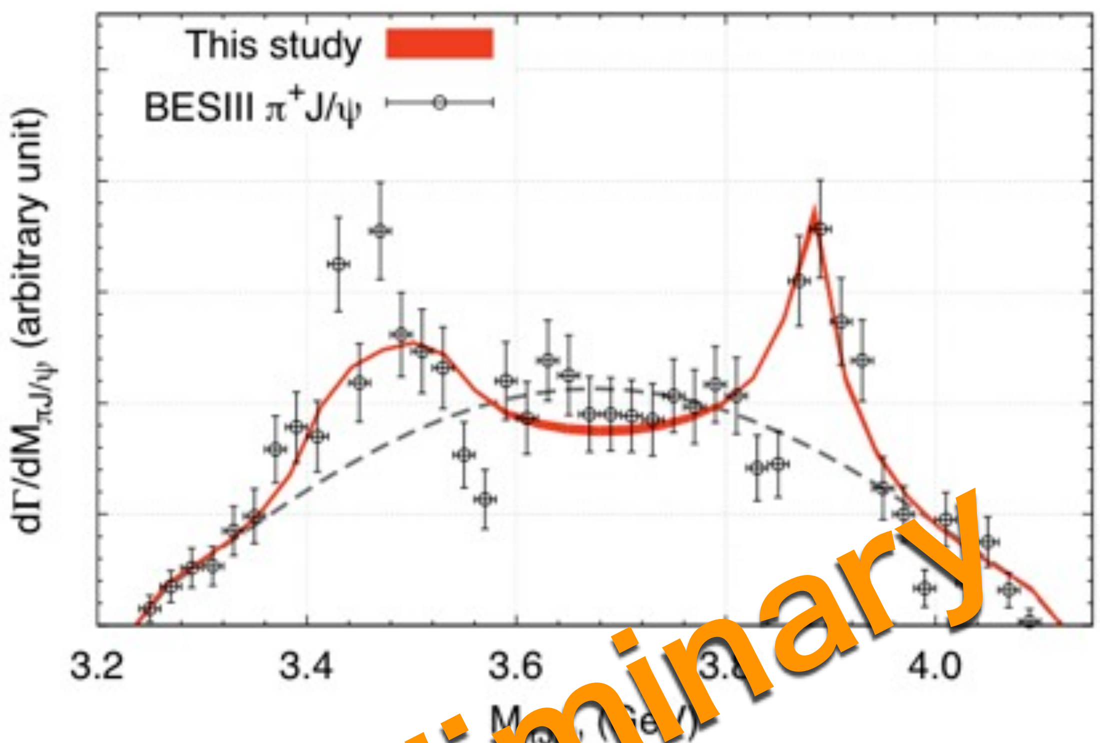
<u>physical hadron masses employed</u> to compare w/ expt. data

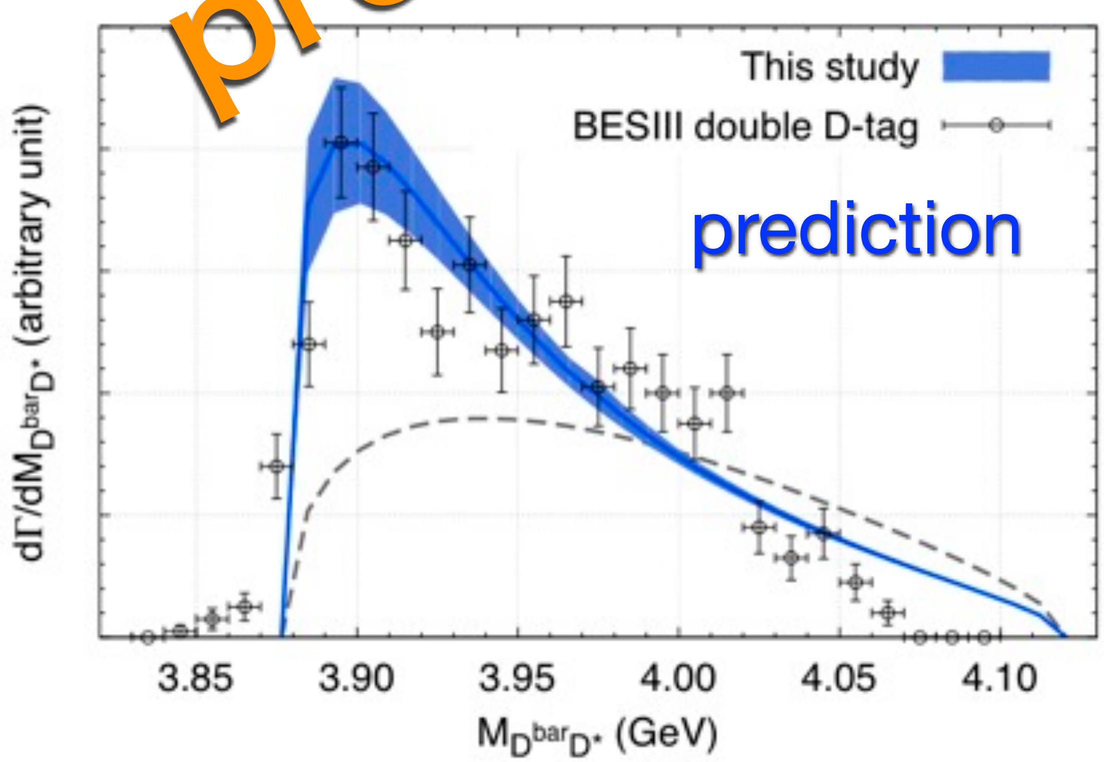
✓ fix decay vertex by Y(4260) --> ππJ/ψ expt. data

✓ predict Y(4260) --> $\pi D^{bar} D^*$ decay spectrum

Mass spectra (πJ/ψ & $D^{bar}D^*$ w/ relativistic dispersion)

[Y. Ikeda et al., [HAL QCD], PRL117 & in preparation]

