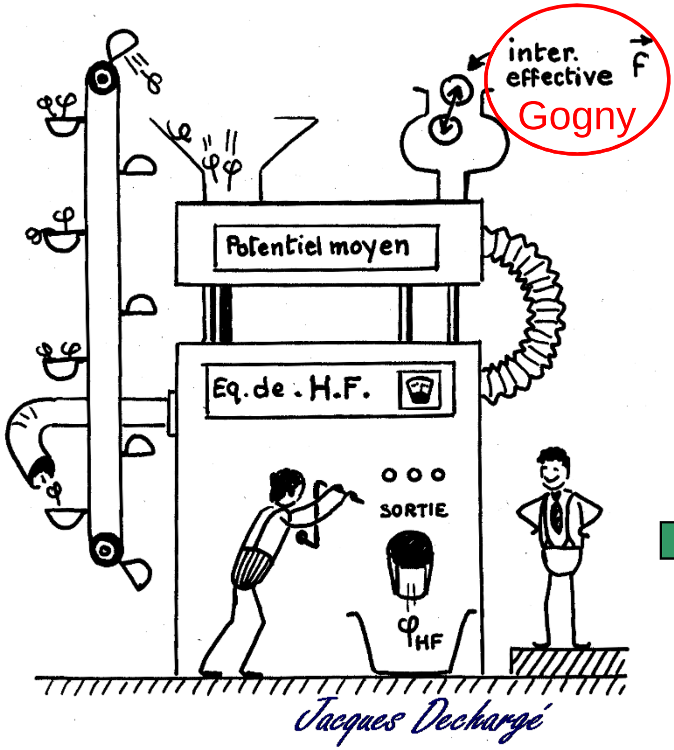
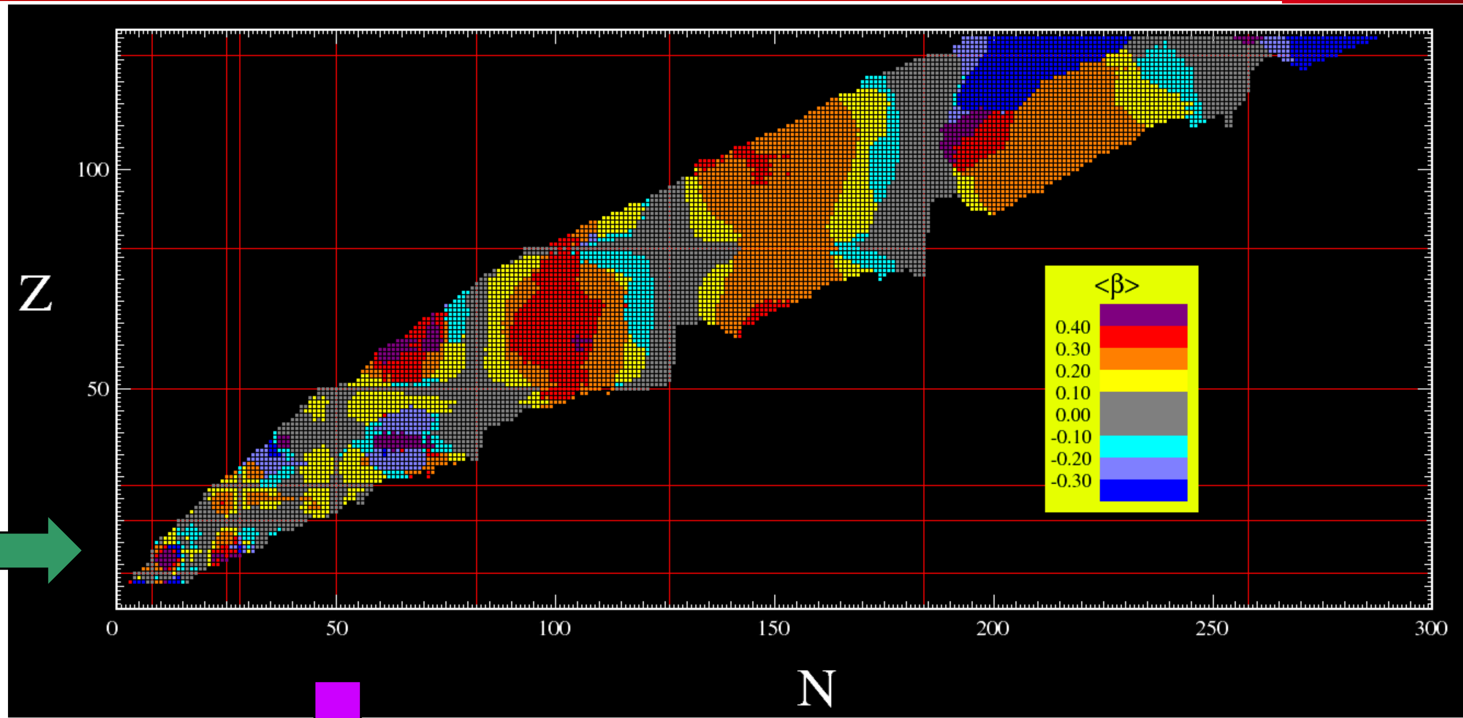
QRPA for low lying excitations

S. Péru (CEA,DAM,DIF)

Reminder





Static mean field (HFB)

for Ground State Properties :

- **Masses**
- **Deformation**
- **(Single particle levels)**

Amedee database :

http://www-phynu.cea.fr/HFB-Gogny_eng.htm

*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

Beyond static mean field approximation (5DCH or QRPA)

for description of Excited State Properties

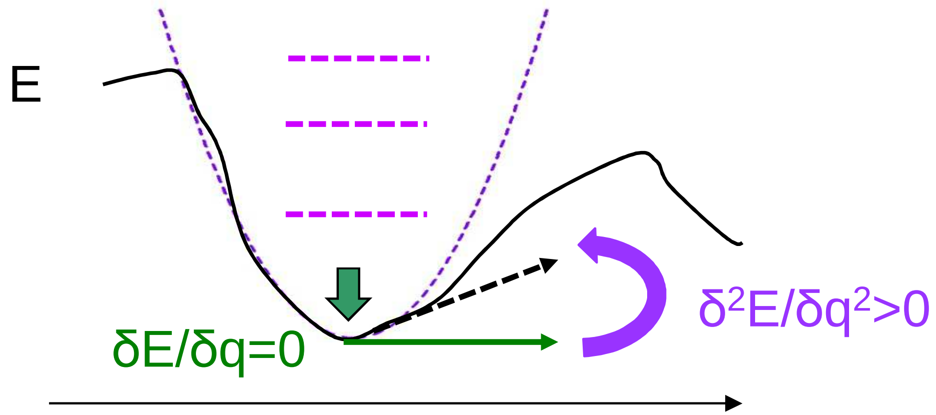
- **Low-energy collective levels**
- **Giant Resonances**

Beyond mean field… with QRPA

RPA approaches describe

all multipolarties and all parities,
collective states and individual ones,
low energy and high energy states
with the same accuracy.

Within the small amplitude approximation, i.e. « harmonic » nuclei



Spherical RPA with Gogny force

J. Dechargé and L.Sips, Nucl. Phys. **A 407**,1 (1983)
J.P. Blaizot, J.F. Berger, J. Dechargé, M. Girod, Nucl. Phys. A 591, 435 (1995)
S. Péru, JF. Berger, PF. Bortignon, Eur. Phys. J. A **26**, 25-32, (2005)

Axially symetric deformed QRPA with Gogny force

S. Péru, H. Goutte, Phys. Rev. C **77**, 044313, (2008)
M. Martini, S. Péru and M. Dupuis, Phys. Rev. C **83**, 034309 (2011)
S. Péru *et al*, Phys. Rev. C **83**, 014314 (2011)
M. Martini et al, PRC 94, 014304 (2016)

RPA approaches are well adapted for describing giant resonances

HFB formalism

$$F(\rho,\kappa) = \sum_{\alpha\beta} t_{\alpha\beta}\rho_{\beta\alpha} + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle\rho_{\gamma\alpha}\rho_{\delta\beta} + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle\kappa^*_{\beta\alpha}\kappa_{\gamma\delta}$$

$$\delta F = \sum_{\alpha\beta}\frac{\partial F}{\partial\rho_{\beta\alpha}}\delta\rho_{\alpha\beta} + \frac{1}{2}\sum_{\alpha\beta}\left(\frac{\partial F}{\partial\kappa_{\beta\alpha}}\delta\kappa_{\alpha\beta} + \frac{\partial F}{\partial\kappa^*_{\beta\alpha}}\delta\kappa^*_{\alpha\beta}\right)$$

$$H_B = \begin{pmatrix} e & \Delta \\ -\Delta^* & -e^* \end{pmatrix} \qquad e_{\alpha\beta} = \frac{\partial F}{\partial\rho_{\beta\alpha}} \qquad \Delta_{\alpha\beta} = \frac{\partial F}{\partial\kappa^*_{\alpha\beta}}$$

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & (1-\rho^*) \end{pmatrix}$$

$$[H_B, \mathcal{R}] = 0$$

(Q)RPA formalism 1/2

$$F(\rho,\kappa) = \sum_{\alpha\beta} t_{\alpha\beta}\rho_{\beta\alpha} + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle\rho_{\gamma\alpha}\rho_{\delta\beta} + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}(\rho)|\widetilde{\gamma\delta}\rangle\kappa^*_{\beta\alpha}\kappa_{\gamma\delta}$$

$$\delta F_2 = \frac{1}{2}\sum_{\alpha\beta}\left[\delta\rho_{\alpha\beta}\sum_{\gamma\delta}\left(V^{CM}_{\beta\alpha,\delta\gamma}\delta\rho_{\gamma\delta} + V^{M}_{\beta\alpha,\delta\gamma}\delta\kappa_{\gamma\delta}\right) + \delta\kappa_{\alpha\beta}\sum_{\gamma\delta}\left(V^{M*}_{\beta\alpha,\delta\gamma}\delta\rho_{\gamma\delta} + V^{P}_{\beta\alpha,\delta\gamma}\delta\kappa_{\gamma\delta}\right)\right]$$

$$V^{CM}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\rho_{\alpha\beta}\partial\rho_{\gamma\delta}}$$

$$V^{M}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\rho_{\alpha\beta}\partial\kappa_{\gamma\delta}}$$

$$V^{M*}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\partial\kappa_{\alpha\beta}\rho_{\gamma\delta}}$$

$$V^{P}_{\beta\alpha,\gamma\delta} = \frac{1+\delta_{\alpha\beta}}{2}\frac{1+\delta_{\gamma\delta}}{2}\frac{\partial^2 F}{\partial\kappa_{\alpha\beta}\partial\kappa_{\gamma\delta}}$$

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + \frac{\partial^2 F}{\partial\rho_{hp}\partial\rho_{p'h'}}$$

$$B_{ph,p'h'} = \frac{\partial^2 F}{\partial\rho_{hp}\partial\rho_{h'p'}}$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \omega_n \begin{pmatrix} X_n \\ -Y_n \end{pmatrix}$$

RPA approaches describe all multipolarties and all parities

A. Corsi et al PLB 743 (2015) 451-455

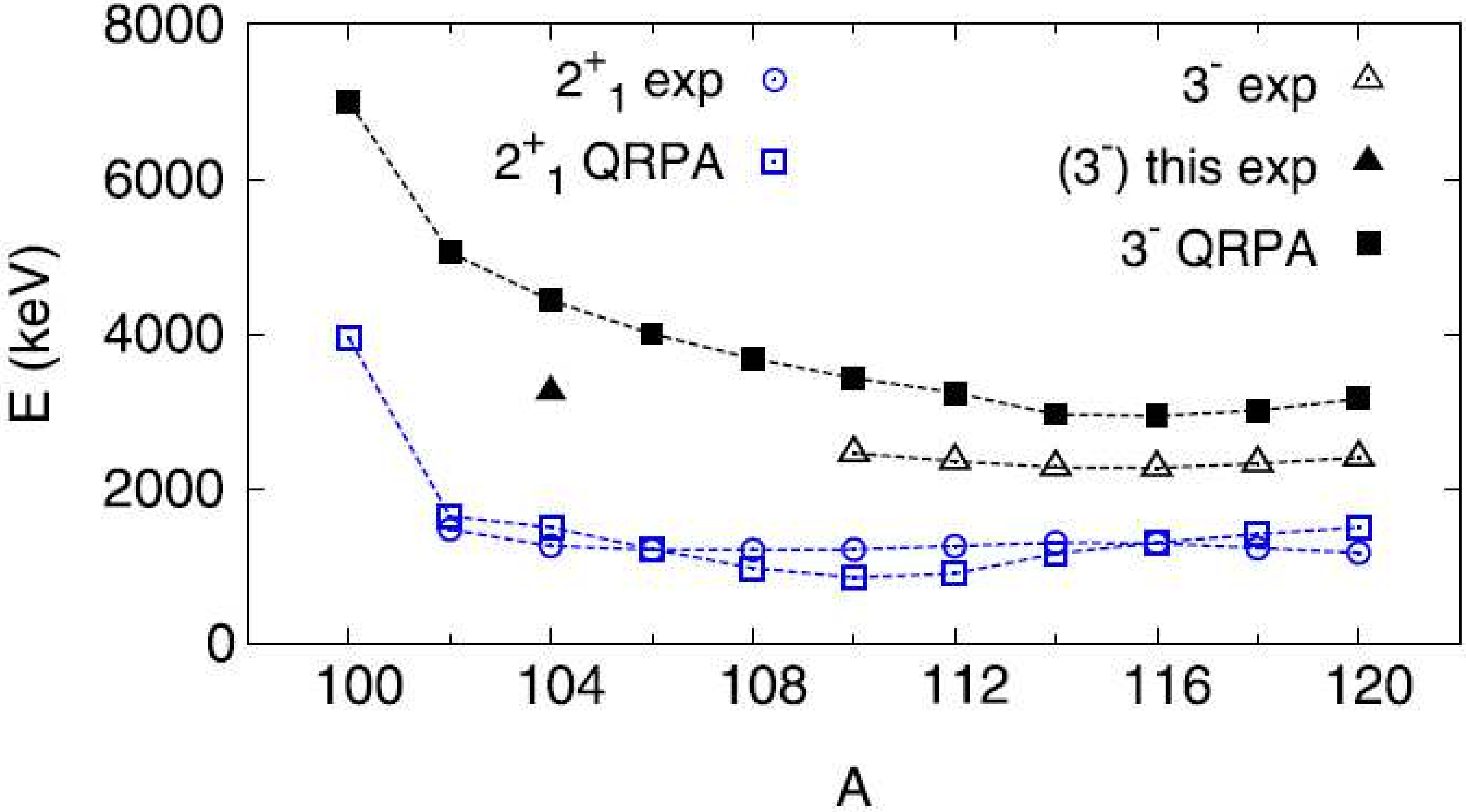


Fig. 3. (Color online.) Systematics of 2^+ and 3^- excitation energies in tin isotopes from experiment and HFB + QRPA calculations using the Gogny D1M interaction.

