

PRAGMATIC VS RIGOROUS VIEW ON CHIRAL EFT(-BASED) INTER-NUCLEON INTERACTIONS

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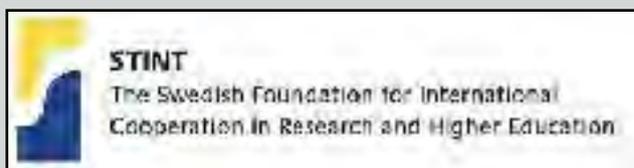
ESNT workshop, CEA Saclay,
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MANY THANKS TO MY COLLABORATORS

- ❖ **Boris Carlsson, Andreas Ekström**, Dag Strömberg, Oskar Lilja, Mattias Lindby, Björn Mattsson (Chalmers)
- ❖ Kai Hebel (TU Darmstadt), Kyle Wendt (ORNL, LLNL)

and

- ❖ Gaute Hagen, Gustav Jansen, Thomas Papenbrock (ORNL/UT), Morten Hjorth Jensen (UiO, MSU), Petr Navrátil (TRIUMF), Witek Nazarewicz (ORNL, MSU)



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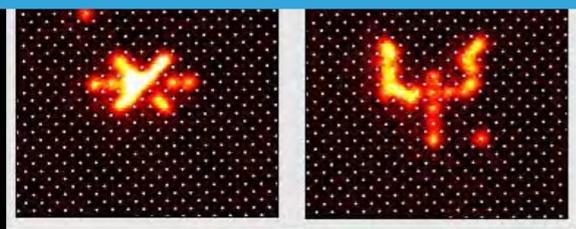
MODERN NUCLEAR PHYSICS

INTRODUCTION

PROFOUND INTERSECTIONS

- Can we solve QCD to describe hadronic structures and interactions?
- Can we employ the separation of scales to build successful effective field theories?
- What is the new standard model of particle physics?

- What controls nuclear saturation?
- What are the properties of nuclei with extreme neutron/proton ratios?
- Can we predict useful cross sections that cannot be measured?
- Can nuclei provide precision tests of fundamental symmetries?



QUANTUM MANY-BODY PHYSICS

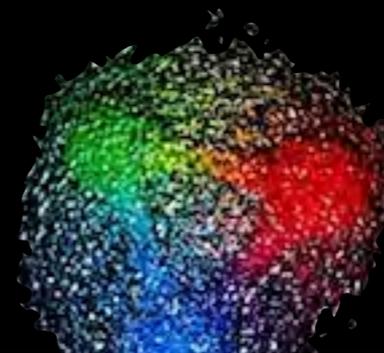
femtophysics



PHYSICS OF NUCLEI

- How do nuclei shape the physical universe?
- What is the origin of the elements?
- What is the interaction between baryonic and dark matter?

subfemto...



giga...

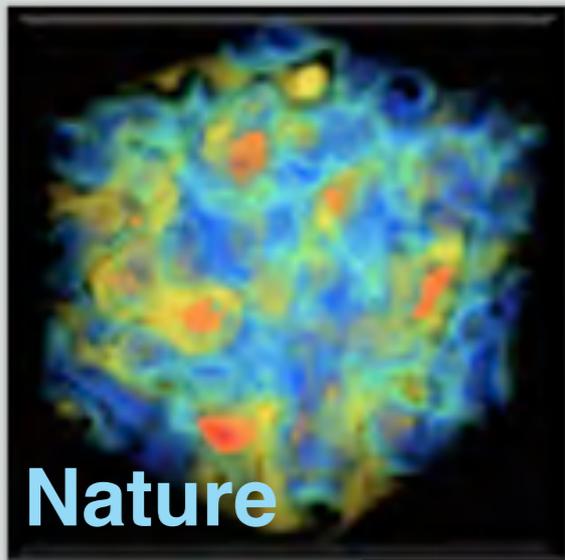


PHYSICS AND COSMOLOGY

ARTICLES

- How do collective phenomena emerge from simple constituents?
- How can complex systems display astonishing simplicities?
- What are unique properties of open quantum systems?

THE SCIENTIFIC METHOD

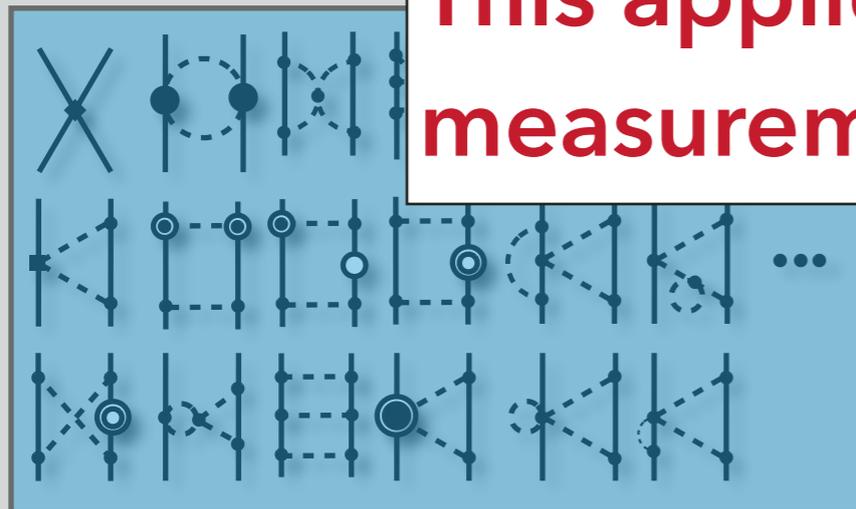


Manifests itself in



The quantification of uncertainties is absolutely critical for progress. This applies BOTH to the measurement and to the theory.

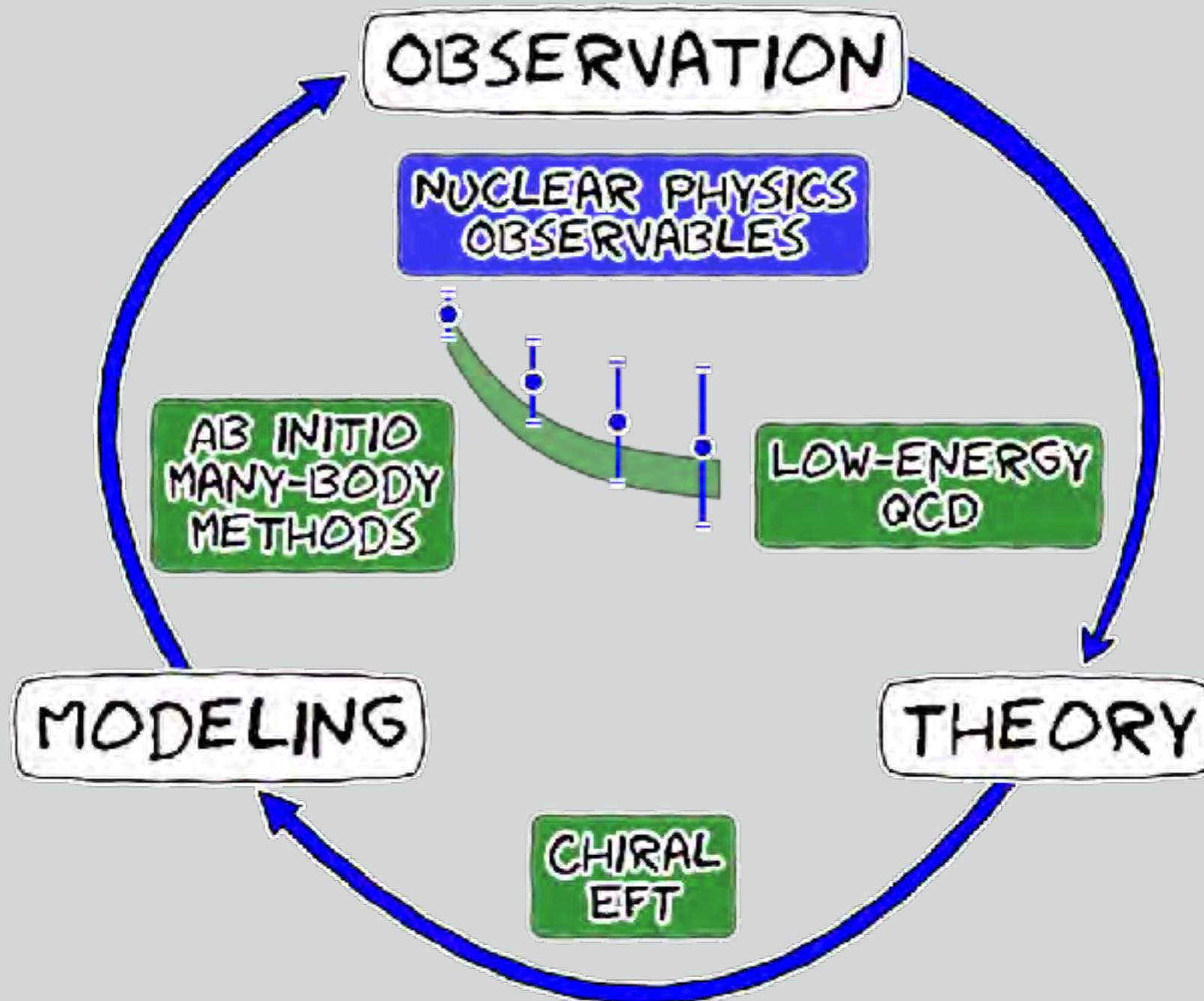
accessible to



inspire



THE SCIENTIFIC METHOD: NUCLEAR PHYSICS



An EFT approach offers many nice features;
No free lunch: there are a number of parameters;
How do we determine those? Does this EFT deliver?

CHIRAL EFT BASED NUCLEON-NUCLEON INTERACTIONS

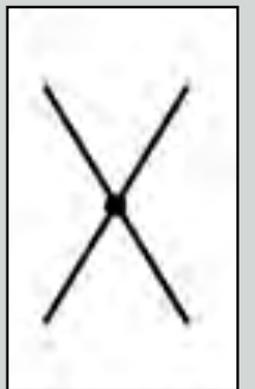
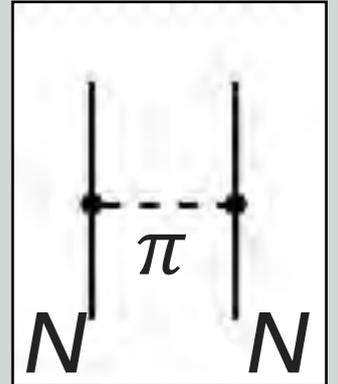
**“BLACK BOX” =
CHIRAL EFT FOR THE
MANY-NUCLEON SECTOR**

See work by: Weinberg, van Kolck,
Epelbaum, Meissner, Krebs, Entem,
Machleidt...

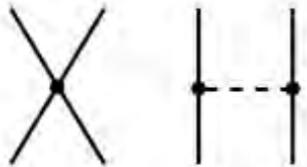
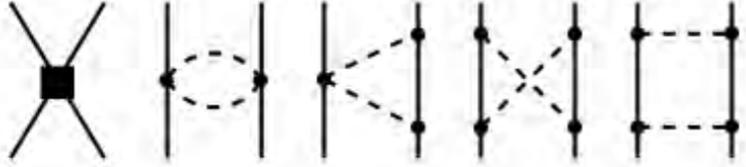
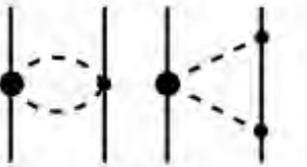
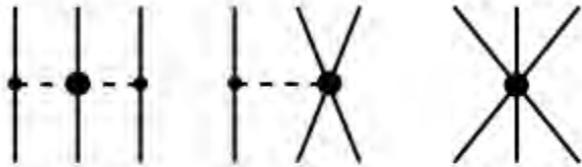
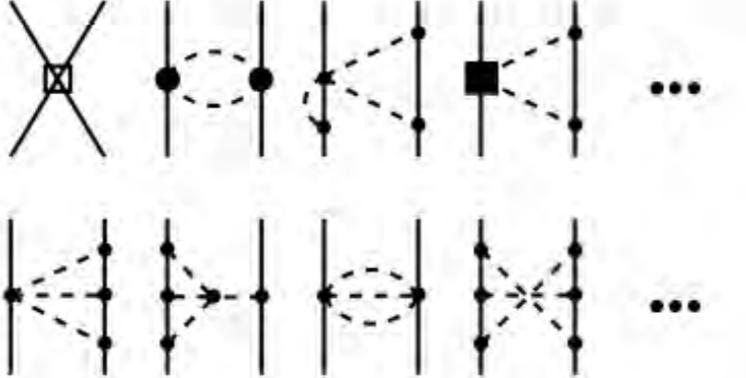
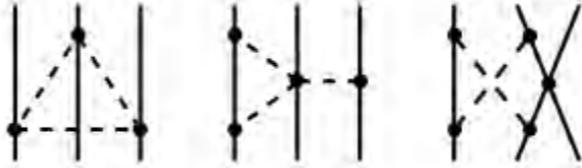
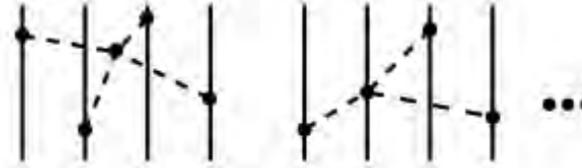


... however, with different
instructions

- ▶ Separation of scales in nuclear physics.
- ▶ Pions (π) and nucleons (N) as relevant degrees of freedom.
- ▶ One-pion exchange = long-range physics
- ▶ Contact interactions capture physics at very short distances



CHIRAL EFT BASED NUCLEON-NUCLEON INTERACTIONS

	2N force	3N force	4N force
LO			
NLO			
N²LO			
N³LO			

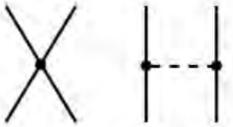
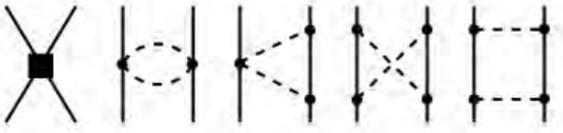
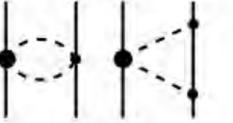
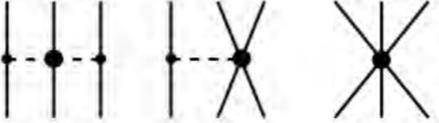
Chiral EFT

- E. Epelbaum, H. Hammer, U. Meissner Rev. Mod. Phys. **81** (2009) 1773
- R. Machleidt, D. Entem, Phys. Rep. **503** (2011) 1

* For non-germans:
add an extra 'N'

EXPECTATIONS

- ▶ Should simultaneously give a good description of πN , NN , and many-nucleon observables.
- ▶ LECs should be fitted to low-energy data (uncertainties will propagate)
- ▶ Fits and predictions should improve with increasing order in the expansion.
- ▶ We should be able to estimate the systematic model error.