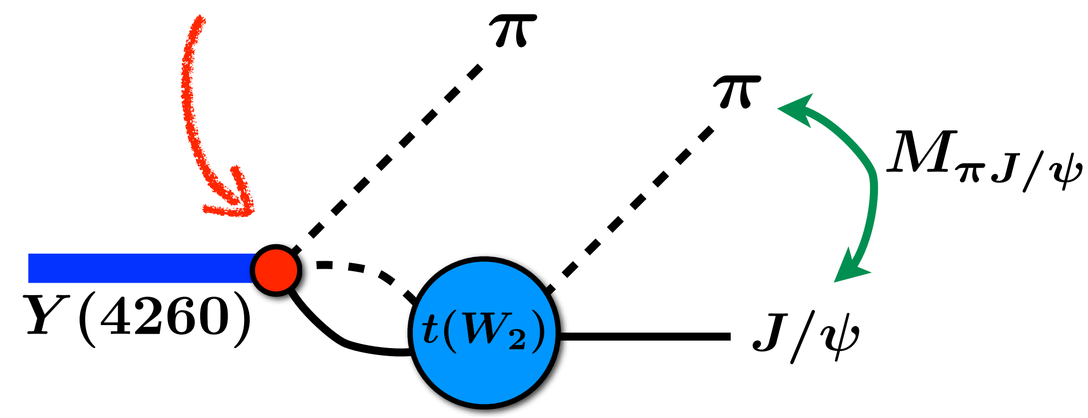




- Good agreement w/ expt data

  (2-parameter fit works well)

- Primary vertex is fixed



➡ predict another decay mode (Y-->π$D^{bar}D^*$)

- **Predicted line shape agrees very well with expt. data**

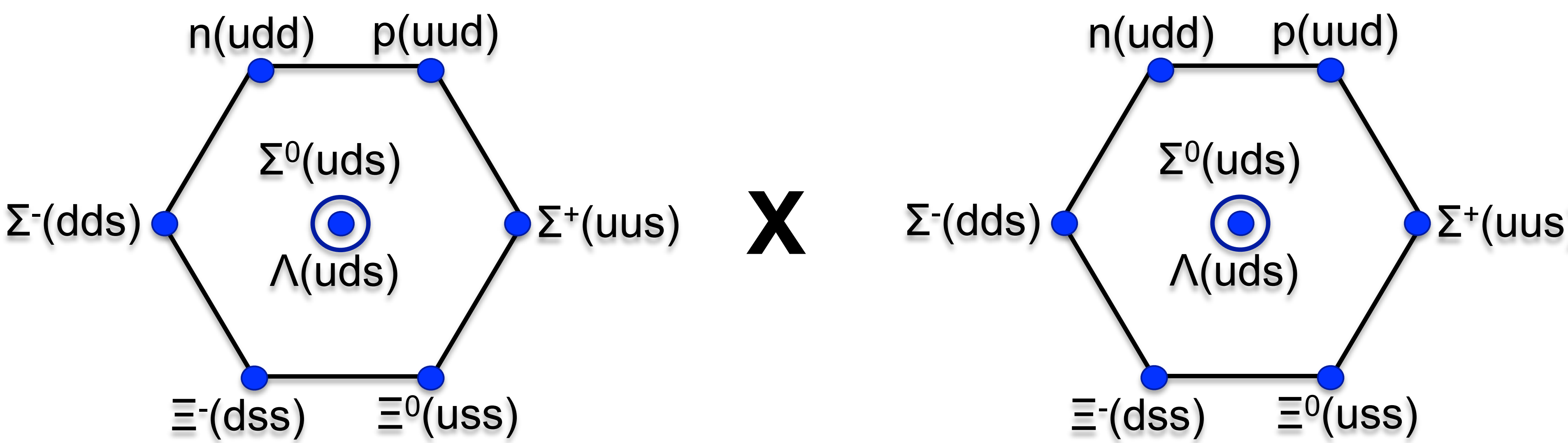
- no peak structure w/o $V^{\pi J/\psi,\, DbarD^*}$ (dashed curve)

Conclusion: $Z_c(3900)$ is threshold cusp induced by strong $V^{\pi J/\psi,\, DbarD^*}$

Fate of H-dibaryon

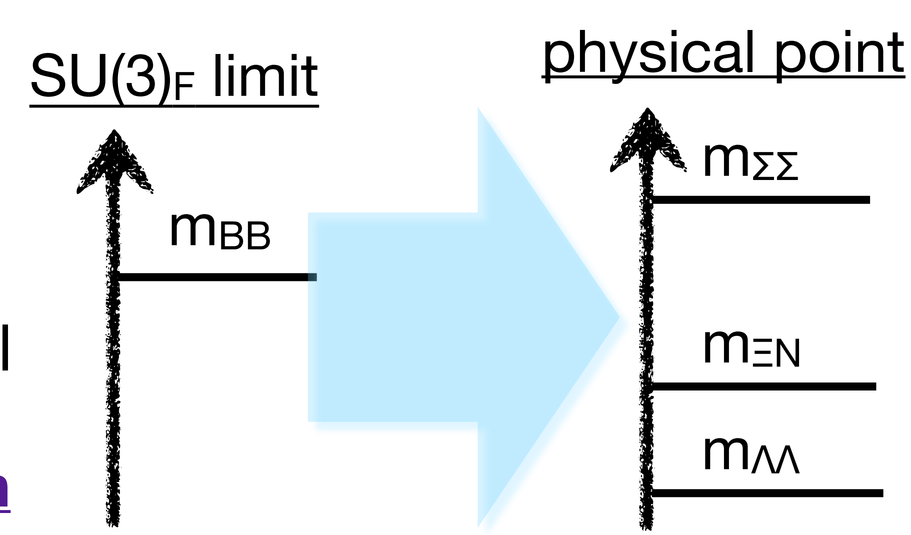
- **Generalized baryon-baryon force @SU(3)$_F$ limit**

Inoue et al. (HAL QCD), PRL106 (2011), NPA881 (2012).



