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Mechanisms of pion double charge reaction and search of trineutron and tetraneutron

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Workshop
Dynamics of highly unstable exotic light nuclei and few-body systems

$$\pi^{\pm} + A(Z,N) \Rightarrow \pi^{\mp} + B(Z \pm 2, N \mp 2)$$

Double charge exchange of pions on nuclei occupies a particular position among all known nuclear reactions. It is unique because through the reaction one can obtain nuclei for which the Z component of the isospin differs by two units from that of the original nuclei. This is possible by double isospin flip of the pion.

- **de Shalit, Drell, and Lipkin in 1962 predicted existence of DCX**

A. de Shalit, Seminar at Saclay, 1962.

- **Experimentally this process was discovered in the Laboratory of Nuclear Problems at the JINR in 1963**

Yu. A. Batusov, et al.Preprint R-1474, JINR, Dubna, 1963; Sov. Phys. JETP 19, 557 (1964).

- **During the 54 years after the discovery the pion double charge exchange reaction has generated a significant amount of theoretical and experimental work**

For a comprehensive review, please refer to Refs

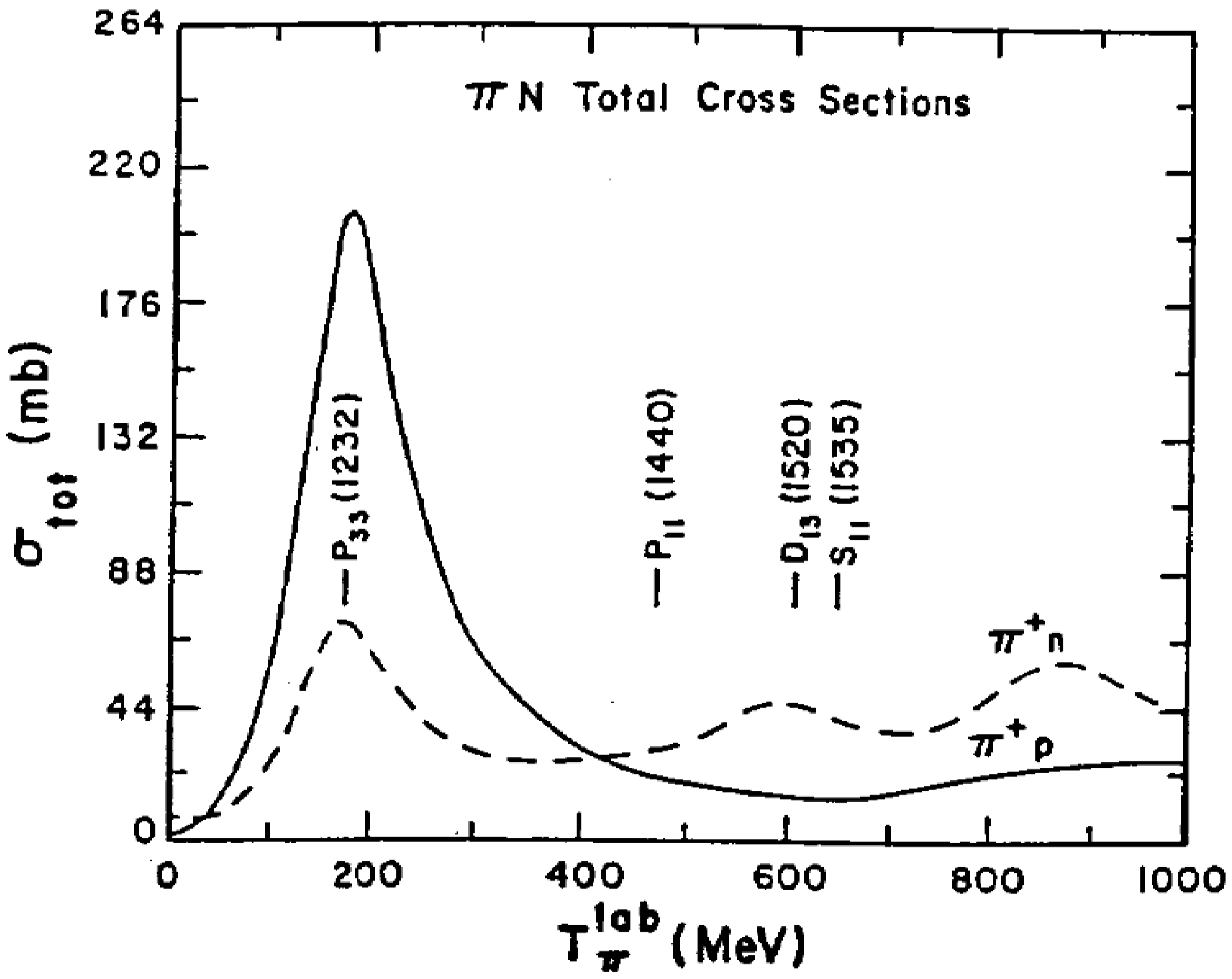
- F. Backer and Yu.A. Batusov, Riv. Nuovo Cimento **1**, 309 (1971).
- R.I. Dzhbuti and R. Ya. Kezerashvili, J. Part and Nuclei, 16, 519 (1986).
- W. R. Gibbs and B. F. Gibson, Annu. Rev. Nucl. Part. Sci. **37**, 411 (1987).
- H. Clement, Prog. Part. Nucl. Phys. 29, 175 (1992).
- M.B. Johnson and C.L. Morris, Ann. Rev. Nucl. Part. Sci. 43, 165 (1993)

What makes it attractive to study pion DCX reaction on nuclei

- In DCX at least two nucleons must participate in order to conserve the electric charge.
- DCX reaction is more sensitive to the two-nucleon effects, manifested here in the first order, than reactions in which there is no need to consider two nucleons and in which the effects of the two nucleons dynamics are manifested indirectly.
- Pion DCX can give direct information on the two nucleons aspect of nuclear dynamics
- Short-range two nucleons correlations.
- Meson exchange current.
- In pion DCX we can obtain and study nuclei far from the stability region.
- To obtain information about double-isobar states of nuclei.
- We hope of studying the expected deference between the neutron and proton densities in nuclei.
- Pion DCX reaction is sensitive to the details of the pion-nucleus interactions.

Pion DCX and Few-body System

- ^{3}**He and** ^{4}**He** possess the minimal number of nucleons for the realization of the DCX process.
- When study the DCX reaction on a few-body system we arrive at more or less unambiguous conclusions on the reaction mechanism.
- To describe a few-body system in discrete and continuum spectra the well-known different ab-initio techniques as Faddeev equations, Faddeev-Yakubovsky equations, Hyperspherical Harmonics, Gaussian expansion method can be successfully used.
- The DCX reaction on a few-body nuclei provides a unique opportunities to study three- and four-neutron system, explore and understand the structure of the most neutron-rich light nuclei.
- The microscopic model-independent description of the initial and final states in the DCX reaction on a few –body system using realistic NN potential allow us to connect the experimentally established characteristics of the pion DCX reaction with the properties of two-body and three-body interactions and to come more or less unambiguous conclusions concerning the latter.



π-nucleon total cross sections as a function of energy

π^- DCX on ^3He

$$\pi^- + {}^3He \rightarrow {}^3n + \pi^+$$

$$\pi^- + {}^3He \rightarrow n + n + n + \pi^+$$

Is trineutron ^3n a bound state of three neutron?

three-neutron resonance?

three neutron in continuum?

Experiment

J. Sperrinde, et al., *Phys. Lett. B* ***32***, *185 (1970).*

A. Stez, et al., *Nucl. Phys. A* ***457***, *669 (1986).* Beam of π^- : 140, 200, 295 MeV

Pion double-charge-exchange on ^{4}He

- The earliest inclusive DCX measurements were made at the JINR in Dubna
  Yu. A. Batusov, et al.Preprint R-1474, JINR, Dubna, 1963; Phys. JETP 1964.
- Davis et al., Bull. Am. Phys. Soc. 9, 627 1964.

DOUBLE CHARGE EXCHANGE WITH NEGATIVE PIONS
SEARCH FOR TETRANEUTRON

Gilly, et al., Phys. Lett., 19, 335 (1965).

CERN, and *Laboratoire de Physique Nucléaire, Faculté des Sciences, Orsay*

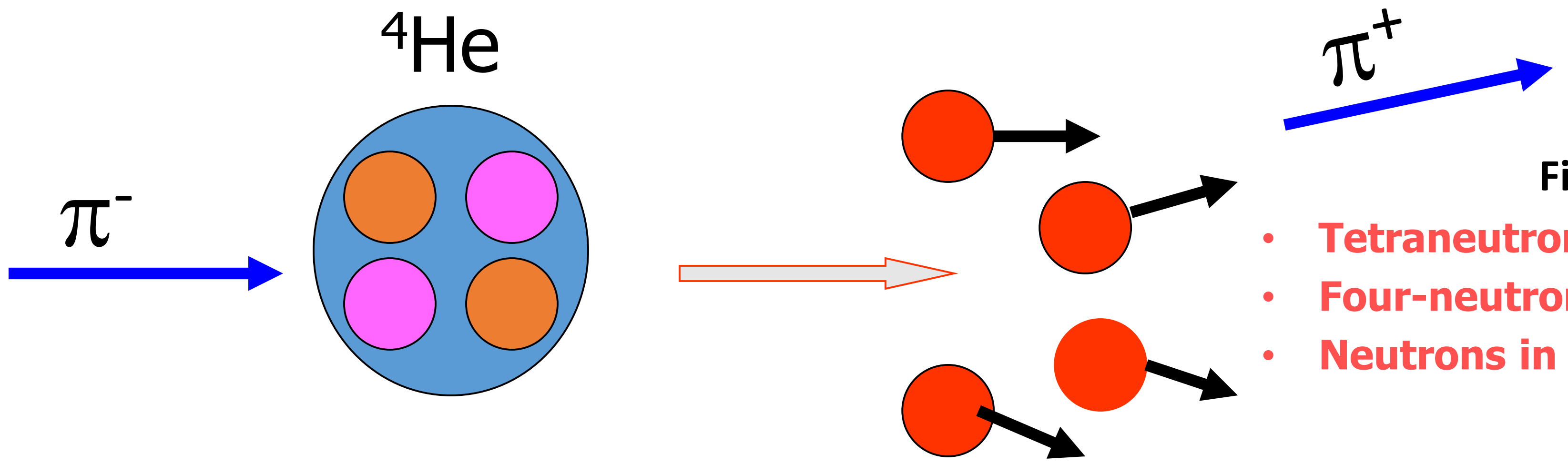
120 - 280 MeV

DOUBLE CHARGE EXCHANGE IN π^+–HELIUM SCATTERING

Experiment performed at the Argonne National Laboratory with beam of π^+ 610 MeV/c

