Many-body correlations: the relative nature of their definition and the non-observable character of their value

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Outline

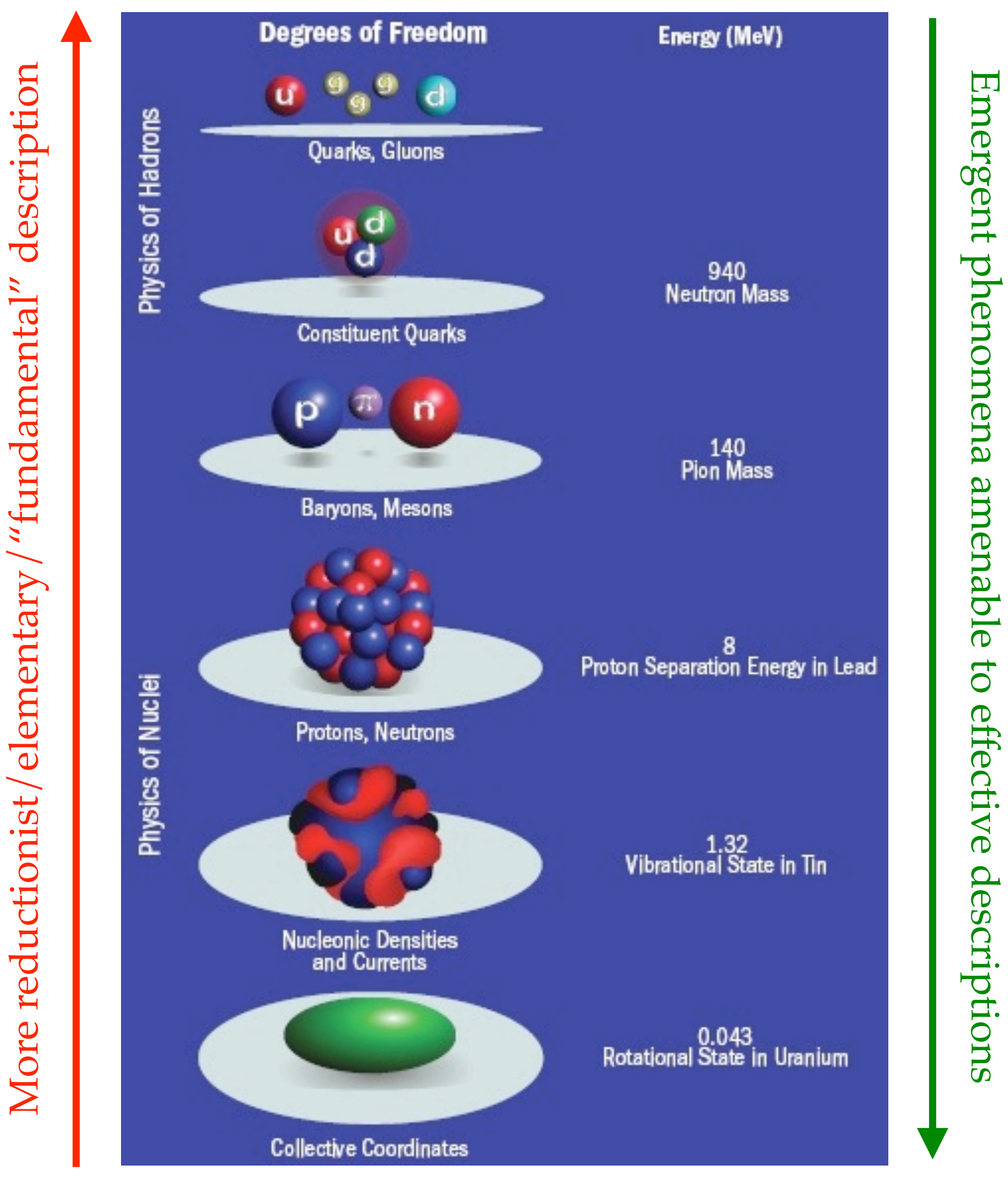
- **Part I: Relative nature of many-body correlations**
  - Introduction: quasiparticles, correlations & many-body methods
  - Examples in nuclei and nuclear matter
  - Nuclear Hamiltonians & similarity renormalisation group techniques
  - Correlations and resolution scale

- **Part II: Non-observable character of the nuclear shell structure**
  - Single-nucleon shells $\leftrightarrow$ correlated nucleon dynamics
  - Definition & properties of effective single-particle energies
  - Scale dependence & non-observability of effective single-particle energies
  - Fermi gaps & spectroscopic factors

- **Conclusions**

Part I
Relative nature of many-body correlations

Physical systems as a many-body problem



- **Quantum/mesoscopic system as many-body problem**
- **Choice of degrees of freedom**

↓

Physical system in terms of correlations between d.o.f.

- **Many-body Schrödinger equation**
  - Exact solution for A=2, 3, 4
  - Approximated solution for $A \gtrsim 5$

↓

Accuracy / difficulty depend on correlations

- **At a given A, how to minimise correlations?**
- **When increasing A, how to monitor the accuracy?**

Quasiparticles

⦿ Difficulty as the number of particles increases → how to picture/model many-body correlations?

⦿ Easy to deal with independent particles → reformulate in terms of $A \times$ one-body problems

⦿ Can we change the (nature of the) chosen degrees of freedom & eliminate many-body correlations?

⦿ Concept of (Landau) quasiparticles

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Entities with modified (in-medium, renormalised, …) properties w.r.t. the bare d.o.f.

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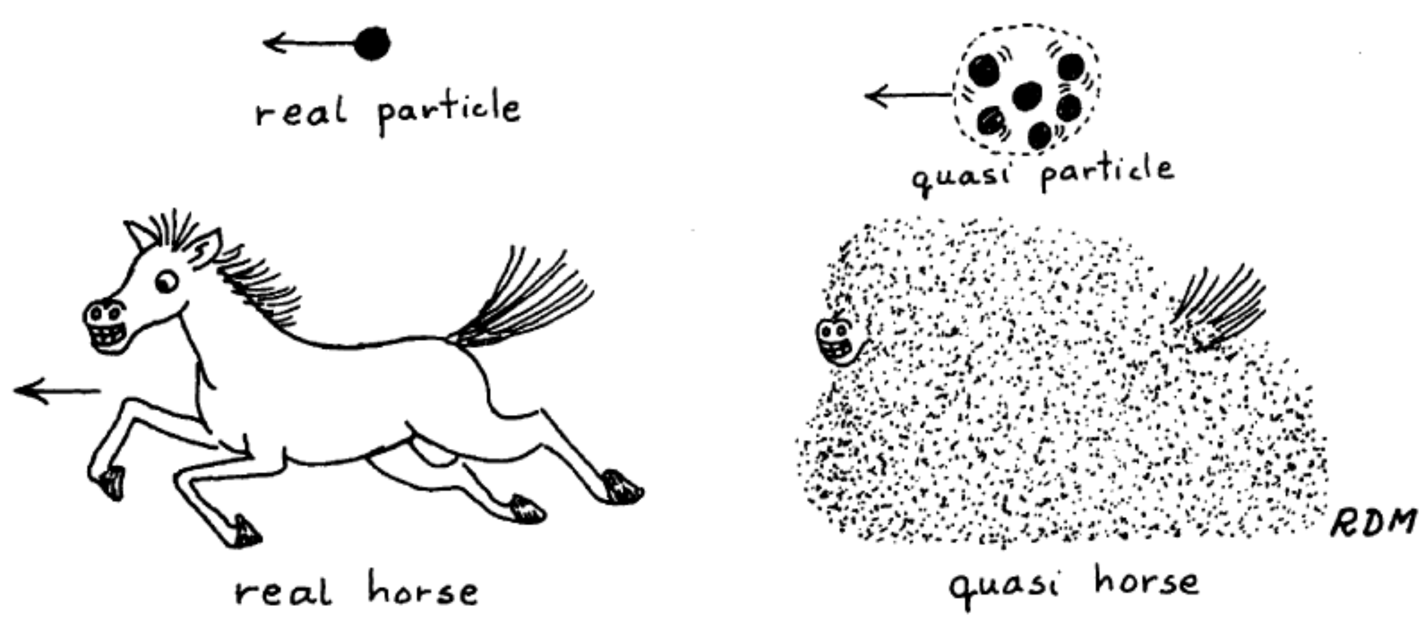
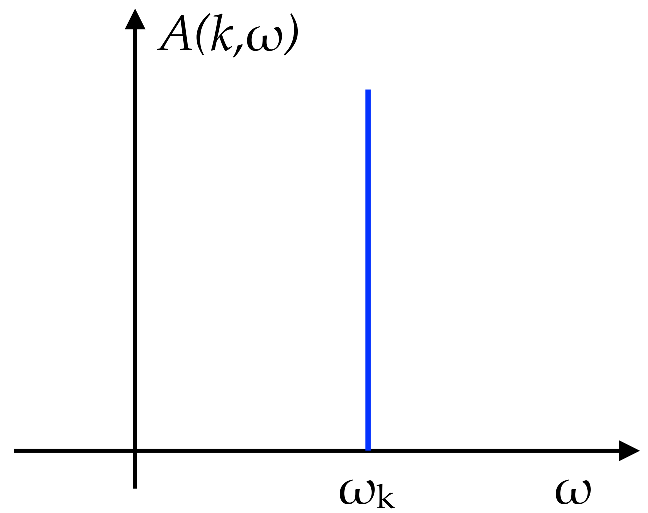
Many-body problem of interacting particles → one-body problem of (independent) quasiparticles



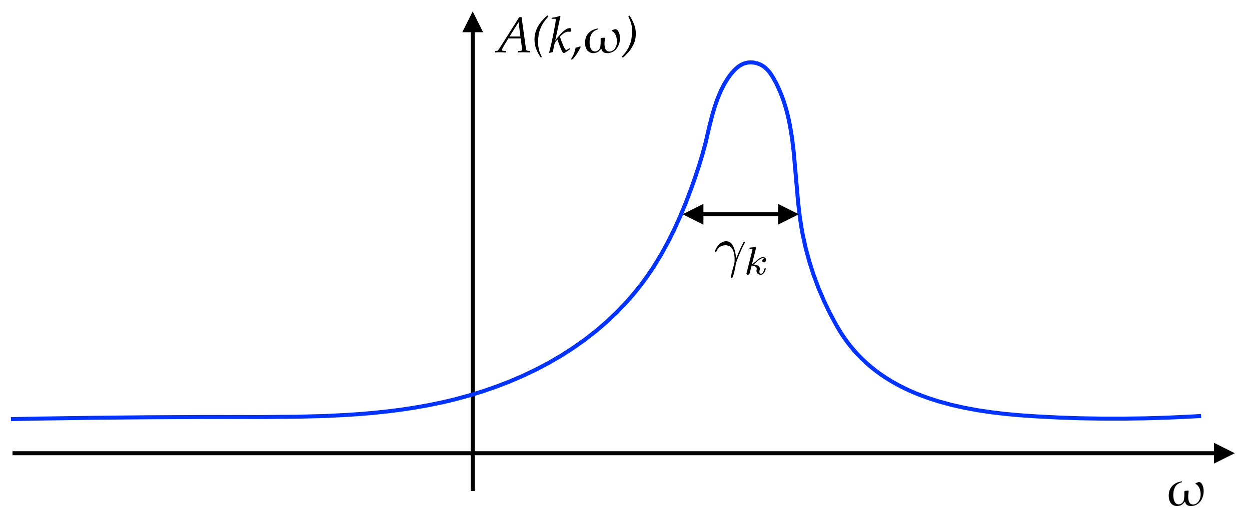
[figure from R. D. Mattuck]

Interacting quasiparticles

- In some cases, quasiparticles can be constructed explicitly
- In most cases, quasiparticles-like excitations emerge from the many-body dynamics
  - Spectral function $A(k,\omega)$ embodies quasiparticle features & many-body correlations
  - For free particles $A(k,\omega) = \delta(\omega - k^2/2m)$



Infinitely-lived (=independent) quasiparticle



Decaying (=interacting) quasiparticle

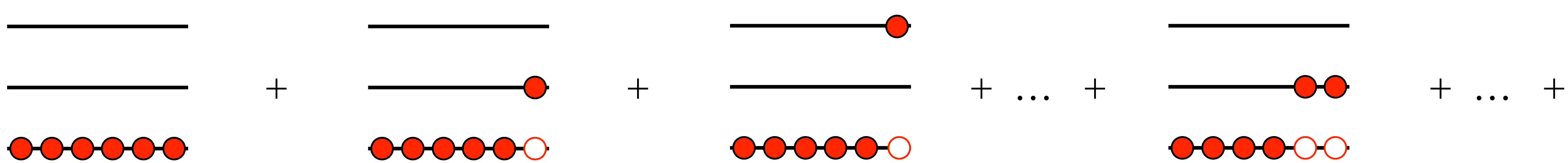
- Quasiparticles with finite lifetime → departure from independent (quasi)particle picture
- Many-body correlations as residual interactions between quasiparticles

Particle-hole expansions

- **Independent-particles as 0th-order tenet of numerous many-body methods**
  - Perturbation theory
  - Density functional theory
  - Nuclear shell model

- **Hartree-Fock method as an optimised independent-particle description**
  - (Many-body) correlations: everything beyond Hartree-Fock

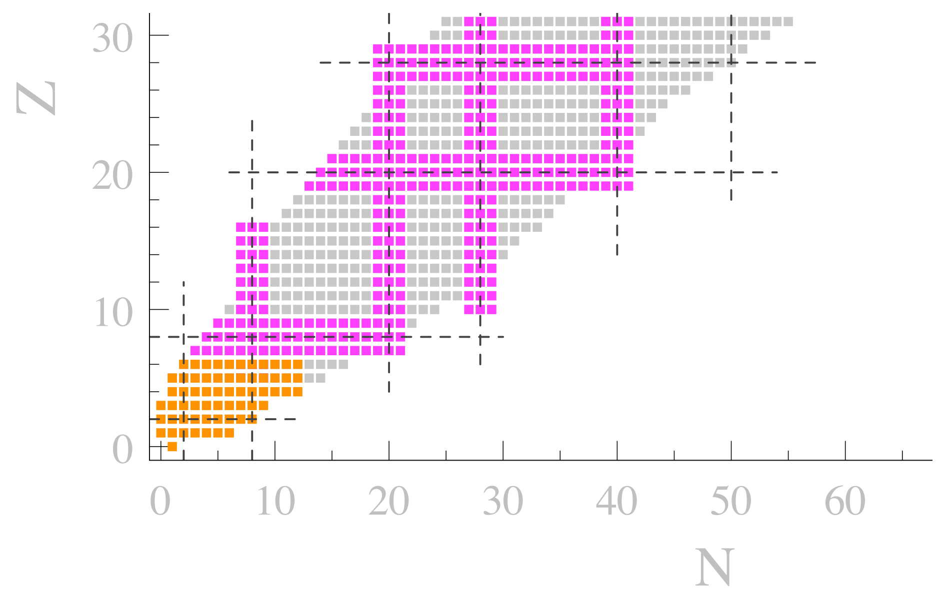
- **Beyond-Hartree-Fock methods as expansions in particle-hole excitations**



  - Simplest: MBPT
  - Exact (= whole expansion): Configuration interaction / No-core shell model
  - Freedom to choose the interaction such that HF is the closest to the exact solution?

Different schemes for different correlations

- **Methods based on particle-hole expansions face severe scaling**



- **Why don't include some correlation in the interaction itself? → effective interactions**
  - One aims at limiting the complications of ph expansions
  - Interaction traditionally phenomenological, possible to derive one ab initio?

- **Why don't limit ourselves to part of the Hilbert space → valence space methods**
  - One aims at the exact solution in the limited Hilbert space
  - Interaction traditionally phenomenological, recently also ab initio

Correlations in different schemes will be different by construction

Correlations via symmetry breaking & restoration

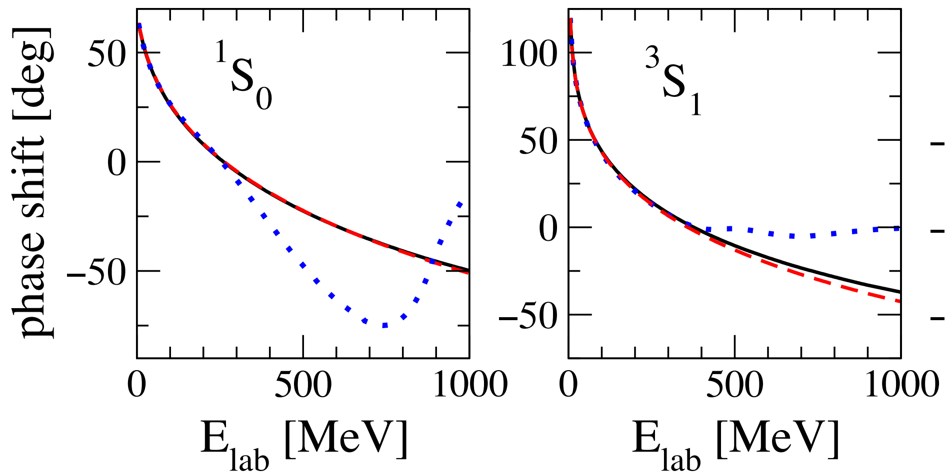
- **Correlations can be grasped by exploiting (breaking & restoration of) symmetries**
  - For near-degenerate systems essential to expand around a symmetry-breaking reference
  - In nuclear physics: U(1) ↔ pairing correlations; SU(2) ↔ quadrupole correlations
- **Can the two types be related?**
  - Correlations included via symmetry breaking might be very hard to get via ph expansion
  - And viceversa
- **Can the two types be combined?**
  - Gorkov Green's functions [Somà, Duguet, Barbieri 2011]
  - Multi-reference IM-SRG [Hergert *et al.* 2013]
  - Symmetry broken & restored MBPT and CC [Duguet 2015, Duguet, Signoracci 2016]
  - Many-body driven EDF [Duguet *et al.* 2015]
  - Symmetry breaking & restoration + truncated CI [Ripoche *et al.* 2017]