	2N force	3N force
LO		
NLO		
N ² LO		

higher-order corrections:

$$+\mathcal{O}(q/\Lambda)$$

$$+\mathcal{O}((q/\Lambda)^3)$$

$$+\mathcal{O}((q/\Lambda)^4)$$

THE NUCLEAR MANY-BODY PROBLEM

**“BLACK BOX” =
SOLVING THE MANY-NUCLEON
PROBLEM**

- ▶ Strongly-interacting \Rightarrow
Strongly correlated
- ▶ Fermionic \Rightarrow
Exchange (a)symmetry
- ▶ Quantum mechanical
many-body \Rightarrow
Many-dimensional
coupled differential
equations
- ▶ The solution of this many-
body problem used to be
the bottleneck

AB INITIO METHODS

- ▶ Consider an A-body system described by a well- defined microscopic Hamiltonian ($A = \#$ of particles)
- ▶ Ab initio methods solve the relevant QM many-body equations without uncontrolled approximations
- ▶ Controlled approximations are allowed as they can be systematically improved.
- ▶ Converged results are considered precise ab initio results.
- ▶ Ab initio methods: No-Core Shell Model, Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT

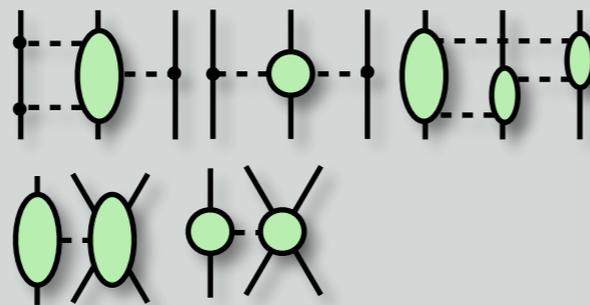
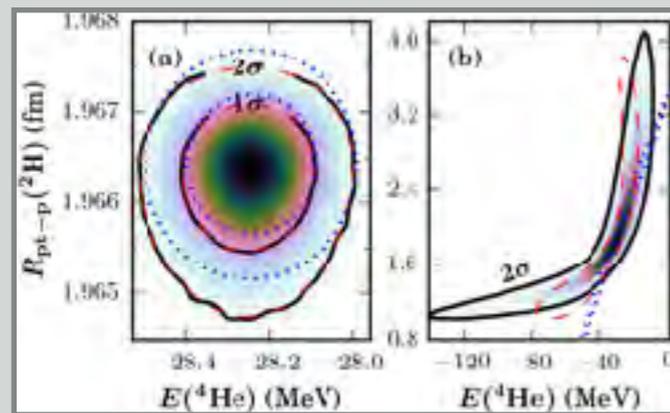
“PRAGMATIC” VS “RIGOROUS” VIEW

**FROM EFT-BASED NUCLEAR INTERACTIONS
TO EMERGENT PHENOMENA**

Overview of our research efforts

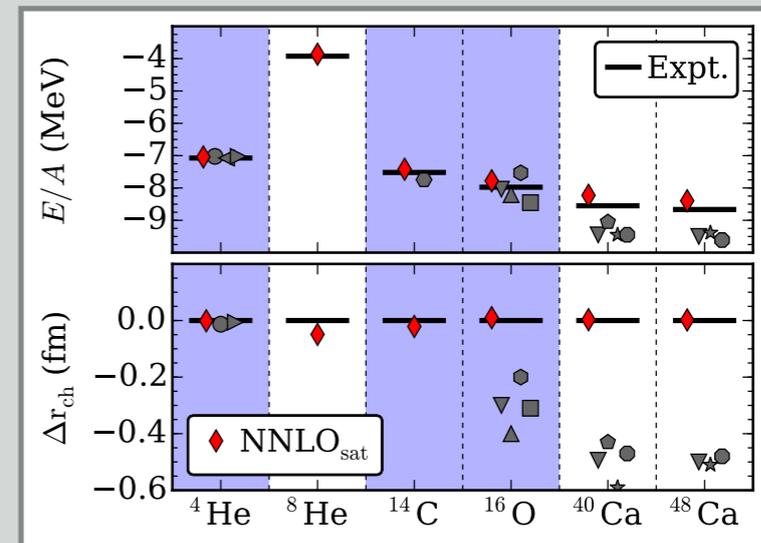
We aim to develop the technology and ability to:

Diversify and extend the **statistical analysis** of chiral-EFT based nuclear interactions in a **data-driven** approach.



- ▶ Does nuclear-physics phenomena emerge in a "from few to many" ab initio approach?
- ▶ Is available few-body data sufficient to constrain this model? Does the model become fine-tuned?

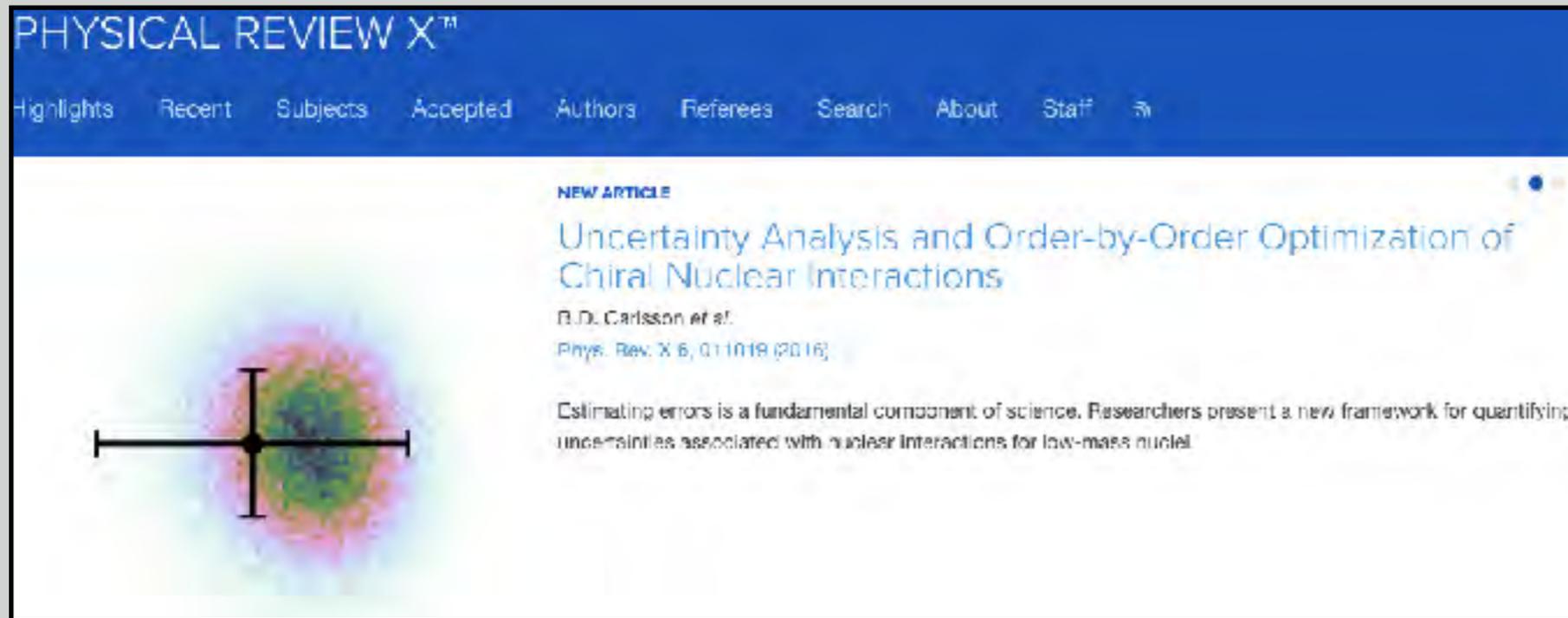
Explore **alternative strategies of informing** the model about low-energy many-body observables.



- ▶ Can/should emergent phenomena be used to constrain the model?
- ▶ How to quantify model uncertainties in such an approach?

Based on: B.D. Carlsson, A. Ekström, C. Forssén et al, Phys. Rev. X **6** (2016) 011019

B. D. Carlsson et al., In preparation



THEORETICAL UNCERTAINTY QUANTIFICATION

FROM NN TO $A=4$ WITH CHIRAL EFT AND ERROR ANALYSIS

OPTIMIZATION STRATEGY

Low-energy constants (LECs) are the parameters of the EFT. In practice they need to be fitted to experimental data.

$$\chi^2(\vec{p}) \equiv \sum_i \left(\frac{O_i^{\text{theo}}(\vec{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2 \equiv \sum_i r_i^2(\vec{p})$$

Historic approach:

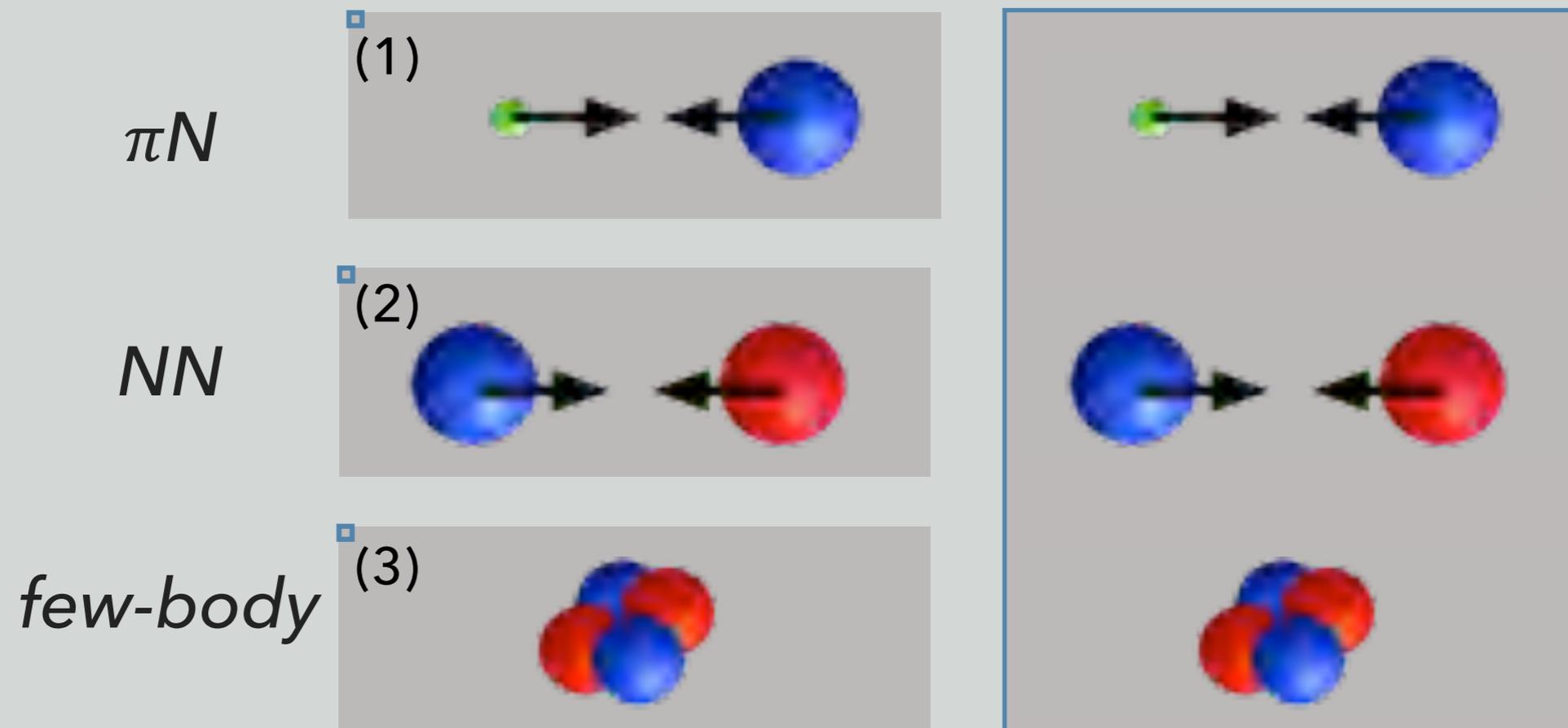
- nN LECs determined first;** either from Pion-Nucleon scattering phase shifts or from NN phase shifts in peripheral waves
- (NN-only) objective function based on Nijmegen phase shift analysis**
 - Chi-by-eye optimization; "it's an art" (Machleidt)
 - N³LO needed for high-accuracy fit up to $T_{\text{lab}}=290$ MeV
- NNN LECs determined at the end** given the NN part. Usually at NNLO. First results at N³LO are coming.

