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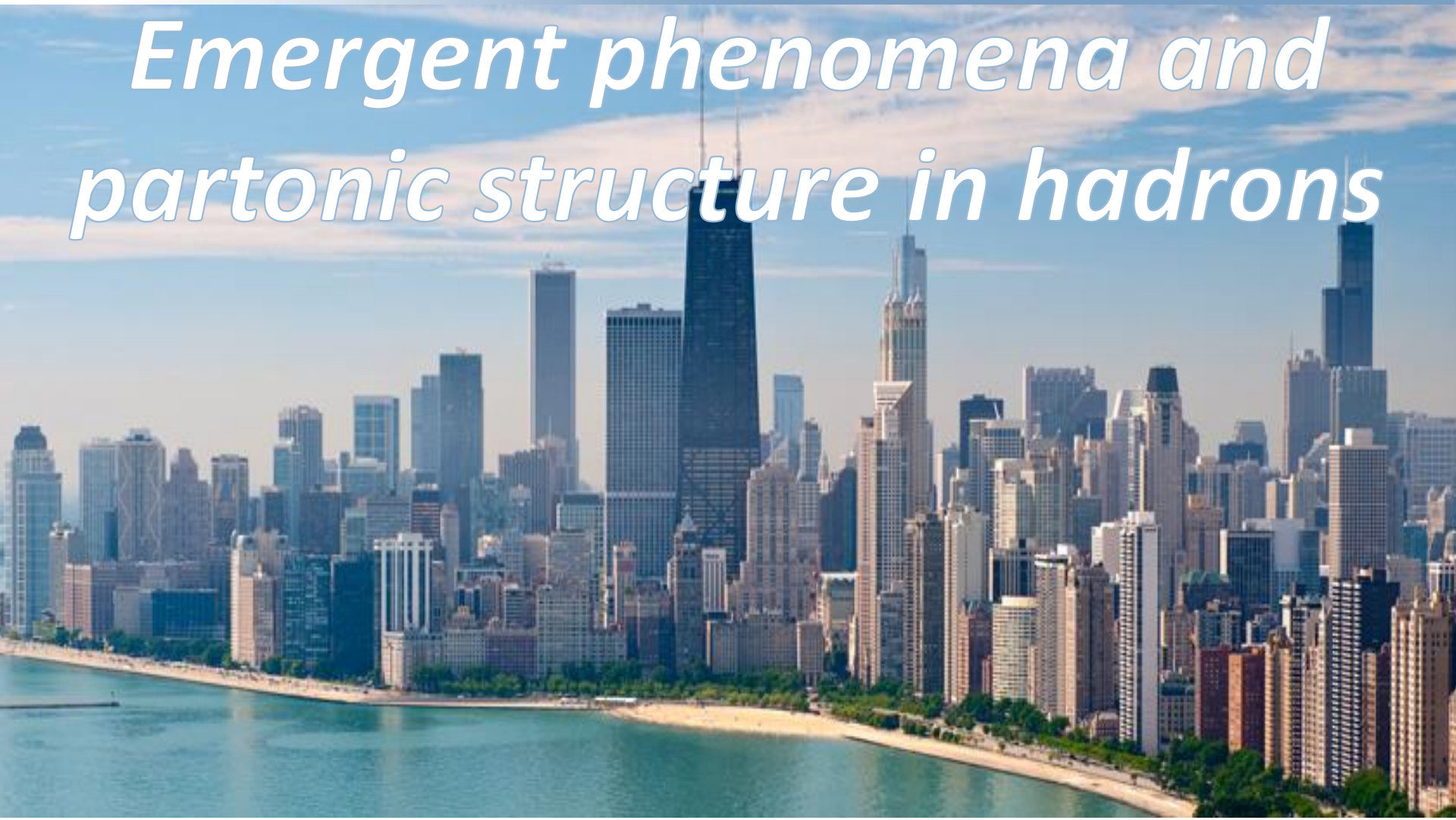
Emergent phenomena and partonic structure in hadrons



Craig Roberts, Physics Division

U.S. DEPARTMENT OF ENERGY

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L'ANIMAL-MACHINE

L'**animal-machine** est une hypothèse éthologique selon laquelle les animaux sont des machines. Comme les machines, les animaux seraient des assemblages de pièces et rouages, dénués de conscience ou de pensée. D'un point de vue religieux, l'application du mécanisme à la vie revient à nier l'âme des bêtes qui périssent donc entièrement au moment de leur mort. Poussée à l'extrême, notamment par Nicolas Malebranche, cette conception implique que leurs cris et gémissements ne peuvent être que le reflet de dysfonctionnements dans les « rouages » plutôt que l'expression d'une souffrance. Même si cette vision du problème est complètement décalée par rapport à la vision moderne, elle peine à être délogée par des conceptions plus en adéquation avec les avancées scientifiques récentes.

In the nineteenth century, Descartes was revered for his mechanistic physiology and theory that animal bodies are machines (that is, are constituted by material mechanisms, governed by the laws of matter alone).

Reductionist Perspective

Strong Interactions in the Standard Model of Particle Physics

- Chromodynamics = non-Abelian, relativistic gauge field theory

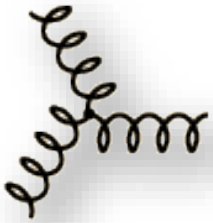
$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

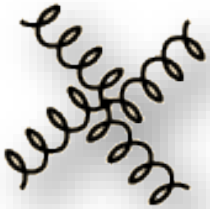
Quark fields here (material)

Gluon fields here (Force field)

- Problems
  - Quantise the theory
  - Solve for the spectrum of supported states
  - Elucidate their internal structure, *i.e*. expose & explain how the states are built from the gluon & quark fields used to express the Lagrangian
- Special features of chromodynamics
  - $\bar{\psi}\, \gamma\cdot D\, \psi$ involves gluon-quark interactions, a normal part of theories since Maxwell, *i.e*. matter-field interactions
  - $G_{\mu\nu}^a\, G_{\mu\nu}^a$ involves gluon self-interactions = field-field interactions

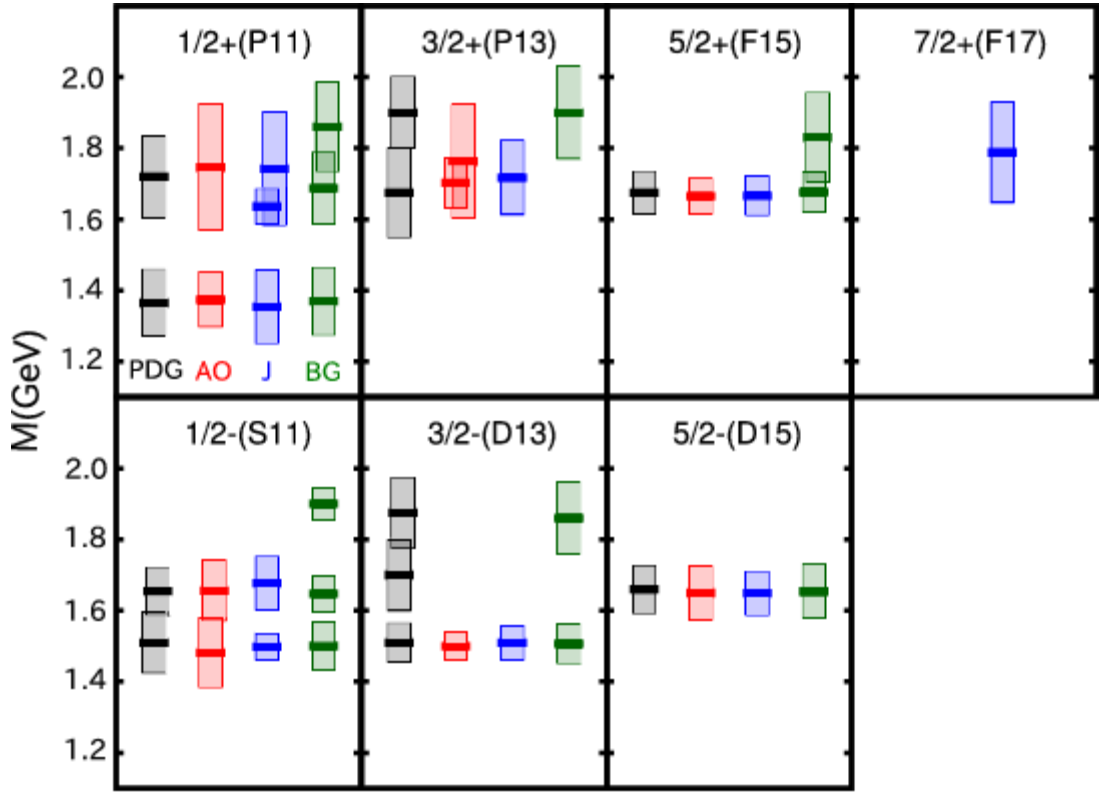
    … *Essentially New Feature*





Strong Interactions in the Standard Model of Particle Physics

- Extract from spectrum of nucleon states (resonances) with mass less-than 2GeV
- Experiment (PDG) compared with theory (AO, J, BG)
- Theory results are outcome of massive computational effort, analysing *22,348 independent data points*, representing *complete array of partial waves*
- *Partly Emergent* … AO & J computations use meson-baryon degrees-of-freedom



Nature's scale for visible, strongly-interacting matter = 1 GeV = 1.783×10^{-27} kg ≈ 2000 × m_e

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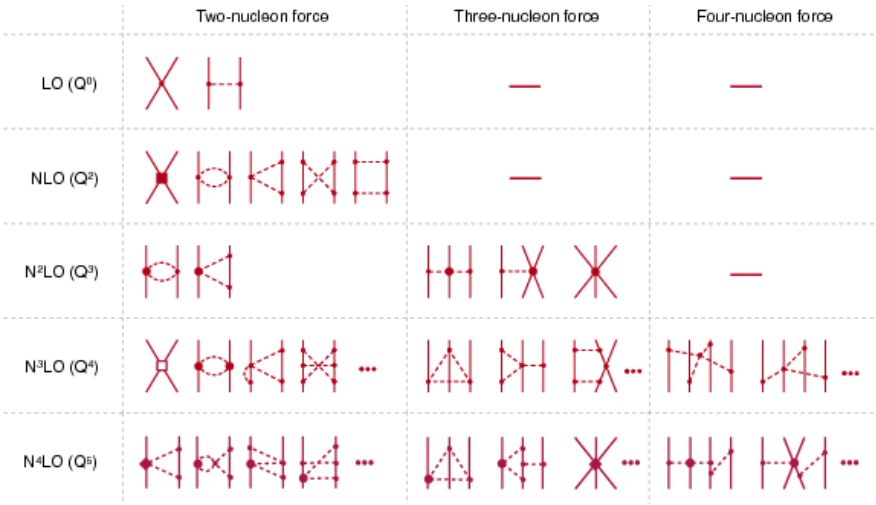
- Only apparent scale in chromodynamics is mass of the quark field
- In connection with everyday matter, that mass is 1/250$^{\rm th}$ of the natural (empirical) scale for strong interactions,
  viz. more-than two orders-of-magnitude smaller
- Reductionist:
  - Quark mass is said to be generated by Higgs boson.
- Plainly, however, that mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $\mathcal{L}_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $\mathcal{L}_{\rm QED}$

Nuclear Models & EFTs

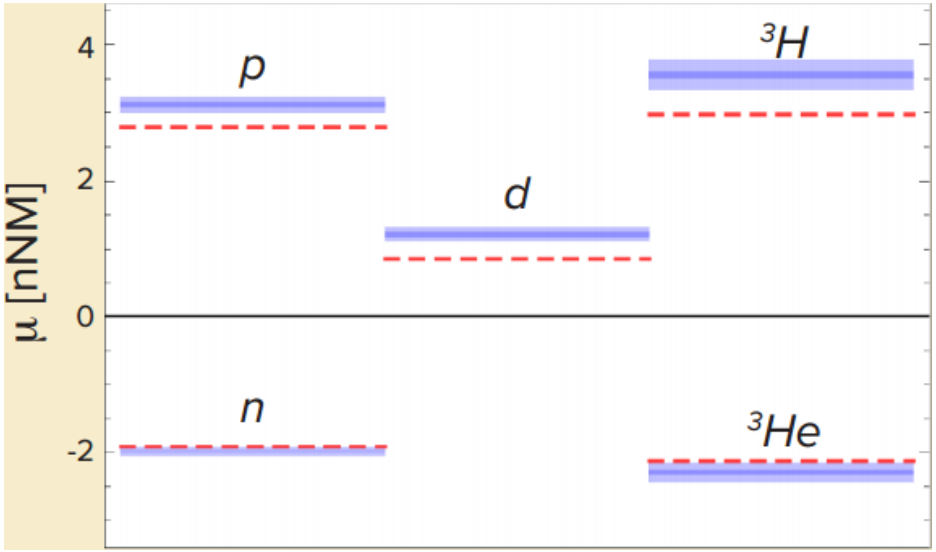
- Models and EFTs for nuclear physics typically assume and accept the existence of the natural mass scale, $m_p \approx 1$ GeV
- Assume and accept, too, the reality of effectively pointlike nuclear constituents (proton, neutron, etc.) and force carriers (pion and, perhaps, other meson-like entities)



- Issue ≠ elucidate their internal structure
  Instead, develop systematically improvable techniques that can reliably describe & predict the number & nature of (atomic) nuclei
- *Reductionism built on an emergent plateau*
- Basic reductionist question here:
  - Can the plateau upon which the nuclear model/EFT paradigm is built be constructed from chromodynamics?
- If "yes", then all parameters used and fitted in nuclear theories will be confronted with *ab initio* predictions

Nuclear Models & EFTs

- Suppose the answer is "yes" or, at least, that the conjecture is plausible
- Why?
- Example, magnetic moments of light nuclei
  nuclear shell model ⇔ lattice-regularised QCD
  - ✓ $d = p + n \Rightarrow \mu_d \approx \mu_p + \mu_n$
  - ✓ $^3H = p^{\uparrow}\,(1s) + n^{\uparrow}\,n^{\downarrow}\,(1s)$
    $\Rightarrow \mu(^3_H) \approx \mu_p + \mu_n - \mu_n = \mu_p$
  - ✓ $^3He = p^{\uparrow}\,p^{\downarrow}\,(1s) + n^{\uparrow}\,(1s)$
    $\Rightarrow \mu(^3_{He}) \approx \mu_n + \mu_p - \mu_p = \mu_n$



2015 NSAC Long Range Plan, Nuclear Science

What & where is mass?

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromdynamics … non-Abelian local gauge theory
- Remove the current mass … no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* … normal matter is constituted from light-quarks & the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*
  … becomes … Where did it all come from?

Whence Mass?

