

# PRAGMATIC VS RIGOROUS VIEW ON CHIRAL EFT(-BASED) INTER-NUCLEON INTERACTIONS

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CHRISTIAN FORSSÉN

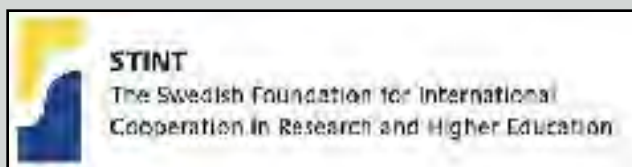
Department of Physics, Chalmers,  
Sweden

ESNT workshop, CEA Saclay,  
France, Jan. 16-20, 2017

# MANY THANKS TO MY COLLABORATORS

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- ❖ **Boris Carlsson, Andreas Ekström**, Dag Strömberg, Oskar Lilja, Mattias Lindby, Björn Mattsson (Chalmers)
  - ❖ Kai Hebel (TU Darmstadt), Kyle Wendt (ORNL, LLNL)
- and
- ❖ Gaute Hagen, Gustav Jansen, Thomas Papenbrock (ORNL/UT), Morten Hjorth Jensen (UiO, MSU), Petr Navrátil (TRIUMF), Witek Nazarewicz (ORNL, MSU)



Research funded by:

- STINT
- European Research Council

**MODERN NUCLEAR PHYSICS**

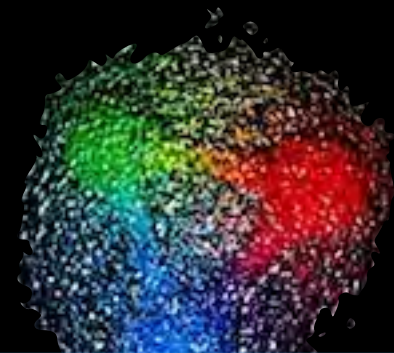
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# **INTRODUCTION**

# PROFOUND INTERSECTIONS

- Can we solve QCD to describe hadronic structures and interactions?
- Can we employ the separation of scales to build successful effective field theories?
- What is the new standard model of particle physics?

subfemto...



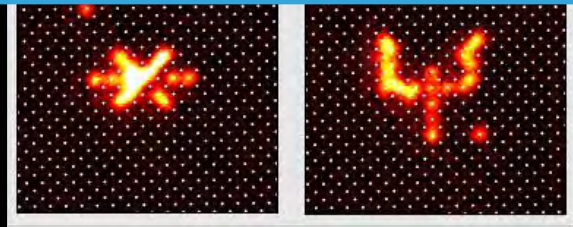
- What controls nuclear saturation?
- What are the properties of nuclei with extreme neutron/proton ratios?
- Can we predict useful cross sections that cannot be measured?
- Can nuclei provide precision tests of fundamental symmetries?

femtophysics



- How do nuclei shape the physical universe?
- What is the origin of the elements?
- What is the interaction between baryonic and dark matter?

ARTICLES



QUANTUM MANY-BODY PHYSICS

PHYSICS OF NUCLEI

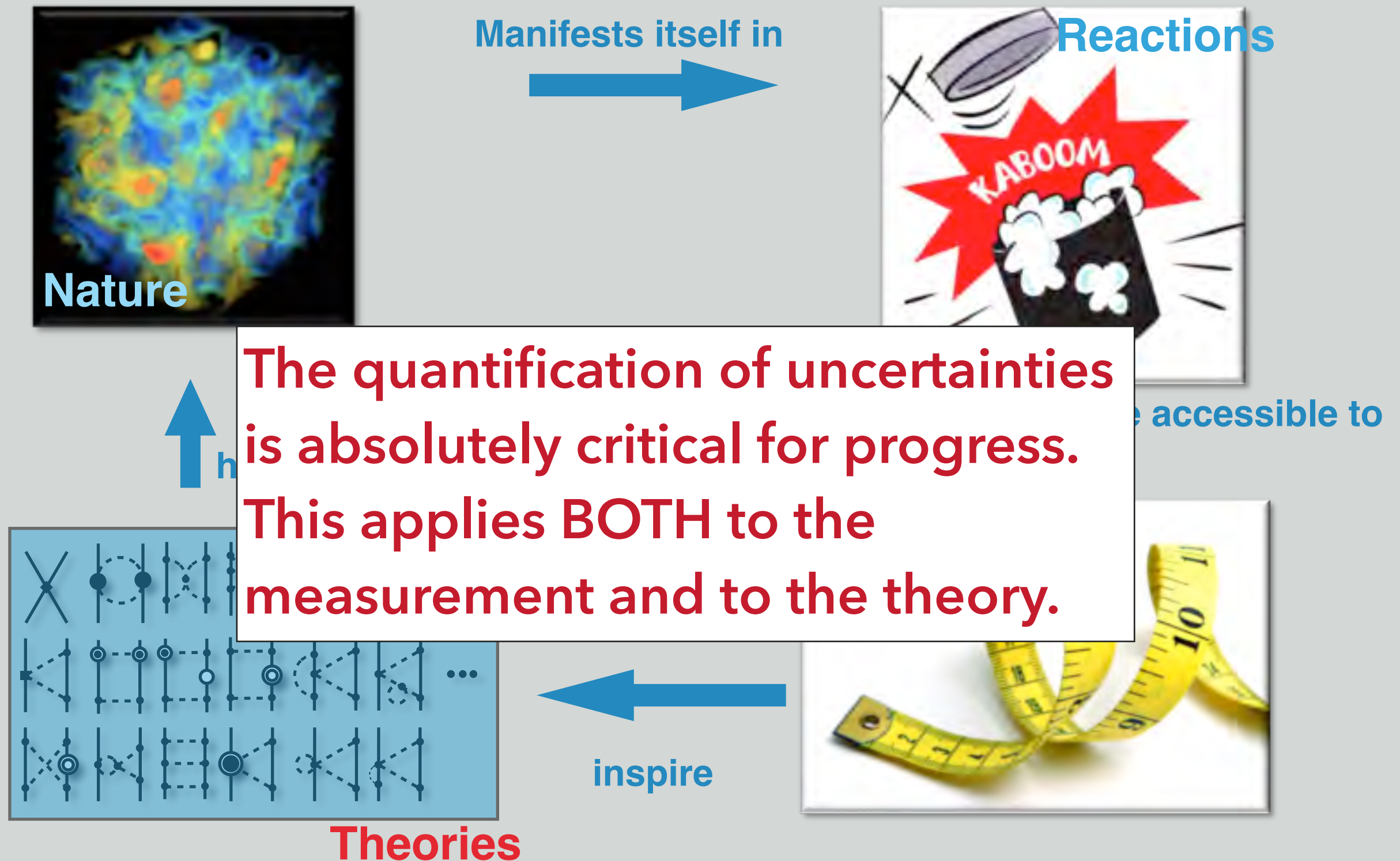
giga...



- How do collective phenomena emerge from simple constituents?
- How can complex systems display astonishing simplicities?
- What are unique properties of open quantum systems?

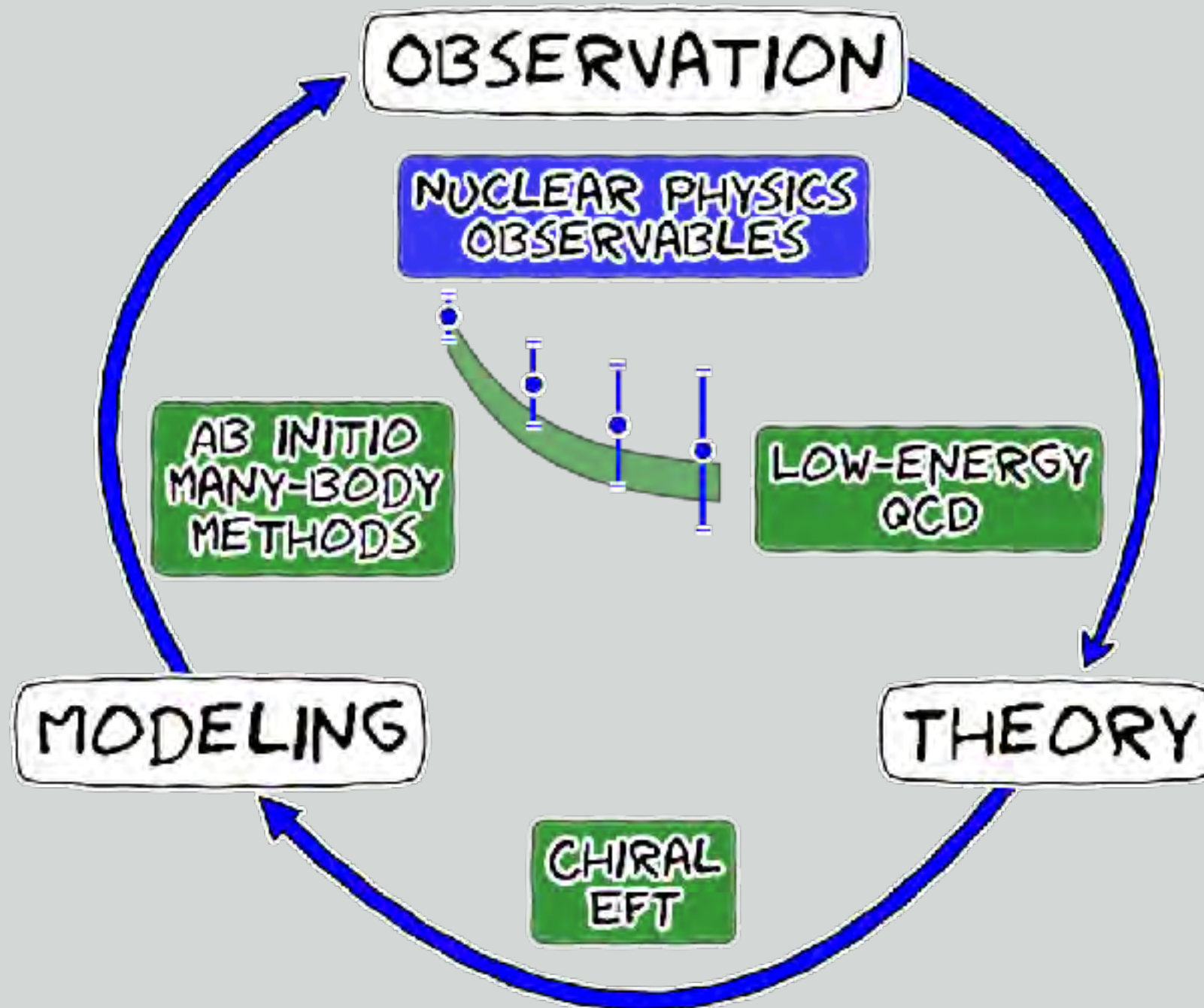
PHYSICS AND COSMOLOGY

# THE SCIENTIFIC METHOD





# THE SCIENTIFIC METHOD: NUCLEAR PHYSICS



An EFT approach offers many nice features;  
No free lunch: there are a number of parameters;  
How do we determine those? Does this EFT deliver?

# CHIRAL EFT BASED NUCLEON-NUCLEON INTERACTIONS

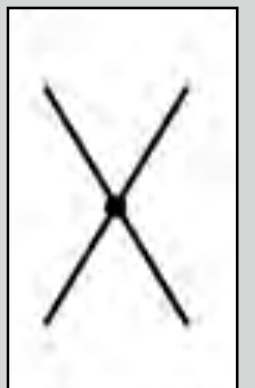
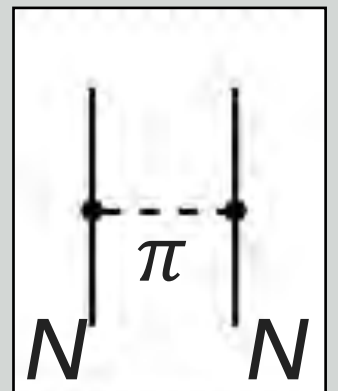
**“BLACK BOX” =  
CHIRAL EFT FOR THE  
MANY-NUCLEON SECTOR**

See work by: Weinberg, van Kolck,  
Epelbaum, Meissner, Krebs, Entem,  
Machleidt...