OPTIMIZATION STRATEGY

**Low-energy constants (LECs) are the parameters of the EFT.
In practice they need to be fitted to experimental data.**

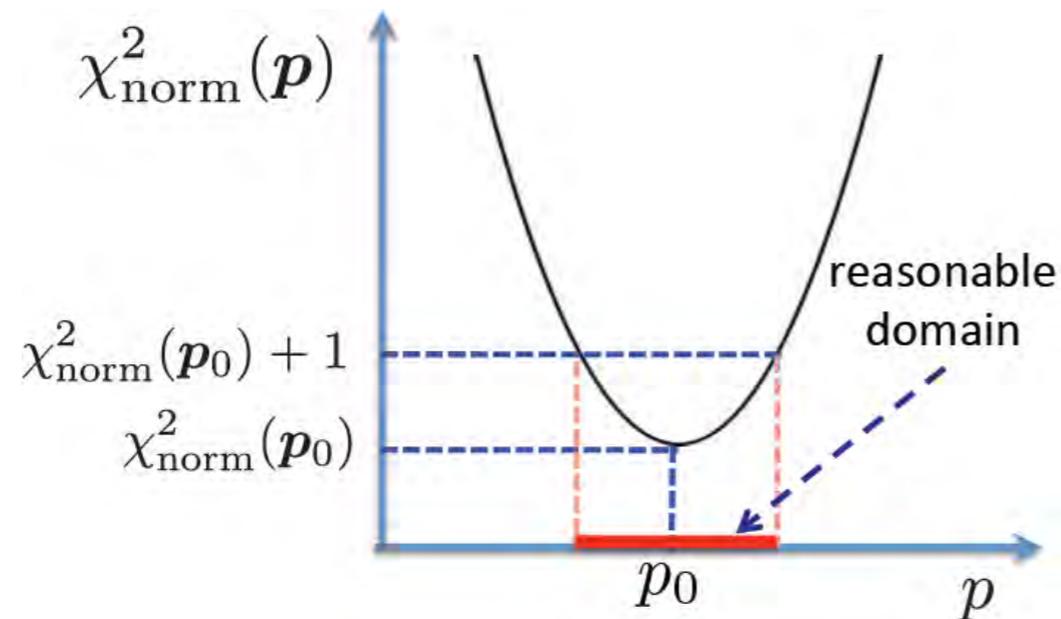
$$\chi^2(\vec{p}) \equiv \sum_i \left(\frac{O_i^{\text{theo}}(\vec{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2 \equiv \sum_i r_i^2(\vec{p})$$

Sequential (historic) approach: **Simultaneous** approach:

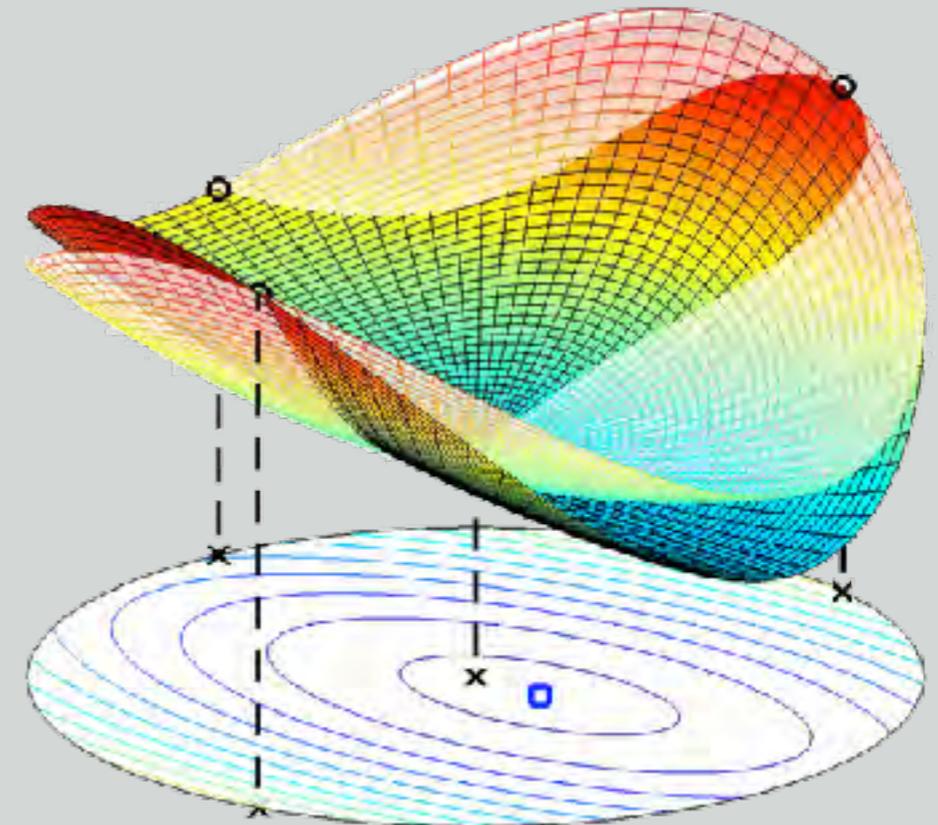


Statistical error analysis

- ▶ In a minimum there will be an **uncertainty in the optimal parameter values** \mathbf{p}_0 given by the χ^2 surface.¹



- ▶ From the hessian at \mathbf{p}_0 we can calculate a **covariance matrix** and from that a **correlation matrix**.



¹J Dobaczewski et al 2014 J. Phys. G: Nucl. Part. Phys. 41 074001

HESSIAN

$$H_{ij} = \frac{1}{2} \left. \frac{\partial^2 \chi^2}{\partial x_i \partial x_j} \right|_{\mathbf{x}=\mathbf{x}_\mu}$$

COVARIANCE MATRIX

$$\Sigma = \frac{\chi^2}{N_{df}} \mathbf{H}^{-1}$$

CORRELATION MATRIX

$$R_{ij} = \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii}\Sigma_{jj}}}$$

Input and technology

πN scattering

- WI08 database
- T_{lab} between 10-70 MeV
- $N_{\text{data}} = 1347$
- $\chi^{\text{EFT}}(Q^4)$ to avoid underfitting

NN scattering

- SM99 database (+Granada)
- T_{lab} between 0-290 MeV
- $N_{\text{data}} = 2400(\text{np}) + 2045(\text{pp})$
- $\chi^{\text{EFT}}(Q^0, Q^2, Q^3, Q^4)$

All 6000 residuals computed on 1 node in ~90 sec.

A=3 bound states

- ${}^3\text{H}, {}^3\text{He}$ (binding energy, radius, ${}^3\text{H}$ half life)

On 1 node in ~10 sec

+ derivatives! ($\times 2-20$ cost)

Total error budget

$$\chi^2(\vec{p}) \equiv \sum_i \left(\frac{O_i^{\text{theo}}(\vec{p}) - O_i^{\text{expr}}}{\sigma_{\text{tot},i}} \right)^2$$

- ▶ The total error budget is

$$\sigma_{\text{tot}}^2 = \sigma_{\text{exp}}^2 + \sigma_{\text{method}}^2 + \sigma_{\text{numerical}}^2 + \sigma_{\text{model}}^2$$

E.g., NCSM Neglected

- ▶ At a given chiral order ν , the omitted diagrams should be of order

$$\mathcal{O} \left((Q/\Lambda_\chi)^{\nu+1} \right)$$

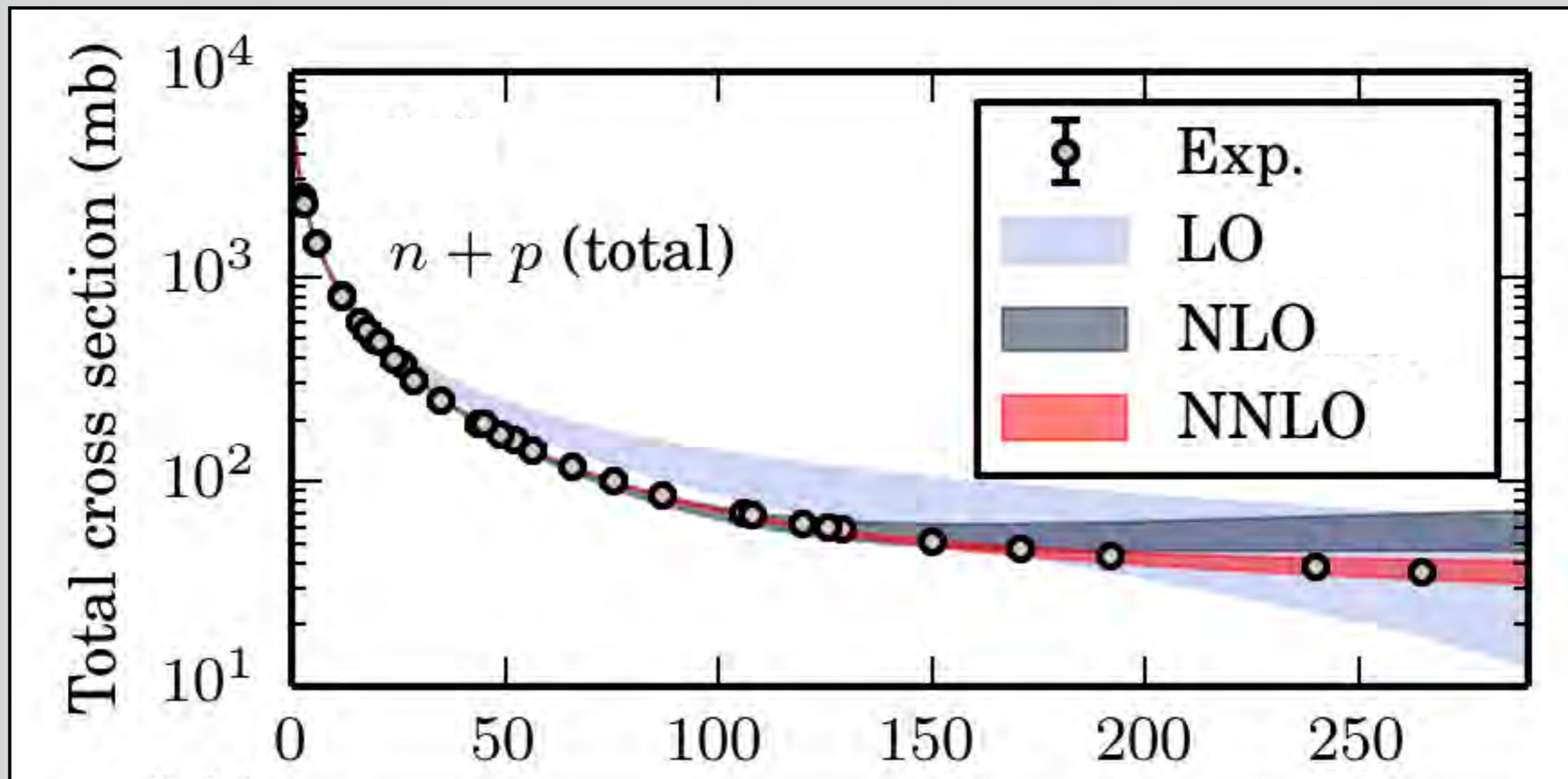
- ▶ Still needs to be converted to actual numbers σ_{model}

- ▶ We translate this EFT knowledge into an error in the scattering amplitudes

$$\sigma_{\text{model},x}^{(\text{amp})} = C_x \left(\frac{Q}{\Lambda_\chi} \right)^{\nu+1}, \quad x \in \{NN, \pi N\}$$

