Three-body exotic atoms

Jean-Marc Richard

Institut de Physique Nucléaire de Lyon
Université Claude Bernard (Lyon 1)–IN2P3-CNRS
Villeurbanne, France

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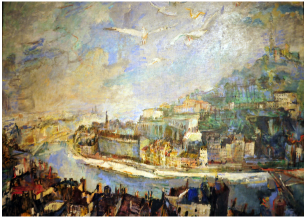

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Collaboration with Claude Fayard (IPNL, Lyon)

Preliminary version: eprint arXiv:1609.09350

Two-body exotic atoms

Exotic atoms: Plateaux and rearrangement

- History: Coulomb + short-range, Zel'dovich, Shapiro, etc.

$$-1/r + \lambda\, V(r)$$

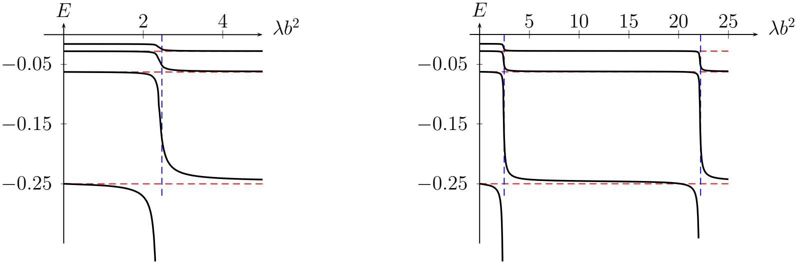
- Where $V(r)$ is very strong but with very short-range
- Energy shift δE small but non perturbative,
- Rearrangement if $V(r)$ attractive, when $\lambda \to$ coupling threshold for binding,
- Pattern explained by the Trueman–Deser formula

$$\delta E \propto a|\phi(o)|^2$$

- Holds beyond the case of a Coulomb interaction

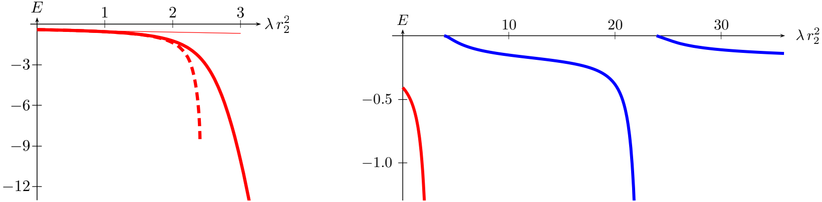
Exotic atoms in 3D

Coulomb + Square well



Results 3D

Sum of two square wells



Theory 3D

- Pattern recovered with a point interaction
- See Albeverio et al., Combescure et al.
- But only one “nuclear” bound state in this model

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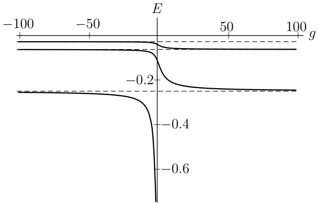

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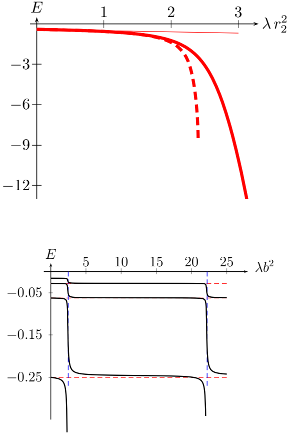


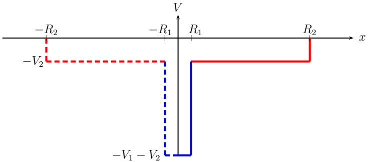
Trueman-Deser

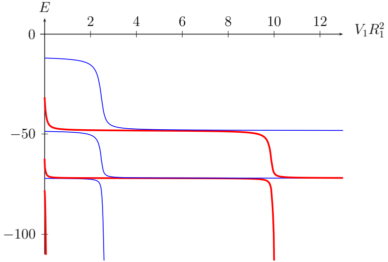
$$\delta E \propto a|\phi(o)|^2$$

- improves ordinary perturbation
- works for small a,
- indicates the trend for large a,
- works again when a becomes small (but positive)
- many corrections and generalisations
  - Coulomb-corrected scattering length
  - Effective range terms
  - P-wave analogues (Partensky + Ericson)