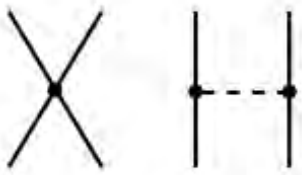


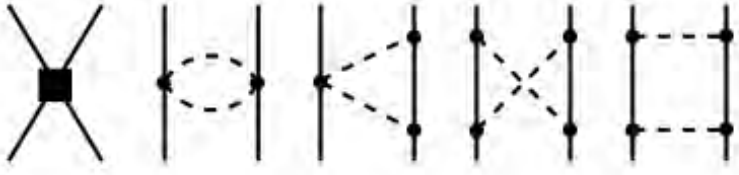


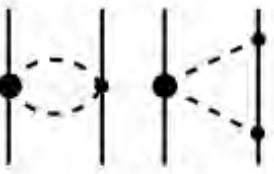
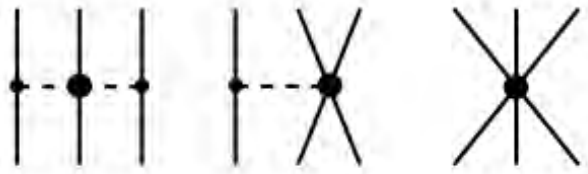

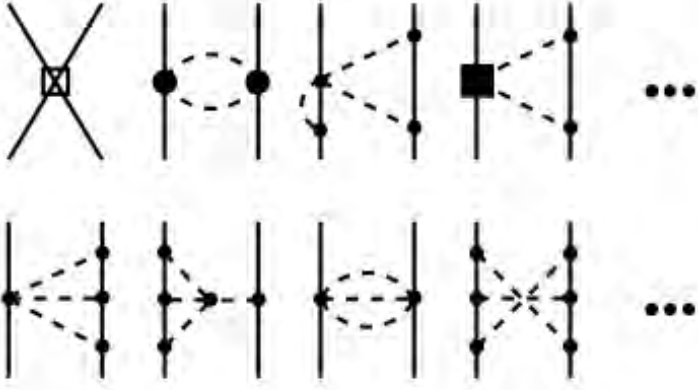
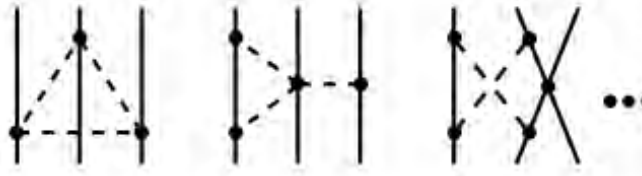
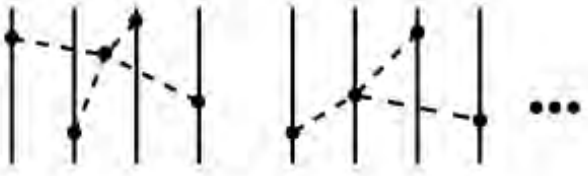


... however, with different  
instructions

- ▶ Separation of scales in nuclear physics.
- ▶ Pions ( $\pi$ ) and nucleons ( $N$ ) as relevant degrees of freedom.
- ▶ One-pion exchange = long-range physics
- ▶ Contact interactions capture physics at very short distances



# CHIRAL EFT BASED NUCLEON-NUCLEON INTERACTIONS

	2N force	3N force	4N force
<b>LO</b>			
<b>NLO</b>			
<b>N<sup>2</sup>LO</b>			
<b>N<sup>3</sup>LO</b>			

## Chiral EFT

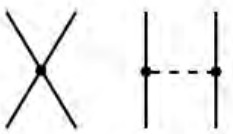

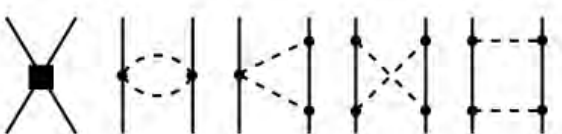

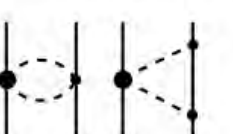
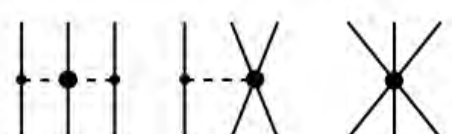
- E. Epelbaum, H. Hammer, U. Meissner Rev. Mod. Phys. **81** (2009) 1773
- R. Machleidt, D. Entem, Phys. Rep. **503** (2011) 1

\* For non-germans:  
add an extra 'N'



# EXPECTATIONS

- ▶ Should simultaneously give a good description of  $\pi N$ ,  $NN$ , and many-nucleon observables.
- ▶ LECs should be fitted to low-energy data (uncertainties will propagate)
- ▶ Fits and predictions should improve with increasing order in the expansion.
- ▶ We should be able to estimate the systematic model error.

	2N force	3N force
LO		
NLO		
N <sup>2</sup> LO		

higher-order corrections:

$$+\mathcal{O}(q/\Lambda)$$

$$+\mathcal{O}((q/\Lambda)^3)$$

$$+\mathcal{O}((q/\Lambda)^4)$$

# THE NUCLEAR MANY-BODY PROBLEM

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**“BLACK BOX” =  
SOLVING THE MANY-NUCLEON  
PROBLEM**

- ▶ Strongly-interacting  $\Rightarrow$  Strongly correlated
- ▶ Fermionic  $\Rightarrow$  Exchange (a)symmetry
- ▶ Quantum mechanical many-body  $\Rightarrow$  Many-dimensional coupled differential equations
- ▶ The solution of this many-body problem used to be the bottleneck

# AB INITIO METHODS

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- ▶ Consider an A-body system described by a well- defined microscopic Hamiltonian ( $A = \#$  of particles)
- ▶ Ab initio methods solve the relevant QM many-body equations without uncontrolled approximations
- ▶ Controlled approximations are allowed as they can be systematically improved.
- ▶ Converged results are considered precise ab initio results.
- ▶ Ab initio methods: No-Core Shell Model, Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT

**“PRAGMATIC” VS “RIGOROUS” VIEW**

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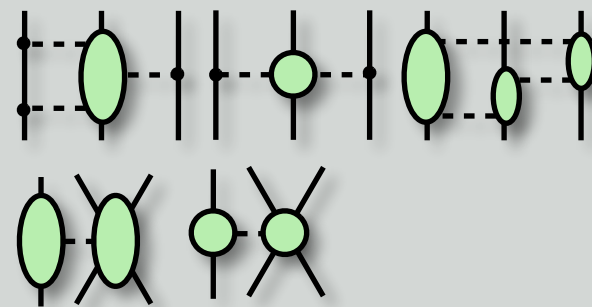
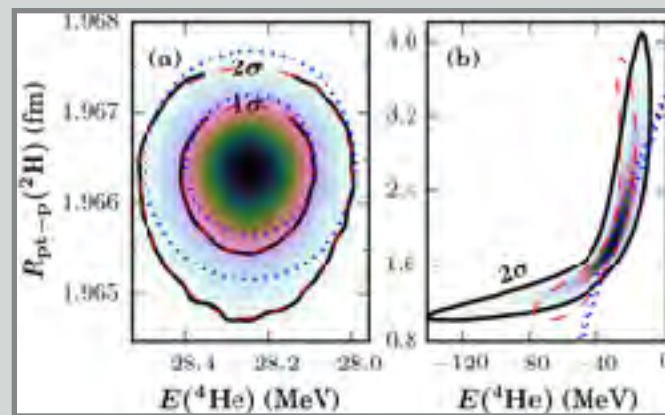
**FROM EFT-BASED NUCLEAR INTERACTIONS  
TO EMERGENT PHENOMENA**



# Overview of our research efforts

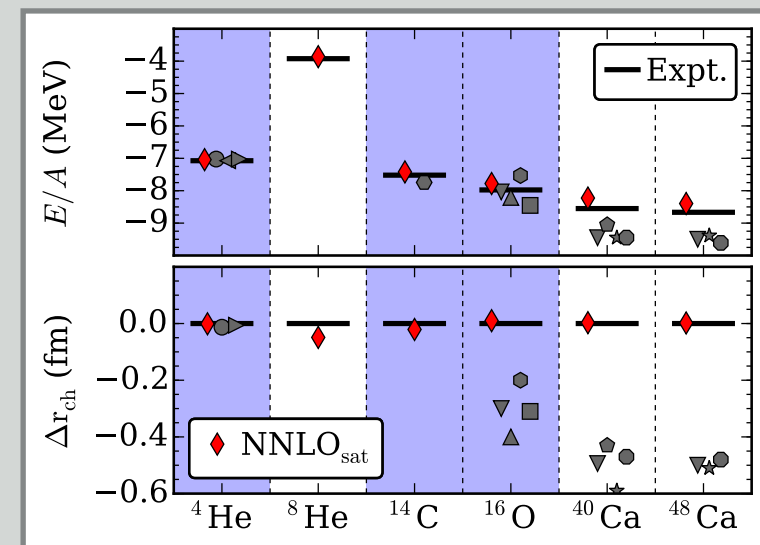
We aim to develop the technology and ability to:

Diversify and extend the **statistical analysis** of chiral-EFT based nuclear interactions in a **data-driven** approach.



- Does nuclear-physics phenomena emerge in a “from few to many” ab initio approach?
- Is available few-body data sufficient to constrain this model? Does the model become fine-tuned?

Explore **alternative strategies of informing** the model about low-energy many-body observables.



- Can/should emergent phenomena be used to constrain the model?
- How to quantify model uncertainties in such an approach?

Based on: B.D. Carlsson, A. Ekström, C. Forssén et al, Phys. Rev. X **6** (2016) 011019

B. D. Carlsson et al., In preparation



# THEORETICAL UNCERTAINTY QUANTIFICATION

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## FROM NN TO $A=4$ WITH CHIRAL EFT AND ERROR ANALYSIS

# OPTIMIZATION STRATEGY

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**Low-energy constants (LECs) are the parameters of the EFT.**  
**In practice they need to be fitted to experimental data.**

$$\chi^2(\vec{p}) \equiv \sum_i \left( \frac{O_i^{\text{theo}}(\vec{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2 \equiv \sum_i r_i^2(\vec{p})$$

**Historic approach:**

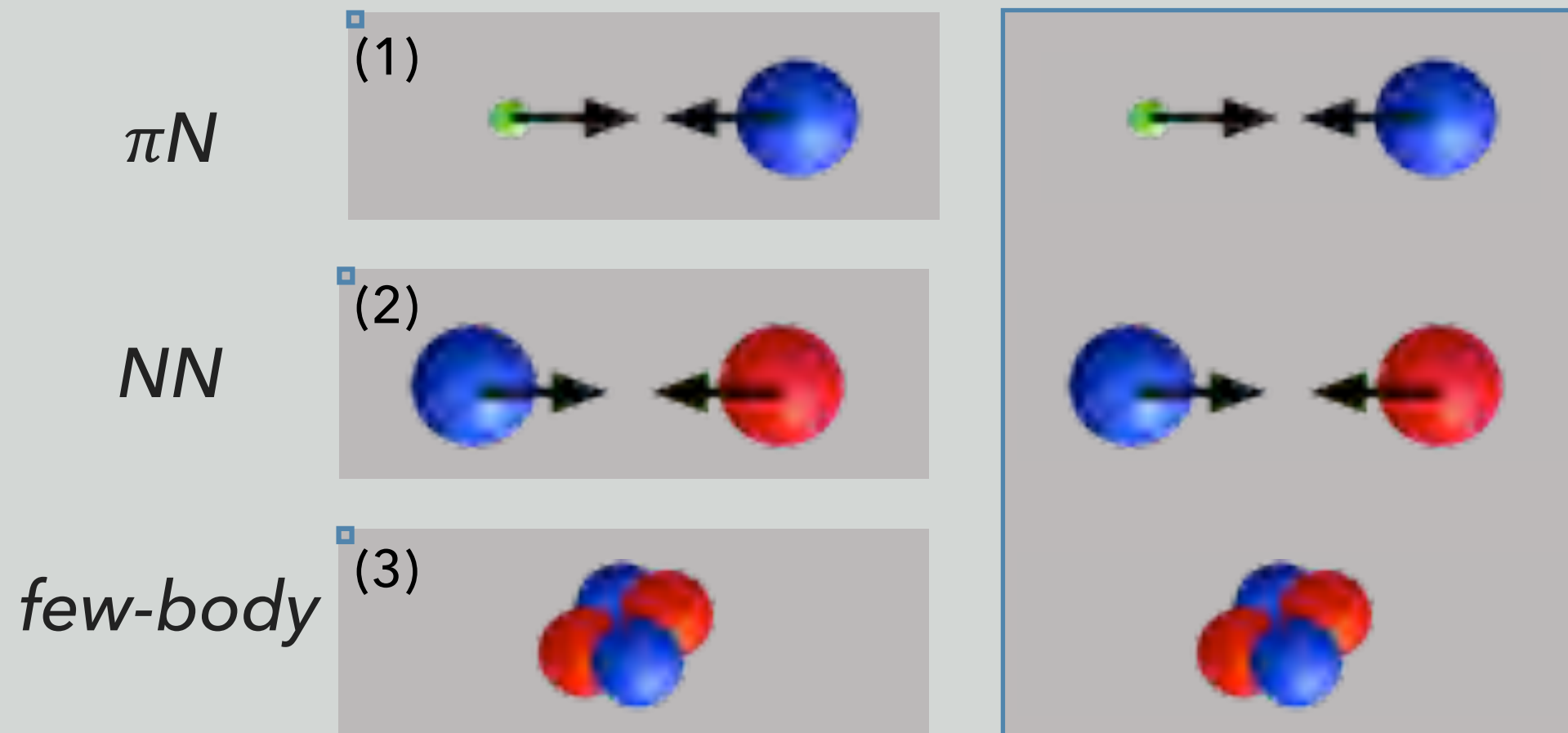
1. **nN LECs determined first**; either from Pion-Nucleon scattering phase shifts or from NN phase shifts in peripheral waves
2. **(NN-only) objective function based on Nijmegen phase shift analysis**
  - Chi-by-eye optimization; “it’s an art” (Machleidt)
  - N<sup>3</sup>LO needed for high-accuracy fit up to T<sub>lab</sub>=290 MeV
3. **NNN LECs determined at the end** given the NN part. Usually at NNLO. First results at N<sup>3</sup>LO are coming.