- Poincaré invariance entails that the Energy-Momentum Tensor is divergence-free, *i.e*. it defines a conserved current:

$$\partial_\mu T_{\mu\nu} = 0$$

$T_{\mu\nu}$ can *always* be made symmetric

- Noether current associated with a global scale transformation:

  $x \to e^{-\sigma} x$

  is the dilation current: $D_{\mu\nu}=T_{\mu\nu}\, x_\nu$

- In a scale invariant theory, the dilation current is conserved

$$\partial_\mu \mathcal{D}_\mu = 0 = [\partial_\mu T_{\mu\nu}]x_\nu + T_{\mu\nu}\delta_{\mu\nu}$$
$$= T_{\mu\mu}\,,$$

- Consequently, in a scale invariant theory

  the *energy-momentum tensor must be traceless:* $T_{\mu\mu} \equiv 0$

Trace Anomaly

- Classical chromodynamics is meaningless … must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale

  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ
- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, $\mathcal{L}(m=0)$

  QCD β function

  Under a scale transformation $\zeta \to e^{\sigma}\zeta$, then $\alpha \to \sigma\, \alpha\beta(\alpha)$

  $$\mathcal{L} \to \sigma\, \alpha\beta(\alpha)\, d\mathcal{L}/d\alpha$$

  Trace anomaly

  $$\Rightarrow \partial_\mu \mathcal{D}_\mu = \delta\mathcal{L}/\delta\sigma = \alpha\beta(\alpha)\, d\mathcal{L}/d\alpha = \boxed{\beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0}$$
- Straightforward, nonperturbative derivation, without need for diagrammatic analysis …

  quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal … indicates only that a mass-scale exists
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

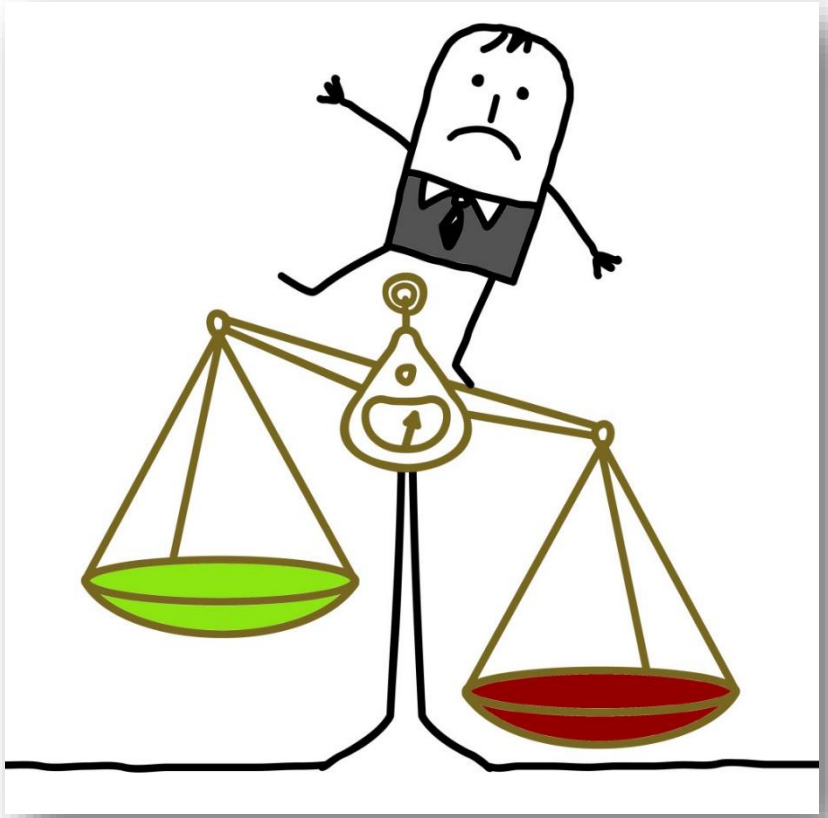
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0

  … In QCD, Θ_0 measures the strength of gluon self-interactions

  … so, from one perspective, m_p is completely generated by glue.



On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- That is a difficult way to obtain "zero"
- Easier, perhaps, to imagine that "zero" owes to cancellations between different operator-component contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

"Whence the proton's mass?"

is complete without the additional clause

"Whence the absence *of a pion mass?"*

- Natural nuclear-physics mass-scale must emerge simultaneously with the apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = m_p
- Is there a reductive explanation?

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Both statements are Poincaré invariant
- In connection with any bound-state, the only things that any two observers can certainly agree upon are the eigenvalues of the two Casimir operators of the Poincaré group evaluated in that state:
  - $M^2 \to m^2$ & $W^2 \to m^2\, j(j+1)$

    … the mass and total spin
- No decomposition of these quantities into separate contributions from constituents can ever be Poincaré-invariant or scale-invariant
  - This fact lies at the heart of the so-called "spin-crisis", which could therefore have been avoided

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Even using light-front quantisation, both the *natures* of and *contributions* from constituents changes with resolving scale, ζ

  In fact, the *meaning* of *constituent* changes with ζ

- Can it be sensible to attempt an expression of these trace anomaly statements in a particular frame, *e.g*. a hadron's rest-frame?
- *Difficulty*

  … a massless particle doesn't have a rest frame

- For a unified, simultaneous description, seems that a Poincaré-invariant analysis would be advantageous

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence?

- Classical chromodynamics … non-Abelian local gauge theory
- Local gauge invariance; but there is no confinement without a mass-scale
  - Three quarks can still be colour-singlet
  - Colour rotations will keep them colour singlets
  - But they need have no proximity to one another
    … proximity is meaningless in a scale-invariant theory
- Whence mass … equivalent to whence a mass-scale … equivalent to whence a confinement scale
- *Understanding the origin and absence of mass in QCD is quite likely inseparable from the task of understanding confinement.*

Existence alone of a scale anomaly answers neither question

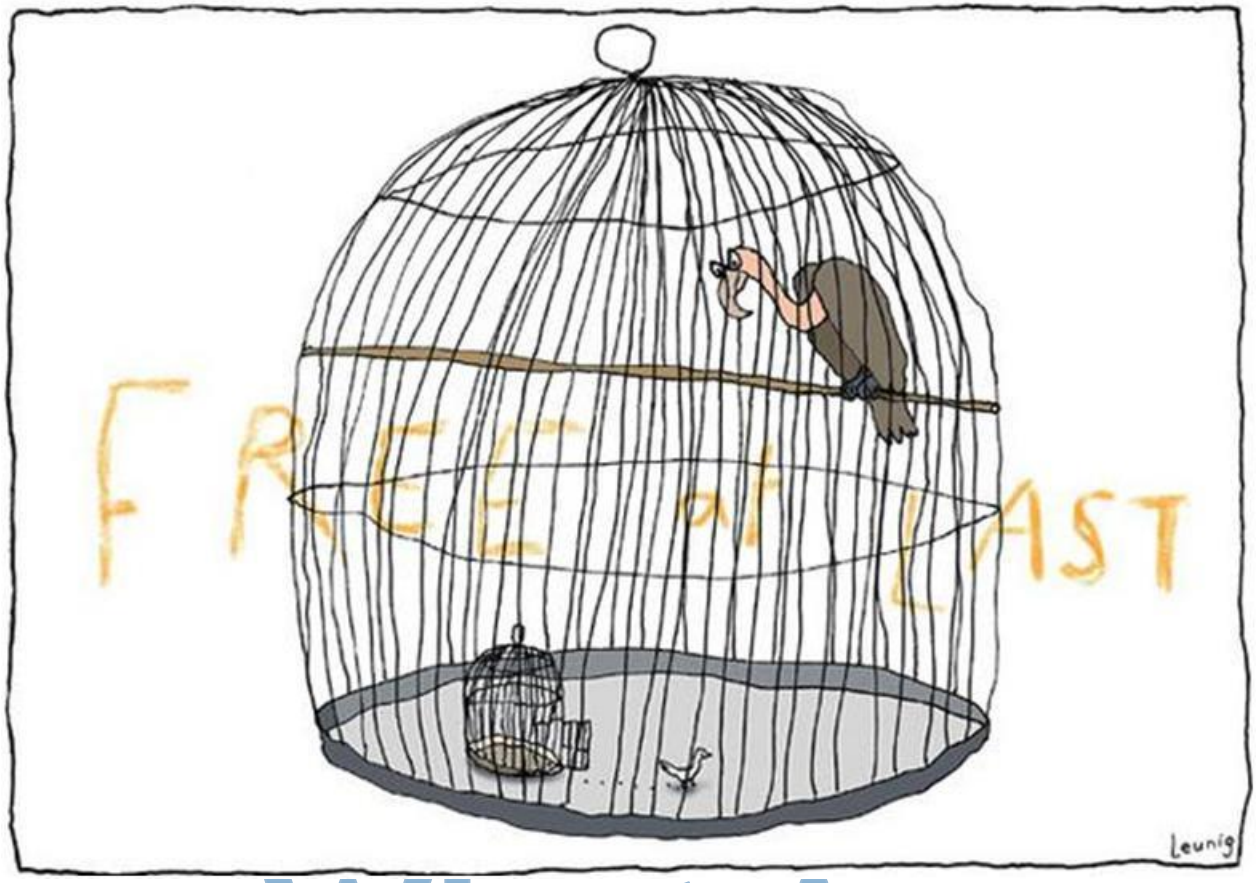
A New Era for hadro-particle physics

Overarching Science Challenges for the coming decade

- What is origin of mass in our Universe?
- What is the nature of confinement in real (dynamical-quarks) QCD?
- How are they connected?
- How can any
  - answers,
  - conjectures
  - and/or conclusions

  be empirically verified?

Physics is an Empirical Science



What is Confinement?

CLAY MATHEMATICS INSTITUTE

Millennium prize of $1,000,000 for proving that $SU_c(3)$ gauge theory is mathematically well-defined, which will necessarily prove or disprove a confinement conjecture

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x} - \vec{y}|), \tag{2}$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Confinement?

Craig Roberts. Emergence of Partonic Structure (80p)

CLAY MATHEMATICS INSTITUTE

142 MILLENNIUM PRIZE PROBLEMS

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Confinement?

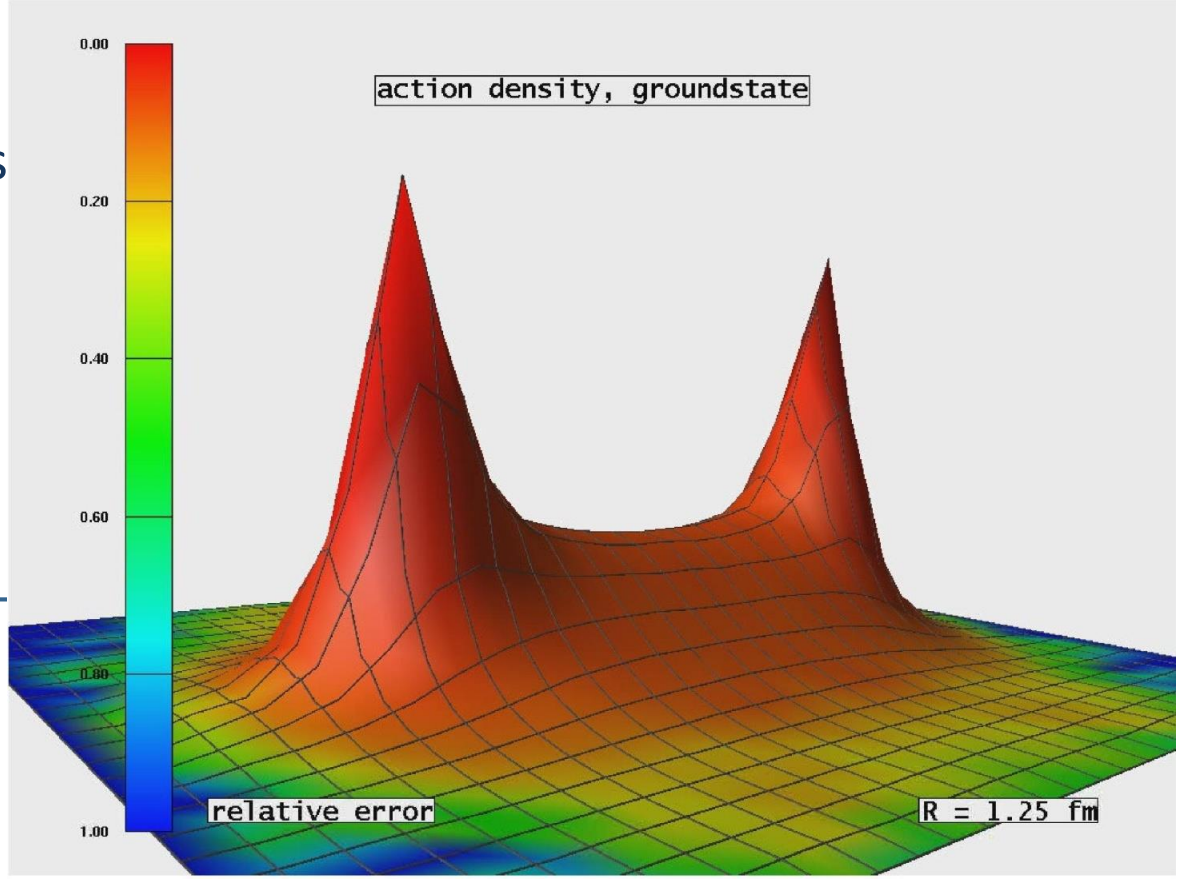
Light quarks & Confinement

- Folklore … ***Hall-D Conceptual Design Report(5)***

  "The color field lines between a quark and an anti-quark form flux tubes.

  A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

  This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."



Light quarks & Confinement

- Problem:

  16 tonnes of force

  makes a lot of pions.

Light quarks & Confinement

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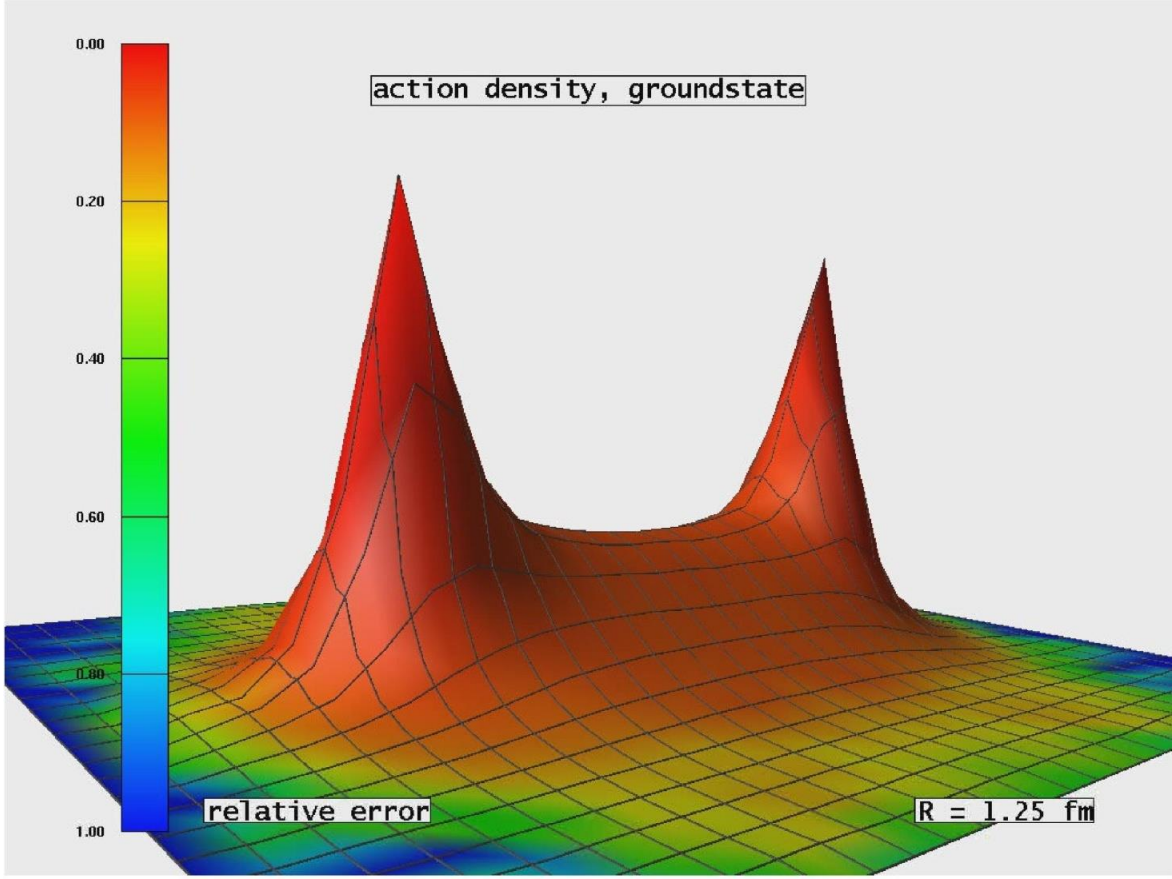
G. Bali et al., PoS LAT2005 (2006) 308

Light quarks & Confinement

- In the presence of light quarks, *pair creation seems to occur non-localized and instantaneously*
- No flux tube in a theory with light-quarks.
- ***Flux-tube is not the correct paradigm for confinement in hadron physics***



