Emergent Symmetry Breaking in Deformed Atomic Nuclei

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Hans A. Weidenmüller
Max-Planck-Institut für Kernphysik
Heidelberg

Work done in collaboration with T. Papenbrock,
University of Tennessee and ORNL

Contents

1. Emergent versus Spontaneous Symmetry Breaking.

Spontaneous symmetry breaking:

Ground state of an infinite nonrelativistic quantum system may spontaneously break symmetry of the Hamiltonian. Example: Ferromagnet. Hamiltonian is rotationally invariant but ground state is not. (All spins point in same direction.) Hilbert spaces built on ground states with different spin orientations are not unitarily equivalent (no unitary transformation connects ground states with different spin orientations). Infinite ferromagnet cannot be rotated because the overlap of two differently oriented ground states is zero (infinitely many spins involved).

Effective Field Theory: Spontaneously broken symmetry causes Nambu-Goldstone modes. These describe low-lying excited states. Except for multiplicative constants, dynamics of the system is determined entirely by the broken symmetry. No additional dynamical information required. In the ferromagnet: long-wavelength spin waves. H. Leutwyler, Phys. Rev. D 49 (1994) 3033; J. M. Roman and J. Soto, Int. J. Mod. Phys. B 13 (1999) 755.

Emergent symmetry breaking: C. Yannouleas and U. Landmann, Rep. Prog. Phys. 70 (2007) 2067.

Precursor of spontaneous symmetry breaking in a finite system. A ferromagnet of finite size may rotate and, thus, realize different spin orientations. With increasing size the moment of inertia grows. Rotational modes become ever more degenerate. In the infinite-size limit, rotation is no longer possible. Rotational modes are truly degenerate. Their superpositions define different (localized) direction of spins in ferromagnet.

Symmetry breaking emerges because low-energy spectrum is governed by combination of Nambu-Goldstone modes and rotations (provided the system is sufficiently large). Then rotational energy small compared to energy of Nambu-Goldstone excitations.

Our approach to deformed atomic nuclei:

T. Papenbrock and H. A. Weidenmüller, Phys. Rev. C 89 (2014) 014334; J. Phys. G: Nucl. Part. Phys. 42 (2015) 105103; Phys. Scr. 91 (2016) 053004.

We work close to the infinite-size limit and consider rotations as the leading correction to that limit. Nuclear vibrations as Nambu-Goldstone modes. These survive the infinite-size limit. In the finite-size nucleus, combine these with rotational degrees of freedom.

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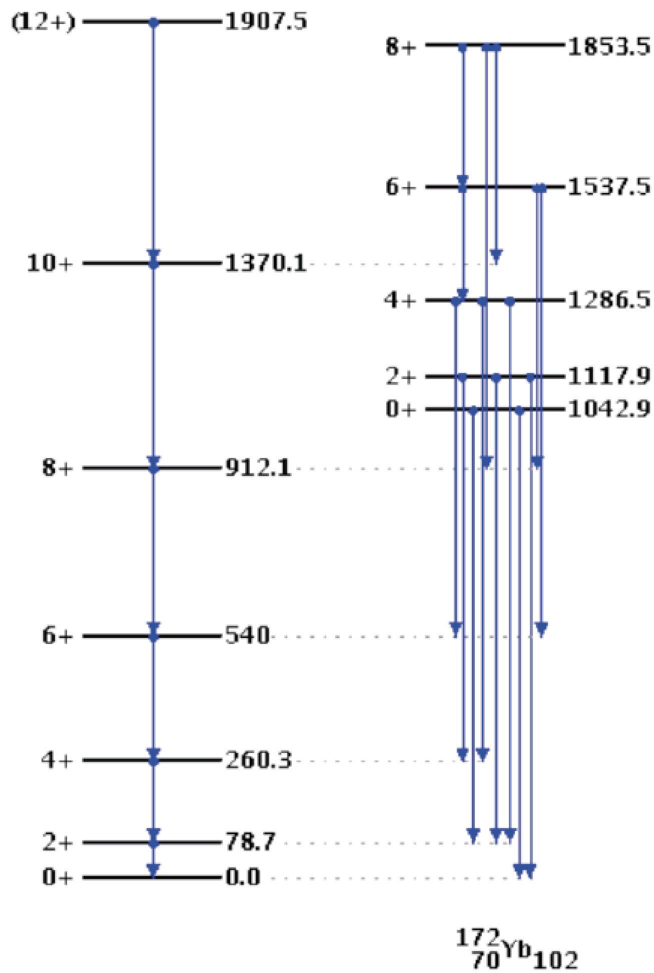
Spectrum of rotational nucleus / energy scales



Breakdown
$\Lambda \approx 2.5$ MeV

Vibration
$\Omega \approx 1$ MeV

Rotation
$\xi \approx 0.1$MeV

338,7 x 190,5 mm

2. "Effective Theory"

T. Papenbrock, Nucl. Phys. A 852 (2011) 36; J. Zhang and T. Papenbrock, Phys. Rev. C 87 (2013) 034323; E. A. Coello Perez and T. Papenbrock, Phys. Rev. C 92 (2015) 014323.