Exotic atoms in 1D

Symmetric case





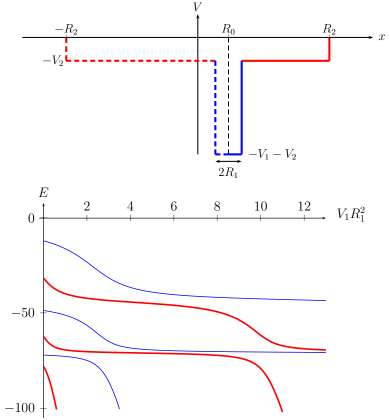
- Odd sector: analogue to 3D
- Even sector: evolves to the odd one.
- Trueman–Deser formula now

$$\delta E \propto \frac{|\phi(0)|^2}{a}$$

- as $\lambda = V_2 - V_1 \nearrow$, the ground-state drops immediately

Exotic atoms in 1D

Asymmetric case

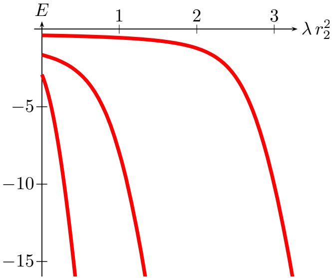


- as $\lambda = V_2 - V_1 \nearrow$, the ground-state drops immediately
- The spectrum evolves from one square well
- to two square wells
- and then can get rearranged

Exotic atoms in 2D

Comparison 1D–2D–3D

$$V = V_{LR} + \lambda\, V_{SR}$$

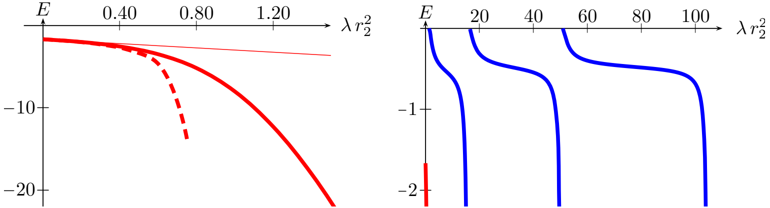


$d = 2$ last dimension for which a potential with $\int V(r)\mathrm{d}^d\vec{r} < 0$, however weak, binds

Exotic atoms in 2D

Patterns in 2D

$$V = V_{LR} + \lambda\, V_{SR}$$



Left: Trueman–Deser, Exact, Simple perturbation

Exotic atoms in 2D

Trueman–Deser in 2D

3D

$$\delta E \propto |\phi(0)|^2\, a + \cdots$$

from, e.g., a Fermi effective contact interaction. n dependence.
a defined as zero of the $E = 0$ asymptotic wave function

$$k \cot\delta(k) = -\frac{1}{a} + \frac{1}{2} r_0^2\, k^2 + \cdots$$

1D

$$\delta E \propto \frac{|\phi(0)|^2}{a} + \cdots$$

2D

$$\delta E \simeq -\frac{|\phi(0)|^2}{\ln(a/\tilde{A}_0)}$$

where $\tilde{A}_0$ deriv. $/E$ of $\sqrt{r}$ coeff. in the ext. pot.

Exotic atoms in 2D

Trueman–Deser in 2D

2D

$$\phi(r) = \frac{1}{\sqrt{2\,\pi}}\frac{u(r)}{\sqrt{r}}~, \qquad -u''(r) - \frac{u(r)}{4\,r^2} + V(r)\,u(r) = E\,u(r)~,$$

at large r

$$u_{\rm as} = \sqrt{r}\,\ln(r/a)~,~\text{at } E = 0$$

$$\cot\delta(k) = \frac{2}{\pi}\,[\ln(a\,k/2) + \gamma] + \frac{1}{2} r_0\,k^2 + \cdots$$

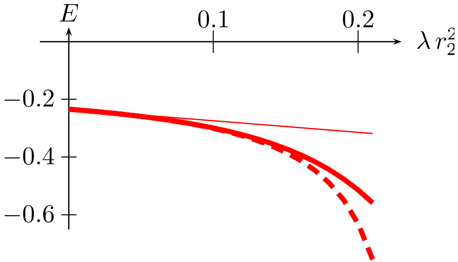
where r_0 is given by an integral similar the the 3D formula.

at small r

$$h(E,r) = B(E)\,\sqrt{r}\,\ln r + A(E)\,\sqrt{r} + \cdots~,$$

$$\tilde{A}_0 = A'(E_0)$$

Example



Exponential well supplemented by another exponential of shorter range. Thick line: exact, dashed line: DT formula, thin line: perturbation theory. We use here
$V = -g_1 \exp(-r/r1) - \lambda \exp(-r/r_2)$ with $r_1 = 2$, $g_1 = 1$ and $r_2 = 0.02$.

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3-body exotic atoms

Motivations

- Many stable atomic ions: $(p, p, \bar{p})$, (p, p, K^-), etc.
- With interesting hadronic dynamics
- Is there in the atomic spectrum any signature of the hadronic interaction?
- Is there an analogue of level rearrangement?
- How many bound states below the $2 + 1$ breaking threshold?
- Preliminary investigation for 3 bosons with LR + SR

Model

$$m = 1\ , \qquad V = \sum_{i<j} v(r_{ij})$$

with

$$v(r) = g_1\, v_e(\mu_1, r) + g_2\, v_e(\mu_2, r)\ , \quad v_e(\mu, r) = -1.4458\, \mu^2\, \exp(-\mu\, r)\ ,$$

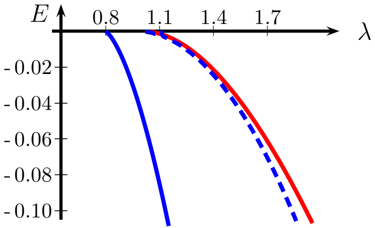
The normalisation is such that v_e at the edge of binding for $g = 1$.