(Q)RPA Formalism 2/2

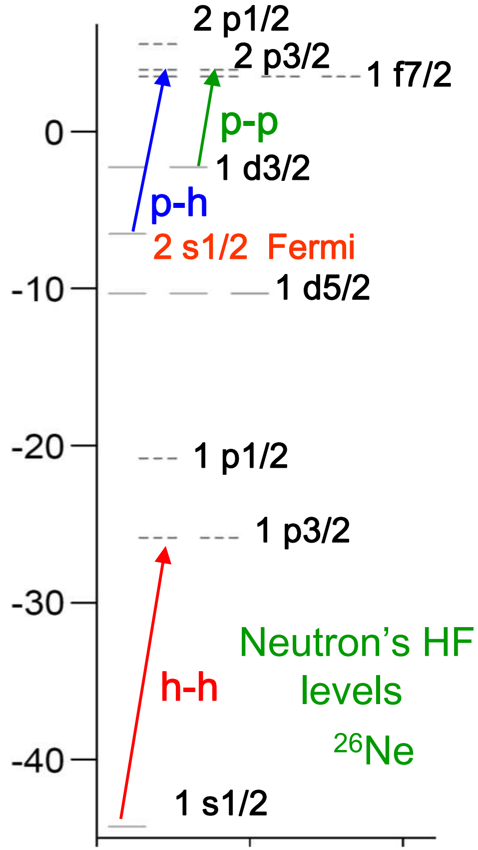
$$H|\nu\rangle = E_\nu|\nu\rangle \qquad Q_\nu^\dagger|0\rangle = |\nu\rangle \qquad Q_\nu|0\rangle = 0$$

Particle-hole excitations: RPA

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu a_p^\dagger a_h - Y_{ph}^\nu a_h^\dagger a_p$$

2 quasi-particles excitations: QRPA

$$Q_\nu^+ = \sum_{ij} X_{ij}^\nu \eta_i^+ \eta_j^+ + Y_{ij}^\nu \eta_j \eta_i \qquad \eta_i^+ = \sum_\alpha u_{i\alpha} a_\alpha^+ - v_{i\alpha} a_\alpha$$



$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ -Y^\nu \end{pmatrix}$$

Hartree-Fock Bogoliubov: ε, u, v ⟶ **Ground state properties**

QRPA: ω, X, Y ⟶ **Excited states properties**

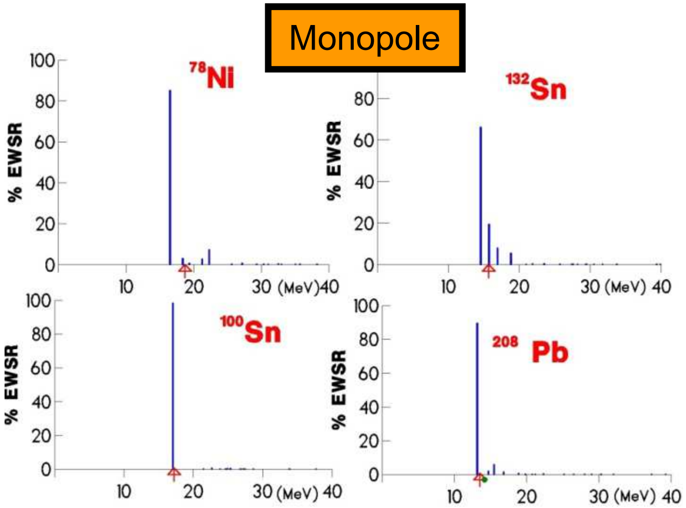
RPA approaches are well adapted for describing giant resonances

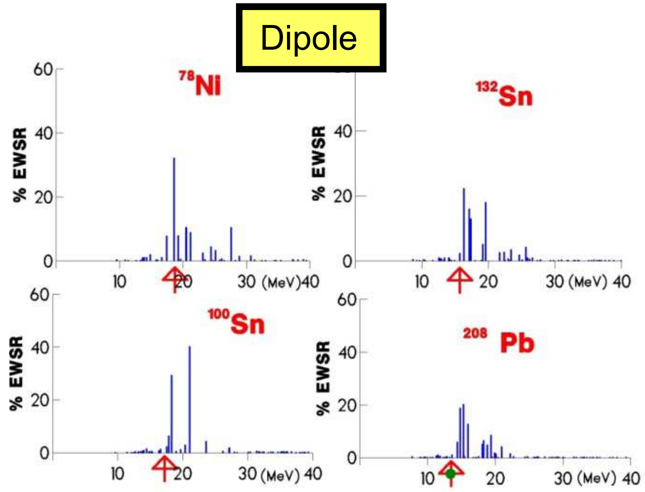
RPA in spherical symmetry

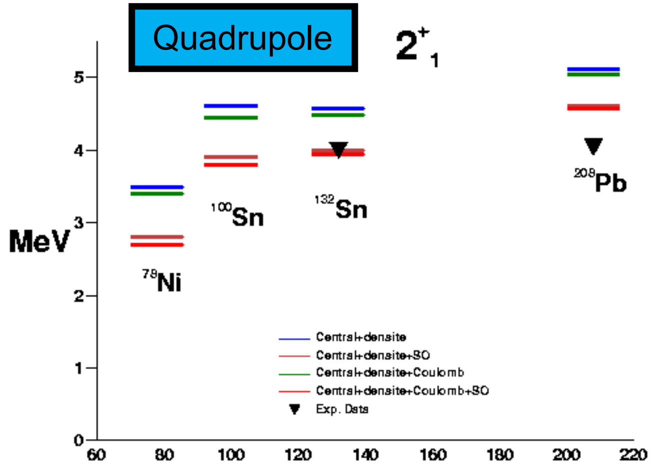
Giant resonances in exotic nuclei:

^{100}Sn, ^{132}Sn, ^{78}Ni; S. Péru, J.F. Berger, and P.F. Bortignon, Eur. Phys. Jour. A 26, 25-32 (2005)

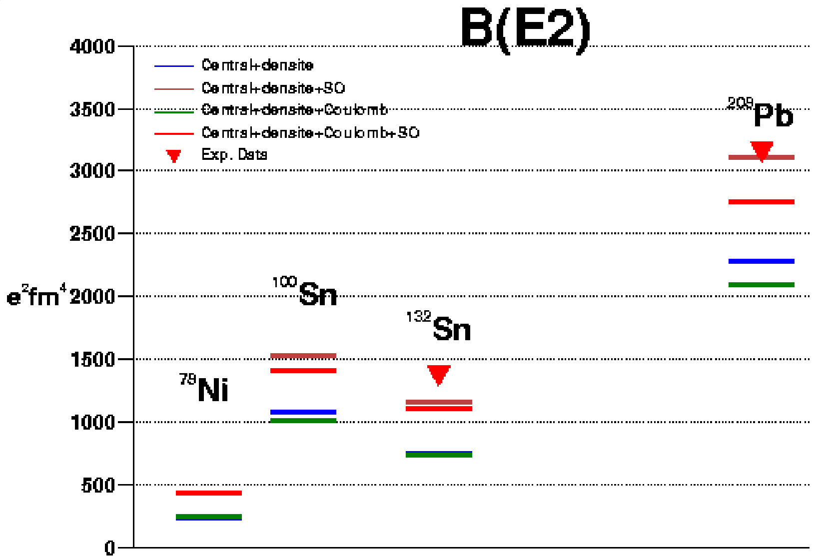
Approach limited to Spherical nuclei with no pairing







→ **Such study have shown the role of the consistence between mean field and RPA matrix.**



$$V(1,2) = \sum_{j=1,2} e^{-\frac{(\vec{r}_1-\vec{r}_2)^2}{\mu_j^2}} (W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau)$$ **central finite range**

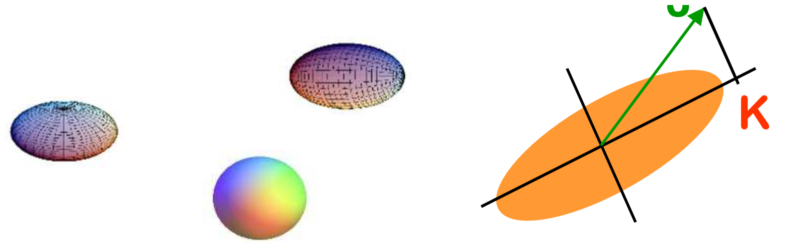
$$+t_0 (1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \left[\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\right]^\alpha$$ **density dependent**

$$+iW_{ls} \overleftarrow{\nabla}_{12} \delta(\vec{r}_1 - \vec{r}_2) \times \overrightarrow{\nabla}_{12} . (\vec{\sigma}_1 + \vec{\sigma}_2)$$ **spin-orbit**

Axially-symmetric deformed QRPA

$$|\alpha, K\rangle = \theta^{+}_{\alpha,K}|0\rangle \qquad \theta^{+}_{n,K} = \sum_{i<j} X^{ij}_{n,K}\eta^{+}_{i,k_i}\eta^{+}_{j,k_j} - (-)^{K}\, Y^{ij}_{n,K}\eta_{j,-k_j}\eta_{i,-k_i}$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K}\begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$



- **Possibility to treat axially-symmetric deformed nuclei**

Restoration of rotational symmetry for deformed states

$$|JM(K)\rangle = \frac{\sqrt{2J+1}}{4\pi}\int d\Omega D^{J}_{MK}(\Omega)\mathrm{R}(\Omega)|\theta_K\rangle + (-)^{J-K} D^{J}_{M-K}(\Omega)\mathrm{R}(\Omega)|\overline{\theta}_K\rangle$$

to calculate: $\langle\tilde{0}|\hat{Q}_{\lambda\mu}|JM(K)\rangle$ **for all QRPA states (K ≤ J)**

$$\hat{Q}_{\lambda\mu} \propto \sum r^{\lambda}(Y_{\lambda\mu}) \qquad r^2 Y_{\lambda\mu} = \sum_{\upsilon} D^{\lambda}_{\mu\upsilon} r^2 Y_{\lambda\upsilon}$$ **In intrinsic frame**

We use rotational approximation and relations for 3j symbols

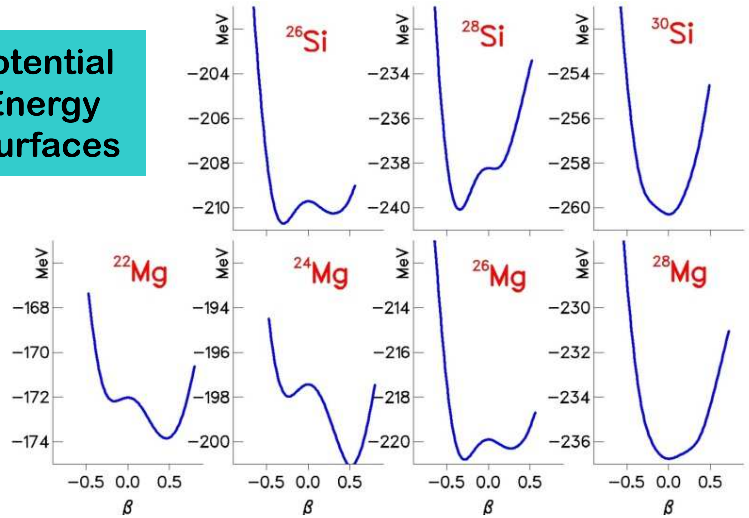
For example: $J^{\pi} = 2^{+}$

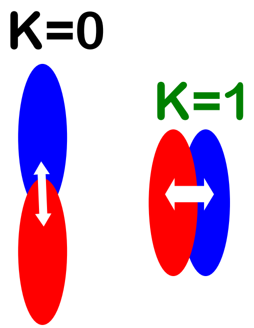
$$\langle\tilde{0}|\hat{Q}_{20}|JM(K)\rangle = \frac{1}{\sqrt{5}}\langle 0|\hat{Q}_{20}|\theta_K\rangle\delta_{K,0} + \frac{\sqrt{3}}{\sqrt{5}}\langle 0|\hat{Q}_{2-1}|\theta_K\rangle\delta_{K,\pm1} + \frac{\sqrt{3}}{\sqrt{5}}\langle 0|\hat{Q}_{22}|\theta_K\rangle\delta_{K,\pm2}$$

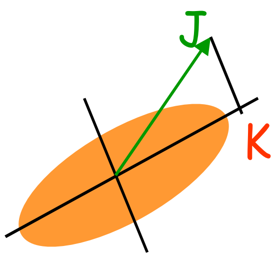
Using time reversal symmetry, three independent calculations **($K^{\pi} = 0^{+}, 1^{+}, 2^{+}$) are needed.**