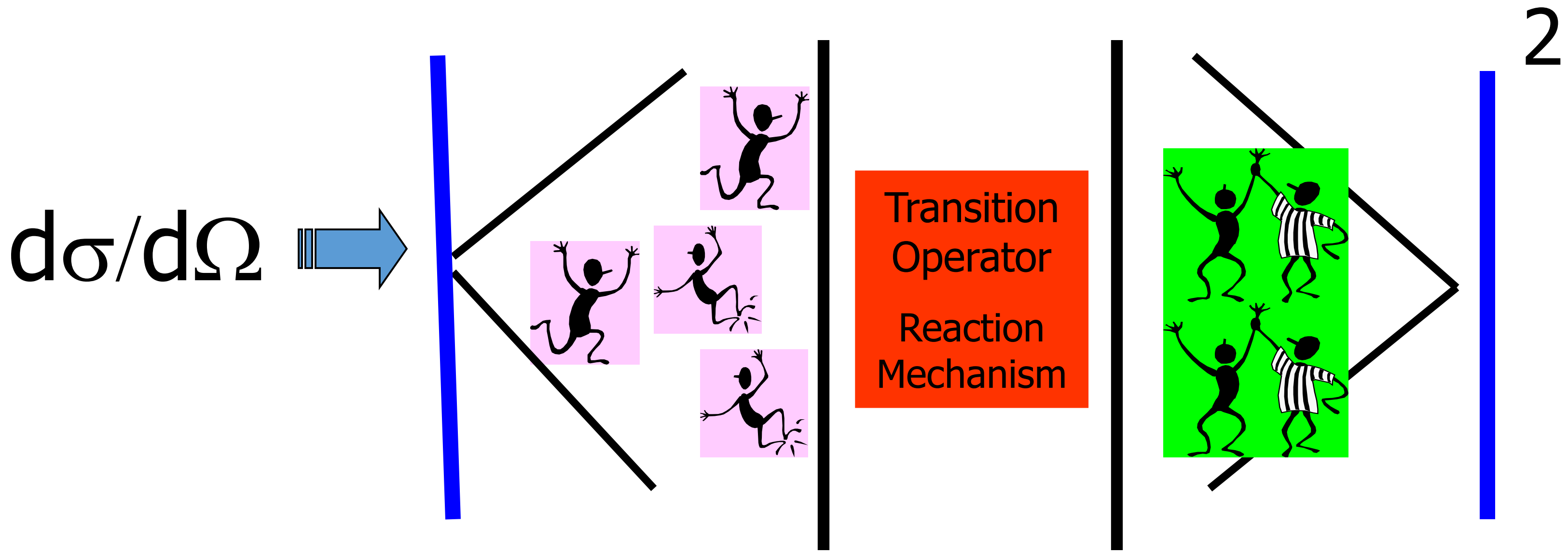
Carayannopoulos, Phys. Rev. Lett. 20, 1215 (1968)

- Falomkin, et al., Nuovo Cimento A 22, 333(1974); Lett. Nuovo Cimento 16, 525 1976. <300 MeV
- Kaufman, et al., Phys. Rev. 175 (1968): Beam of π^- 140 MeV
- *Jeanneret, et al., Nucl. Phys. A 350 (1980):* Beam of π^+ 1.7 GeV/c
- Stetz, et al., *Phys. Rev. Lett. 47, 782 (1981); Nucl. Phys. A 457, 669 1986:* Beam of π^+ : 140, 200, 295 MeV
- Ungar et al. , Phys. Lett. 144B, 333 (1984): Beam of π^- 165 MeV
- *Kinney et al., Phys. Rev. Lett. 57, 3152 (1986).* Measurement has been undertaken over a broad range of pions incident energies encompassing the $\Delta(1232)$ resonance: 120, 150, 180, 240, 270 MeV.



Final State:

- **Tetraneutron ?**
- **Four-neutron resonance?**
- **Neutrons in continuum spectrum?**



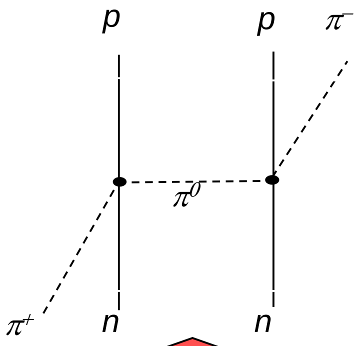
Mechanisms of DCX of Pions

When the pion energy is below the pion-production threshold, normally, the reaction is dominated by the sequential mechanism, In the region of energies around the resonance, analysis of the reaction is very complicated. This is due to the strong distortion of the pion waves at resonance on the one hand and a variety of mechanisms that play a significant role on the other. Explaining the energy dependence of DCX has led to a diversity of mechanisms proposed, including:

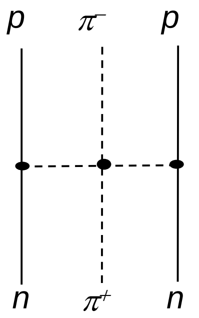
- the successive delta interaction mechanism Johnson, *M. B. Johnson, et al., Phys.Rev. Lett.* ***52****, 593 (1984). Oset, et al., Nucl. Phys.* ***A483****, 514 (1988).*
- the meson exchange current mechanism *Germond, Wilkin, Lett. Nuovo Cim.* ***13****, 605 (1975); J. Phys. G* ***4****, L115 (1978).*
- the six-quark bag mechanism, *Miller, Phys. Rev. Lett.* ***53****, 2008 (1984).*
- Dibaryon mechanisms assuming the production of the hypothetical *d'* dibaryon, a resonance with baryon number B = 2 in the πNN subsystem, Clement, et al Phys. Lett. **B337**, 43 (1994); J. Grater *et al.*, Phys. Rev. C **58**,
- DCX in a composite-meson model, Kezerahshvili, *Phys Rev. C* ***75****, 015203 (2007); Nucl. Phys A790, 366 (2007)*
- absorption mechanisms and DCX that involves more than two nucleons, *Koltun M. K. Singham,* Phys. Rev. C **41**, 2266 (1990; Jeanneret, et al., Nucl. Phys A 350, 345 (1980).

DCX Reaction Mechanisms

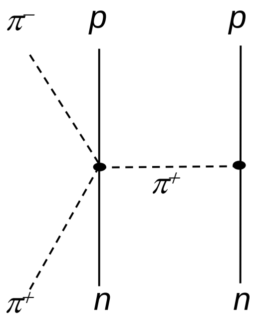
Two-Step Mechanism



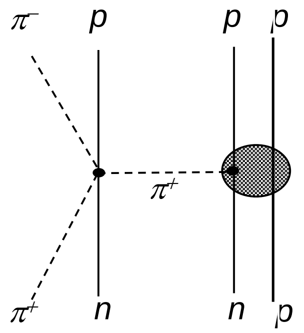
Meson Exchange Current Mechanism



Pole Diagram



Seagull Diagram



Absorption Mechanism

The incoming pion undergoes two sequential single charge exchange scatterings on nucleons within a nucleus

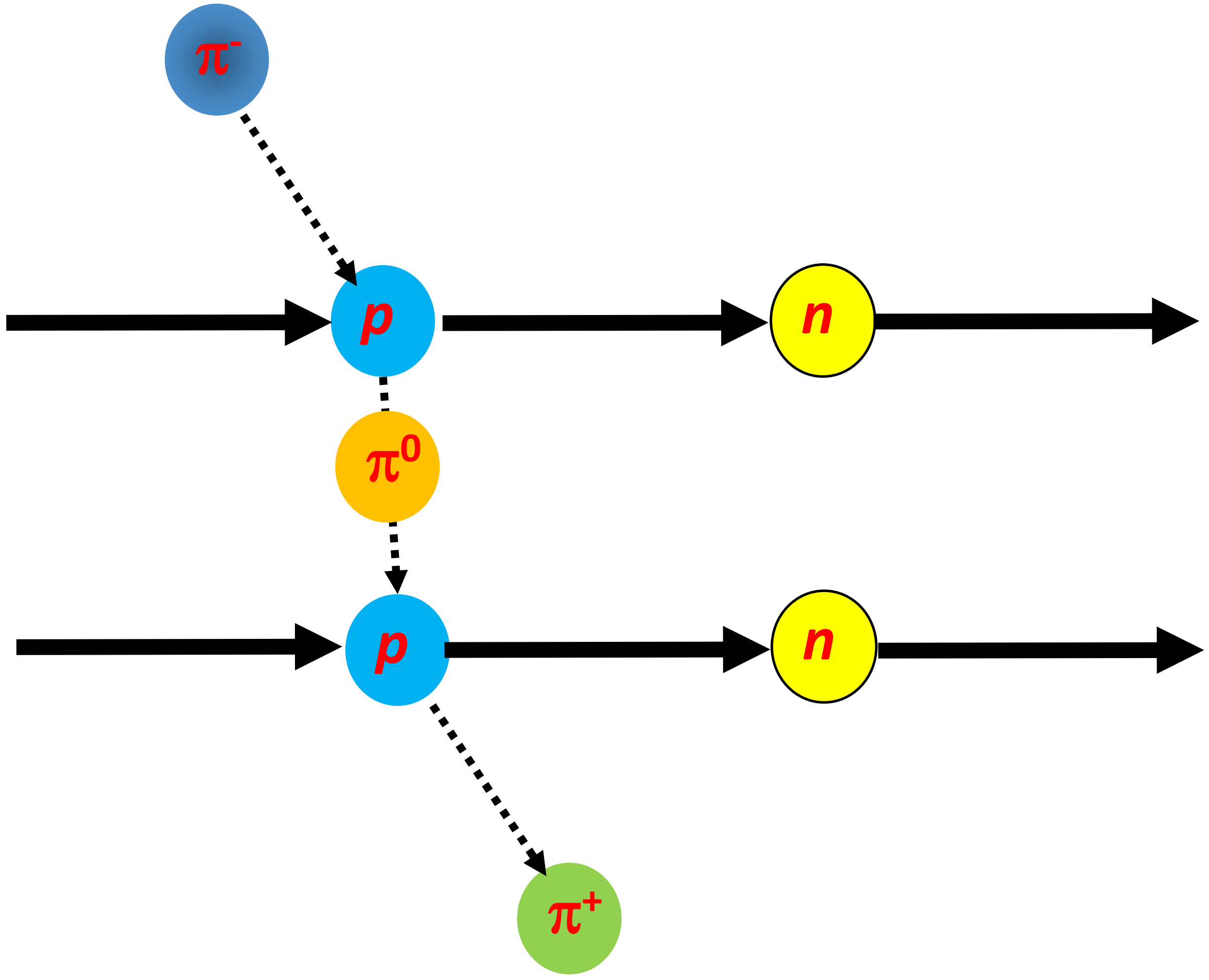
The incoming pion scatters wih a virtual pion of opposite charge in the "cloud" surrounding the target nucleon of the nucleus, and is itself absorbed on another nucleon.

Germond and Wilkin, 1975
Robilotta and Wilkin, 1978

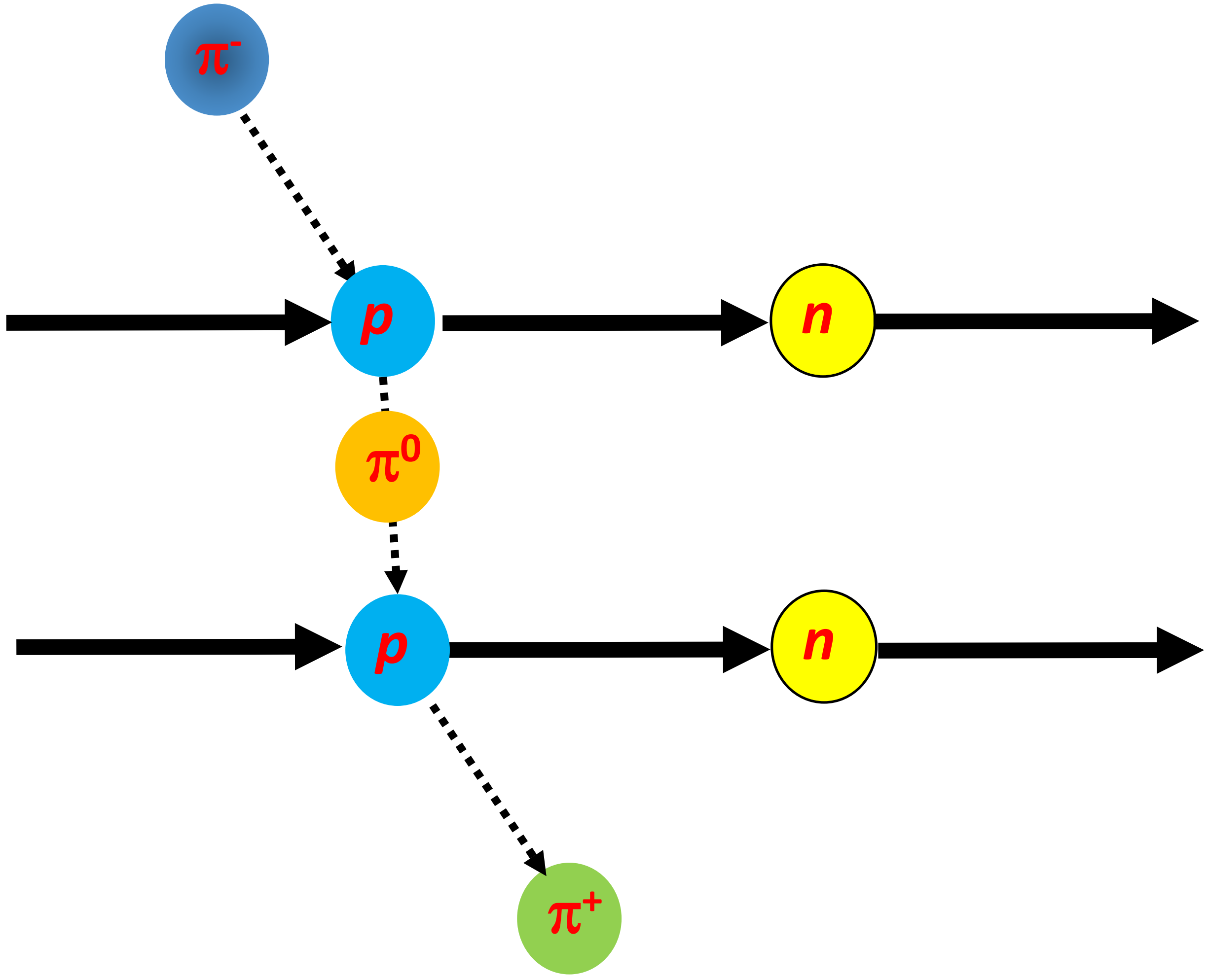
Pion-induced pion production is followed by two-nucleon absorption of one of the two final pions

