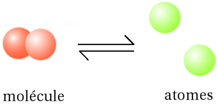
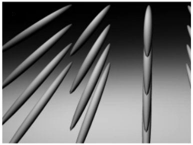
Few atoms in atomic waveguides





Ludovic Pricoupenko

Laboratoire de Physique Théorique de la Matière Condensée
Université Pierre et Marie Curie – Paris

Saclay - 03 February 2017

Outline

- **Introduction:**

Few-body problems: a common interest in nuclear and ultracold physics

- **Context:**
  - Atomic gases near *s*-wave magnetic Feshbach resonances
  - Three-body resonant problem for bosons → Efimov physics
  - Atomic gas in low dimensions

    → the 1D Bose gas and the Lieb Liniger model
- **2- and 3-body problem in 1D atomic waveguides**
  - Two-channel model
  - Confined induced dimers
  - Some results of the Skorniakov Ter Martirosian equation
- **Summary & perspectives**

Deep links between nuclear physics and ultracold atoms

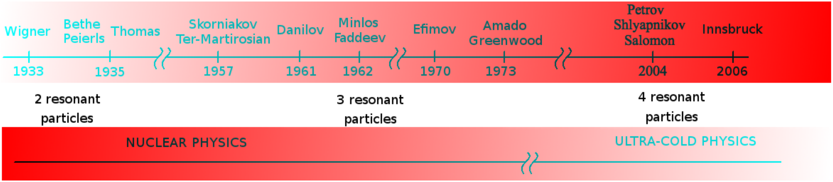
Two different energy scales ($\sim 10^{-13}$ ev $\equiv$ 1 nK $\rightarrow \sim$ 10 Mev)

... but a lot of things to share:

- common interest on the few-body problem and the limit toward many-body states (ex: few n $\rightarrow$ neutron star)
- importance of resonant phenomena
- search for exotic quantum states (ex: 4n, Efimov ...)

⋮

Example : contact models & universality in few-body physics



Some key features of ultracold-atoms not present in nuclear physics

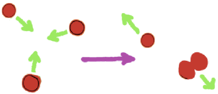
- Tunable effective interactions
- Tunable dimensionality 3D $\leftrightarrow$ 2D, 3D $\leftrightarrow$ 1D
- Dilute limit $nb^3 \ll 1$ & possible large correlations $na^3 \gtrsim 1$ (unitary limit)
- Two-body microscopic potentials well known and sufficient for quantitative calculations

Few-body properties : at the heart of ultracold gases experiments

The N-body ground state at T=0K is a solid 

$\Longleftrightarrow$ many deep bound states for $2, 3, \ldots N$-atoms

Three-body recombination processes



- Bose gas : large losses for $na^3 > 1$
  $\Longrightarrow$ No thermodynamical equilibrium at resonance!
- Two-component Fermi gas : stability at resonance !
  (Petrov-Shlyapnikov-Salomon 2004)

$\Longrightarrow$ Experimental study of the BEC-BCS crossover & unitary Fermi gas

Order of magnitude in ultracold physics

- Typical trap size: few μ m
- Number of atoms: 10^5
- Atomic density: $10^{13} \lesssim n \lesssim 10^{15}$ atoms/cm^3
- Temperature: 1 nK $\lesssim T \lesssim \mu$ K
- Range of the interaction: $b \sim R_{\mathrm{vdW}} = \frac{1}{2}\left(\frac{mC_6}{\hbar^2}\right)^{1/4}$ ~10 nm

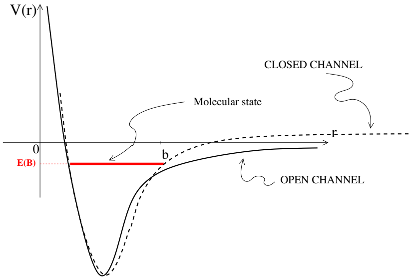
Scale separation:

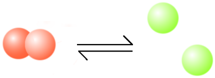
$$\frac{\hbar^2 n^{2/3}}{m}, k_B T, \mu \ldots \ll \frac{\hbar^2}{mb^2}$$

- "Low" energy: collective modes, bound states near scattering resonances ...

  "High" energy: usual molecules, clusters ...