First study with QRPA in axial symmetry

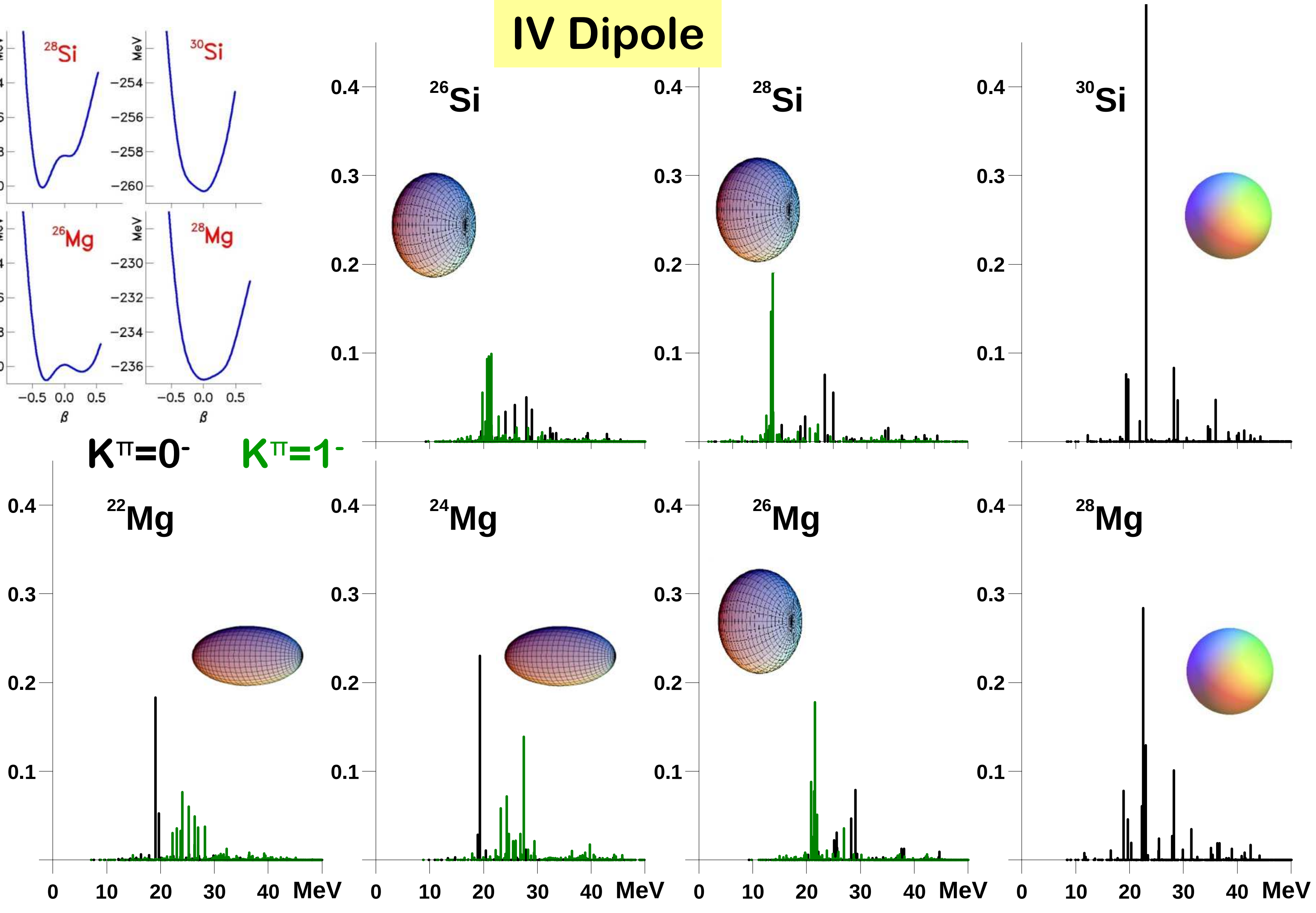
Potential Energy Surfaces





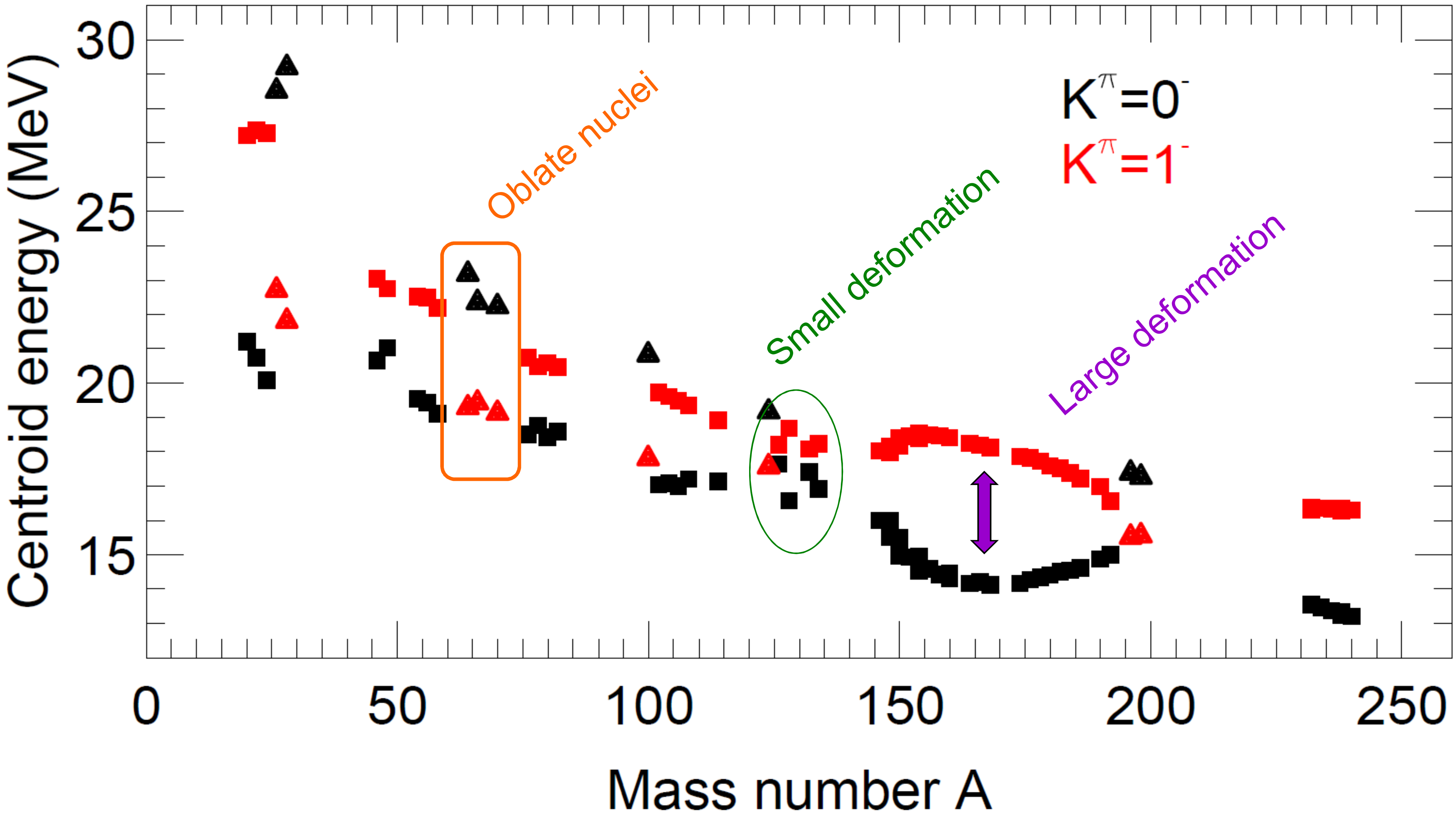


IV Dipole



S. Péru and H. Goutte, Phys. Rev. C 77, 044313 (2008).

Impact of the deformation



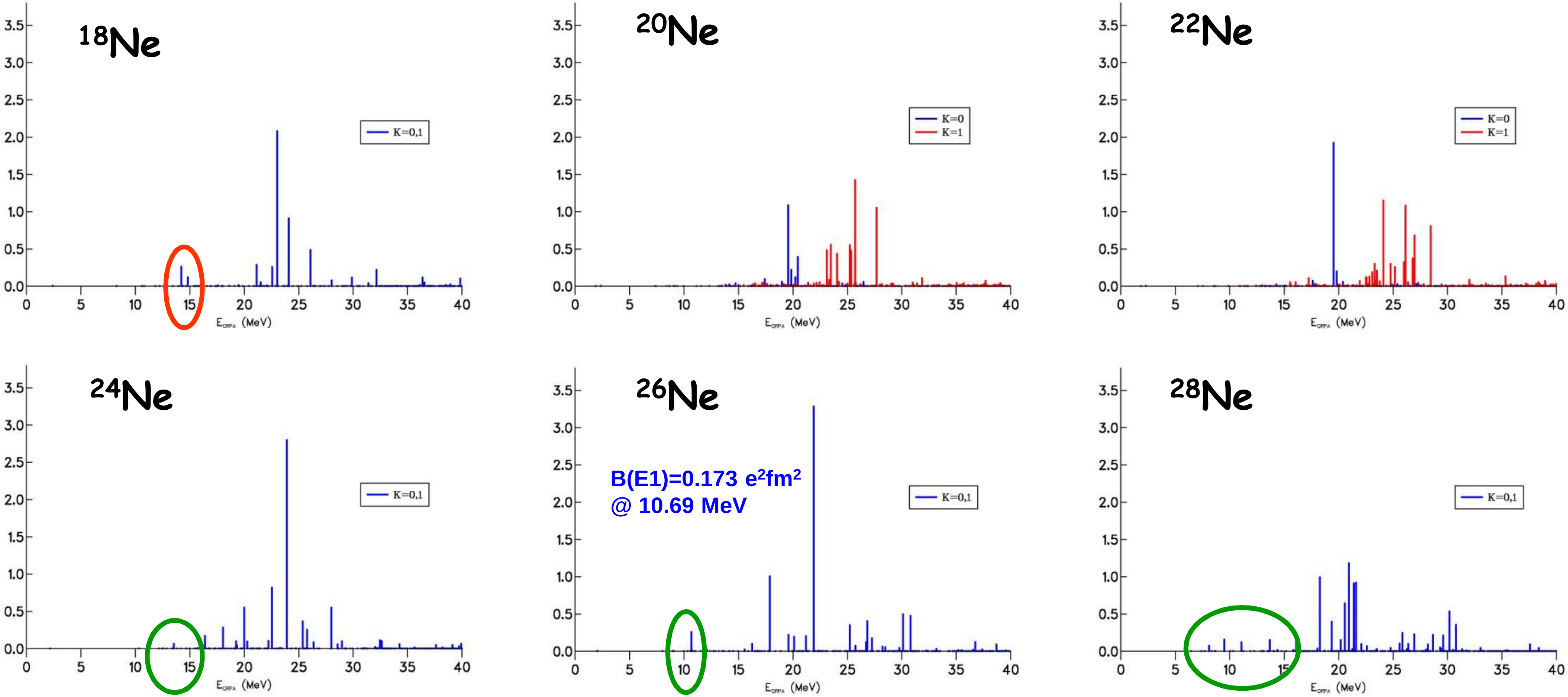
M. Martini et al, PRC 94, 014304 (2016)

Dipole response for Neon isotopes

Increasing neutron number

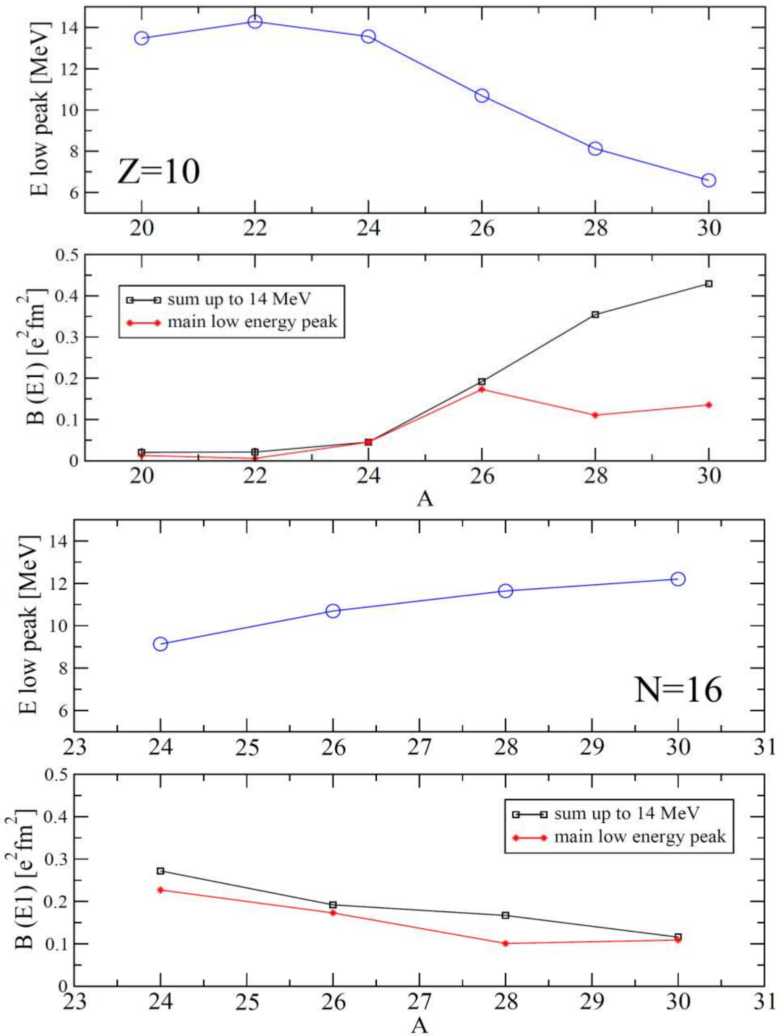
- Low energy dipole resonances and shift to low energies
- Increasing of fragmentation

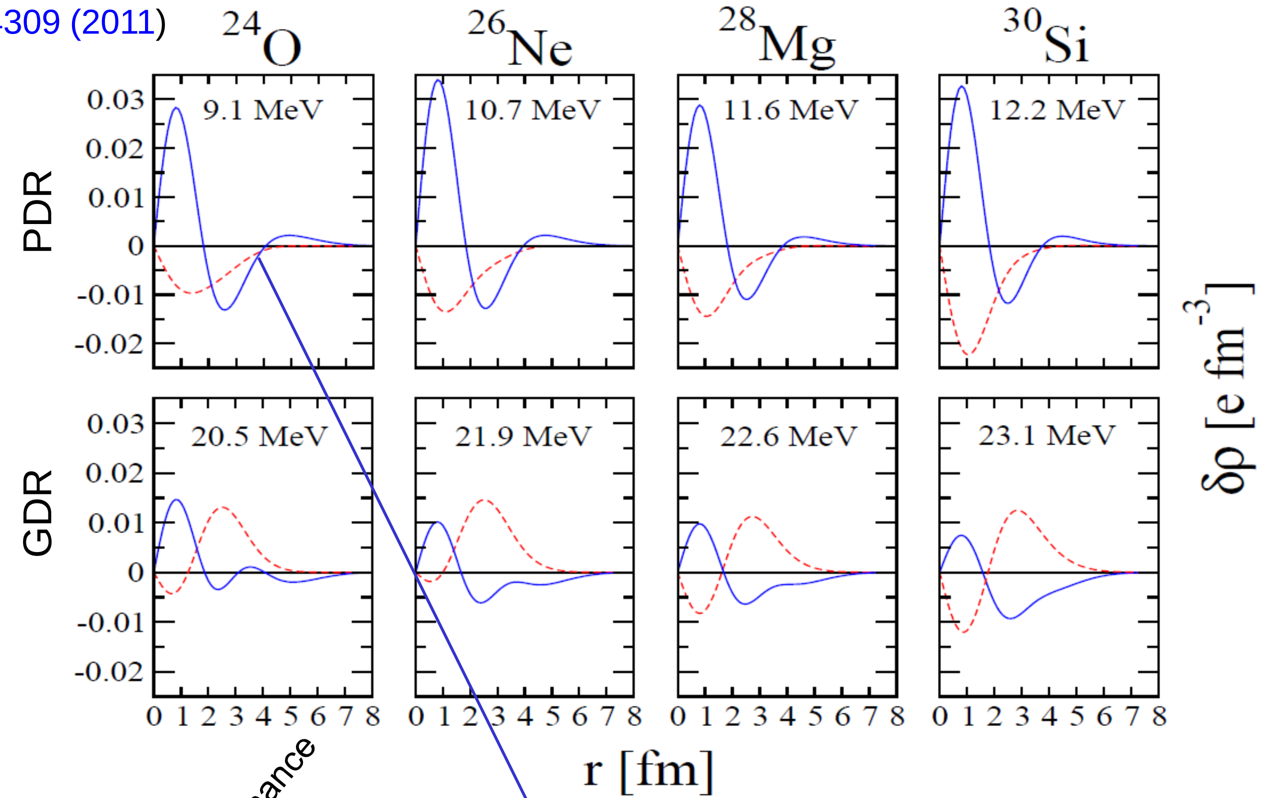
^{26}Ne : B(E1) = 0.49 ± 0.16 e^2 fm^2 %STRK = 4.9 ± 1.6 @ 9 MeV
J. Gibelin et al, PRL 101, 212503 (2008)