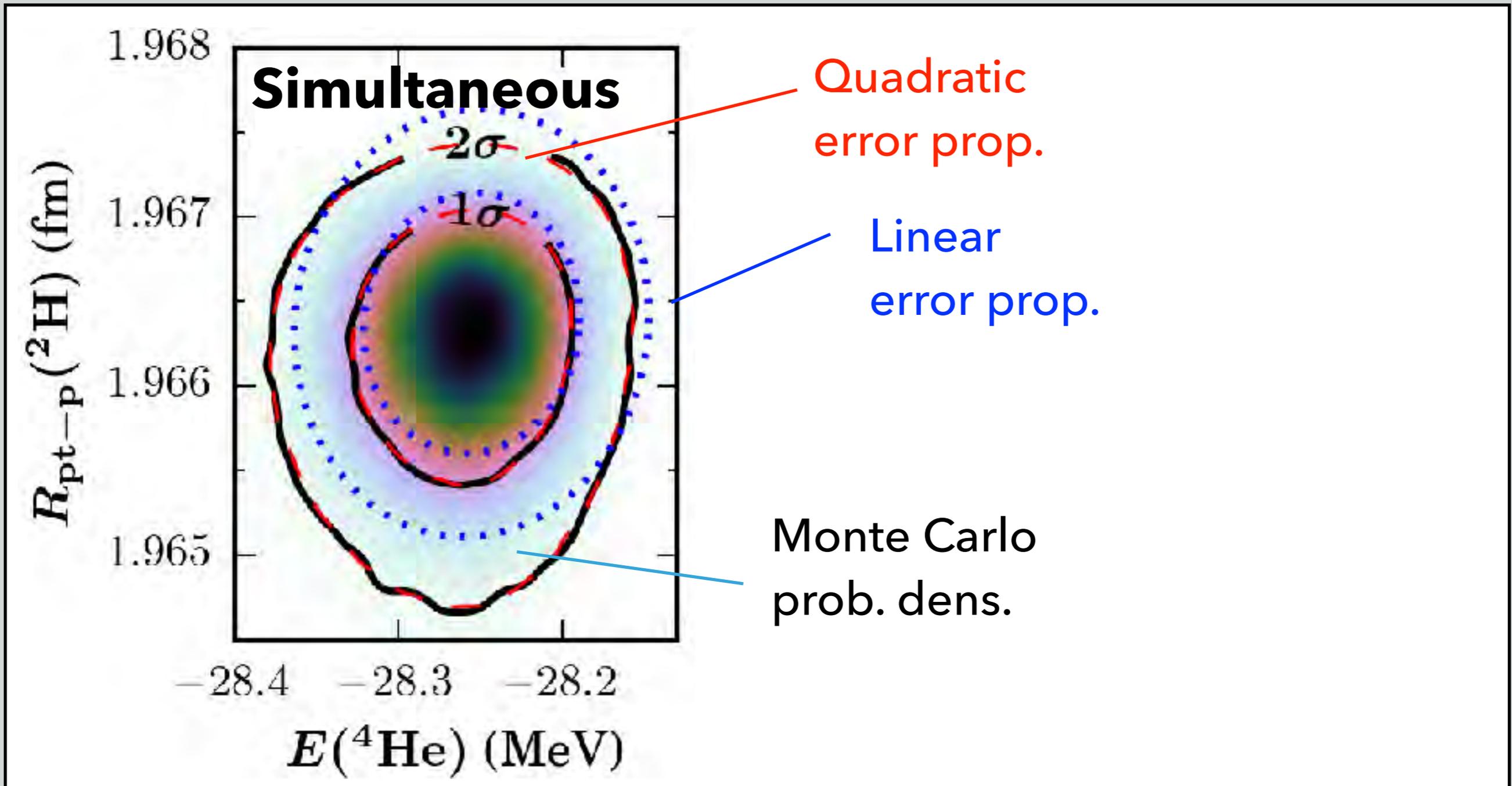
- ▶ which is then propagated to an error in the observable.

TOTAL NP CROSS SECTION



Quadratic error propagation vs Brute force sampling

$$O(\mathbf{p}) \approx O(\mathbf{p}_0) + J_O \Delta \mathbf{p} + \frac{1}{2} \Delta \mathbf{p}^T H_O \Delta \mathbf{p}$$



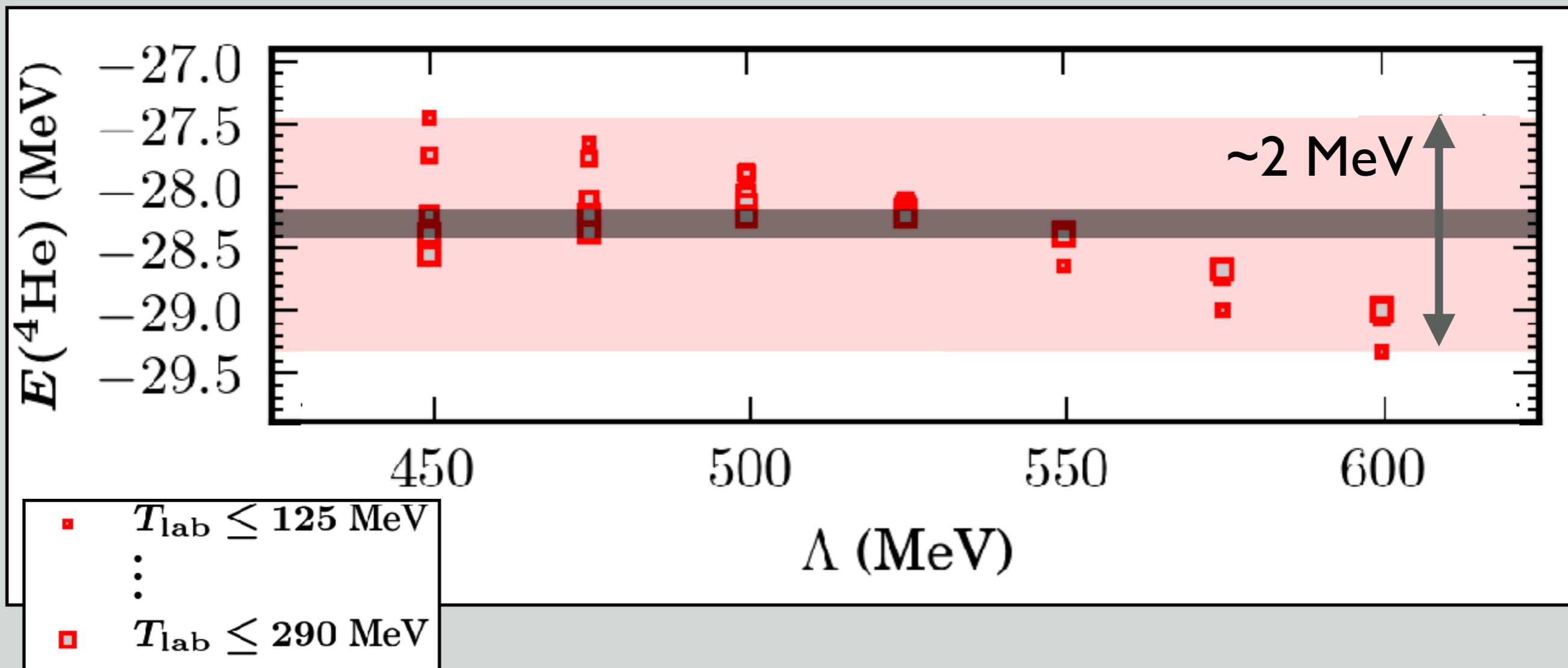
$$E(^4\text{He}) = -28.24_{-11}^{+9} \text{ (MeV)}$$

Statistical uncertainty

EXPLORING FURTHER SYSTEMATIC UNCERTAINTIES

- ▶ So far, all results have been obtained with a non-local regulator with cutoff $\Lambda=500$ MeV.
 - ▶ A subset of systematic uncertainties can be probed by varying Λ .
- ▶ The bulk of input data comes from NN scattering. We have truncated the data base at $T_{\text{lab}}=290$ MeV
 - ▶ Always with model error that gives more weight to low E .
 - ▶ A subset of systematic uncertainties can be probed by varying the truncation $\max(T_{\text{lab}})$
- ▶ Reoptimizing with different Λ and T_{lab} and will give us a **family** of models.
- ▶ **All of them will reproduce the same few-body physics.**

Systematic uncertainties: input data, regulator cutoff



- ▶ 7 different regulator cutoffs:
 $\Lambda=450, 475, \dots, 575, 600$ MeV
- ▶ 6 different *NN*-scattering datasets
 $T_{\text{lab}} \in [0, T_{\text{lab,max}}]$, with
 $T_{\text{lab,max}}=125, \dots, 290$ MeV

Do-it-yourself

All 42 different sim/sep potentials, as well as the respective covariance matrices are available as supplemental material.

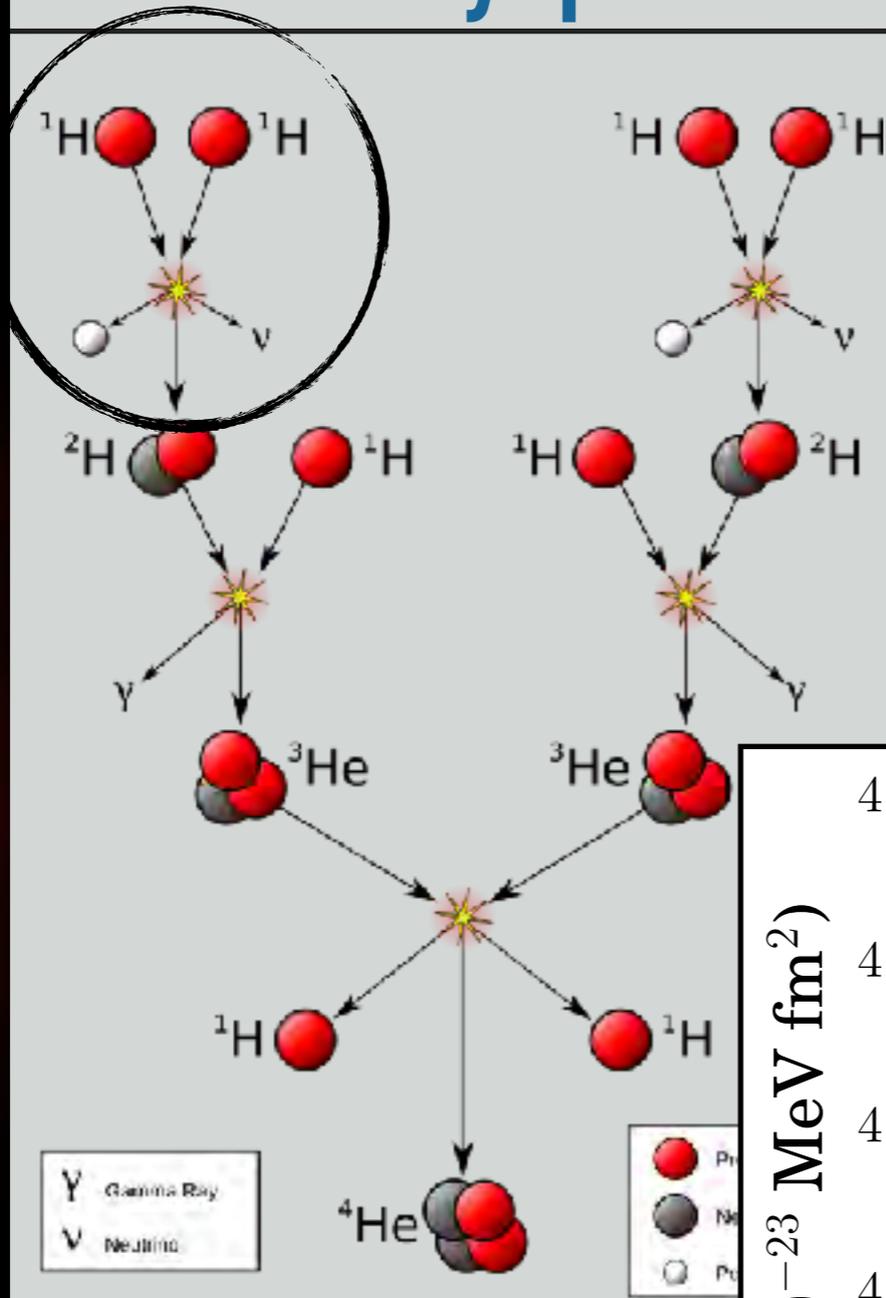
- ▶ LO-NLO-NNLO
- ▶ with 7 different cutoffs:
450,475,...,600 MeV
- ▶ from 6 different NN-scattering datasets

$$\begin{aligned}\text{Cov}(\mathbf{A}, \mathbf{B}) &\equiv \mathbb{E}[(\mathcal{O}_A(\mathbf{p}) - \mathbb{E}[\mathcal{O}_A(\mathbf{p})])(\mathcal{O}_B(\mathbf{p}) - \mathbb{E}[\mathcal{O}_B(\mathbf{p})])] \\ &\approx \mathbb{E}\left[\left(\tilde{\mathbf{J}}_{A,i}x_i + \frac{1}{2}\tilde{H}_{A,ij}x_ix_j - \frac{1}{2}\tilde{H}_{A,ii}\sigma_i^2\right)\right. \\ &\quad \left.\times \left(\tilde{\mathbf{J}}_{B,k}x_k + \frac{1}{2}\tilde{H}_{B,kl}x_kx_l - \frac{1}{2}\tilde{H}_{B,kk}\sigma_k^2\right)\right] \\ &= \tilde{\mathbf{J}}_A^T \boldsymbol{\Sigma} \tilde{\mathbf{J}}_B + \frac{1}{2}(\boldsymbol{\sigma}^2)^T (\tilde{\mathbf{H}}_A \circ \tilde{\mathbf{H}}_B) \boldsymbol{\sigma}^2,\end{aligned}$$

compute the derivatives of your own observables wrt LECs, then explore:

