(Q)RPA calculations with the Gogny force for charge exchange excitations in deformed nuclei and in infinite nuclear matter

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Calculations with the Gogny force

I) pnQRPA in deformed nuclei

M. Martini, S. Péru, S. Goriely, Phys. Rev. C 89, 044306 (2014)

II) RPA in infinite nuclear matter A. De Pace, M. Martini, Phys. Rev. C 94, 024342 (2016)

From ORPA (S. Péru talk) to pnORPA

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K} \begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$

$$\stackrel{\theta^+_{n,K}}{=} \sum_{i < j} X^{ij}_{n,K} \eta^+_{i,ki} \eta^+_{j,kj} - (-)^K Y^{ij}_{n,K} \eta_{j,-kj} \eta_{i,-k_i}$$

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Nuclear excitations







Charge exchange excitation operators

Isobaric Analog Resonance (or Fermi)

$$\hat{O}_{IAR} = \sum_{i=1}^{A} \tau_{-}(i)$$

isospin flip τ





Gamow Teller $\hat{O}_{GT} = \sum_{i=1}^{A} \vec{\sigma}(i) \ \tau_{-}(i)$ isospin flip τ

spin flip σ S=1 T=1 J^{π}=1⁺



pnQRPA IAR Strength Distributions with Gogny (D1M and D1S) force



- Both interactions give quite similar results
- The energy position of the experimental IAR is quite well reproduced
- The single narrow state results reflect the right contribution of the pp channel to the proton-neutron residual interaction, without which the response function will be fragmented

pnQRPA GT Strength Distributions with Gogny (D1M and D1S) force



- Strength located at lower energies for D1M with respect to D1S
- Tendency for the GT energy to increase with increasing $W_{\mbox{\tiny LS}}$, g'

	W _{LS} [MeV fm ⁵]	g'
D1M	115.4	0.71
D1S	130.0	0.61

- The systematic D1S overestimate with respect to D1M seems to suggest that W_{LS} plays the major role
- A small but systematic overestimate of the GT peak is found.
 Particle-vibration coupling as well as tensor interaction contribution, absent in our approach, have been shown to lead to a small shift towards lower energies.

An example of deformed nucleus : ⁷⁶Ge

GT J^{π}=1⁺ distributions obtained by adding twice the K^{π}=1⁺ result to the K^{π}=0⁺ one





Quadrupole deformation parameter

 $\beta_2 \propto \langle HFB | 3z^2 - r^2 | HFB \rangle$



- The deformation tends to increase the fragmentation
- Displacements of the peaks
- Deformation effects also influence the low energy strength

The folded results for the ⁷⁶Ge



p.s.

The pnQRPA calculation provides a discrete strength distribution.

In order to derive a smooth continuous strength function, the pnQRPA GT strength is folded with a Lorentz function $L(E, \omega)$ of width Γ

$$L(E,\omega) = \frac{1}{\pi} \frac{\Gamma/2}{(E-\omega)^2 + \Gamma^2/4} \qquad S_{GT}^{fold}(E) = \sum_n L(E,\omega_n) S_{GT}(\omega_n)$$



In the allowed GT decay approximation the β^- decay half-life T_{1/2} can be expressed in terms of the **GT strength function S_{GT}**

$$\frac{\ln 2}{T_{1/2}} = \frac{(g_A/g_V)_{\text{eff}}^2}{D} \sum_{\substack{E_{ex}=0}}^{Q_\beta} f_0(Z, A, Q_\beta - E_{ex}) S_{GT}(E_{ex})$$

$$f_0(Z, A, \omega) = \int_{m_e c^2}^{\omega} p_e E_e(\omega - E_e)^2 F_0(Z, A, E_e) dE_e$$
Lepton phase-space volume Coulomb and finite nuclear size corrections
$$D = 6163.4 \pm 3.8 \text{ s} \left[\left(\frac{g_A}{g_V} \right)_{eff} = 1 \right]$$