$mu_1 = 1$ fixes the scale in r.

We will adopt typically $\mu_2 = 20$ and study the spectrum as a function of g_2.

Before, we show the spectrum is the external potential alone.

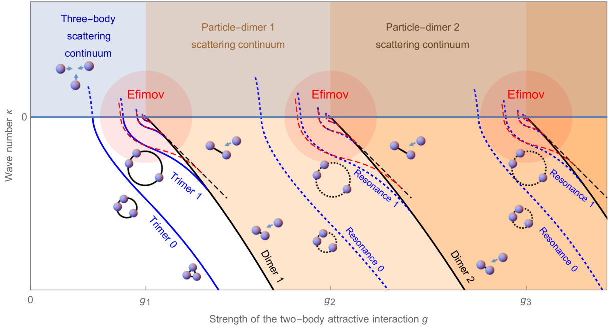
Case of a single potential



- Borromean bound state starts near $g = 0.8$ for monotonic potentials
- First excited: Efimov-like, as it starts just below $g = 1$, but Efimov-unlike, as it often never joins the $2 + 1$ threshold.
- For larger g, more stable states show up.

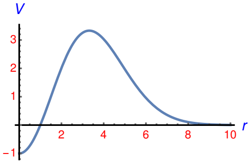
Remark on the large coupling limit

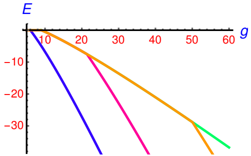
- Much attention is paid to the upper part of the spectrum, i.e., the Efimov sector

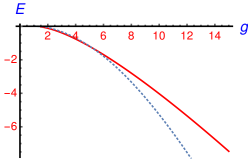


Remark on the large coupling limit

- At large coupling, more and more stable states (but not Borromean)
- Solvable model: $g\,V$, where V = HO shifted downwards, and damped at very large r
- Exponential pot. (right)



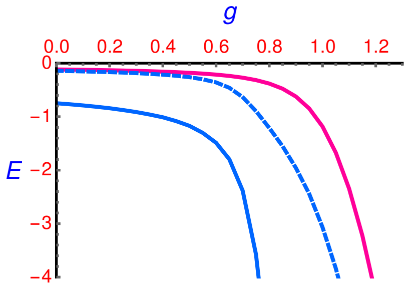


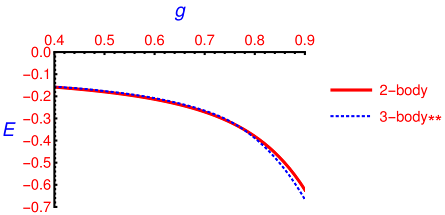


Results for two exponential pot.

$\mu_1 = 1$ and $\mu_2 = 20$

$$V = 2\, v_e(1, r) + g\, v_e(20, r)$$



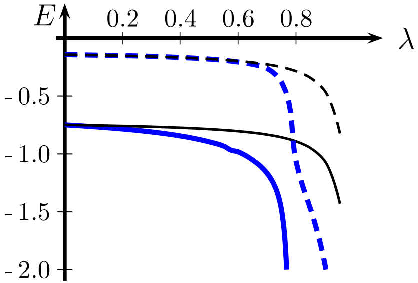


Generalisation of Trueman-Deser?

- At small g

$$\delta E \simeq 3\, A\, a$$

  where
  - a is the 2-body scattering length
  - $A = \langle \delta^{(3)}(\vec{r}_{12}) \rangle$
- But this does not account for δE blowing up near $g = 0.8$ instead of $g = 1$
- We thus need a three-body term



Generalisation of Trueman-Deser?

- The pattern looks like

$$\delta E = A_{\mathrm{LR}}\, B_{\mathrm{SR}} + A'_{\mathrm{LR}}\, B'_{\mathrm{SR}} + \cdots$$

where

- $A_{\mathrm{LR}} = \langle \delta^{(3)}(\vec{r}_{12}) \rangle$
- B_{SR} = 2-body scat. length
- $A'_{\mathrm{LR}} \propto |\Phi_{123}(0,0)|^2$ or some derivative
- B'_{SR} some (connected) three-body “scattering length” that becomes infinite for $g \to g_3^{\mathrm{cr}} \simeq 0.8$

Summary

- $V(r) = V_{\rm ext} + V_{\rm int}$
- 2-body
  - 2-body energy shift understood in terms of scattering length of the inner potential
  - rearrangement = sudden change when the inner potential develops its own spectrum
- 3-body
- restricted to three bosons with simple potentials
- calculations delicate with a superposition of LR and SR
- the onset of Borromean 3-body binding in SR is seen in the "atomic" spectrum
- aim: calculate $K^- pp$ to "see" the hadronic molecule
- many open questions remaining

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