- ▶ cutoff variations
- ▶ order-by-order evolution
- ▶ LEC UQ/correlations

Uncertainty quantification applied to pp fusion

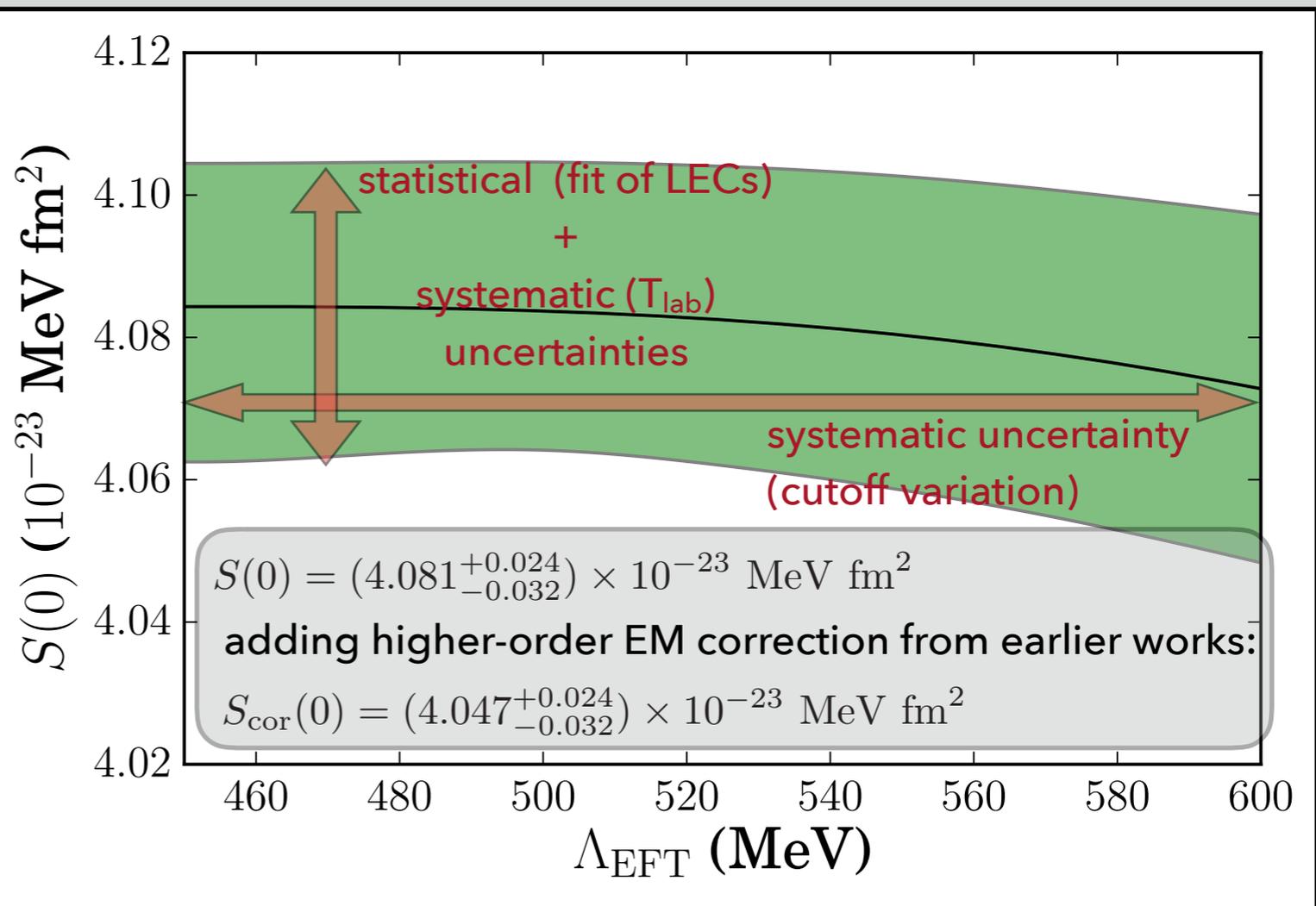


L. E. Marcucci et al PRL 110, 192503 (2013)
 R. Schiavilla et al PRC 58, 1263 (1998)
 J-W. Chen et al. PLB 720, 385 (2013)

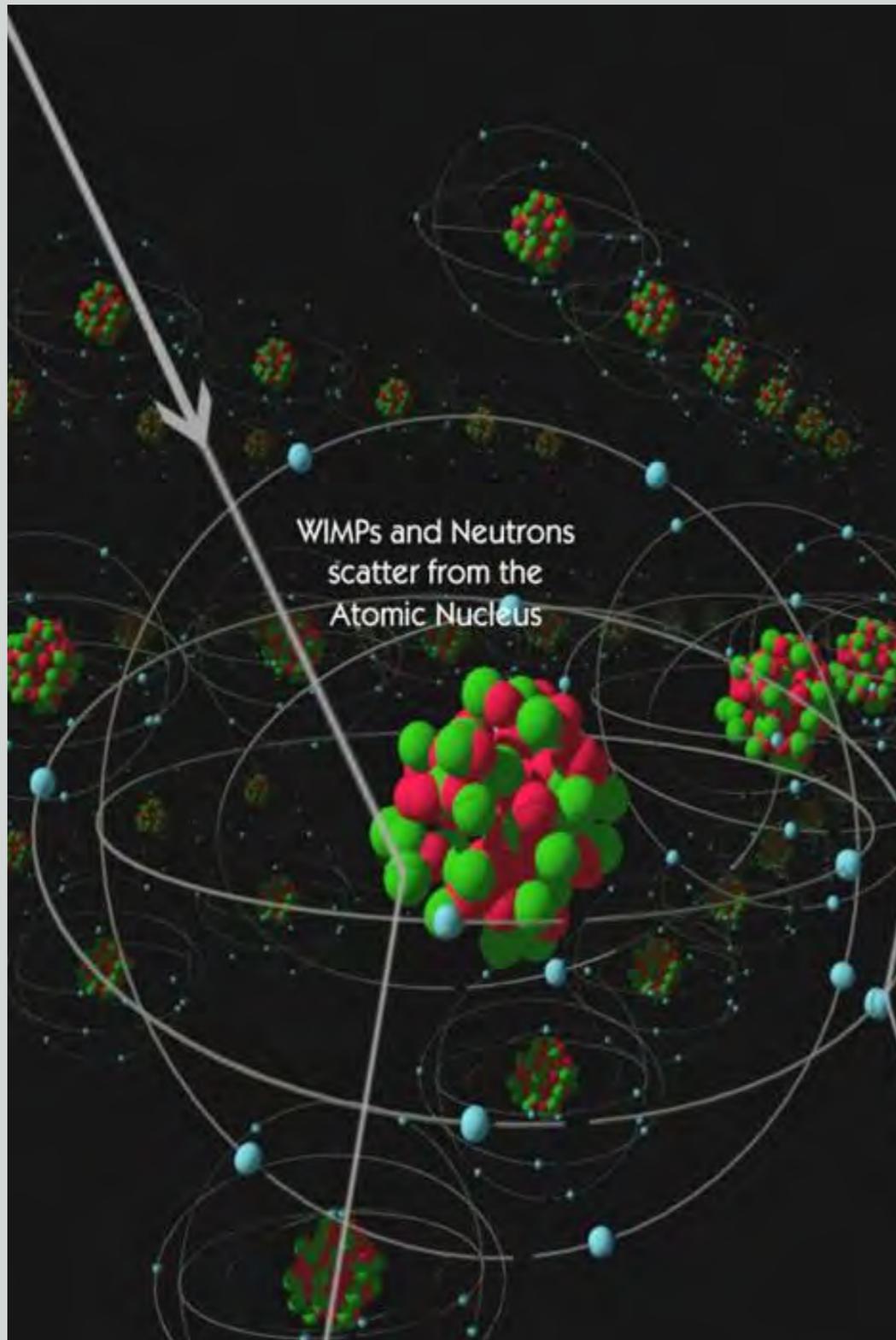
$$p + p \rightarrow d + e^+ + \nu_e$$

$$S(E) = \sigma(E) E e^{2\pi\eta}$$

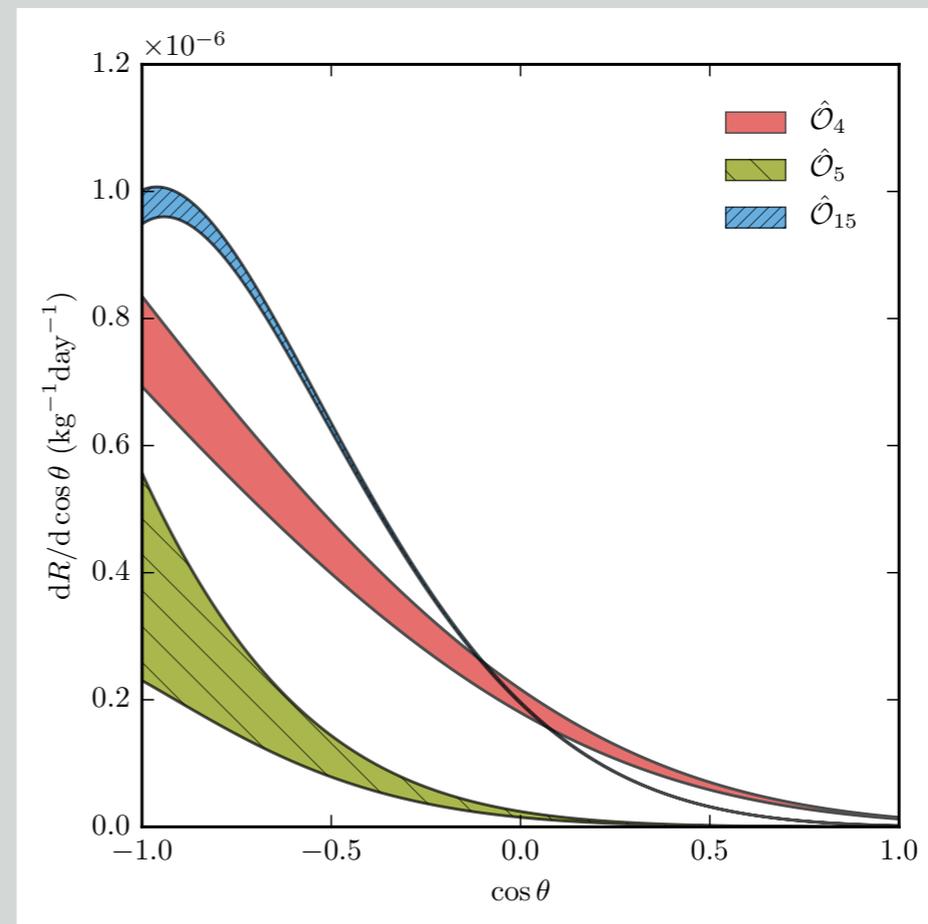
B. Acharya et al, Phys. Lett. B 760 (2016) 584



Uncertainty quantification applied to dark-matter nucleus scattering



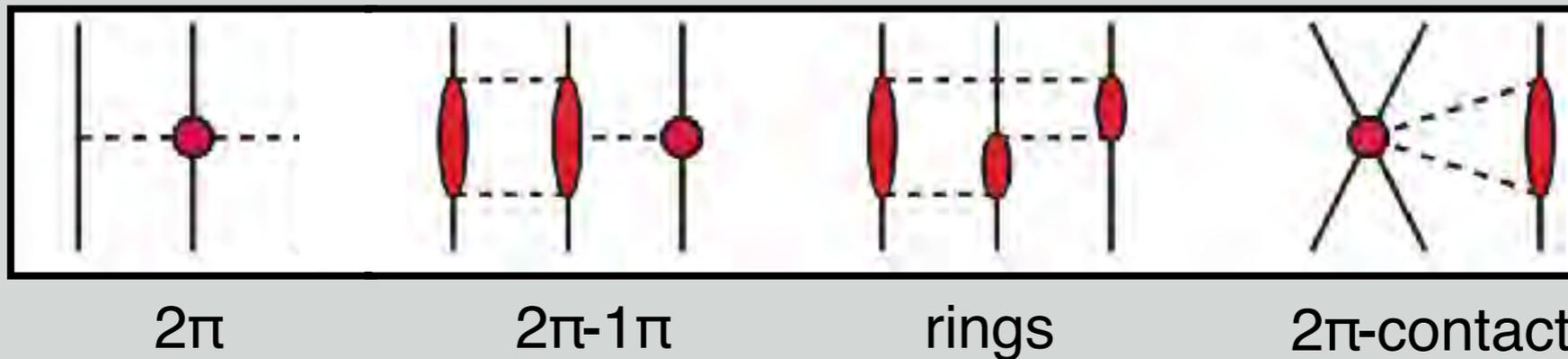
- ▶ WIMP scattering off $^3,^4\text{He}$ described in NR-EFT
- ▶ Nuclear response functions from NCSM wave functions
- ▶ Studied rates of dark matter-nucleus scattering events



D. Gazda et al, arXiv:1612.09165

Work in progress: N3LO

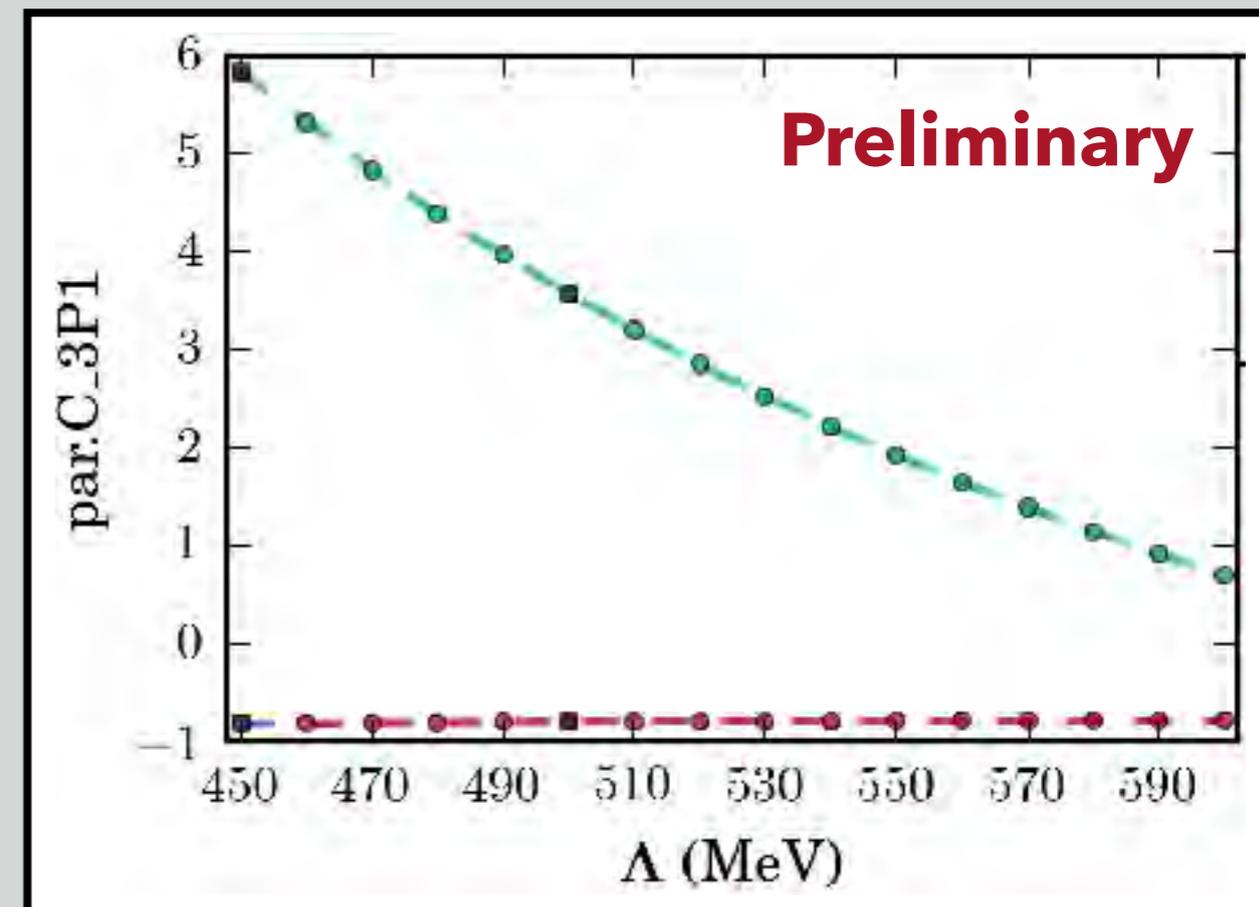
N3LO optimizations are challenging



41 parameters to optimize,
3NF matrix elements recently made
available (K. Hebeler)

