

An aerial photograph of Jerusalem, showing the Old City with its dense, light-colored buildings and the Dome of the Rock with its prominent golden dome. The city is surrounded by hills and modern urban areas.

Effective field theory for lattice nuclei

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The Racah institute for Physics
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CEA/SPhN

The Tower of effective (field) theories and the emergence of nuclear phenomena
16-20 January, 2017

האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem





האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

Jerusalem, Israel

D. Gazit, J. Kirscher, E. Pazy



UNIVERSITY
OF TRENTO - Italy

Trento, Italy

F. Pederiva, L. Contessi

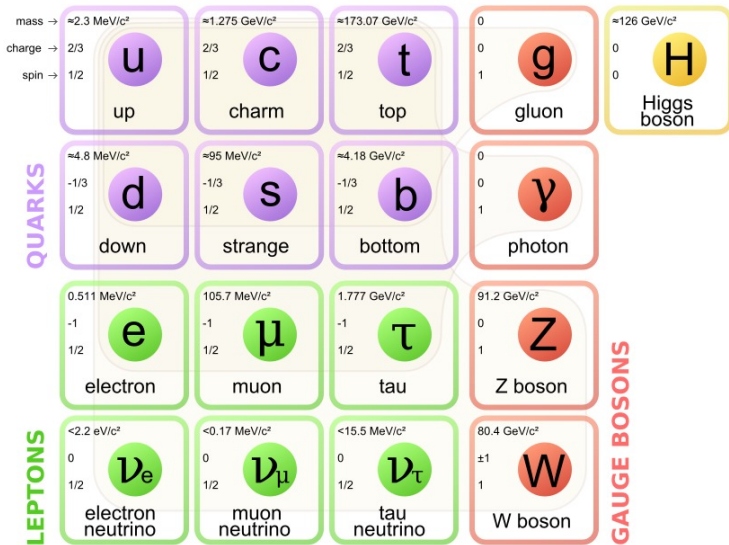


Orsay, France

U. van Kolck

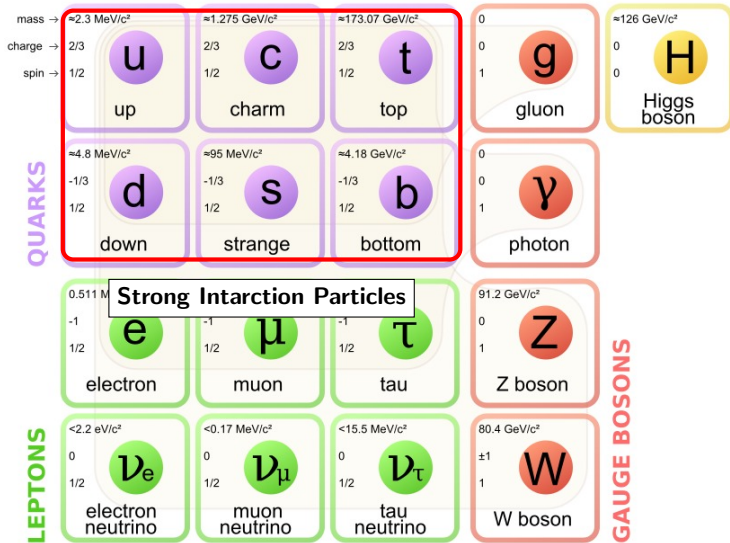
Introduction - the elementary particles

Figure thanks to WIKIPEDIA



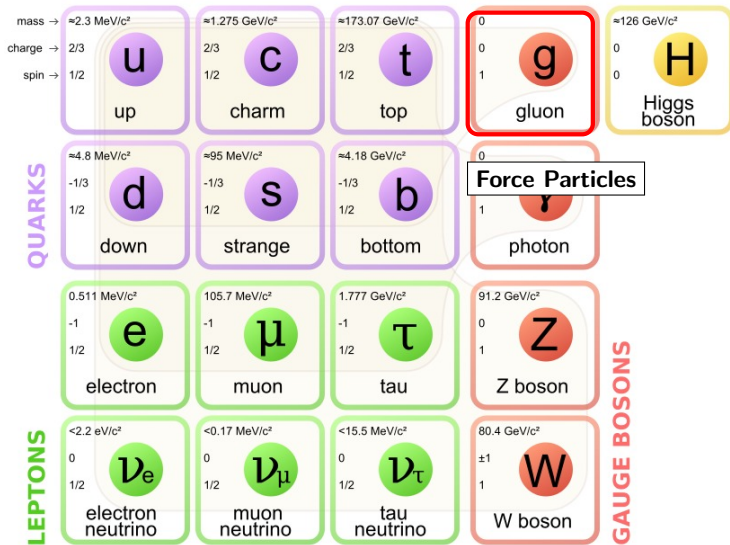
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The QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{a\mu\nu} G^{a\mu\nu}$$

Some things you (didn't) know about quarks

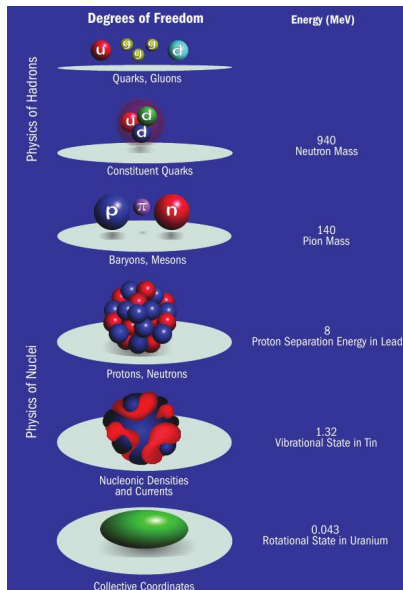
- Quarks don't roam freely on campus
- They usually come in groups of 2 and 3
 - qqq - Baryons (neutron - udd , proton - uud , ..)
 - $\bar{q}q$ - Mesons (π -meson, ρ -meson, ...)
- They carry electro-weak and **COLOR** charges
- The **COLOR** interaction is better known as **Q**uantum **C**hromo **D**ynamics
- QCD is **very weak** at high energy, but **very strong** at low energies