$$= 27 + 8 + 1 + \overline{10} + 10 + \bar{8}$$

- effect of Pauli blocking
- H-dibaryon in SU(3)$_F$



- **Fate of H-dibaryon w/ physical quark mass**
  - ΛΛ - ΞN -ΣΣ coupled-channel problem in $^1S_0$ channel

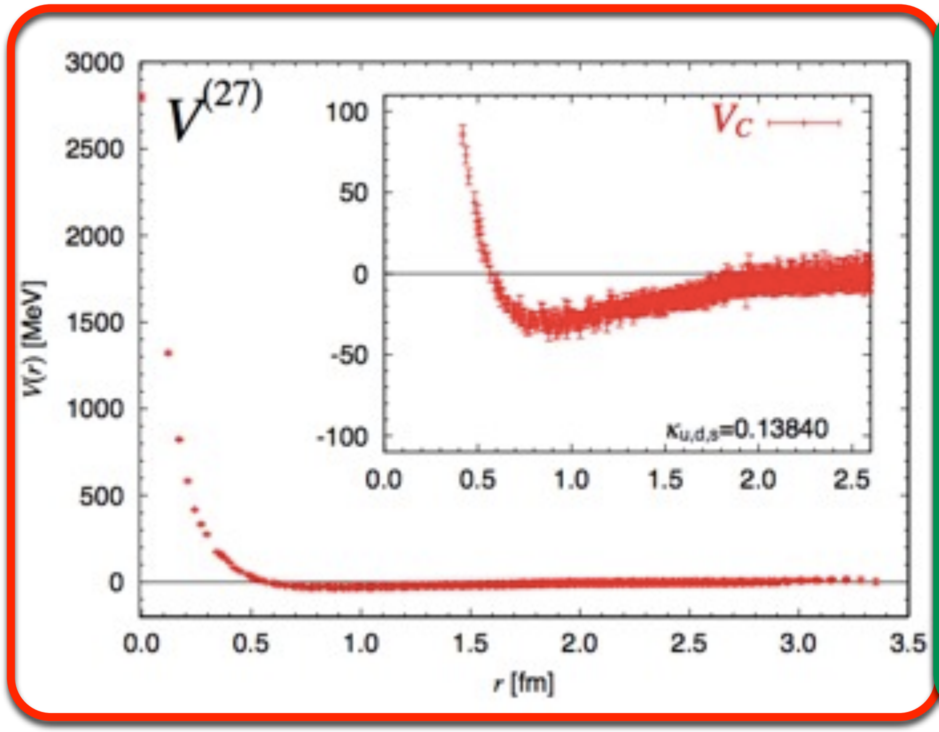
Sasaki et al. (HAL QCD), in preperation

Generalized BB potentials in SU(3)$_F$ limit

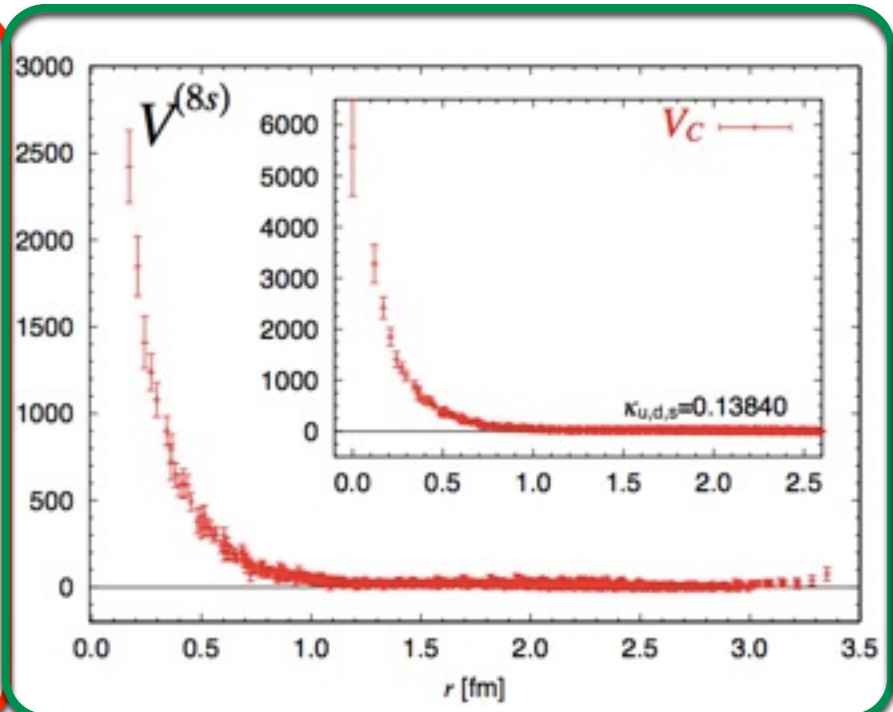
❖ Full QCD in SU(3)$_F$ limit : m_π~0.47GeV, L=3.9 fm