# OPTIMIZATION STRATEGY

**Low-energy constants (LECs) are the parameters of the EFT.  
In practice they need to be fitted to experimental data.**

$$\chi^2(\vec{p}) \equiv \sum_i \left( \frac{O_i^{\text{theo}}(\vec{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2 \equiv \sum_i r_i^2(\vec{p})$$

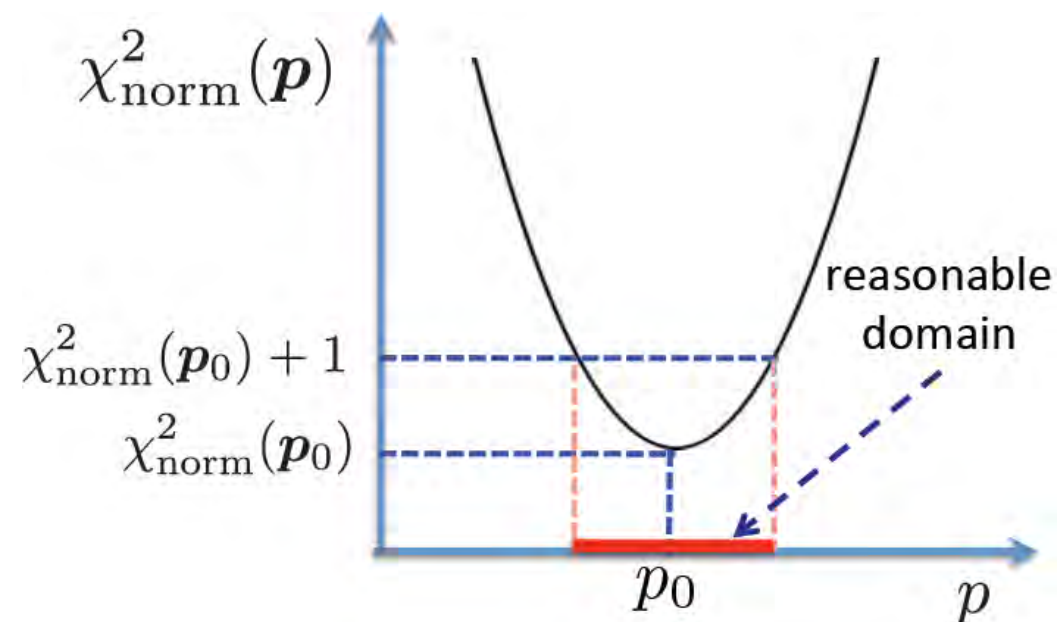
**Sequential** (historic) approach:      **Simultaneous** approach:





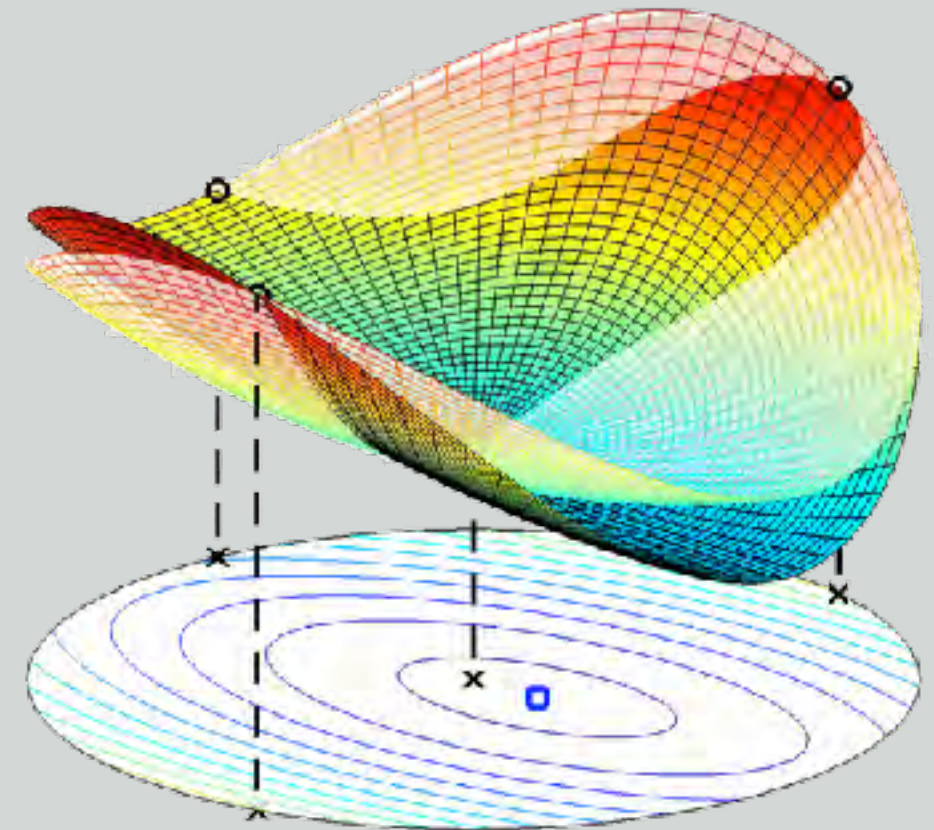
# Statistical error analysis

- In a minimum there will be an **uncertainty in the optimal parameter values**  $\mathbf{p}_0$  given by the  $\chi^2$  surface.<sup>1</sup>



- From the hessian at  $\mathbf{p}_0$  we can calculate a **covariance matrix** and from that a **correlation matrix**.

<sup>1</sup>J Dobaczewski et al 2014 J. Phys. G: Nucl. Part. Phys. 41 074001



## HESSIAN

$$H_{ij} = \frac{1}{2} \left. \frac{\partial^2 \chi^2}{\partial x_i \partial x_j} \right|_{\mathbf{x}=\mathbf{x}_\mu}$$

## COVARIANCE MATRIX

$$\Sigma = \frac{\chi^2}{N_{df}} \mathbf{H}^{-1}$$

## CORRELATION MATRIX

$$R_{ij} = \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii} \Sigma_{jj}}}$$

# Input and technology

## $\pi N$ scattering

- WI08 database
- $T_{\text{lab}}$  between 10-70 MeV
- $N_{\text{data}} = 1347$
- $\chi\text{EFT}(Q^4)$  to avoid underfitting

## NN scattering

- SM99 database (+Granada)
- $T_{\text{lab}}$  between 0-290 MeV
- $N_{\text{data}} = 2400(\text{np}) + 2045(\text{pp})$
- $\chi\text{EFT}(Q^0, Q^2, Q^3, Q^4)$

**All 6000 residuals computed on 1 node in ~90 sec.**

## A=3 bound states

- ${}^3\text{H}, {}^3\text{He}$  (binding energy, radius,  ${}^3\text{H}$  half life)

**On 1 node in ~10 sec**

**+ derivatives! ( $\times 2$ -20 cost)**

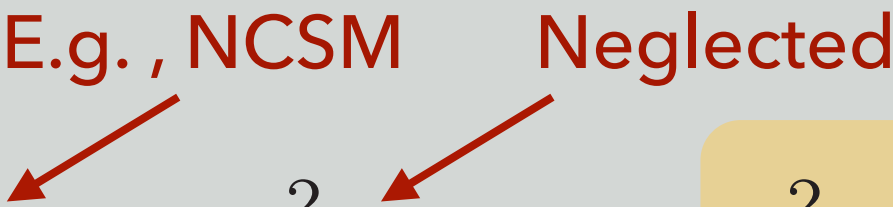
# Total error budget

$$\chi^2(\vec{p}) \equiv \sum_i \left( \frac{O_i^{\text{theo}}(\vec{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2$$

- ▶ The total error budget is

$$\sigma_{\text{tot}}^2 = \sigma_{\text{exp}}^2 + \sigma_{\text{method}}^2 + \sigma_{\text{numerical}}^2 + \sigma_{\text{model}}^2$$

E.g., NCSM      Neglected



- ▶ At a given chiral order  $\nu$ , the omitted diagrams should be of order

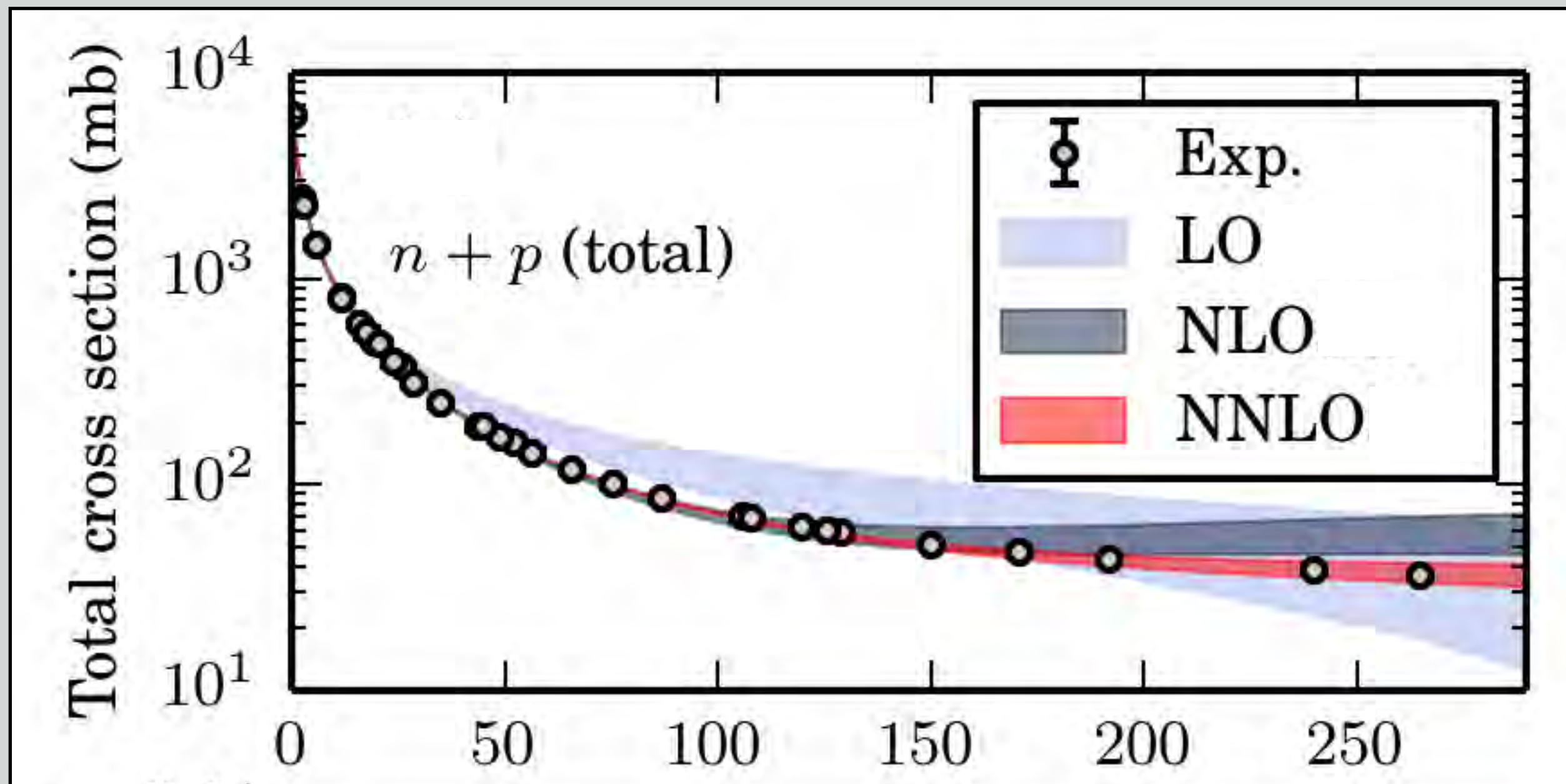
$$\mathcal{O} \left( (Q/\Lambda_\chi)^{\nu+1} \right)$$

- ▶ Still needs to be converted to actual numbers  $\sigma_{\text{model}}$
- ▶ We translate this EFT knowledge into an error in the scattering amplitudes

$$\sigma_{\text{model},x}^{(\text{amp})} = C_x \left( \frac{Q}{\Lambda_\chi} \right)^{\nu+1}, \quad x \in \{NN, \pi N\}$$

- ▶ which is then propagated to an error in the observable.