Energy Scales

The emergence of nuclear phenomena

The ladder of nuclear phenomena

- The QCD energy scale
 $\Lambda_{QCD} \approx M_p, M_n \approx 1000 \text{ MeV}$
dof: quarks and gluons
- The pion is the lightest meson
 $m_\pi \approx 100 \text{ MeV}$
- Nuclear binding energy $B/A \approx 10 \text{ MeV}$
dof: nucleons and pions
- Nuclear vibrations $E_v \approx 1 \text{ MeV}$
- Rotational excitations $E_r \approx 0.1 \text{ MeV}$
dof: collective excitations



The QCD Lagrangian

The zero mass limit, $\Lambda_{\text{QCD}} \gg m_u, m_d \approx 0$

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{a\mu\nu} \mathcal{G}^{a\mu\nu}$$

Equal masses $m_u \approx m_d \implies$ **isospin symmetry** between u and d quarks

$$SU(2)_I$$

Neglecting the mass terms \mathcal{L}_{QCD} is invariant under separate rotations of **left** and **right** quarks \implies **chiral symmetry**

$$SU(2)_L \times SU(2)_R$$

In nature the chiral symmetry breaks down

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_I$$

The pions π_0, π_{\pm} are the resulting Goldstone bosons

The QCD Lagrangian

The zero mass limit, $\Lambda_{QCD} \gg m_u, m_d \approx 0$

$$\mathcal{L}_{QCD}^0 = \sum_{f=u,d} \bar{q}_f i \not{D} q_f - \frac{1}{4} \mathcal{G}_{a\mu\nu} \mathcal{G}^{a\mu\nu}$$

Equal masses $m_u \approx m_d \implies$ **isospin symmetry** between u and d quarks

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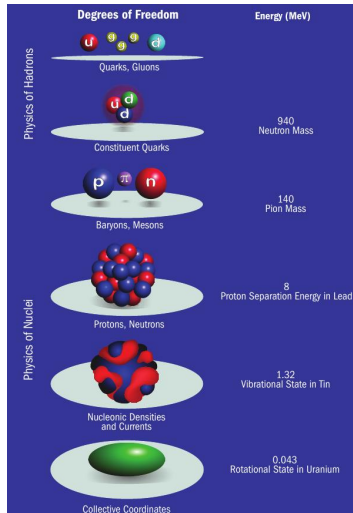
The χ EFT Revolution

Weinberg 1990, and many others

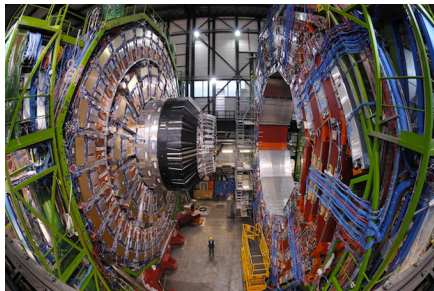
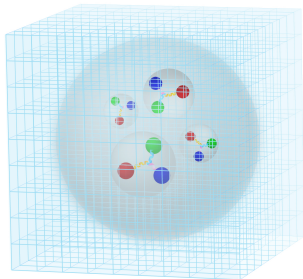
- Nuclear chiral EFT (χ EFT) is an Effective Field Theory of **nucleons and pions**
- Realizes chiral symmetry breaking
- The lagrangian is an expansion in

$$\frac{Q}{M}, \frac{m_\pi}{M}$$

- Leading Order (LO) – $(Q/M)^0$
- Next to Leading Order (NLO) – $(Q/M)^2$
- ...

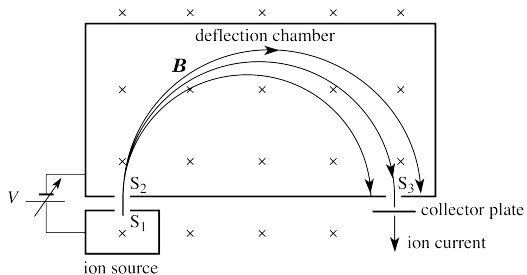


LQCD - a new type of experiment?



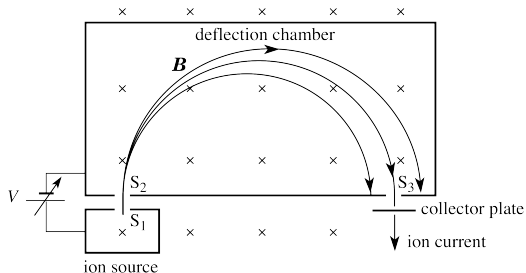
Nuclear amplitudes from lattice QCD

HAL QCD, Yamazaki et al., NPLQCD, CalLat (Figure thanks to J. Kirscher)



Nuclear amplitudes from lattice QCD

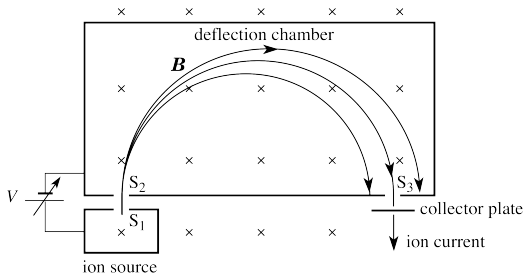
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$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{O} e^{-\int d^4x (\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \sum_f \log(\text{Det} M_f))}$$