Parenthesis: g_v and g_A coupling constant $\frac{G}{\sqrt{2}} l^{\mu} h_{\mu}$

• The matrix element in **neutron** β decay

$$G\langle p | h_{\mu} | n \rangle = \overline{u}_{p} \gamma_{\mu} (g_{V} - g_{A} \gamma_{5}) u_{n}$$

$$V-A$$

$$\frac{g_{V}}{G} = 1 \qquad \frac{g_{A}}{g_{V}} = 1.26$$



Forty(?)-year old problem: GT Matrix elements and related observables are over-predicted by the theory

Since
$$[\tau_{1/2}]^{-1} \propto g_A^2 |M_{GT}|^2$$

Typical practice Typical practice



$\Delta M_{Z,Z+1} = Q_{\beta} = B_{nucl}(Z,A) - B_{nucl}(Z+1,A) + m(nH)$

For the Q_{β} mass differences, we take experimental (and recommended) masses when available or the D1M mass predictions , otherwise.

Nuclear Masses



β^{-} decay half-life T_{1/2}

Comparison with experimental data for 145 even-even nuclei



- Deviation with respect to data rarely exceeds one order of magnitude
- Larger deviations for nuclei close to the valley of β -stability, as found in most models
- An A dependence of g_A^{eff} (*e.g.* $g_A^{eff} = 1.26 A^{-\alpha}$) could further improve our predictions

Our model

Other models



Prog. Theor. Phys., 84, 641 (1990)

FRDM: Moller et al., ADNDT, 66,131 (1997)

β^{-} decay half-lives of deformed isotopic chains



β^{-} decay half-lives of deformed isotopic chains



β^{-} decay half-lives of the N=82, 126, 184 isotones

Relevance for the r-process nucleosynthesis



Shell Model: Martinez-Pinedo et al., PRL 83, 4502 (1999)

Possible origins of differences: GT Strengths, estimation of Q_{β} values, ...

Questions:

How well constrained are the Gogny D1*-type forces in the spin-isospin sector?
How well constrained are the recently introduced Gogny forces including tensor terms?



- An important tool which helps to answer to these questions: the infinite nuclear matter response functions

RPA with Gogny in infinite nuclear matter: Tensor terms and instabilities

A. De Pace, M. Martini, Phys. Rev. C 94, 024342 (2016)

Nuclear response function



Response function:

$$\Pi_X(q,\omega) = \frac{1}{V} \sum_n |\langle n | \mathcal{Q}^{(X)} | 0 \rangle|^2 \left(\frac{1}{\omega - E_{n0} + i\eta} - \frac{1}{\omega + E_{n0} - i\eta} \right)$$

Poles: energies of the excites states

$$R_X(q,\omega) = -\frac{V}{\pi} \mathrm{Im} \Pi_X(q,\omega)$$

Response of non interacting systems

Ext. perturbation

- (ω, \vec{q})
 - Free nucleon at rest: Response function $\propto \delta(\omega-q^2/2m_N)$
 - Non interacting nuclear matter

Fermi momentum $\mathbf{k}_{\mathbf{F}}$ spreads δ distribution (Fermi Gas)

Pauli blocking cuts part of the response

N

Ν







Non interacting nuclear matter: Fermi Gas

$$G_{\alpha\beta}(\vec{k},\omega) = \delta_{\alpha\beta} \left[\frac{\theta(k-k_{\rm F})}{\omega - \omega_{k} + i\eta} + \frac{\theta(k_{\rm F}-k)}{\omega - \omega_{k_{\rm s}} - i\eta} \right] \qquad \omega_{k} = \frac{k^{2}}{2m}$$

$$\Pi(Q) = -2 \ i \int \frac{\mathrm{d}^{4}K}{(2\pi)^{4}} [G(K)G(K+Q)]$$

$$\Pi^{0}(\vec{q},\omega) = g \int \frac{\mathrm{d}\vec{k}}{(2\pi)^{3}} \left[\frac{\theta(|\vec{k}+\vec{q}| - k_{F})\theta(k_{F}-k)}{\omega - (\omega_{\vec{k}+\vec{q}} - \omega_{\vec{k}}) + i\eta} - \frac{\theta(k_{F}-|\vec{k}+\vec{q}|)\theta(k-k_{F})}{\omega + (\omega_{\vec{k}} - \omega_{\vec{k}+\vec{q}}) - i\eta} \right] \qquad \mathbf{Q}$$