Jeanneret et. Al. 1980

Two Step Mechanism or the Sequential Mechanism



Two-Step Mechanism or the Sequential Mechanism



Two-Step Mechanism

When the incident pion energy is below the pion-production threshold, normally, the reaction is dominated by the sequential mechanism, in which the incoming pion undergoes two sequential single charge exchange scatterings on nucleons within a nucleus.

A double-scattering amplitude obtained from two single-scattering *p-N* charge-exchange amplitudes with ***intermediate off-shell propagation***:

$$F(\vec{k},\vec{k}') = \frac{4\pi}{(2\pi)^3}\int d\vec{p}\, f_2(\vec{p},\vec{k}')\,\frac{e^{i\vec{p}\cdot\vec{r}_{21}}e^{i\vec{k}\cdot\vec{r}_1}e^{-i\vec{k}'\cdot\vec{r}_2}f_1(\vec{k},\vec{p})}{p^2-k^2-i\eta}$$

Separable *s* and *p*-wave amplitudes having off-shell form factors

$$f_i(\vec{p},\vec{q}) = \frac{2\pi}{ik}\sum_{lm} f_i^l(p,q)\,Y_{lm}^*(\hat{p})\,Y_{lm}(\hat{q})\,,\quad \text{where}\quad f_i^l(p,q) = \lambda_i^l(\omega)\,v_l(p)\,v_l(q)\,.$$

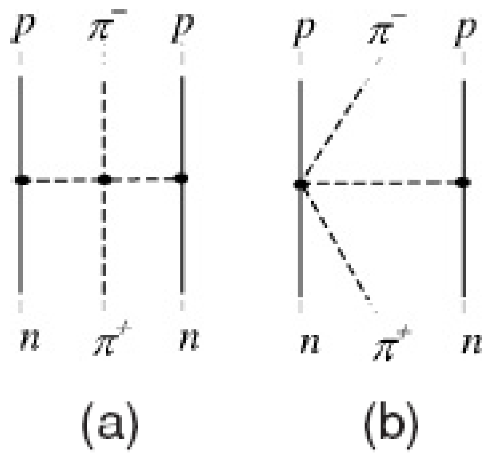
The functions $\boxed{v_l(q)}$ describe the off-shell extension of the *t* matrix and go to unity on shell (q→k) as they should.

The $\lambda_i^l(\omega)$ is a function of the π-N phase shifts

$\lambda_i^l(\omega) = \{\exp[2i\delta_l(\omega)] - 1\}\,k/\kappa$, where $\omega = (\kappa^2 + \mu^2)^{1/2}$ is the pion energy in the π-N center of mass frame

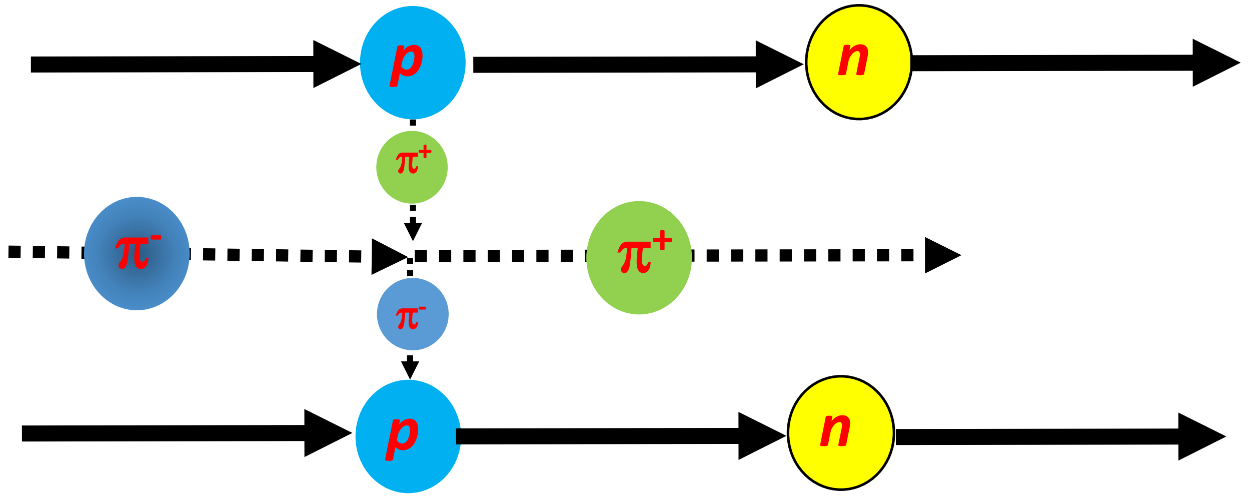
Meson Exchange Current Mechanism

The MEC mechanism, in which the incoming pion scatters with a virtual pion of opposite charge in the "cloud" surrounding the target nucleon of the nucleus and is itself absorbed on another nucleon, was first proposed by Germond and Wilkin [*Lett. Nuovo Cimento (1975)*] and consists of the assumption that the incident pion is scattered on the off-shell pion exchanged by the nucleons within the nucleus and the DCX takes place at the $\pi\pi$ vertex. The pole diagram in Fig. (a) corresponds to that mechanism. Later, Robilotta and Wilkin [J. Phys. G, 1978)] introduced an additional diagram shown in Fig. (b) and concluded that the MEC effects would be small for an analog DCX because this diagram partially cancels the contribution of the pole term and, as a result, reduces the effects of MEC in the reaction.

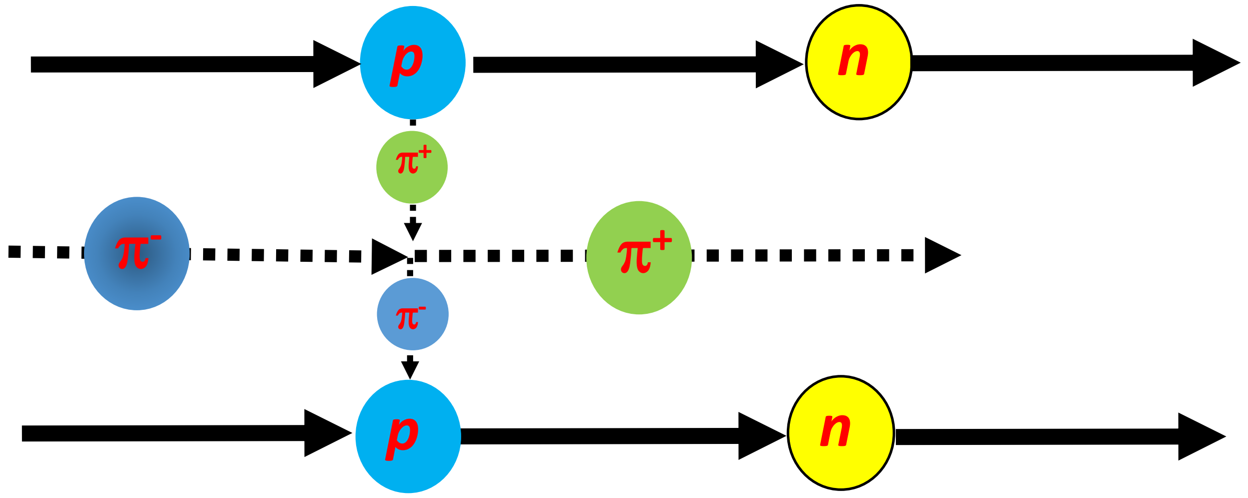


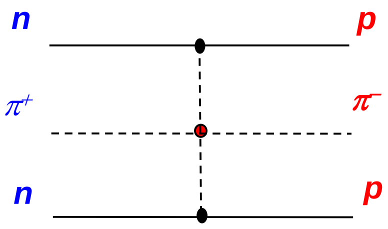
Diagrams corresponding to the MEC mechanism in the Born approximation: (a) pole diagram and (b) contact diagram.

Meson Exchange Current Mechanism



Meson Exchange Current Mechanism





$$T = (i\pi g)^2 \frac{(\vec{\sigma}_1 \cdot \vec{q}_1)(\vec{\sigma}_2 \cdot \vec{q}_2)}{4m^2} \frac{M_{\pi\pi}(\vec{k}_i, \vec{q}_1; \vec{k}_i, \vec{q}_2)}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)}$$

Effective Lagrangian Formalism

Forward scattering DCX

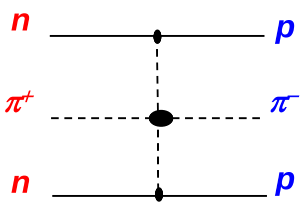
$$T = \frac{g^2}{2m^2} \frac{m_\pi^2}{f_\pi^{\,2}} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(q^2 + m_\pi^2)^2} (\xi - 2)$$

Jiang and Koltun, 1990

ξ is the chiral symmetry breaking parameter

In PWA the forward scattering DCX cross section does not depend on the pion energy and depends on ***the chiral symmetry breaking parameter***

The parameter β takes different values, depending on the mechanism of chiral symmetry breaking. For example, in the Weinberg model (PRL 18 (1967) 188) $\beta = 1/2$, in the Schwinger model (PL B 24, 473, 1967) $\beta = 1/4$, and in the exponential model (Chang and F. Gursey, PR 164 1752, 1968) $\beta = 1/3$.



$$M_{\pi\pi} = -\frac{m_\pi^2}{f_\pi^2} + \frac{(s+t)}{f_\pi^2}$$

Weinberg Lagrangian

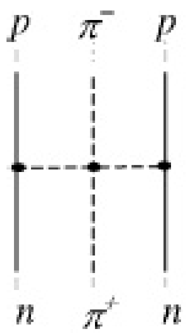
In effective Lagrangian formalism only the contribution of the "tree" diagrams are included and no pionic or baryonic closed loop diagrams. The tree diagrams correspond to the Born approximation for ππ scattering, and their contribution is defined by the first term of the expansion of the ππ-amplitude in terms of $\frac{1}{f_\pi^2}$

$$T \propto \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(q^2 + m_\pi^2)^2}(2q^2 - m_\pi^2) \Rightarrow \mathrm{A}V(\vec{r}) + \mathrm{B}\frac{\partial}{\partial m_\pi}V(\vec{r})$$