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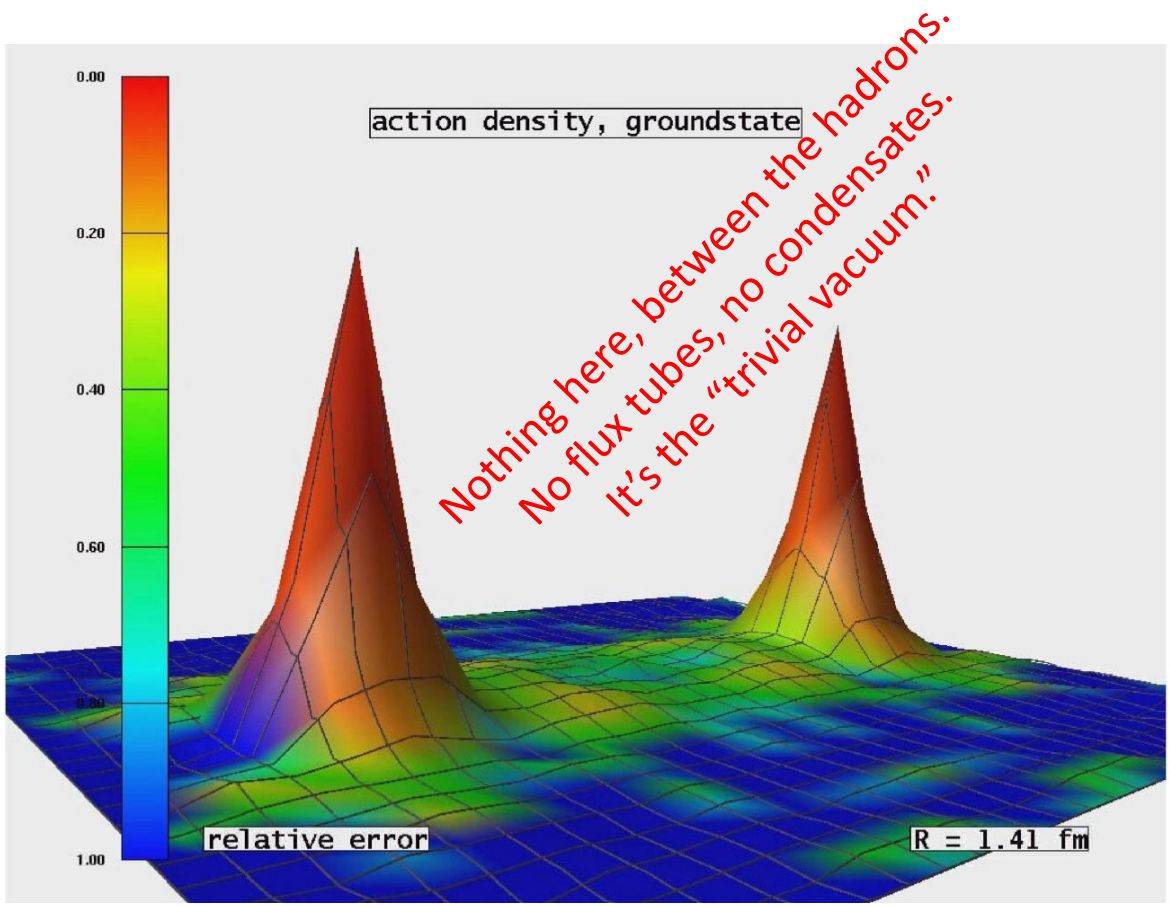
Confinement contains condensates
Brodsky, Roberts, Shrock,Tandy
arXiv:1202.2376 [nucl-th], Phys. Rev. C**85** (2012) 065202



- *Existence of mass-gap in pure-gauge theory*
- Strong evidence supporting this conjecture: lQCD predicts $\Delta \sim 1.5$ GeV
- But $\Delta^2/m_\pi^2 > 100$,

  So, can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?
- Conjecture: If *answer is* not *simply* ***no***, then it is probable that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.
- Conjecture: *Pion must play critical role in any explanation of real-world confinement. Any discussion that omits reference to the pion's role is possibly irrelevant.*

142 MILLENNIUM PRIZE PROBLEMS

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Reductive explanation of emergent phenomena?

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- Confinement and DCSB are emergent phenomena
  - Not revealed by any amount of staring at $\mathcal{L}_{\rm QCD}$
  - Yet, arguably, they determine the character of the QCD's spectrum and the structure of bound states
- Can one understand confinement and DCSB reductively,

  i.e. in terms of properties of the degrees-of-freedom used to formulate QCD?
- <u>OR</u> … Does the complexity of strong interaction phenomena make prediction and explanation impractical?
  - *E.g*., is it pointless to attempt to predict the nucleon's form factor on a domain that is not yet empirically accessible?

Nonperturbative QCD

If YES:

I. Must rely on the vast array of effective field theories, developed for different systems, in order, *e.g*. to express and understand the consequences of confinement & DCSB, without identifying their source

If NO

II. Must develop nonperturbative calculational methods to define and tackle QCD

   i. Lattice-regularised QCD

   ii. Continuum methods in quantum field theory

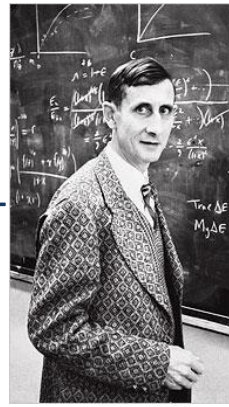
      - A collection of models and schemes, each with varying degrees of separation from QCD

   iii. Combinations of <u>all</u> the above

Currently, each approach has strengths and weaknesses, so (iii) is probably the best: combine all available methods to the fullest extent tha is reasonably possible

Dyson-Schwinger equations

- Generalisation of Euler-Lagrange equations to quantum field theory
- Continuum method for computing Schwinger functions = Euclidean Green functions
  - Schwinger functions are the same things computed using lattice-regularised QCD (lQCD)
  - Opportunities for cross-fertilisation, especially at the level of two-point functions for elementary excitations = gluon and quark propagators
- Challenge = Tower of equations coupling n-point to (n+1)-point functions truncations ⇒ truncation necessary in order to define tractable problem
- Systematic, symmetry preserving truncation schemes exist
  - Comparisons between schemes and orders within schemes are used to identify robust outcomes
  - Developments can be refined by comparisons with lQCD
  - Predictions can be tested by comparison with experiment



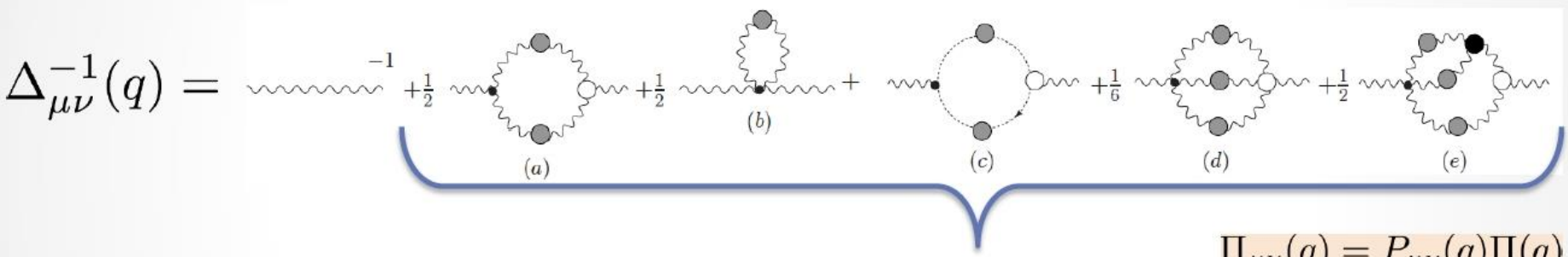
Dyson-Schwinger equations

DSE analyses can be into three classes:

A. Model-independent statements about QCD

B. Illustrations of such statements using well-constrained model elements and possessing a traceable connection to QCD

C. Studies that can fairly be described as QCD-based but whose elements have not been computed using a truncation that preserves a systematically-improvable connection with QCD



$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

Gluon Gap Equation

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons become massive!

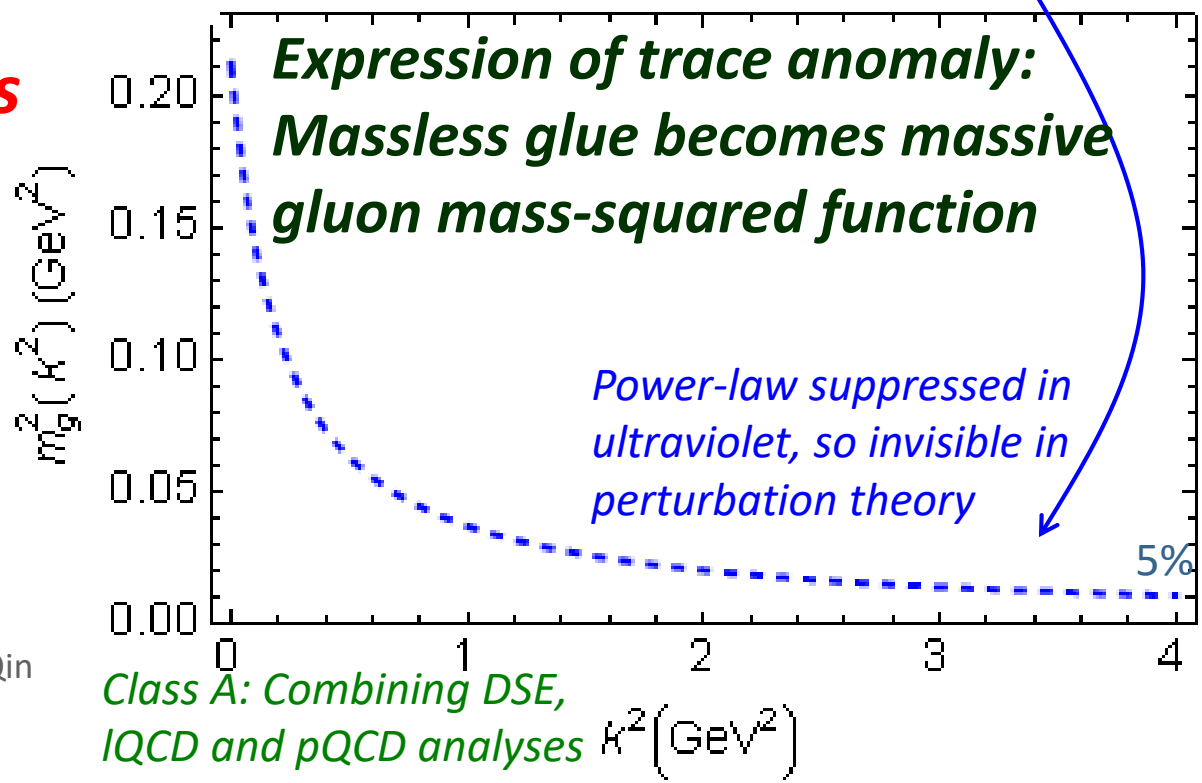
- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\ m_g^2(0) = (0.46\,\mathrm{GeV})^2$$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!



Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge sector

Massive Gauge Bosons!

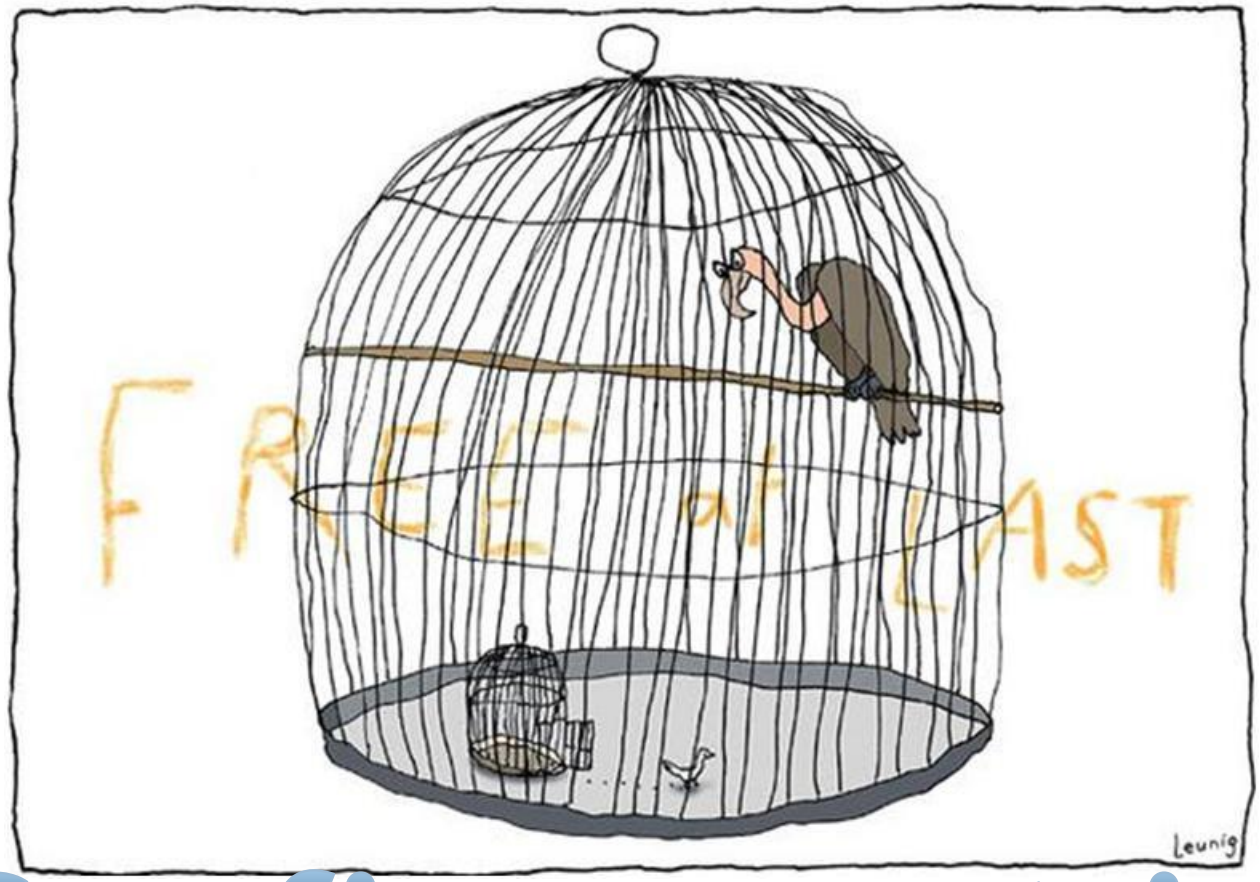
- Gauge boson cannibalism

  … a new physics frontier … within the Standard Model
- Asymptotic freedom means
  … ultraviolet behaviour of QCD is controllable
- Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs
  - Gluons and quarks with

    wavelength $\lambda > 2/\text{mass} \approx 1$ fm

    decouple from the dynamics … ***Confinement?!***
- How does that affect observables?
  - It will have an impact in any continuum study
  - Possibly (probably?) plays a role in gluon saturation … In fact, could be a harbinger of gluon saturation?

Electron Ion Collider: The Next QCD Frontier



Confinement is dynamical

Confinement

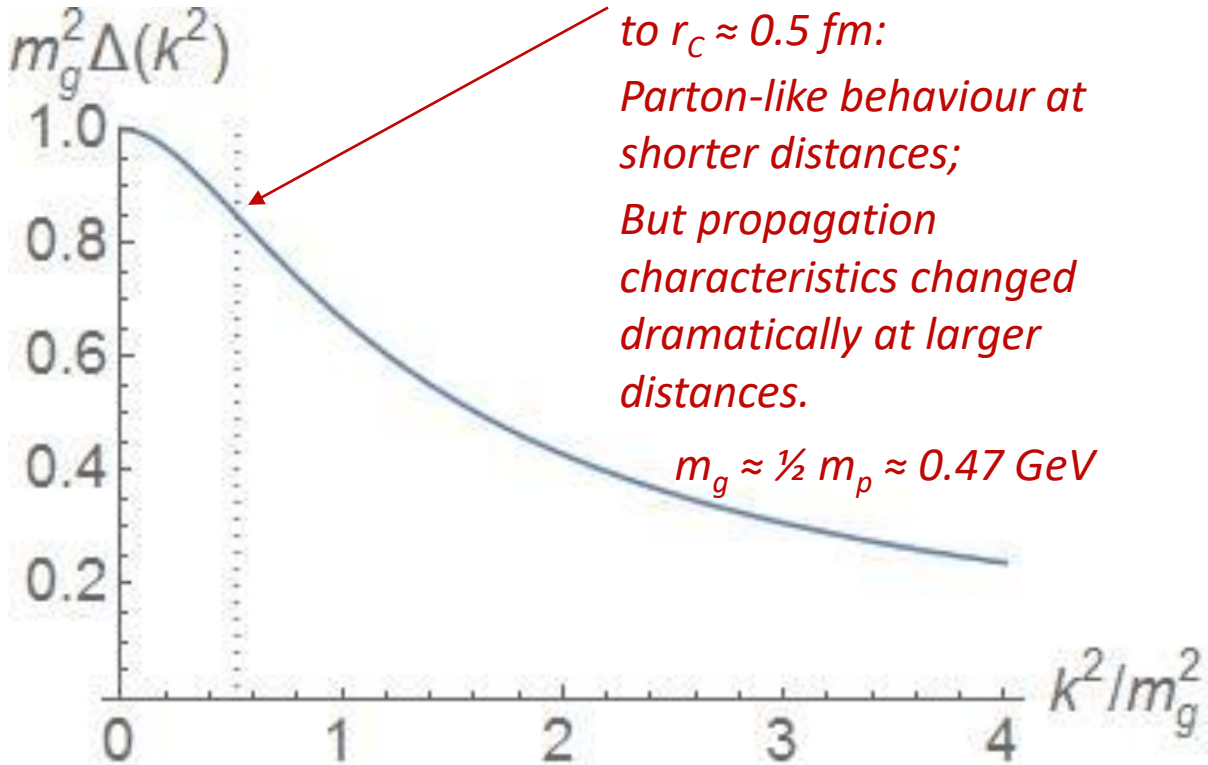
- All continuum and lattice solutions for Landau-gauge gluon & quark propagators exhibit an inflection point in k^2