Considers only physics of rotations, and adds vibrations as new degrees of freedom. (In the EFT vibrations are Nambu-Goldstone modes.)

A precursor to effective field theory: Construct low-energy Hamiltonian from an effective theory (only time derivatives) with axial symmetry. 5 real quadrupole degrees of freedom ϕ_μ (time-reversal invariance) . $\phi_{\pm 1}$ define Euler angles of rotational motion, remainder defines vibrational modes. ϕ_0 has nonzero expectation value and defines deformed ground state. Use methods of effective field theory to construct rotationally invariant Lagrangean. Power counting: $\xi \approx 100$ keV is scale of rotational motion, $\Omega \approx 500$ keV is scale of vibrational motion, $\Lambda \approx 1-2$ MeV is breakdown scale. With $\xi \ll \Omega \ll \Lambda$ effective Lagrangean can be arranged into leading-order, next-to-leading order, … terms. Form of terms is fixed, only dimensionless constants as parameters. Leading order: Vibrational states that are bandheads of rotational spectra (Bohr model). All moments of inertia are equal. Higher-order terms used to predict inverse moments of inertia A. J. Zhang and T. Papenbrock, Phys. Rev. C 87 (2013) 034323.

	^{168}Er			^{166}Er			^{232}Th		
E	0	821	2056	0	786	2028	0	785	1414
K	0	2	4	0	2	4	0	2	4
A	13.17	12.33	11.37	13.43	12.25	10.56	8.23	7.38	7.27
A_{theo}	13.17	12.33	11.49	13.43	12.25	11.07	8.23	7.38	6.53

	^{162}Dy			
E	0	888	1536	1400
K	0	2	4	0
A	13.45	12.34	9.87	8.87
A_{theo}	13.45	12.34	11.23	11.23

E2 Transitions:

Bohr model has problems. For instance, overprediction by factors 2 to 10 of E2 transitions between the rotational band on top of the O_2^+ vibrational bandhead and ground-state band in Bohr model. Can effective theory do better? Use gauge invariance to construct interaction terms. That generates both minimal and non-minimal coupling terms (presence of composite objects). Use power counting to group these into leading order terms, next to leading order terms, etc. Scheme allows prediction of theoretical uncertainties.

Example: Transitional nuclei (non-rigid rotors with $E_{4^+}/E_{2^+} \approx 3$).

E. A. Coello Perez and T. Papenbrock, Phys. Rev. C 92 (2015) 014323.

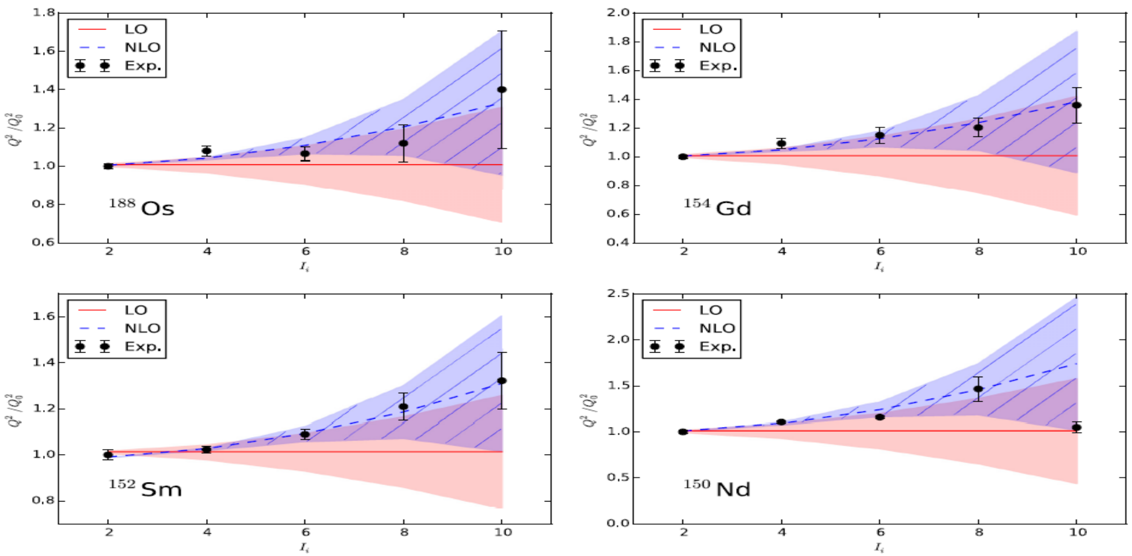


FIG. 6. (Color online) Experimental data (black data points with error bars) for decays within the ground band of ^{188}Os (top left) [81], ^{154}Gd (top right) [17], ^{152}Sm (bottom left) [14], and ^{150}Nd (bottom right) [16] are compared against LO (red line and corresponding uncertainty band) and NLO (blue dashed line with corresponding uncertainty band) calculations of the effective theory. At NLO, the quadratic deviation (in spin I_i) from the LO rigid-rotor result is described well by the effective theory.