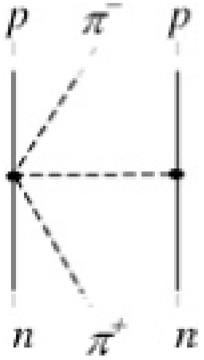
$$T \propto \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(q^2 + m_\pi^2)^2}\left(1.369\frac{q^2}{m_\rho^2 - m_\pi^2}(1 - 1.423\frac{q^2}{m_\rho^2 - m_\pi^2})\right) \Rightarrow$$

Lovelace-Veneziano model

Oset et.al., 1983



$$L_{NN\pi} = \frac{g}{2m}\bar{\psi}\gamma^{\mu}\gamma_5\tau\psi\cdot(\partial_{\mu}\pi) \quad T_1 = \frac{g^2}{4m^2}\frac{2}{F_{\pi}^2}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{q}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{q}_2)(2\mathbf{q}_1\cdot\mathbf{q}_2-\mathbf{q}_1^2-\mathbf{q}_2^2-2m_{\pi}^2)}{(\mathbf{q}_1^2+m_{\pi}^2)(\mathbf{q}_2^2+m_{\pi}^2)}$$



$$L_{NN\pi} = \frac{g}{2m}\bar{\psi}\gamma^{\mu}\gamma_5\tau\psi\cdot(\partial_{\mu}\pi),$$

$$L_{NN\pi\pi\pi} = -\frac{g}{2m}\frac{1}{4F_{\pi}^2}\bar{\psi}\gamma^{\mu}\gamma_5\tau\psi\cdot(\partial_{\mu}\pi)\pi^2$$

$$T_2 = -\frac{g^2}{4m^2}\frac{2}{F_{\pi}^2}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{q}_1)\left[\boldsymbol{\sigma}_2\cdot\mathbf{q}_1-\boldsymbol{\sigma}_2\cdot\mathbf{q}_2\right]}{(\mathbf{q}_1^2+m_{\pi}^2)}$$

One pion exchang NN potential

$$\boxed{\text{For forward scattering } \mathbf{q}_1 = \mathbf{q}_2 \equiv \mathbf{q}}$$

$$T^B = -\frac{g^2}{4m^2}\frac{2}{F_{\pi}^2}\frac{2(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})m_{\pi}^2}{(\mathbf{q}^2+m_{\pi}^2)^2}. \quad T^B = \frac{2}{F_{\pi}^2}\frac{1}{m_{\pi}}\frac{\partial}{\partial m_{\pi}}V(r)$$

$$V(r) = \frac{1}{4\pi}\left(\frac{gm_{\pi}}{2m}\right)^2\left\{\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\frac{1}{3}S_{12}\times\left[1+\frac{3}{m_{\pi}r}+\frac{3}{(m_{\pi}r)^2}\right]\right\}\frac{e^{-m_{\pi}r}}{r},$$

$$S_{12} = \frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

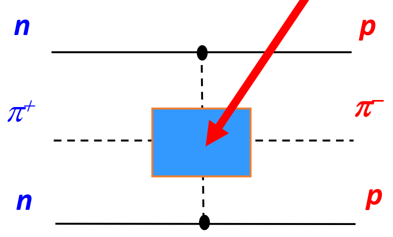
Thus, the sum of contributions of the pole and the contact diagrams in the Born approximation is proportional to the derivative of the one-pion exchange potential with respect to the pion mass.

Pion Double Charge Exchange in Composite-Meson Model

The pion double charge exchange amplitude is evaluated in a composite-meson model based on the four-quark interaction. The model assumes that the mesons are two-quark systems and can interact with each other only through quark loops. To evaluate the meson exchange current contribution, the form factors of the two-pion decay modes of the *ρ, σ*, and *f*0 mesons have been used in the calculations. The contribution of the four-quark box diagram has been taken into account as well as a contact diagram. The contributions of the *ρ, σ*, and *f*0 mesons increase the forward scattering cross section, which depends weakly on the energy.

The vertex box corresponding to ππ scattering can be considered at the quark level and includes the quark diagrams which successfully describe ππ scattering

$\pi^+ = u\bar{d}$



$\pi^- = d\bar{u}$

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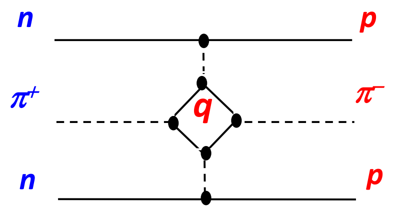

Kezerashvili, Nucl. Phys. A, 2007

The quark-box diagram corresponds to the Pole diagram and represents the Born approximation in the effective Lagrangian method.

The choice of the other diagram is based on the probability of the two pion decay of mesons:

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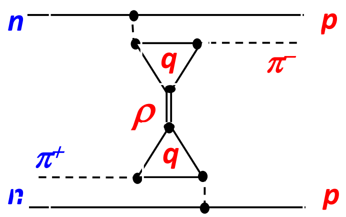

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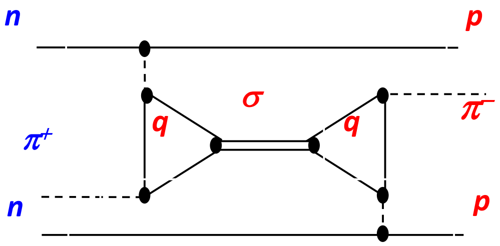

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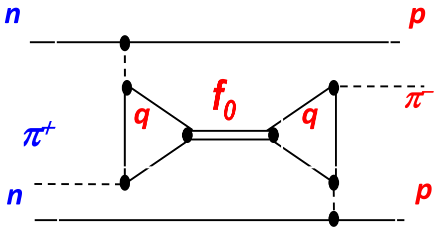


ρ (770 MeV) → $\pi\pi$ (100%)

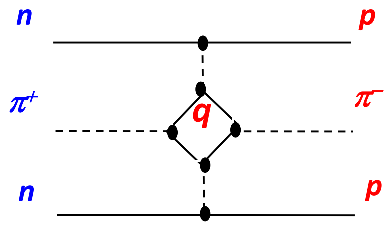
σ (600 MeV) → $\pi\pi$ (>90%)

f_0 (975 MeV) → $\pi\pi$ (>90%)

$$L = \bar{q}\left| i\partial - m_q + \frac{m_q}{f_\pi}\sigma\cos\alpha + f_0\sin\alpha + i\gamma_5\tau\cdot\pi + \frac{g_\rho}{2}\tau\cdot\rho \right| q$$

Weinberg amplitude

$$M_{\pi\pi} = -\frac{m_\pi^2}{F_\pi^2} + \frac{(s+t)}{F_\pi^2}$$



$$M_{\pi^+\pi^-} = -\frac{m_\pi^2}{F_\pi^2} + \frac{m_q^2}{(\pi F_\pi)^2}\left[\frac{s+t}{F_\pi^2} - \frac{m_q^2}{(4\pi F_\pi)^2}\frac{s+t}{F_\pi^2}\right] +$$

$$\frac{4m_q^2}{m_\sigma^2}\left(\frac{s}{(m_\sigma^2-s)} + \frac{t}{(m_\sigma^2-t)}\right)\sin^2\alpha +$$

$$\frac{4m_q^2}{F_\pi^2}\left\{\frac{4m_q^2}{m_{f_0}^2}\left(\frac{s}{(m_{f_0}^2-s)} + \frac{t}{(m_{f_0}^2-t)}\right)\cos^2\alpha\right\} +$$

$$g_\rho\left\{\frac{t-u}{m_\rho^2-s}\left(1+\frac{s-m_\rho^2}{8\pi^2F_\pi^2}\right)^2 + \frac{s-u}{m_\rho^2-t}\left(1+\frac{t-m_\rho^2}{8\pi^2F_\pi^2}\right)^2\right\}$$

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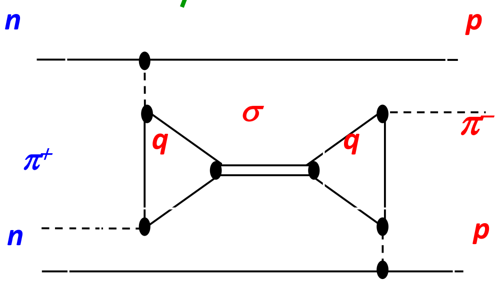

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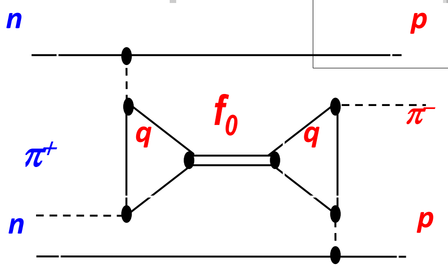

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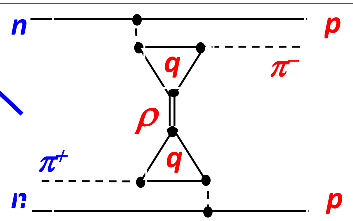
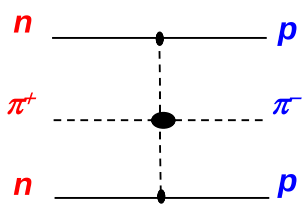
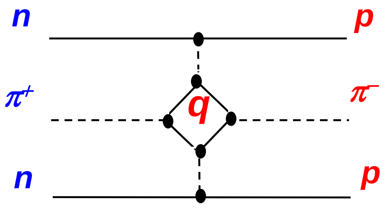


$$M_{\pi\pi} = -\frac{m_\pi^2}{f_\pi^2} + \frac{(s+t)}{f_\pi^2}$$



$$T \propto \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(q^2 + m_\pi^2)^2}(2q^2 - m_\pi^2) \Rightarrow \boxed{\mathrm{A}V(\vec{r}) + \mathrm{B}\frac{\partial}{\partial m_\pi}V(\vec{r})}$$



$$M_{\pi\pi} = -\frac{m_\pi^2}{f_\pi^2} + 0.973\frac{(s+t)}{f_\pi^2}$$

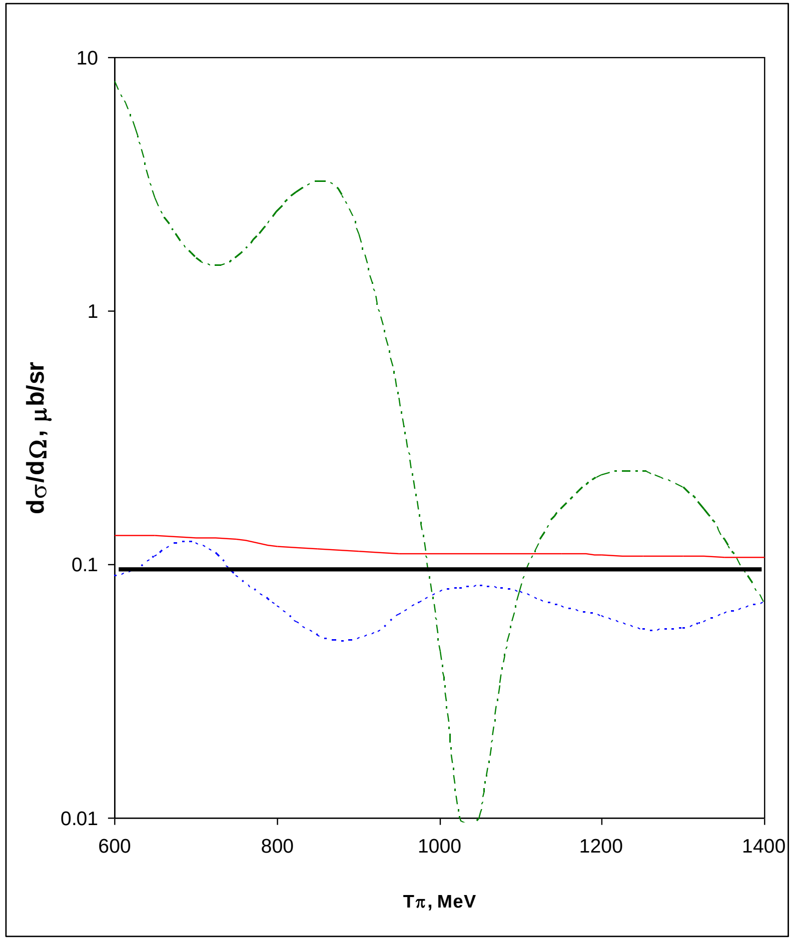
The contribution of the diagrams representing ε and S^* mesons are proportional to $\frac{1}{f_\pi^2}$