⇒ Violate reflection positivity = sufficient for confinement

⇒ Such states have negative norm

⇒ All observable states of a physical Hamiltonian have positive norm

⇒ Negative norm states are not observable



Inflexion point corresponds to $r_C \approx 0.5$ fm:

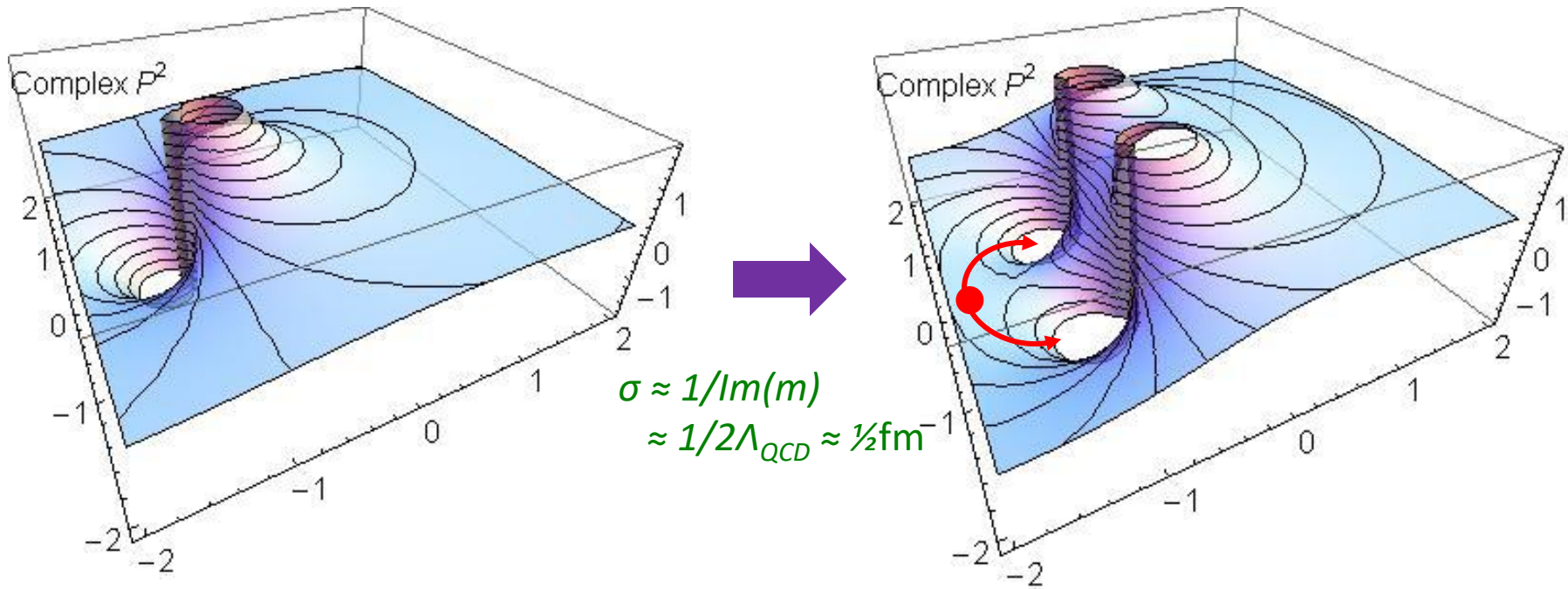
Parton-like behaviour at shorter distances;

But propagation characteristics changed dramatically at larger distances.

$m_g \approx ½\, m_p \approx 0.47$ GeV

Confinement

- Meaning (illustration) …



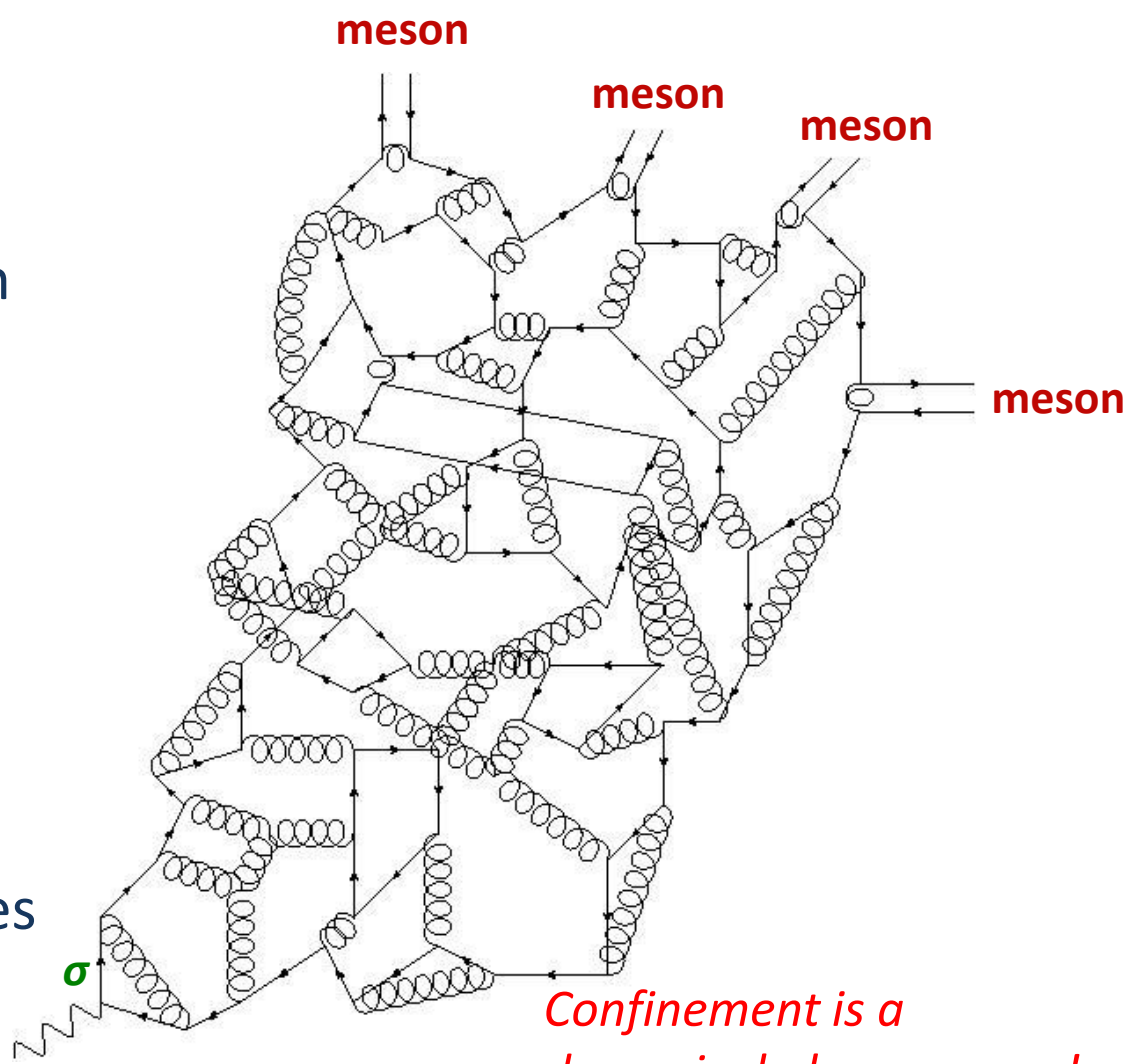
$\sigma \approx 1/\mathrm{Im}(m)$
$\approx 1/2\Lambda_{QCD} \approx ½\mathrm{fm}$

Real-particle mass-pole splits, moving into pair(s) of complex conjugate singularities, (or qualitatively analogous structures chracterised by a dynamically generated mass-scale)

Propagation described by rapidly damped wave & hence state cannot exist in observable spectrum

Quark Fragmentation

- A quark begins to propagate
- But after each "step" of length ***σ***, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



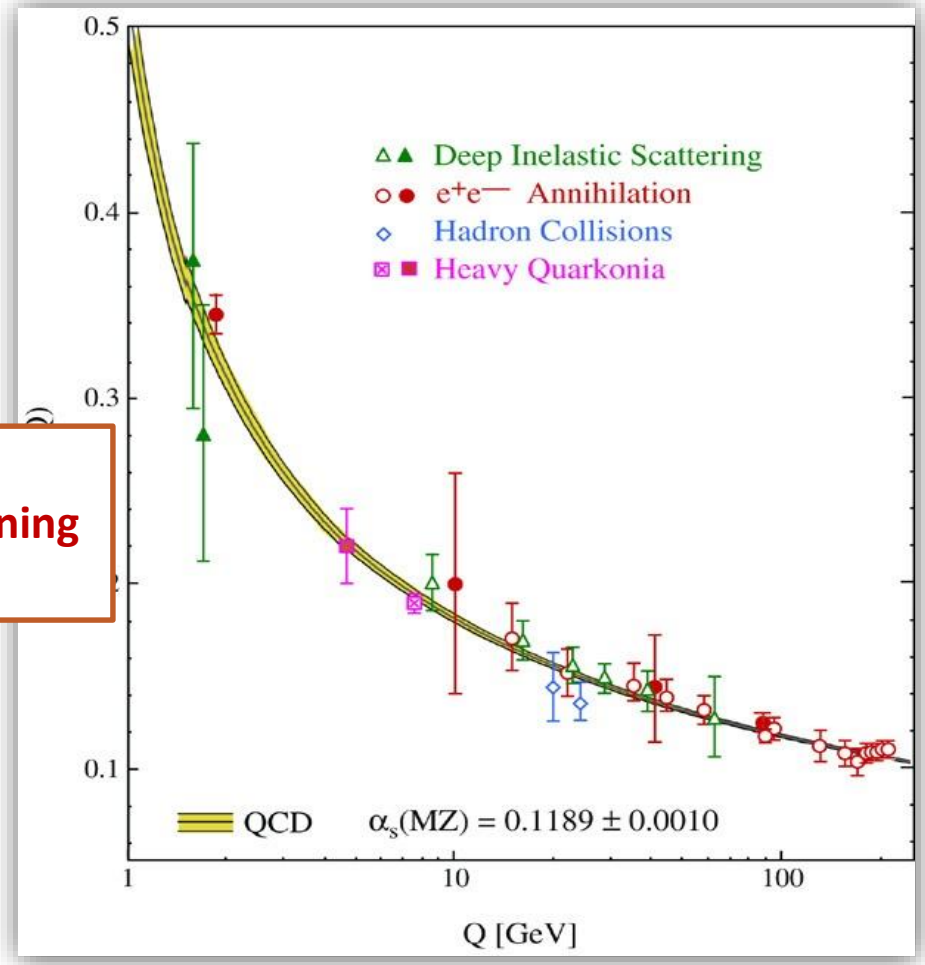
Confinement is a dynamical phenomenon!

Confinement in Thessaloniki

Outcome of discussions at Confinement XII

– Agreed position of Bali, Brambilla, Petreczky, Roberts:

- The flux tube measured in numerical simulations of lQCD with static quarks has *zero* relevance to confinement in the purely light-quark realm of QCD.
- There is *zero* knowledge of the strength or extension of a flux tube between a static-quark and any light-quark. Indeed, it is impossible to define such a flux tube. It is impossible to compute or even define a flux-tube between a light-quark source and light-quark sink.
- Since the vast bulk of visible matter is constituted from light valence quarks, with no involvement of even an accessible heavy quark, then the common flux tube picture is not the correct paradigm for confinement in hadron physics. (Refinements *might* be part of the story.)
- Confinement in hadron physics is largely a dynamical phenomenon, intimately connected with the fragmentation effect. It is unlikely to be comprehended without simultaneously understanding dynamical chiral symmetry breaking, which is the origin of a near-zero mass hadron (pion).



⇐
What's happening out here?!

QCD's Running Coupling

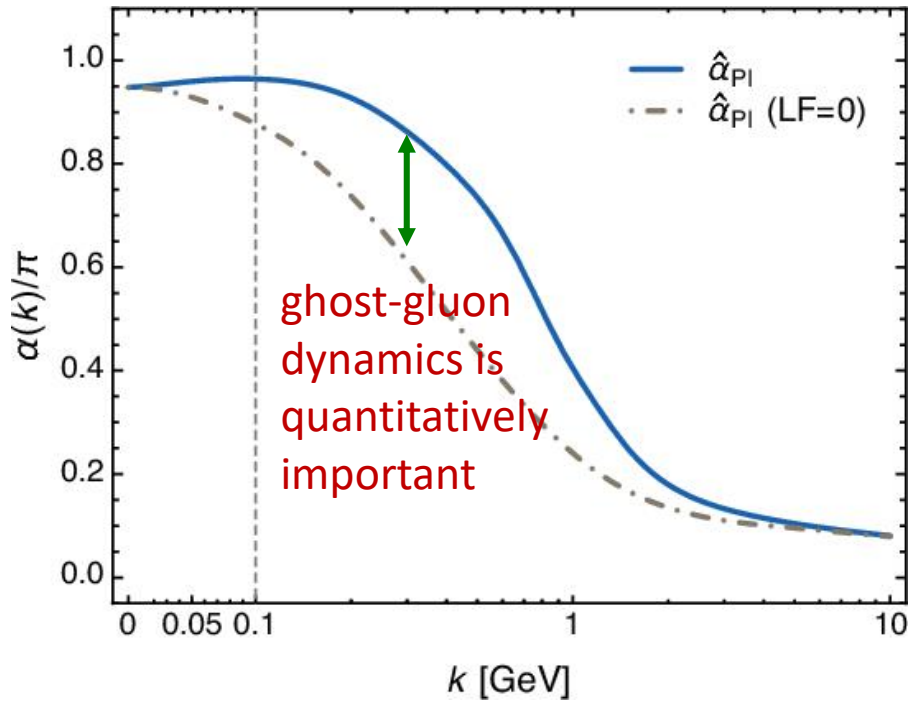
QCD Running Coupling

- Four individual, apparently UV-divergent interaction vertices in perturbative QCD ⇒ possibly four distinct IR couplings.
  - Naturally, if nonperturbatively there are two or more couplings, they must all become equivalent on the perturbative domain.
- Questions:
  - How many distinct running couplings exist in nonperturbative QCD?
  - How can they be computed?
- ***Claim*:**

  Nonperturbatively, too,

  QCD possesses a unique running coupling.
- Alternative
  - Possibly an essentially different RGI intrinsic mass-scale for each coupling
  - Then BRST symmetry irreparably broken by nonperturbative dynamics
  - Conclusion: QCD non-renormalisable owing to IR dynamics.
- No empirical evidence to support such a conclusion: QCD does seem to be a well-defined theory at all momentum scales, possibly owing to dynamical generation of gluon and quark masses, which are large at IR momenta.

Process-independent QCD Effective Charge



- Parameter-free prediction: curve is completely determined by results obtained for gluon and ghost two-point functions using continuum and lattice-regularised QCD.
- Physical, in the sense that there is no Landau pole, and saturates in the IR: $\hat{\alpha}(0) \approx 0.9\pi$, *i.e.* the coupling possesses an infrared fixed point
- Prediction is equally sound at all spacelike momenta, connecting the IR and UV domains
  - no need for *ad hoc* "matching procedure," such as that employed in models
- Essentially nonperturbative: combination of self-consistent solutions of gauge-sector gap equations and lattice simulations

Process-dependent (emergent) Effective Charge

- [Grunberg:1982fw]: process-dependent procedure

$$\int_0^{1^-} dx_{Bj}\left(g_1^p\left(x_{Bj},Q^2\right)-g_1^n\left(x_{Bj},Q^2\right)\right)\equiv\frac{g_A}{6}\left[1-\frac{\alpha_{g_1}\left(Q^2\right)}{\pi}\right]$$

- an effective running coupling defined to be completely fixed by leading-order term in the perturbative expansion of a given observable in terms of the canonical running coupling.
  - Obvious difficulty/drawback = process-dependence itself.
  - Effective charges from different observables can in principle be algebraically connected to each other via an expansion of one coupling in terms of the other.
  - But, any such expansion contains infinitely many terms; and connection doesn't provide a given process-dependent charge with ability to predict another observable, since the expansion is only defined after both effective charges are independently constructed.