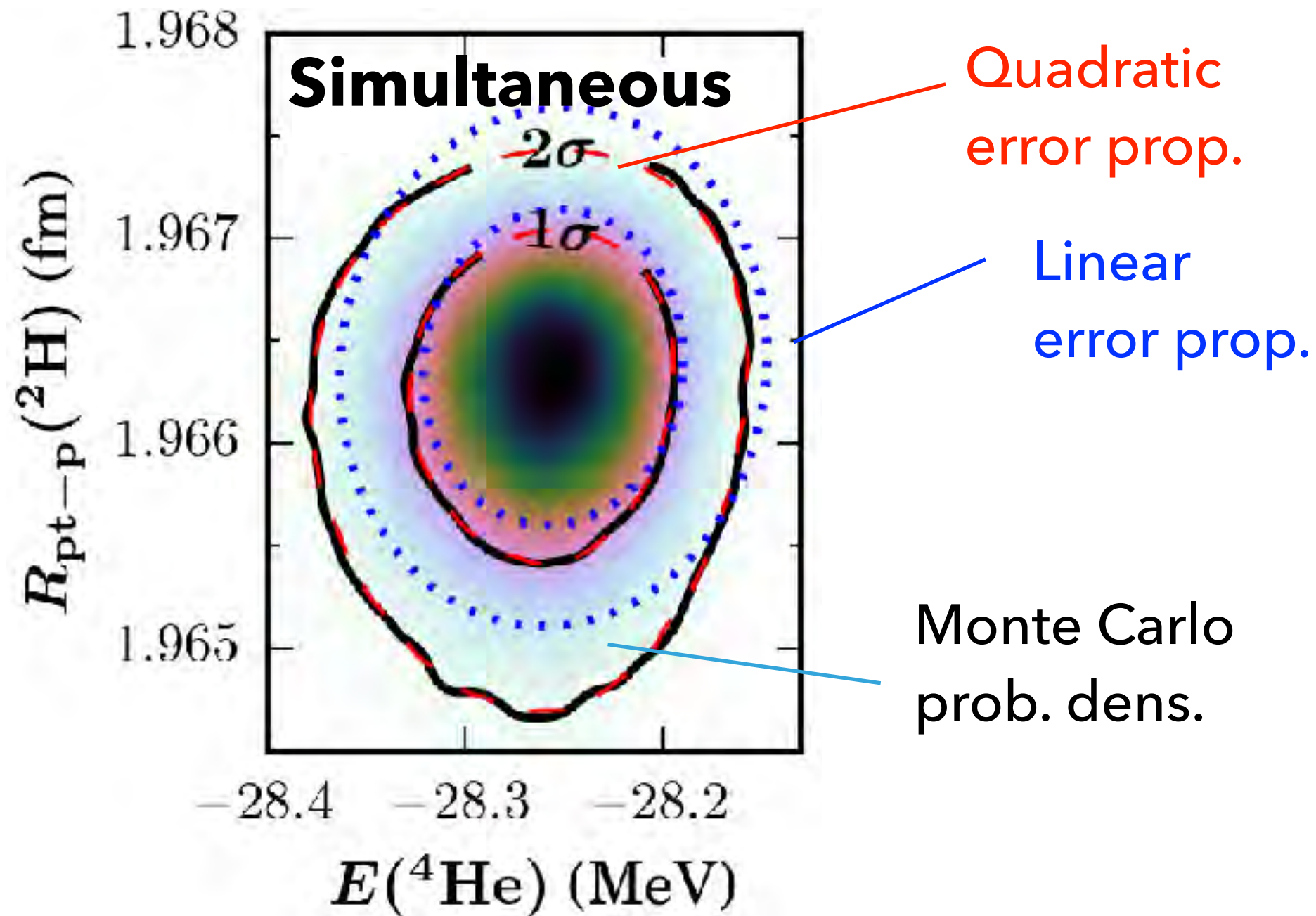
# TOTAL NP CROSS SECTION





# Quadratic error propagation vs Brute force sampling

$$O(\mathbf{p}) \approx O(\mathbf{p}_0) + J_O \Delta \mathbf{p} + \frac{1}{2} \Delta \mathbf{p}^T H_O \Delta \mathbf{p}$$



$$E(^4\text{He}) = -28.24^{+9}_{-11} \text{ (MeV)}$$

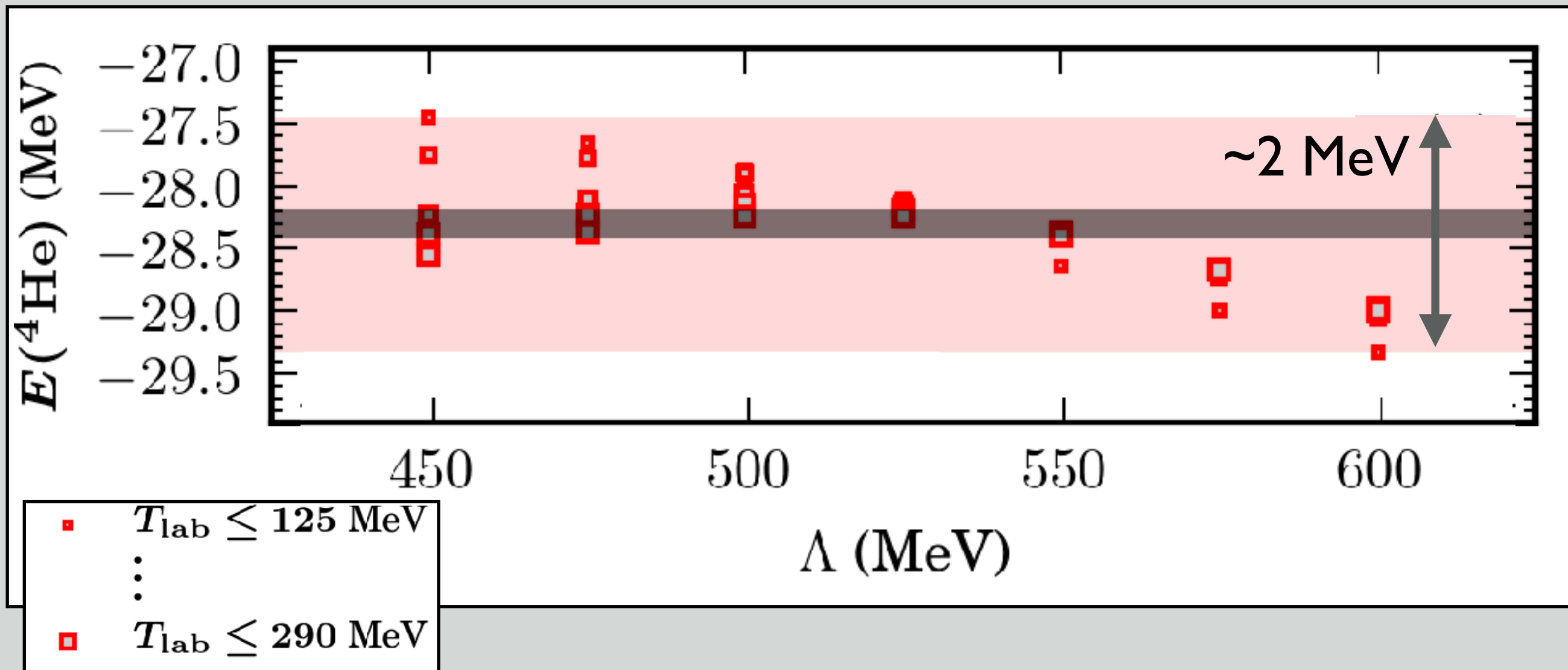
Statistical uncertainty

# EXPLORING FURTHER SYSTEMATIC UNCERTAINTIES

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- ▶ So far, all results have been obtained with a non-local regulator with cutoff  $\Lambda=500$  MeV.
  - ▶ A subset of systematic uncertainties can be probed by varying  $\Lambda$ .
- ▶ The bulk of input data comes from  $NN$  scattering. We have truncated the data base at  $T_{\text{lab}}=290$  MeV
  - ▶ Always with model error that gives more weight to low  $E$ .
  - ▶ A subset of systematic uncertainties can be probed by varying the truncation  $\max(T_{\text{lab}})$
- ▶ Reoptimizing with different  $\Lambda$  and  $T_{\text{lab}}$  and will give us a **family** of models.
- ▶ **All of them will reproduce the same few-body physics.**

# Systematic uncertainties: input data, regulator cutoff



- ▶ 7 different regulator cutoffs:  
 $\Lambda=450, 475, \dots, 575, 600$  MeV
- ▶ 6 different *NN*-scattering datasets  
 $T_{\text{lab}} \in [0, T_{\text{lab,max}}]$ , with  
 $T_{\text{lab,max}}=125, \dots, 290$  MeV

# Do-it-yourself

All 42 different sim/sep potentials, as well as the respective covariance matrices are available as supplemental material.

- ▶ LO-NLO-NNLO
- ▶ with 7 different cutoffs: 450,475,...,600 MeV
- ▶ from 6 different NN-scattering datasets

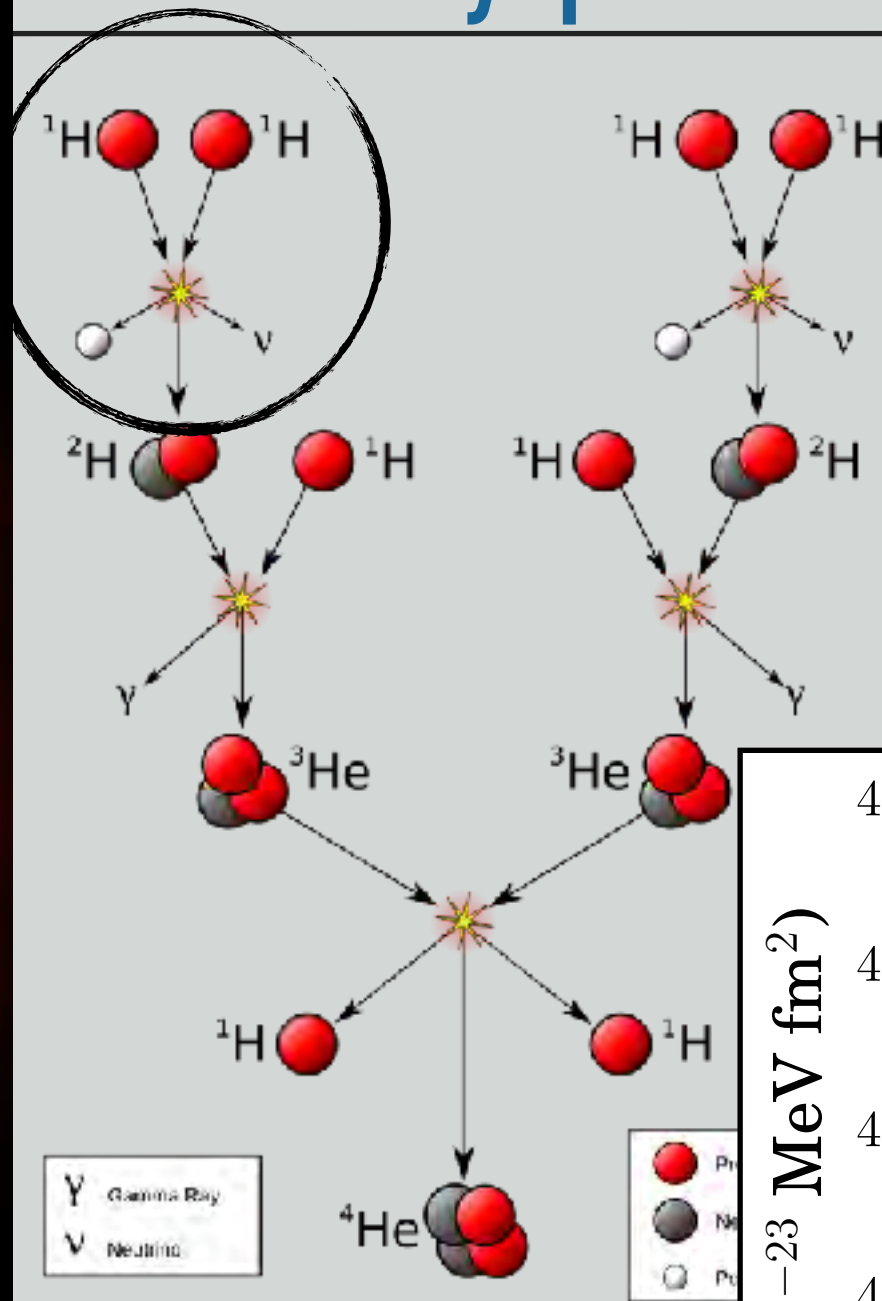
$$\begin{aligned}\text{Cov}(\mathbf{A}, \mathbf{B}) &\equiv \mathbb{E}[(\mathcal{O}_A(\mathbf{p}) - \mathbb{E}[\mathcal{O}_A(\mathbf{p})])(\mathcal{O}_B(\mathbf{p}) - \mathbb{E}[\mathcal{O}_B(\mathbf{p})])] \\ &\approx \mathbb{E}\left[\left(\tilde{J}_{A,i}x_i + \frac{1}{2}\tilde{H}_{A,ij}x_ix_j - \frac{1}{2}\tilde{H}_{A,ii}\sigma_i^2\right)\right. \\ &\quad \times \left.\left(\tilde{J}_{B,k}x_k + \frac{1}{2}\tilde{H}_{B,kl}x_kx_l - \frac{1}{2}\tilde{H}_{B,kk}\sigma_k^2\right)\right] \\ &= \tilde{\mathbf{J}}_A^T \Sigma \tilde{\mathbf{J}}_B + \frac{1}{2}(\sigma^2)^T (\tilde{\mathbf{H}}_A \circ \tilde{\mathbf{H}}_B) \sigma^2,\end{aligned}$$

compute the derivatives of your own observables wrt LECs, then explore:

- ▶ cutoff variations
- ▶ order-by-order evolution
- ▶ LEC UQ/correlations



# Uncertainty quantification applied to pp fusion

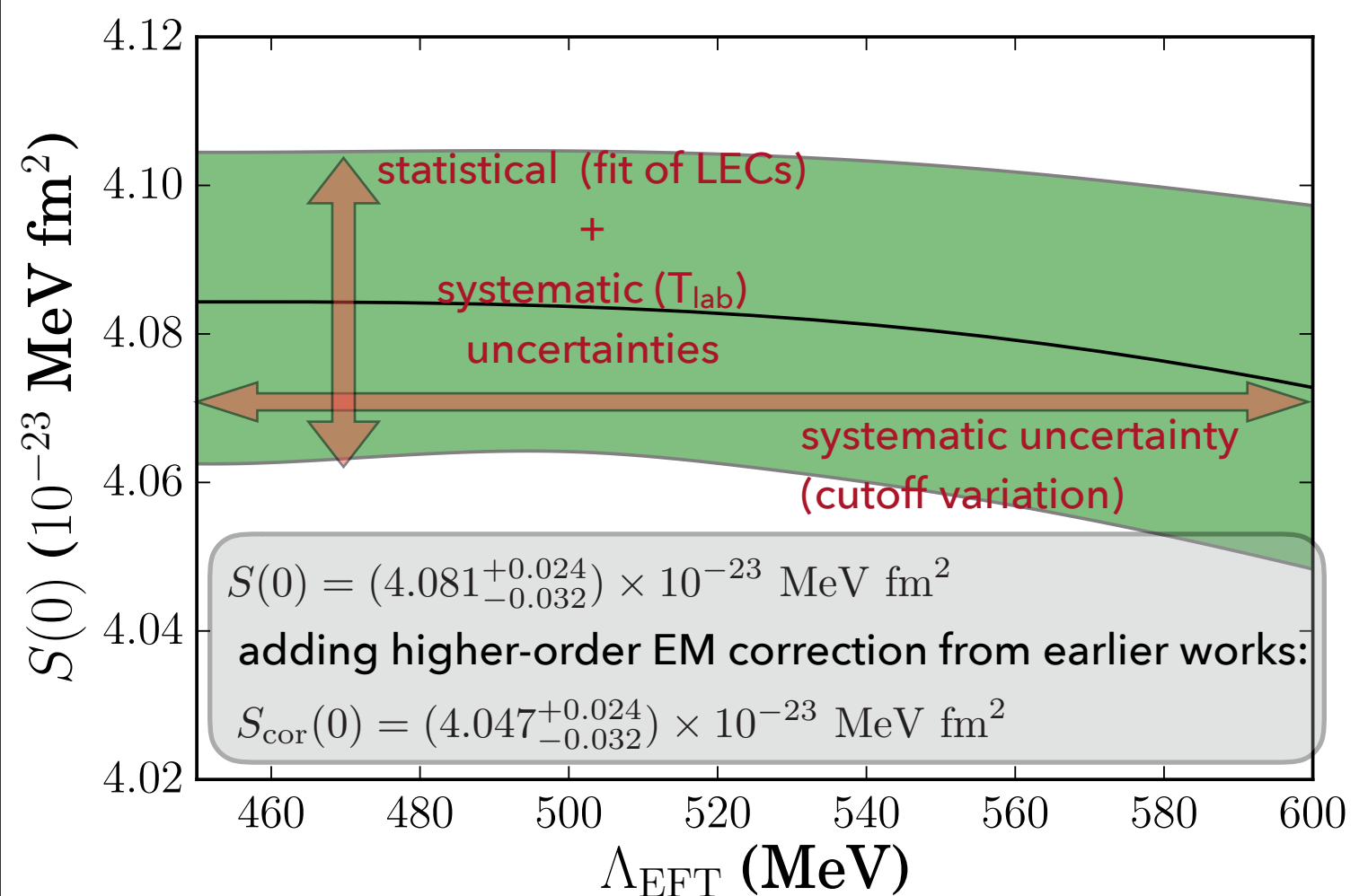


L. E. Marcucci et al PRL 110, 192503 (2013)  
 R. Schiavilla et al PRC 58, 1263 (1998)  
 J-W. Chen et al. PLB 720, 385 (2013)

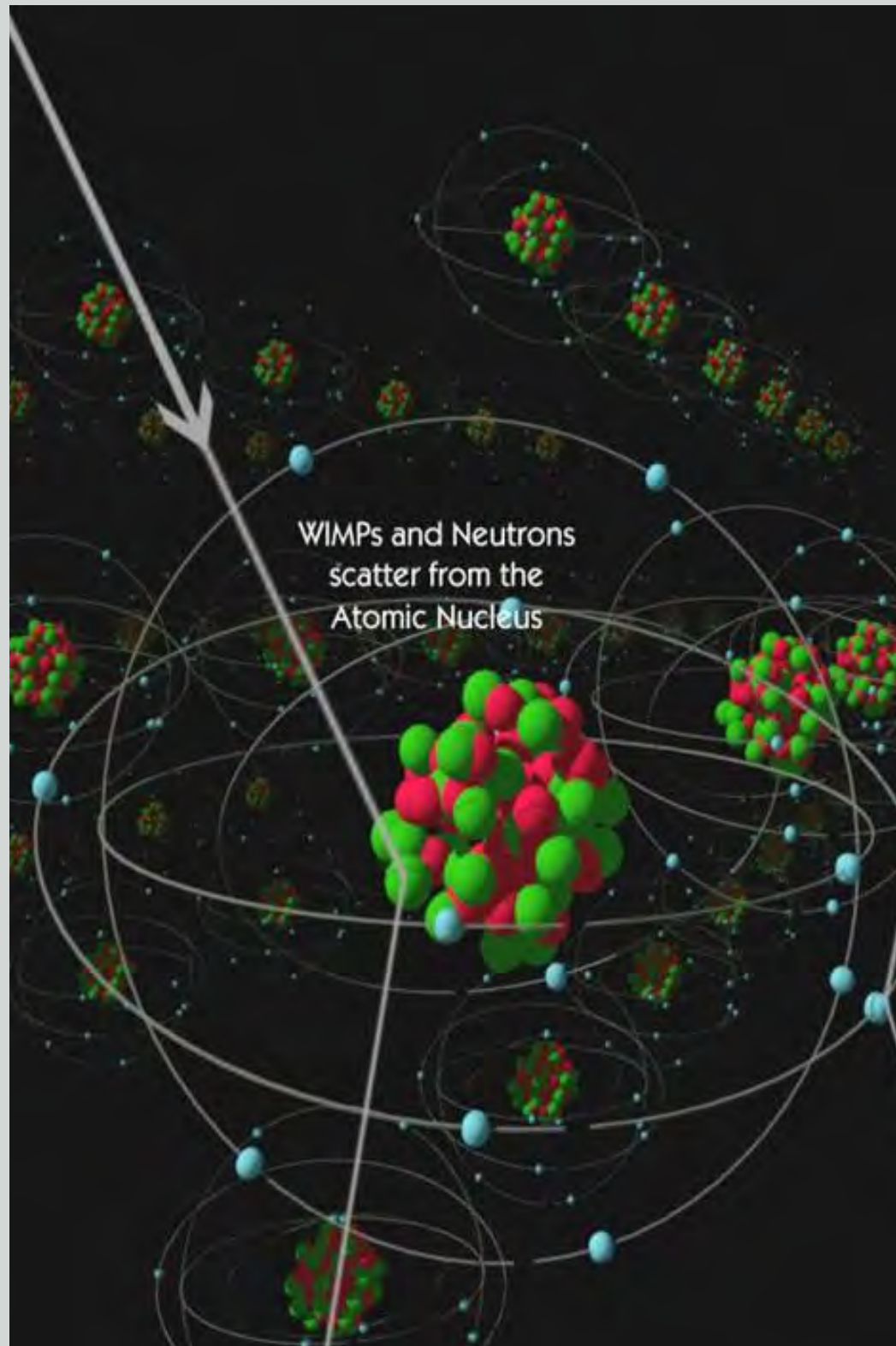
$$p + p \rightarrow d + e^+ + \nu_e$$

$$S(E) = \sigma(E) E e^{2\pi\eta}$$

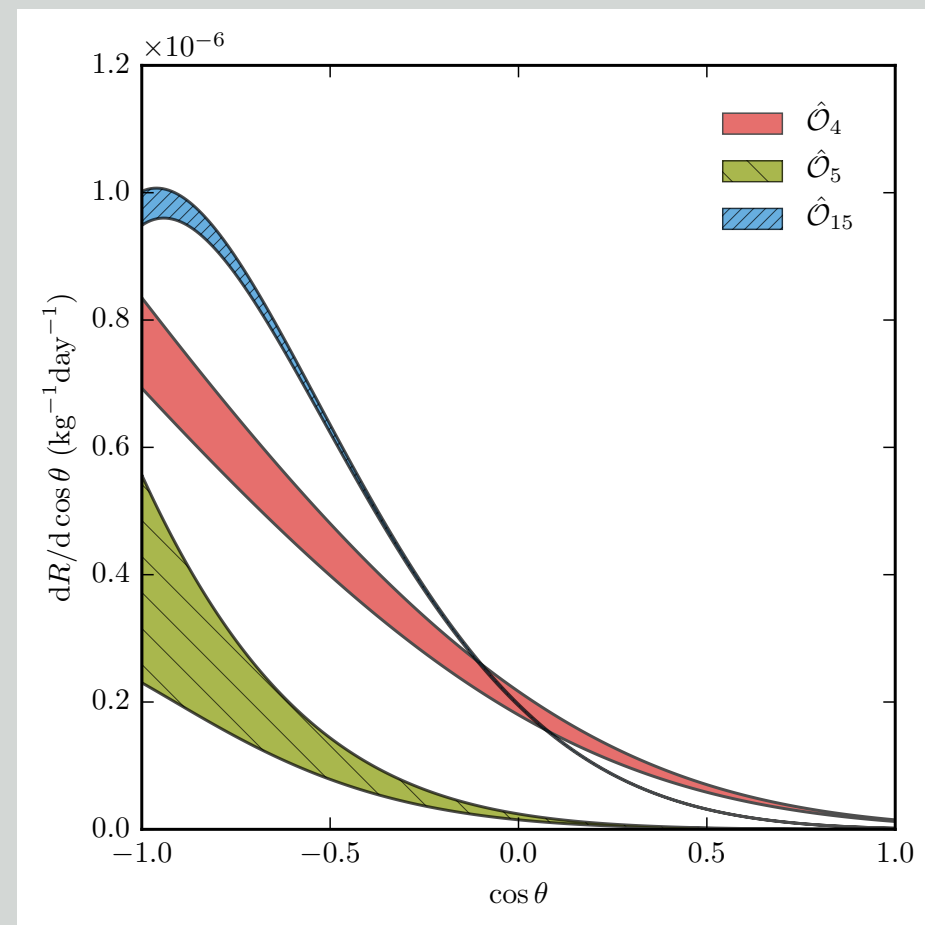
B. Acharya et al, Phys. Lett. B 760 (2016) 584



# Uncertainty quantification applied to dark-matter nucleus scattering



- ▶ WIMP scattering off  $^3,^4\text{He}$  described in NR-EFT
- ▶ Nuclear response functions from NCSM wave functions
- ▶ Studied rates of dark matter–nucleus scattering events

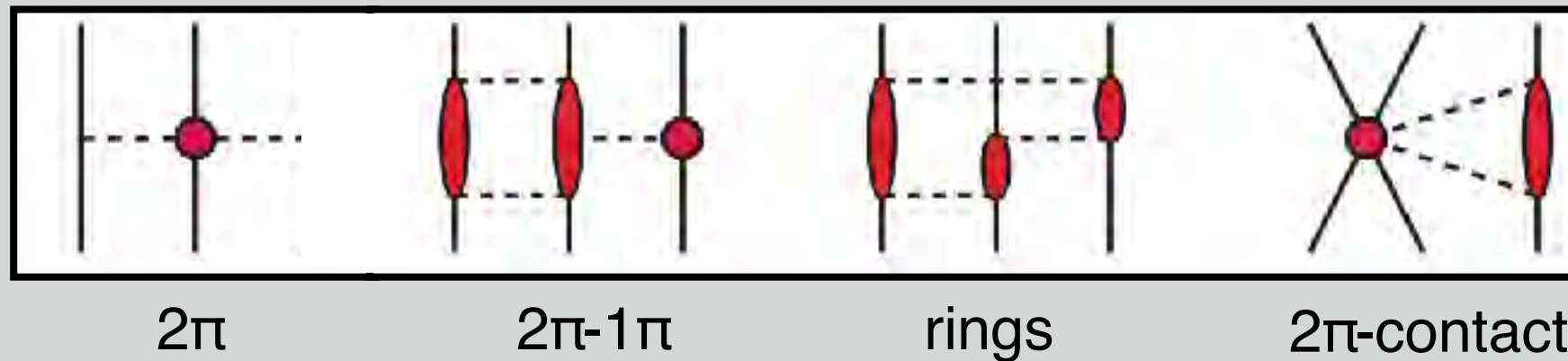


D. Gazda et al, arXiv:1612.09165

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**Work in progress: N3LO**

# N3LO optimizations are challenging

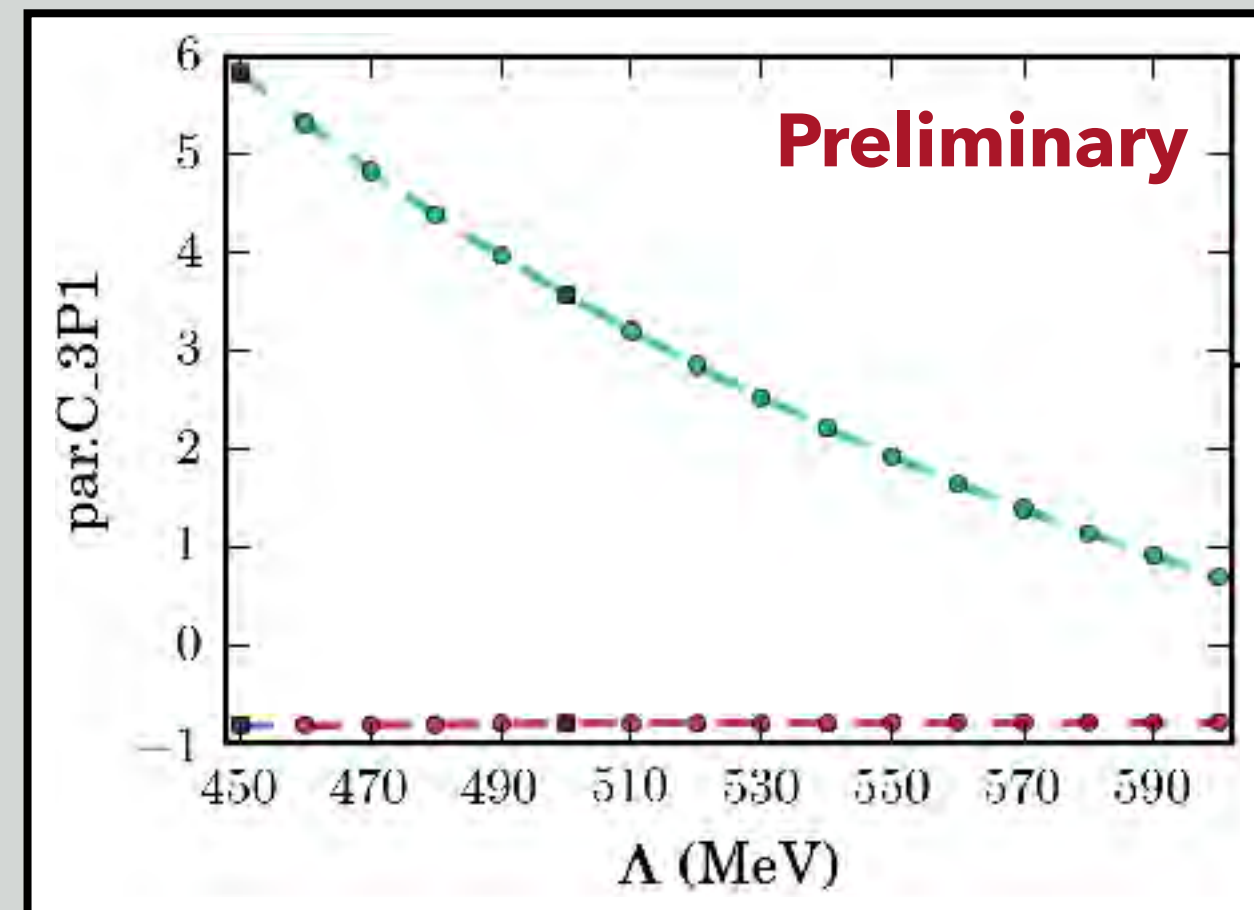


+ rel. corr.