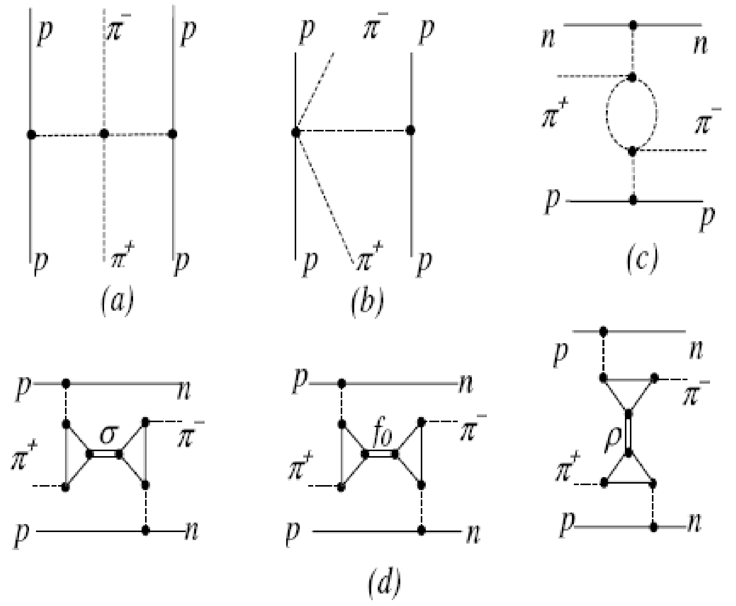
The contribution of the ρ meson has the terms proportional to $\frac{1}{f_\pi^2}$ $\left(\frac{1}{f_\pi^2}\right)^2$ and $\left(\frac{1}{f_\pi^2}\right)^4$

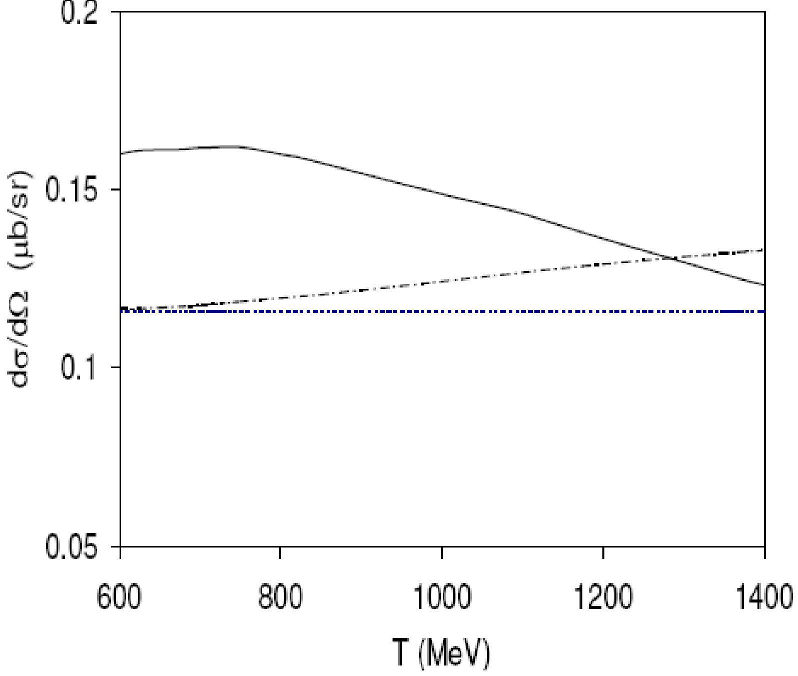
$$^{18}O(\pi^+, \pi^-)^{18}Ne$$

Vicente Vagas, 1996

Composite Meson Model







Forward scattering cross section for the reaction ^{18}O(p$^-$, p$^+$)^{18}Ne as a function of the incident pion energy.

The dotted line represents the contributions of the MEC in the Born approximation (sum of the diagrams in Fig. (a) and (b), the solid curve shows the contributions of the MEC in the composite meson model with the pole and contact diagrams (sum of the diagrams in Fig. (a}, (b) and (d) and dash-dotted curve is the result of unitarization of the Born approximation (sum of the diagrams in Fig. (a, (b) and (c).

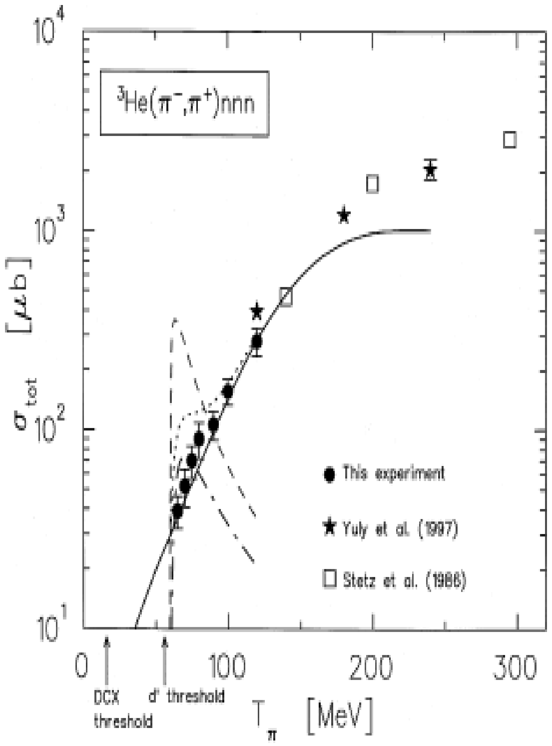
Thus in the Born approximation the forward scattering pion DCX amplitude for the meson exchange current mechanism does not depend on the energy.

The unitarization of the amplitude as well as the contributions of the *ρ, σ*, and f_0 mesons into MEC mechanism based on the composite meson model increase the cross section and lead to its energy dependence.

Pion double-charge-exchange

$\pi^- + {}^3He \rightarrow \pi^+ + {}^3n(3n?)$

In the first experiment only one spectrum was measured, for 140 MeV incident pions at an average scattering angle of about 30°, with detected outgoing pions having kinetic energies between 40 and 125 MeV. The measured spectrum differed significantly from pure four-body phase space. *Sperrinde, et al., Phys. Lett. B 32 (1970).*



The second experiment was performed *at* LAMPF using the EPICS spectrometer. These results consist of doubly differential cross sections at 140 MeV for laboratory angles of 20°, 30°, 50°, and 80°, at 200 MeV for 23.5°, and at 295 MeV for 30° and 120°. *A. Stetz et al., Nucl. Phys. A457 (1986).*

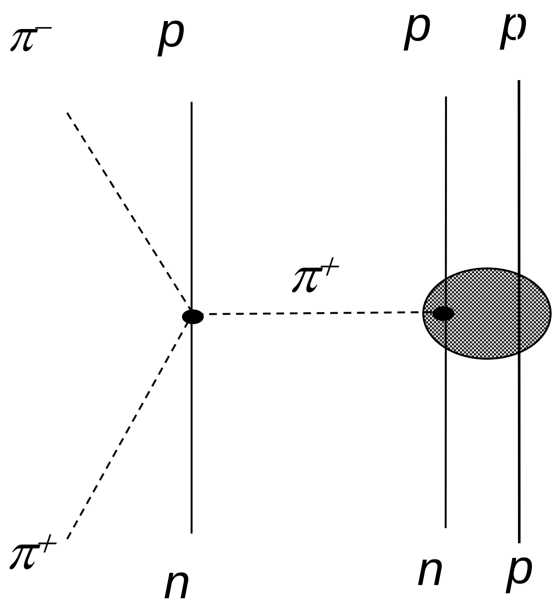
The DCX spectra exhibit a double-peaked structure at forward angles that can be understood as a consequence of a sequential single charge exchange mechanism. *Yuly et al., Phys. Rev. C 55(1997).*

Double differential and total cross sections were obtained for kinetic pion energies from 65 to 120 MeV at TRIUMF *Grater, et al., Phys. Lett. B 471, 113 (1999).*

$\pi^+ + {}^4He \rightarrow \pi^- + 4p$.

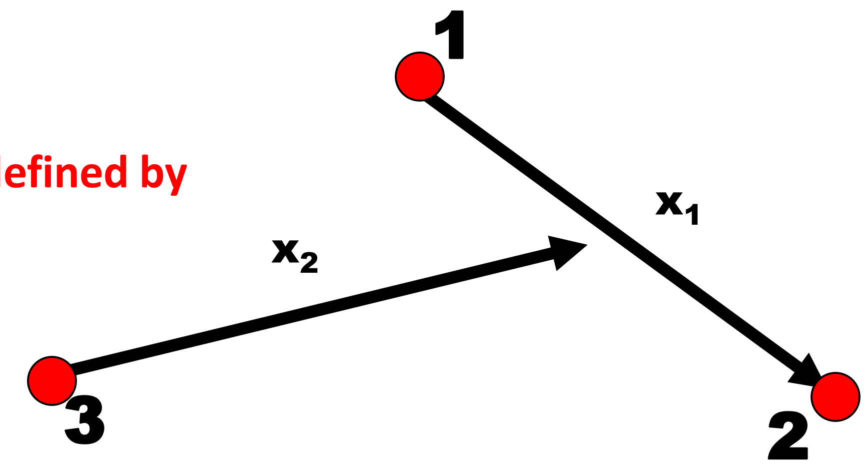
Double charge exchange and one-pion production in collisions at 1.7 GeV/c was studied by *Jeanneret et al., Nucl. Phys. 350, 345 (1980).*

The Authors presented the results of a study of the exclusive DCX reaction **$\pi^+ + {}^4He \rightarrow \pi^- + 4p$.** The experiment was performed with the helium bubble chamber irradiated with a 1.7 GeV/$c\pi^+$ beam. The experimental results at other energies are discussed in the light of existing models. None of the existing theoretical models is compatible with data at 1.7 GeV/*c*. It was propose a new mechanism for DCX, involving three nucleons, which reproduces the integrated as well as the differential cross section.