Process-dependent Effective Charge

S.J. Brodsky, H.J. Lu, Phys. Rev. D 51 (1995) 3652
S.J. Brodsky, G.T. Gabadadze, A.L. Kataev, H.J. Lu, Phys. Lett. B 372 (1996) 133
A. Deur, V. Burkert, Jian-Ping Chen, Phys.Lett. B 650 (2007) 244-248

- α_{g1} – Bjorken sum rule

$$\int_0^{1^-} dx_{Bj}\left(g_1^p\left(x_{Bj},Q^2\right)-g_1^n\left(x_{Bj},Q^2\right)\right)\equiv\frac{g_A}{6}\left[1-\frac{\alpha_{g_1}\left(Q^2\right)}{\pi}\right]$$

$g_1^{p,n}$ are spin-dependent proton and neutron structure functions

g_A is the nucleon flavour-singlet axial-charge

- Merits, *e.g.*
  - Existence of data for a wide range of k^2
  - Tight sum-rules constraints on the behaviour of the integral at the IR and UV extremes of k^2
  - isospin non-singlet ⇒ suppression of contributions from numerous processes that are hard to compute and hence might muddy interpretation of the integral in terms of an effective charge
    - Δ resonance
    - Disconnected (gluon mediated) diagrams

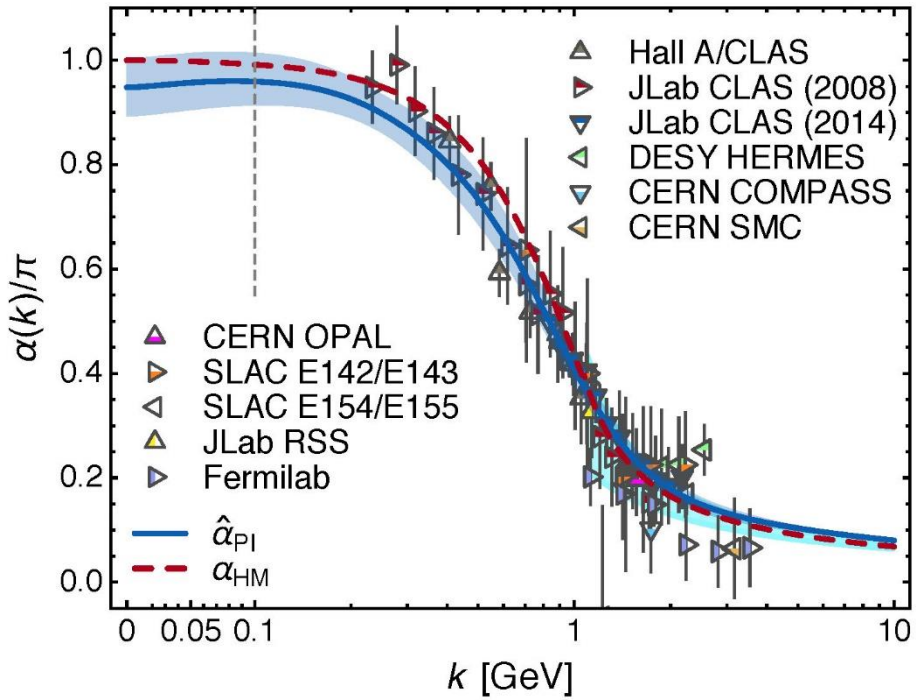
QCD Effective Charge

- Near precise agreement between process-independent $\hat{\alpha}_{PI}$ and α_{g1}
- Perturbative domain:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.14\,\alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$
$$\widehat{\alpha}_{\rm PI}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\,\alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$

Just 4% difference

- Parameter-free prediction:
  - curve completely determined by results obtained for gluon and ghost two-point functions using continuum and lattice-regularised QCD.

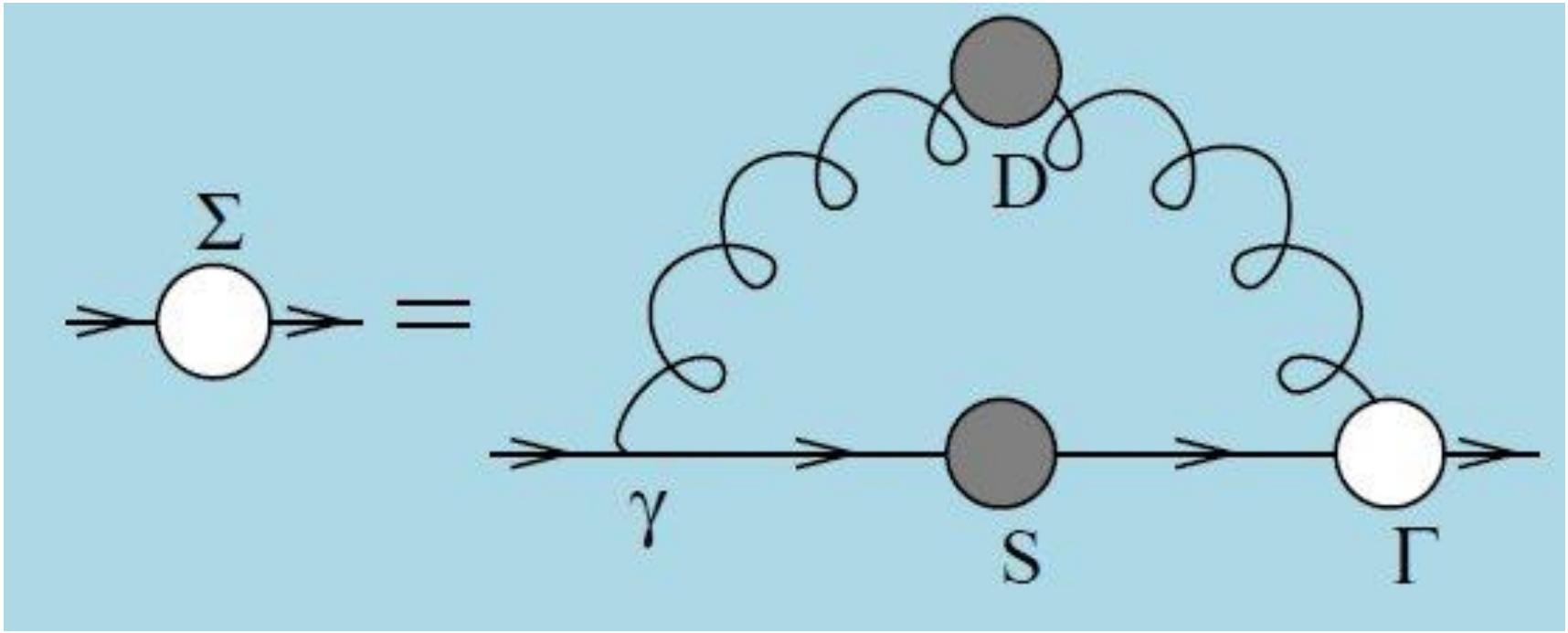


- Ghost-gluon scattering contributions are critical for agreement between the two couplings at intermediate momenta … omit them, and disagreement by factor of ~ 2 at intermediate momenta

QCD Effective Charge

- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, being completely determined by the gauge-boson two-point function.
- Prediction for $\hat{\alpha}_{PI}$ is parameter-free
  - combines completely self-consistent solution of a set of Dyson-Schwinger equations with results from lattice-QCD
- Prediction for $\hat{\alpha}_{PI}$ smoothly unifies the nonperturbative and perturbative domains of the strong-interaction theory.
- $\hat{\alpha}_{PI}$ is process-independent and known to unify a vast array of observables
- Existence of infrared-stable fixed-point in QCD supports efforts to use, *e.g*. extended technicolour scenarios for extension of Standard Model

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

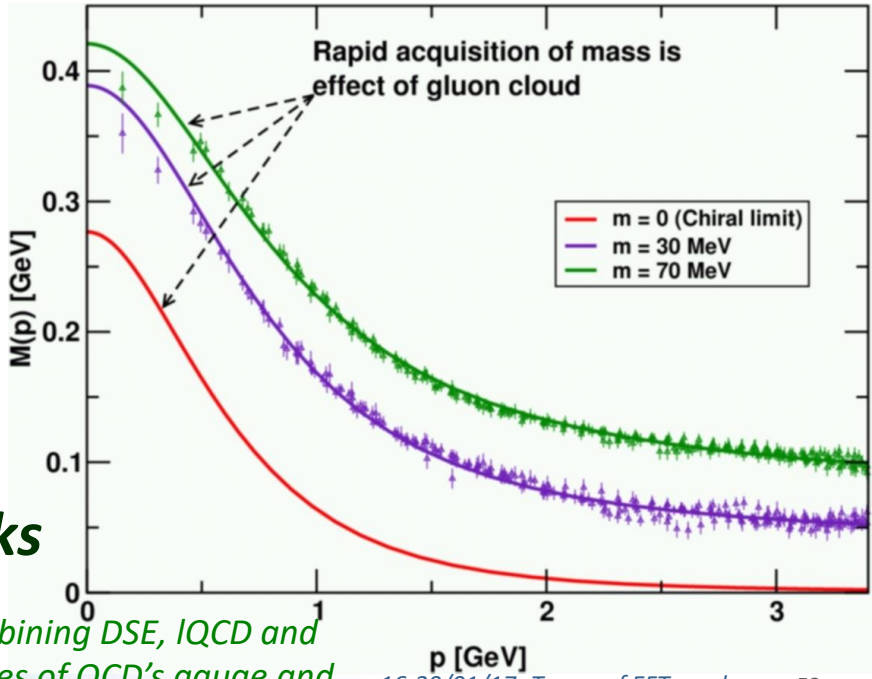


Quark Gap Equation

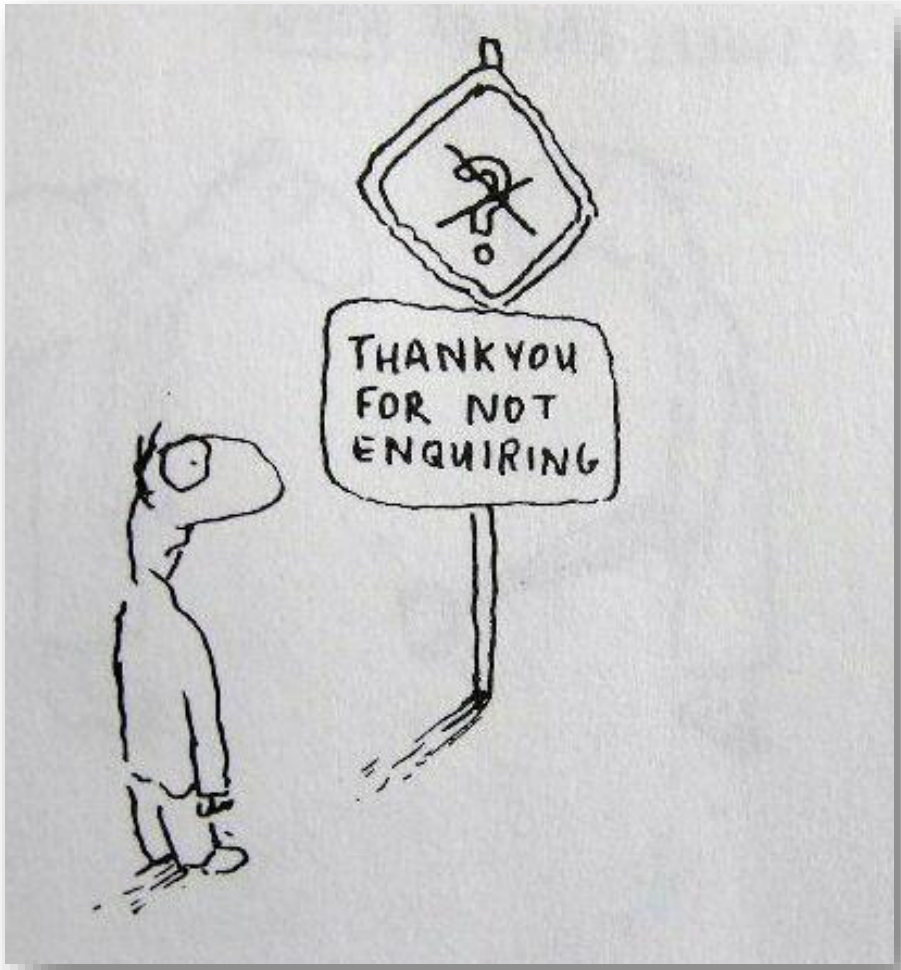
DCSB

- Dynamical chiral symmetry breaking (DCSB) is a critical emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that DCSB is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.
- ***Dynamical***, not spontaneous
  - Add nothing to *QCD* ,

    No Higgs field, nothing!
    Effect achieved purely through quark+gluon dynamics.
  - ✓ ***Trace anomaly: massless quarks become massive***



Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge and matter sectors



Enigma of Mass

Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

Pion exists if, and only if, mass is dynamically generated

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

This is why $m_\pi = 0$ in the absence of a Higgs mechanism

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

Keystone that supports the success of chiral effective field theories

$f_\pi E_\pi(p^2) = B(p^2)$

Enigma of mass

This algebraic identity is why QCD's pion is massless in the chiral limit

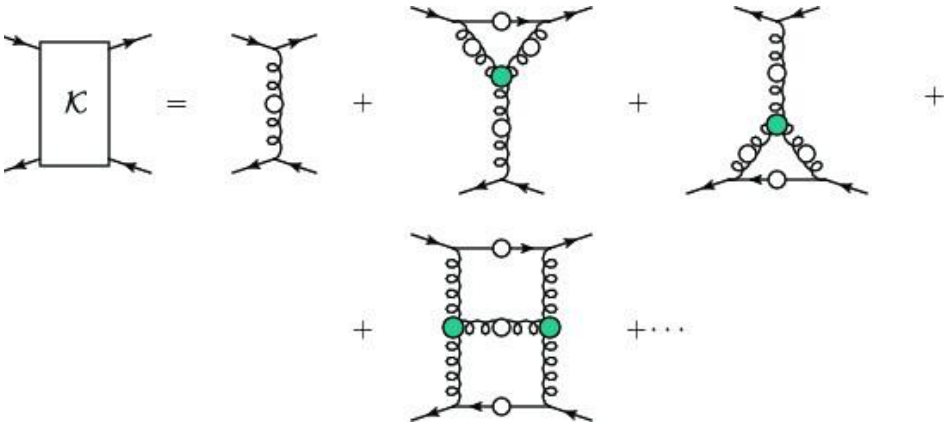
- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

Pion masslessness

- Renormalisation scale: ζ = 2GeV =: ζ_2
- Pion's Poincaré-invariant mass and Poincaré-covariant wave function are obtained by solving a Bethe-Salpeter equation.
- This is a scattering problem
- In chiral limit
  - two massless fermions interact via exchange of massless gluons
    … initial system is massless;
    … and it remains massless at every order in perturbation theory
- But, complete the calculation using an enumerable infinity of dressings and scatterings



Munczek, H. J., Phys. Rev. D 52 (1995) pp. 4736-4740
Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B 380 (1996) pp. 7-12
Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B 420 (1998) pp. 267-273

Pion masslessness

- Produces a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these "basis vectors"
  - Chiral limit:
    - Algebraic proof that, at any finite order in a symmetry-preserving construction of the kernels for the gap (quark dressing) and Bethe-Salpeter (bound-state) equations, there is a precise cancellation between the mass-generating effect of dressing the valence-quarks and the attraction introduced by the scattering events
    - Cancellation guarantees that the simple system, which began massless, becomes a complex system, with a nontrivial bound-state wave function attached to a pole in the scattering matrix, which remains at $P^2=0$ … remains massless
- Quantum field theory statement: in the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$

Pion masslessness

DCSB Paradigm

$$\langle \pi(q)|\theta_0|\pi(q)\rangle \overset{\zeta\gg\zeta_2}{=} \langle \pi(q)|\tfrac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}|\pi(q)\rangle$$

$$\overset{\zeta\simeq\zeta_2}{\to} \langle \tilde{\pi}(q)| \sum_{f=u,d} M_f(\zeta)\,\bar{\mathcal{Q}}_f(\zeta)\mathcal{Q}_f(\zeta) + \tfrac{1}{4}[\beta(\alpha(\zeta))\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu}]_{2\mathrm{PI}}|\tilde{\pi}(q)\rangle$$

- Parton-basis chiral-limit expression of the expectation-value of the trace-anomaly in the pion at $\zeta \gg \zeta_2$
- Metamorphoses into a new expression, written in terms of a nonperturbatively-dressed quasi-particle basis and associated, evolved wave functions
  - 1st term = *positive* = one-body dressing content of the trace anomaly … Plainly, a massless valence-quark acquiring a large mass through interactions with its own gluon field is an expression of the trace-anomaly in the one-body subsector of the complete pion wave function
  - 2nd term = *negative* (attraction) = 2-particle-irreducible scattering event content of the scale-anomaly … Plainly, acquires a scale because the couplings, and the gluon- and quark-propagators in the 2PI processes have all acquired a mass-scale
- Away from the chiral limit, and in other channels, the cancellation is incomplete.