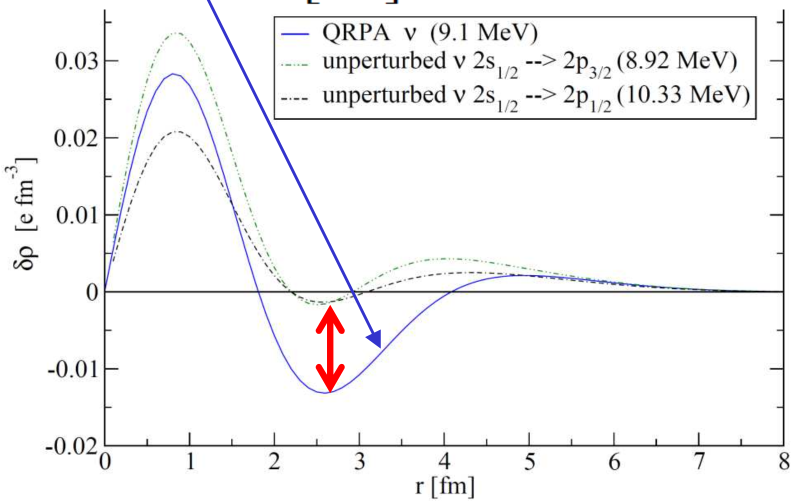
Dipole response for Neon isotopes and N=16 isotones

M. Martini, S. Péru and M. Dupuis, Phys. Rev. C **83**, 034309 (2011)





PDR : piccola dipole resonance

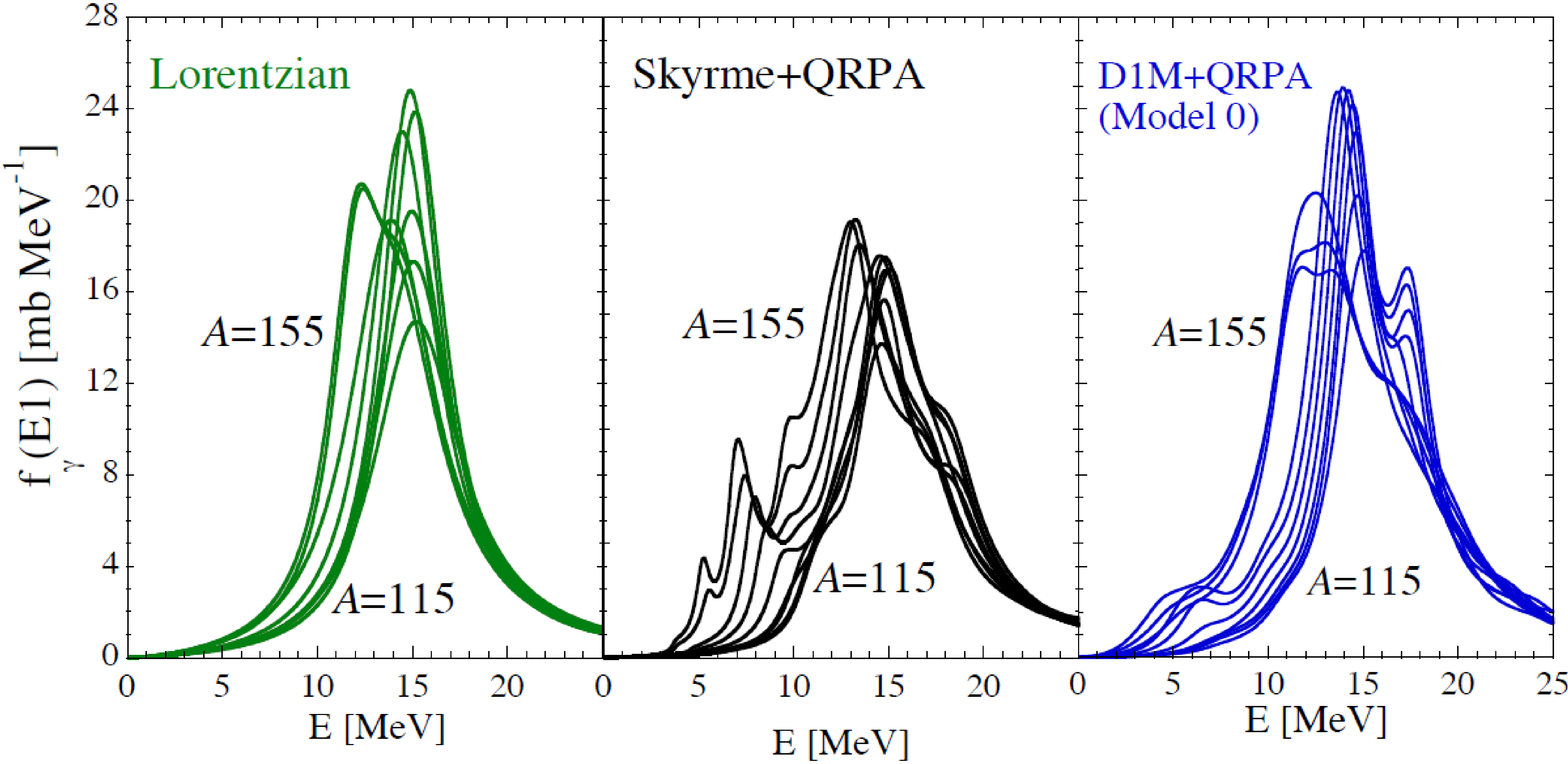


Increasing |N-Z| number :

- Low energy dipole resonances shift to low energies
- Increasing of fragmentation and collectivity