41 parameters to optimize,  
3NF matrix elements recently made  
available (K. Hebeler)

Initialize by computing phase shifts for  $10^5$   
random contact LEC values for each  
partial wave and select the  $\sim 1000$  best  
values and optimize. This leads to  
[5x2x2x2x2=160] different optima (for  
cutoff 500 MeV) with respect to phase  
shifts. (pi-N LECs from sep-optimization).

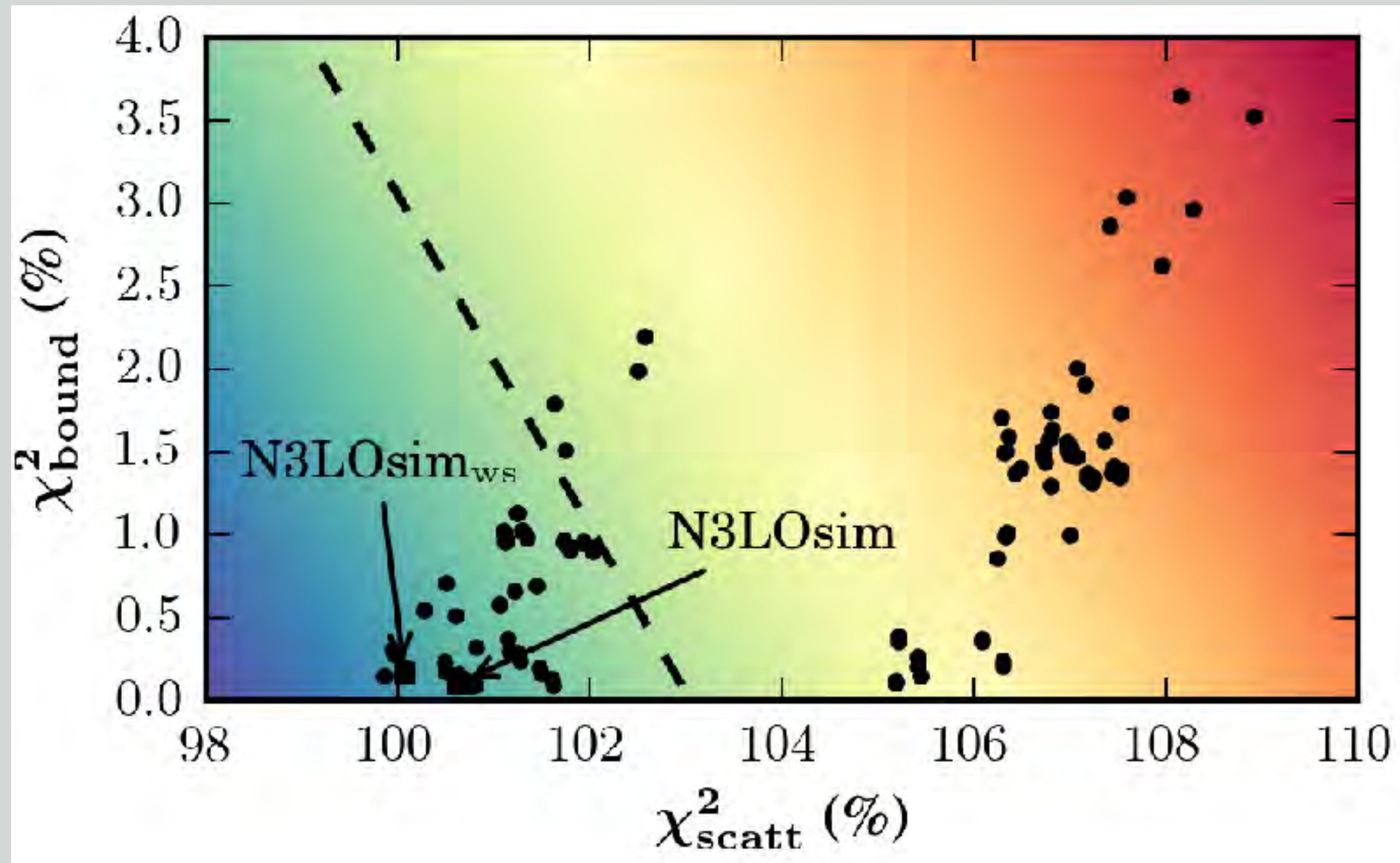
These minima perform equally well in the  
NN sector. But the LECs display rather  
different  $\Lambda$ -dependence.





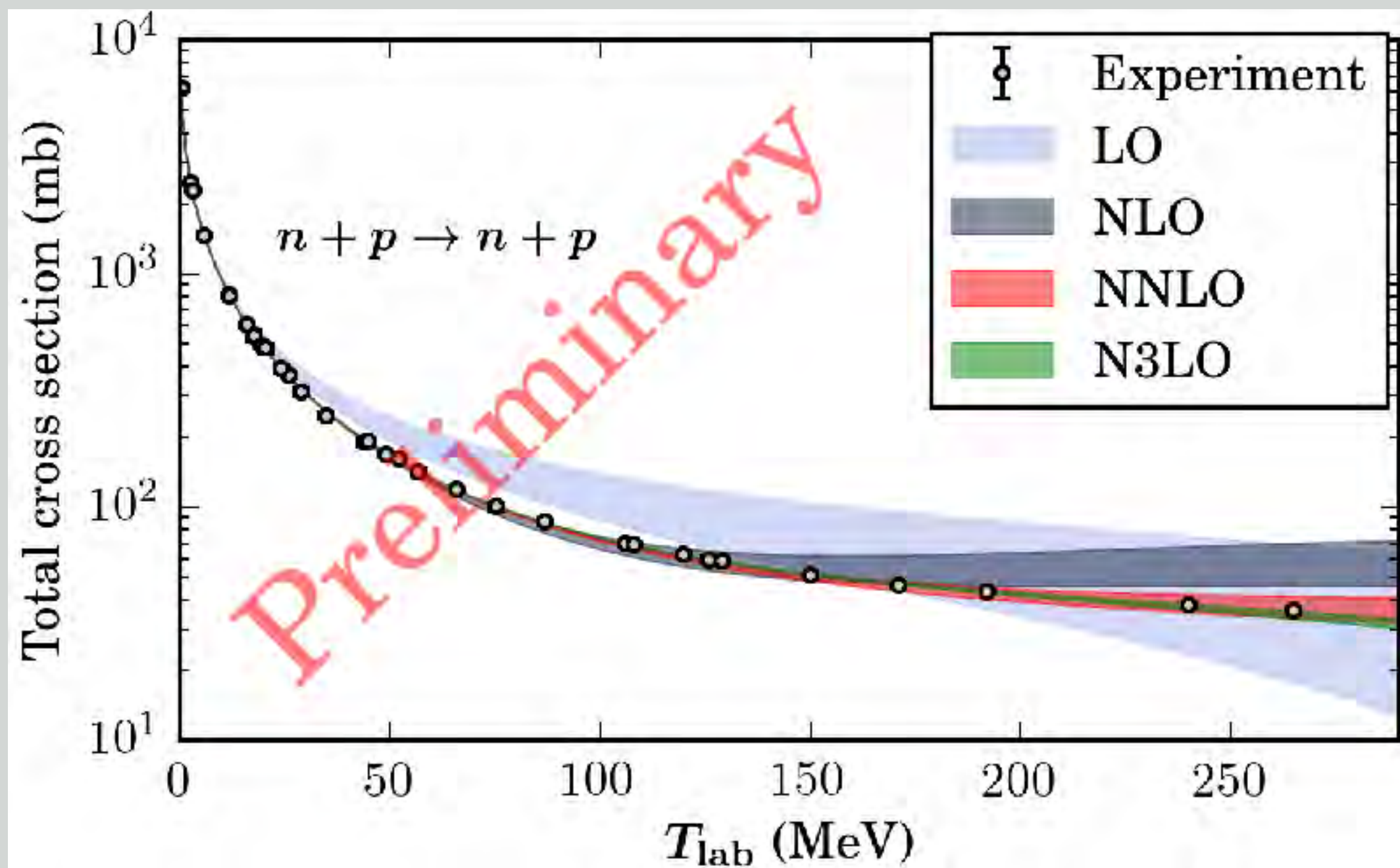
# N3LO MINIMA

$\approx 100$  optima remain after performing simultaneous optimizations from these starting points

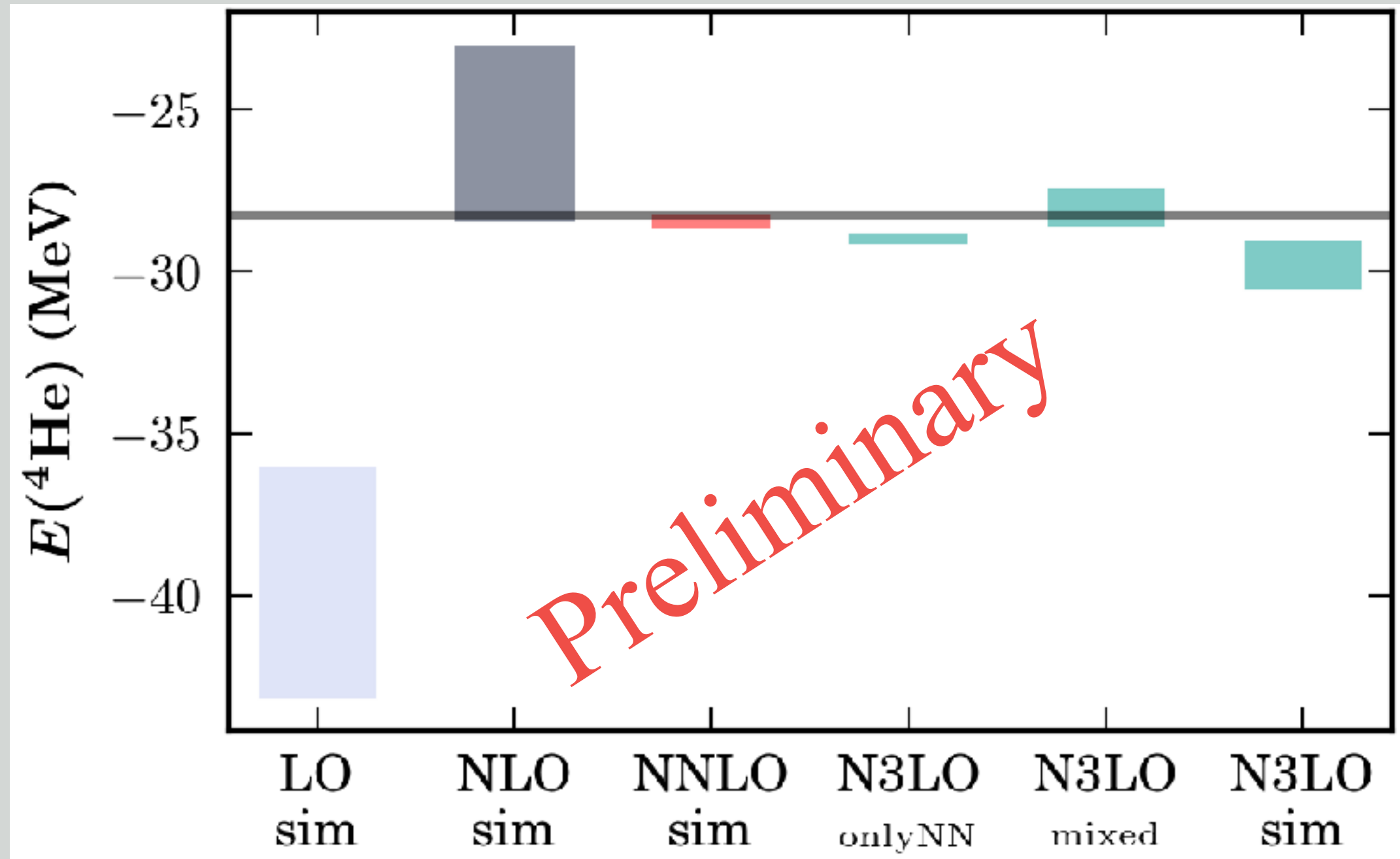




# N3LO SCATTERING



# SYSTEMATIC UNCERTAINTIES: INPUT DATA, REGULATOR CUTOFF



Based on: A. Ekström et al, Phys. Rev. C **91** (2015) 051301(R)