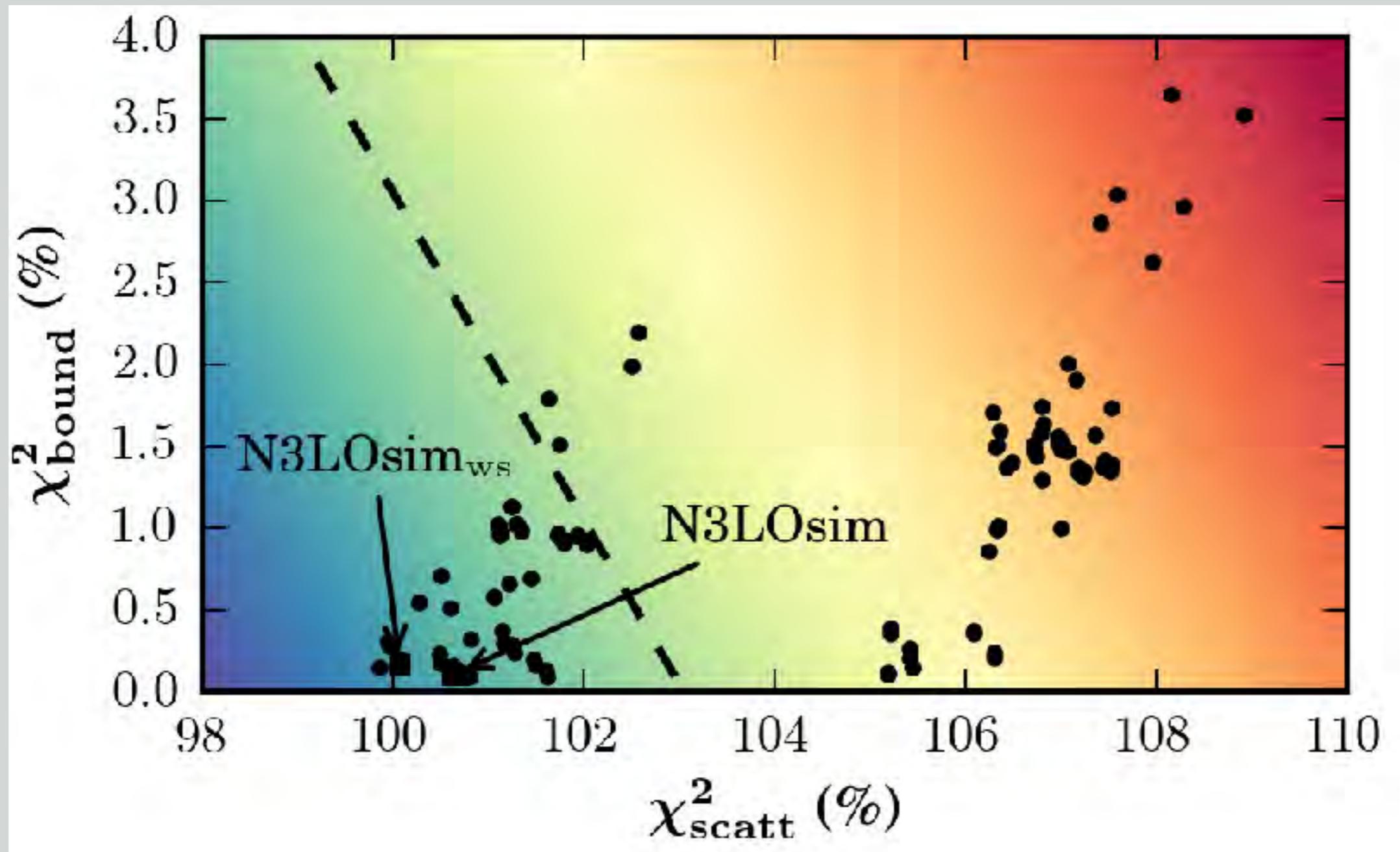
Initialize by computing phase shifts for 10^5
random contact LEC values for each
partial wave and select the ~ 1000 best
values and optimize. This leads to
[5x2x2x2x2=160] different optima (for
cutoff 500 MeV) with respect to phase
shifts. (π -N LECs from sep-optimization).

These minima perform equally well in the
NN sector. But the LECs display rather
different Λ -dependence.

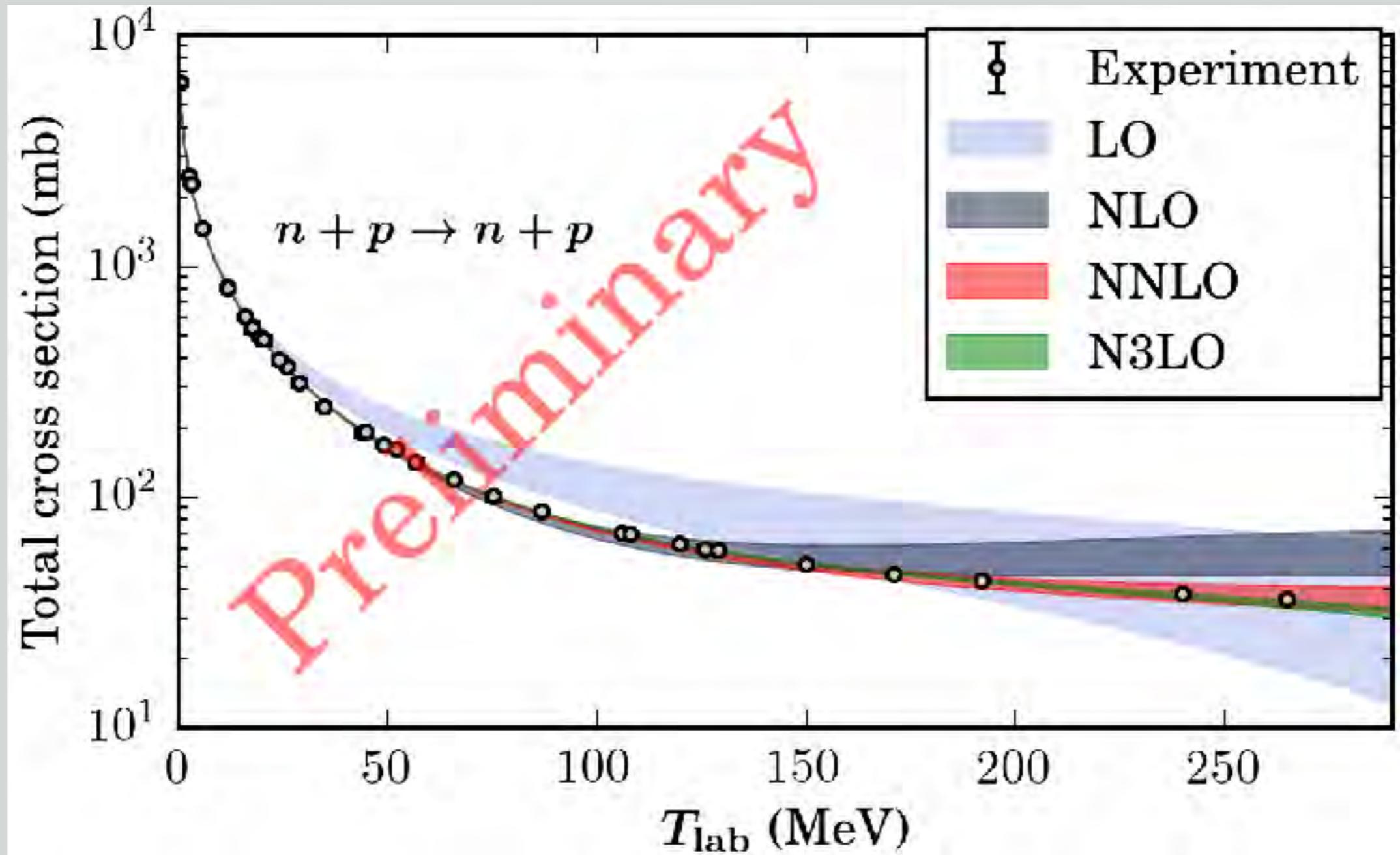


N3LO MINIMA

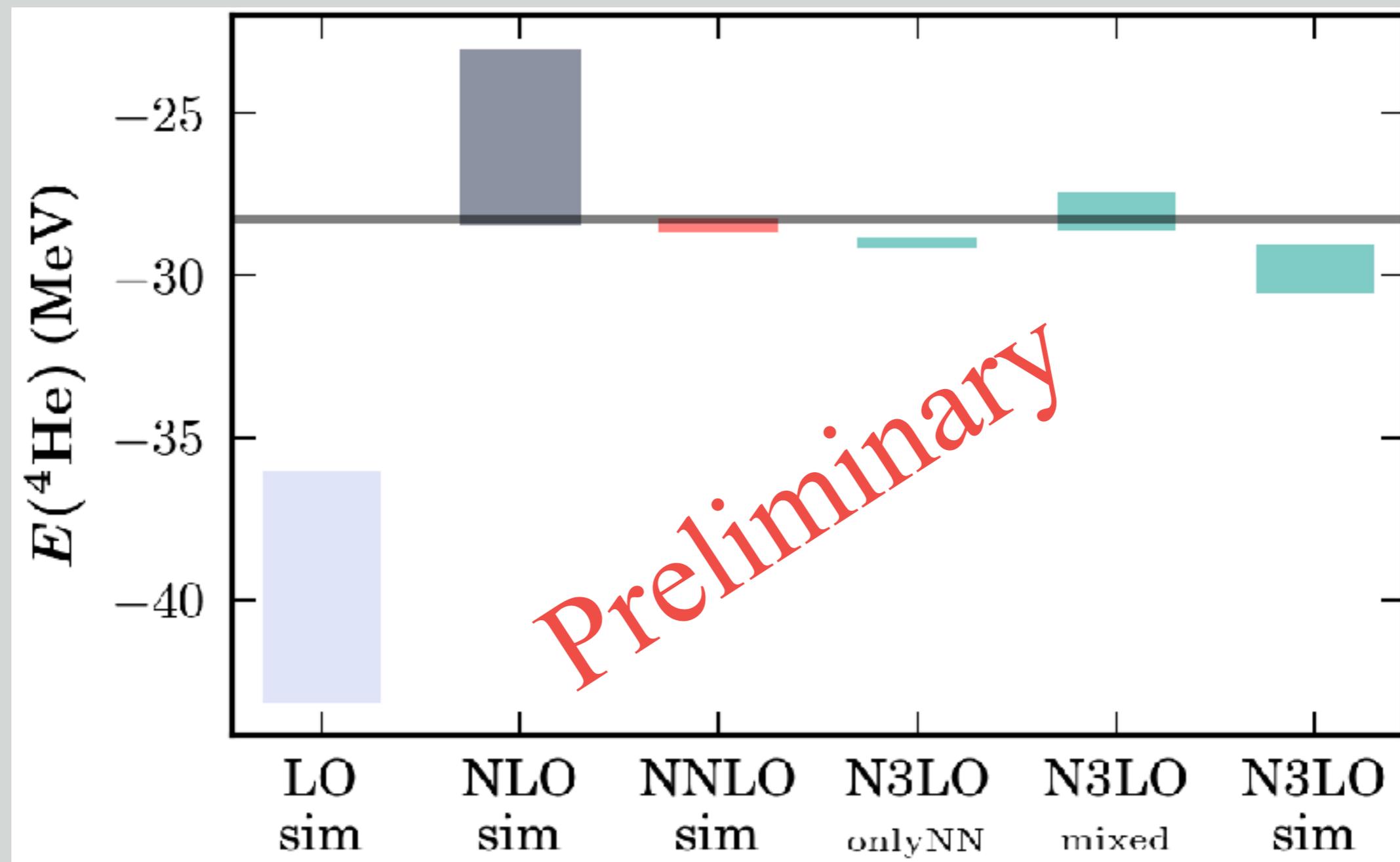
≈ 100 optima remain after performing simultaneous optimizations from these starting points