γ- ray strength functions predictions for exotic nuclei

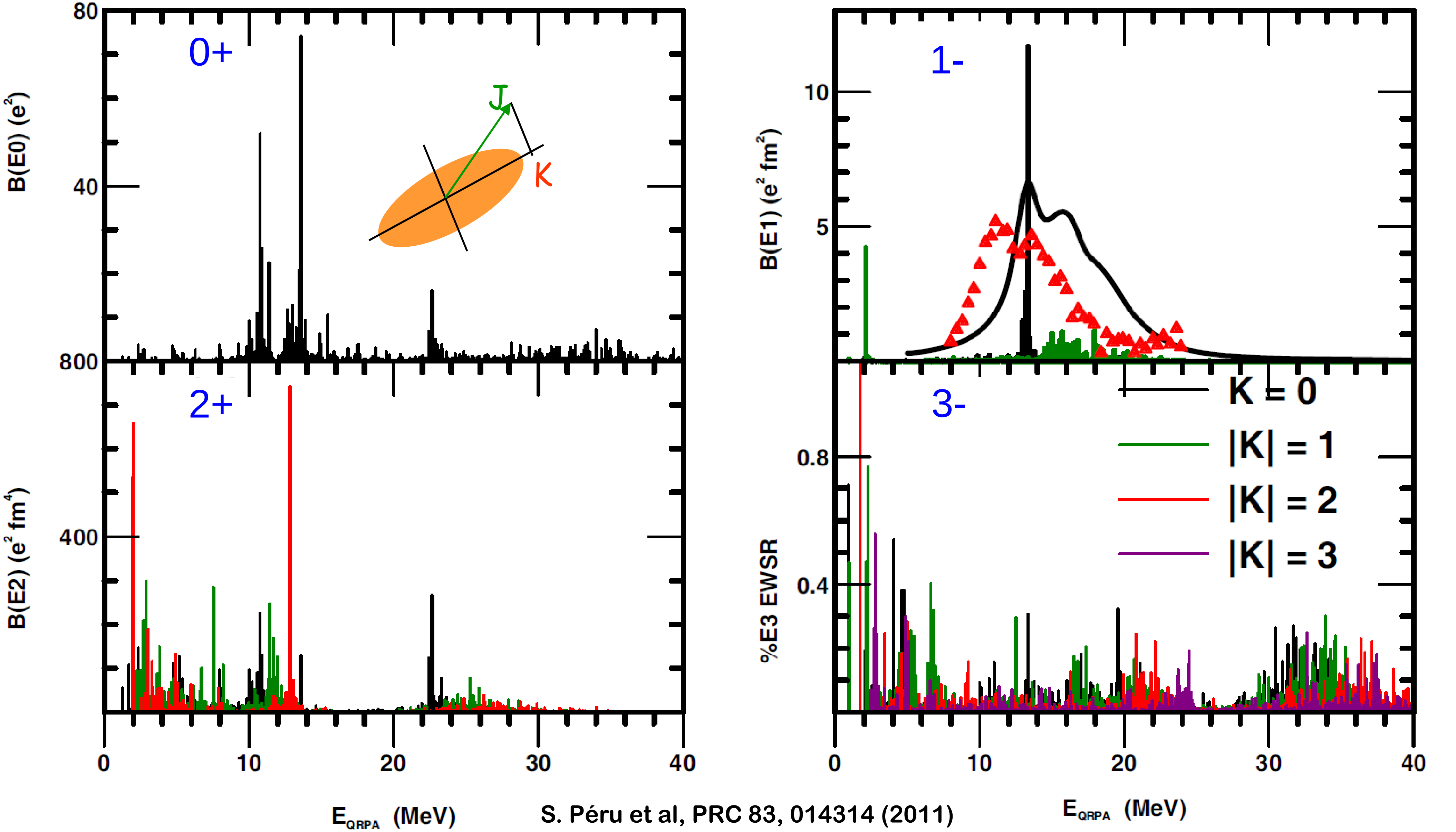
e.g. Sn isotopes



M. Martini et al, PRC 94, 014304 (2016)

Multipolar responses for ^{238}U

Heavy deformed nucleus ⟶ massively parallel computation

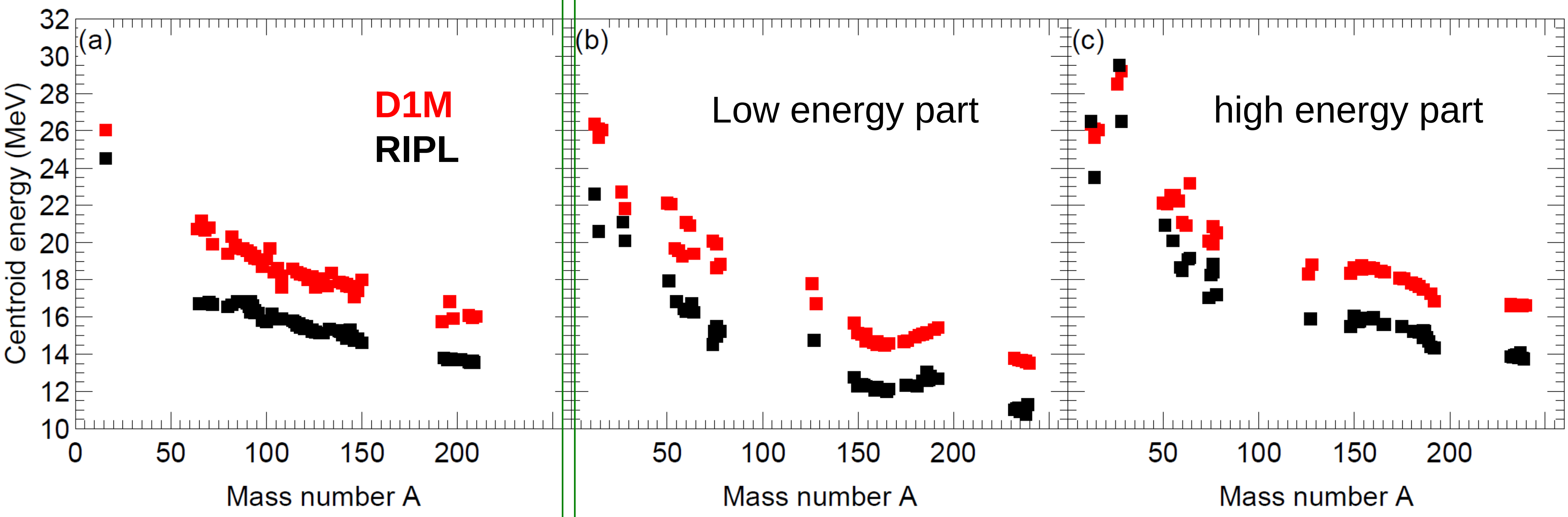


S. Péru et al, PRC 83, 014314 (2011)

Comparison with experimental data

One Lorentzian in RIPL

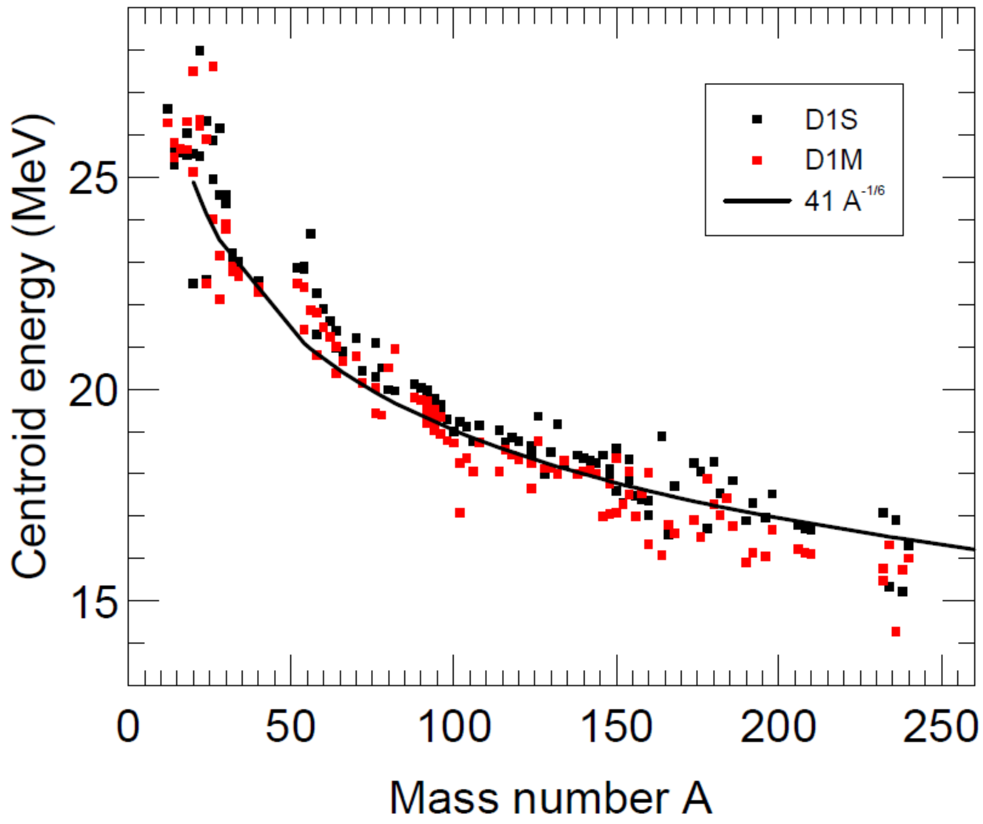
Two Lorentzians in RIPL

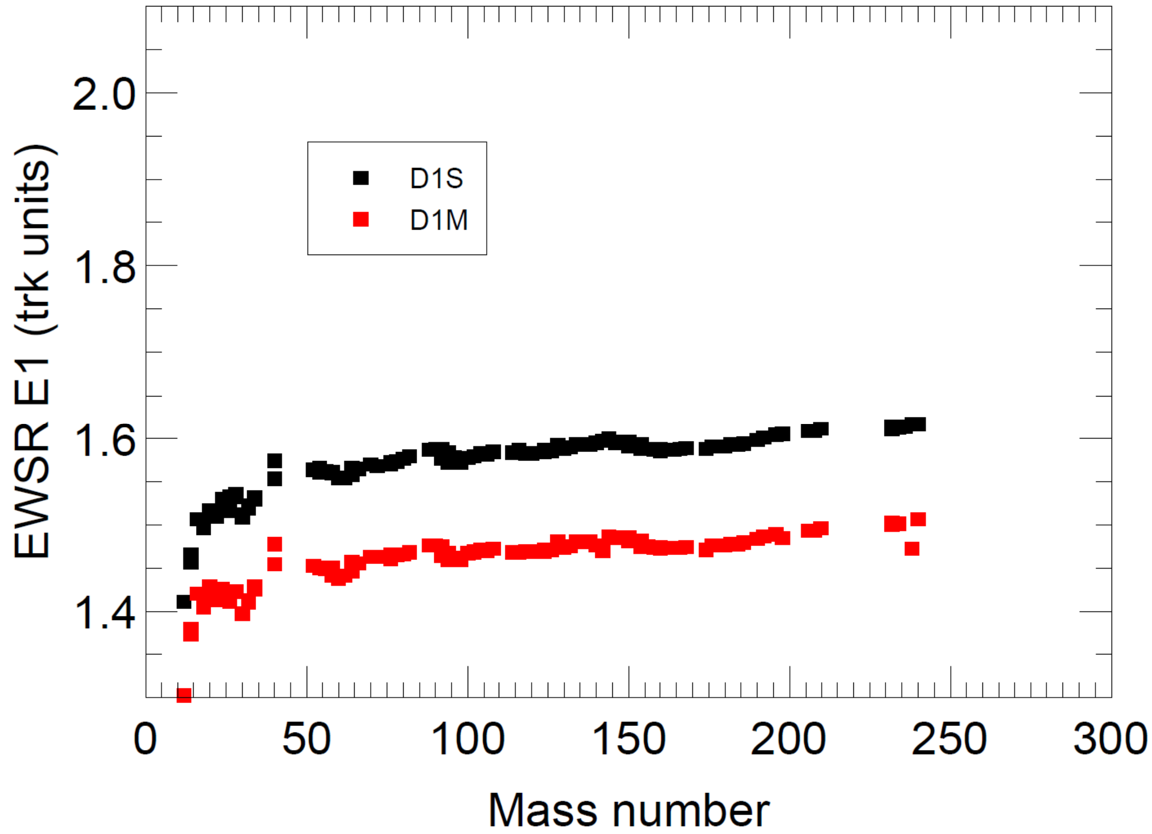


Systematic overestimation of the centroid energies :~ 2MeV

M. Martini et al, PRC 94, 014304 (2016)

Global trend : D1S versus D1M





A few 100 keV overestimation of the D1S centroid energies with respect to D1M ones leads to a 0,2 shift of the EWSR (in TRK units).

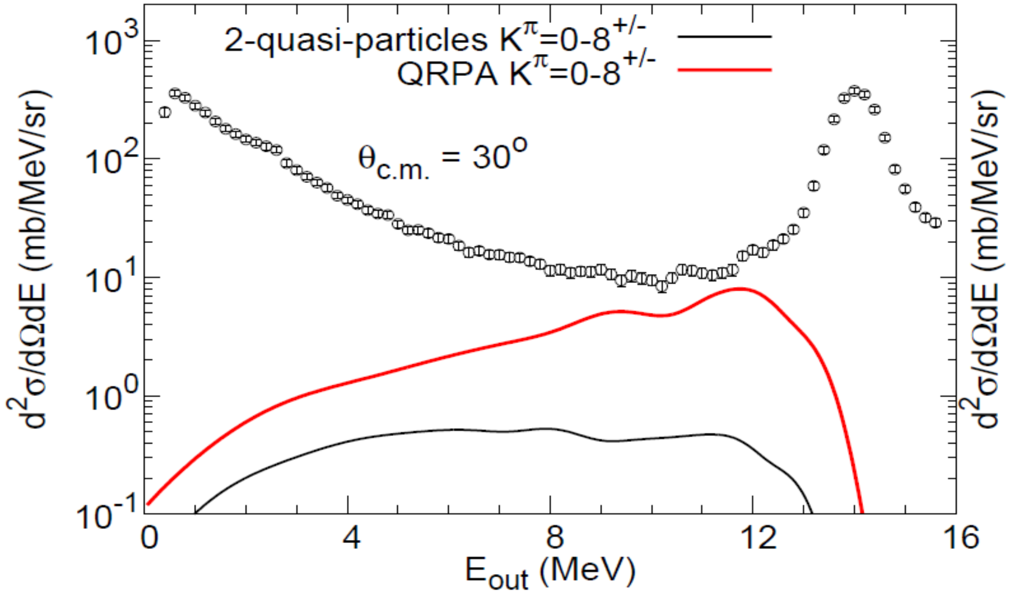
M. Martini et al, PRC 94, 014304 (2016)

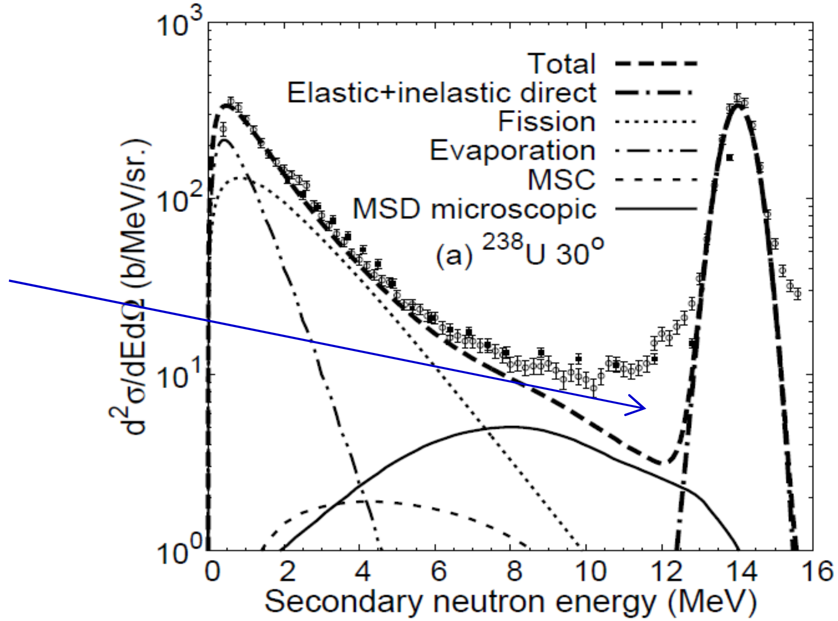
Beyond the nuclear structure

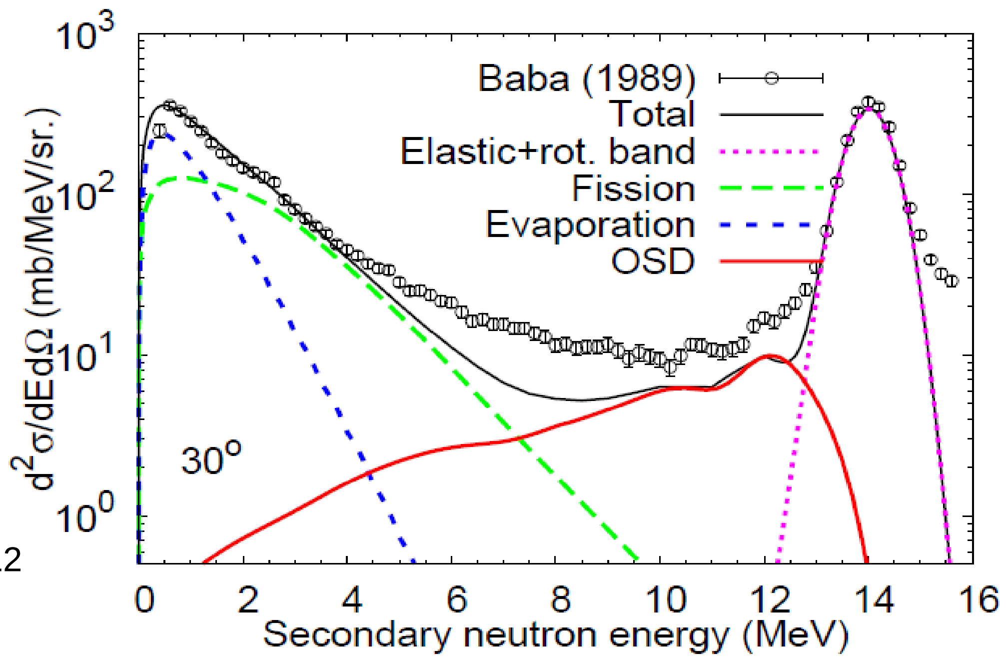
(n, *x* n) cross section on ^{238}U

Problem of underestimation of n emission cross section at high energy

QRPA provides enough collective contribution







M. Dupuis, S.Péru, E. Bauge and T. Kawano,
13th International Conference on Nuclear Reaction Mechanisms, Varenna 2012
CERN-Proceedings-2012-002, p 95