3. Effective Field Theory.

Symmetry of Hamiltonian is SO(3), deformed ground state is invariant under SO(2). Coset space SO(3) / SO(2) parametrized by space-dependent fields $\psi_x(\vec{x},t), \psi_y(\vec{x},t)$ and by purely time-dependent angles $\phi(t), \theta(t)$. Consider

$$\begin{aligned} U &= g(\phi,\theta)u(\psi_x,\psi_y) \ , \\ g(\phi,\theta) &= \exp\{-i\phi J_z\}\exp\{-i\theta J_y\} \ , \\ u(\psi_x,\psi_y) &= \exp\{-i\psi_x J_x - i\psi_y J_y\} \ . \end{aligned}$$

Here J_x, J_y, J_z are components of usual angular momentum.

S. Weinberg, Phys. Rev. 166 (1968) 1568; S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2239; C. G. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2247.

Physical picture: Choose z-axis to coincide with nuclear symmetry axis. Apply U = g u. Since ψ_x, ψ_y depend on space, operator $u(\psi_x,\psi_y)$ distorts every volume element. Followed by g which generates rotation of distorted nucleus. Identify fields ψ_x, ψ_y with Nambu-Goldstone modes (these survive the infinite-volume limit, rotations do not). Small-amplitude approximation for ψ_x, ψ_y (harmonic approximation). Rotations are fully taken into account. Disregard of pairing and shell structure defines breakdown scale Λ . Invariants of the theory constructed from $U^\dagger \partial_t U$ and from $U^\dagger \vec{\nabla} U$. The infinitely many invariants are ordered using power counting.

Construction of invariants: With $\mu = t, x, y, z$ write

$$U^{-1} i\partial_\mu U = a_\mu^x J_x + a_\mu^y J_y + a_\mu^z J_z$$

Coefficients $a_\mu^x, a_\mu^y, a_\mu^z$ are worked out from U and are building blocks of (rotational) invariants.

Power counting:

Expand in powers of $\psi_x, \psi_y \sim \varepsilon \ll 1$. Use $\phi, \theta \sim \mathcal{O}(1)$, $\dot{\phi}, \dot{\theta} \sim \xi$, $\dot{\psi}_x, \dot{\psi}_y \sim \Omega\varepsilon$. Establish transformation properties of resulting terms under rotations. Construct low-order invariants. Each is part of effective Lagrangean.

Kinetic terms (time derivatives):

$$\begin{aligned}
\mathcal{L}_{1a} &= \dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \ , \\
\mathcal{L}_{1b} &= \dot{\psi}_x^2 + \dot{\psi}_y^2 + 2(\psi_x \dot{\psi}_y - \psi_y \dot{\psi}_x)\dot{\phi}\cos\theta \ , \\
\mathcal{L}_{1c} &= (\psi_x \dot{\psi}_y - \psi_y \dot{\psi}_x)^2 \ , \\
\mathcal{L}_{1d} &= (\psi_x^2 + \psi_y^2)\mathcal{L}_{1b} \ .
\end{aligned}$$

These are multiplied by constants and added to yield kinetic part of effective classical Lagrangean. Analogously for potential part (spatial derivatives). Use power counting to determine order of multiplicative constants.

Classical Lagrangean defines classical field theory. Expand ψ_x, ψ_y in spherical harmonics. Use Legendre transformation to define classical Hamiltonian. Quantization yields quantum Hamiltonian. Not trivial because of curvilinear coordinates. Terms of leading order are

$$H = \frac{1}{2C_a}(Q^2 - K^2) + \frac{1}{2}\sum_{L\mu}\left[\frac{1}{C_b}\left((p^x_{L\mu})^2 + (p^y_{L\mu})^2\right) + (\psi^2_{xL\mu} + \psi^2_{yL\mu})(DL(L+1) + D'\mu^2)\right].$$

That justifies effective theory as a low-energy approximation. Computation of terms of higher order is possible.

Nambu-Goldstone modes are (infinitely many) quantized vibrations. In terms of higher order, these interact. In addition, two degrees of freedom to describe rotations coupled to vibrations.

Small parameters: ratio ξ/Ω of energies of rotational motion and of vibrational motion; ratio Ω/Λ where Λ is breakdown parameter; parameter ε that measures anharmonicity.

4. Summary

Spontaneous symmetry breaking in infinite nonrelativistic quantum systems: Described in terms of Nambu-Goldstone modes.

Emergent symmetry breaking in finite system as precursor of spontaneous symmetry breaking in infinite system. Additional degrees of freedom required.

In atomic nuclei with deformed ground states: In the infinite-size limit, Nambu-Goldstone modes are vibrations. Rotational degrees of freedom provide first-order correction to infinite-size limit. Hence $\xi \ll \Omega \ll \Lambda$.

Systematic construction of effective Lagrangen and quantized Hamiltonian using methods of quantum field theory. Power counting leads to identification of terms of leading order, next to leading order, etc. Construction of higher-order terms has no ambiguities.

Justification of results obtained in effective theory. Terms of higher order provide corrections to phenomenological models (Bohr-Mottelson, IBM). Good agreement with data. Estimates of uncertainties.