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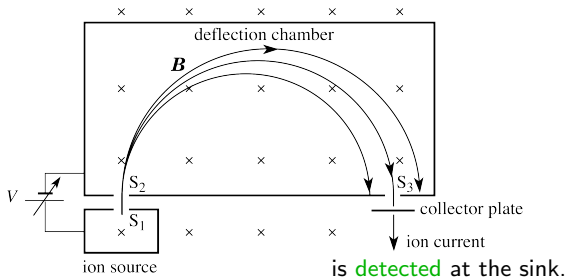
A hadron **prepared** at the source

$$\overline{N}_{\text{source}}^{\alpha}(\mathbf{0}, t_0) = \epsilon_{abc} (u^{a,T} C \gamma_5 d^b) u^{c,\alpha}(\mathbf{0}, t_0)$$

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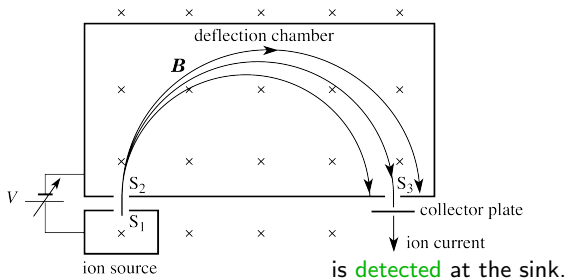
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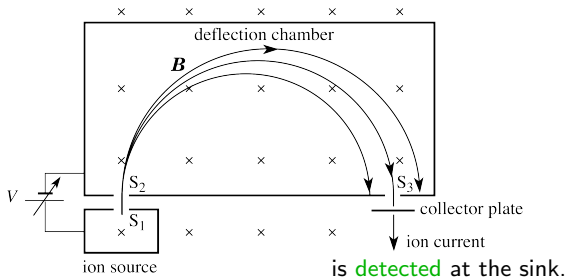
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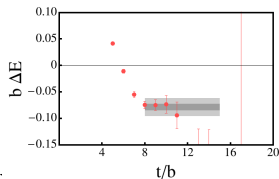
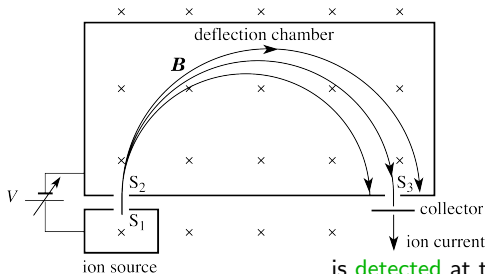
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is **detected** at the sink.

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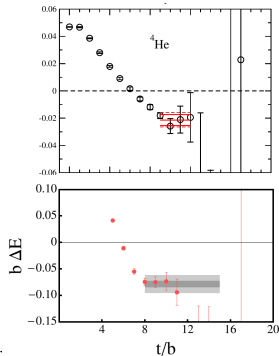
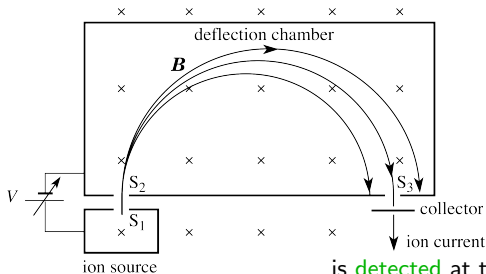
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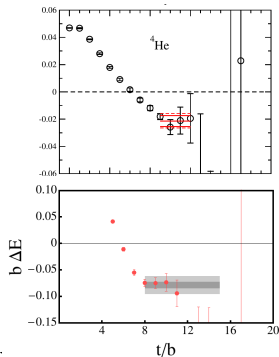
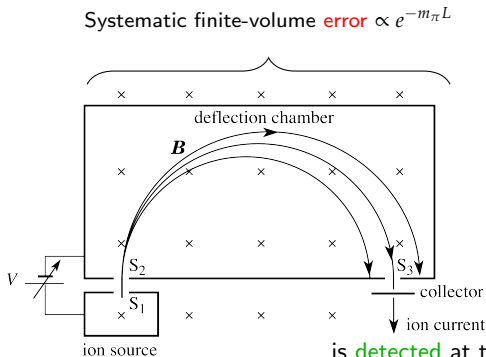
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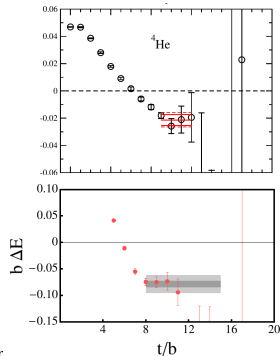
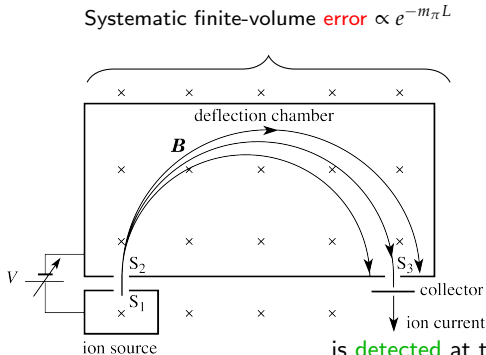
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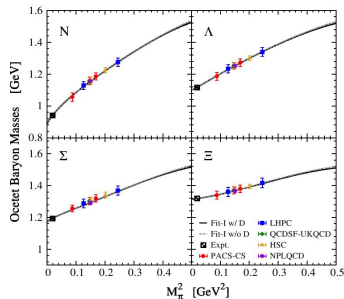
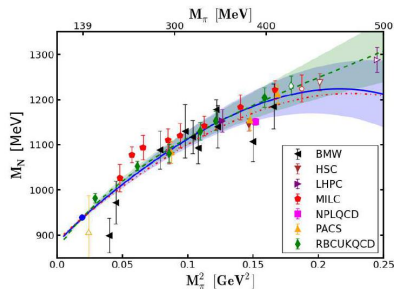
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LQCD - The Single Baryon Case