⇓

see Thomas', Denis', … talks

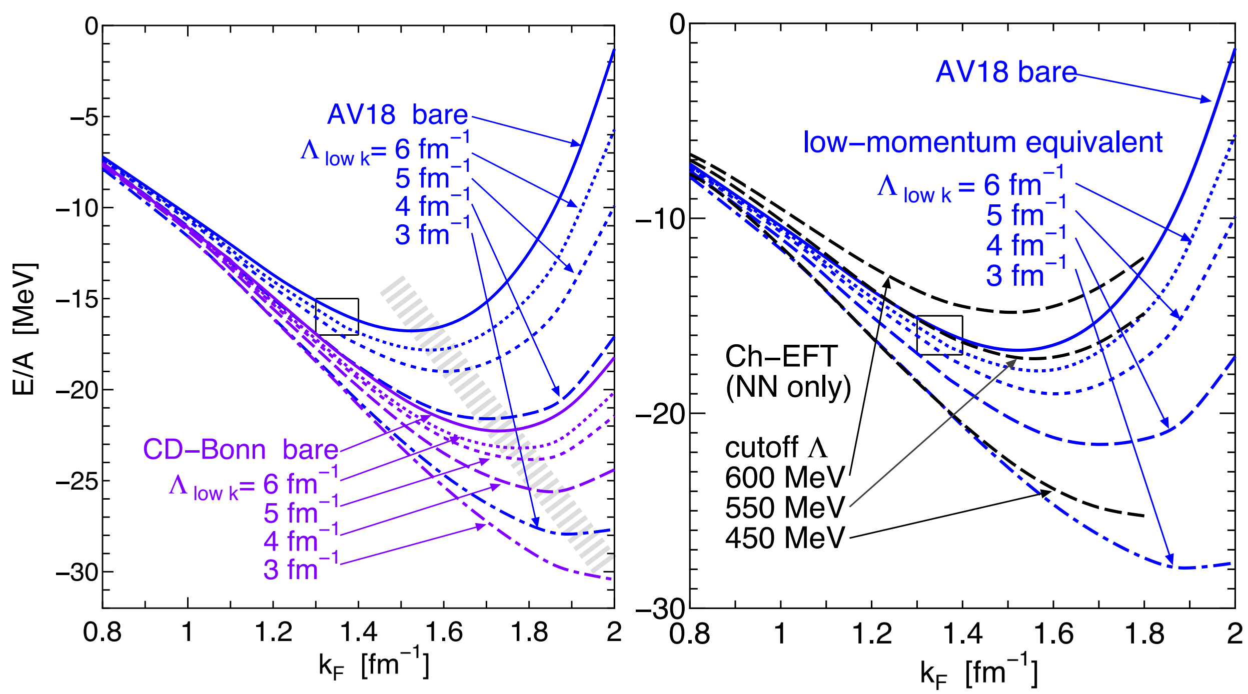
Nuclear Hamiltonians

- **Early Hamiltonians** (60's & 70's)
  - Soft core
  - Could not reproduce nuclear saturation
- **Phenomenological Hamiltonians** (80's & 90's)
  - Hard core
  - Three-body forces?
- **Chiral EFT interactions** (from 00's)
  - Softer core
  - Three-body forces consistent
- **SRG techniques**
  - Unitary transformation of the Hamiltonian
  - Trade hard core for higher-body forces
  - Universality at low energy scales



[Bogner *et al.* 2010]

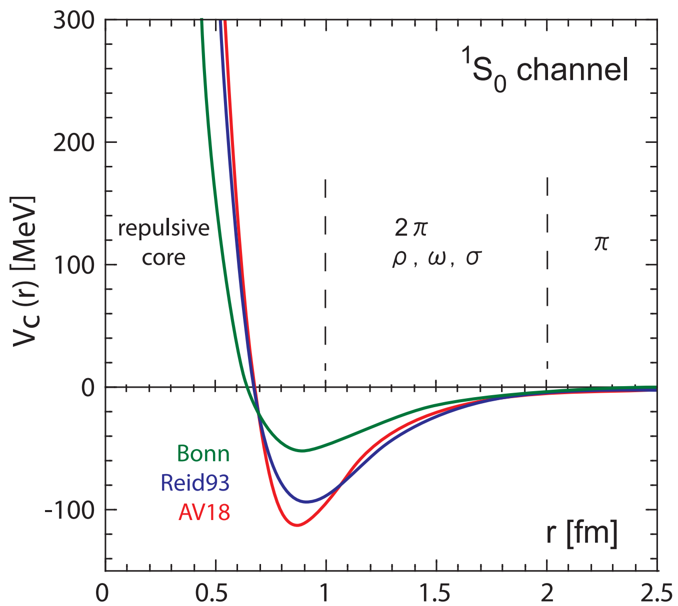
Coester band



[Kohno 2015]

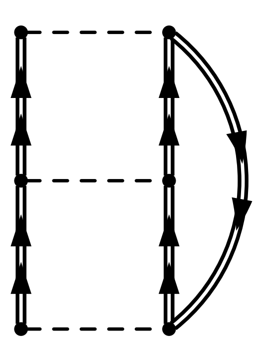
Long- vs short-range correlations

- **Hard core induces strong short-range correlations**

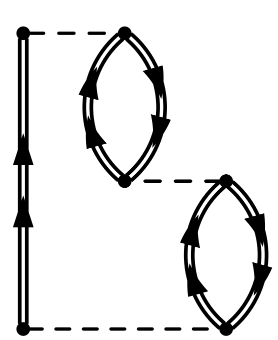


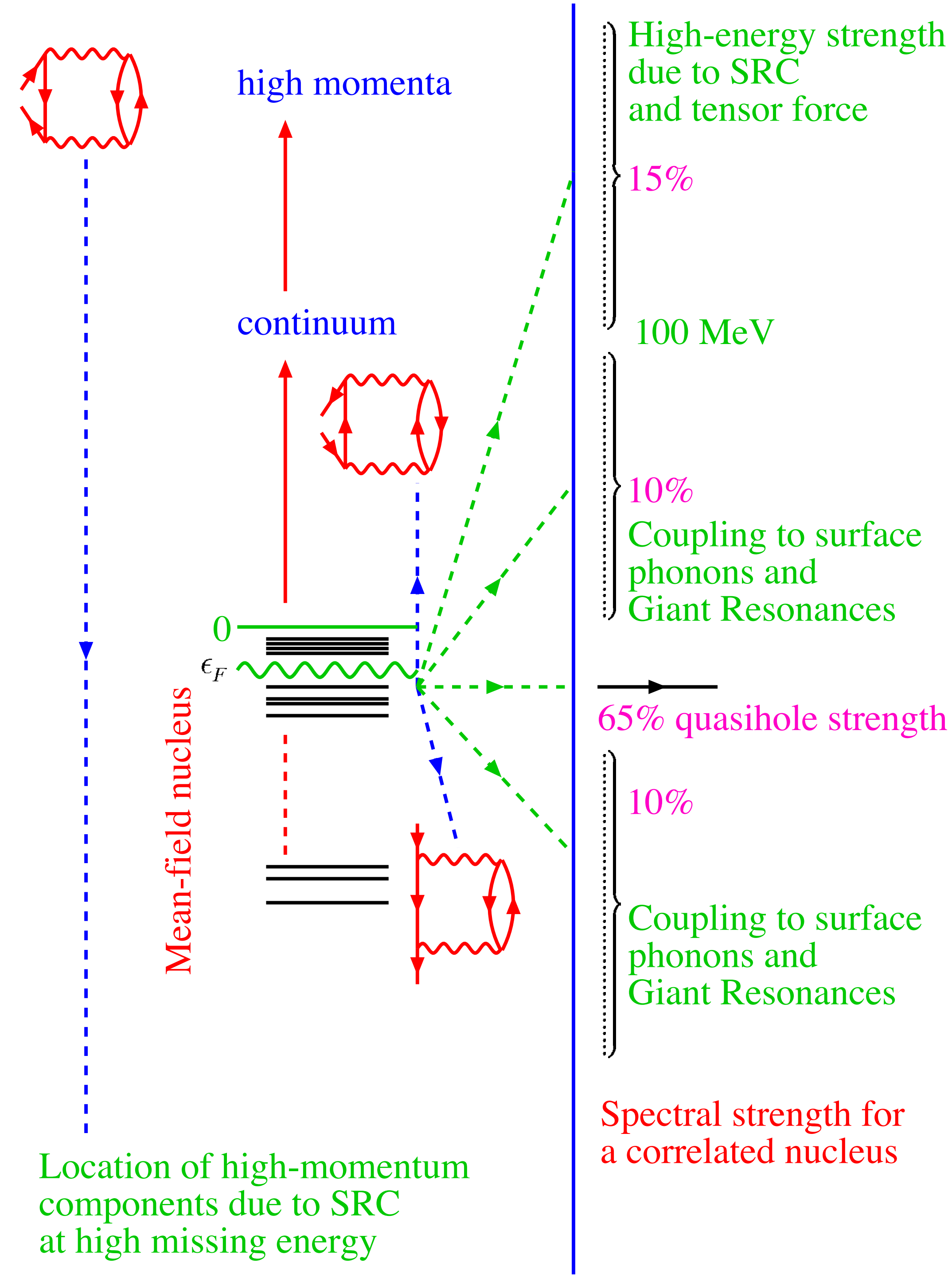
- ○ Sophisticated many-body methods needed
- ○ Strong correlations fragmentation of s.p. strength
- ○ pp/ph excitation ↔ short-/long-range physics

Short-range



Long-range

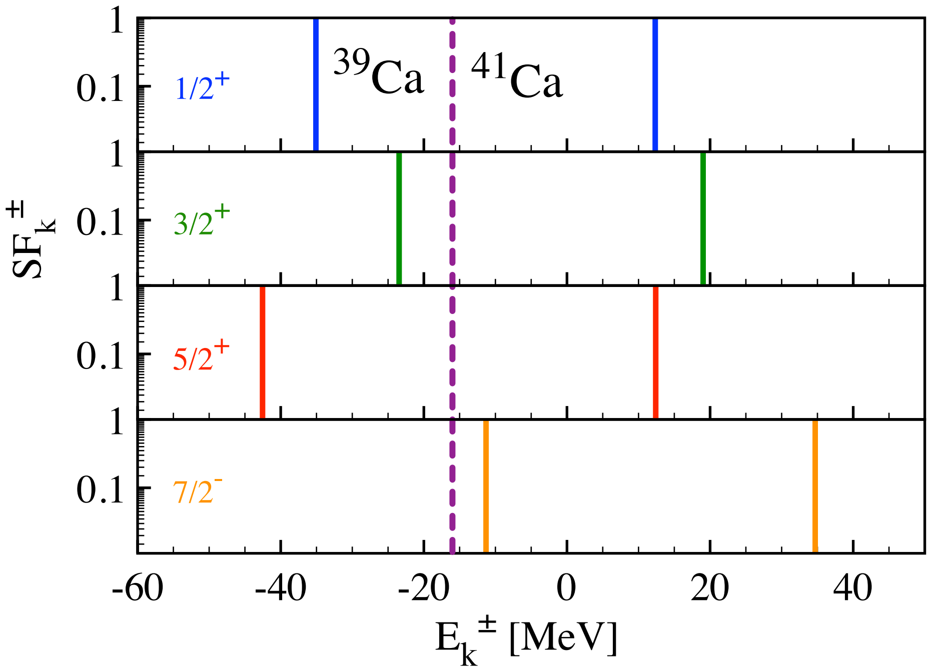




[Dickhoff, Barbieri 2004]

Fragmentation of single-particle strength in nuclei

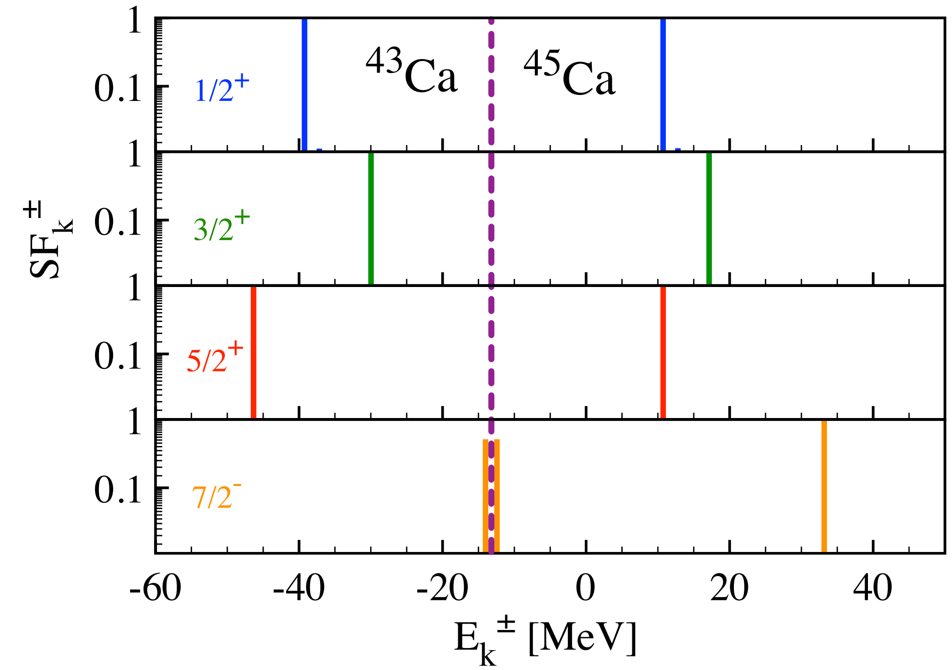
Dyson 1st order (HF)



Fragmentation

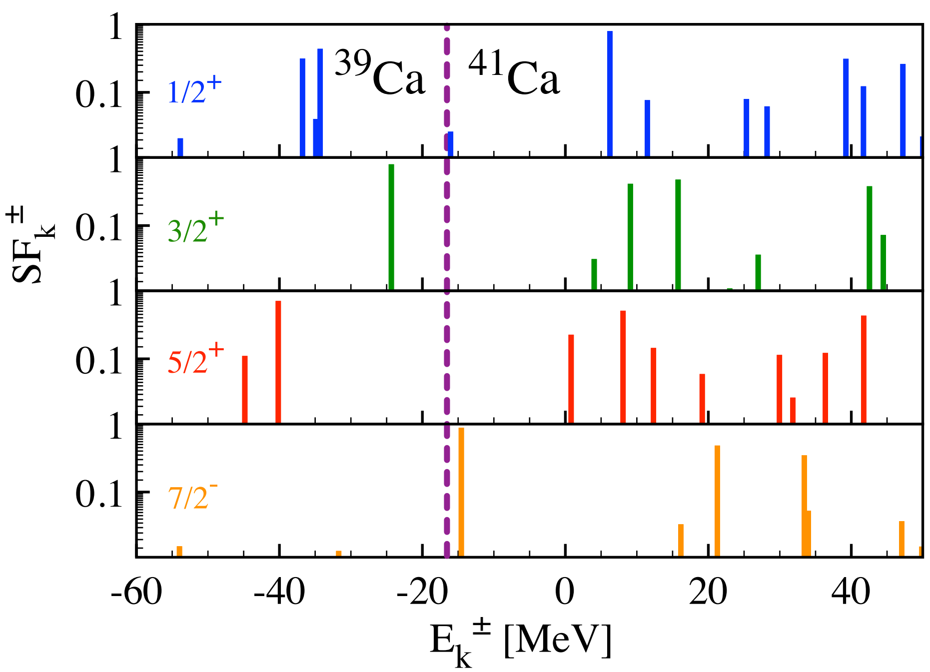
Static pairing →

Gorkov 1st order (HFB)