Resonance and Feshbach mechanism



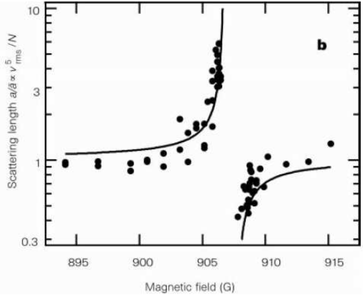


+ external magnetic field $\mathcal{B}$

$$E_{\rm mol} \sim E^0_{\rm mol} + \delta\mathcal{M} \times \mathcal{B} \quad \Longrightarrow \quad a = a_{\rm bg}\left(1 - \frac{\Delta\mathcal{B}}{\mathcal{B} - \mathcal{B}_0}\right)$$

s-wave scattering length near the resonance

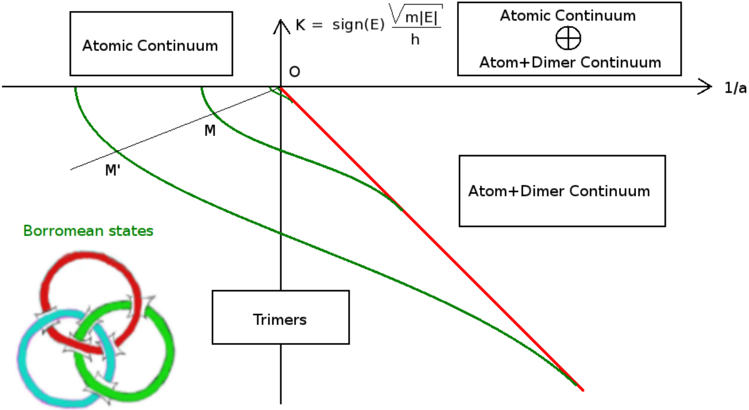
First achievement: group of Ketterle - 1998



$$a = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

- Feshbach resonance at : $B = B_0$
- Shape resonance: $|a_{\rm bg}| \gg R_{\rm vdW}$
- Away from a shape resonance: $a_{\rm bg} = \mathcal{O}(R_{\rm vdW})$

Efimov spectrum



Discrete scaling symmetry $E(M') = \lambda^2 \times E(M)$ where $\lambda = \exp(\pi/|s|)$.

Evidence of the Efimov effect in ultracold atoms

Studies in the vicinity of magnetic Feshbach resonances

Three-bosons

- Group at Innsbrück university: experiments using ^{133}Cs
- Group at Firenze university: experiments using ^{39}K
- Group at Bar-Ilan University: experiments using ^{7}Li
- Group at Rice university: experiments using ^{7}Li

Three-spin-component fermions ^{6}Li

- Group of the university of Penn-State
- Group of the university of Heidelberg
- Group of the university of Tokyo

Hetero-nuclear systems

Group of Firenze : KKRb and KRbRb Efimov trimers.

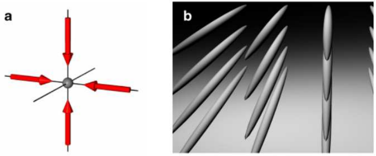
...

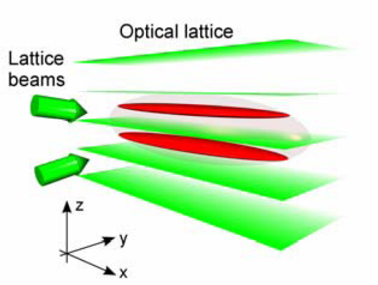
See review : F. Ferlaino, R. Grimm, Physics **3**, 9 (2010)

Atoms in Low dimensions

Possible to tune the external potential

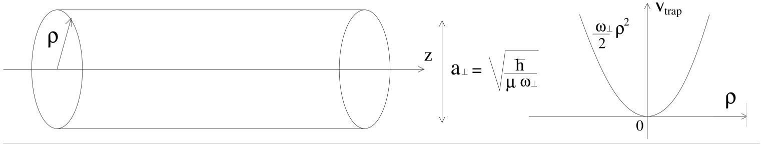
$\Longrightarrow$ studies of atoms in quasi-1D or quasi-2D geometries