Observing Mass

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- 2012 … methods were developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k)\Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with the DCSB-improved DSE kernel, which unifies matter & gauge sectors

$$\varphi_\pi(x) \propto x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5$$

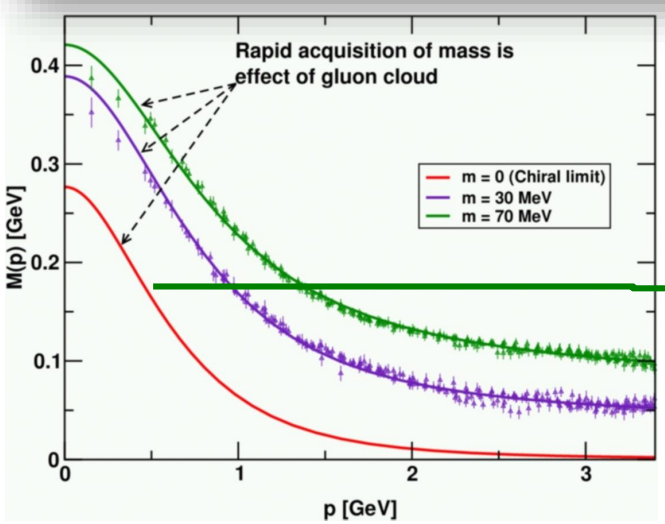
Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*, arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

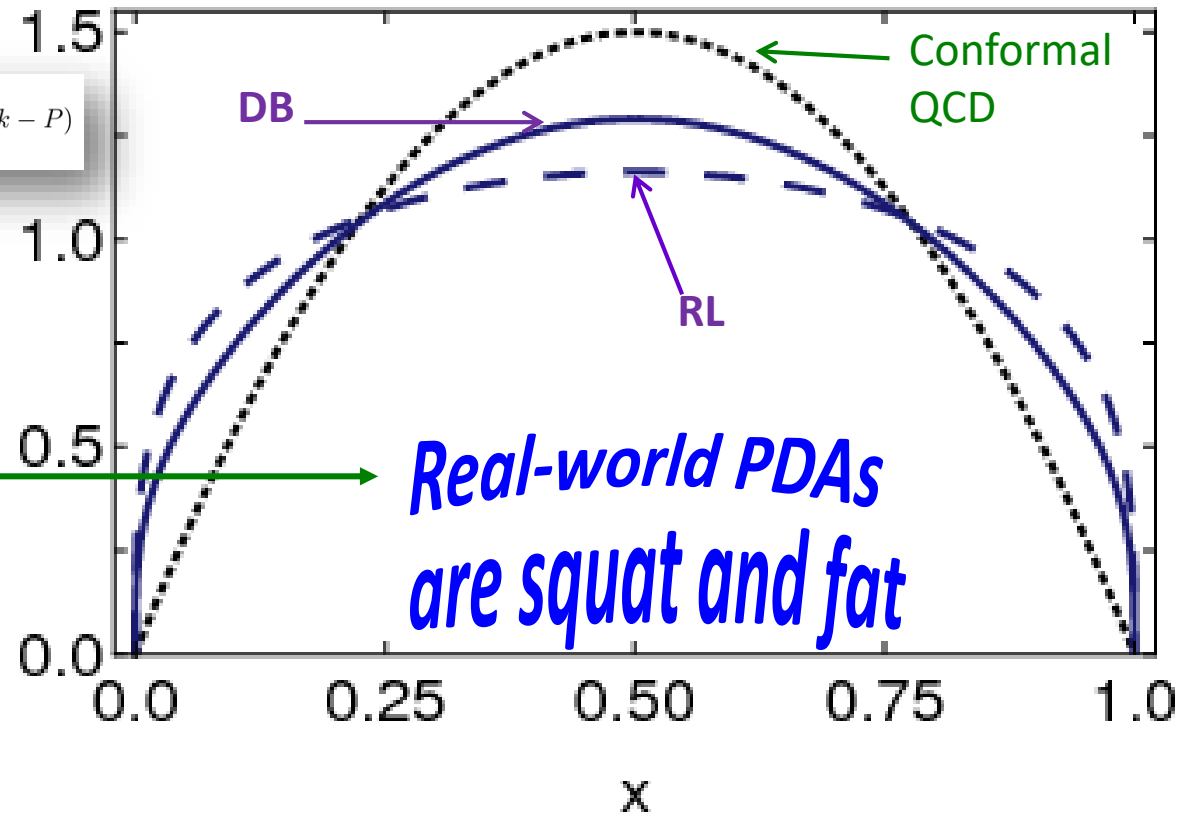
Pion's valence-quark Distribution Amplitude

- Continuum-QCD prediction:

  marked broadening of $\varphi_\pi(x)$, which owes to DCSB

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$



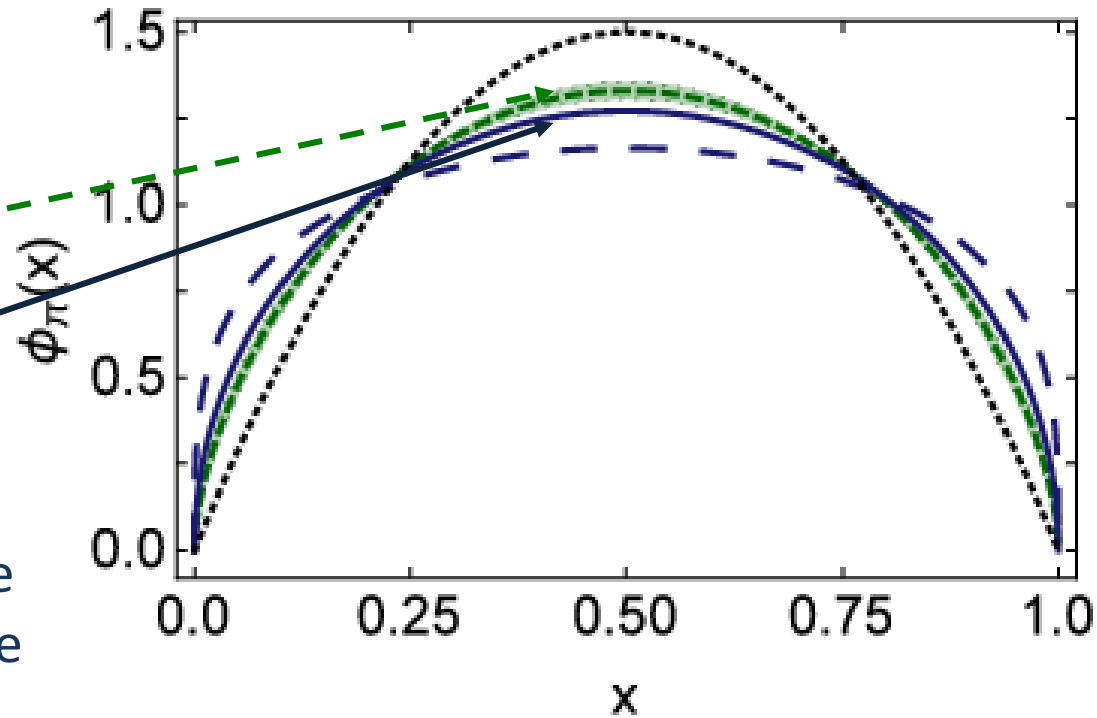


Pion distribution amplitude from lattice-QCD
I. C. Cloët *et al., arXiv:1306.2645 [nucl-th], Phys. Rev. Lett. 111 (2013) 092001 [5 pages]*

Flavour symmetry breaking in the kaon parton distribution amplitude, Chao Shi *et al., arXiv:1406:3353 [nucl-th], Phys. Lett. B 738 (2014) pp. 512–518*

Lattice-QCD & Pion's valence-quark PDA

- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Blue solid curve = DSE prediction obtained with DB kernel
- DSE & lQCD predictions are practically indistinguishable



Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$

A: Internally-consistent DSE prediction

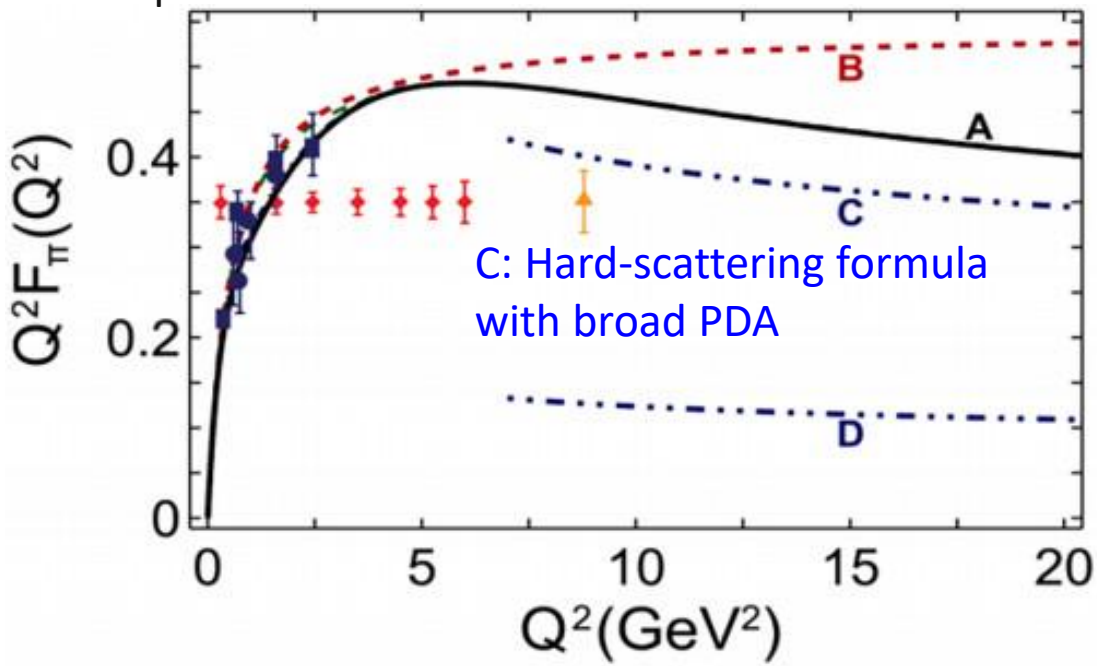
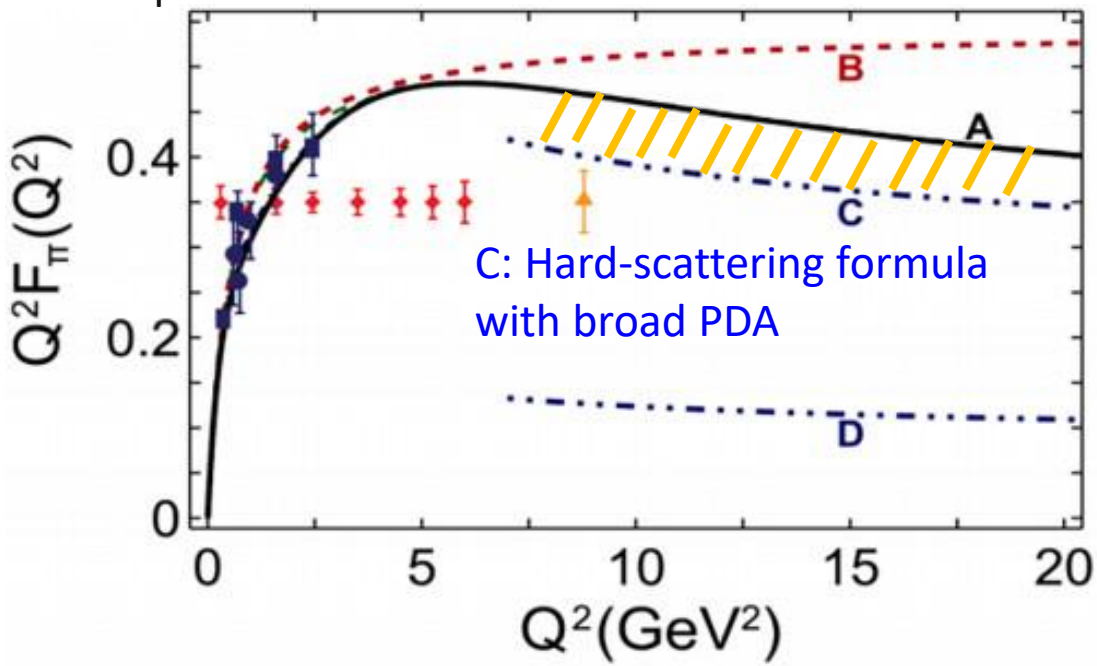


Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Appears that JLab12 is within reach of first verification of a QCD hard-scattering formula

A: Internally-consistent DSE prediction



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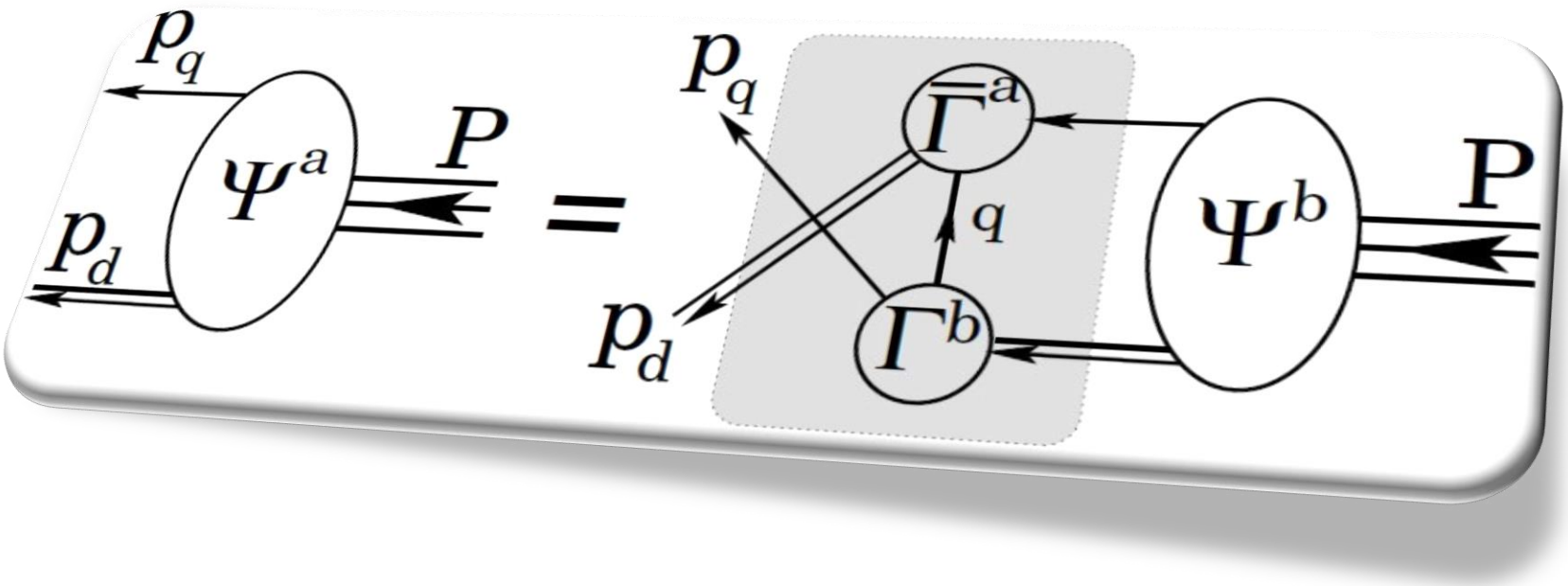
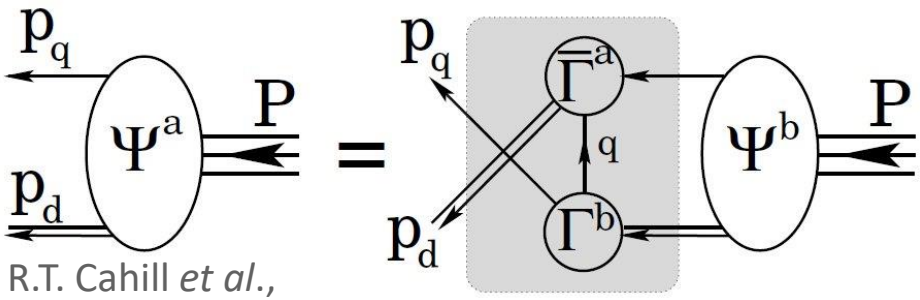