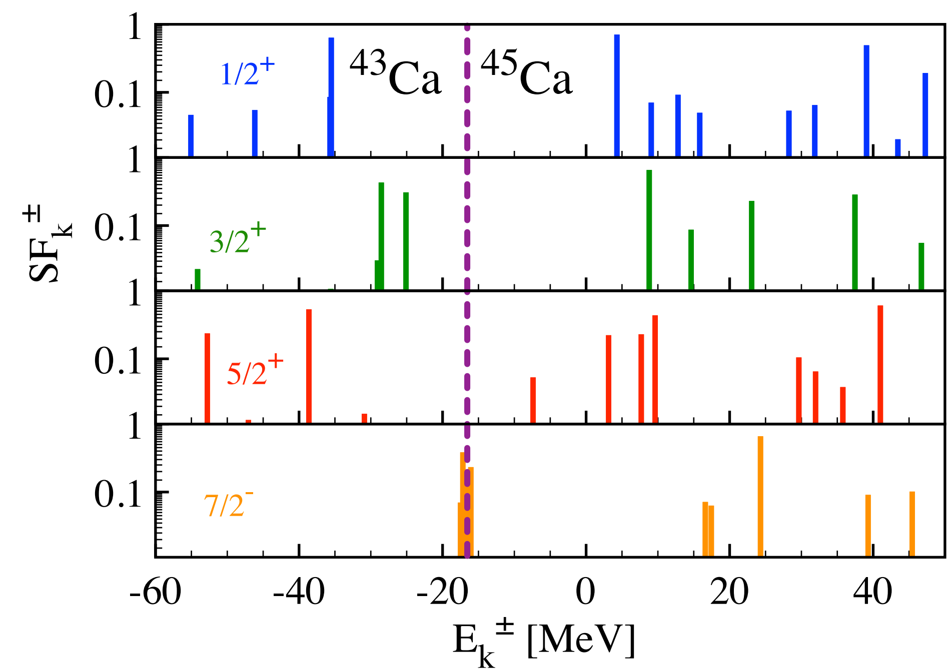


Dynamical fluctuations ↓

Dyson 2nd order

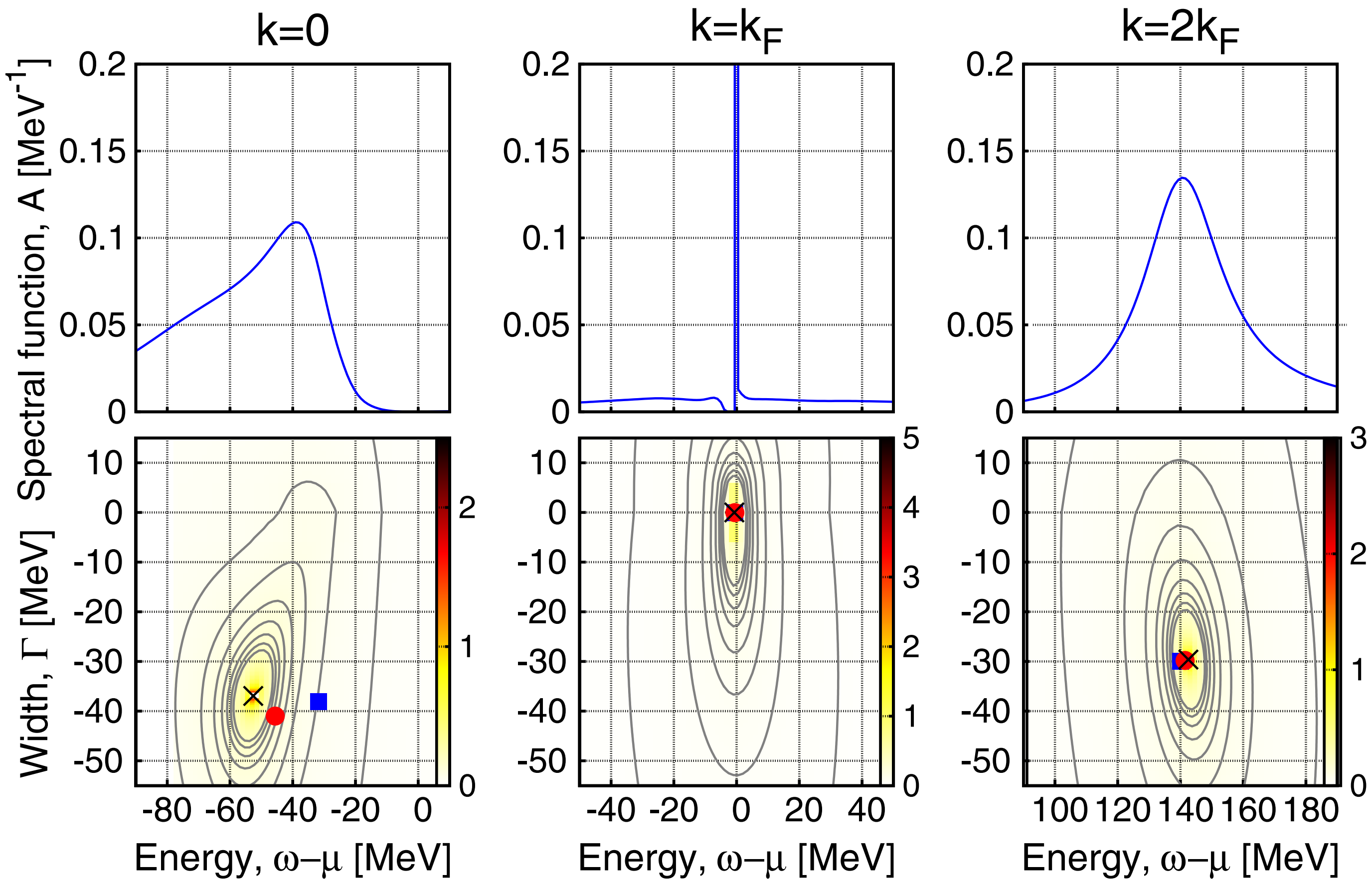


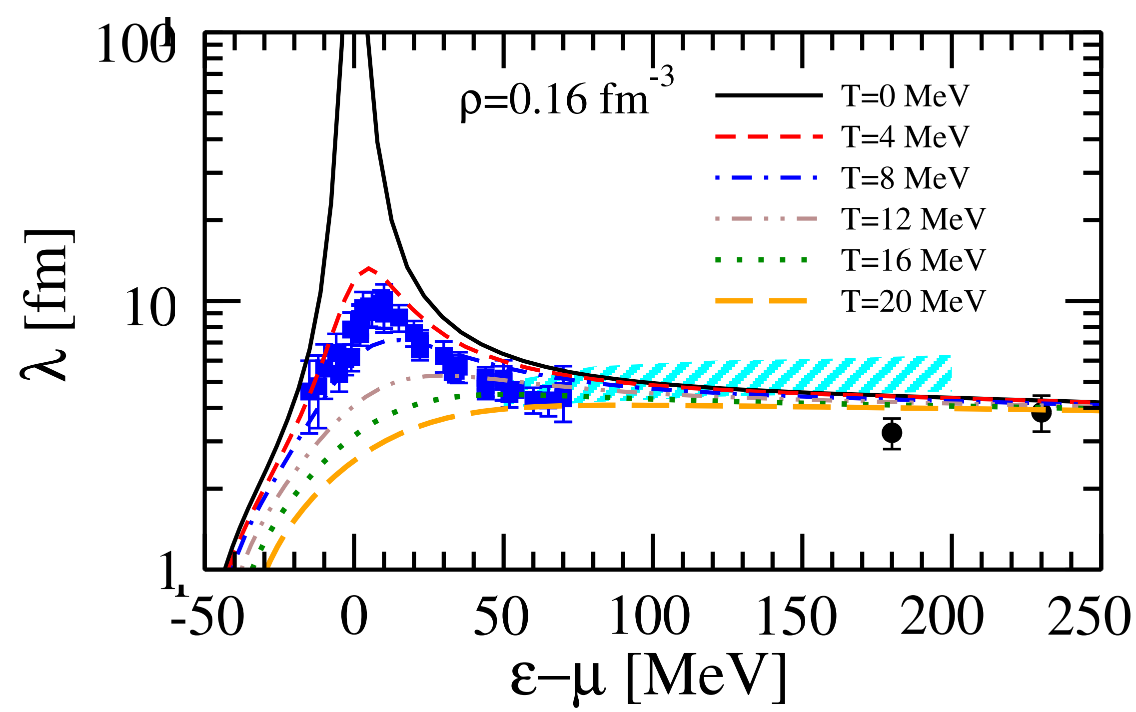
Gorkov 2nd order



Fragmentation of single-particle strength in infinite matter

- **Spectral function depicts correlations**
  - Broad peak signals depart from
  - Well-defined (long-lived) quasiparticles at the Fermi surface
  - Long mean free path for $E < E_F$





[Rios, Somà 2012]

Renormalisation-group techniques for nuclear forces

⦿ **SRC generated by couplings between low and high momenta**

- ○ Large model spaces needed to converge → applicability limited to light nuclei

⦿ Are high momenta, i.e. high resolution, necessary to compute **low-energy observables**?

- ○ Interested in long-wavelength information
- ○ Small-distance details irrelevant
- ○ **Change the resolution → "integrate out" unnecessary information**

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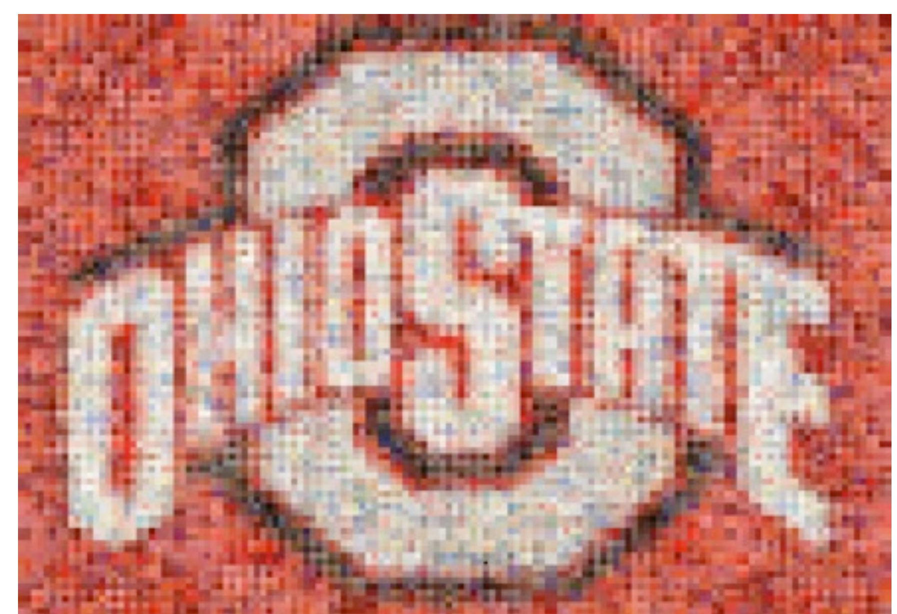
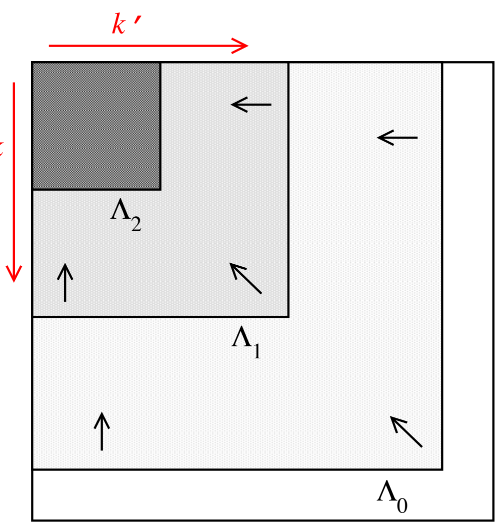


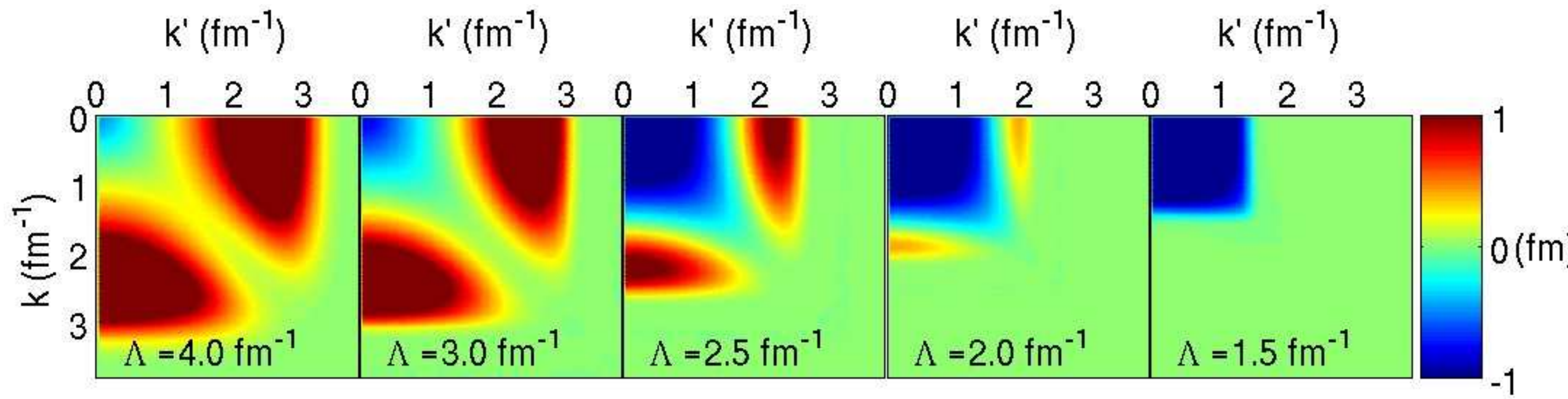
[figures from K. Hebeler]

Low-momentum evolutions

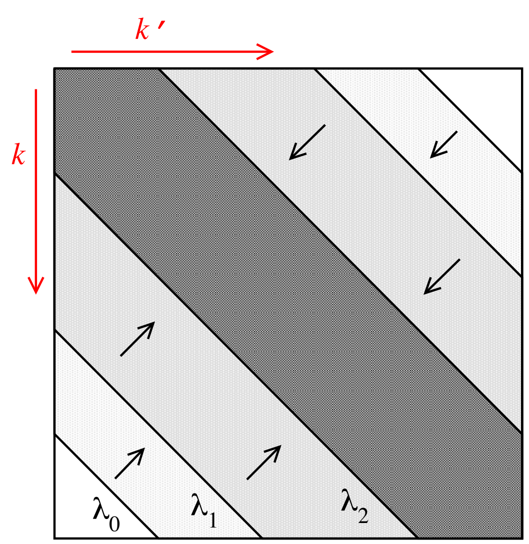
- (Unitary) transformation to **change the resolution scale** of the Hamiltonian
- Two main types of transformation

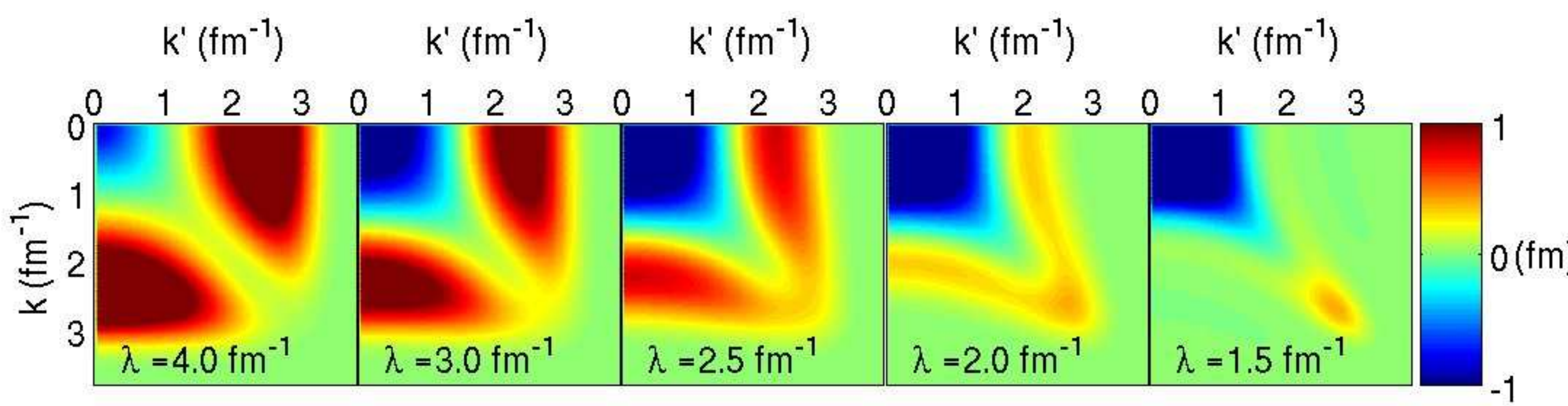
$V_{\text{low-k}}$





V_{SRG}



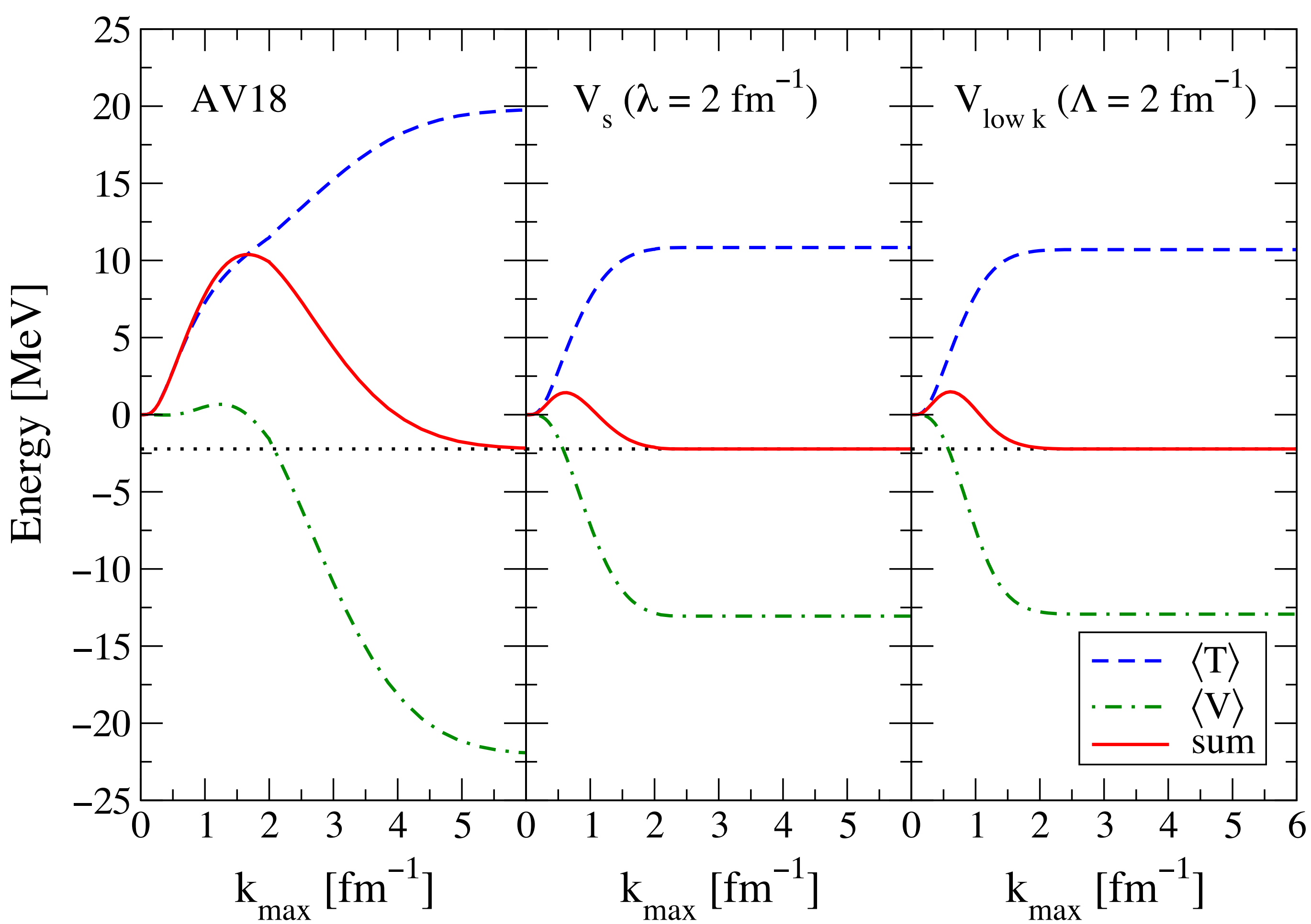


[Bogner *et al.* 2010]

Example: deuteron binding energy

- Performing RG changes weights of different parts of the Hamiltonian
- Observable binding energy remains unchanged
- High momenta not needed for softened interactions
- Simply cutting off high momenta doesn't work

$$E_d(k < k_{\max}) = \int_0^{k_{\max}} d\mathbf{k} \int_0^{k_{\max}} d\mathbf{k}' \, \psi_d^\dagger(\mathbf{k};\lambda) \left(k^2\delta^3(\mathbf{k}-\mathbf{k}') + V_s(\mathbf{k},\mathbf{k}')\right) \psi_d(\mathbf{k}';\lambda)$$