✓ **potentials in flavor symmetric channels --> 27 + 8s + 1**

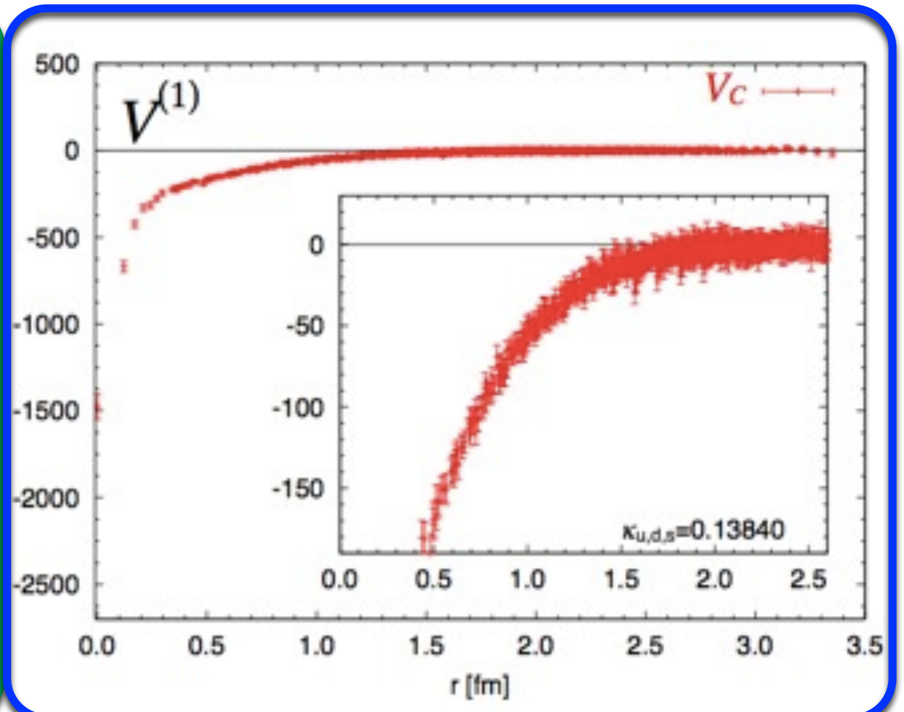
★NN $^1S_0$ channels
(partially Pauli blocked)



★8s channel
(Pauli forbidden)



★H-dibaryon channel
(Pauli allowed)



bound H-dibaryon?

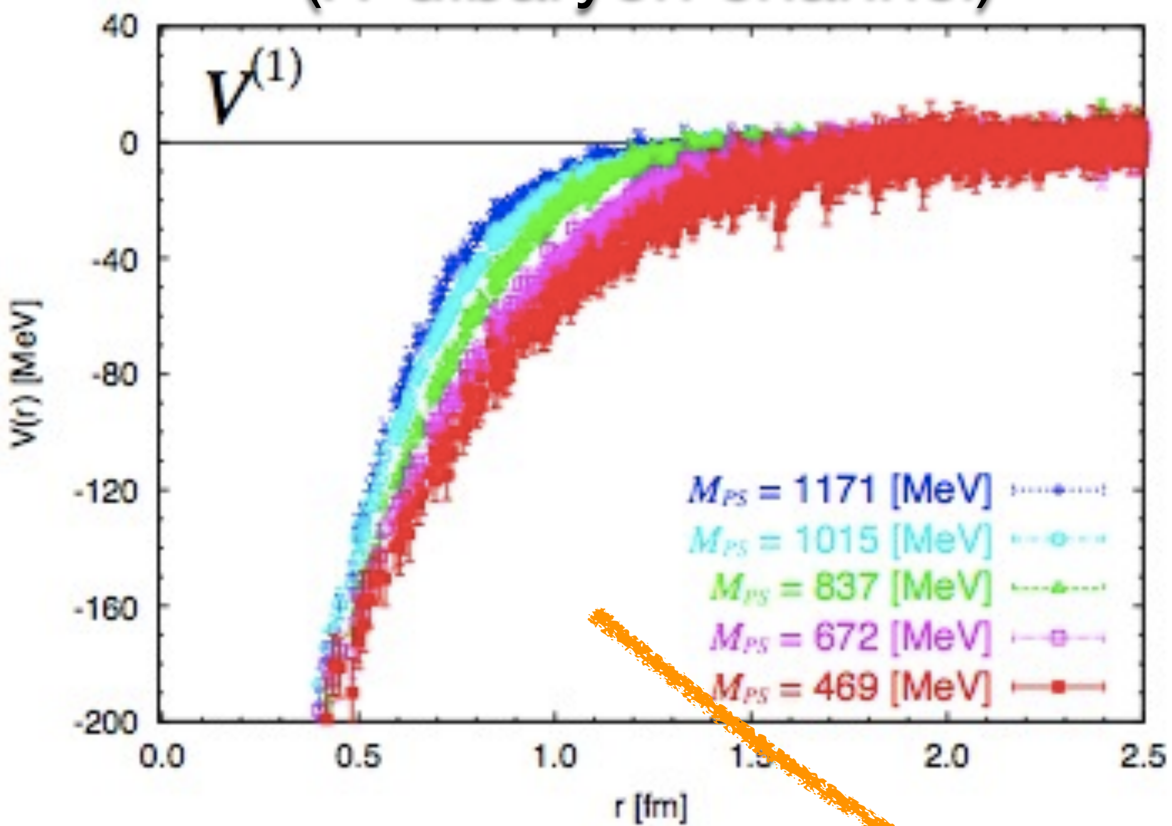
✦ **origin of repulsive cores <--> Pauli principle**

(+ magnetic gluon coupling)