Structure of Baryons



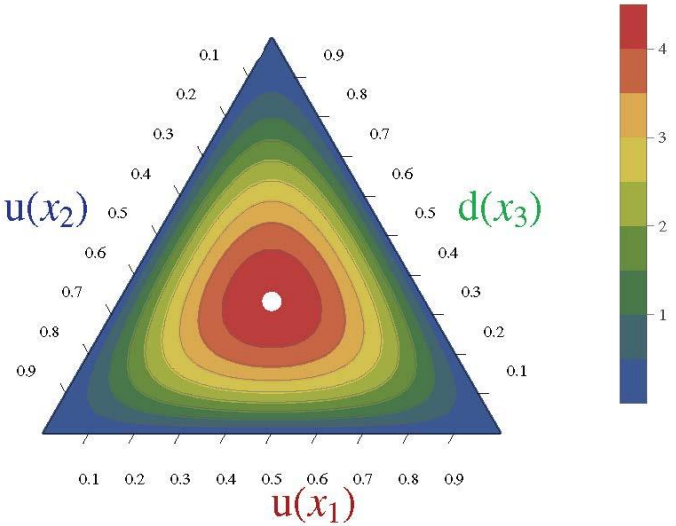
R.T. Cahill *et al.*, Austral. J. Phys. 42 (1989) 129-145

Baryon Structure

- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Confinement and DCSB are readily expressed
- ***Prediction***: owing to *DCSB in QCD, strong diquark correlations exist within baryons*
- Diquark correlations are not pointlike
  - Typically, $r_{0+} \sim r_\pi$ & $r_{1+} \sim r_\rho$ (actually 10% larger)
  - They have soft form factors

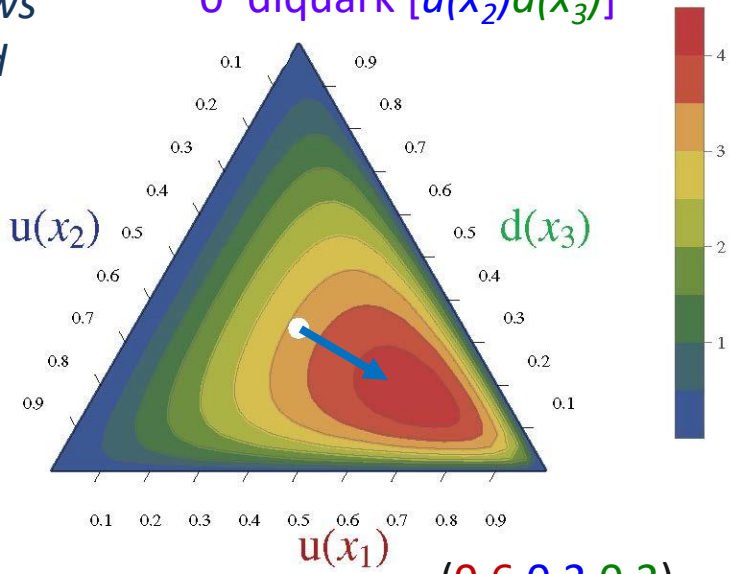
Nucleon Parton Distribution Amplitudes

- Computations underway. First results available.



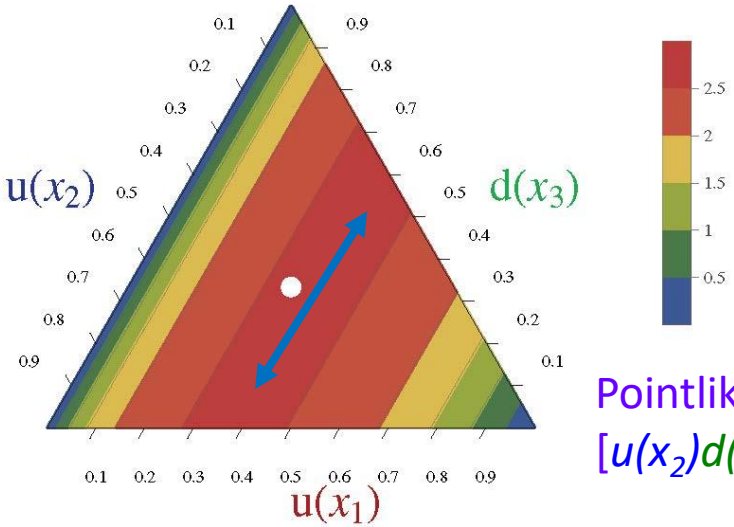
Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum

Realistic, finite size (0.7fm)
0^+ diquark [$u(x_2)d(x_3)$]



(0.6,0.2,0.2)

conformal limit:
120 $x_1x_2x_3$
$\langle x_i \rangle = ⅓$ … peak of the distribution



Pointlike 0^+ diquark
[$u(x_2)d(x_3)$]

Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD
V. M. Braun *et al.*, Phys. Rev. D 89 (2014) 094511

Light-cone distribution amplitudes of the baryon octet
G. S. Bali *et al.* JHEP 1602 (2016) 070

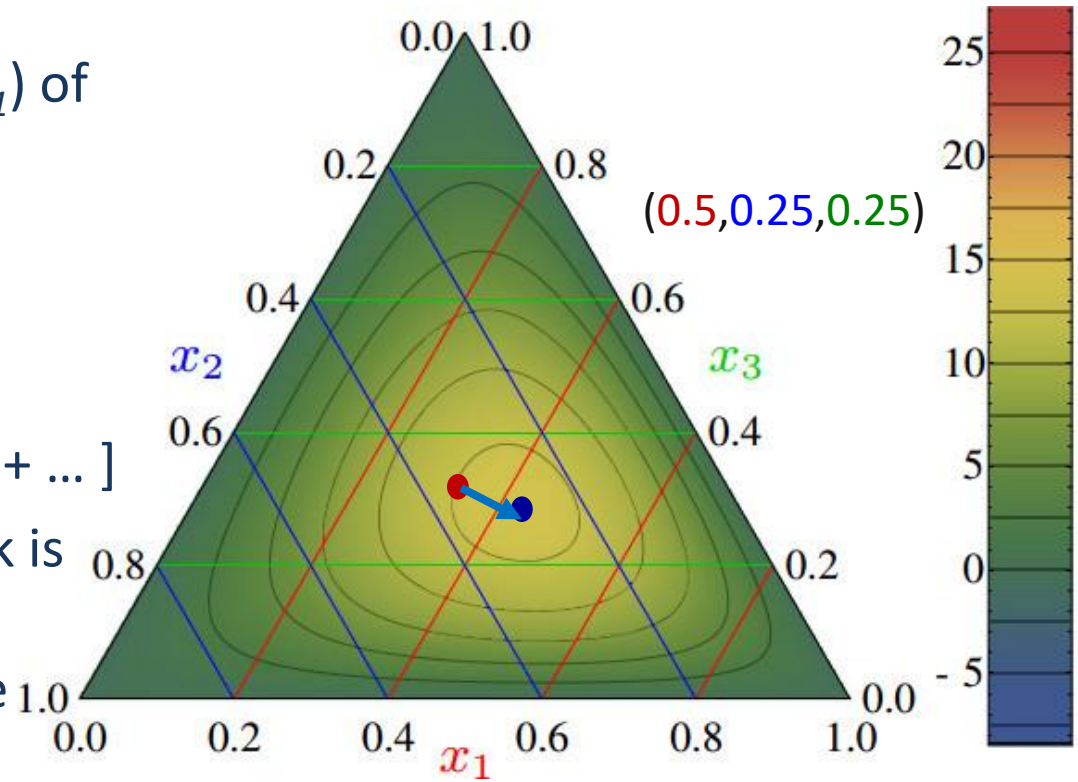
Nucleon PDAs & lQCD

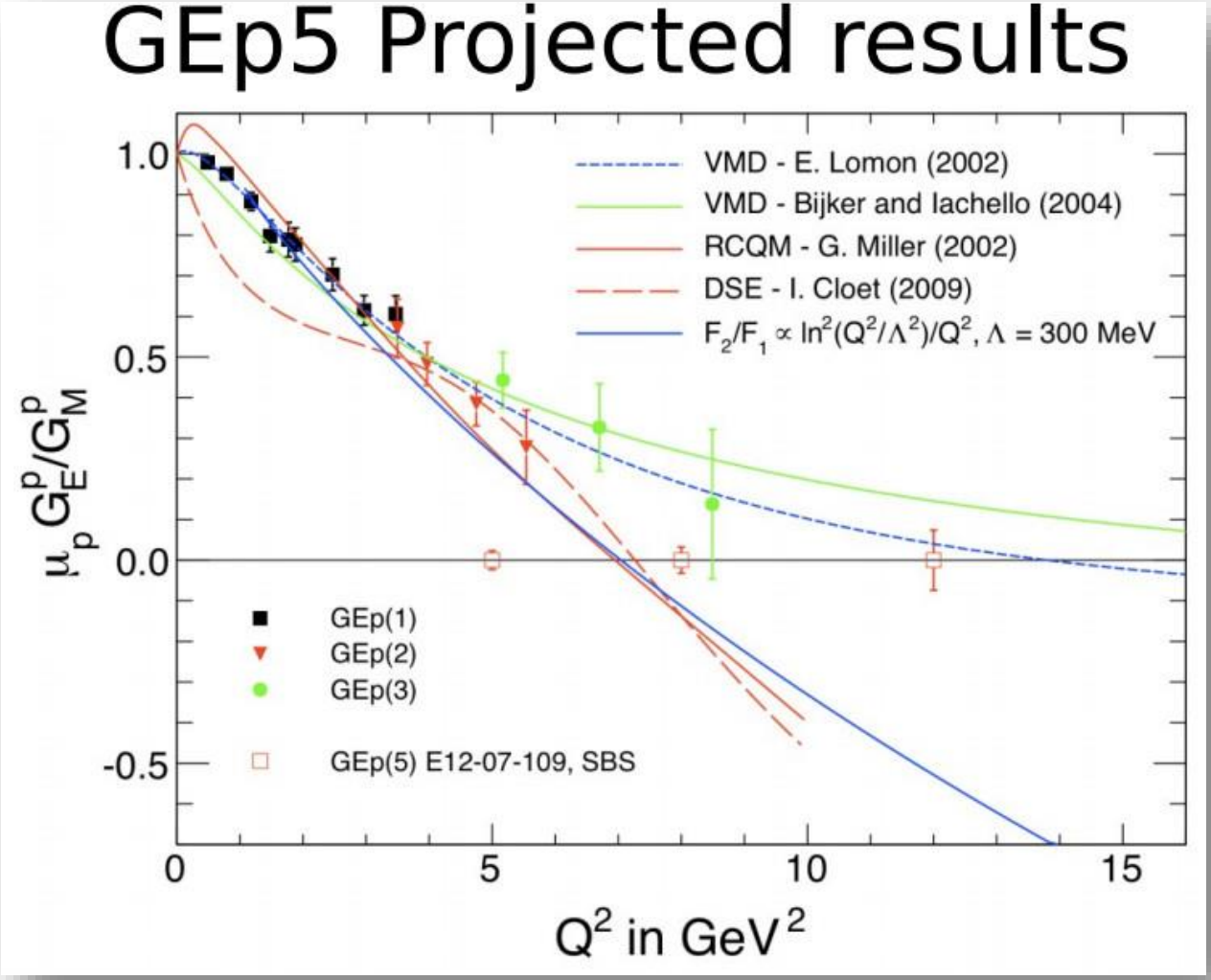
- First lQCD results for n=0, 1 moments of the leading twist PDA of the nucleon are available
- Used to constrain strength (a_{11}) of the leading-order term in a conformal expansion of the nucleon's PDA:

$\Phi(x_1,x_2,x_3)$

$= 120\, x_1\, x_2\, x_3\, [\, 1 + a_{11}\, P_{11}(x_1,x_2,x_3) + \ldots \,]$

- Shift in location of central peak is consistent with existence of diquark correlations within the nucleon

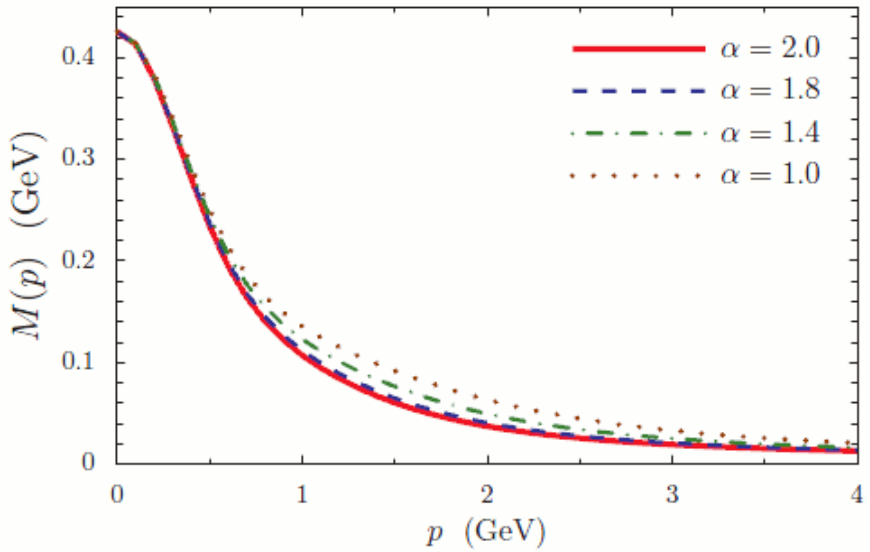




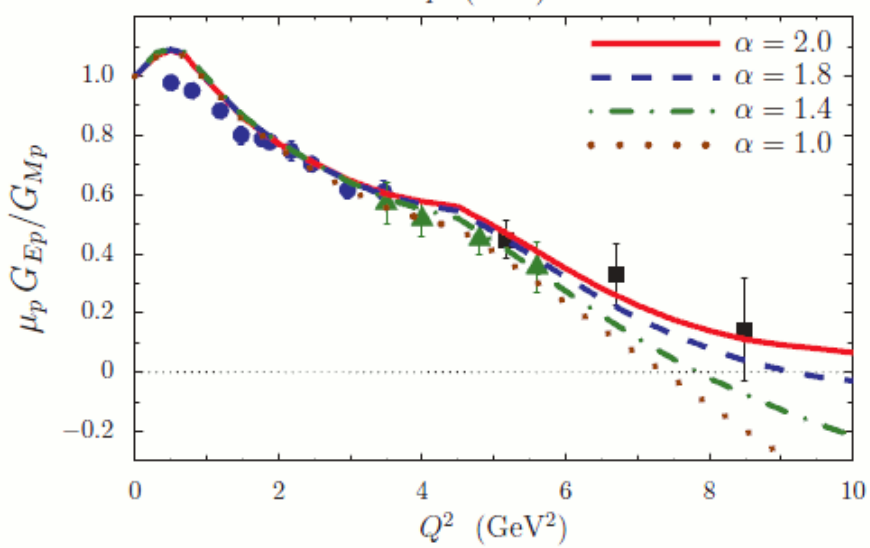
Nucleon Elastic FFs

I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution, arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Visible Impacts of DCSB



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



- Possible existence and location of a zero in the ratio of proton elastic form factors