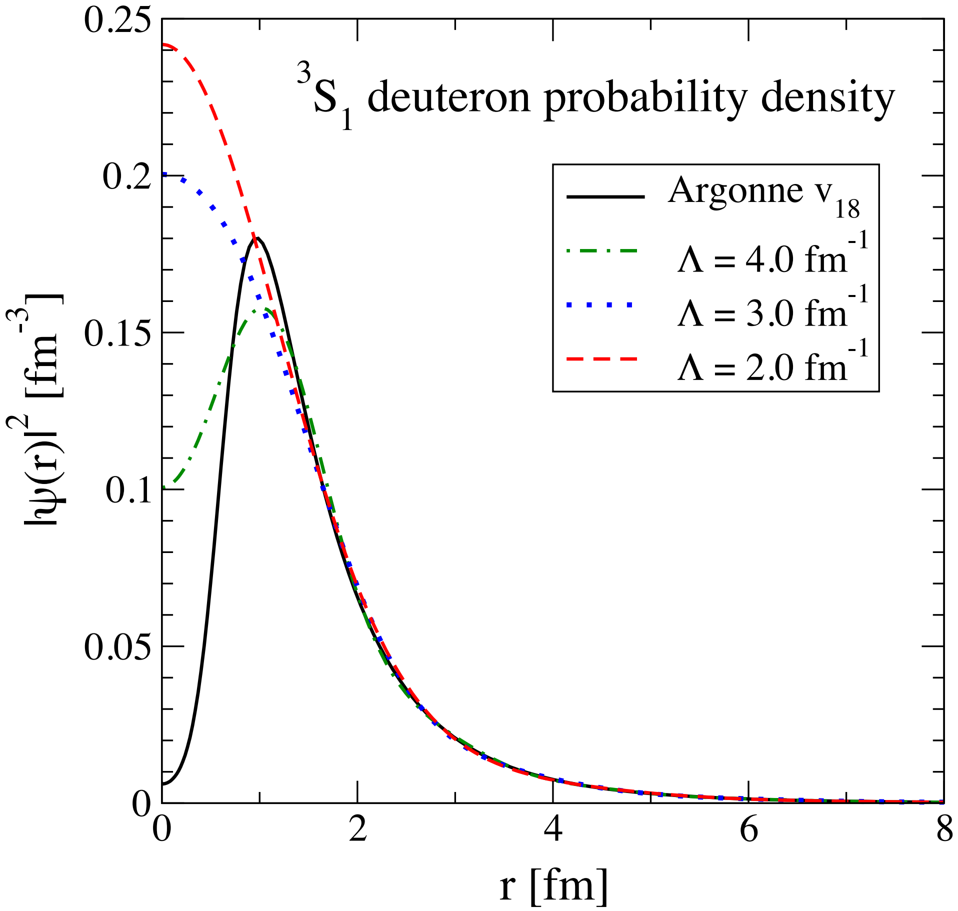


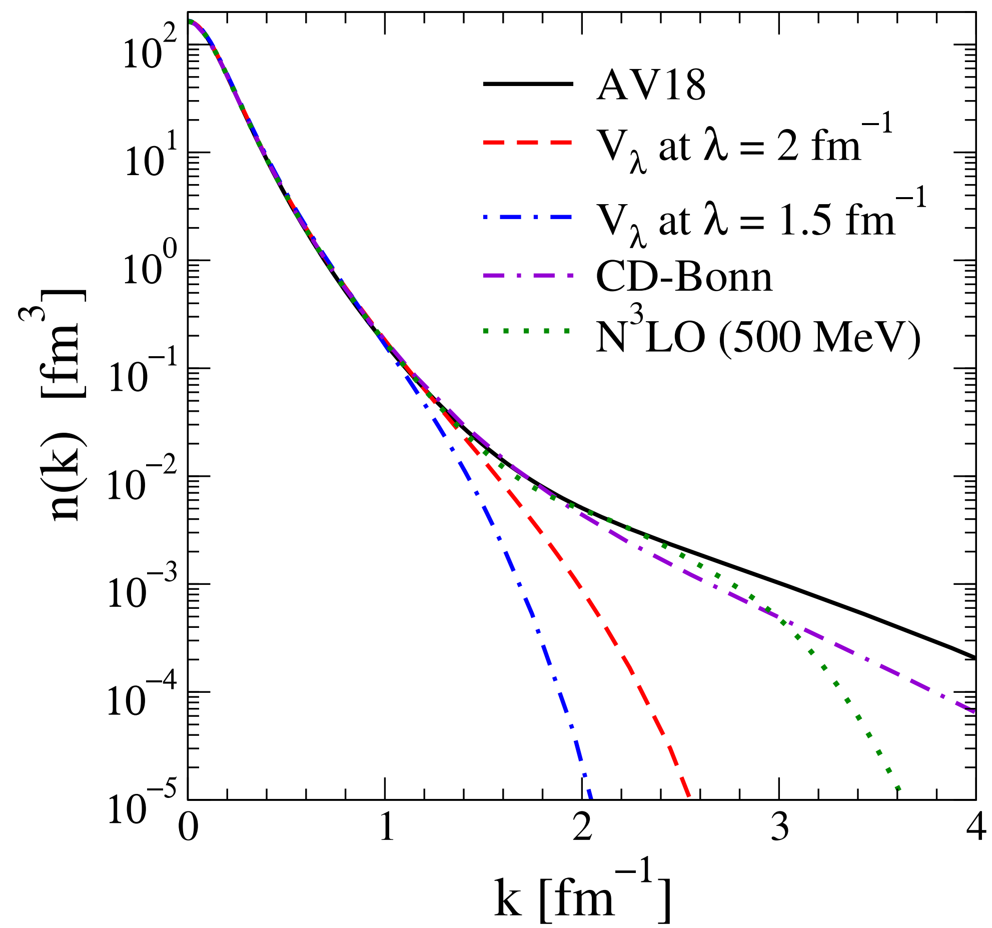
[Bogner *et al.* 2010]

Short-range correlations & momentum distribution

- **Short-range correlations change drastically with resolution scale**

[Bogner *et al.* 2010]



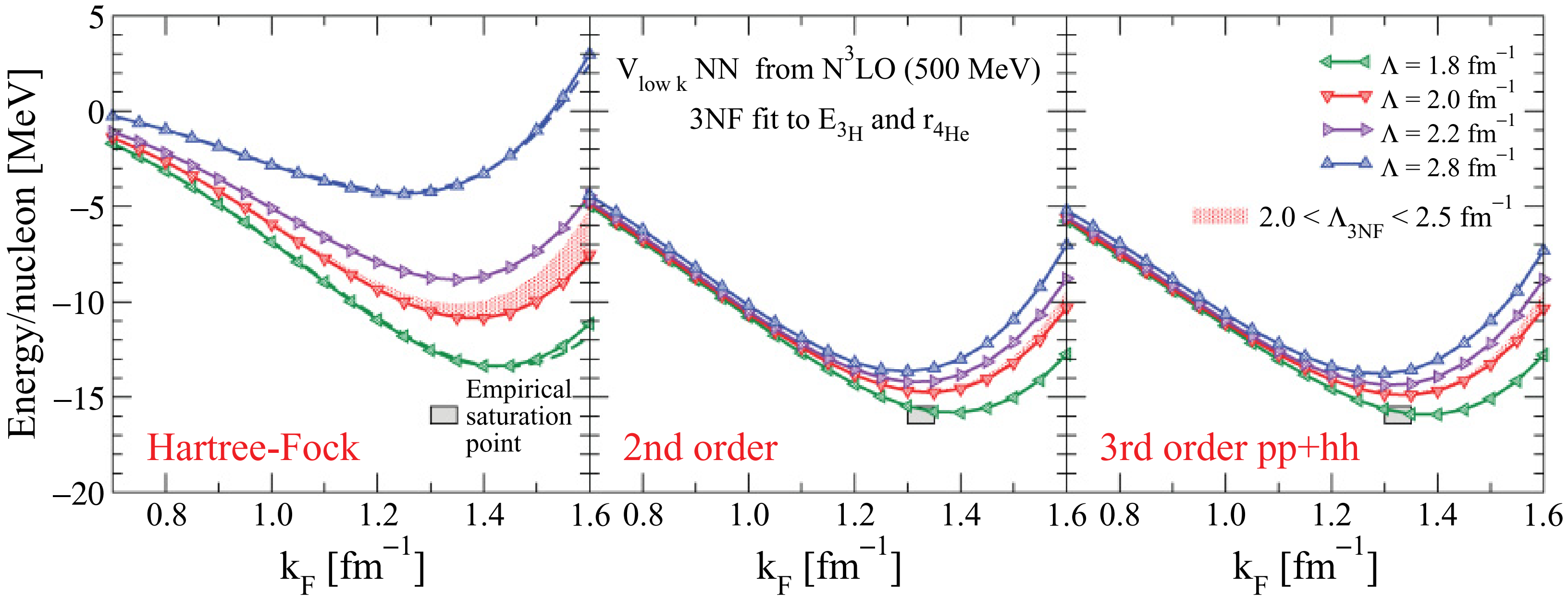


[More *et al.* 2015]

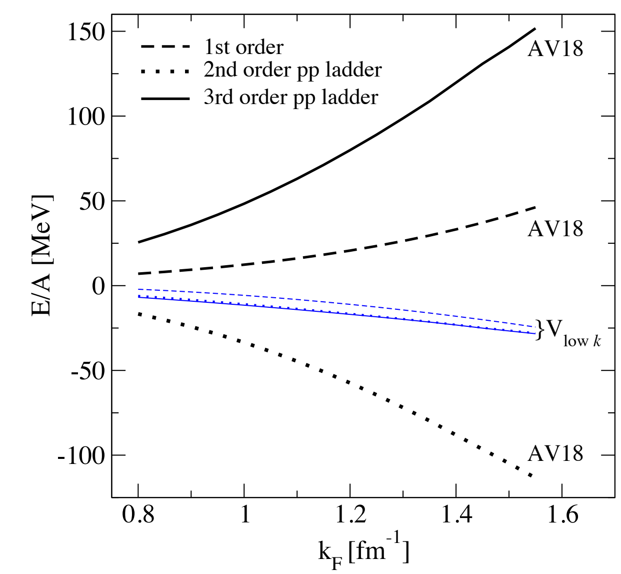
- **How to explain the momentum distribution "extracted" from experiment?**
  - Separation between structure and reaction is scale-dependent
  - Operators & currents have to evolved consistently with the Hamiltonian
  - E.g. what is a one-body current at one scale, gets shifted in two-body currents at another

Benefits in many-body systems

- **Improved convergence of many-body calculations**
  - Smaller **model spaces** & less refined **many-body truncations** needed

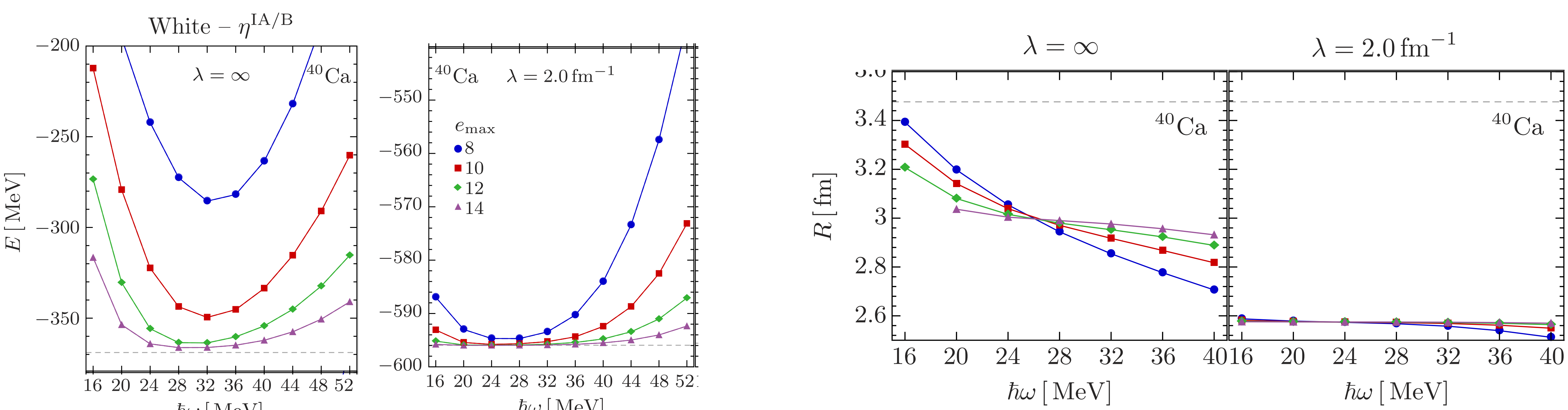


[Hebeler *et al.* 2011]



[Bogner *et al.* 2010]

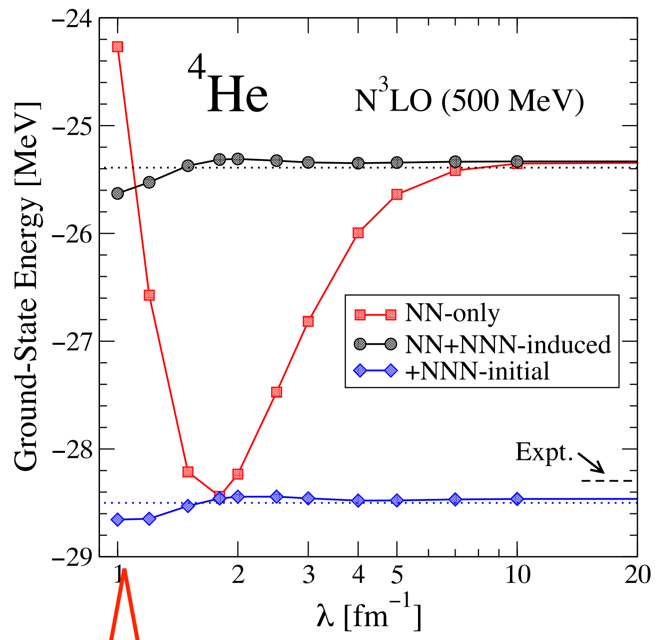
- **Drawback**: additional many-body forces generated through unitary transformation



[Hergert *et al.* 2016]

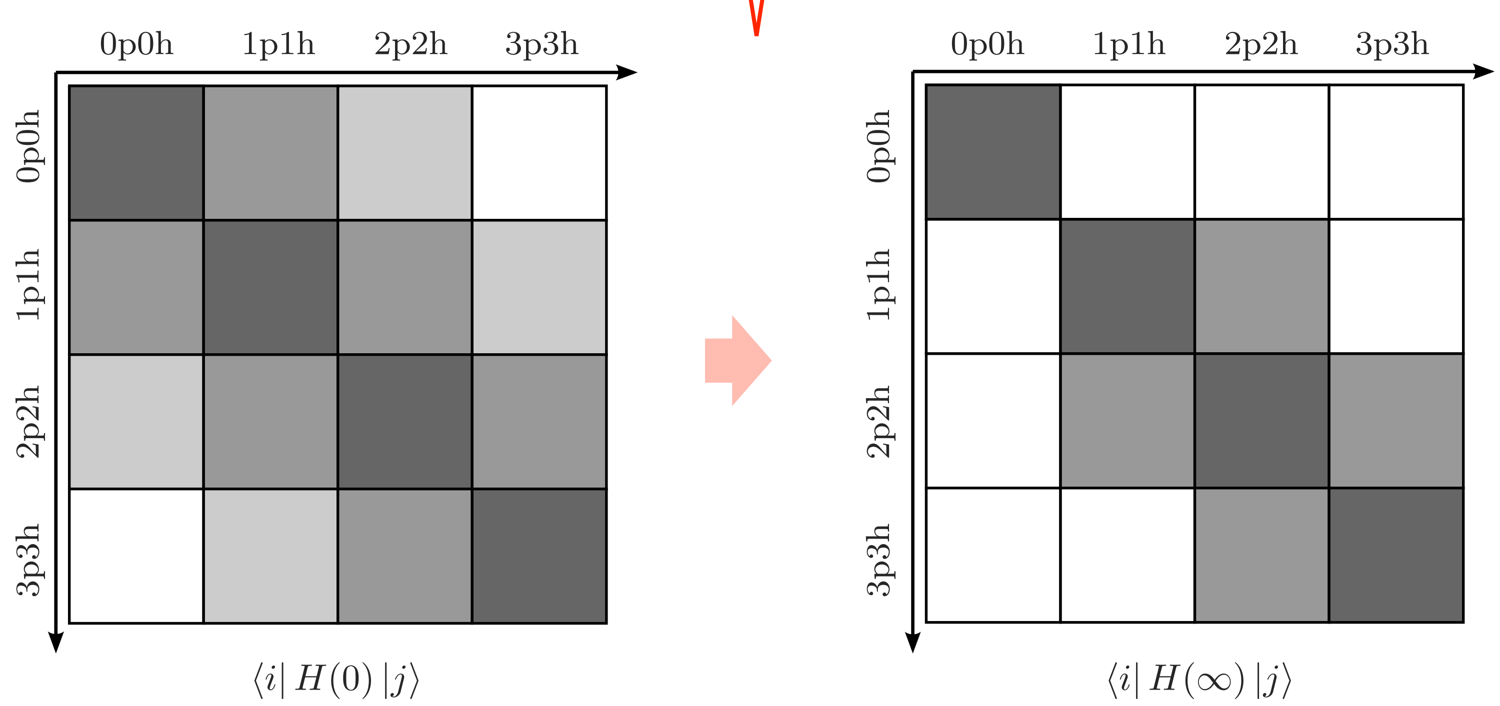
From free space to in medium

- Why don't evolve to the point where correlations have disappeared?



At very low scales, many-body (A>3) forces explode

However, if done step by step keeping normal-ordered parts at each step…



In-medium Similarity Renormalisation Group

- Unpractical to evolve in medium every operator we are interested in
- Combine with another many-body method (e.g. NCSM) to access wide range of observables

Part II
Non-observable character of the nuclear shell structure

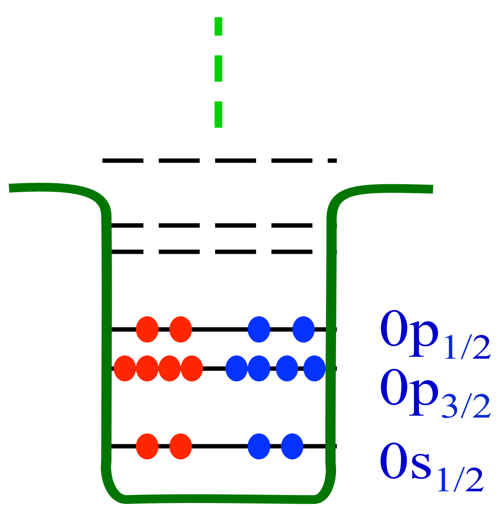
T. Duguet, H. Hergert, J.D. Holt, V. Somà, Phys. Rev C 92 034313 (2015)

Single-nucleon shell structure

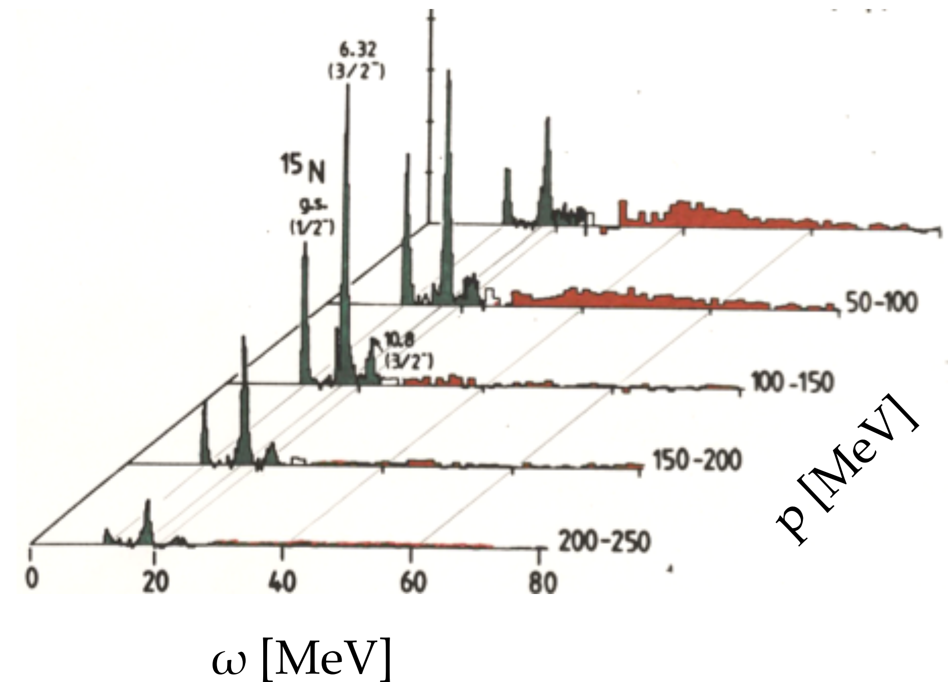
⦿ **Correlated many-body system ↔ description in terms of independent particles**

○ Can a one-to-one correspondence be established?

^{16}O



? ↔



[Mougey *et al.* 1980]

⦿ **Concept of single-nucleon shells**

○ Basic pillar of the shell model

○ Provides interpretation of nuclear (low-energy) observables

○ Leads to considering a single-particle spectrum (magicity, shell evolution, …)

Useful interpretation, but which degree of reality?