Differential Equation in 6-D Space

We introduce the Jacobi coordinates, defined by



$$\vec{x}_1 = \left(\frac{2m_2 m_3}{m_2 + m_3} \right)^{1/2} \left(\vec{r}_2 + \vec{r}_3 \right),$$

$$\vec{x}_2 = \left(\frac{2m_1 \left(m_2 + m_3 \right)}{m_1 + m_2 + m_3} \right)^{1/2} \left(\frac{m_2 \vec{r}_2 + \vec{r}_3 m_3}{m_2 + m_3} \vec{r}_2 - \vec{r}_1 \right),$$

Equation for three body in Euclidean 3-D space and a rectangular coordinate system

$$\left[-\nabla_{x_1}^2-\nabla_{x_2}^2+V(\vec{x}_1,\vec{x}_2)\right]\Psi(\vec{x}_1,\vec{x}_2)=E\Psi(\vec{x}_1,\vec{x}_2)$$

Let us introduce hyperspherical coordinate in Euclidian Six dimensional space as

$$\rho^2=x_1^2+x_2^2;\quad x_1=\rho\cos\alpha;\quad x_1=\rho\sin\alpha.\quad \Omega\equiv(\alpha,\hat{x}_1,\hat{x}_2)$$

$$-\nabla_{x_1}^2-\nabla_{x_2}^2=-\left[\frac{d^2}{d\rho^2}+\frac{5}{\rho}\frac{d}{d\rho}-\frac{1}{\rho^2}\hat{K}(\Omega)\right]\quad \text{where}$$

$$\hat{K}(\Omega)=-\frac{\partial^2}{\partial\alpha^2}-4\cot 2\alpha\frac{\partial}{\partial\alpha}+\frac{1}{\cos^2\alpha}\Delta_{x_1}+\frac{1}{\sin^2\alpha}\Delta_{x_2}$$

Let us introduce hyperspherical functions Φ_K as eigenfunctions of the angular part of the six dimensional Laplace operator

$$\hat{K}(\Omega)\Phi_K(\Omega)=K(K+4)\Phi_K(\Omega),\quad K\text{ is a positive integer number}$$

Let expand the function $\Psi(\vec{x}_1, \vec{x}_2)$ by a complete set of hyperspherical functions

$$\Psi(\vec{x}_1, \vec{x}_2) = \sum_{K\alpha} \frac{u_K(\rho)}{\rho^2} \Phi_K^\alpha(\Omega)$$

This expansion is substituted into previous equation and differential equation is separated into the system of differential equations for hyperspherical function and the system of second order differential equations for hyperradial functions

$$\frac{d^2 u_K(\rho)}{d\rho^2} + \frac{1}{\rho}\frac{du_K(\rho)}{d\rho} + \left[E - \frac{(K+2)^2}{\rho^2}\right] u_K(\rho) = \sum_{K'} W_{KK'}(\rho) u_{K'}(\rho)$$

We shell seek the solution of this system of differential equations in the form

$$u(\rho) = \left(J(\rho)V^{-1}(\rho) - N(\rho)V^{-1}T(\rho)\right)A(\rho)$$

where $J(\rho)$ and $N(\rho)$ are diagonal matrices constructed from Bessel and Neumann function

Substituting this expression into the system of differential equations we obtain the nonlinear first order matrix differential equations for the phase functions and amplitude function

Nonlinear system of differential equations for phase functions

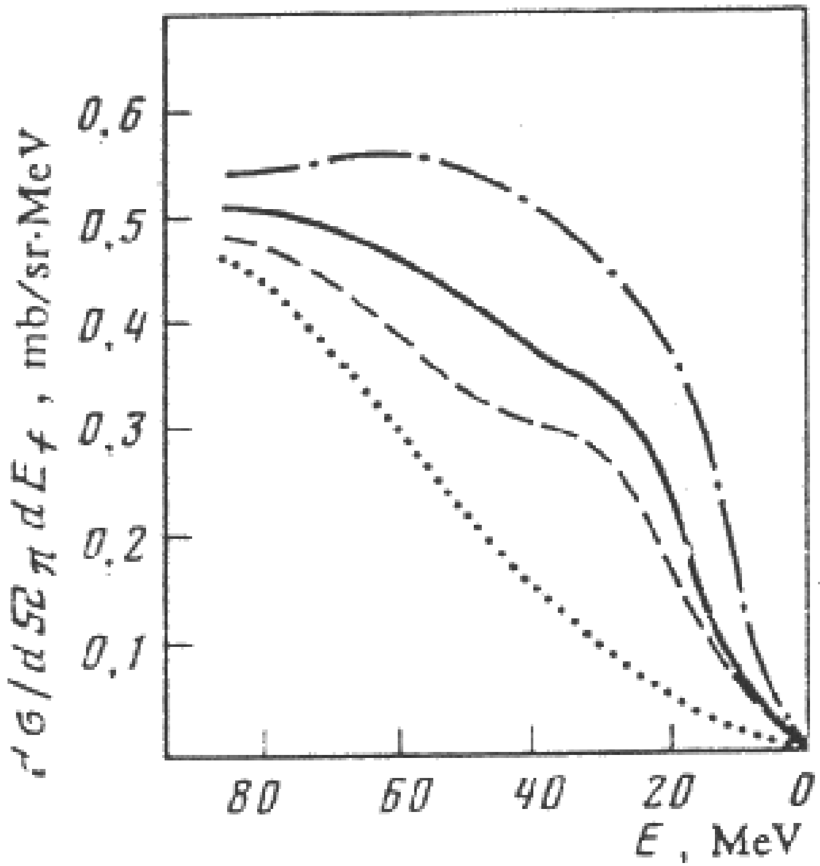
$$\frac{dT(\rho)}{d\rho}-\left[T(\rho)\frac{dU^{-1}(\rho)}{d\rho}U^{-1}(\rho)\right]=-\frac{\pi}{2}\rho\big(U(\rho)J(\rho)-T(\rho)U(\rho)N(\rho)W(\rho)\big)$$

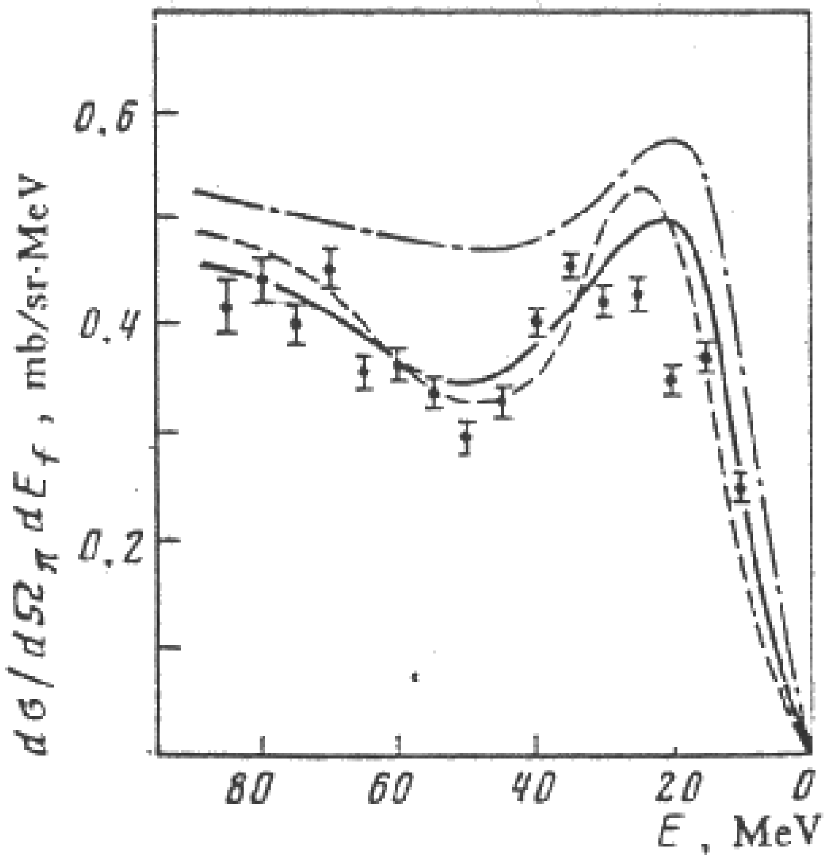
$$\times\big(J(\rho)U^{-1}(\rho)-N(\rho)U^{-1}(\rho)T(\rho)\big)$$

Amplitude function

$$A(\rho)=U(\rho)\exp\left\{-\frac{\pi}{2}\int_0^{\rho}\rho d\rho N(\rho)W(\rho)\times\left[J(\rho)U^{-1}(\rho)-N(\rho)U^{-1}(\rho)T(\rho)U(\rho)\right]\right\}$$

Double Charge Exchange of the π mesons on ^{3}He





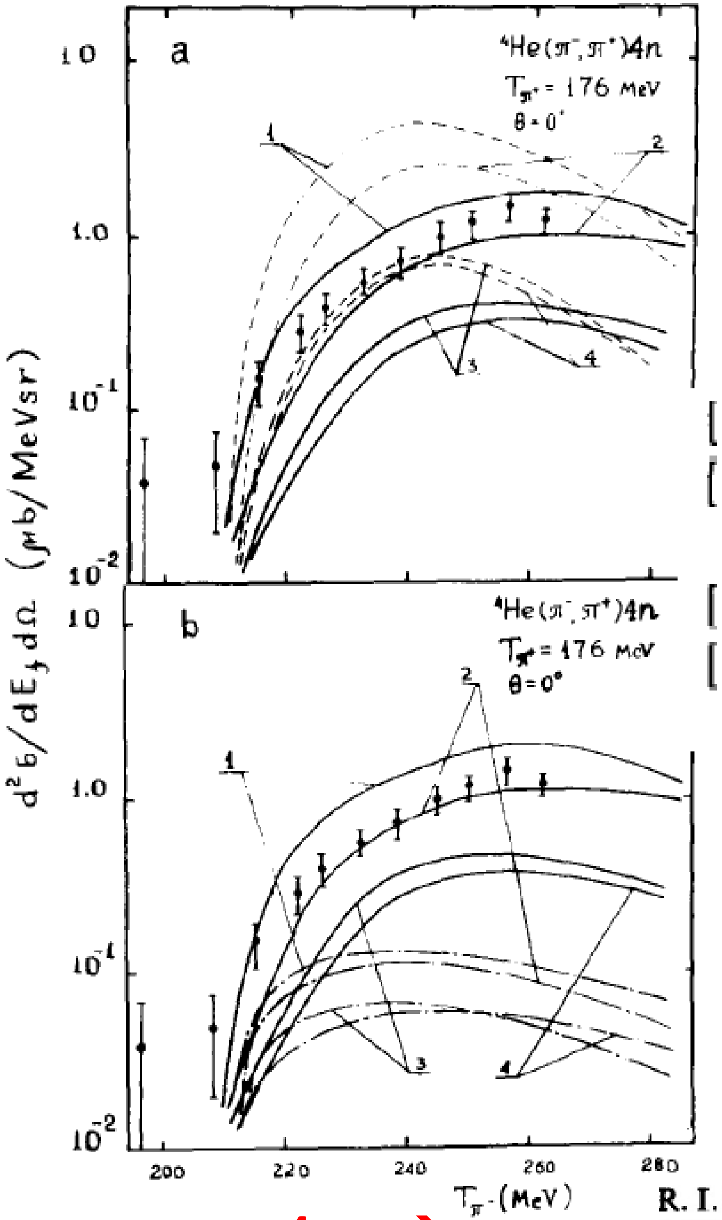


Fig. 1. Differential cross section of the reactions $^4He(\pi^-,\pi^+)4n$. The results of a calculation taking into account FSI in the state with $L = 0$ [Solid curve in (a)], in the state with $L = 2$ (dashed–dotted curve) and in the plane wave approximation (dashed curve). Curves 1, 2, 3 and 4 show results obtained for the potentials [7], [6], [9] and [8], respectively.

$\pi^- + {}^4He \rightarrow \pi^+ + 4n$

Potentials

[6] A. Volkov, Nucl. Phys. 74 (1965) 33.

[7] D. Gogny, P. Pires and R. De Tourrell, Phys. Lett. 3 (1970) 591.

[8] G.A. Becher et al., Phys. Rev. 125 (1962) 1754.

[9] L. Hultnen and M. Sugawara, Handbűch der Physik S. Flüge, Bd. XXXIX.