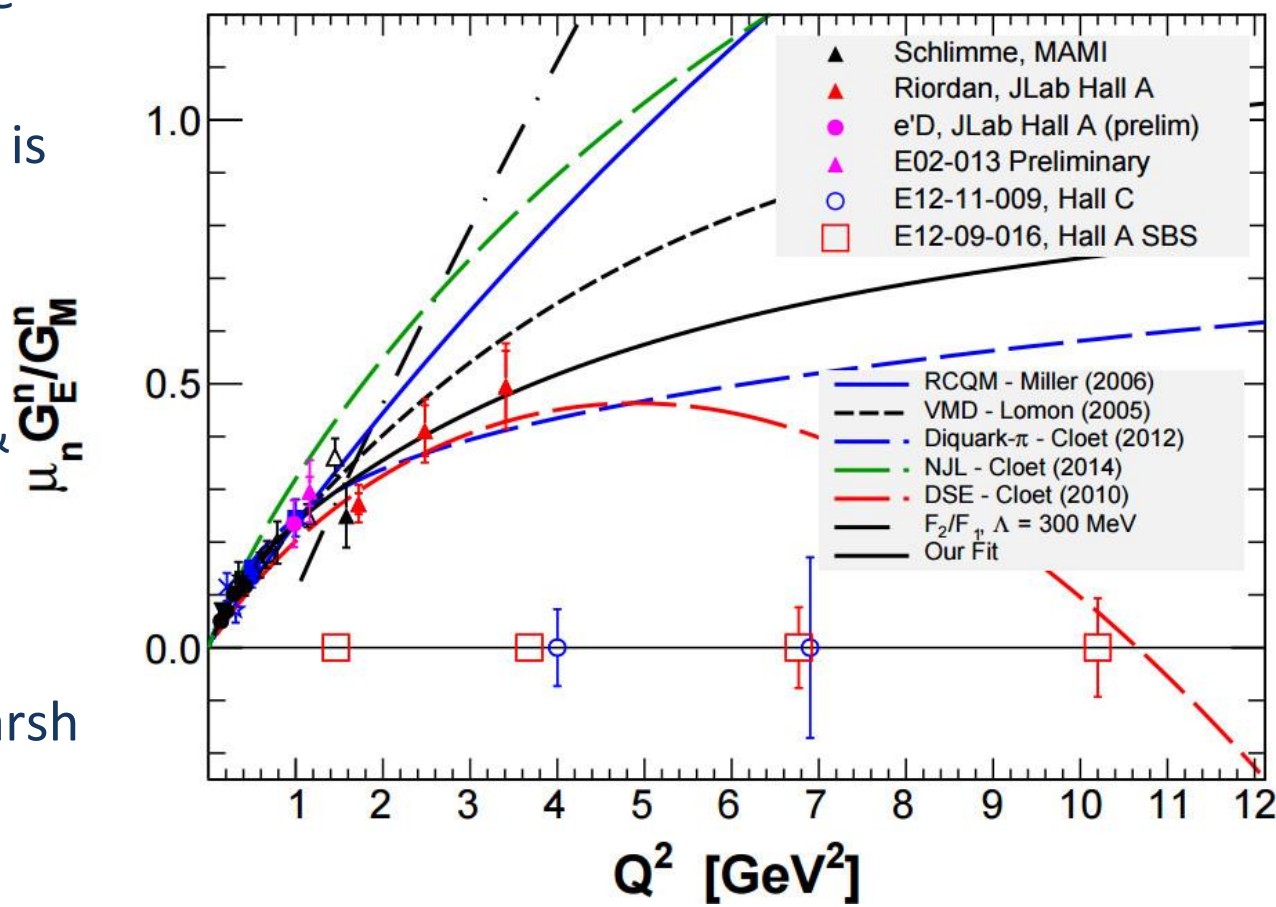
  $[\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)]$

  are a direct measure of the rate at which dressed-quarks become partons again,

  i.e. character of strong interactions in the Standard Model.

G_{En}

- Numerous calculations on this figure; but only one viable prediction
- DSE result (2008/2010) is not fitted to any data
  - Predicts zero in G_{En}
  - Owes to presence of running quark-mass & strong diquark correlations
  - Verifiable at JLab12
- G_{En} promises to be a harsh discriminator between descriptions of nucleon structure



Seamus Riordan, ECT, April 2016*

Epilogue

Epilogue

- Emergence:
  - Confinement and dynamical chiral symmetry breaking in the Standard Model
    - Are they related?
    - Are they the same?
      - Role of the pion seems to be key in answering these questions
  - Conformal anomaly
    - Can have neither confinement nor DCSB if scale invariance of (classical) chromodynamics is not broken by quantisation
    - Know a mass-scale must exist, but only experience/experiment informs us of its value
    - Once size known, continuum and lattice-regularised *quantum* chromodynamics ⇒ *gluons and quarks acquire momentum-dependent masses*
      - Values are large in the infrared $m_g \propto 500$ MeV ≈ $m_p/2$ & $M_q \propto 350$ MeV ≈ $m_p/3$
      - Seem to be the foundation for DCSB
      - And can be argued to explain confinement as a dynamical phenomenon, tied to fragmentation functions

Epilogue

- Reductive explanation
  - Fundamental equivalence of the one- and two-body problems in the matter-sector
    - Quark gap equation ≡ Pseudoscalar meson Bethe-Salpeter equation
  - Entails that properties of the pion – Nature's lightest observable strong-interaction excitation – are the cleanest means by which to probe the origin and manifestations of mass in the Standard Model
  - Numerous predictions that can be tested at contemporary and planned facilities
    - JLab 12GeV
    - EIC
  - Refining those predictions *before experiments begin* will require combination of all existing nonperturbative approaches to strong interaction dynamics in the Standard Model

Epilogue

- Reductive explanation
  - Fundamental ms in the matter
    - Quark ga
  - Entails th ble strong- ich to probe Mod
  - Nun nd pla
    -
    -
  - Refining those predictions by equire combination of all existing nonperturbative approaches to strong interaction dynamics in the Standard Model

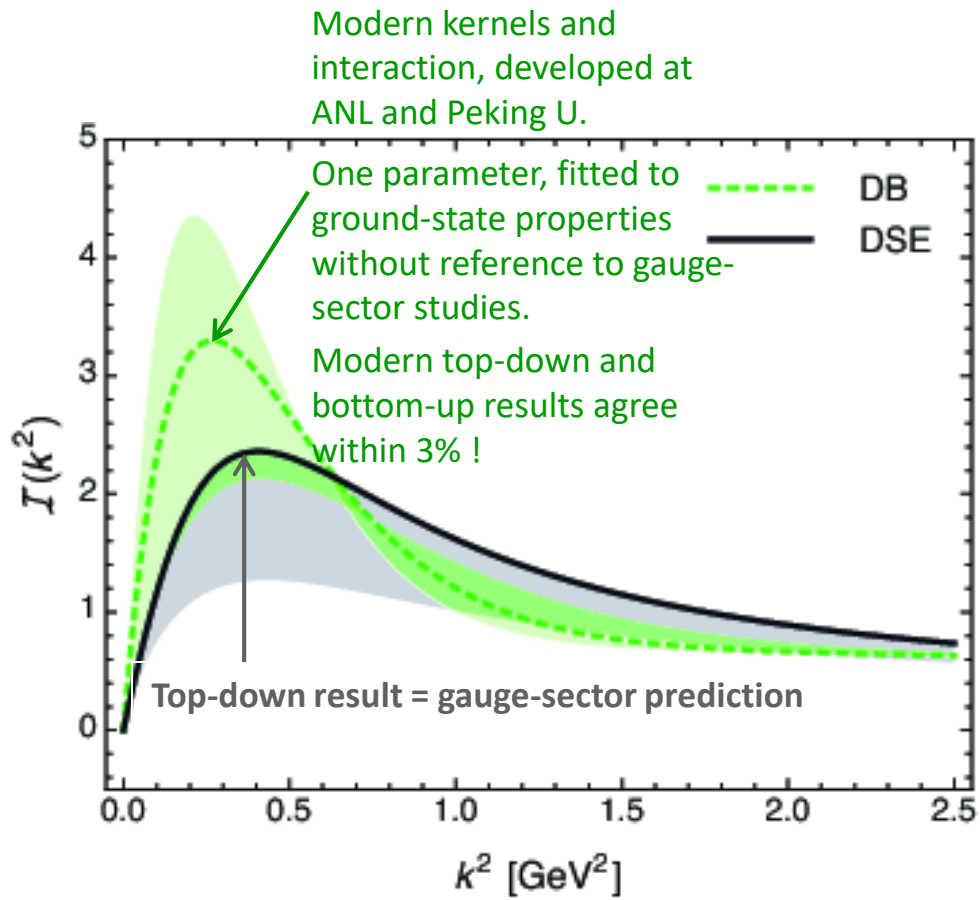
Continuum-QCD & *ab initio* predictions

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

Top down & Bottom up

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain), C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

- *Top-down approach* – ab initio *computation of the interaction via direct analysis of the gauge-sector gap equations*
- *Bottom-up scheme – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *Serendipitous collaboration, conceived at one-week ECT* Workshop on* DSEs in Mathematics and Physics, *has united these two approaches*



– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using a sophisticated matter-sector DSE truncation

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

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- *Bottom-up scheme – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *Top-down*
  computat
  direct ana
  equations
- *Serendipit*
  at one-we
  in Mathematics and Physics, *has united these two approaches*

Modern kernels and interaction, developed at ANL and Peking U.

One parameter, fitted to ground-state properties without reference to gauge-sector studies.

Modern top-down and
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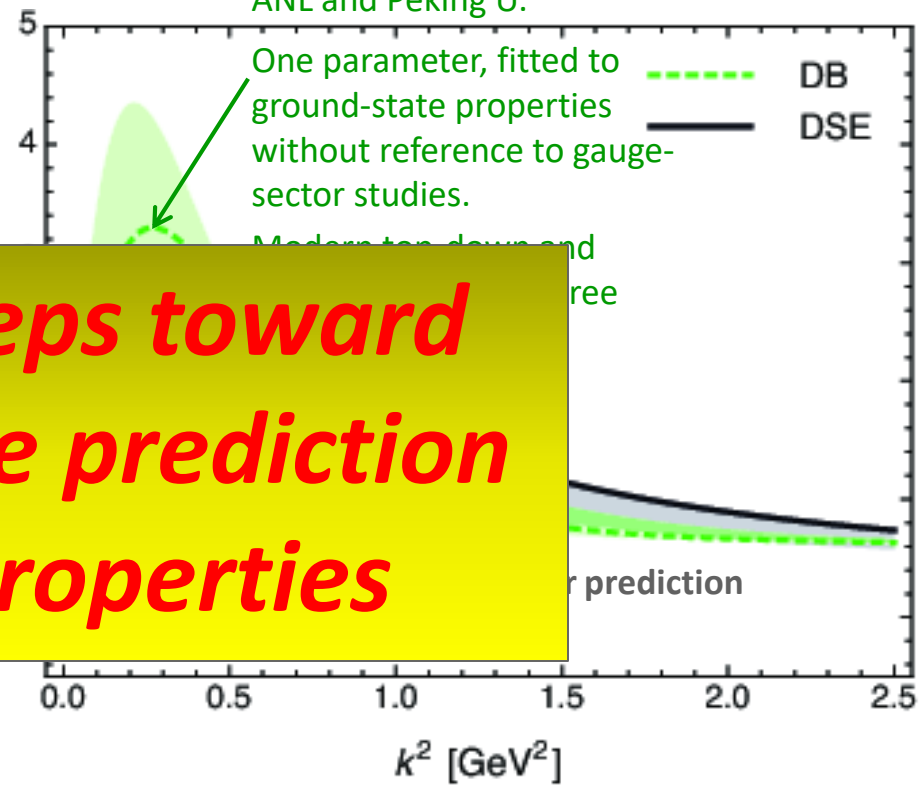


Significant steps toward parameter-free prediction of hadron properties

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using a sophisticated matter-sector DSE truncation

Natural constraints on the gluon-quark vertex,
Binosi, Chang, Papavassiliou, Qin & Roberts,
nearing completion

Reconciliation demands dressed-gluon-quark vertex

- Significant progress since 2009:
  dressed Γ_μ in gap- and Bethe-Salpeter equations …
- In principle, ∃ unique form of Γ_μ , but it's still obscure.
- To improve this situation, used the top-down/bottom-up RGI running-interaction
  - Computed gap equation solutions with
    1,660,000 distinct *Ansätze* for Γ_μ
- Each one of the solutions tested for compatibility with three physical criteria
- Remarkably, merely 0.55% of solutions survive the test

⇒ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*

Bashir, Bermudez, Chang, Roberts, arXiv:1112.4847 [nucl-th], Phys. Rev. C**85** (2012) 045205 [7 pages]

$$\tau_1^{qk} = a_1 \frac{\Delta_B^{qk}}{q^2+k^2} \qquad \tau_3^{qk} = -a_3 2\Delta_A^{qk}, \qquad T_\nu^1 = \frac{i}{2} t_\nu^{\mathrm{T}}, \qquad T_\nu^3 = \gamma_\nu^{\mathrm{T}},$$

$$\tau_4^{qk} = a_4 \frac{4\Delta_B^{qk}}{t^{\mathrm{T}} \cdot t^{\mathrm{T}}}, \quad \tau_5^{qk} = a_5 \Delta_B^{qk}, \qquad T_\nu^4 = -i T_\nu^1 \sigma_{\alpha\beta} q_\alpha k_\beta, \quad T_\nu^5 = \sigma_{\nu\rho} p_\rho,$$

$$\tau_8^{qk} = a_8 \Delta_A^{qk}, \qquad T_\nu^8 = q_\nu \gamma \cdot k - k_\nu \gamma \cdot q + i \gamma_\nu \sigma_{\alpha\beta} q_\alpha k_\beta, \qquad \text{(A1)}$$

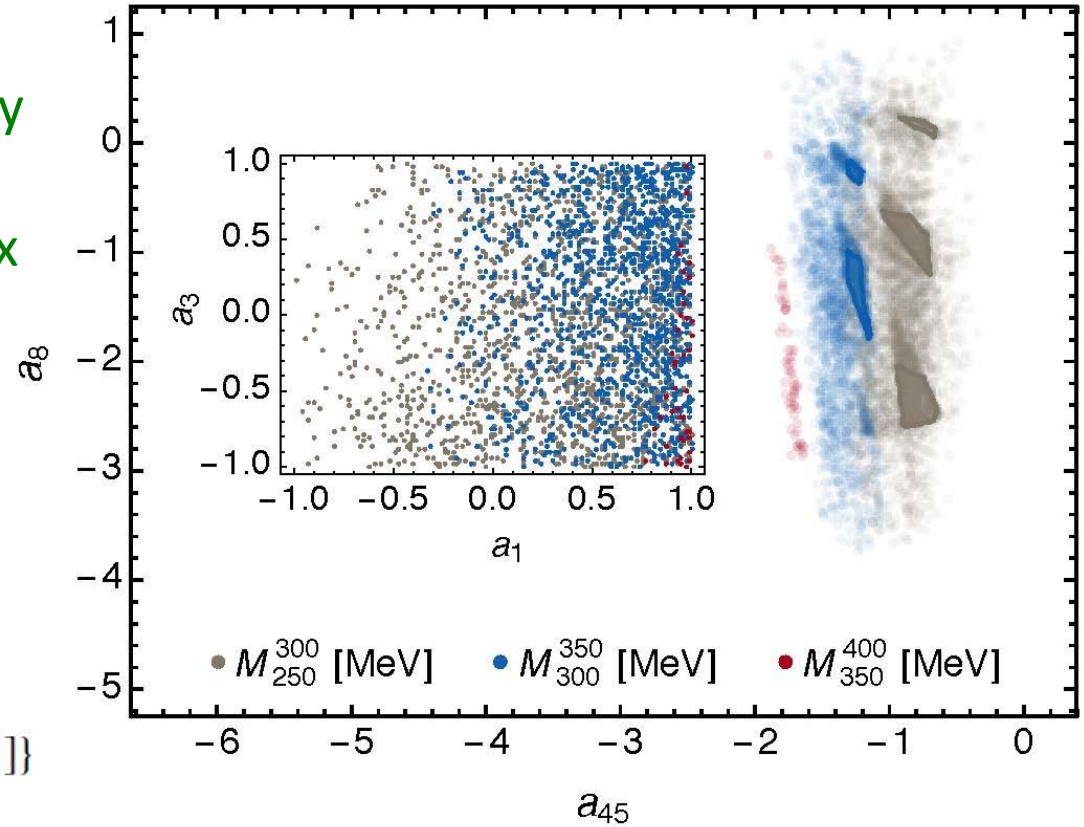
⇒ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*

- Meson spectrum ⇒ $a_{2,6,7} = 0$

  (Sixue Qin *et al.*)

- In $\mathbf{R}^4$ … subset of (almost) zero measure

$$\mathbb{G}_4 \subset \{(a_1, a_3, a_{\hat{45}}, a_8) \,|\, a_1 \in [-0.5, 1], \; a_3 \in [-1, 1], a_{\hat{45}} \in [-2, -0.4], a_8 \in [-4, 1]\}$$



Dressed-gluon-quark vertex

Gap equation only "feels" $a_{45}=a_4-3a_5$