Lattice QCD

- LQCD calculations are done at large m_u, m_d
- parametrized as m_π
- Baryon masses $M_B(m_\pi)$ are predictions
- The limit $m_\pi \rightarrow m_\pi(\text{nature})$ should be taken



Xui-Lei Ren et al., PRD **87** 074001 (2013)

L. Alvarez-Ruso et al., ArXiv hep-ph:

1304.0483 (2013)

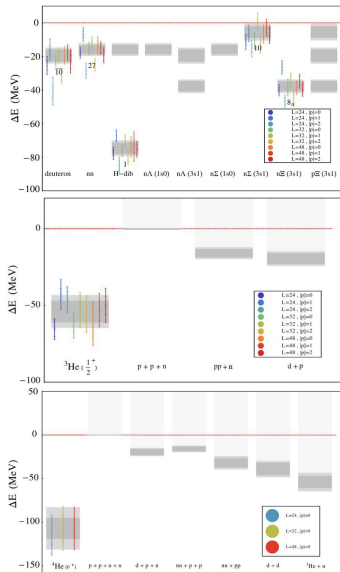
LQCD - Few-Body Baryon Spectra

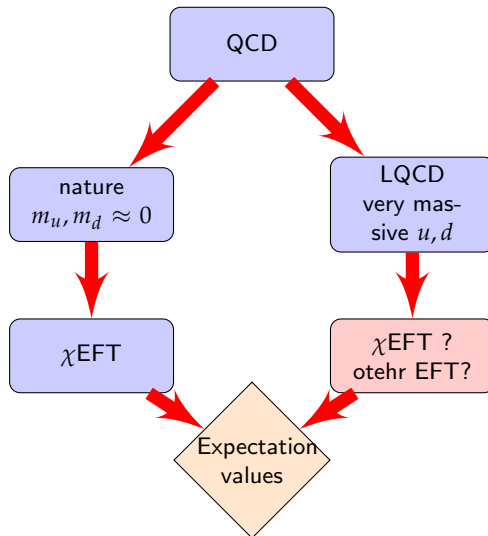
2-body system - Deuteron, dineutron,...

3-body system - ^3He , triton

4-body system - ^4He

NPLQCD Collaboration, PRD **87** 034506 (2013)





Energy Scales

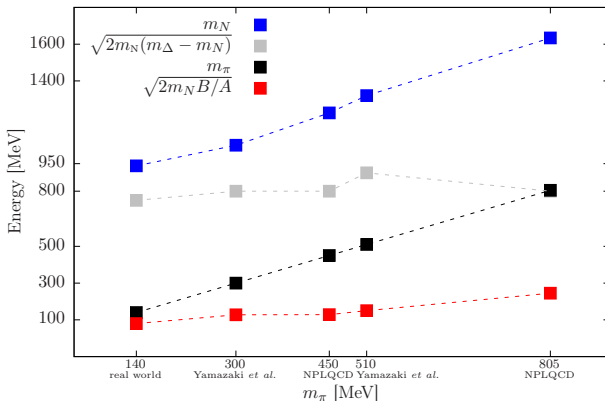
- Baryon mass M_n , pion mass m_π
- Excitations, i.e. the mass difference $\Delta = M_\Delta - M_n$
- Pion exchange momentum $q_\pi = m_\pi/\hbar c$, and energy

$$E_\pi = \frac{\hbar^2 q_\pi^2}{M_n} = \frac{m_\pi}{M_n} m_\pi$$

- Nuclear binding energy B/A

EFT for Lattice Nuclei

scales

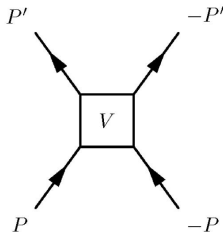


Conclusions

- For the Natural case $\mathcal{L} \rightarrow \mathcal{L}_{EFT}(N, \pi)$
- For lattice nuclei at $m_\pi \geq 300\text{MeV}$ $E_\pi \gg B/A$
- In this case \nexists EFT is the natural theory $\mathcal{L} \rightarrow \mathcal{L}_{EFT}(N)$

The LO Lagrangian for π EFT

$$\begin{aligned}\mathcal{L} = & N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}}{2M} \right) N \\ & - a_1 N^\dagger N N^\dagger N - a_2 N^\dagger \boldsymbol{\sigma} N \cdot N^\dagger \boldsymbol{\sigma} N \\ & - a_3 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N - a_4 N^\dagger \boldsymbol{\sigma} \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\sigma} \boldsymbol{\tau} N - \dots \\ & - d_1 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N N^\dagger N\end{aligned}$$



- Higher order terms include more derivatives.
- Naively, the order goes as the number of derivatives.
- The 3-body term appears at LO to avoid the Thomas collapse.
- Due to Fermi symmetry the number of terms can be cut by half.
- The coefficients depend on the cutoff Λ .