Quasi-1D geometry

- Atoms in a 2D isotropic harmonic trap



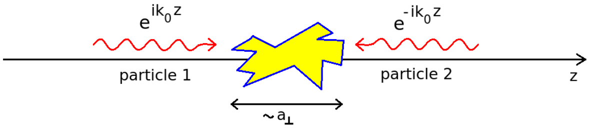
$\longrightarrow$ 1D atomic wave guide $E_{1-\text{particle}} = \frac{\hbar^2 k^2}{2m} + \hbar\omega_\perp(2n + |m| + 1)$

- Low energy monomode regime

Characteristic energies : $k_B T, \frac{\hbar^2}{m}\rho^{2/3} \cdots \ll \hbar\omega_\perp$

1D contact model of Lieb Liniger

- Low energy 1D scattering of two particles $z = z_1 - z_2$



$\langle z|\psi_{k_0}\rangle \sim e^{ik_0 z} + f_{1D}(k_0) e^{ik_0|z|}$ $f_{1D}(k_0) = \frac{-1}{1+ik_{1D}a_{1D}}$

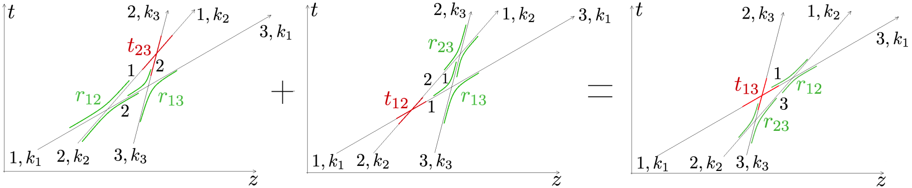
- Lieb-Liniger model : δ interaction

  $\rightarrow$ reproduce exaclty f_{1D} at the 2-body level

- $a_{1D} \rightarrow 0^-$: Tonks Girardeau regime
- $a_{1D} > 0$: 1D N-body bound states $\rightarrow$ solitonic regime:

Energy : $\frac{-\hbar^2 N(N^2-1)}{6m{a_{1D}}^2}$

3-body problem: heart of the integrability in 1D



$r_{12}\ r_{13}\ t_{23} + t_{12}\ r_{13}\ r_{23} = r_{23}\ t_{13}\ r_{12}$

r_{ij} & t_{ij} : reflexion and transmission coefficients

$$r_{ij} = f_{1\mathrm{D}}\left(\frac{k_i - k_j}{2}\right) \ \& \ t_{ij} = 1 + f_{1\mathrm{D}}\left(\frac{k_i - k_j}{2}\right)$$

Purpose of the present study

Three-body bound states for bosons in a 1D waveguide

From the Efimov law $-E_0 \exp(-2n\pi/s_0)$

... to the trimer of the Lieb-Liniger model $-\dfrac{4\hbar^2}{ma_{1\mathrm{D}}^2}$

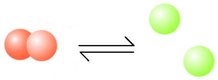
Two-channel modeling of the Feshbach resonance

Atom : $a^\dagger_{\mathbf{k}}|0\rangle$; Molecule : $b^\dagger_{\mathbf{k}}|0\rangle$

$$H = \int \frac{d^3k}{(2\pi)^3} \left[E_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \left(\frac{E_{\mathbf{k}}}{2} + E_{\mathrm{mol}} \right) b^\dagger_{\mathbf{k}} b_{\mathbf{k}} \right]$$

(Kinetic term & $E_{\mathrm{mol}} = E^0_{\mathrm{mol}} + \delta\mathcal{M}\mathcal{B}$)

$$+\Lambda \int \frac{d^3k d^3K}{(2\pi)^6} \left[\langle k|\delta_\epsilon\rangle b^\dagger_{\mathbf{K}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}+\mathbf{k}} + \mathrm{h.c.} \right]$$

()

$$+\frac{g}{2} \int \frac{d^3k d^3K d^3k'}{(2\pi)^9} \langle k'|\delta_\epsilon\rangle \langle \delta_\epsilon|k\rangle a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}}.$$

(atom-atom interaction)

$E_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m}$ & $\langle k|\delta_\epsilon\rangle = \exp\left(-\frac{k^2\epsilon^2}{4}\right)$ cut-off function