Rapid Communication

## Accurate nuclear radii and binding energies from a chiral interaction

A. Ekström, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, B. D. Carlsson, C. Forssén, M. Hjorth-Jensen, F. Navrátil, and W. Nazarewicz

Phys. Rev. C **91**, 051301(R) – Published 1 May 2015

See also: G. Hagen et al, Nat. Phys. **12** (2015) 186



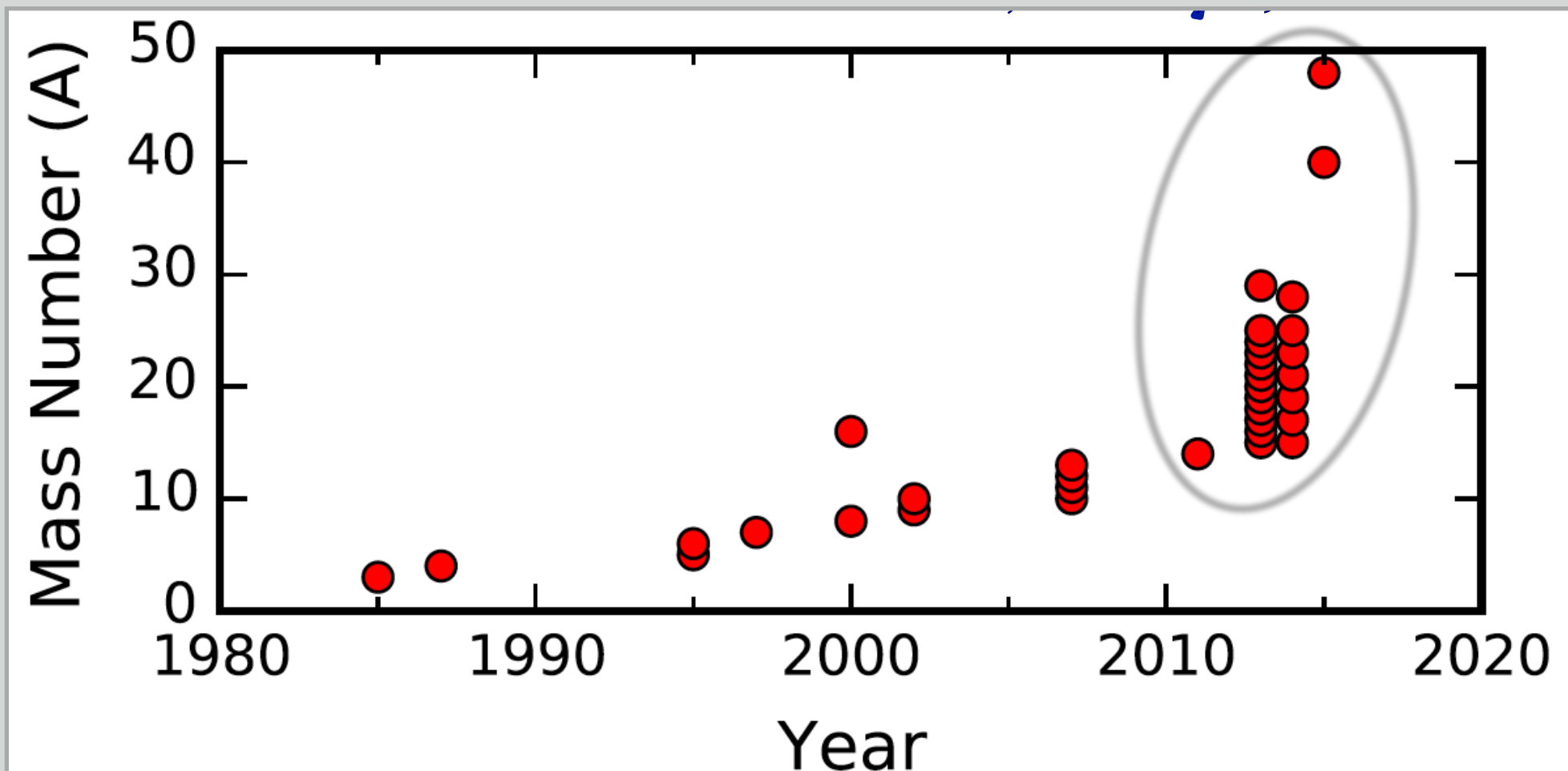
# FROM FEW TO MANY

# IS NUCLEAR SATURATION AN EMERGENT PHENOMENON?

# TREND IN REALISTIC AB INITIO CALCULATIONS

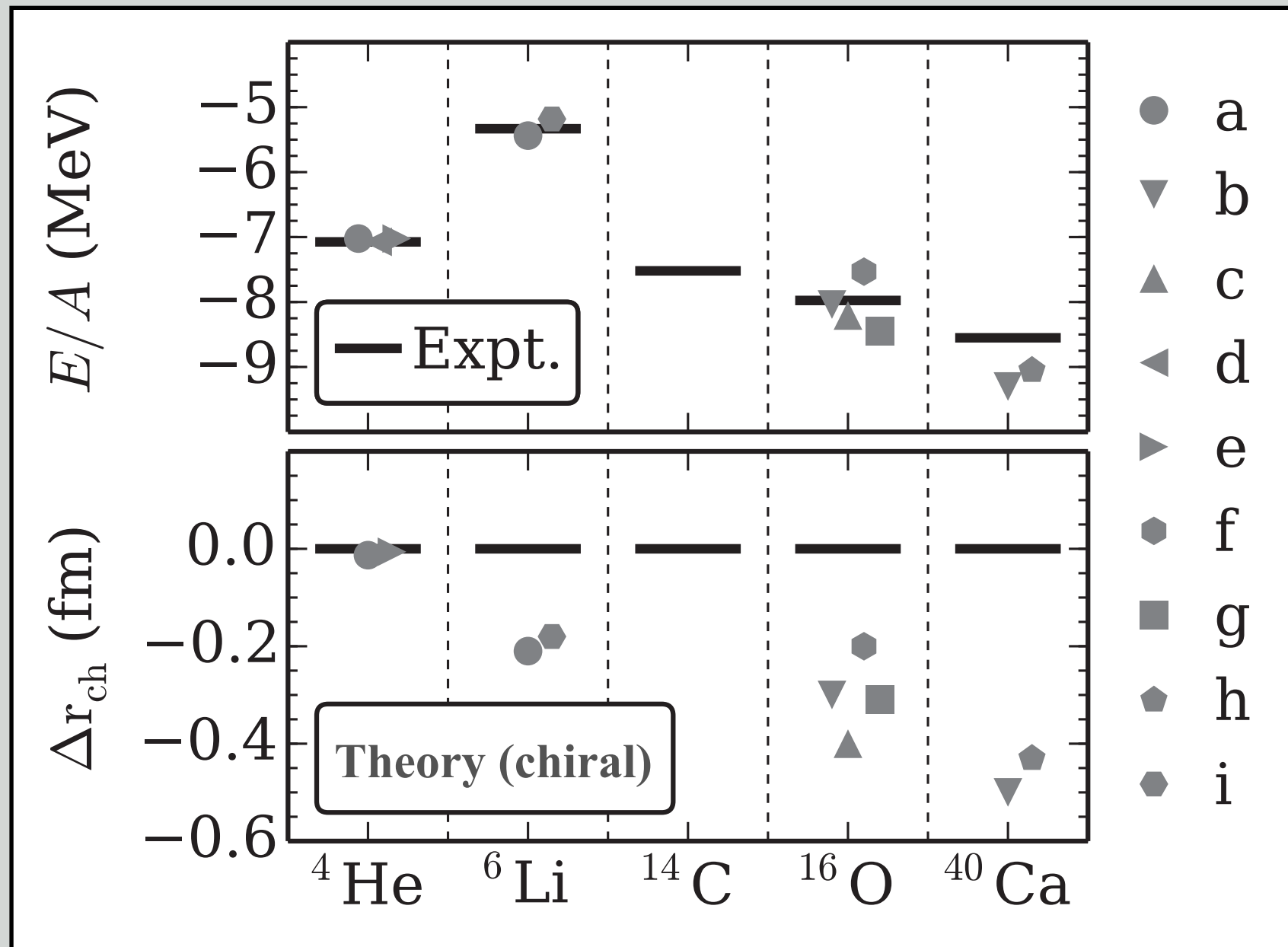
## Explosion of many-body methods

(Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT, No-Core Shell Model, Self-Consistent Green's Function, UMOA, ...)



“Computational capabilities exceed accuracy of available interactions”

# STATUS OF CHIRAL-FORCE PREDICTIONS



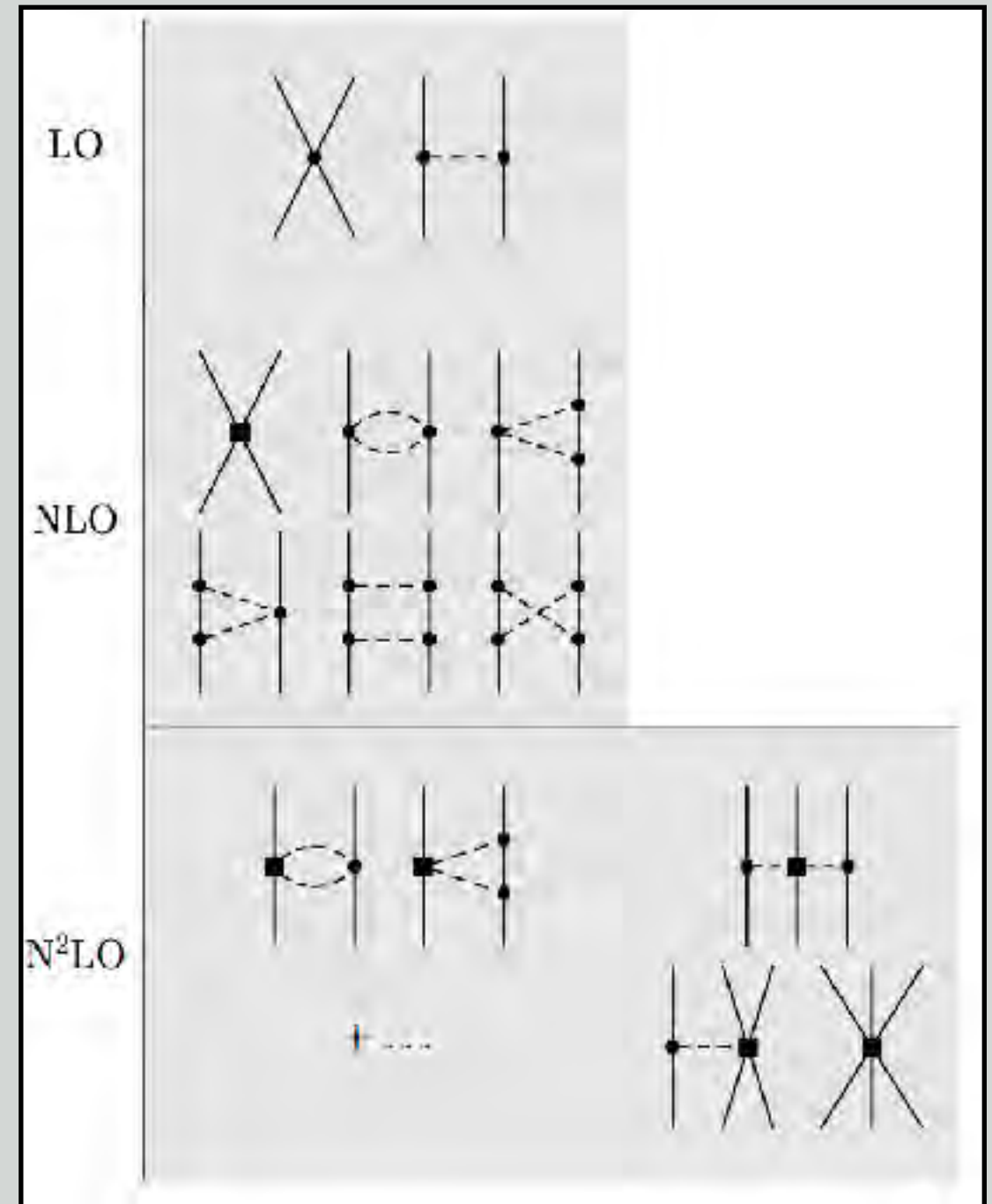
**Ab initio calculations with existing chiral interactions**

- **overbind** medium-mass and heavy nuclei, and
- **underestimate charge radii**.