- At LO the π EFT potential takes the form

$$V_{LO}^{2b} = a_1 + a_2 \sigma_1 \cdot \sigma_2 + a_3 \tau_1 \cdot \tau_2 + a_4 (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$$

- The leading order also contains a 3-body term of the form

$$V_{LO}^{3b} = d_1 \tau_1 \cdot \tau_2 \quad \text{or} \quad V_{LO}^{3b} = d_1$$

- At NLO the π EFT potential takes the form

$$\begin{aligned} V_{NLO}^{2b} = & b_1 q^2 + b_2 q^2 \sigma_1 \cdot \sigma_2 + b_3 q^2 \tau_1 \cdot \tau_2 + b_4 q^2 (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) \\ & + b_5 k^2 + b_6 k^2 \sigma_1 \cdot \sigma_2 + b_7 k^2 \tau_1 \cdot \tau_2 + b_8 k^2 (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) \\ & + b_9 i \frac{1}{2} (\sigma_1 + \sigma_2)(k \times q) + b_{10} \tau_1 \cdot \tau_2 i \frac{1}{2} (\sigma_1 + \sigma_2)(k \times q) \\ & + b_{11} (\sigma_1 \cdot q)(\sigma_2 \cdot q) + b_{12} \tau_1 \cdot \tau_2 (\sigma_1 \cdot q)(\sigma_2 \cdot q) \\ & + b_{13} (\sigma_1 \cdot k)(\sigma_2 \cdot k) + b_{14} \tau_1 \cdot \tau_2 (\sigma_1 \cdot k)(\sigma_2 \cdot k) \end{aligned}$$

- The incoming particle have relative momentum p , the outgoing p' .
- The momentum transfer $q = p' - p$, and $k = (p' + p)$
- Nonlocalities are associated with k .

Kirscher, H. W.
Griesshammer, D. Shukla,
H. M. Hofman, arXiv:
0903.5583
A. Gezerlis, et al., PRL
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- Due to antisymmetrization of the nuclear wave function

$$V_{LO}^{2b} = C_1^{LO} + C_2^{LO} \sigma_1 \cdot \sigma_2$$

- The leading order also contains a 3-body term of the form

$$V_{LO}^{3b} = D_1^{LO} \tau_1 \cdot \tau_2 \quad \text{or} \quad V_{LO}^{3b} = D_1^{LO}$$

- Using the freedom to choose these parameters we set

$$V_{NLO}^{2b} = C_1^{NLO} q^2 + C_2^{NLO} q^2 \sigma_1 \cdot \sigma_2$$

- The antisymmetric potential V_{NLO} contains

- LO 2-body: 2 parameters.
- LO 3-body: 1 parameter.
- NLO 2-body: 2 parameters.

- At the moment we consider only LO.

At leading order the coordinate space Hamiltonian is

$$\begin{aligned} H = & -\sum_i \frac{\hbar^2}{2M_n} \nabla_i^2 + \sum_{i<j} \left(C_1^{LO}(\Lambda) + C_2^{LO}(\Lambda) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) e^{-\Lambda^2 r_{ij}^2/4} \\ & + \sum_{i<j<k} \sum_{cyc} D_1^{LO}(\Lambda) \left(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \right) e^{-\Lambda^2 (r_{ik}^2 + r_{jk}^2)/4} \end{aligned}$$

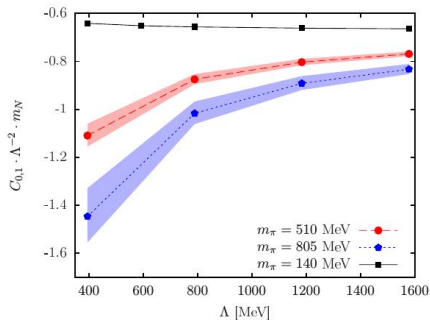
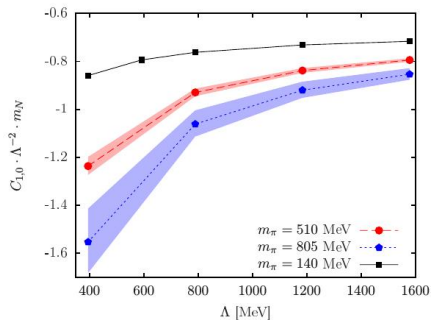
In the 3-body term the notation \sum_{cyc} stands for cyclic permutation of particles (ijk) .

Few-body arsenal

- 1 Numerov, $A = 2$
- 2 The Effective Interaction Hyperspherical Harmonics (EIH) method, $3 \leq A \leq 6$
- 3 The Resonating Group Method (RGM), $A \leq 6$
- 4 The Auxiliary Field Diffusion Monte-Carlo (AFDMC) method, $A \geq 2$
- 5 ...

Predictions for a universe with $m_\pi > 140$ MeV

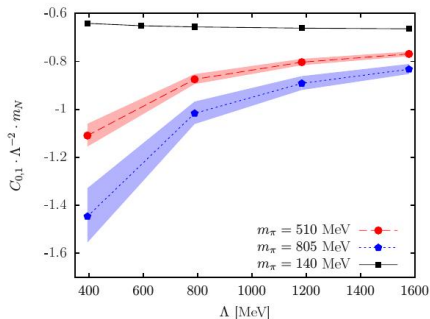
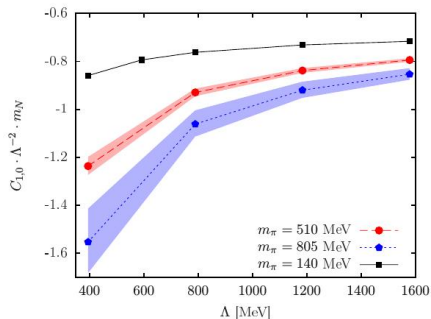
$A = 2, 3$



- i) Low-energy constants scale **natural**.
- ii) Low-energy constants \approx SU(4) **symmetric**.