ϵ : short range parameter $\epsilon \equiv \mathcal{O}(b)$

Parameters obtained from the 2-body properties at low E

Transition operator: $\hat{T}(E) = \frac{|\delta_\epsilon\rangle\langle\delta_\epsilon|}{\left(g+\frac{\Lambda^2}{E-E_{\rm mol}}\right)^{-1} - \langle\delta_\epsilon|\hat{G}_0^{\rm rel}(E)|\delta_\epsilon\rangle},$

$\hat{G}_0^{\rm rel}(E)$: resolvent for the relative particle

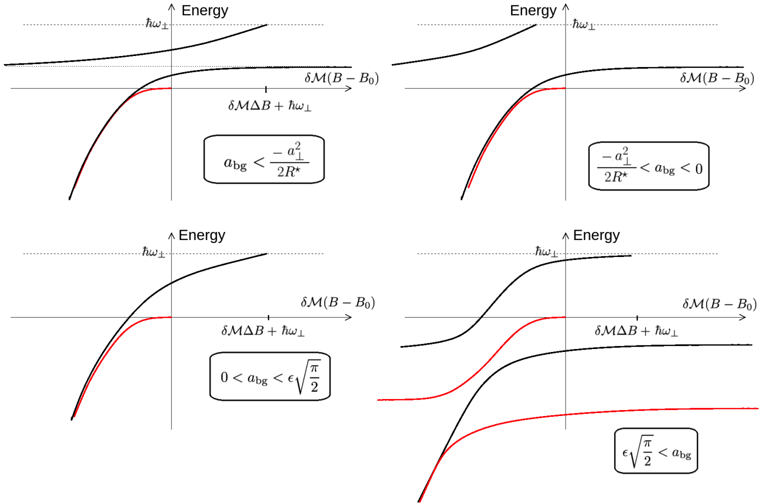
- **background scattering length:** $a_{\rm bg} \leftrightarrow g$
- **scattering length:** $a = a_{\rm bg}\left(1 - \frac{\Delta\mathcal{B}}{\mathcal{B}-\mathcal{B}_0}\right)$
- **width parameter (Petrov 2004) :**

$$R^\star = \frac{\hbar^2}{m a_{\rm bg}\delta\mathcal{M}\Delta\mathcal{B}} \propto \frac{1}{\Lambda^2}.$$

If $R^\star \gg R_{\rm vdw} \equiv$ natural cut-off for low energy properties

Confinement Induced Dimers in the 1D waveguide

- Zero-range model Olshanii (1998)
- Two-channel model:



PRA T. Kristensen, L. Pricoupenko

Study of trimers in the waveguide

- $E = E_{\mathrm{3body}} - E_{\mathrm{Com}} < 0$
- s-wave sector
- Skorniakov Ter Martirosian equation for trimers in the 1D waveguide :

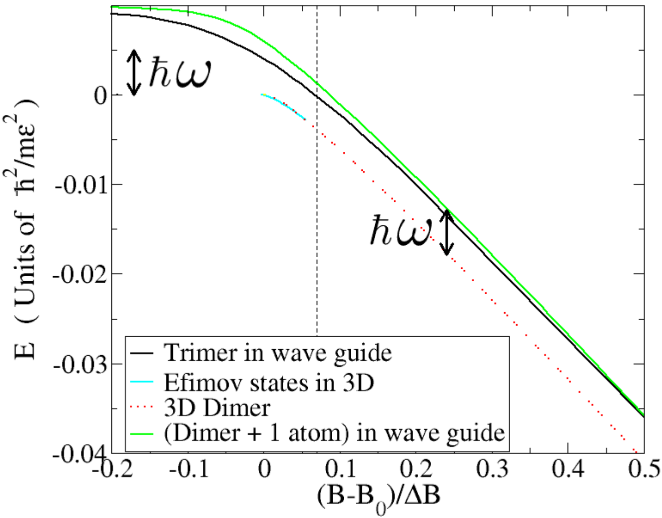
$$D(E^{\mathrm{rel}}) f(\underline{n}, k) - 2 \sum_{\underline{n}'=0}^{\infty} \int \frac{dk'}{2\pi} \langle \underline{n}, k | \mathcal{K}(E) | \underline{n}', k' \rangle f(\underline{n}', k') = 0$$