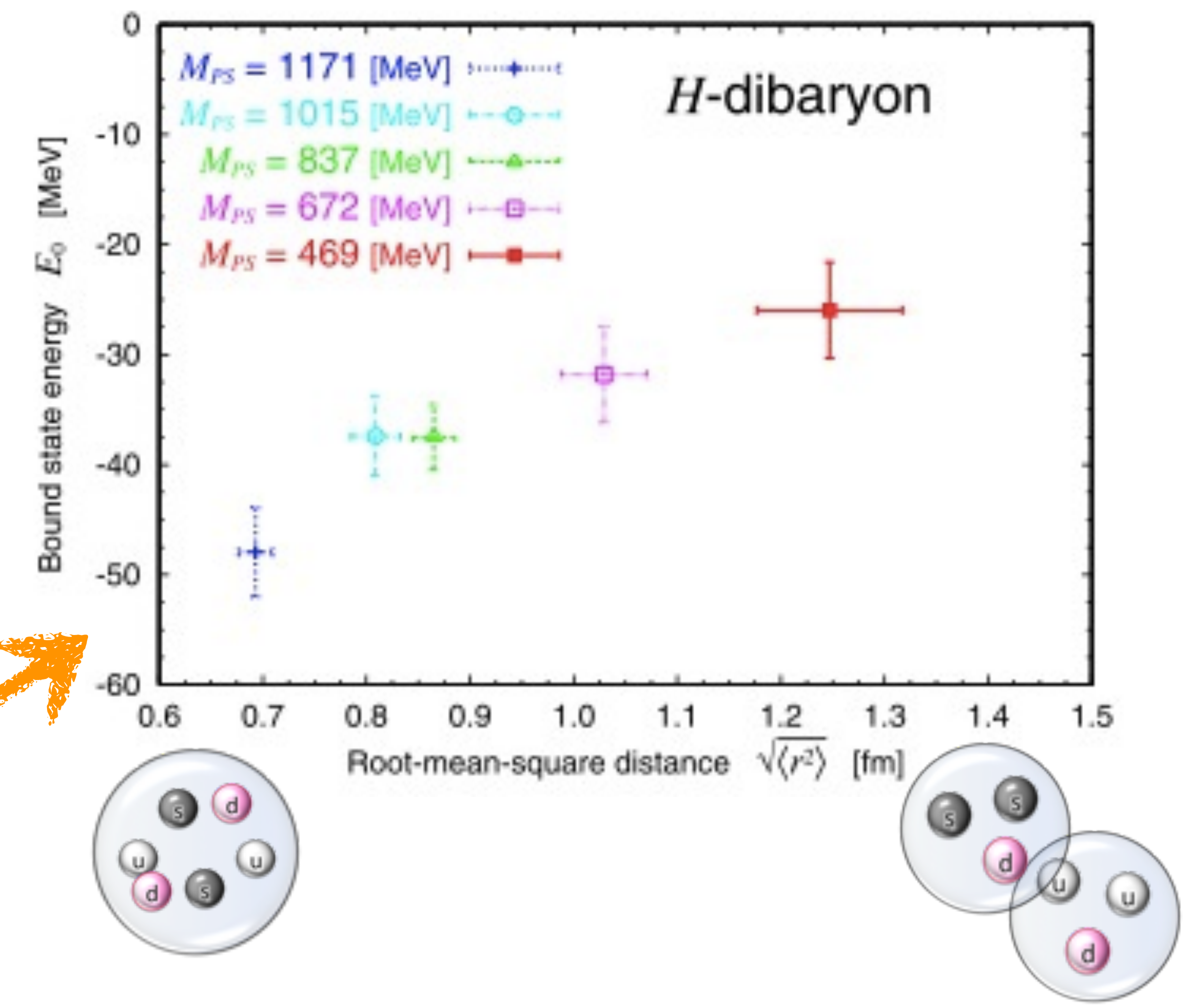
see, Oka & Yazaki, NPA464 (1987)

Structure of H-dibaryon

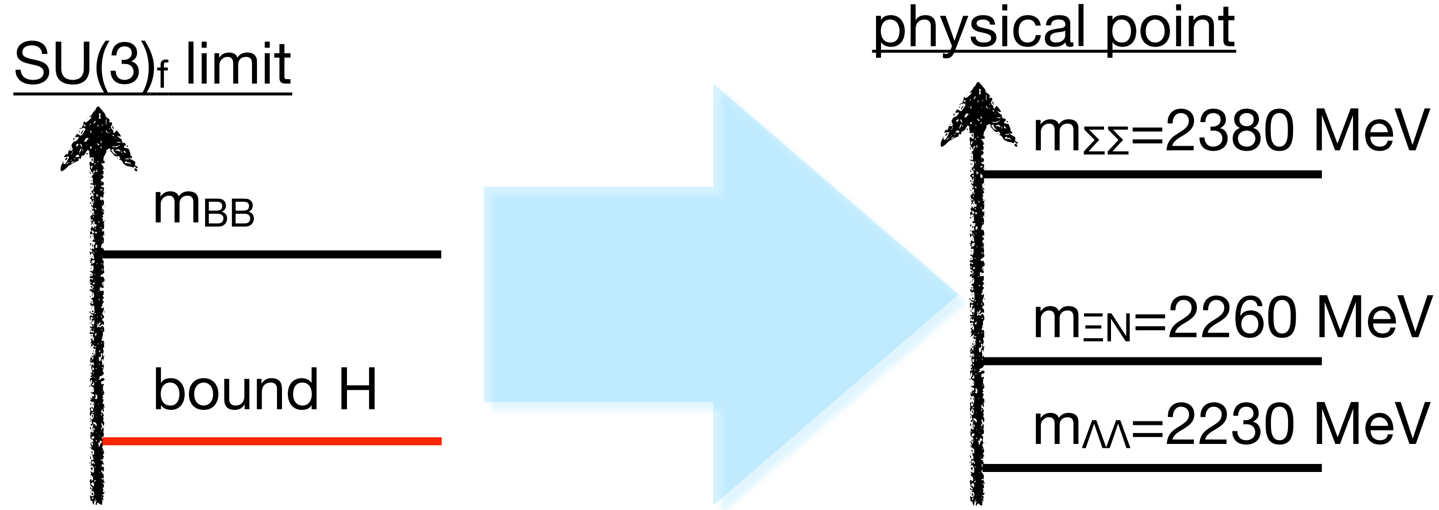
★flavor singlet potential $V^{(1)}$
(H-dibaryon channel)