The incoming pion undergoes two sequential single charge exchange scatterings on nucleons within a nucleus

R. I. Dzhibuti and R. Ya. Kezerashvili, J. Nucl. Phys. 39, 264 (1984)

$\pi^+ + {}^4He \rightarrow \pi^- + 4p$

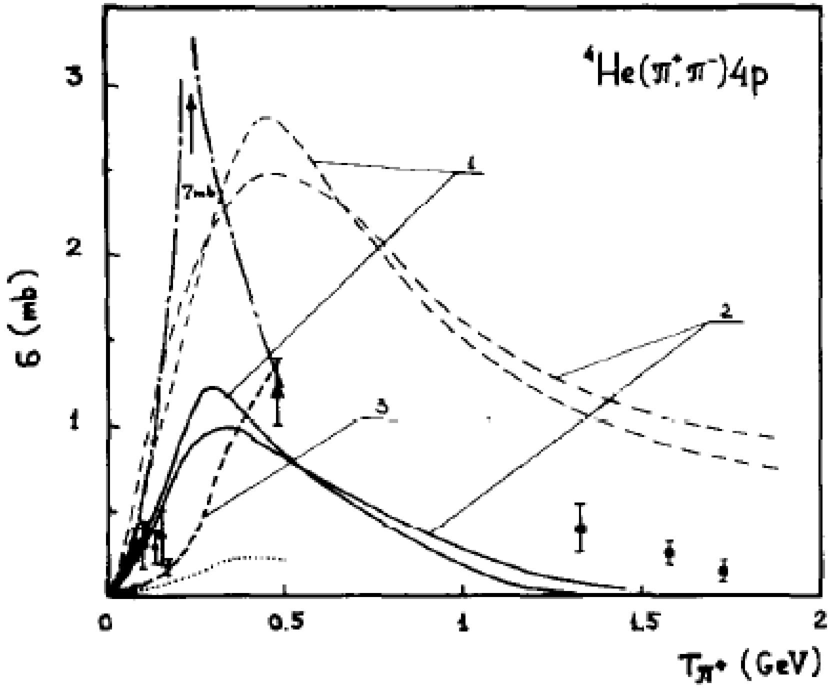


Fig. 3. Total cross section results of Becker and Schmit [5] (dashed–dotted curve), Germond and Wilkin [16] (dashed curve 3), Gibbs et al. [2] (dotted curve) and our results. Curves 1 and 2 show the results of a calculation for the potential [7] and [6], respectively. Experimental data are taken from refs. [13,14] from ref. [12], from ref. [15].

Comparison of mechanisms

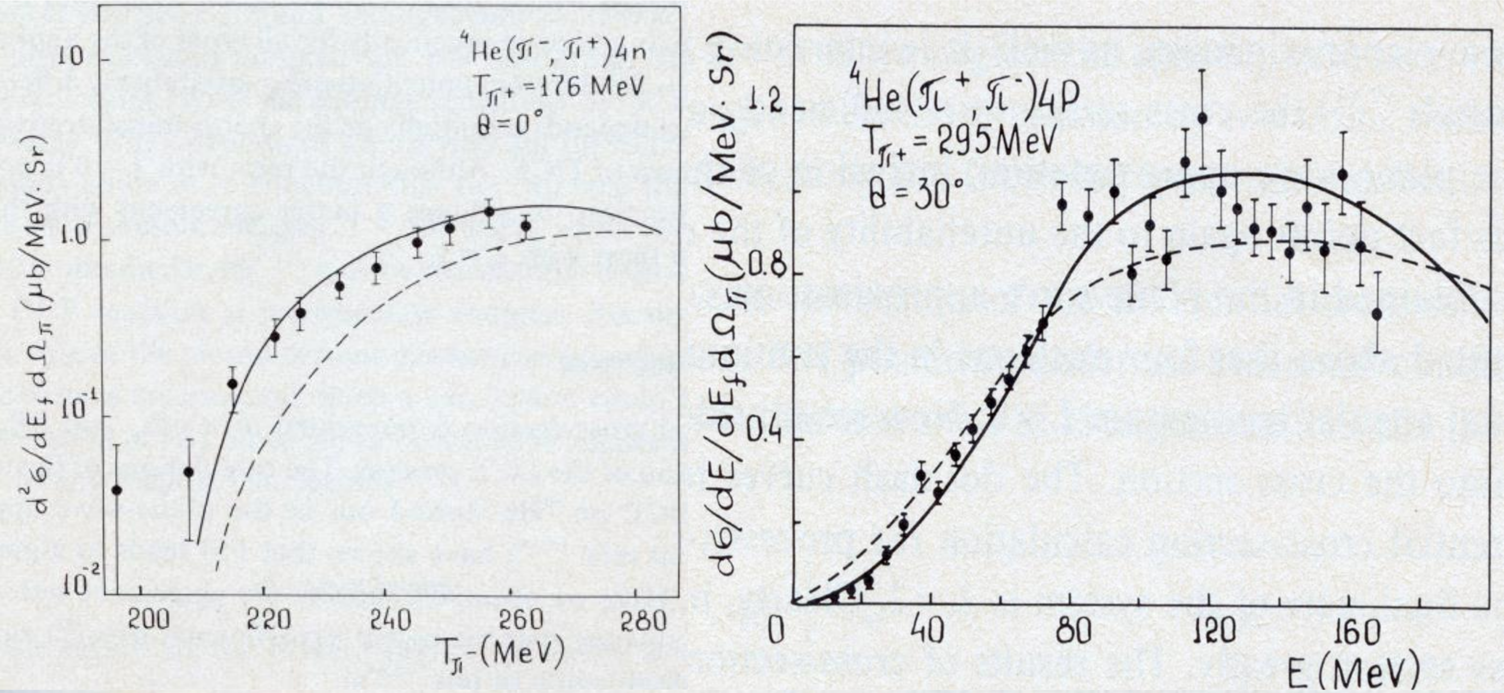
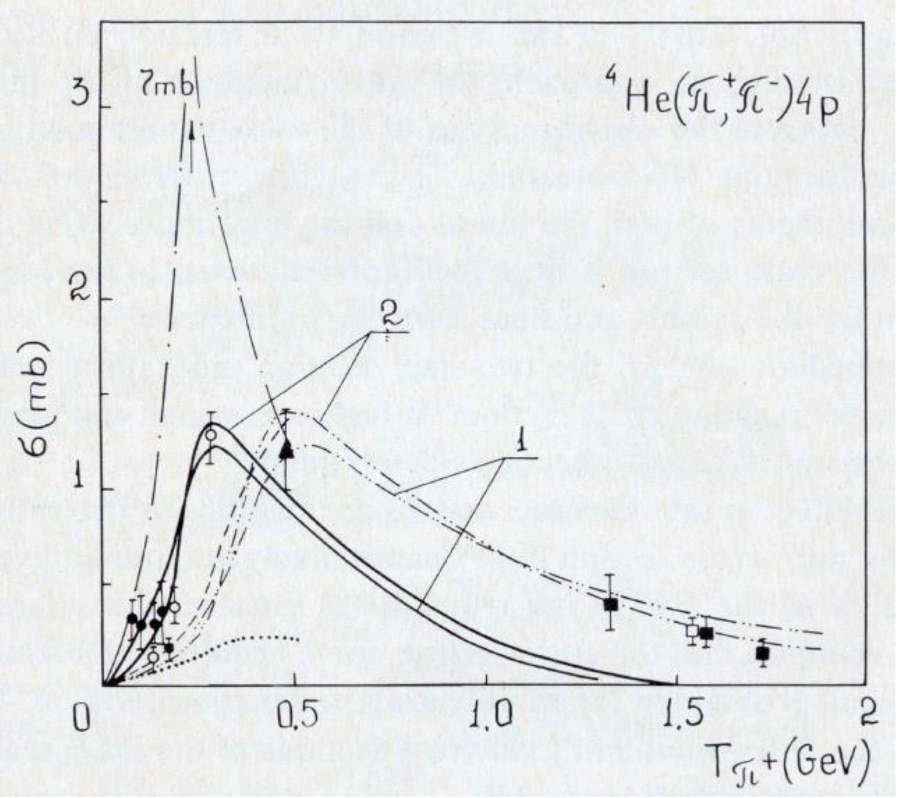


Fig. 9. The differential cross section of reaction (4). The dashed curve shows the meson current mechanism, and the solid curve the two-step mechanism of the DCE reaction. These results are from calculations with the potential TS). Experimental data are taken from ref. [49]).

Gilly, et al., PL 19, (1965) and Stetz et al., PRL 47 (1981)

Total cross-section. Solid curves – two-step mechanism of DCX; dash-double dotted curves –meson current mechanism.
Calculations for GPT (curves1) and de Tourreil Sprung (TS) potentials



R. I. Jibuti and R. Ya. Kezerashvili, Nucl. Phys. A434, 687 (1985).

Contribution of different mechanisms

$\pi^- + {}^4He \rightarrow \pi^+ + 4n$

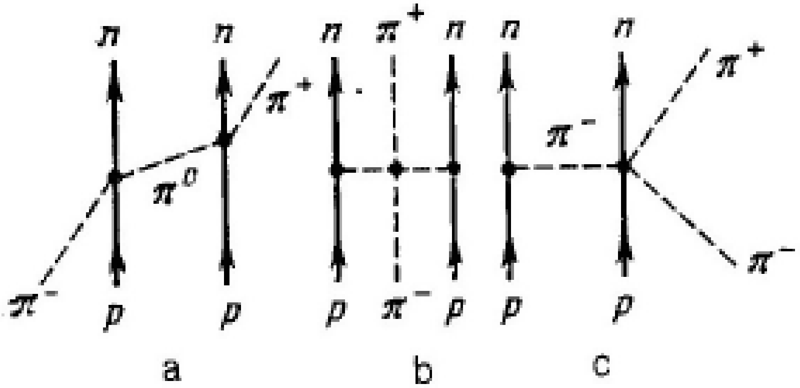
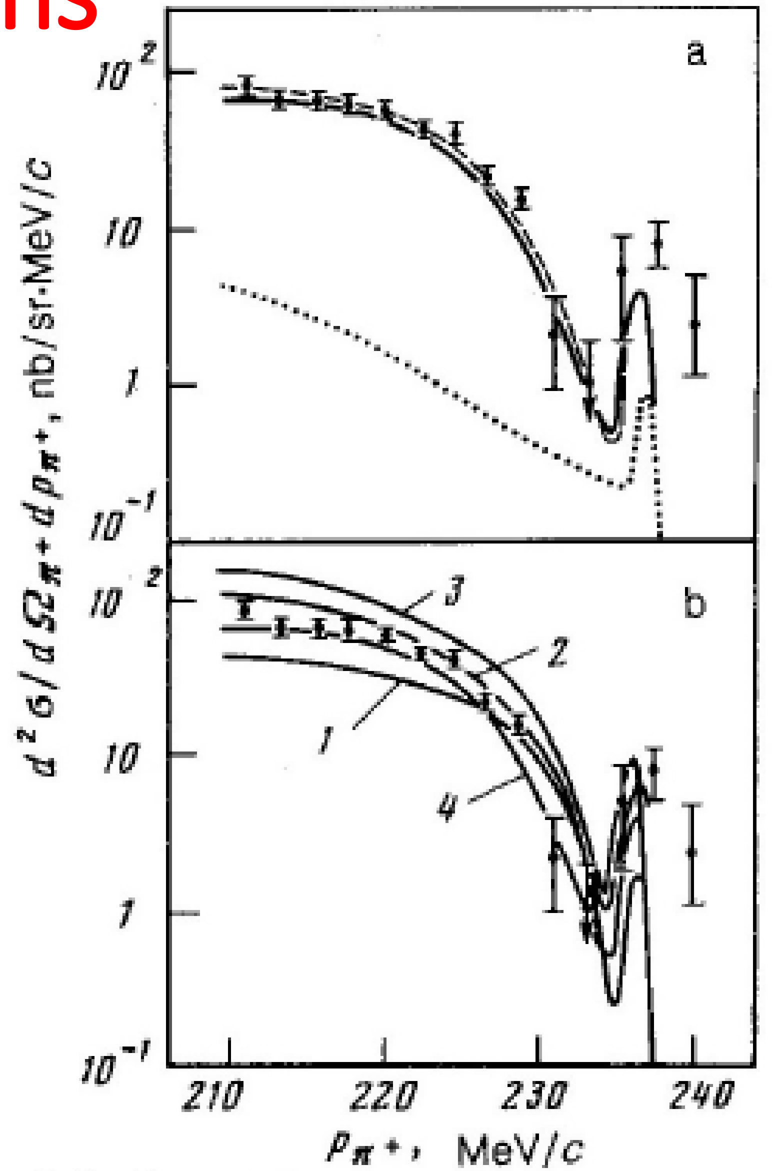


FIG. 2. Differential cross section for the reaction (1) at $T_{\pi^-} = 165$ MeV and $\theta = 0°$. (a) are results of the calculations with TS potential in the assumption of validity of the mechanism of the subsequent charge exchange (dashed curve), of the meson currents (dotted curve) and simultaneously of both mechanisms (solid curve); (b) are results of the calculations with the potentials *B* (curve 1) *V* (curve 2), GPT (curve 3), and TS (curve 4) wit simultaneous inclusion of both mechanisms.