Photoneutron cross sections for Mo isotopes

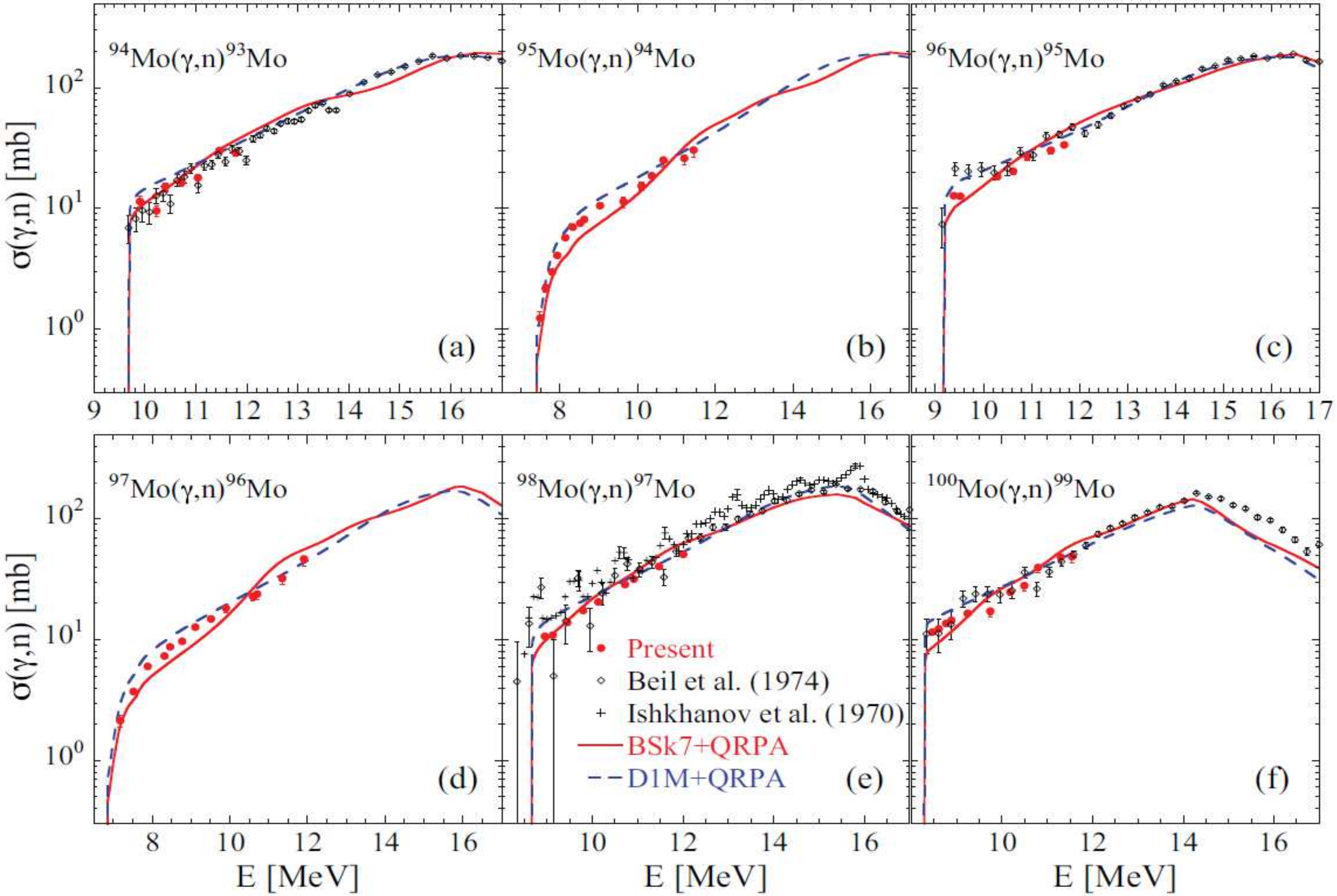
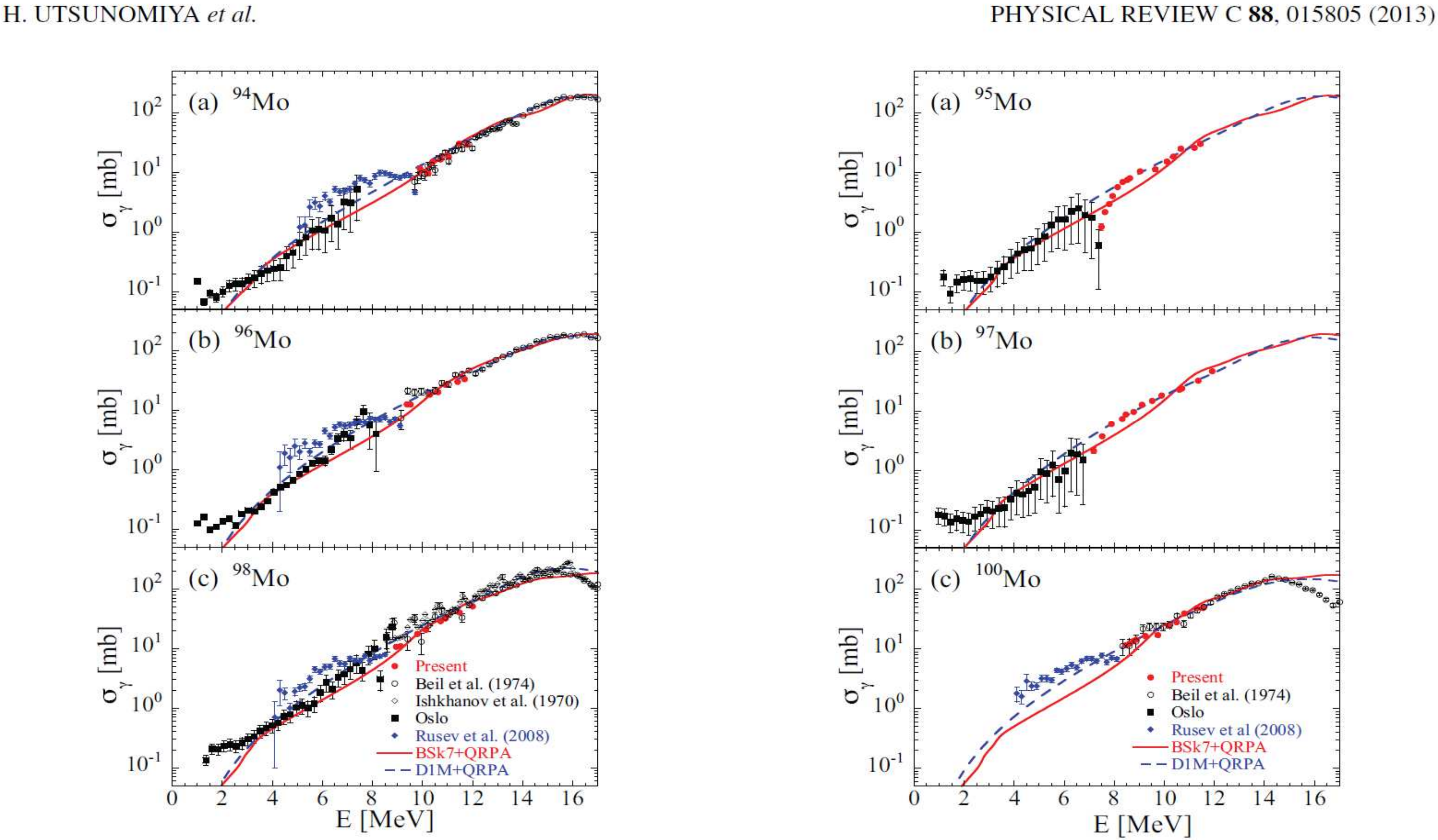
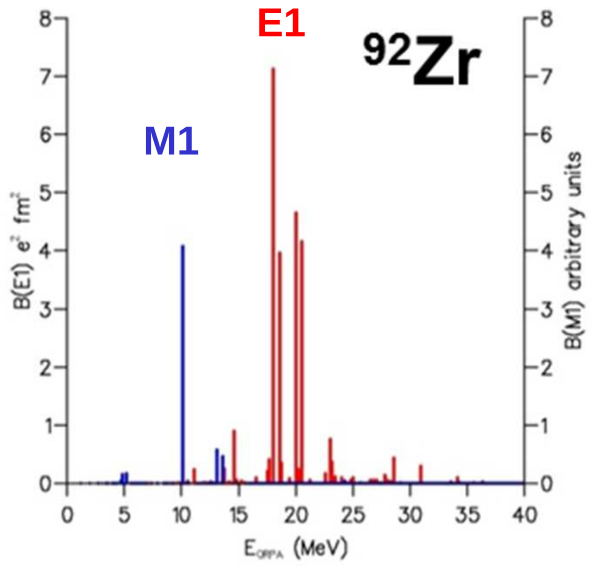


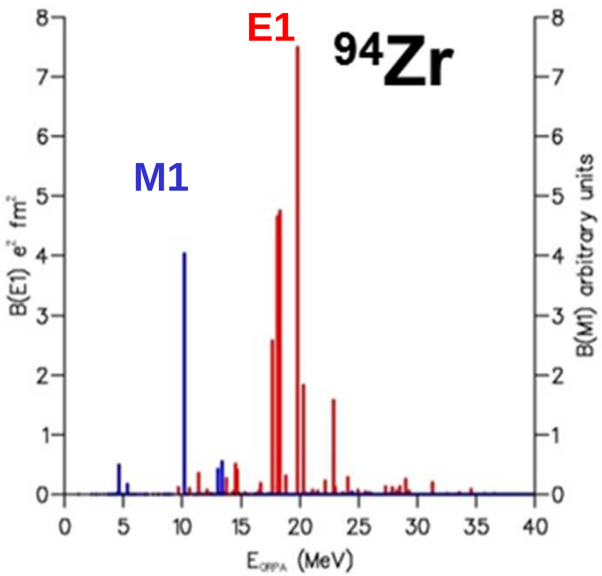
FIG. 3. (Color online) Comparison between the present photoneutron emission cross sections and previously measured ones [17,18] for six Mo isotopes, ^{94}Mo (a), ^{95}Mo (b), ^{96}Mo (c), ^{97}Mo (d), ^{98}Mo (e), and ^{100}Mo (f). Also included are the predictions from Skyrme HFB + QRPA (based on the BSk7 interaction) [20] and axially deformed Gogny HFB + QRPA models (based on the D1M interaction) [23].

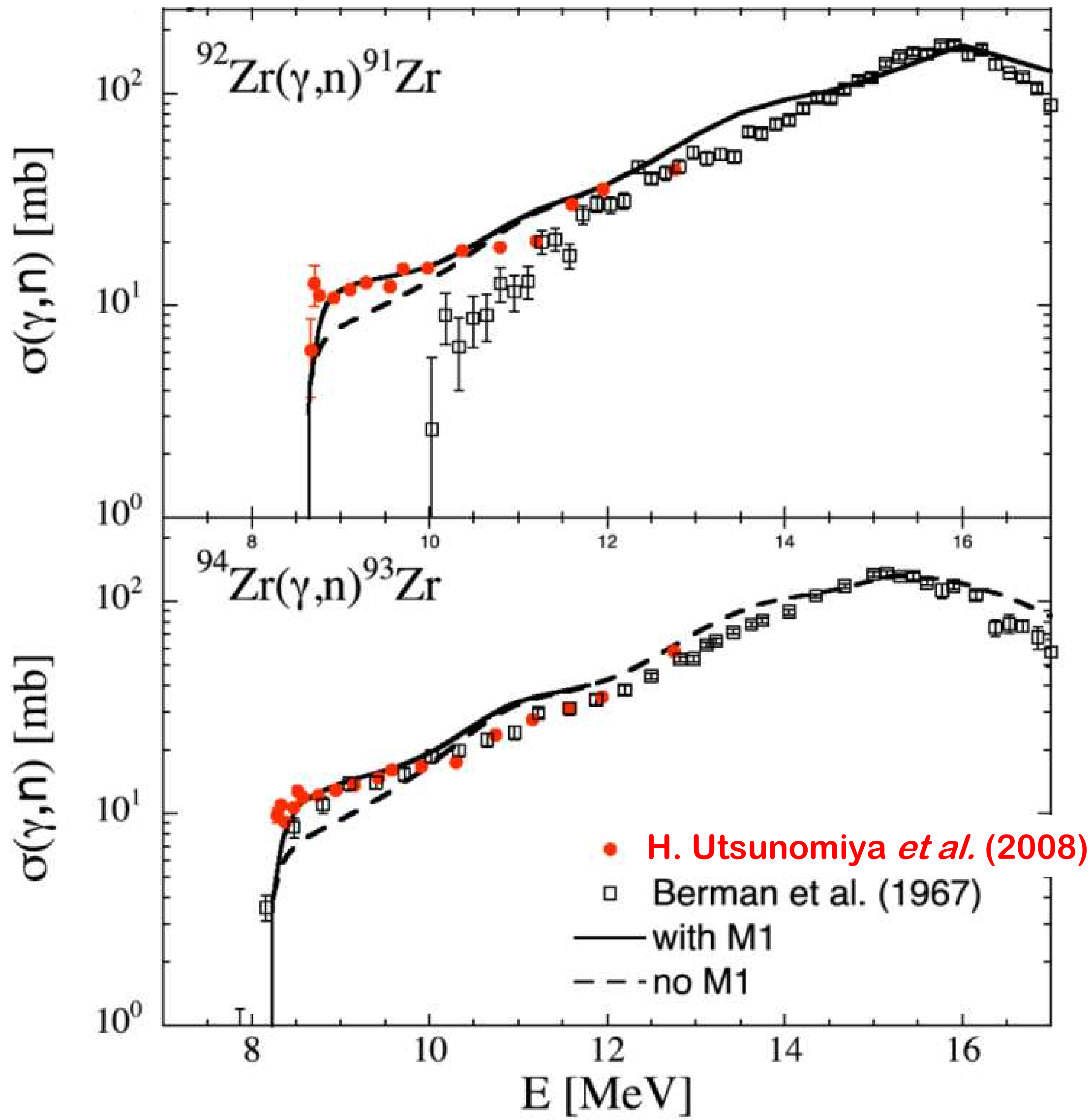
Photo-absorption cross sections for Mo isotopes