N3LO SCATTERING



SYSTEMATIC UNCERTAINTIES: INPUT DATA, REGULATOR CUTOFF



Based on: A. Ekström et al, Phys. Rev. C **91** (2015) 051301(R)

Rapid Communication

Accurate nuclear radii and binding energies from a chiral interaction

A. Ekström, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, B. D. Carlsson, C. Forssén, M. Hjorth-Jensen, F. Navrátil, and W. Nazarewicz

Phys. Rev. C **91**, 051301(R) – Published 1 May 2015

See also: G. Hagen et al, Nat. Phys. **12** (2015) 186



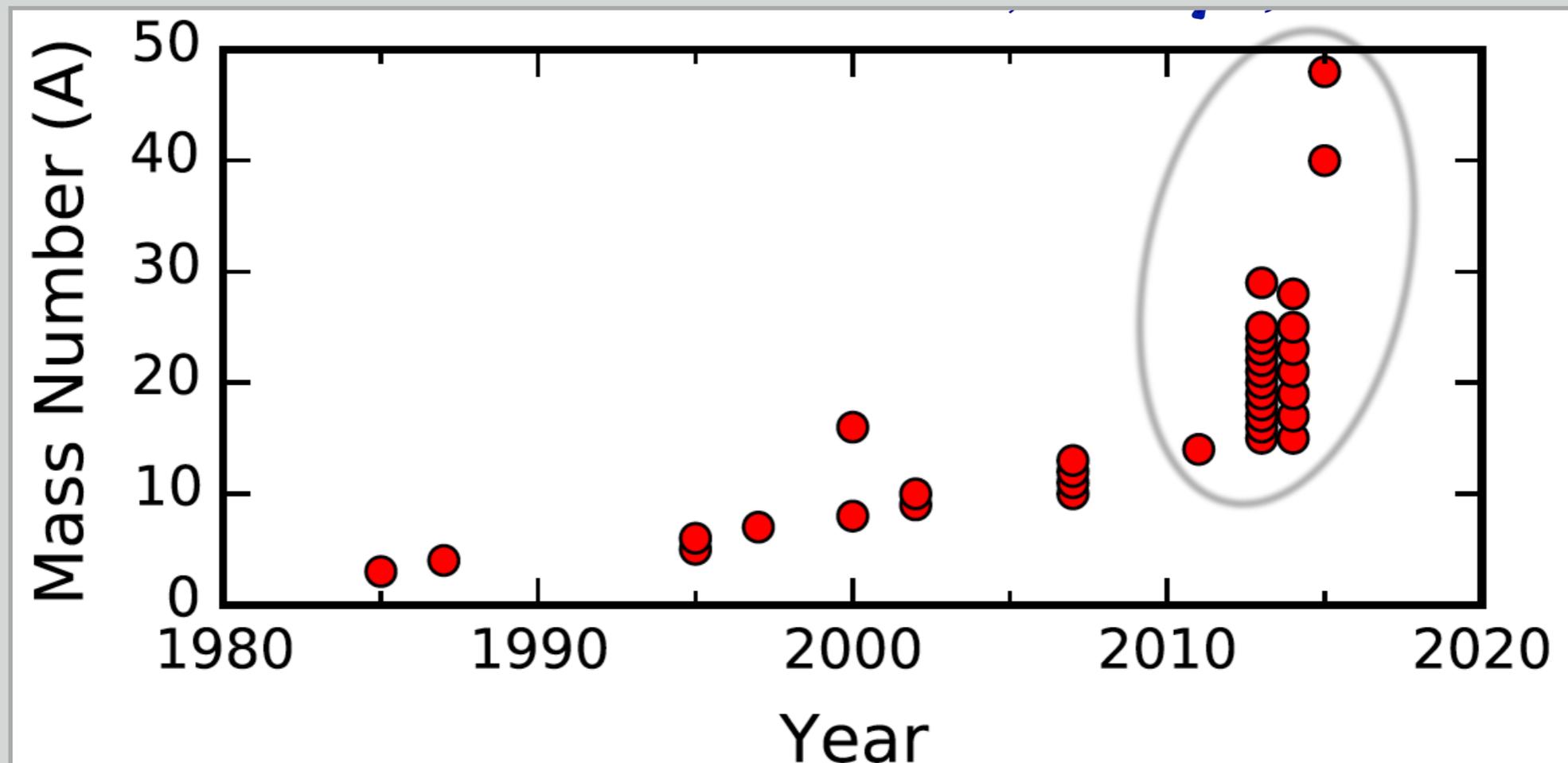
FROM FEW TO MANY

IS NUCLEAR SATURATION AN EMERGENT PHENOMENON?

TREND IN REALISTIC AB INITIO CALCULATIONS

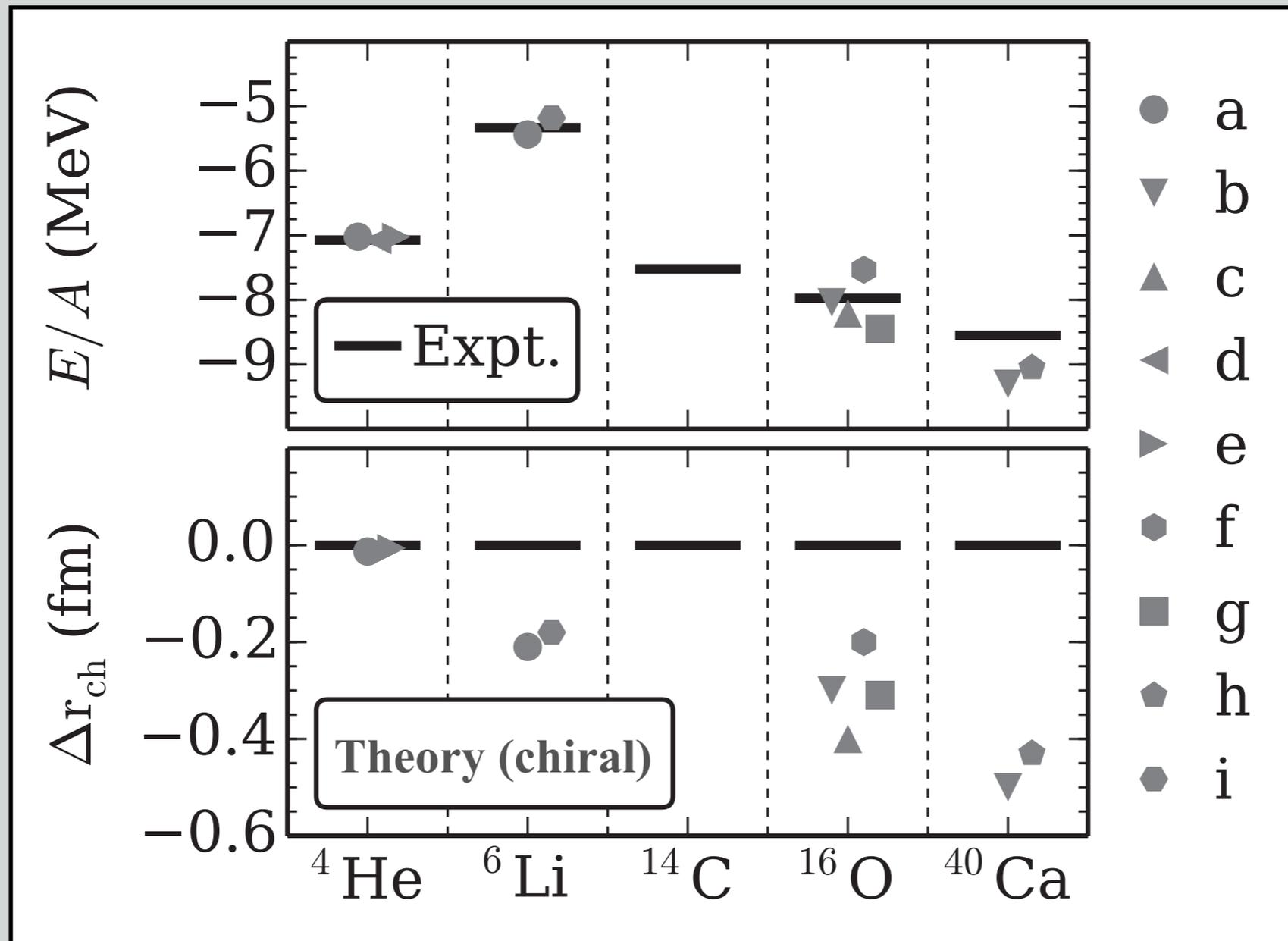
Explosion of many-body methods

(Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT, No-Core Shell Model, Self-Consistent Green's Function, UMOA, ...)



“Computational capabilities exceed accuracy of available interactions”