Mean Field Approximation (Hartree-Fock)



Switching on the p-h interaction



Galitskii-Migdal integral equation

 $+iG(K+Q)G(K)\int \frac{d^4T}{(2\pi)^4}\Gamma^{\rm ph}(K+Q,K;T+Q,T) \ G^{\rm ph}(T+Q,T;P+Q,P)$



The ring approximation



Including the exchange term: the RPA equation



- One still has a closed equation but only for the four-point Green's function G^{ph} and not for Π
- For zero range (Skyrme) interactions the Π^{RPA} results are analytical *Garcia-Recio, Navarro, Van Giai, Salcedo, Ann. Phys. 214, 293 (1992)
 *Margueron, Van Giai, Navarro, Phys. Rev. C 74, 015805 (2006)
 *Davesne, Martini, Bennaceur, Meyer, Phys. Rev. C 80, 024314 (2009)
 + Tensor
- For finite range (Gogny, Yukawa,...) interactions fully analytical calculations for Π^{RPA} no longer possible
- Many papers with zero range forces and with finite range forces but in the Landau Migdal ($q \rightarrow 0$) limit for the exchange
- Few papers considering fully antisymmetrized RPA with finite range forces
- Only 2 papers considering fully antisymmetrized RPA with the Gogny force:

*Margueron, Navarro, Van Giai, Schuck, Phys. Rev. C 77, 064306 (2008) central (D1) based on the *De Pace, Martini, Phys. Rev. C 94, 024342 (2016) central (D1,D1S,D1N,D1M); + Tensor (many) continued fraction technique

Continued fraction



Examples:

 $\pi = 3.14159265359... \qquad \pi = [3,7,15,1,292,1,1,1,2,1,4,1,2,9,1,2]$ $3 + \frac{1}{7} = 3.\overline{142857} \qquad 3 + \frac{1}{7 + \frac{1}{15}} = 3.141509... \qquad 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1}}} = 3.14159292...$ $e = 2.71828182846... \qquad e = [2,1,2,1,1,4,1,1,6,1,1,8,1,1,10,1,1,8,1,6,...]$ $2 + \frac{1}{1} = 3 \qquad 2 + \frac{1}{1 + \frac{1}{2}} = 2.\overline{6} \qquad 2 + \frac{1}{1 + \frac{1}{2}} = 2.75$

In the CF technique there is no general way of estimating the convergence of the series

Continued fraction expansion of the polarization propagator

 $\mathbf{T}(\mathbf{0})$

$$\Pi_{X}^{\text{RPA}} = \frac{\Pi^{(0)}}{1 - \Pi_{X}^{(1)\text{d}} / \Pi^{(0)} - \Pi_{X}^{(1)\text{ex}} / \Pi^{(0)} - \frac{\Pi_{X}^{(2)\text{ex}} / \Pi^{(0)} - \left[\Pi_{X}^{(1)\text{ex}} / \Pi^{(0)}\right]^{2}}{1 - \dots}$$

At infinite order the CF expansion gives the exact result as summation of the perturbative series. When truncated at finite order, it reproduces the standard perturbative series at the same order plus an approximation for each one of the infinite number of higher order contributions.



Continued fraction expansion: second order



Second order: the highest order so far reached for finite-range forces

The calculation of the RPA response at order n in the CF expansion is reduced to the calculation of the exchange contribution $\Pi^{(n)ex}$ up to order n

$$\Pi_{\alpha_{1}...\alpha_{n}}^{(n)\text{ex}}(q,\omega) = -i^{n+1} \int \frac{d^{4}K_{1}}{(2\pi)^{4}} \cdots \frac{d^{4}K_{n+1}}{(2\pi)^{4}} G^{(0)}(K_{1}) G^{(0)}(K_{1}+Q) V_{\alpha_{1}}(\boldsymbol{k}_{1}-\boldsymbol{k}_{2}) \cdots$$