Dipole electric and magnetic excitations for Zr isotopes

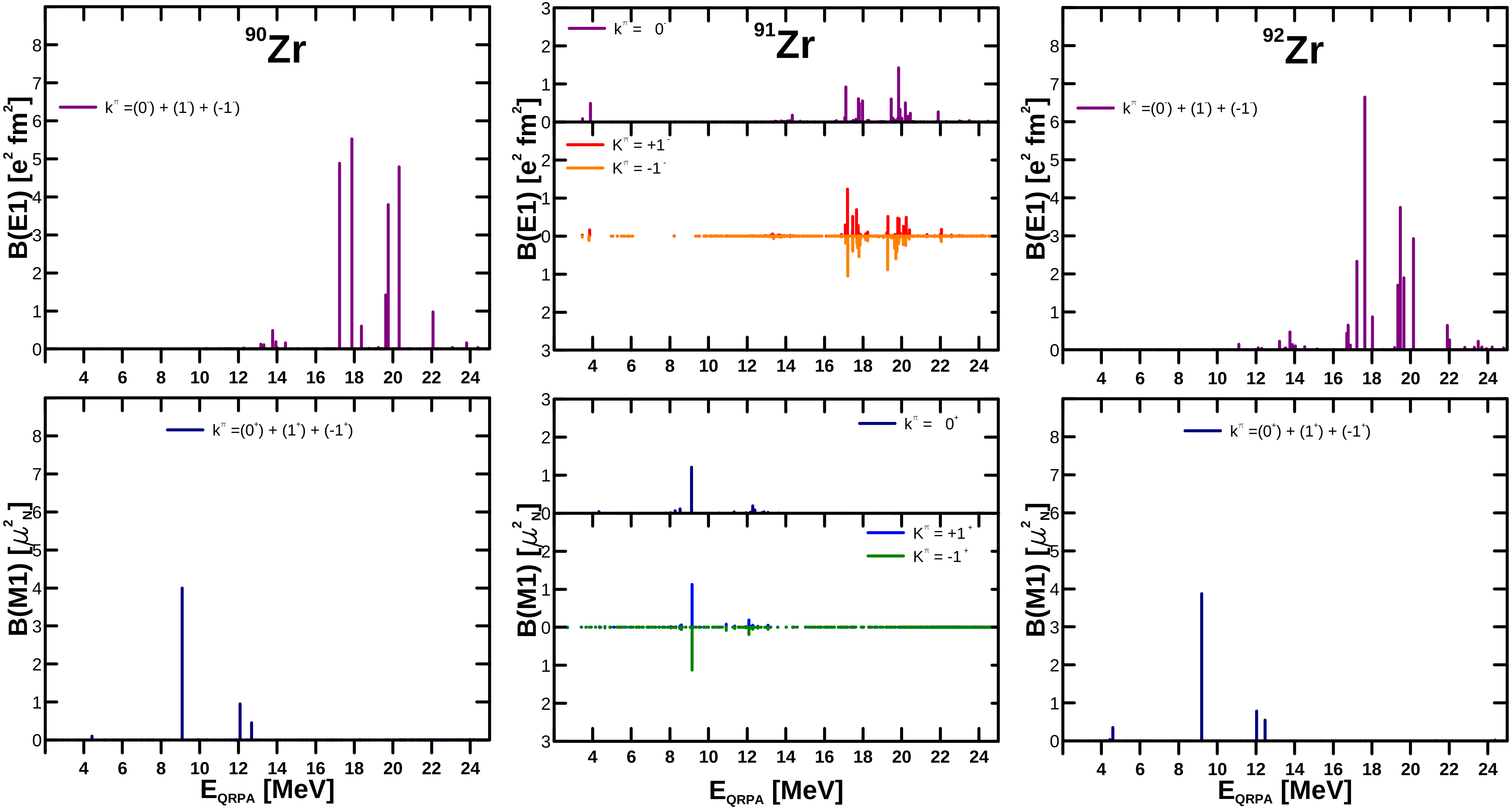






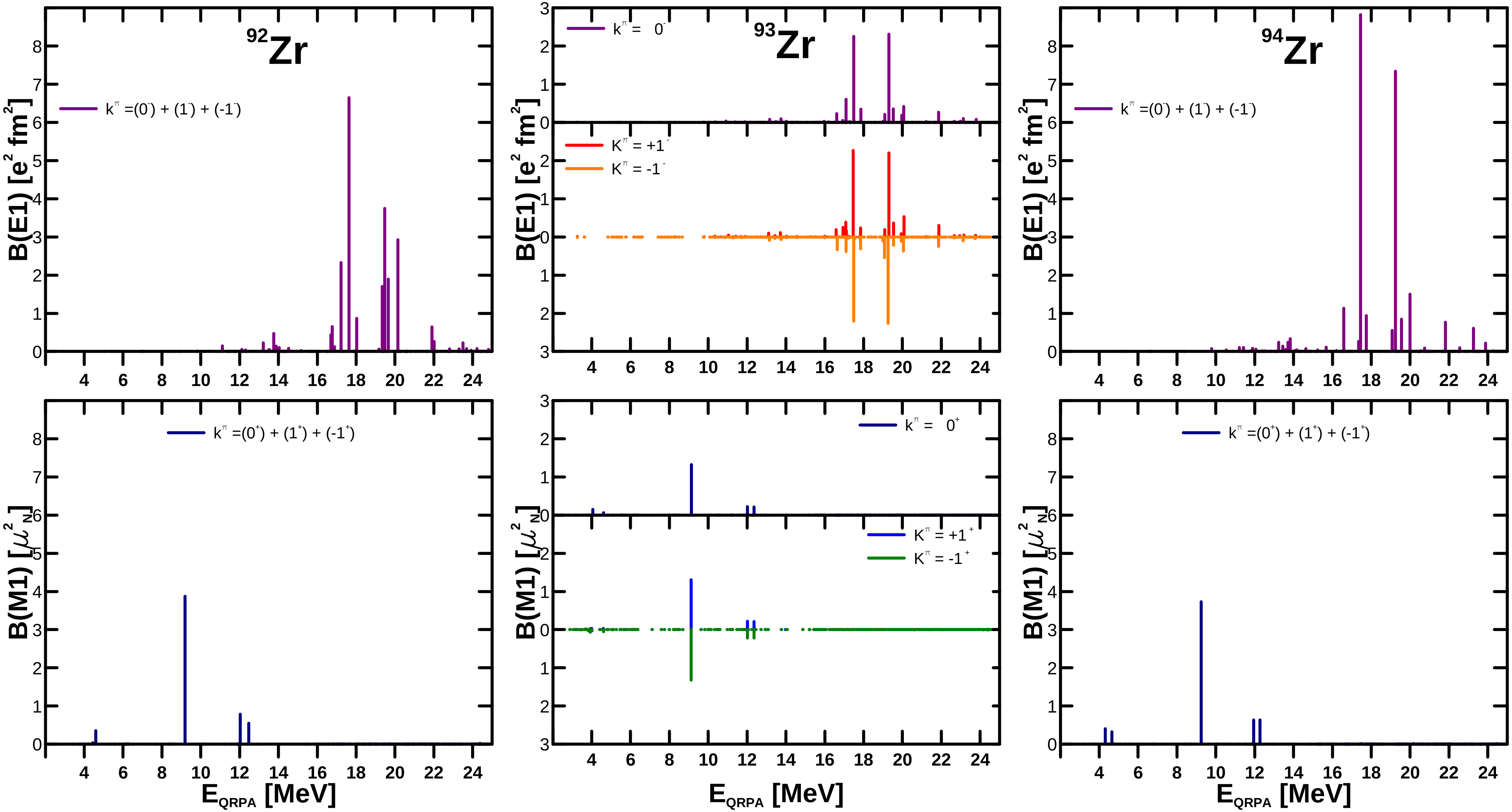
H. Utsunomiya et al, PRL 100, 162502 (2008)

Dipole states in odd and even Zr isotopes



I. Deloncle, S. Péru, M. Martini, EPJA under revision

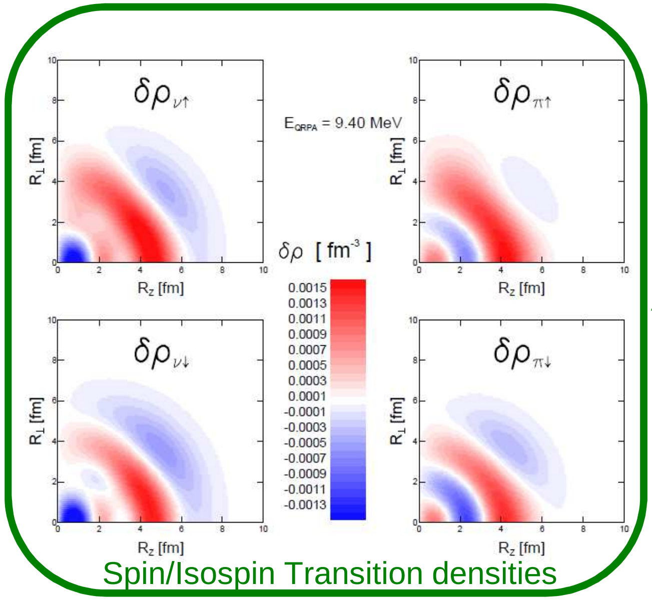
Dipole states in odd and even Zr isotopes

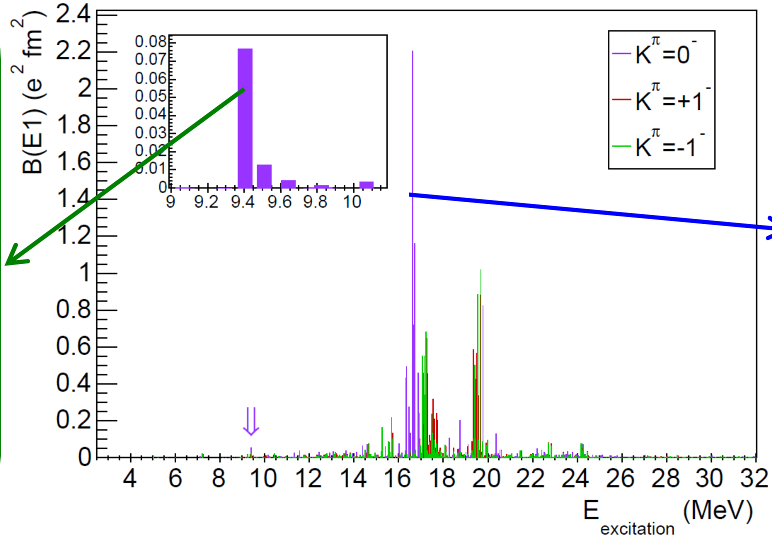


I. Deloncle, S. Péru, M. Martini, EPJA under revision

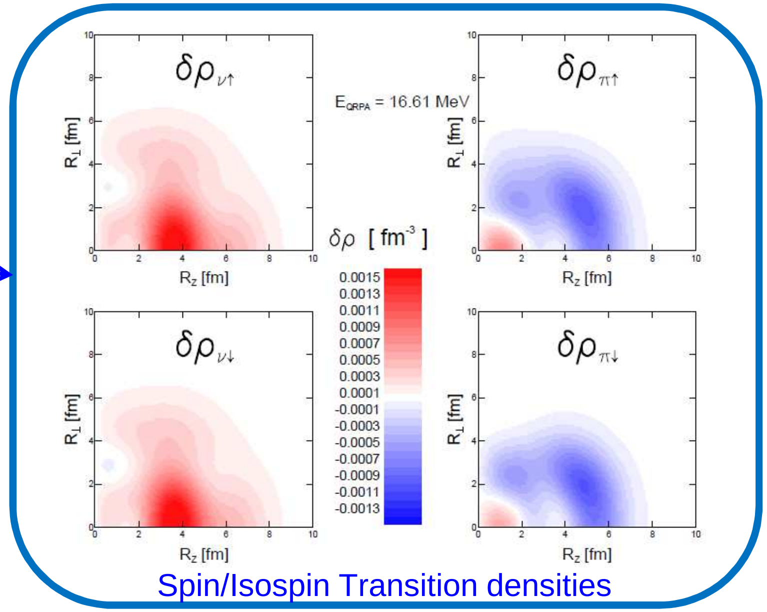
Low Energy Enhancement in the γ Strength of the Odd-Even Nucleus ^{115}In

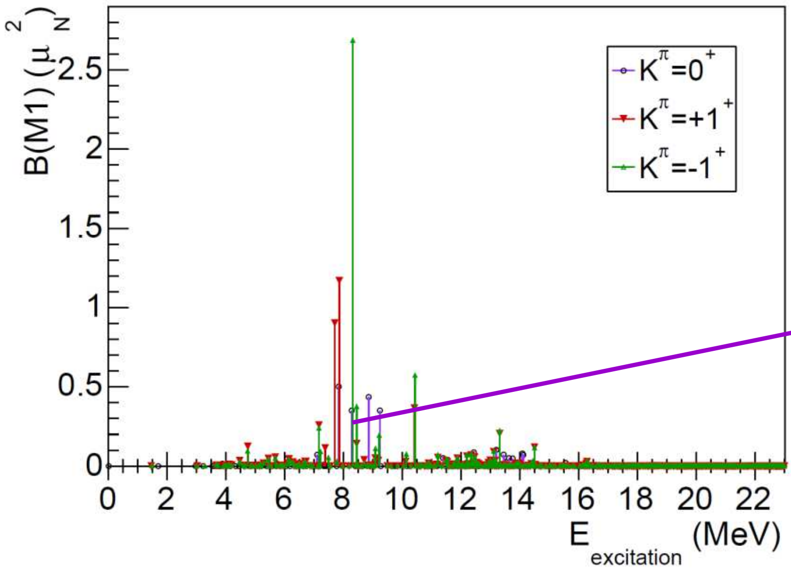
PDR Iso Scalar dipole

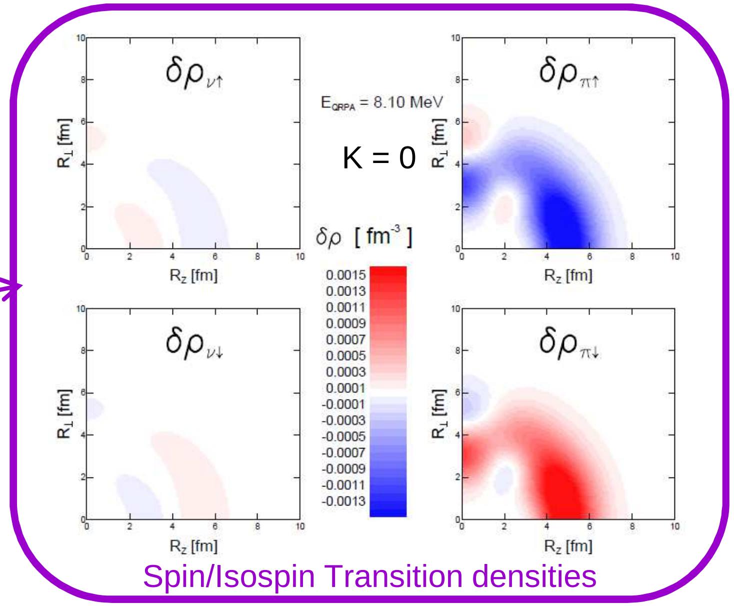




Iso Vector dipole







Spin flip

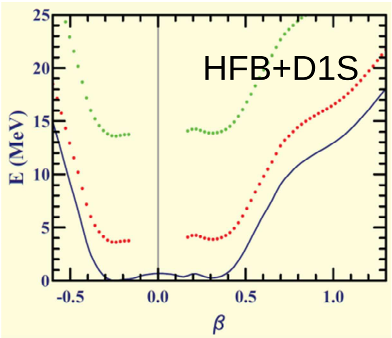
M. Versteegen et al, PRC 94, 044325 (2016)

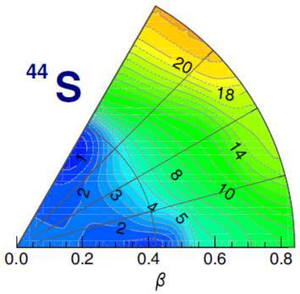
Limits of the QRPA approach
Comparison with 5DCH results

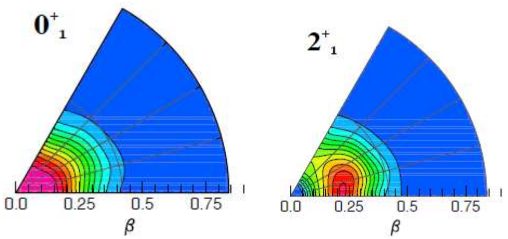
S. Péru and M. Martini, EPJA (2014) 50: 88.

Beyond static mean field … with 5DCH or QRPA

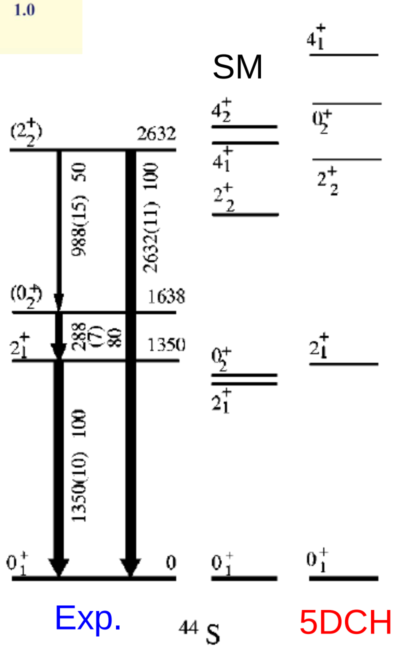
5 Dimension Collective Hamiltonian
describes ground state and excited states within configuration mixing : quadrupole vibration and rotational degrees of freedom.