❖ N_f=3 full QCD : m_π~0.47-1.17GeV, L=3.9 fm



✓ Fate of H-dibaryon

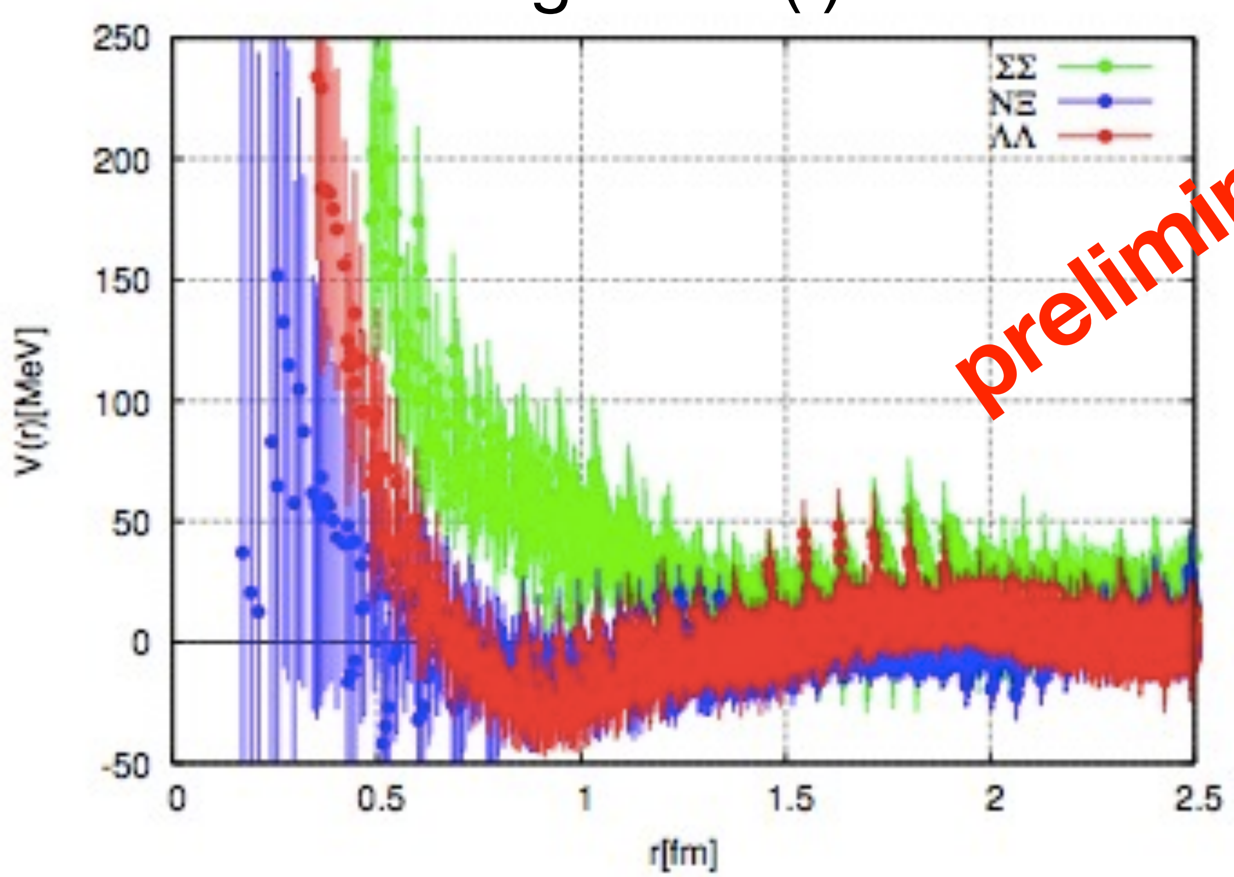


▸ Coupled-channel analysis
on physical point

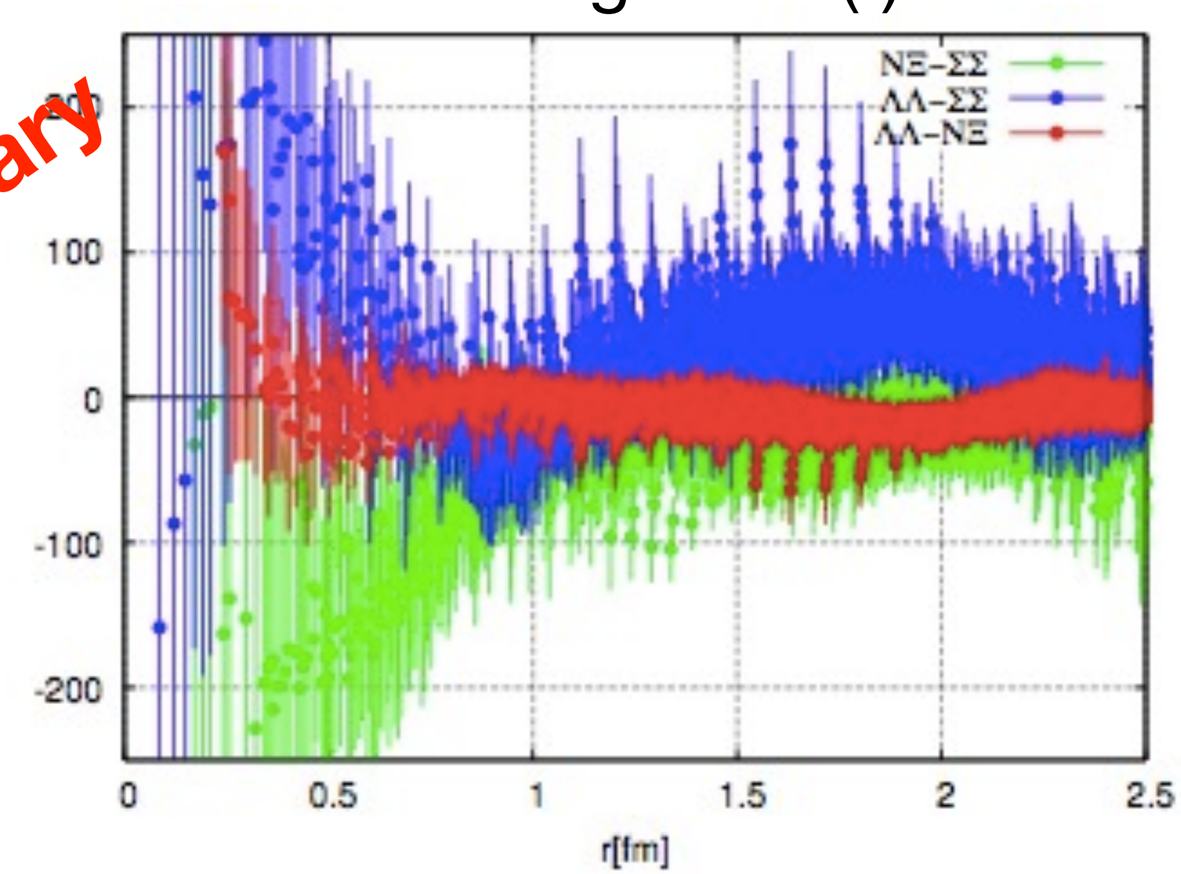
Fate of H-dibaryon @physical point