STATUS OF CHIRAL-FORCE PREDICTIONS

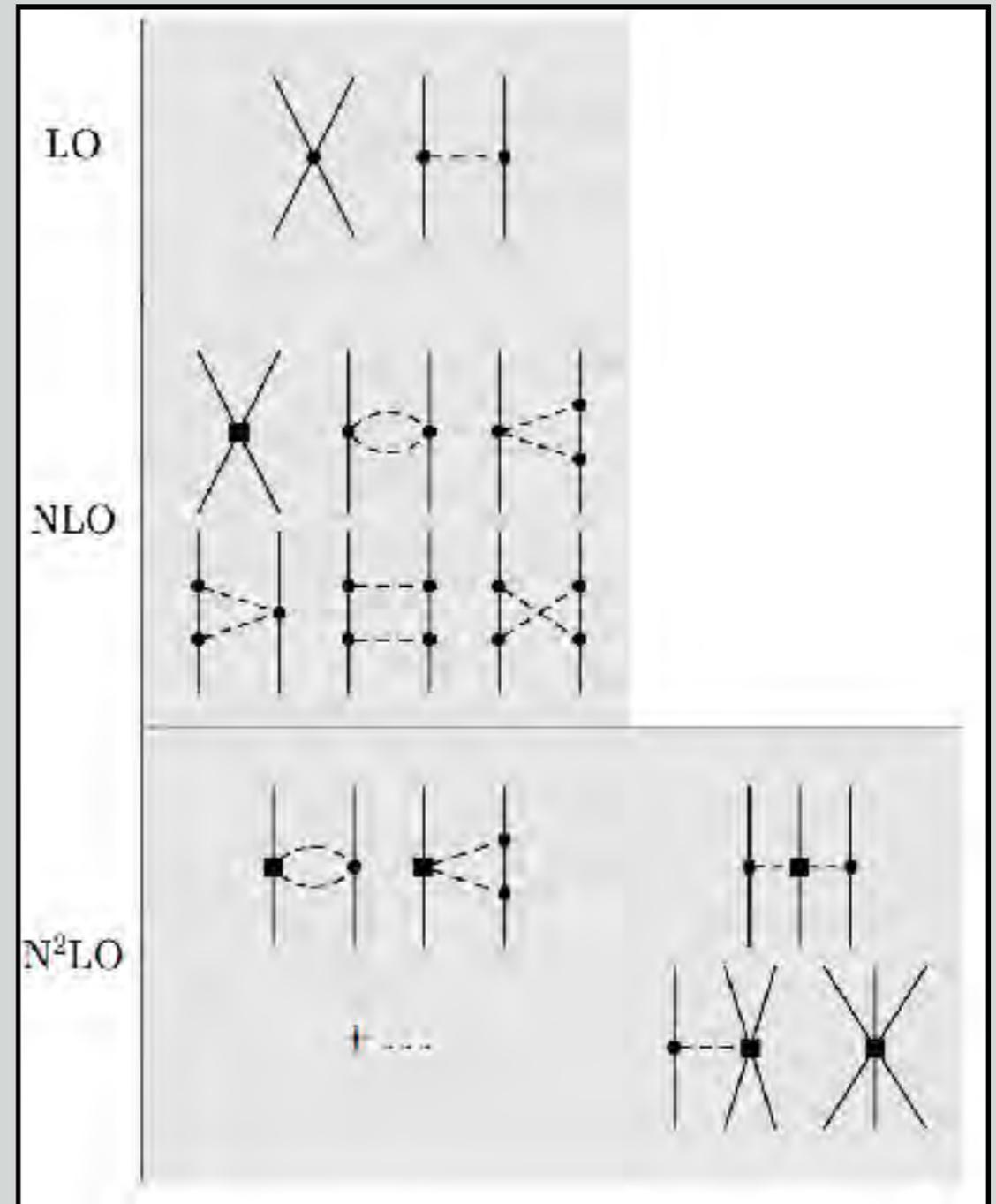


Ab initio calculations with existing chiral interactions

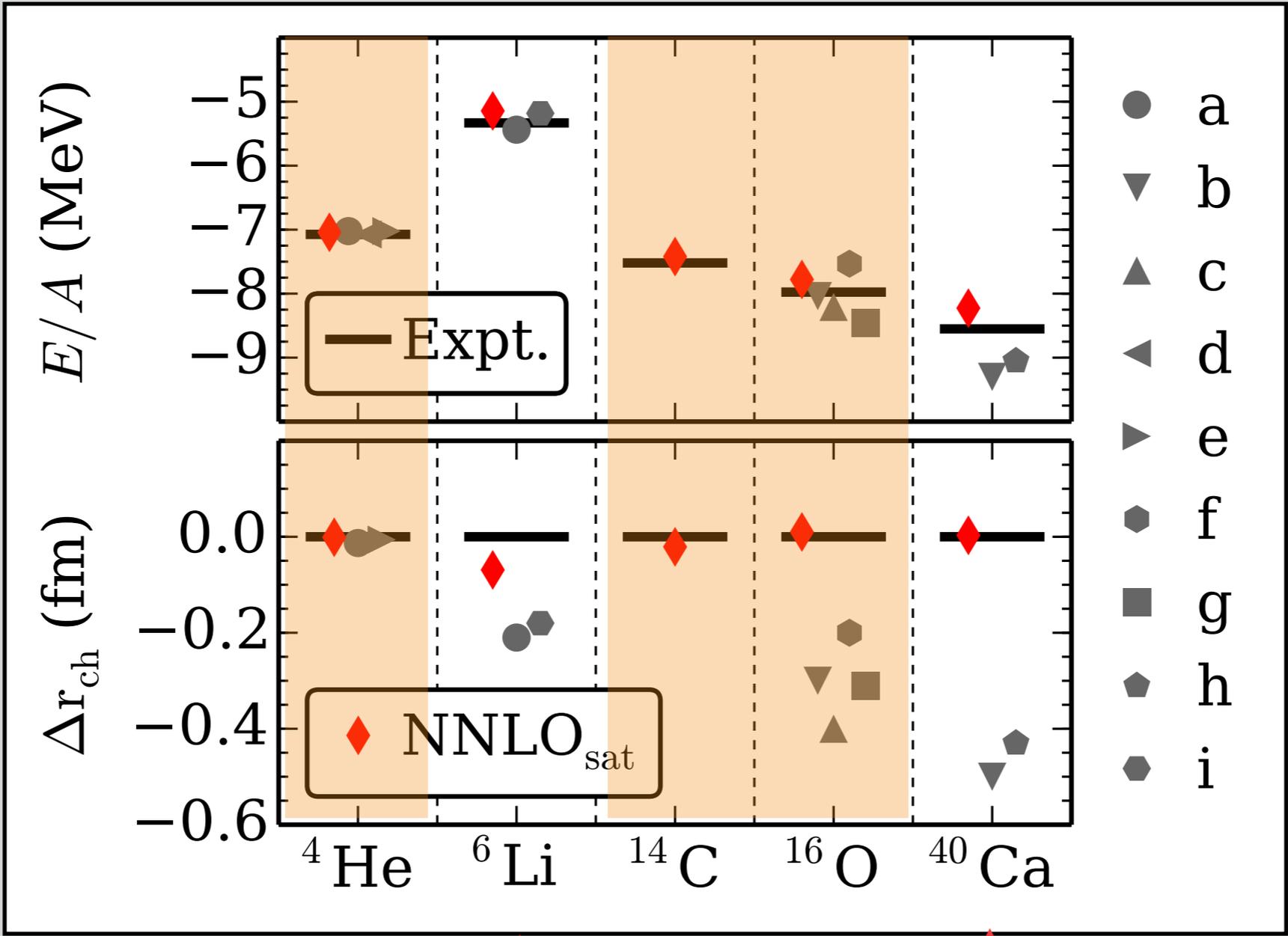
- **overbind** medium-mass and heavy nuclei, and
- **underestimate charge radii.**

PRAGMATIC OPTIMIZATION STRATEGY

- ▶ Simultaneous optimization of NN and NNN LECs at NNLO.
- ▶ NCSM and CC calculations are performed within the optimization
- ▶ Objective function contains:
 - ▶ deuteron properties and NN scattering data ($T_{\text{lab}} < 35$ MeV)
 - ▶ $A=3,4$ binding energies, radii
 - ▶ $^{14}\text{C}, ^{16}\text{O}$ binding energies, radii
 - ▶ $^{22,24,25}\text{O}$ binding energies



CHIRAL INTERACTION WITH ACCURATE SATURATION: N^2LO_{SAT}

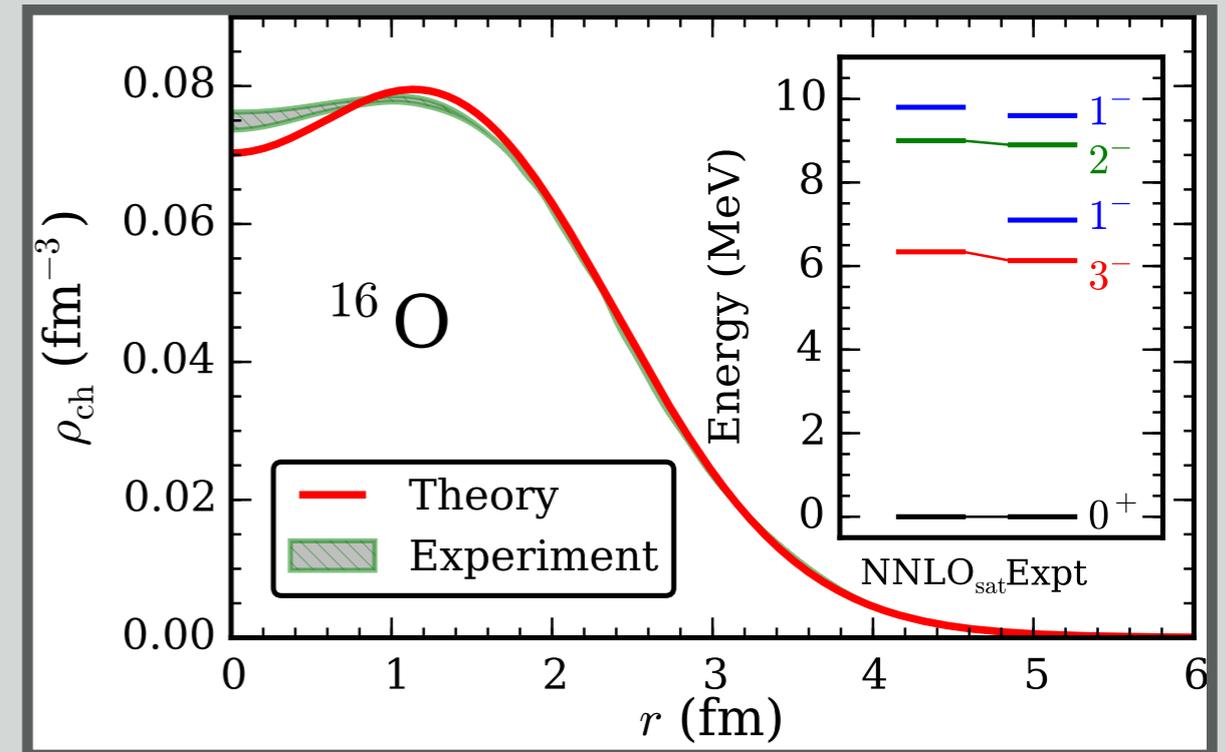
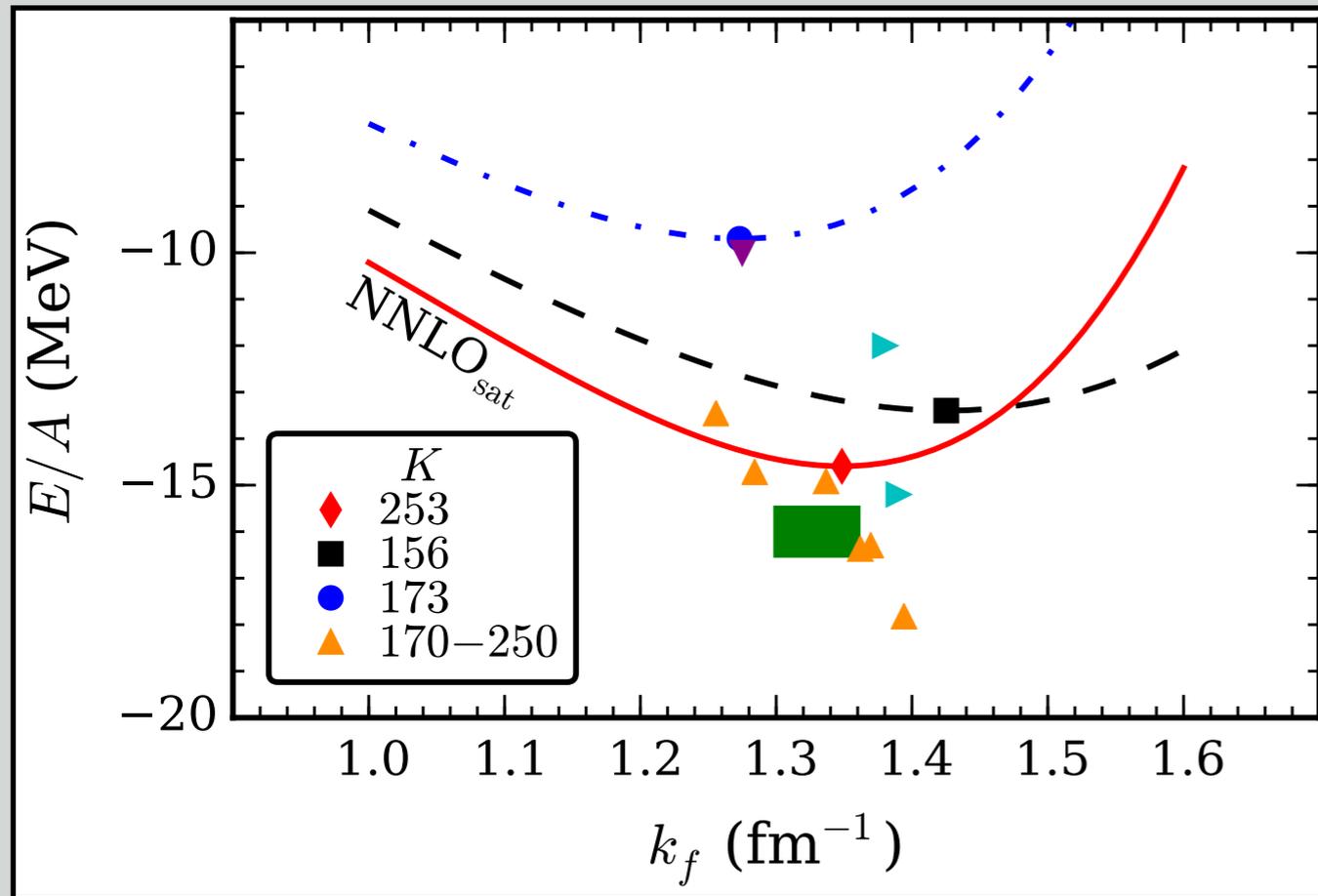


Interpolation

Extrapolation

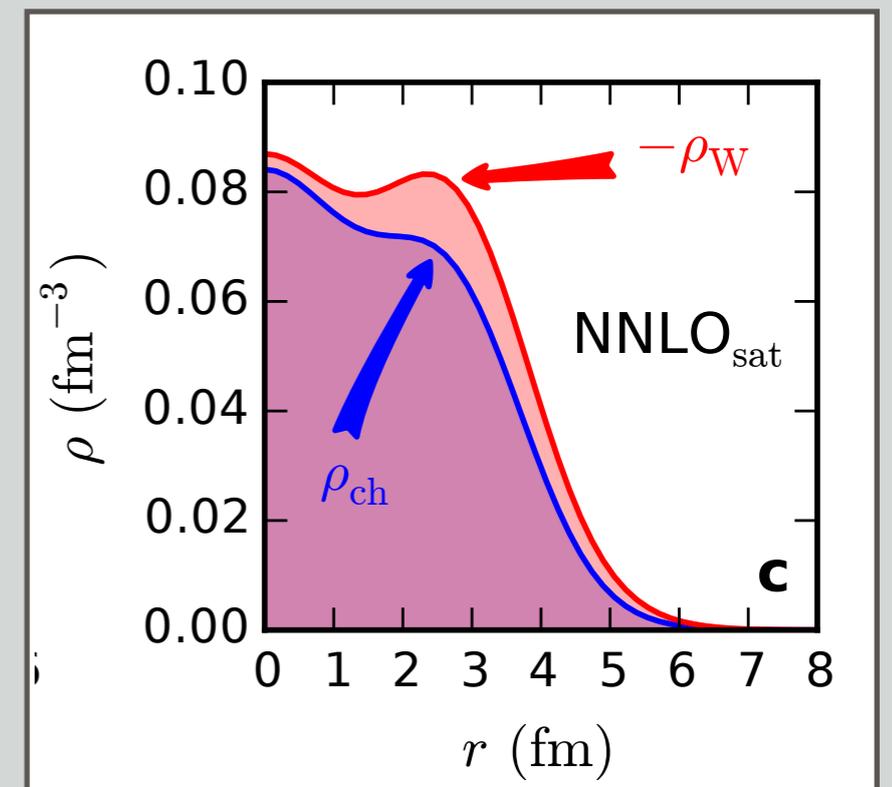
CHARGE, NEUTRON DISTRIBUTION, AND WEAK SIZE OF THE ATOMIC NUCLEUS

Accurate saturation



^{16}O charge radius

Neutron skin of ^{48}Ca



CONCLUSION

Chiral EFT with error analysis

- ▶ **Uncertainty quantification** is a **unique opportunity** when employing **systematic approaches (EFT + ab initio)**.
- ▶ First results for correlations, parameter uncertainties and **error propagation in the few and many-body sectors**.
- ▶ **Simultaneous optimization of all LECs** at LO, NLO, NNLO, N3LO using NN, NNN and piN data is critical in order to:
 - ▶ capture all correlations between the parameters, and
 - ▶ reduce the statistical errors.
- ▶ We find that **statistical errors** are small ($\approx 1\%$), and the total error budget is dominated by **systematic model errors**. Statistical errors increase dramatically for sequentially optimized potentials.