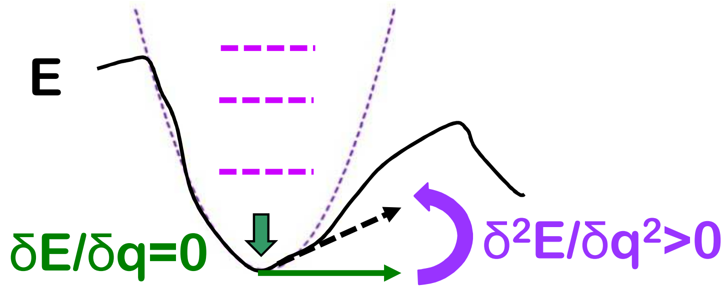
S.Péru and M. Martini, EPJA (2014) 50: 88.

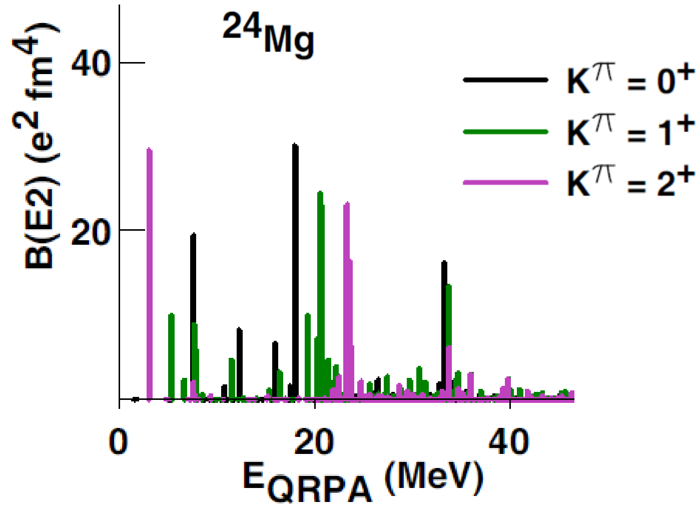


D. Sohler et al, PRC 66, 054302 (2002)

(Q)RPA approaches describe all multipolarties and all parities, collective states and individual ones, low energy and high energy states with the same accuracy.

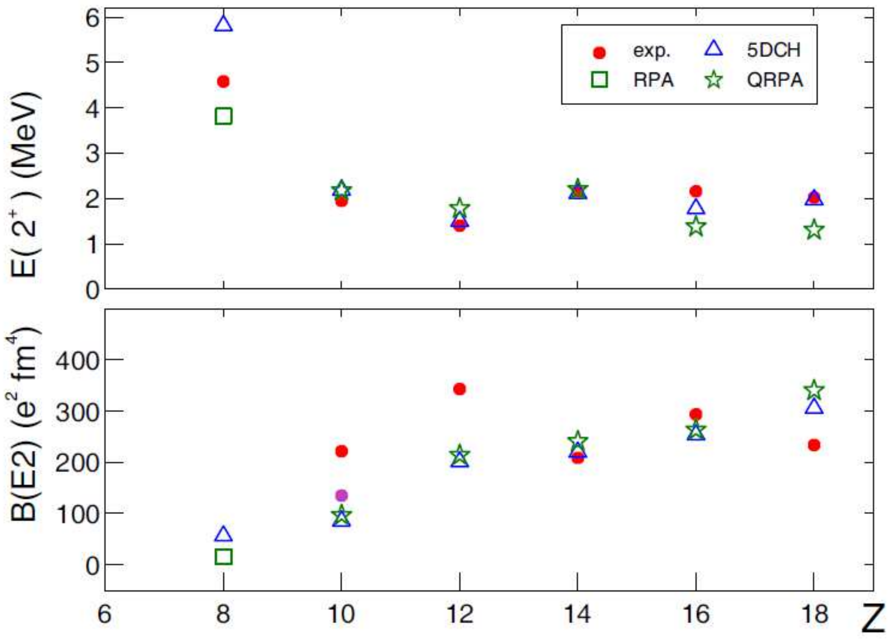
But small amplitude approximation i.e. « harmonic » nuclei



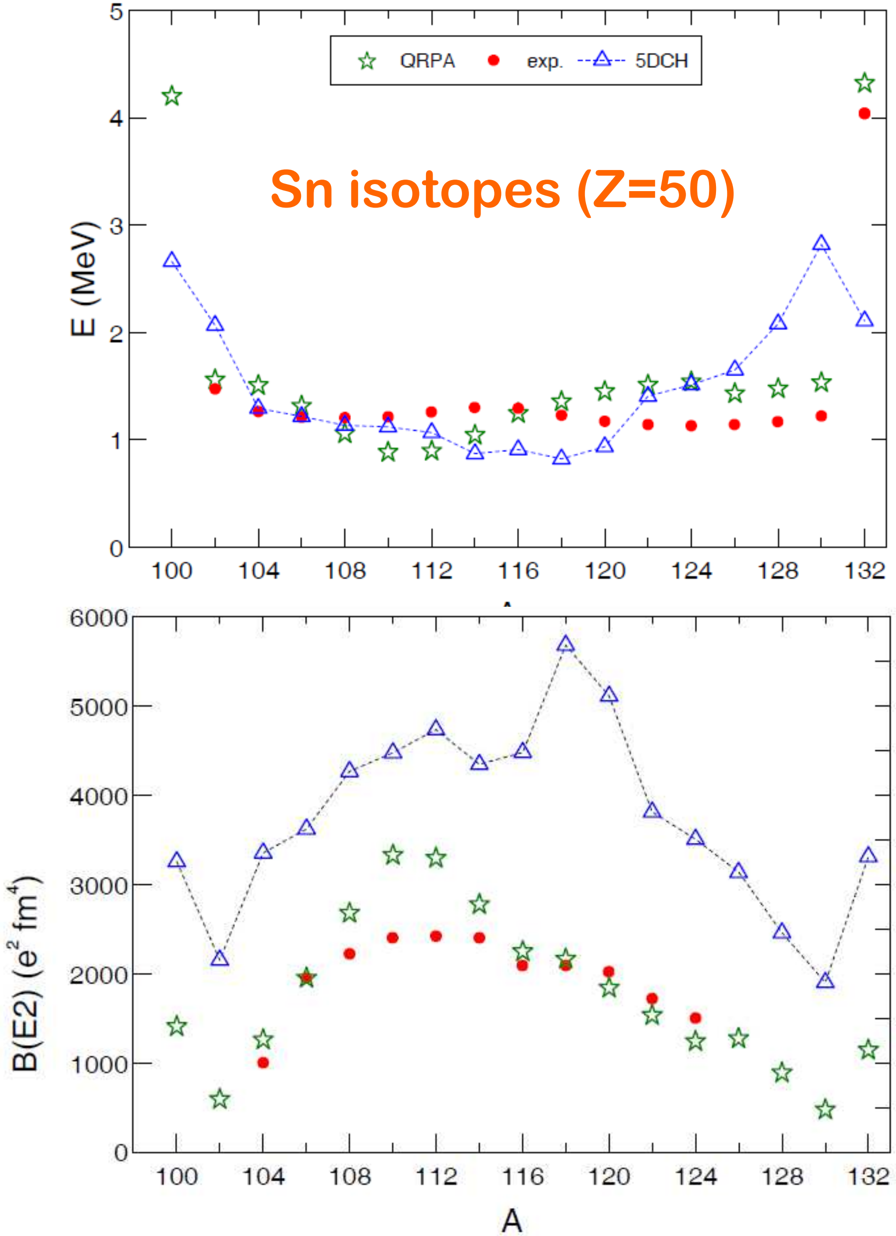


HFB+QRPA versus HFB+5DCH with the same interaction

N=16 isotones



5DCH : A. Obertelli, et al, Phys. Rev. C **71**, 024304 (2005)

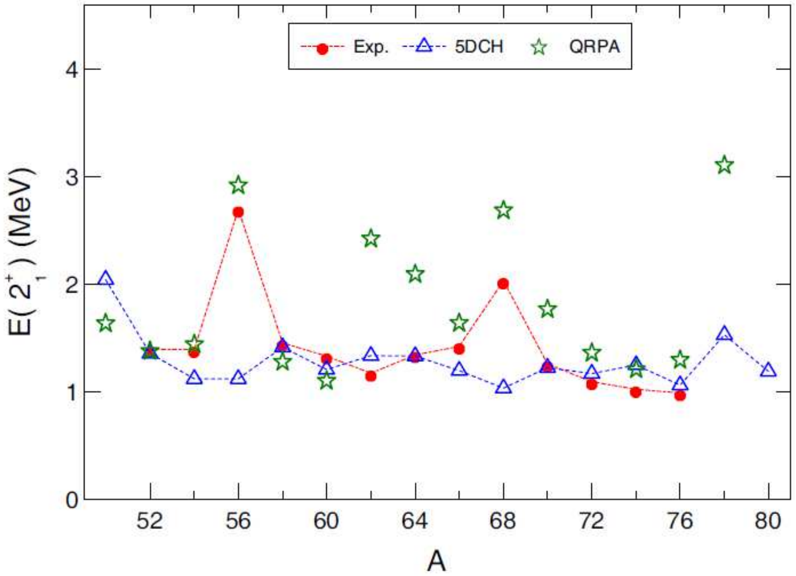


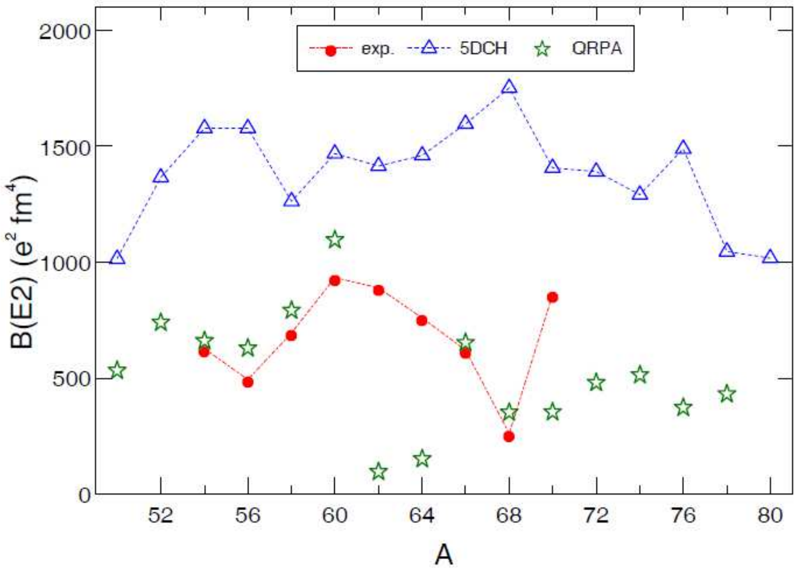
S. Péru and M. Martini, EPJA (2014) 50: 88.

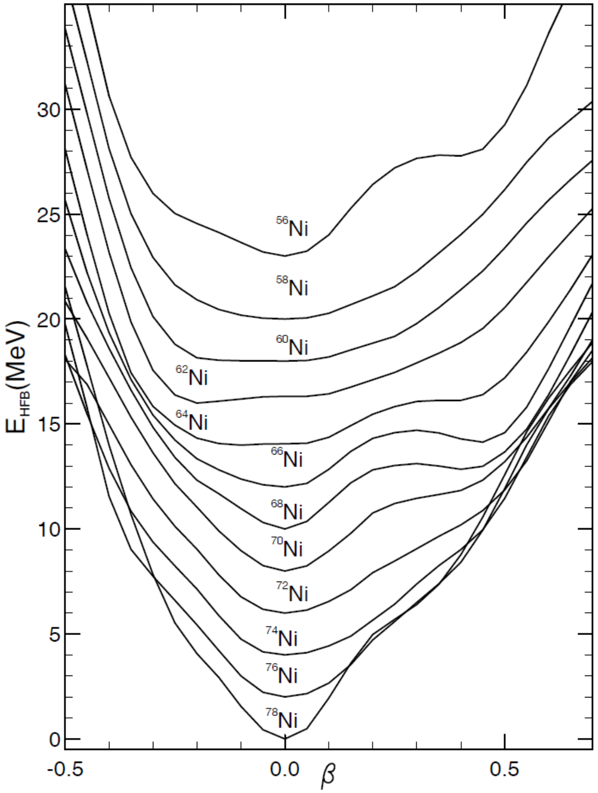
HFB+QRPA versus HFB+5DCH with the same interaction

Ni isotopes (Z=28)

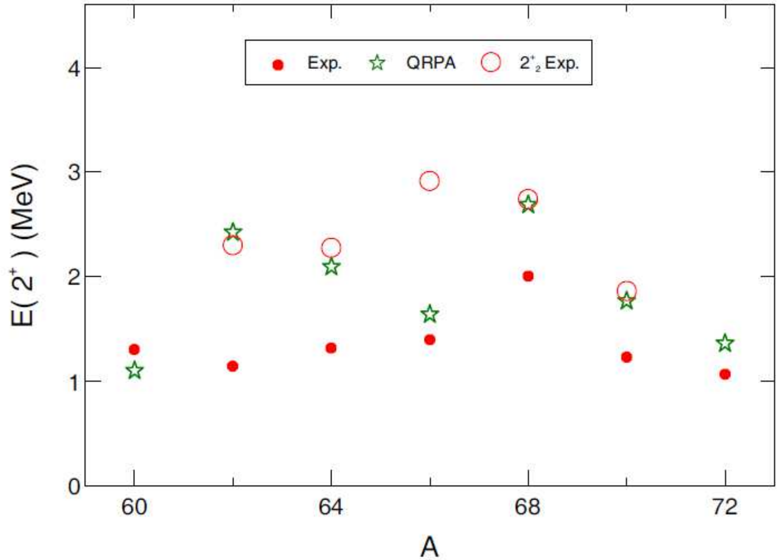
Two shell (N= 28, 50) and one sub-shell (N=40) closures







^{78}Ni is predicted doubly magic



For deformed nuclei the first 2$^+$ state is rotational

S. Péru and M. Martini, EPJA (2014) 50: 88.

Degree of freedom of the present QRPA

- **Which degrees of freedom are considered and at which level of modeling they enter ?**

Quasiparticle and effective interaction: both in underlying HFB and in QRPA

Deformation: only quadrupolar in HFB (intrinsic), all of them in QRPA (dynamic)

Conservation of isospin and parity for QP states and 2QPs excitations

- **Which phenomena these can be expected to be relevant for ?**

Collective as well as individual vibrations

- **Can the present treatment be combined with the one of other degree of freedom ?**

Rotation ? Shape coexistence?

- **Can they be expected to be independent to other degree of freedom ?**

Since pn pairing is not implemented in HFB, no coupling with charge exchange modes.

Thanks for your attention