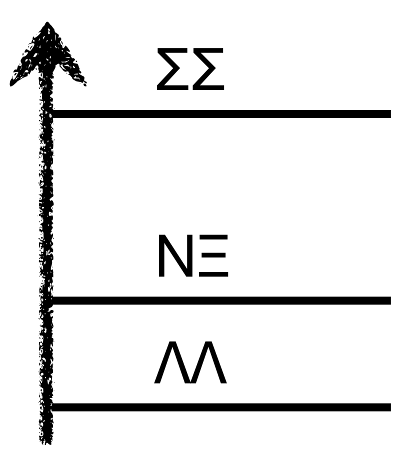
- N_f=2+1 full QCD : m_π~0.14 GeV (physical), L~8 fm (huge volume)

diagonal V(r)



off-diagonal V(r)





- repulsive (8_s)
- attractive (1)
- attractive (27)

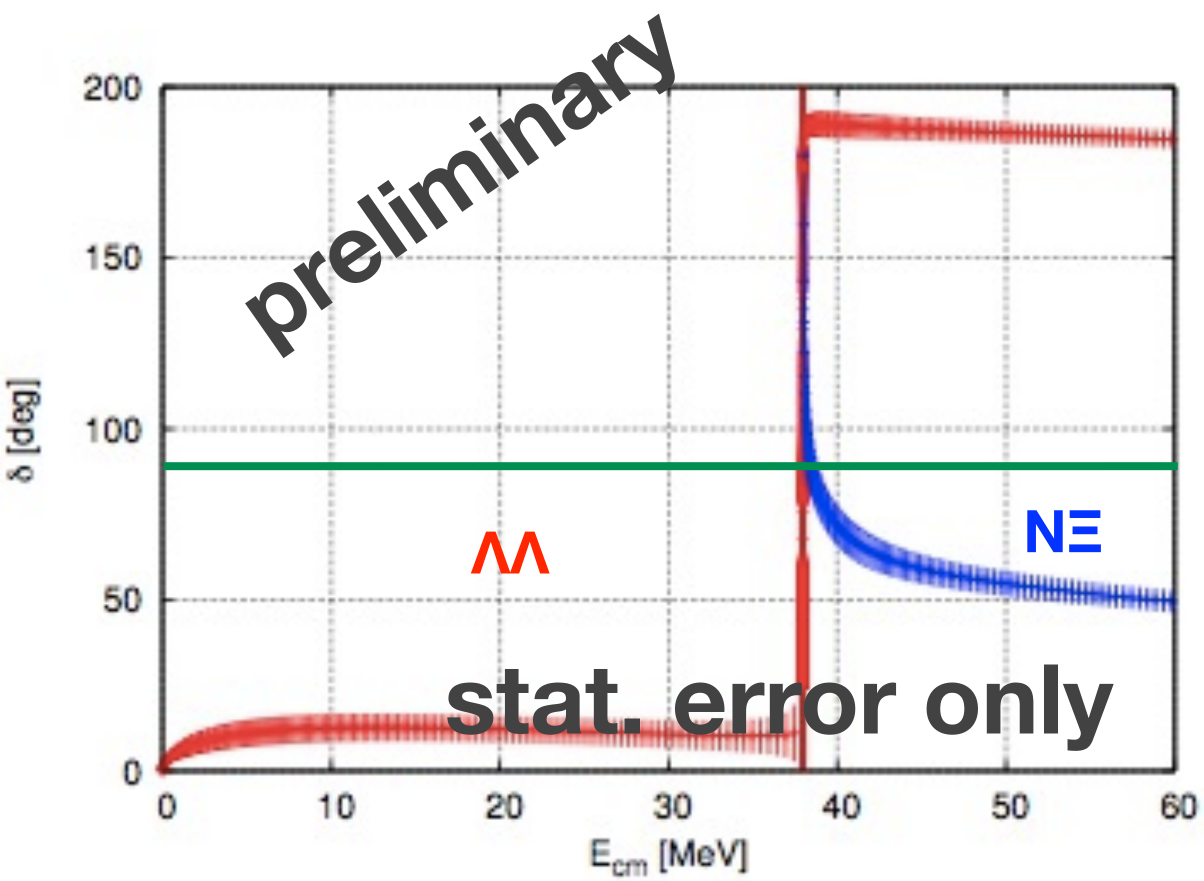
✓ if flavor SU(3) assumed...