Single-nucleon shell structure

- **Quantum mechanical nuclear many-body problem**
  - Many-body Schrödinger equation → one-nucleon addition/removal energies

$$H|\Psi_k^{\mathrm{A}}\rangle = E_k^{\mathrm{A}}|\Psi_k^{\mathrm{A}}\rangle \qquad\qquad E_k^{\pm} \equiv \pm\left(E_k^{\mathrm{A}\pm 1} - E_0^{\mathrm{A}}\right)$$

To what extent the single-particle energy spectrum relates to low-energy observables?

$$\underbrace{E_k^{\pm}}_{\text{Outcome of Schr. equation}} = \underbrace{e_p}_{\text{Ind. particles}} + \underbrace{\Delta E_{p\to k}}_{\text{Correlations}}$$

- **In the following:**
  - Reminder of Green's function theory
  - Is there a proper/unique definition of single-particle energy? → Baranger ESPEs
  - Scale dependence of the above partitioning, i.e. of ESPEs
  - Illustration of the scale dependence form ab initio calculations

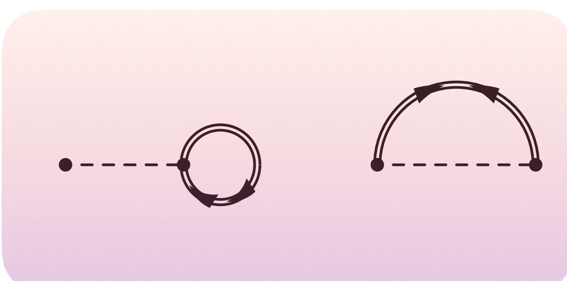
Self-consistent Green's function approach

- **Solution of the *A*-body Schrödinger equation** $H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$ **achieved by**

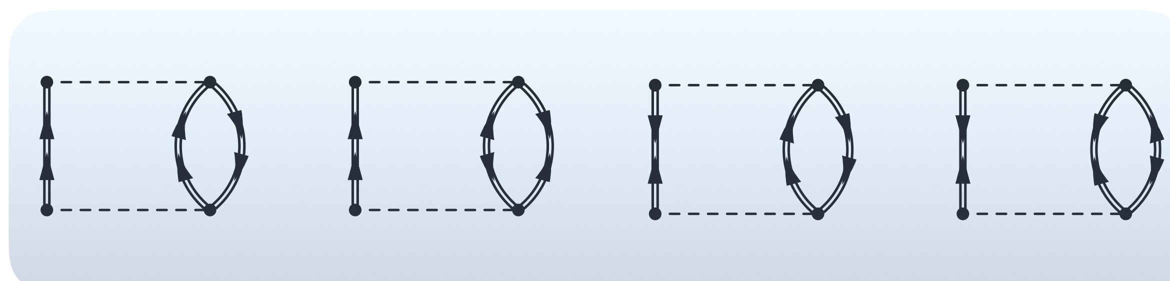
  1) Rewriting it in terms of **1-, 2-, …. *A*-body objects** $G_1 = G$, $G_2, \ldots G_A$ (**Green's functions**)

  2) Expanding these objects in perturbation (in practise only **G** ➟ **one-body observables**)

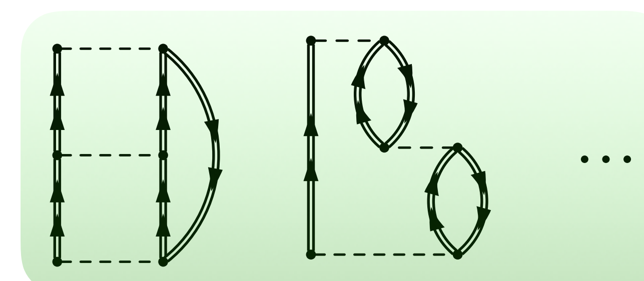
  ➟ **Self-consistent** schemes resum (infinite) subsets of perturbation-theory contributions



ADC(1)=HFB



ADC(2)



ADC(3)

- **Here we employ the Algebraic Diagrammatic Construction (ADC) method**
  - Systematic, improvable scheme for the one-body Green's functions, truncated at order n
  - ADC(1) = Hartree-Fock(-Bogolyubov); ADC(∞) = exact solution
  - At present **ADC(1)**, **ADC(2)** and **ADC(3)** are implemented and used

- **Extension to open-shell nuclei: (symmetry-breaking) Gorkov scheme**
  - Developed at Saclay & Surrey 2010-today [Somà, Duguet & Barbieri 2011]

Spectral representation

- Numerator contains spectroscopic information

$$G_{ab}(z) = \sum_{\mu} \frac{\langle \Psi_0^A | a_a | \Psi_\mu^{A+1} \rangle \langle \Psi_\mu^{A+1} | a_b^\dagger | \Psi_0^A \rangle}{z - E_\mu^+ + i\eta} + \sum_{\nu} \frac{\langle \Psi_0^A | a_b^\dagger | \Psi_\nu^{A-1} \rangle \langle \Psi_\nu^{A-1} | a_a | \Psi_0^A \rangle}{z - E_\nu^- - i\eta}$$

spectroscopic amplitudes

$$U_\mu^b \equiv \langle \Psi_0^A | a_b | \Psi_\mu^{A+1} \rangle$$
$$V_\nu^b \equiv \langle \Psi_0^A | a_b^\dagger | \Psi_\nu^{A-1} \rangle$$

spectroscopic probabilities matrices

$$S_\mu^{+ab} \equiv \langle \Psi_0^A | a_a | \Psi_\mu^{A+1} \rangle \langle \Psi_\mu^{A+1} | a_b^\dagger | \Psi_0^A \rangle$$
$$S_\nu^{-ab} \equiv \langle \Psi_0^A | a_a^\dagger | \Psi_\nu^{A\text{-}1} \rangle \langle \Psi_\nu^{A\text{-}1} | a_b | \Psi_0^A \rangle$$

spectroscopic factors

$$SF_\mu^+ \equiv \mathrm{Tr}_{\mathcal{H}_1}\left[\mathbf{S}_\mu^+\right] = \sum_{a \in \mathcal{H}_1} \left| U_\mu^a \right|^2$$
$$SF_\nu^- \equiv \mathrm{Tr}_{\mathcal{H}_1}\left[\mathbf{S}_\nu^-\right] = \sum_{a \in \mathcal{H}_1} \left| V_\nu^a \right|^2$$

spectral function

$$\mathbf{S}(z) \equiv \sum_{\mu \in \mathcal{H}_{A+1}} \mathbf{S}_\mu^+ \, \delta(z - E_\mu^+) + \sum_{\nu \in \mathcal{H}_{A-1}} \mathbf{S}_\nu^- \, \delta(z - E_\nu^-)$$

spectral strength distribution

$$\mathcal{S}(z) \equiv \mathrm{Tr}_{\mathcal{H}_1}[\mathbf{S}(z)]$$
$$= \sum_{\mu \in \mathcal{H}_{A+1}} SF_\mu^+ \, \delta(z - E_\mu^+) + \sum_{\nu \in \mathcal{H}_{A-1}} SF_\nu^- \, \delta(z - E_\nu^-)$$

Spectral representation

⊙ Combine numerator and denominator of Lehmann representation

$$G_{ab}(z) = \sum_{\mu} \frac{U_a^{\mu} (U_b^{\mu})^*}{z - E_{\mu}^{+} + i\eta} + \sum_{\nu} \frac{(V_a^{\nu})^* V_b^{\nu}}{z - E_{\nu}^{-} - i\eta}$$

denominator

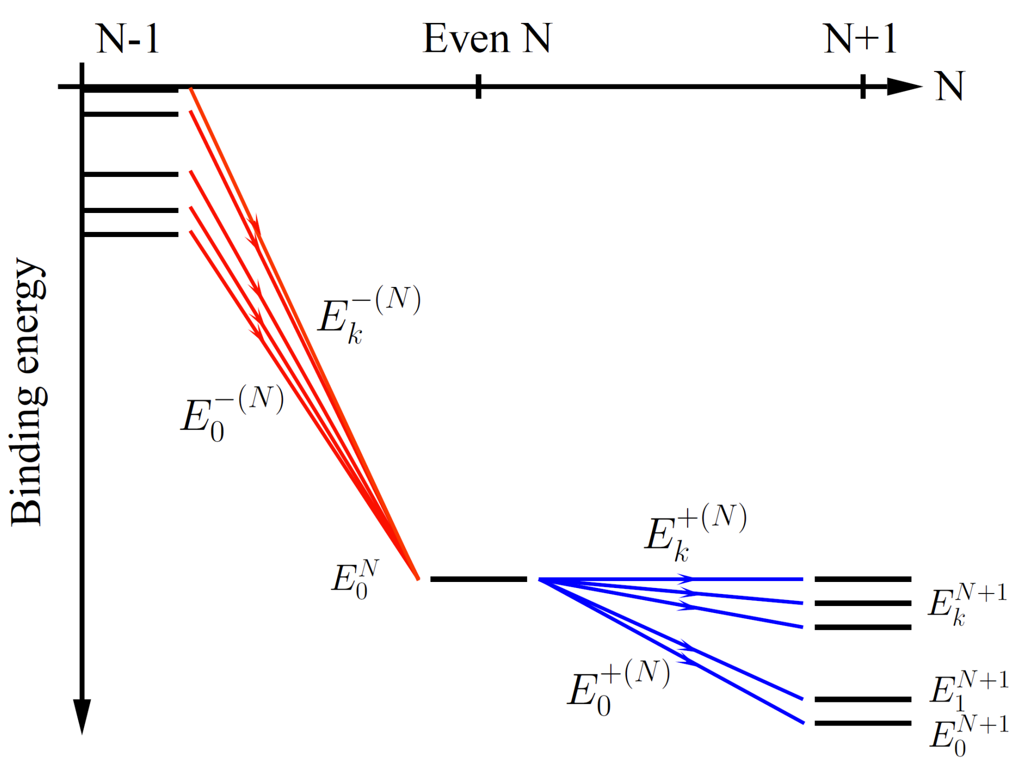
$$E_{\mu}^{+} \equiv E_{\mu}^{N+1} - E_0^{N}$$

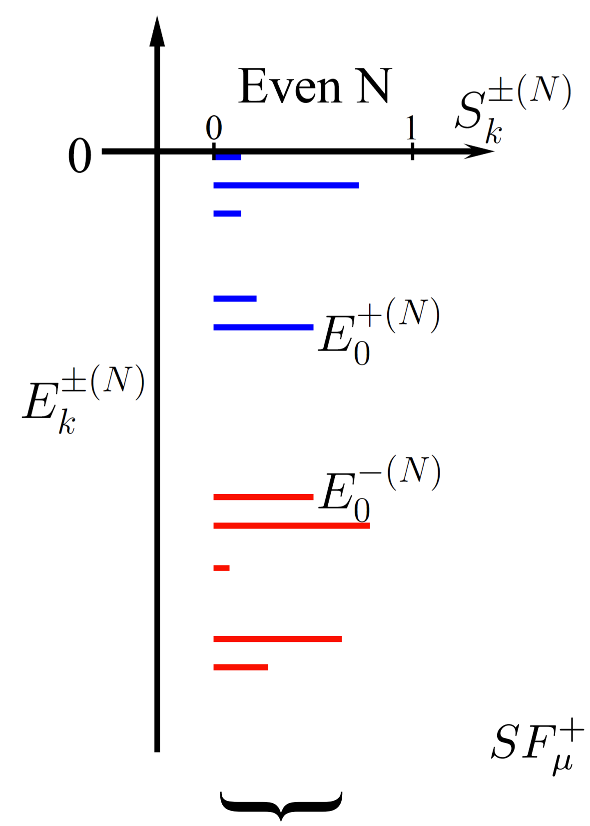
$$E_{\nu}^{-} \equiv E_0^{N} - E_{\nu}^{N-1}$$

+ numerator

spectral strength distribution

$$\mathcal{S}(z) = \sum_{\mu \in \mathcal{H}_{A+1}} SF_{\mu}^{+}\, \delta(z - E_{\mu}^{+}) + \sum_{\nu \in \mathcal{H}_{A-1}} SF_{\nu}^{-}\, \delta(z - E_{\nu}^{-})$$





spectroscopic factors

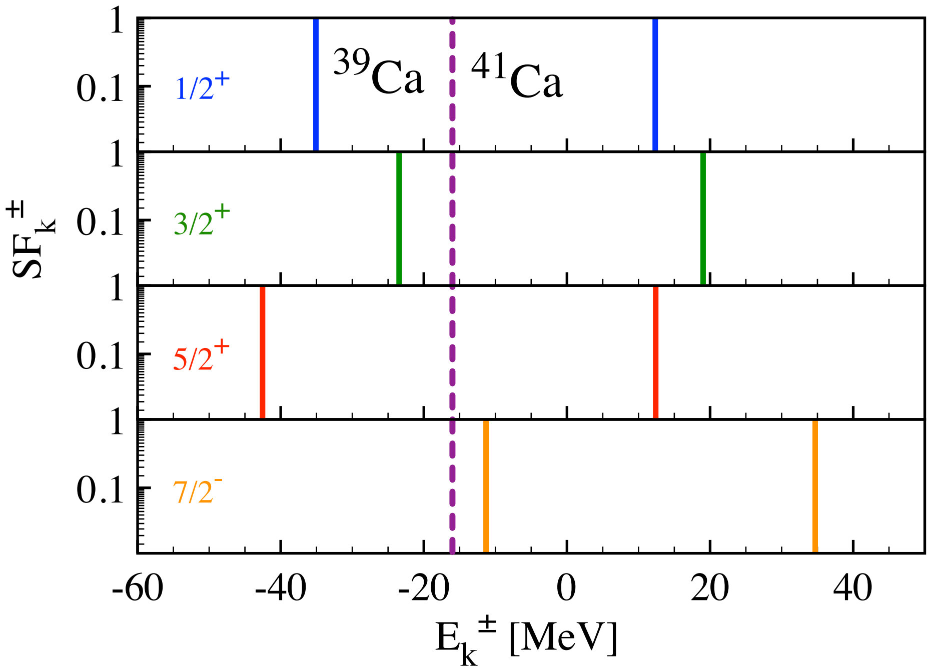
$$SF_{\mu}^{+} \equiv \mathrm{Tr}_{\mathcal{H}_1}\left[\mathbf{S}_{\mu}^{+}\right] = \sum_{a \in \mathcal{H}_1} |U_{\mu}^{a}|^2$$

$$SF_{\nu}^{-} \equiv \mathrm{Tr}_{\mathcal{H}_1}\left[\mathbf{S}_{\nu}^{-}\right] = \sum_{a \in \mathcal{H}_1} |V_{\nu}^{a}|^2$$

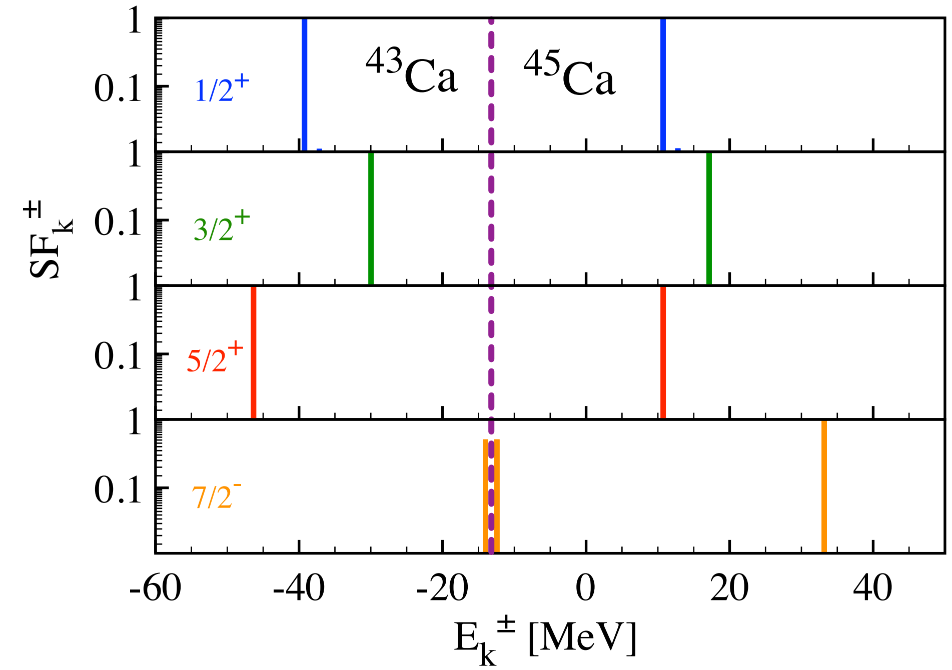
[figures from J. Sadoudi]

Spectral strength distribution

Dyson 1st order (HF)



Gorkov 1st order (HFB)