- Explore the case $a_\perp/\epsilon = 20$ for several resonances
- n_{max} from 100 to 400 $\Longrightarrow (2n_{\mathrm{max}}+1)^3$ values of $d^j_{m,m'}(\theta)$.
- E such that the integral equation has a zero eigenvalue (typical matrix sizes $\sim$ 30 000)

Example of a narrow resonance

^{39}K at $B_0 = 752$ G ; $R^\star = 36.4 R_{\mathrm{vdw}}$

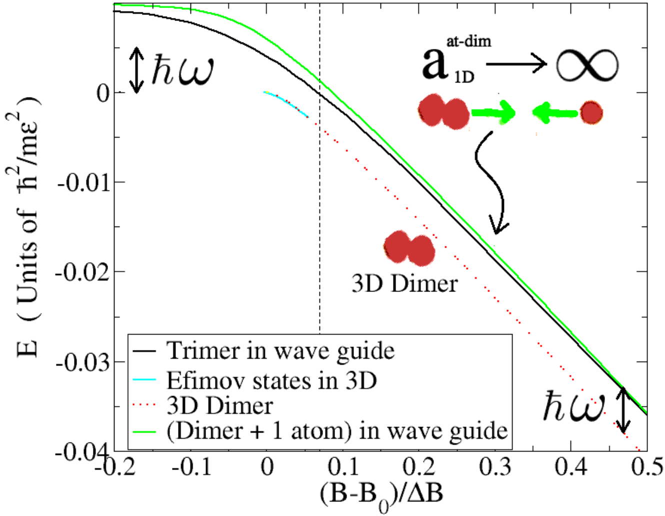
$a_\perp = 20\epsilon$ ($\omega = 2\pi \times 55.4$ kHz)



Example of a narrow resonance

^{39}K at $B_0 = 752$ G ; $R^\star = 36.4 R_{\rm vdw}$

$a_\perp = 20\epsilon$ ($\omega = 2\pi \times 55.4$ kHz)



A direct link with the Efimov 3-body parameter

$$a_{1\mathrm{D}} \sim \frac{-a_\perp}{2}\left[\zeta\left(\frac{1}{2}\right) + \frac{a_\perp}{a_{3\mathrm{D}}}\right]. \quad \text{(Olshanii PRL-1998)}$$

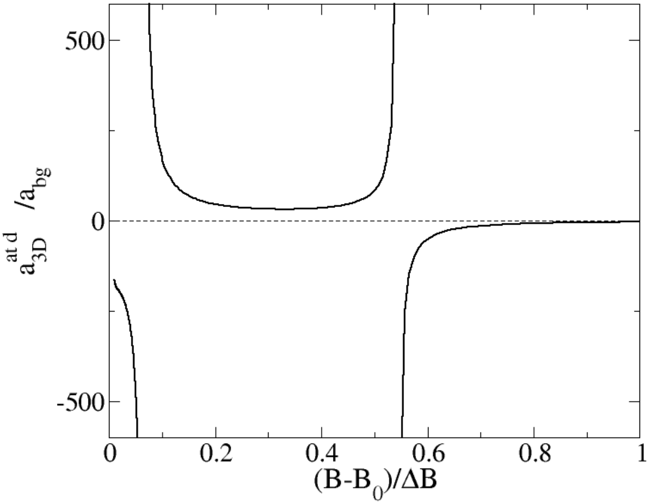
Threshold for the shallow 1D trimer on the 1D dimer line for:

$$a_{1\mathrm{D}}^{\mathrm{at-d}} \to \infty \quad \Longleftrightarrow \quad a_{3\mathrm{D}}^{\mathrm{at-d}} \to 0^+$$

Regime of interest for the dimer binding energy :