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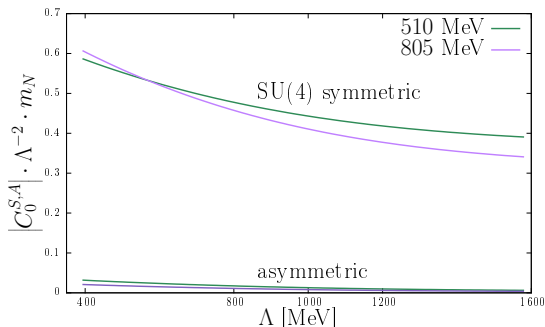
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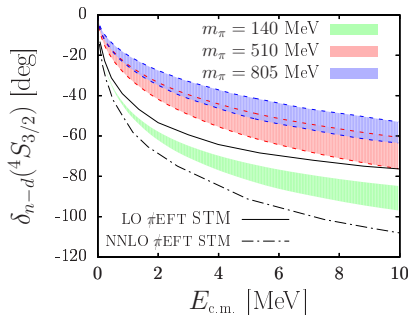
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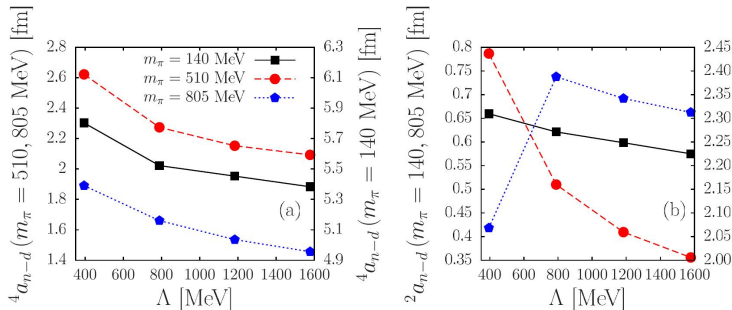


Observations:

- i) No bound $^4S_{3/2}$ 3-nucleon state.
- ii) Scattering lengths run non monotonous with m_π .

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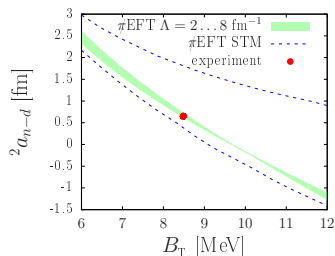


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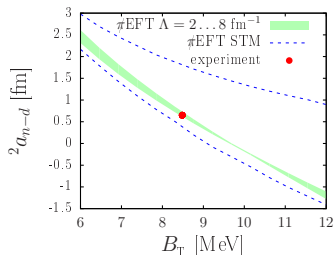
At physical m_π , scattering and bound state are **correlated** (Phillips).

What happens at larger m_π ?

- Peculiar correlation even at larger m_π .
- EFT uncertainty insignificant relative to uncertainty in input data.

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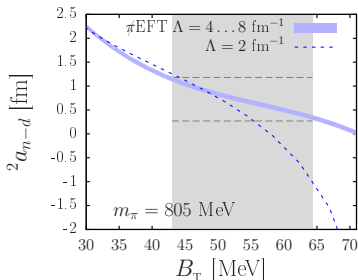
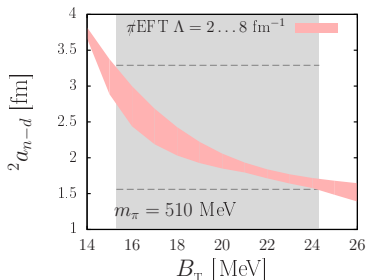
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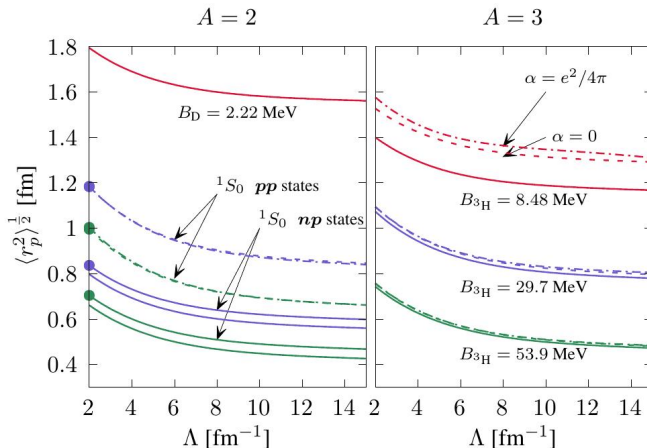
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Predictions for a universe with $m_\pi > 140$ MeV - Radii

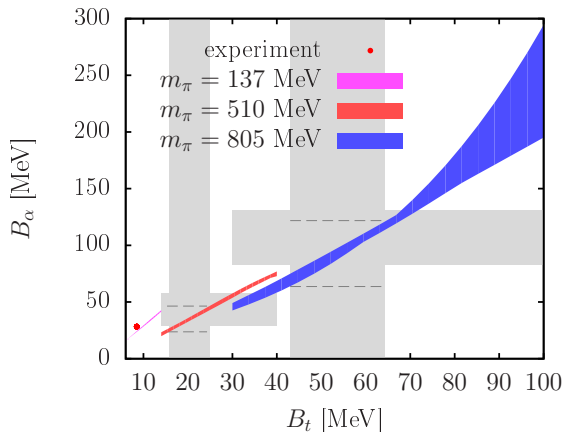
Kirscher, Pazy, Barnea



$m_\pi=140$ MeV, $m_\pi=140$ MeV, $m_\pi=806$ MeV

Predictions for a universe with $m_\pi > 140$ MeV

$A = 4$

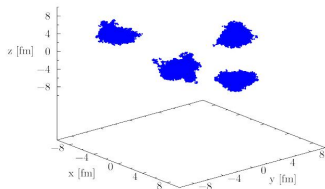
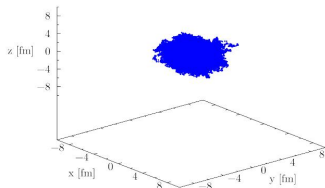


- i) At physical m_π , the 3- and 4-nucleon ground states are **correlated**.
- ii) This correlation is **preserved** at higher m_π .