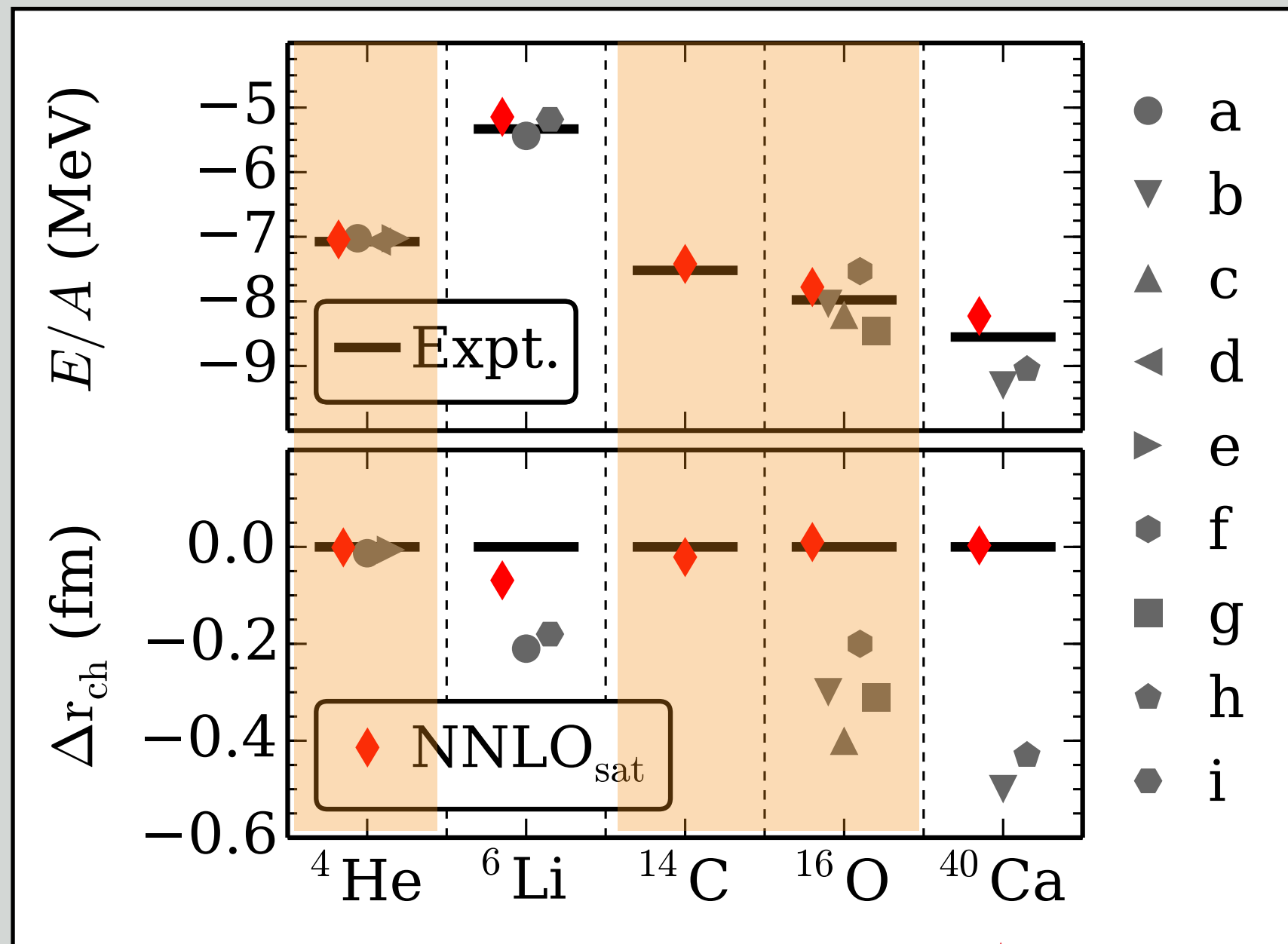


# PRAGMATIC OPTIMIZATION STRATEGY

- ▶ Simultaneous optimization of NN and NNN LECs at NNLO.
- ▶ NCSM and CC calculations are performed within the optimization
- ▶ Objective function contains:
  - ▶ deuteron properties and  $NN$  scattering data ( $T_{\text{lab}} < 35$  MeV)
  - ▶  $A=3,4$  binding energies, radii
  - ▶  $^{14}\text{C}, ^{16}\text{O}$  binding energies, radii
  - ▶  $^{22,24,25}\text{O}$  binding energies



# CHIRAL INTERACTION WITH ACCURATE SATURATION: $N^2LO_{\text{SAT}}$

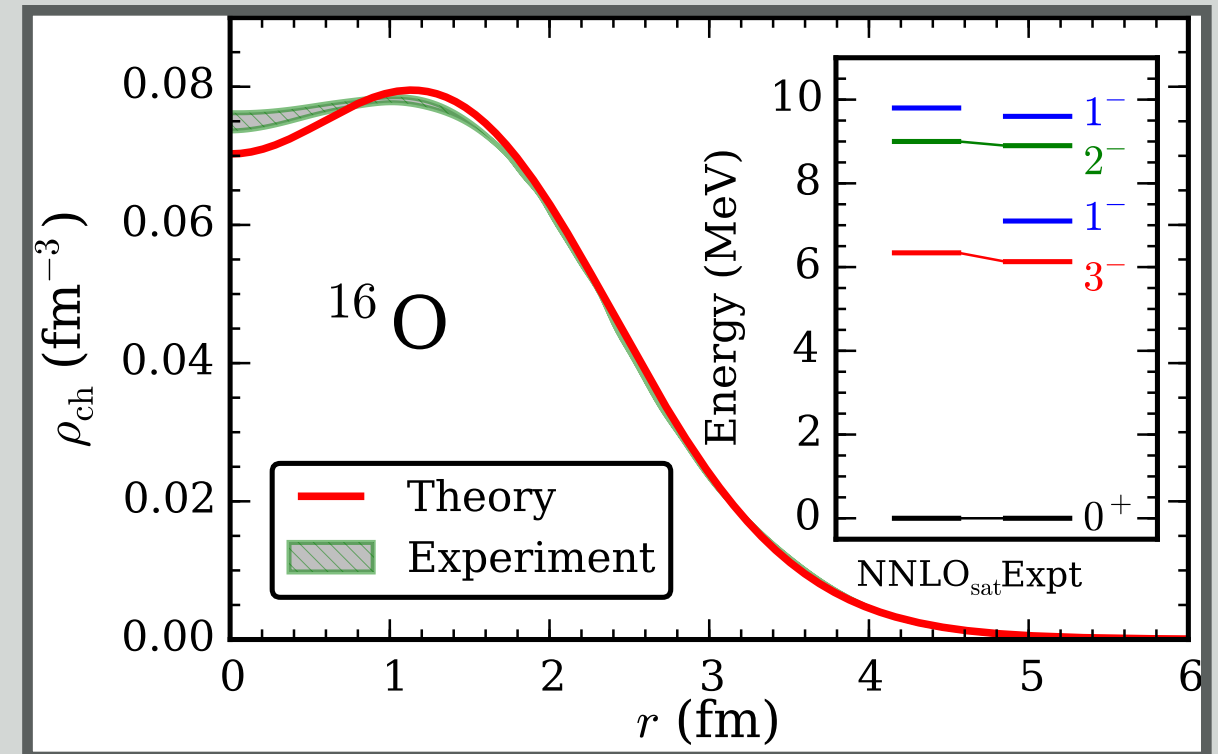
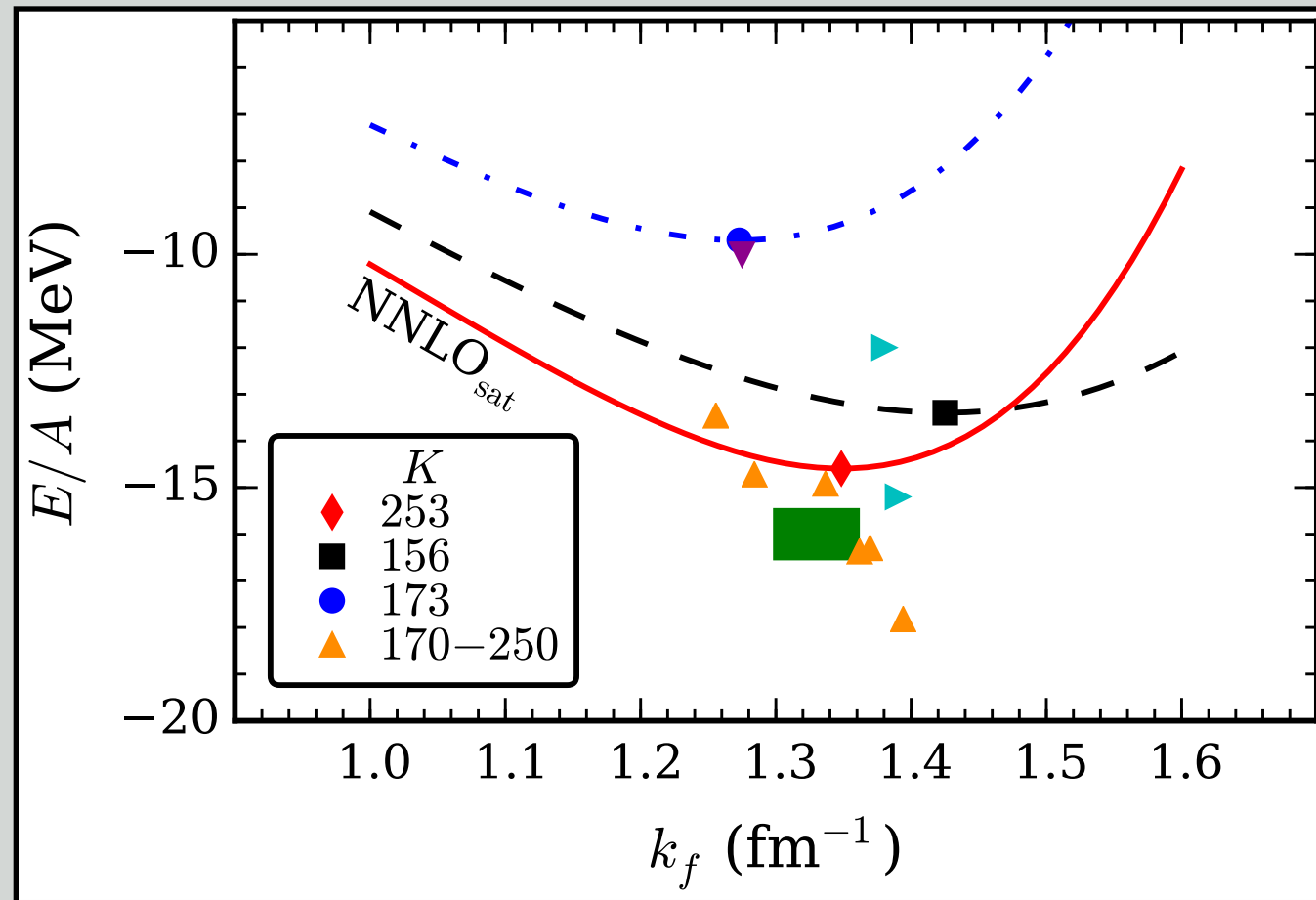


Interpolation

Extrapolation

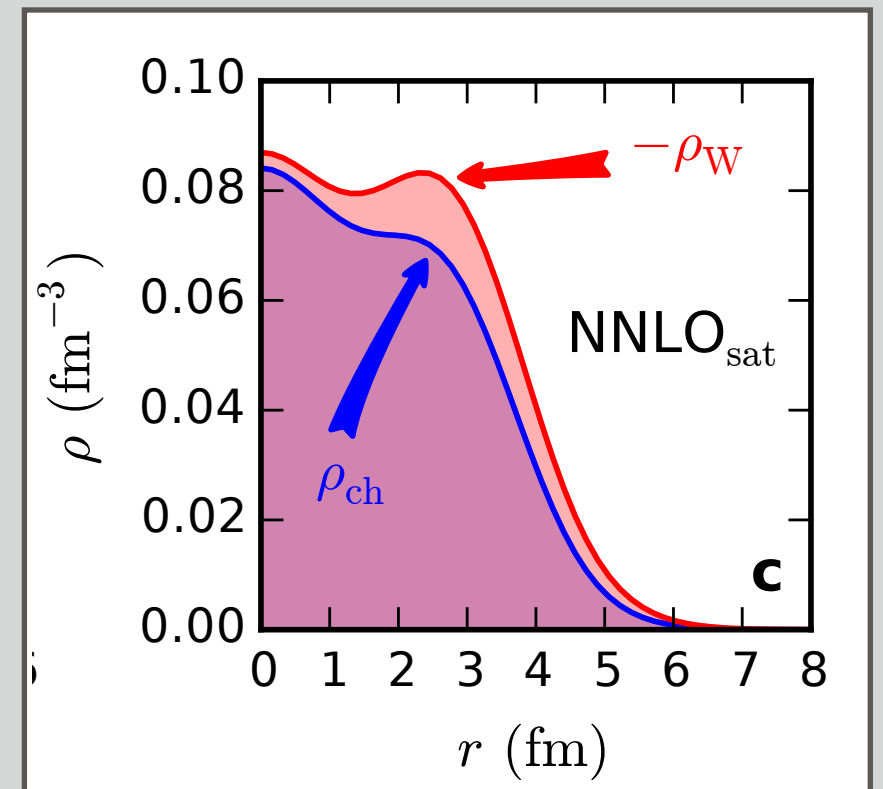
# CHARGE, NEUTRON DISTRIBUTION, AND WEAK SIZE OF THE ATOMIC NUCLEUS

## Accurate saturation



## $^{16}\text{O}$ charge radius

## Neutron skin of $^{48}\text{Ca}$



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# CONCLUSION

## Chiral EFT with error analysis

- ▶ **Uncertainty quantification** is a **unique opportunity** when employing **systematic approaches (EFT + ab initio)**.
- ▶ First results for correlations, parameter uncertainties and **error propagation in the few and many-body sectors**.
- ▶ **Simultaneous optimization of all LECs** at LO, NLO, NNLO, N3LO using NN, NNN and piN data is critical in order to:
  - ▶ capture all correlations between the parameters, and
  - ▶ reduce the statistical errors.
- ▶ We find that **statistical errors** are small ( $\approx 1\%$ ), and the total error budget is dominated by **systematic model errors**. Statistical errors increase dramatically for sequentially optimized potentials.