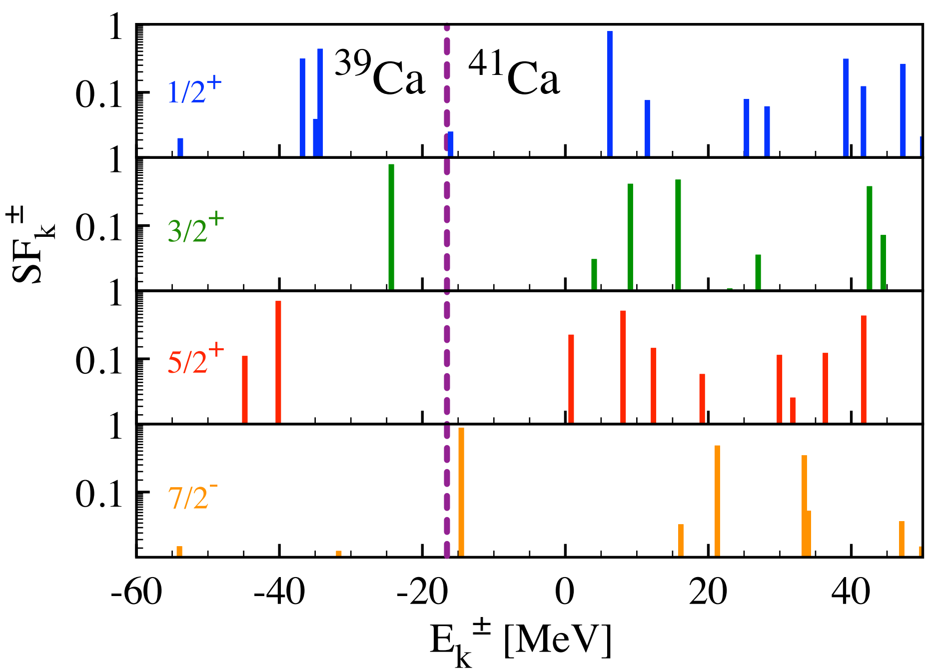


Fragmentation

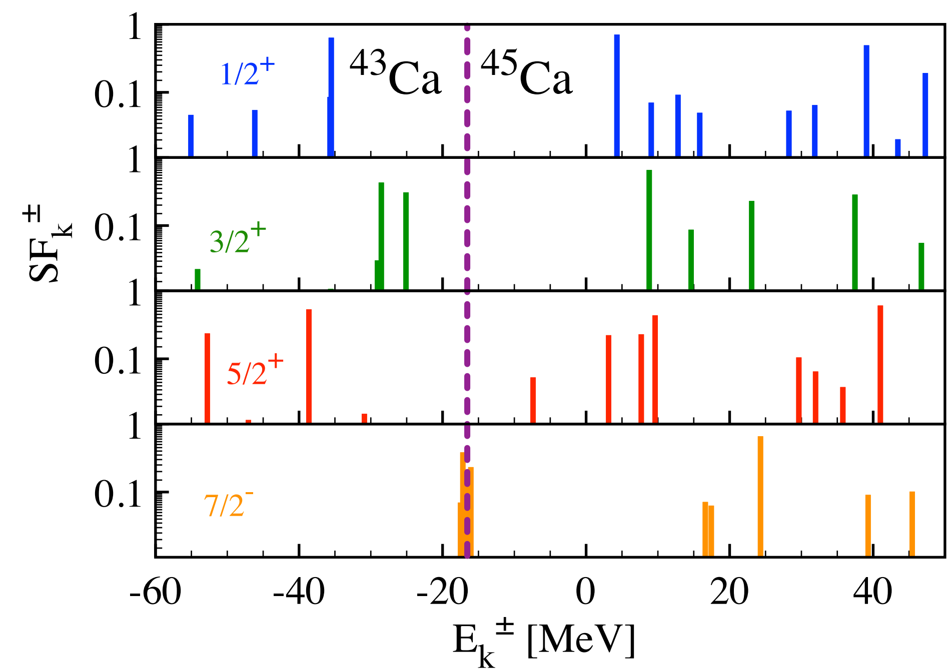
Static pairing

Dynamical fluctuations

Dyson 2nd order

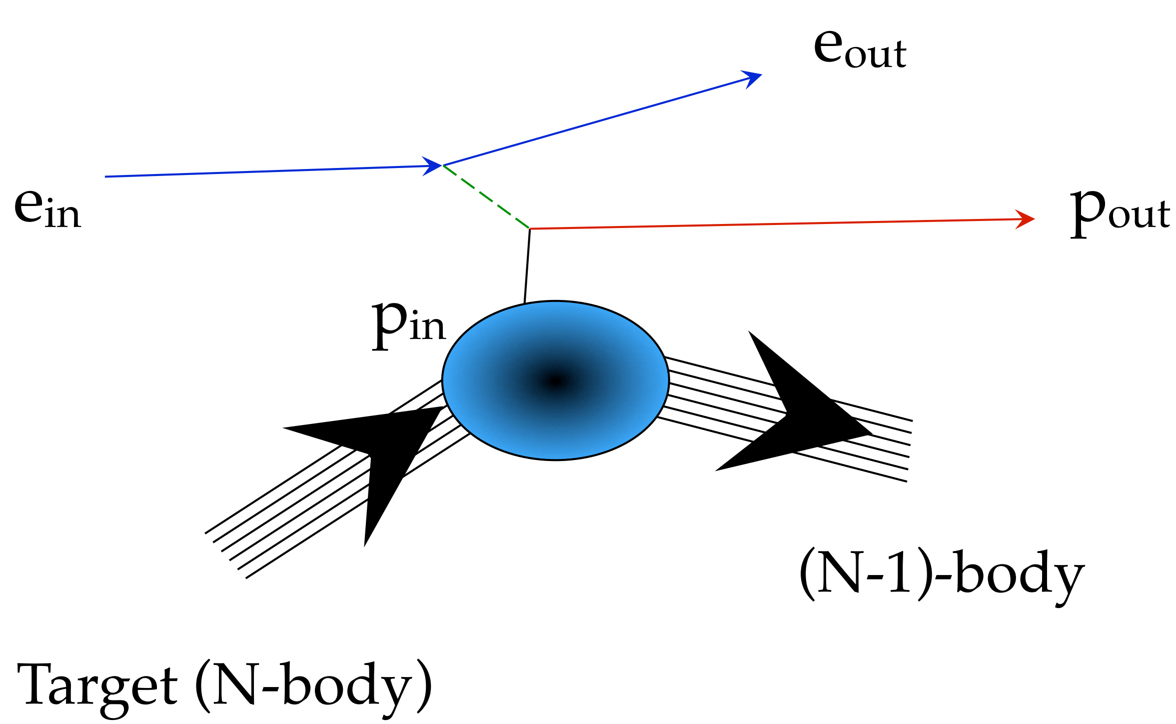


Gorkov 2nd order



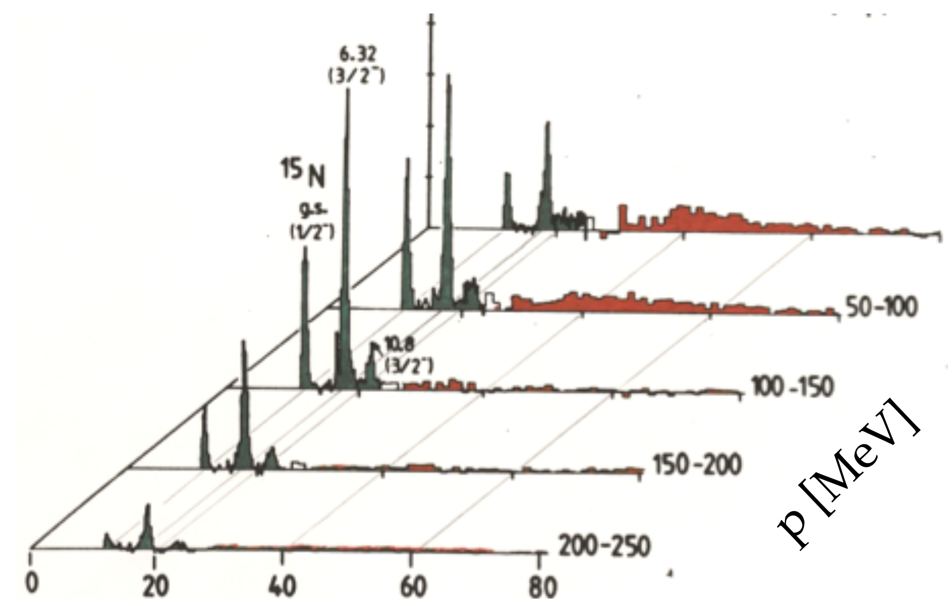
Spectral strength in experiments

⦿ Spectroscopy via **knock-out reactions**



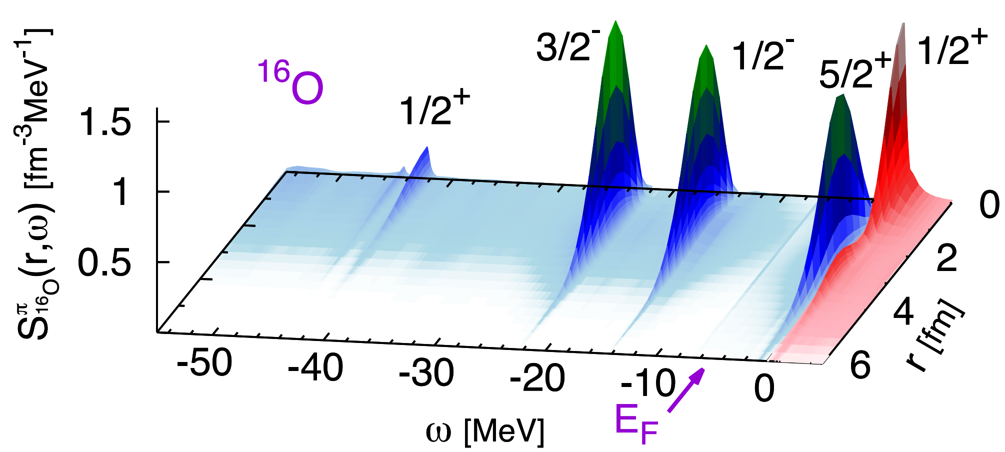
By measuring e_{in}, e_{out} and p_{out}
get information on p_{in}

Results from (e,e′p) on ^{16}O (ALS in Saclay)



[Mougey *et al.* 1980]

SCGF calculations



[Cipollone *et al.* 2015]

Effective Single-Particle Energies (ESPEs)

Spectroscopic probability matrices

$$S_\mu^{+pq} \equiv \langle \Psi_0^{A} | a_p | \Psi_\mu^{A+1} \rangle \langle \Psi_\mu^{A+1} | a_q^\dagger | \Psi_0^{A} \rangle$$

$$S_\nu^{-pq} \equiv \langle \Psi_0^{A} | a_q^\dagger | \Psi_\nu^{A\text{-}1} \rangle \langle \Psi_\nu^{A\text{-}1} | a_p | \Psi_0^{A} \rangle$$

Spectroscopic factors

$$SF_\mu^{+} \equiv \mathrm{Tr}[\mathbf{S}_\mu^{+}]$$

$$SF_\nu^{-} \equiv \mathrm{Tr}[\mathbf{S}_\nu^{-}]$$

[Baranger 1970]

Centroid one-body Hamiltonian

$$\mathbf{h}^{\text{cent}} \equiv \sum_\mu \mathbf{S}_\mu^{+} E_\mu^{+} + \sum_\nu \mathbf{S}_\nu^{-} E_\nu^{-} = \mathbf{T} + \mathbf{\Sigma}(\infty)$$

Self-energy

$$\mathbf{\Sigma}(\omega) = \mathbf{\Sigma}(\infty) + \mathbf{\Sigma}^{\text{dyn}}(\omega)$$

Energy-independent part of the self-energy

Effective single-particle energies

$$\mathbf{h}^{\text{cent}} \, \psi_p^{\text{cent}} = e_p^{\text{cent}} \, \psi_p^{\text{cent}}$$

$$e_p^{\text{cent}} \equiv \sum_{\mu \in \mathcal{H}_{A+1}} S_\mu^{+pp} E_\mu^{+} + \sum_{\nu \in \mathcal{H}_{A-1}} S_\nu^{-pp} E_\nu^{-}$$

Baranger ESPEs

- ○ Defined solely from Schrödinger eq.
- ○ Computable in any many-body scheme
- ○ Relate to the average dynamics of nucleons
- ○ Reduce to HF SPEs in HF approximation

Inverting ESPEs

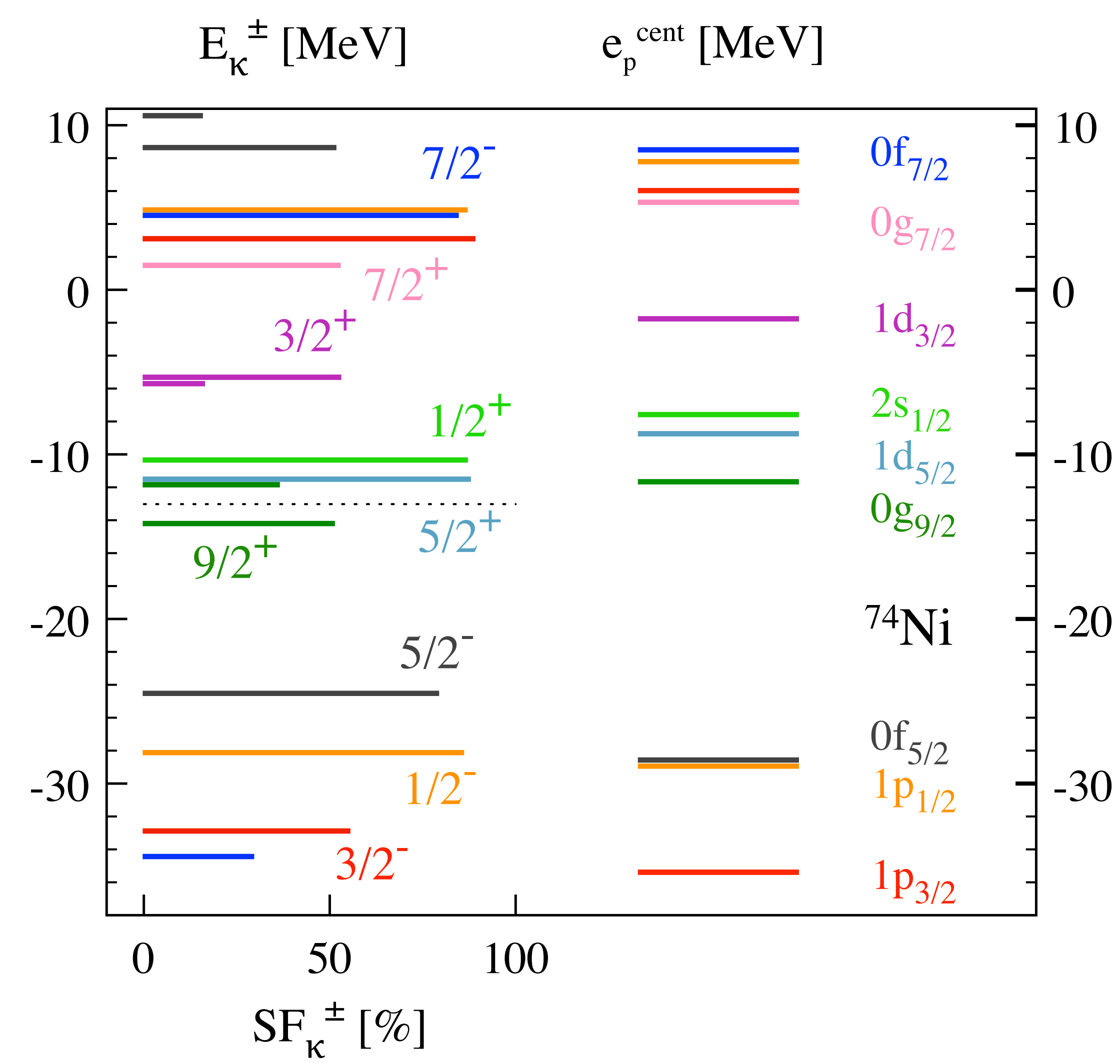
- **Baranger ESPEs in the basis associated to h^cent**

$$e_p^{\text{cent}} \equiv \sum_{\mu \in \mathcal{H}_{A+1}} S_\mu^{+pp} E_\mu^+ + \sum_{\nu \in \mathcal{H}_{A-1}} S_\nu^{-pp} E_\nu^-$$

invert $\downarrow$ (same for E_ν^-)

$$E_\mu^+ = \sum_p s_\mu^{+pp}\, e_p^{\text{cent}} + \sum_{pq} s_\mu^{+pq}\, \Sigma_{qp}^{\text{dyn}}(E_\mu^+)$$

with $\mathbf{s}_\mu^+ \equiv \mathbf{S}_\mu^+ / SF_\mu^+$ & $\mathbf{\Sigma}^{\text{dyn}}(\omega) \equiv \mathbf{\Sigma}(\omega) - \mathbf{\Sigma}(\infty)$



- **Rigorous partitioning into independent-particle + correlation contributions**
  - Exact result, no approximations so far
  - A given one-nucleon addition energy does not relate to a single ESPE
  - Connection between the two spectra is of matrix character

Partitioning & scale dependence

- Nuclear Hamiltonian carries an intrinsic scale resolution Λ_{init}
- One can further apply a unitary transformation U(λ) over Fock space

$$H(\lambda) \equiv U(\lambda) H U^\dagger(\lambda)$$
$$\left|\Psi^A_\mu(\lambda)\right\rangle \equiv U(\lambda)\left|\Psi^A_\mu\right\rangle$$

$$\Rightarrow \quad H(\lambda)\left|\Psi^A_\mu(\lambda)\right\rangle = E^A_k\left|\Psi^A_\mu(\lambda)\right\rangle$$

- Any other operator transforms accordingly

$$O(\lambda) \equiv U(\lambda) O U^\dagger(\lambda) \equiv O^{1N}(\lambda) + O^{2N}(\lambda) + O^{3N}(\lambda) + \cdots$$

- Spectroscopic amplitudes defined at any value of λ

$$U^p_\mu(\lambda) \equiv \left\langle \Psi^A_0(\lambda)\right| a_p \left|\Psi^{A+1}_\mu(\lambda)\right\rangle$$
$$V^p_\nu(\lambda) \equiv \left\langle \Psi^A_0(\lambda)\right| a^\dagger_p \left|\Psi^{A-1}_\nu(\lambda)\right\rangle$$

- Generator of the transformation

$$\eta(\lambda) \equiv \frac{dU(\lambda)}{d\lambda} U^\dagger(\lambda)$$

Partitioning & scale dependence

- Spectroscopic probabilities / factors are **scale-dependent**

$$\frac{d}{d\lambda} V_\nu^p(\lambda) = -\left\langle \Psi_\nu^{A-1}(\lambda) \middle| [\eta(\lambda), a_p] \middle| \Psi_0^A(\lambda) \right\rangle^*$$
$$\frac{d}{d\lambda} U_\mu^p(\lambda) = -\left\langle \Psi_\mu^{A+1}(\lambda) \middle| [\eta(\lambda), a_p^\dagger] \middle| \Psi_0^A(\lambda) \right\rangle^*$$

- ESPEs acquire scale dependence via spectroscopic probabilities

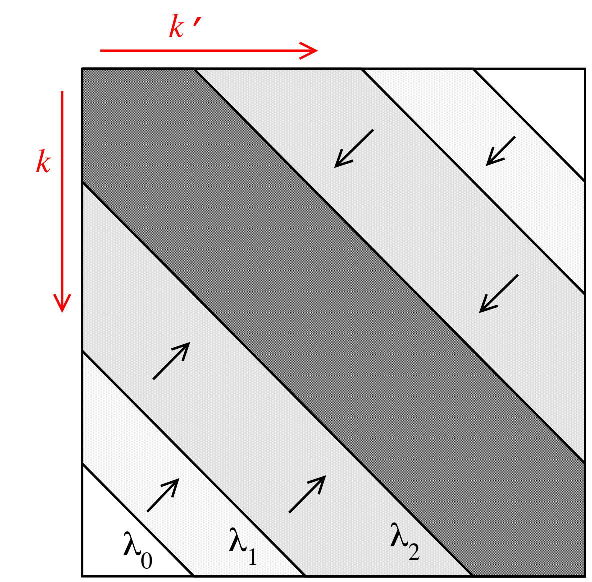
$$e_p^{\text{cent}} \equiv \sum_{\mu \in \mathcal{H}_{A+1}} S_\mu^{+pp} E_\mu^+ + \sum_{\nu \in \mathcal{H}_{A-1}} S_\nu^{-pp} E_\nu^-$$

$$\underbrace{\overset{\text{many-body observable}}{E_\mu^+}}_{\text{invariant under } U(\lambda)} \equiv \underbrace{\overset{\text{single-particle components}}{\sum_p s_\mu^{+pp}(\lambda)\, e_p^{\text{cent}}(\lambda)}}_{\text{varies under } U(\lambda)} + \underbrace{\overset{\text{correlations}}{\sum_{pq} s_\mu^{+pq}(\lambda)\, \Sigma_{qp}^{\text{dyn}}(E_\mu^+; \lambda)}}_{\text{varies under } U(\lambda)}$$