Binding Energies For $A > 4$ and $m_\pi > 140$ MeV

Contessi et al.

AFDMC



upper pannel $\Lambda = 2\text{fm}^{-1}$

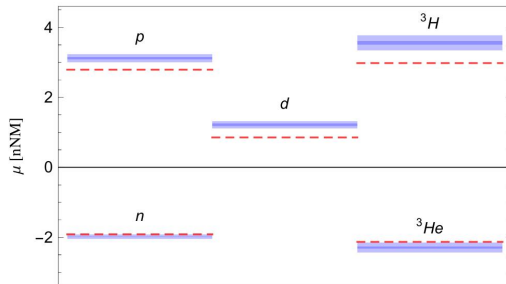
lower pannel $\Lambda = 8\text{fm}^{-1}$

$m_\pi = 140\text{MeV}$

Λ	$m_\pi = 140$ MeV	$m_\pi = 510$ MeV	$m_\pi = 805$ MeV
2 fm^{-1}	-97.19 ± 0.06	-116.59 ± 0.08	-350.69 ± 0.05
4 fm^{-1}	-92.23 ± 0.14	-137.15 ± 0.15	-362.92 ± 0.07
6 fm^{-1}	-97.51 ± 0.14	-143.84 ± 0.17	-382.17 ± 0.25
8 fm^{-1}	-100.97 ± 0.20	-146.37 ± 0.27	-402.24 ± 0.39
$\rightarrow \infty$	$-115^{+1}_{-8}(\text{sys})$	$-151^{+2}_{-10}(\text{sys})$	$-504^{+20}_{-12}(\text{sys})$
Exp.	-127.62	—	—

- The strong 3-body force - a numerical challenge
- $\Lambda \rightarrow \infty$ - **even worse**
- Unbound/barely bound systems-
probably the worst
- The α cluser is **unbound** in \not{t} EFT at LO

Magnetic moments and polarizations



$$\Delta E = \mu B + \frac{1}{2} \beta_M B^2$$

The \not{E} EFT Lagrangian at NLO

$$\mathcal{L} = N^\dagger \left\{ (i\partial_0 - e\hat{Q}A_0) + \frac{1}{2m} (\nabla - ie\hat{Q}\mathbf{A})^2 + \hat{g}_\mu \frac{e}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} \right\} N \\ + C_T (N^\dagger P_i N)^2 + C_S (N^\dagger \bar{P}_3 N)^2 + D_1 (N^\dagger N)^3 + \dots \\ + L_1 (N^\dagger P_i N)^\dagger (N^\dagger \bar{P}_3 N) B_i + L_2 i\epsilon_{ijk} (N^\dagger P_i N) (N^\dagger P_j N) B_k$$

The magnetic current

The one-body current

$$\mu^{(1)} = \sum_{i=1}^A \frac{|e|}{2m} \left[\frac{g_p + g_n}{2} \boldsymbol{\sigma}_i + \frac{g_p - g_n}{2} \boldsymbol{\sigma}_i \boldsymbol{\tau}_{i,z} \right]$$

The two-body current

$$\mu^{(2)} = \sum_{i < j}^A \left[L_1 (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) (\boldsymbol{\tau}_{i,z} - \boldsymbol{\tau}_{j,z}) + L_2 (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \right] \delta_\Lambda(\mathbf{r}_{ij})$$

The magnetic LECs

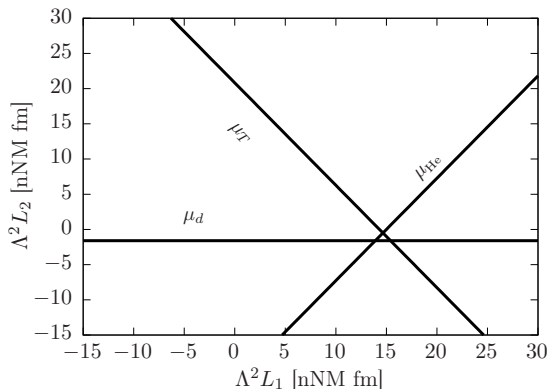
Kirscher, Pazy, Gazit, De-Leon, Barnea

Observables

The deuteron magnetic moment μ_d

The $A = 3$ magnetic moments $\mu_T, \mu_{^3\text{He}}$

The transition matrix element $t_{01} = \langle ^1S_0 | \boldsymbol{\mu} | ^3S_1 \rangle$



Observations

$m_\pi = 140\text{MeV}$ - Consistency between the different observables

$m_\pi = 806\text{MeV}$ - Error bars too large, t_{01} a bit off

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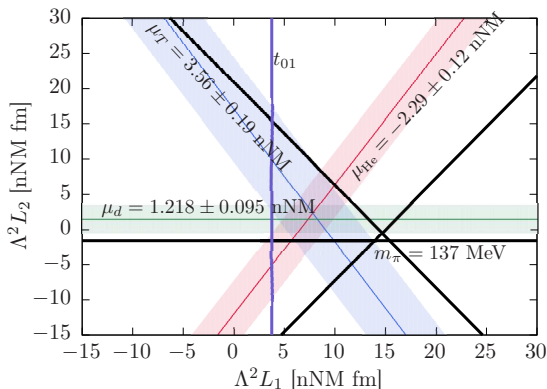
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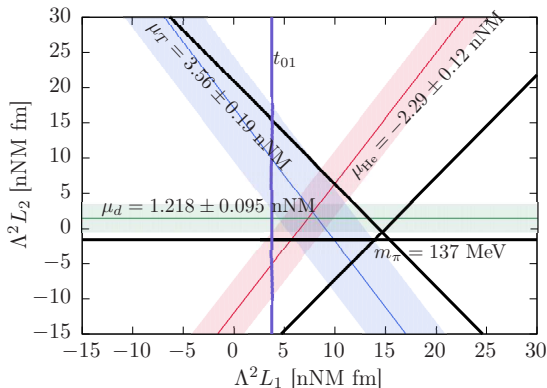
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Short Range Correlations in a many-body system

Heavy Fermions



The Mara river, Kenya (2016).