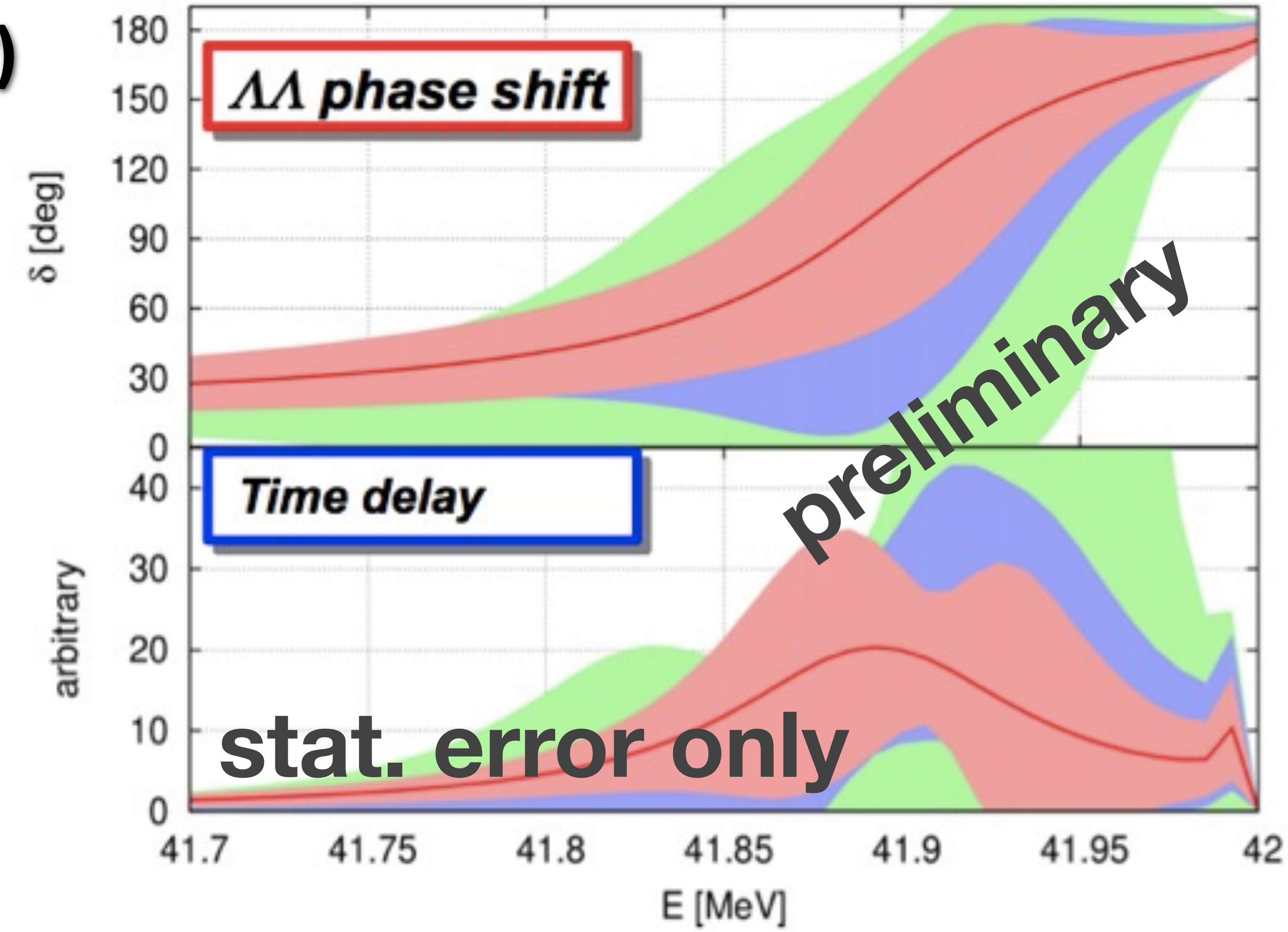
$$\begin{pmatrix} |\Sigma\Sigma\rangle \\ |N\Xi\rangle \\ |\Lambda\Lambda\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -1 & -\sqrt{24} & \sqrt{15} \\ \sqrt{12} & \sqrt{8} & \sqrt{20} \\ \sqrt{27} & -\sqrt{8} & -\sqrt{5} \end{pmatrix} \begin{pmatrix} |27\rangle \\ |8_s\rangle \\ |1\rangle \end{pmatrix}$$

Fate of H-dibaryon @physical point

★ ΛΛ-NΞ phase shift (+ inelasticity)

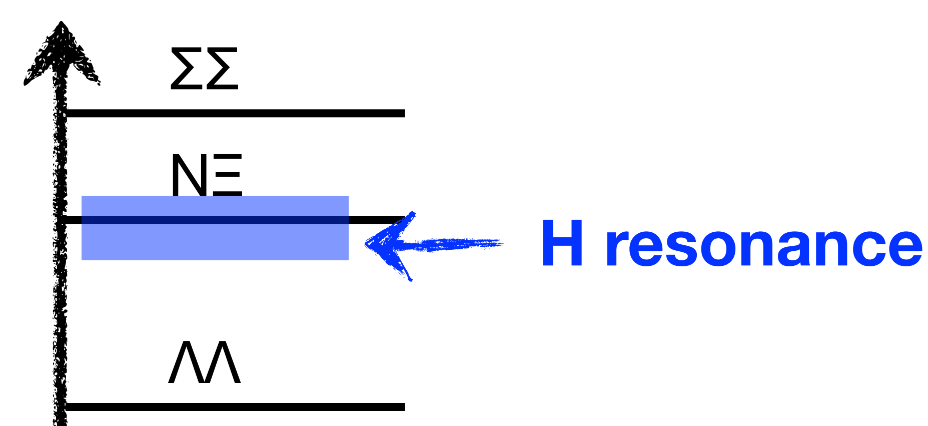
$$S(k) = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$





Answer from QCD for H-dibaryon

"Perhaps a Resonant Dihyperon"



Original prediction of H-dibaryon

Jaffe (1977) based on quark model,

"Perhaps a Stable Dihyperon"

Summary

✦ **HAL QCD method** for coupled-channel hadron interactions

- extraction of “potentials” from equal-time NBS wave functions
- a solution of coupled-channel problems

✤ Charm tetraquark candidate $Z_c(3900)$ in $J^P=1^+$ channel

- **$Z_c(3900)$ is threshold cusp induced by strong $V^{DbarD^*, \pi J/\psi}$**
  - **Virtual pole far from $D^{bar}D^*$ threshold w/ large imaginary part**
  - **Expt. data are well reproduced using coupled-channel HAL QCD method**
  - **No peak structure w/o $V^{DbarD^*, \pi J/\psi}$**

✤ Fate of H-dibaryon

- **Perhaps resonance near ΞN threshold**
  - **No Pauli blocking in flavor singlet channel**
  - **Small ΛΛ-ΞN coupling**
  - **Not a compact hexa-quark**

Thank you for your attention!!