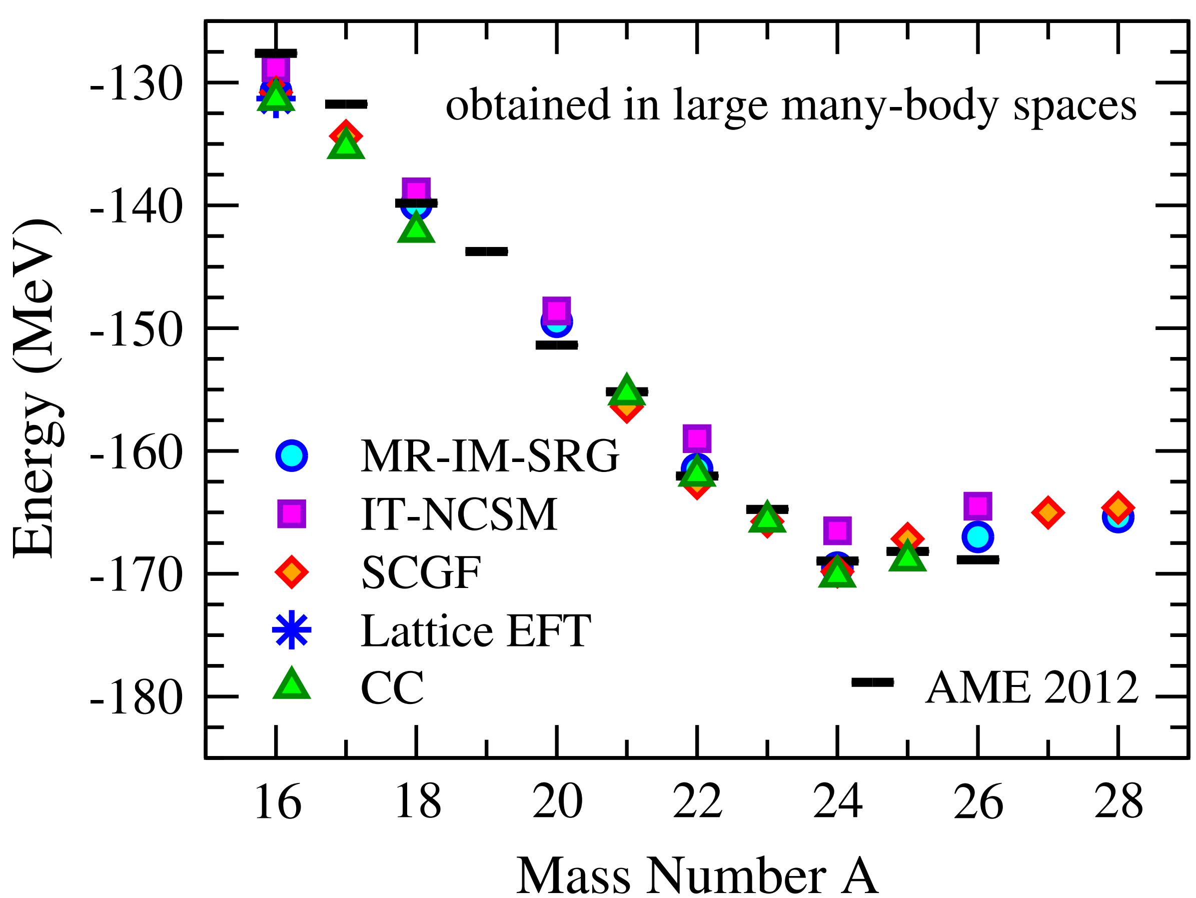
- A convenient choice of λ maximises the ESPE component
- However, **correlations with observables are not absolute**
- Scale must be fixed / specified prior to theoretical / experimental comparisons

SRG transformation & ab initio calculations

- SRG transformations $U(\lambda)$ applied to the starting Hamiltonian $H(\Lambda_{init})$
  - Limited range of variation: $\boldsymbol{\lambda} \in$ **{1.88, 2.0, 2.24} fm^{-1}**



- Two different ab initio methods
  - **Gorkov-Green's functions** [Somà, Barbieri, Duguet 2011, …]
  - **In-medium SRG** [Tsukiyama, Bogner, Schwenk 2010, Hergert *et al.* 2013, …]



[Hergert *et al.* 2013]

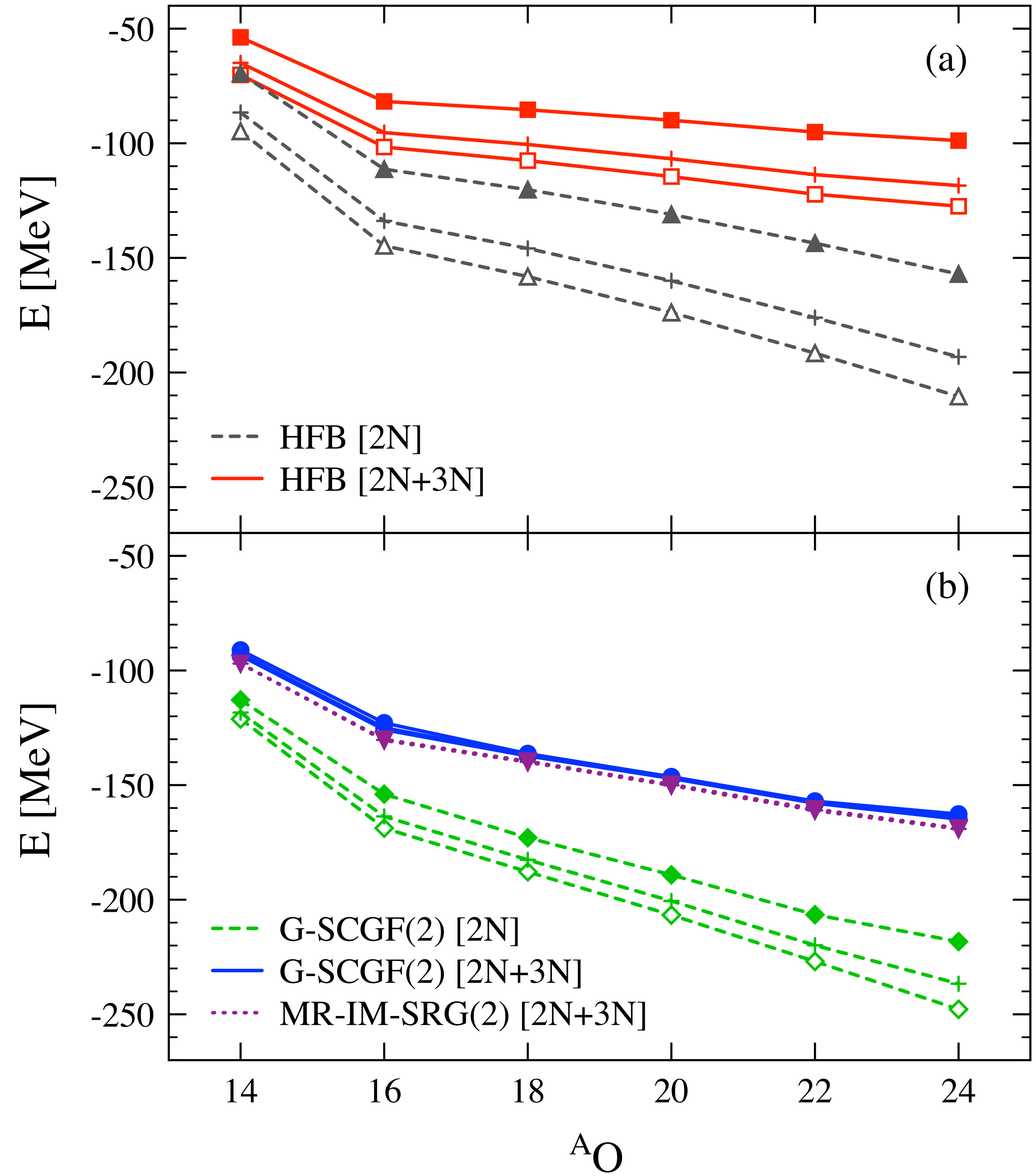
[Cipollone *et al.* 2013]

[Jansen *et al.* 2014]

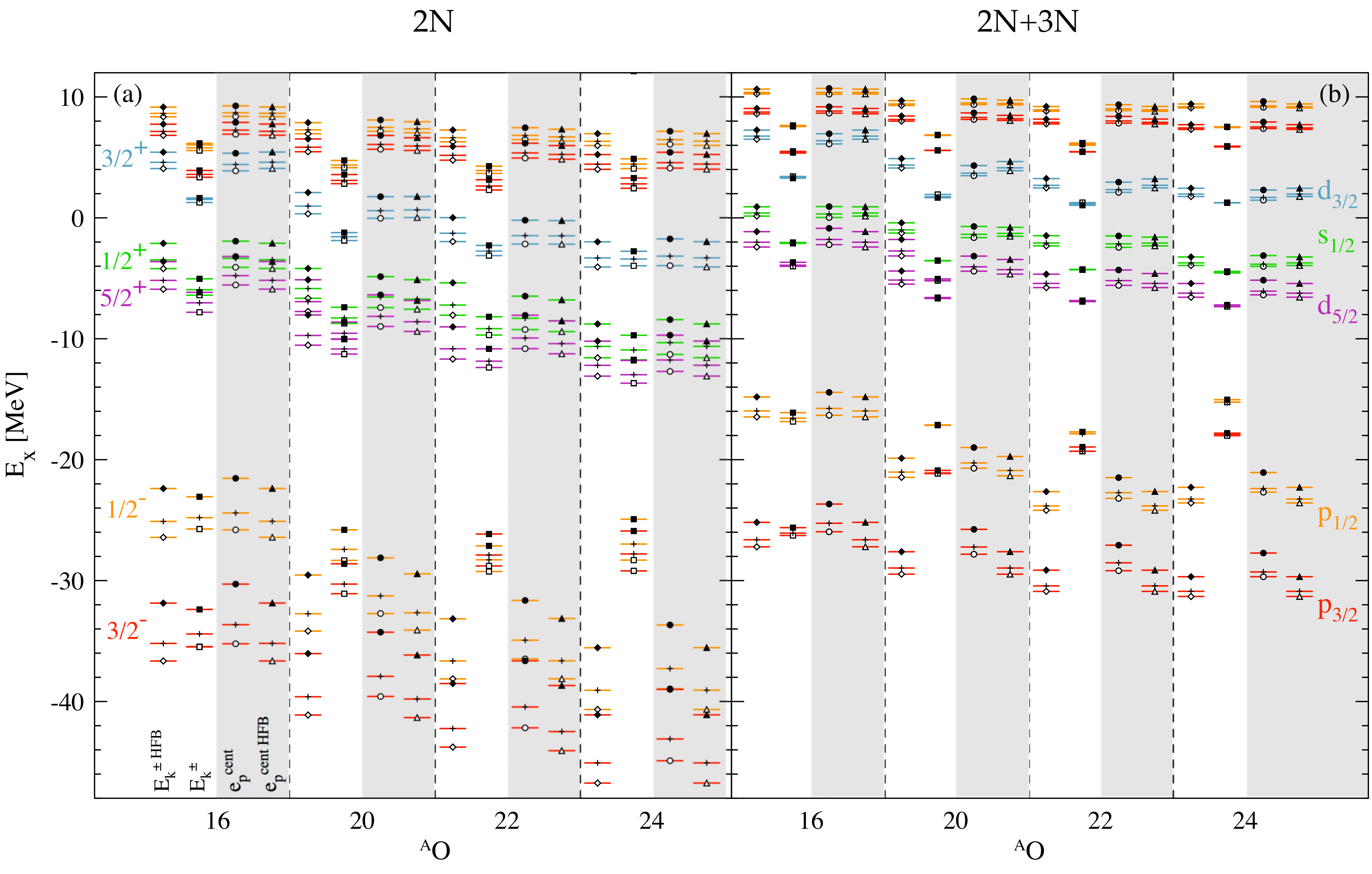
[Lähde *et al.* 2014]

Breaking of unitarity

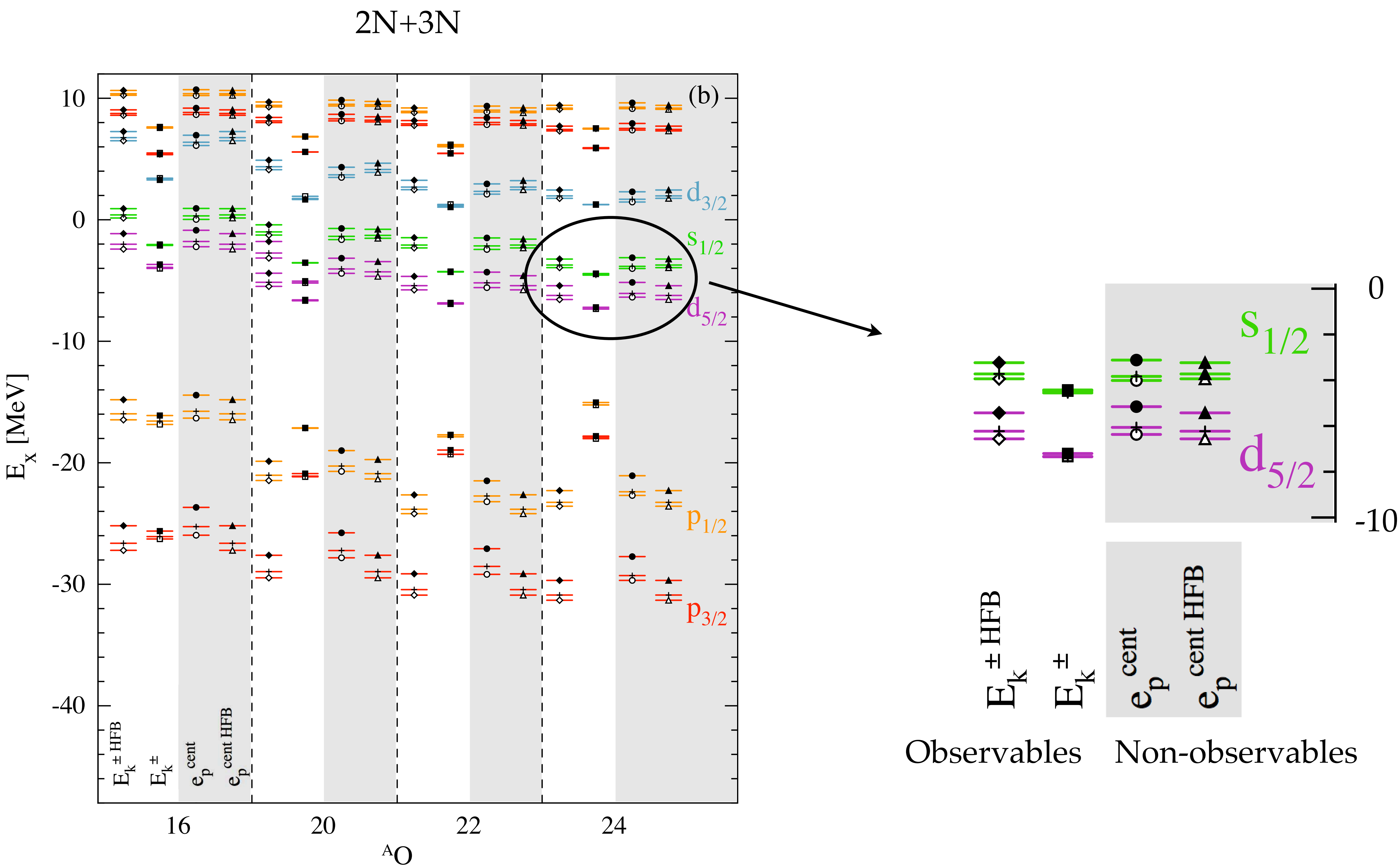
- **Unitarity artificially broken**
  - Omission of *A*-body operators with *A*>3
  - Many-body truncations
- **Breaking can be estimated**
  - Omitting 3-body operators
  - Degrading the many-body truncation
- **Breaking for total energy**
  - Around 1 MeV for GGF
  - Around 100 keV for IM-SRG



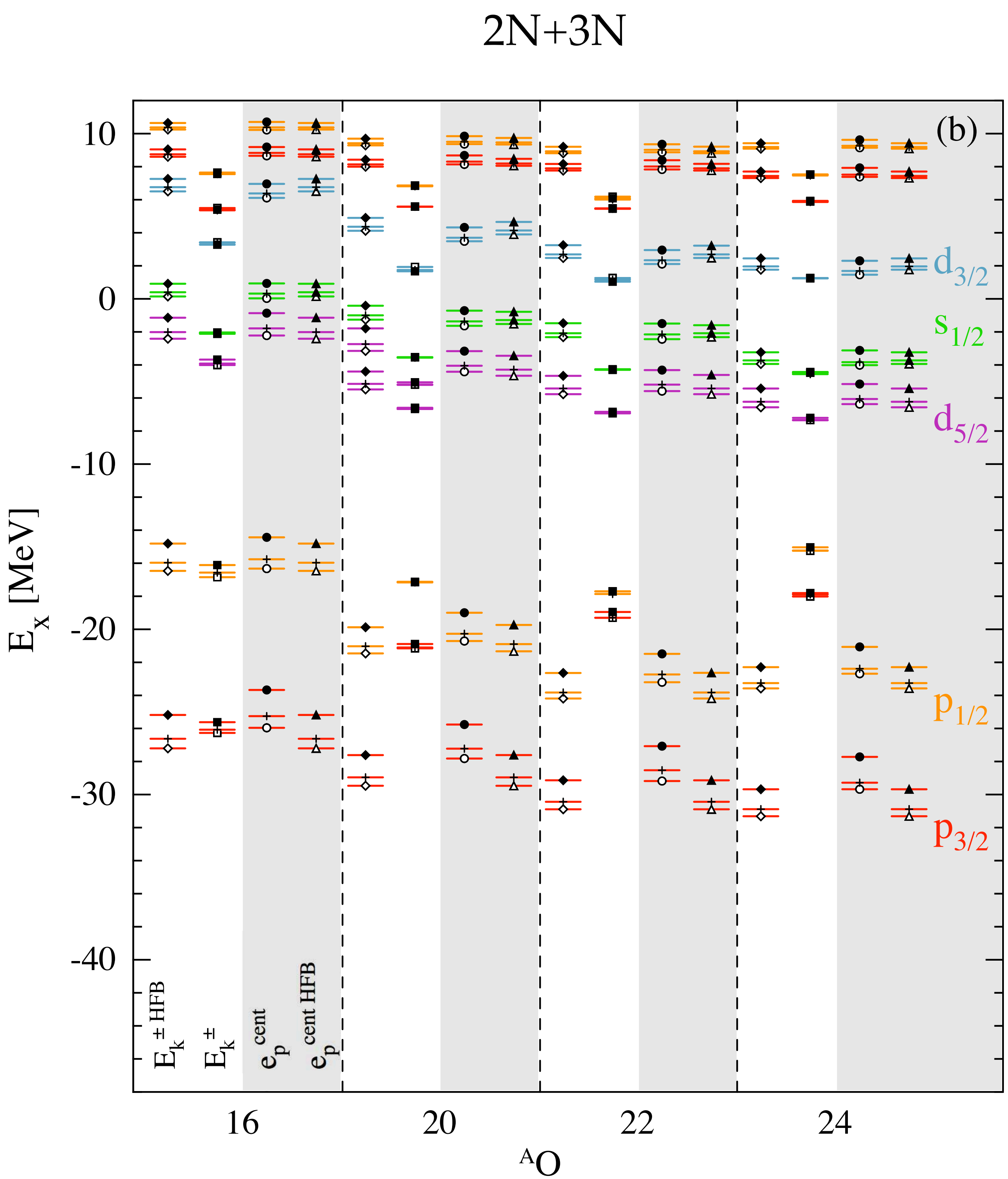
Scale (in)dependence of separation energies & ESPEs