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The short range factorization and Magnetic moments

[Tan, Braatan & Platter, Werner & Castin,...]

- The interaction is represented through the boundary condition

$$\left[\partial \log r_{ij} \Psi / \partial r_{ij} \right]_{r_{ij}=0} = -1/a$$

- Thus, when two particles approach each other

$$\Psi \xrightarrow{r_{ij} \rightarrow 0} \underbrace{(1/r_{ij} - 1/a)}_{\text{universal}} A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j})$$

- The contact C represents the probability of finding an interacting pair within the system

$$C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle$$

- Where

$$\langle A_{ij} | A_{ij} \rangle = \int \prod_{k \neq i,j} d\mathbf{r}_k d\mathbf{R}_{ij} A_{ij}^\dagger(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j}) \cdot A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j})$$

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The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp, nn, pn\}$$

- For each pair there are different $\ell = 0$ channels

$$\alpha = sm$$

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

- For $\ell = 0$ we need consider only **7** contacts

$$\{C_{pp}^{00,00}, C_{nn}^{00,00}, C_{np}^{00,00}, C_{np}^{1m,1m}, C_{np}^{00,10}\}$$

- For short

$$\{C_{pp}^0, C_{nn}^0, C_{np}^0, C_{np}^{1m}, C_{np}^{01}\}$$

- In general we may replace the short range by general form

$$(1/r_{ij} - 1/a_{\alpha})\chi_{\alpha} \longrightarrow \varphi_{\alpha}$$

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A detour

2-body potential energy

$$\begin{aligned}\langle V_{NN} \rangle &= \langle \Psi | \sum_{ij} (A_1 + A_2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \delta(\mathbf{r}_{ij}) | \Psi \rangle \\ &= \frac{C_{pp}^0 + C_{np}^0 + C_{nn}^0}{16\pi^2} \langle \varphi_{00} | (A_1 + A_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \delta(\mathbf{r}) | \varphi_{00} \rangle \\ &\quad + \sum_m \frac{C_{np}^{1m}}{16\pi^2} \langle \varphi_{1m} | (A_1 + A_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \delta(\mathbf{r}) | \varphi_{1m} \rangle\end{aligned}$$

Using the relations between A_i and the scattering length the blue parts can be related directly to experimental 2-body data

Tan, Braatan & Platter, Werner & Castin,...

The Contact and the Nuclear Magnetic moments

Magnetic moments at NLO

The 2-body contribution to the magnetic moment

$$\begin{aligned}\langle \mu^{(2)} \rangle &= \langle \Psi | \sum_{ij} \left(L_1(\sigma_i - \sigma_j)(\tau_{i,z} - \tau_{j,z}) + L_2(\sigma_i + \sigma_j) \right) \delta(r_{ij}) | \Psi \rangle \\ &= \sum_m \frac{C_{np}^{1m}}{16\pi^2} \langle \varphi_{1m} | L_2(\sigma_1 + \sigma_2) \delta(r) | \varphi_{1m} \rangle \\ &\quad + \frac{C_{np}^{01}}{16\pi^2} \langle \varphi_{00} | L_1(\sigma_i - \sigma_j)(\tau_{i,z} - \tau_{j,z}) \delta(r) | \varphi_{10} \rangle\end{aligned}$$

We note that

$$\Delta\mu_d = (\mu_d - \mu_n - \mu_p) = \langle d | \mu^{(2)} | d \rangle = \langle \varphi_{1m=1} | L_2(\sigma_1 + \sigma_2) \delta(r) | \varphi_{1m=1} \rangle$$

$$t_{01} = \langle \varphi_{00} | L_1(\sigma_i - \sigma_j)(\tau_{i,z} - \tau_{j,z}) \delta(r) | \varphi_{10} \rangle$$

Therefore

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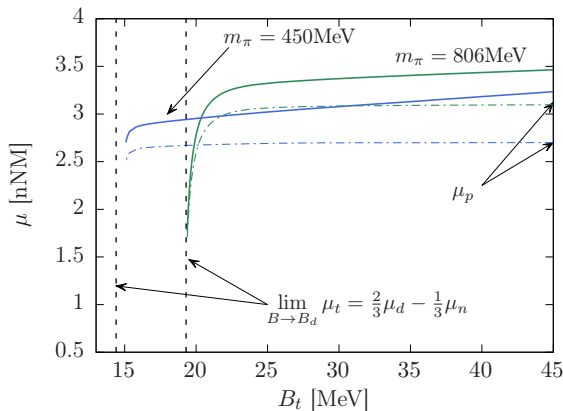
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The running of the magnetic moment with B.E.



The Triton - LO

Tight system -
 $|t\rangle \approx |n \uparrow n \downarrow p \uparrow\rangle$ therefore
 $\mu_t = \mu_p$

Loosly bound - $|t\rangle \approx |d\rangle + |n\rangle$
 therefore $\mu_t = \frac{2}{3} \mu_d - \frac{1}{3} \mu_n$

Fundamental theory = Theology

There can still be hierarchy between theories



Give me an EFT and I shall move the earth !