[4]G. A. Baeker *et al.*, Phys. Rev. **125**, 1754 (1962).
[5]A. Volkov, Nucl. Phys. **74**, 33 (1965).
[6]D. Gogni, P. Pires, and R. de Tourreil, Phys. Lett. **32B**, 591 (1970).
[7]R. de Tourreil and D. W. Sprung, Nucl. Phys. **A201**, 193 (1973).



R. Ya. Kezerashvili Sov. J. Nucl. Phys. **44** (3), September 1986

Pion double-charge-exchange

$\pi^- + {}^4He \rightarrow \pi^+ + {}^4n(4n?)$

A systematic experimental study of inclusive DCX in He, has been undertaken over a broad range of incident energies encompassing the $\Delta(1232)$ resonance. The reaction He(π^+ , π^-)4p and He(π^- , π^+)4n was observed at incident energies 120, 150, 180, 240, and 270 MeV and 5 angles from 25 to 130

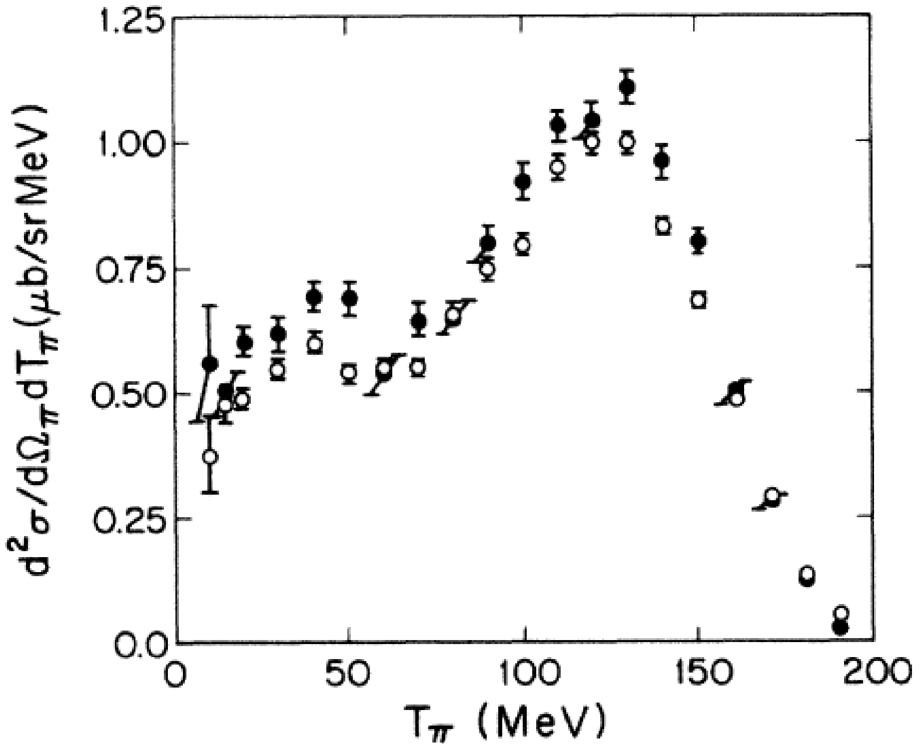
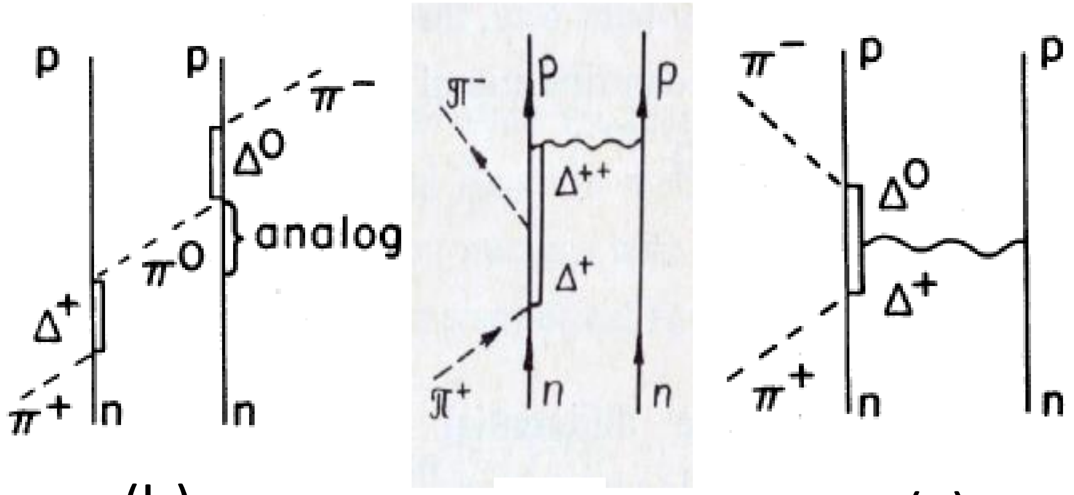


FIG. 4. Doubly differential cross sections for the reactions ${}^4He(\pi^+,\pi^-)4p$ (solid circles) and ${}^4He(\pi^-,\pi^+)4n$ (open circles) at incident energy 240 MeV and laboratory angle 50°.

The earliest inclusive DCX measurements were made at the JINR in Dubna by Batusov et al., Phys. JETP 1964.

Gilly, PRL 11 (1964); PRL 19 (1965)
Carayannopoulos, PRL 20 (1968)
Falomkin, et al., Nuove Cim. A 22 (1974)
Kaufman, et al., PR 175 (1968)
Jeanneret, et al., Nucl. Phys. A 350 (1980)
Ungar et al. , Phys. Lett. 144B, 333 (1984).
Kinney et al., Phys. Rev. Lett. 57, 3152 (1986).

Δ_{33} Dynamics in Pion DCX



(b) (a) (c)

Reaction mechanisms. Dashed lines are pions and solid lines are nucleons. (b) The sequential model. (c) The delta-nucleus charge exchange. The wiggly line indicates the delta-neutron charge exchange interaction.

Johnson et al. calculate effects of direct meson- Δ_{33} interactions on pion DCX to isobaric analog states in **^{18}O**. Data at the higher energies cannot be explained by a simple combination of sequential pion-nucleon scattering plus direct meson- Δ_{33} terms.

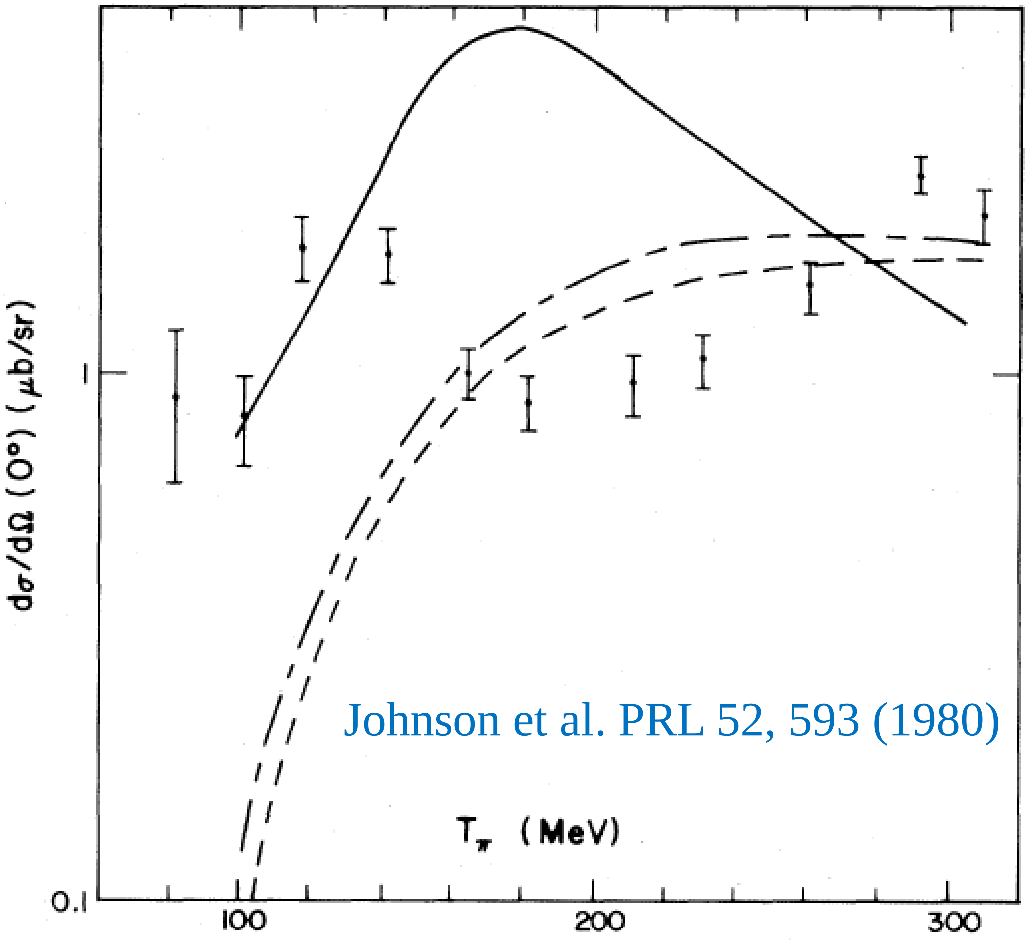


FIG. 2. Cross section for $^{18}O(\pi^+,\pi^-)^{18}Ne$ to the double isobaric analog state as a function of energy compared to the experiment of Ref. 15. The long/short-dashed curve is the sequential process. The short-dashed curve is the sequential plus terms of type in Fig. 1(a). The solid curve is the sequential plus Fig. 1(c).

Let's Discuss all "Yes" or "No"

Should be it interesting to study the DCX reaction of π^- on ^{6}He or ^{8}He?

$$\begin{aligned}\pi^- + {}^6\mathrm{He} &\to \pi^+ + 6n \ (\text{or } {}^6\mathrm{n});\\ &\to \pi^+ + {}^3\mathrm{n} + {}^3\mathrm{n};\\ &\to \pi^+ + {}^3\mathrm{n} + 3\mathrm{n};\\ &\to \pi^+ + {}^4\mathrm{n} + 2\mathrm{n}.\end{aligned}$$

$$\begin{aligned}\pi^- + {}^8\mathrm{He} &\to \pi^+ + 8n \ (\text{or } {}^8\mathrm{n});\\ &\to \pi^+ + {}^4\mathrm{n} + {}^4\mathrm{n};\\ &\to \pi^+ + {}^4\mathrm{n} + 4\mathrm{n};\\ &\to \pi^+ + {}^6\mathrm{n} + 2\mathrm{n}.\end{aligned}$$

The halo nuclei ^{6}He and ^{8}He have a strongly bound core ^{4}He and two or four weakly bound neutrons, respectively.

It seems that the most preferable kinematics should be a forward scattering DCX reaction and any observed channel can be interesting.

Conclusions

- Forward scattering pion DCX within MEC mechanism in Born approximation does not depend on the energy when the pion distortion is neglected.
- The unitarization of the amplitude as well as the contributions of the ρ, σ, and f_0 mesons into MEC mechanism based on the composite meson model increase the cross section and lead to its energy dependence.
- ρ, ε and S^* mesons contribution, appreciably changes the cross section, and it shows the importance of these mesons at the energies above 600 MeV.
- MEC mechanism can reveal in the pion DCX because it has a substantial contribution at the considered energy region
- Interference of MEC with the two step mechanism can dramatically change the forward scattering DCX cross section.

Thank you for your attention