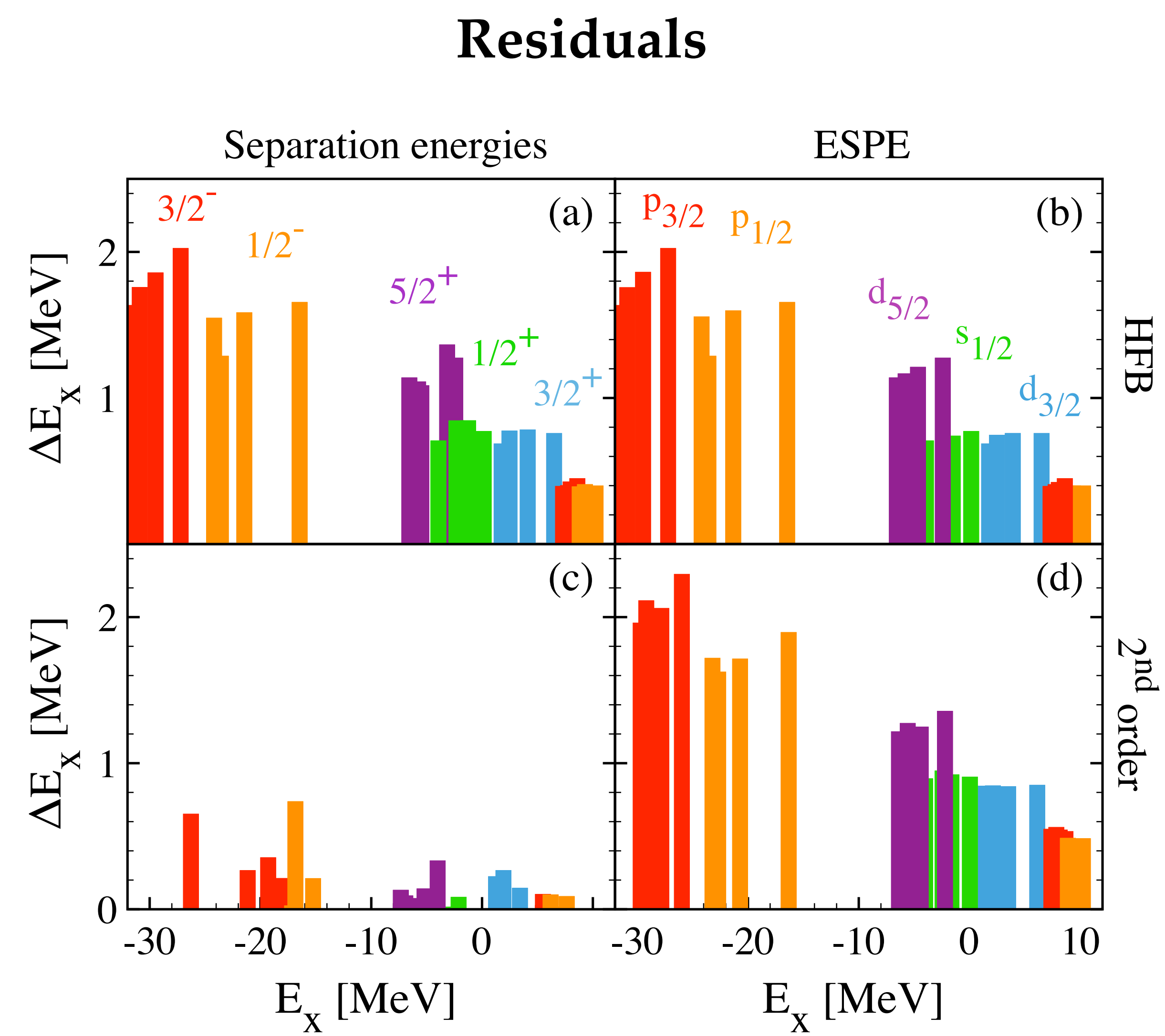


Scale (in)dependence of separation energies & ESPEs



Scale (in)dependence of separation energies & ESPEs

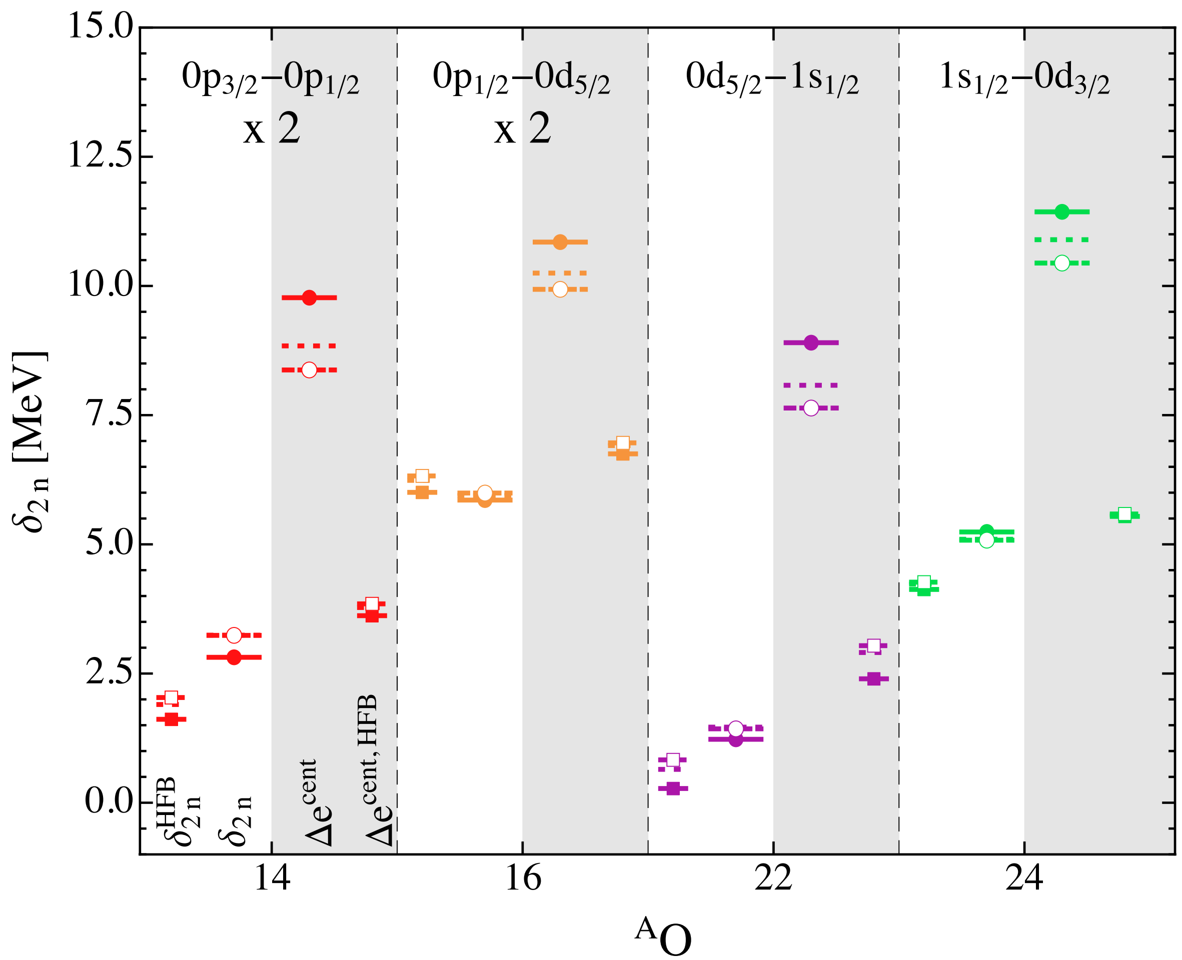




- ○ Spread of sep. en. reduced significantly
- ○ Spread of ESPEs unchanged
- ○ ESPEs less sensitive to correlations

Shell gaps

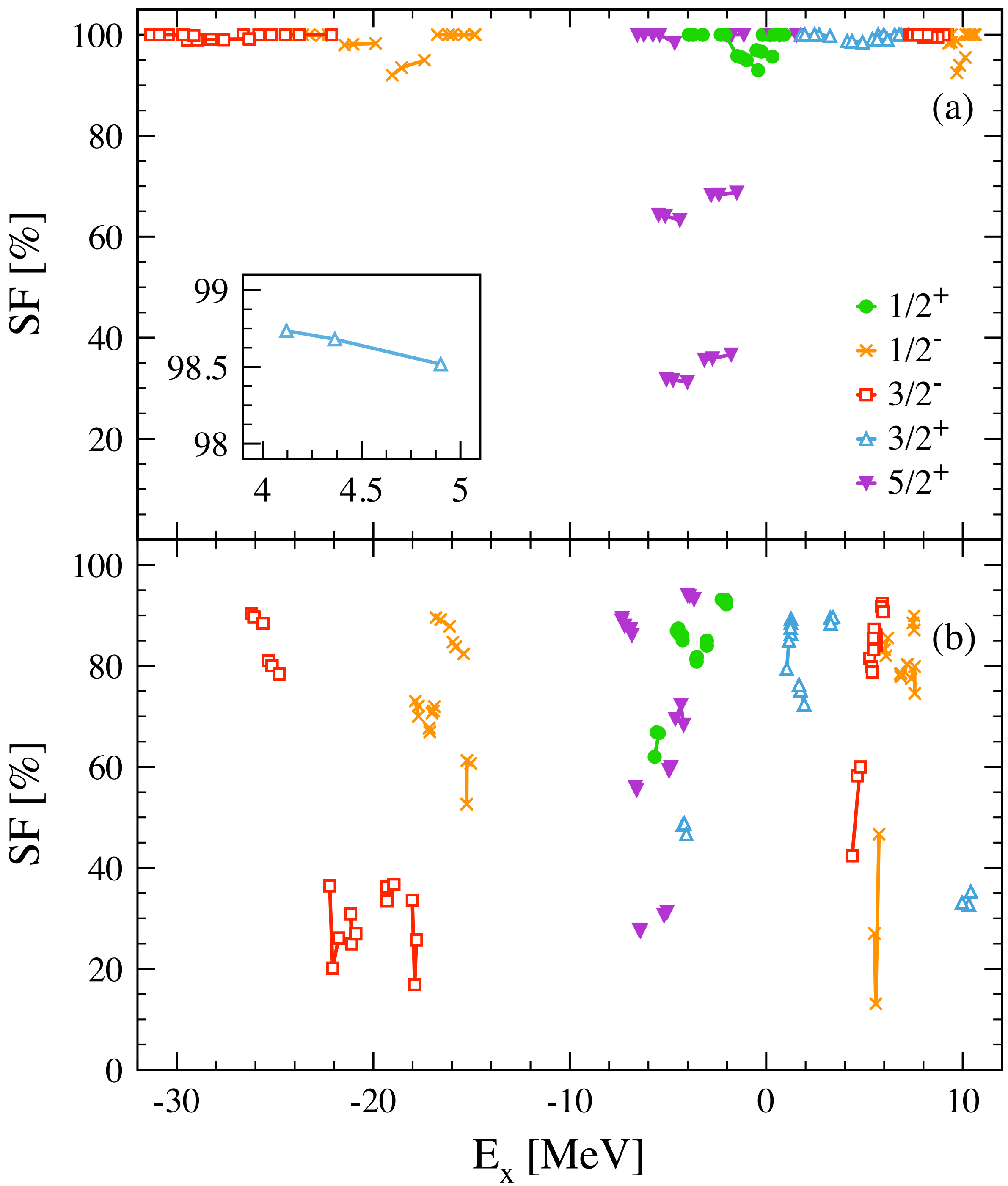
- **Gaps across the Fermi energy (equal in the HF limit)**
  - (Observable) two-neutron shell gap $\delta_{2n}(N,Z) \equiv \frac{1}{2}\left[E(N+2,Z) - 2E(N,Z) + E(N-2,Z)\right]$
  - (Non-observable) ESPE Fermi gap $\Delta e_{\mathrm{F}}^{\mathrm{cent}}(N,Z) \equiv e_p^{\mathrm{cent}}(N,Z) - e_h^{\mathrm{cent}}(N,Z)$



- **HFB level**: ESPE & 2N gaps similar, the former well captures the latter
- **Correlated calculation:** scale dependence of ESPE gaps is systematically large

Spectroscopic factors

- **Compilation of SF for one-neutron addition/removal on $^{14-24}O$**
  - Limited running with $\lambda \in \{1.88, 2.0, 2.24\}$ fm^{-1} !



HFB

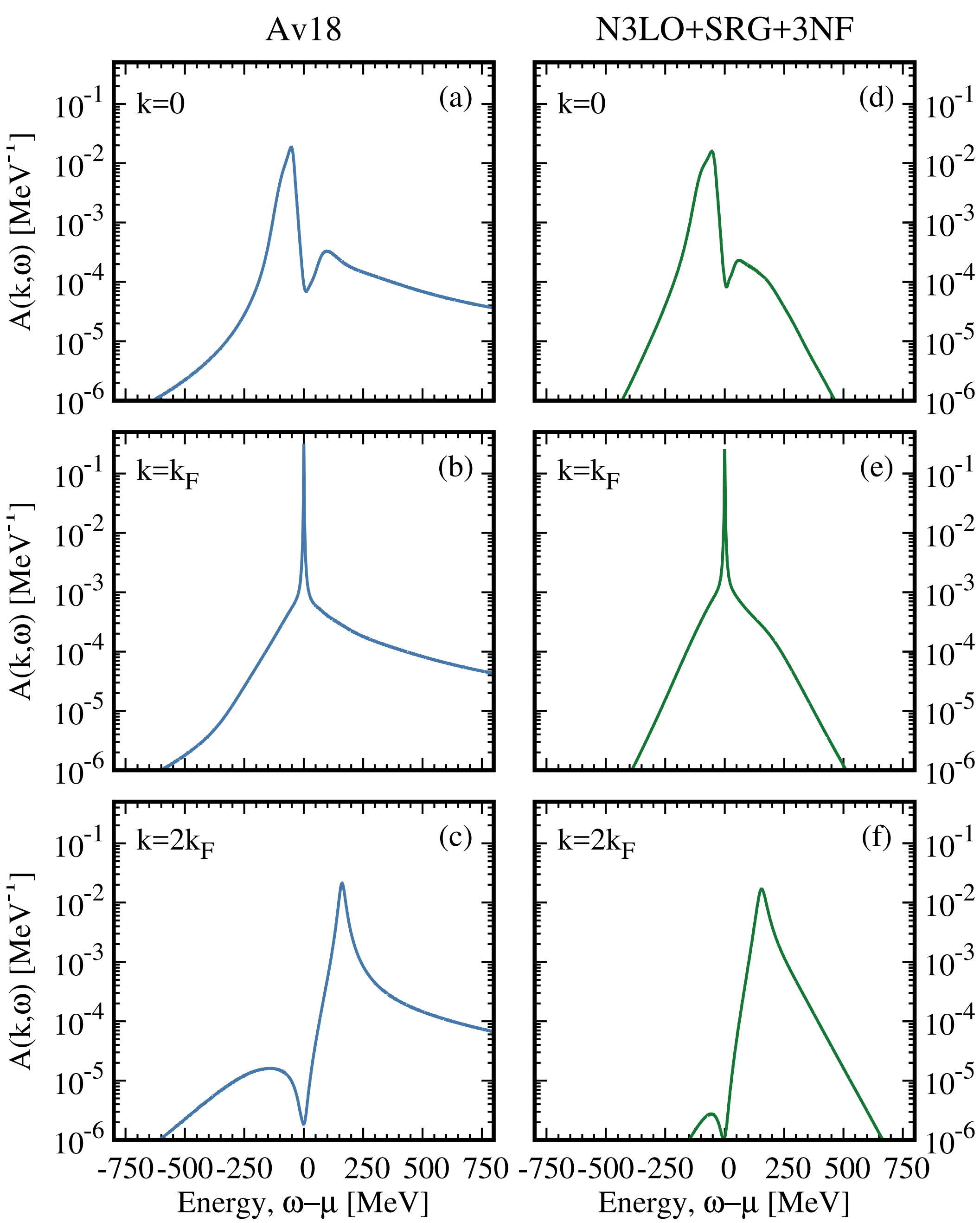
- Independent (quasi-)particle picture
- SFs ~ 1
- Spread is horizontal

Correlated

- Horizontal spread minimised
- Spectroscopic strength now fragmented
- Some scale dependence of SFs appears

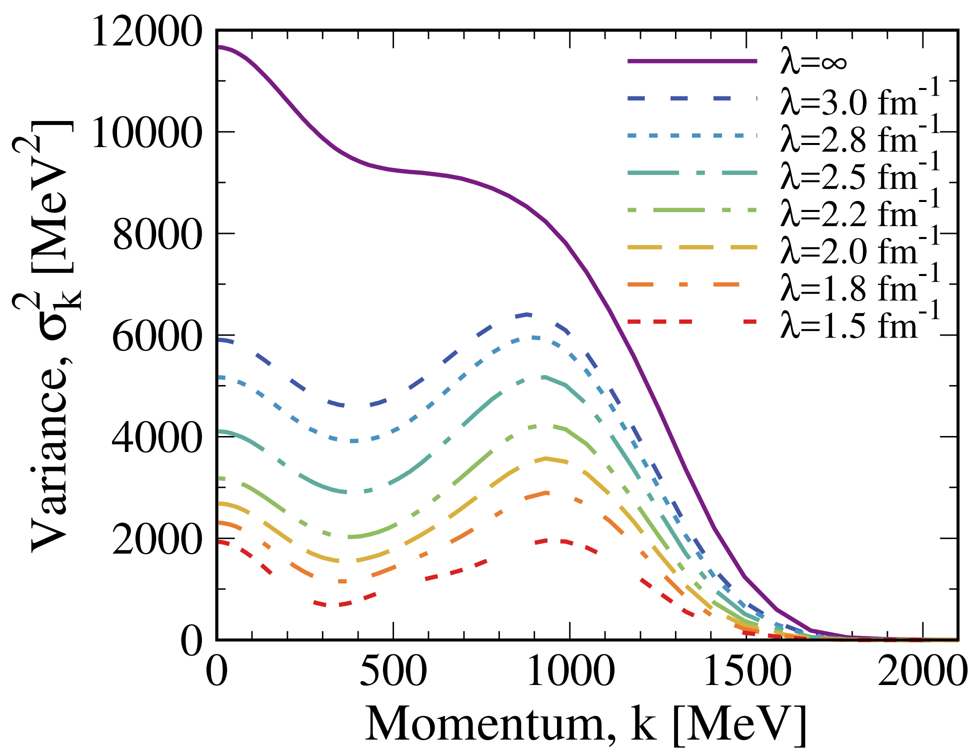
SRG & correlations in infinite matter

- **Larger range of scales can be explored in infinite nuclear matter**
  - Momentum tails in spectral function depend on the interaction
  - Variance depicts amount of correlations



$$\sigma_k^2 = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} [\omega - m_k^{(1)}]^2 \mathcal{A}_k(\omega) = m_k^{(2)} - (m_k^{(1)})^2$$

$$= -\int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \text{Im } \Sigma_k(\omega)$$



[Rios, Carbone, Polls 2017]

Summary

- **Part I**
  - **Correlations are scheme and scale dependent**
  - What balance between different ways of accounting for correlations?
  - Similarity renormalisation group as a knob for (short-range) correlations

- **Part II**
  - **Non-observability of shell structure** formally revisited
  - Ab initio calculations corroborate formal analysis
  - Correlations between observables & shell structure **depend on the resolution scale**
  - Scale/scheme dependence should be explicit & consistent

- **Perspectives**
  - **Quantification of scale dependence** interesting from a pragmatic point of view
  - Focus should be on **consistency** to combine structure & reactions