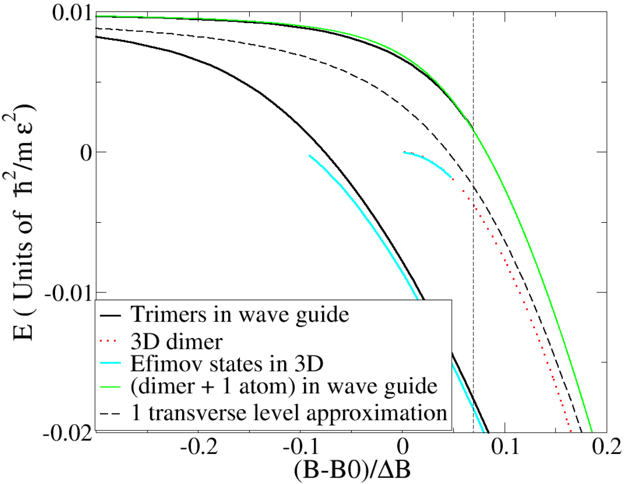
- Structureless 3D dimer in the waveguide
  $$\Longrightarrow |E_{\mathrm{dimer}}| \gg \hbar\omega$$
- $a_{3\mathrm{D}}^{\mathrm{at-d}}$ function of the three-body parameter
  $$\Longrightarrow |E_{\mathrm{dimer}}| \ll \frac{\hbar^2}{m\epsilon^2}$$

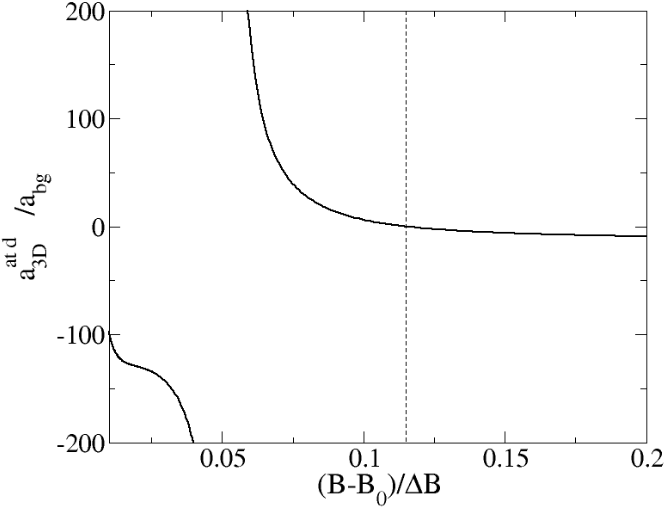
A way to measure the three-body parameter for different values Energy/magnetic field than in 3D ?


500
0
-500
0
0.2
0.4
0.6
0.8
1
(B-B0)/ΔB
a3D at d /abg

Example of a broad resonance

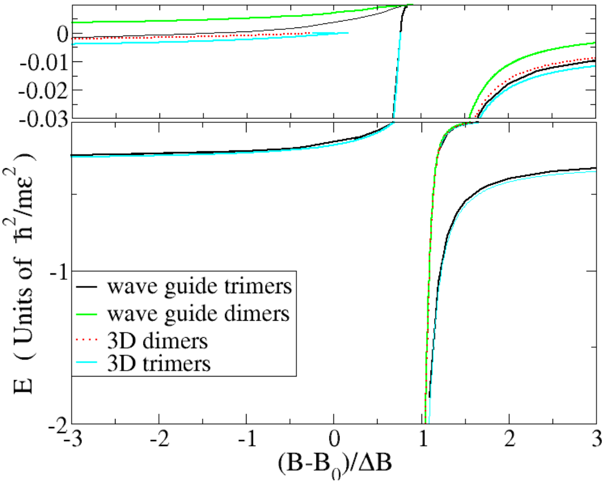
^{7}Li at $B_0 = 737$ G, $R^\star = 1.2 R_{\rm vdw}$
$a_\perp = 20\epsilon$ ($\omega = 2\pi \times 2.7 \times 10^6$ Hz)

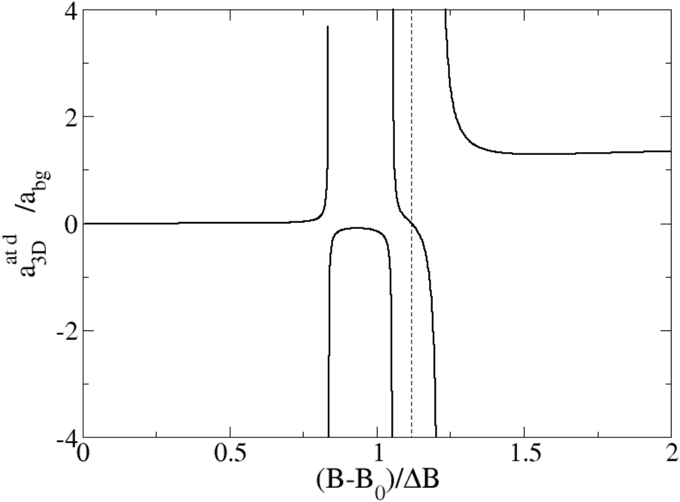



200
100
0
-100
-200
0.05
0.1
0.15
0.2
$a_{3D}^{at\,d}/a_{bg}$
$(B-B_0)/\Delta B$

Feshbach resonance in the vicinity of a shape resonance

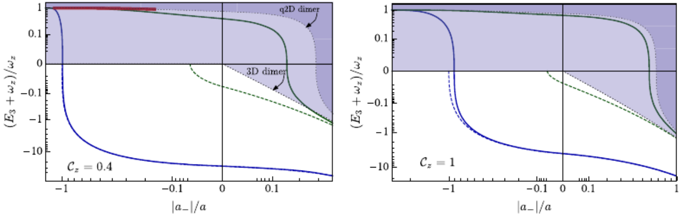
^{133}Cs at $B_0 = -12$ G ; $R^\star = 1.3 \times 10^{-3} R_{\rm vdw}$
$a_\perp = 20\epsilon$ ($\omega = 2\pi \times 6.7$ kHz)




4
2
0
-2
-4
0
0.5
1
1.5
2
$a_{3D}^{at\,d}/a_{bg}$
$(B-B_0)/\Delta B$

Comparison with a recent 2D study

2D waveguide & single-channel separable model



Jesper Levinsen, Pietro Massignan, and Meera M. Parish, Phys. Rev. X **4**, 031020 (2014).

Summary & perspectives

- Model including the Feshbach coupling for the 3-body problem in wave guides

Summary & perspectives

- Model including the Feshbach coupling for the 3-body problem in wave guides
- Regime where a_{1D} is large and positive:

1D Lieb-Liniger trimer continuously connected with the dimer branche

Summary & perspectives

- Model including the Feshbach coupling for the 3-body problem in wave guides
- Regime where a_{1D} is large and positive:

1D Lieb-Liniger trimer continuously connected with the dimer branche

- Lieb-Liniger trimer continuously connect to the atom-dimer branche

other quasi-degenerate trimers exists continuously connected with Efimov trimers for large $a_\perp/\epsilon$

Summary & perspectives

- Model including the Feshbach coupling for the 3-body problem in wave guides
- Regime where a_{1D} is large and positive:

1D Lieb-Liniger trimer continuously connected with the dimer branche

- Lieb-Liniger trimer continuously connect to the atom-dimer branche

other quasi-degenerate trimers exists continuously connected with Efimov trimers for large $a_\perp/\epsilon$

- Trimer threshold at the Atom-dimer spectrum

Systematic shift from $a_{3\mathrm{D}}^{\mathrm{at-dim}} \to \infty$ to $a_{3\mathrm{D}}^{\mathrm{at-dim}} \to 0$ for the deepest trimer branches & large $a_\perp/\epsilon$

Summary & perspectives

- Model including the Feshbach coupling for the 3-body problem in wave guides
- Regime where a_{1D} is large and positive:

1D Lieb-Liniger trimer continuously connected with the dimer branche

- Lieb-Liniger trimer continuously connect to the atom-dimer branche

other quasi-degenerate trimers exists continuously connected with Efimov trimers for large $a_\perp/\epsilon$

- Trimer threshold at the Atom-dimer spectrum

Systematic shift from $a_{\rm 3D}^{\rm at-dim} \to \infty$ to $a_{\rm 3D}^{\rm at-dim} \to 0$ for the deepest trimer branches & large $a_\perp/\epsilon$

- Near future :
  - Explore larger aspect ratio
    $\Longleftrightarrow$ More insights in the confined induced trimers
  - 3-body collisions in 1D waveguides
    $\Longleftrightarrow$ violation of integrability