$$\cdots V_{\alpha_{n}}(\boldsymbol{k}_{n}-\boldsymbol{k}_{n+1}) G^{(0)}(K_{n+1}) G^{(0)}(K_{n+1}+Q)$$

$$V(\boldsymbol{k}) = V_{0}(k) + V_{\tau}(k) \boldsymbol{\tau}_{1} \cdot \boldsymbol{\tau}_{2} + V_{\sigma}(k) \boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2} + V_{\sigma\tau}(k) \boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2} \boldsymbol{\tau}_{1} \cdot \boldsymbol{\tau}_{2}$$

$$+ V_{t}(k) S_{12}(\hat{\boldsymbol{k}}) + V_{t\tau}(k) S_{12}(\hat{\boldsymbol{k}}) \boldsymbol{\tau}_{1} \cdot \boldsymbol{\tau}_{2}$$

RPA response with Gogny at <u>first</u> and <u>second</u> order in CF



- Quenching or enhancement depending on the repulsive or attractive character of the force
- Appearence of **collective modes** in (0,1) and (1,1)
- Differences between 1° and 2° CF order in (0,0) and (1,1)

Collective modes with D1S and D1M



Lower energy collective modes for D1M when compared to D1S

Nuclear responses for Gogny D1, D1S, D1N and D1M

q=27 MeV/c

q=270 MeV/c



- The responses calculated with the different parametrizations can show important differences
- The convergence of the CF expansion strongly depends on the force parameters in the different (S,T) channels

Including tensor terms

- The tensor terms of the effective nuclear interaction are usually neglected in MF calculations
- In Skyrme and Gogny interactions tensor terms have been considered only in the last years

Skyrme

Finite nuclei

G.Colo', H.Sagawa, S.Fracasso, and P.F.Bortignon, Phys.Lett. B646, 227 (2007) T.Lesinski, M.Bender, K.Bennaceur, T. Duguet, and J.Meyer, Phys. Rev. C76, 014312 (2007)

Nuclear matter

D. Davesne, M. Martini, K. Bennaceur, J. Meyer, Phys. Rev. C 80, 024314 (2009)
A. Pastore, D. Davesne, Y. Lallouet, M. Martini, K. Bennaceur, and J. Meyer, Phys. Rev. C85, 054317 (2012)
A. Pastore, M. Martini, V. Buridon, D. Davesne, K. Bennaceur, and J. Meyer, Phys. Rev. C86, 044308 (2012)
D.Davesne, A.Pastore, and J.Navarro, Phys. Rev. C89,044302 (2014)

Gogny

Finite nuclei

- GT2: Gaussian Tensor-isovector; refitting of all the parameters T. Otsuka, T. Matsuo, D. Abe, PRL 97, 162501 (2006)
- Adding tensor component to D1S and D1M without changing the central parameters
 - D1ST, D1MT: Tensor-isovector based on AV18; spin-orbit modified Anguiano, et al PRC83, 064306 (2011)
 - D1ST2a, D1ST2b:Gaussian Tensor-isovector and Tensor-isoscalar Anguiano et al. PRC86, 054302 (2012)
 - D1ST2c, D1MT2c: Gaussian Tensor-isovector and Tensor-isoscalar; S.O modified De Donno et al. PRC 90 (2014)

Nuclear matter

A. De Pace, M. Martini, Phys. Rev. C 94, 024342 (2016)

From the (S,T) results without tensor...

D1S



... to the (S,M,T) results with tensor

D1ST



RPA responses with Gogny+tensor at first and second order in CF



• Homogeneity of results in the S =0 channels and heterogeneity in the S =1 ones

- The differences between the results obtained at 1st and 2nd order in the CF expansion are more pronounced for the forces including tensor terms
- Some unphysical results (R<0) with GT2 \leftrightarrow lack of convergence of the CF method with GT2

RPA responses with Gogny+tensor at first and second order in CF



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RPA Response functions as tools to detect finite-size instabilities



Qualitative link proposed in T. Lesinski, K. Bennaceur, T. Duguet, J. Meyer, PRC 74, 044315(2006)

Finite nuclei and nuclear matter instabilities

Quantitative analysis of the connection between finite nuclei and nuclear matter instabilities:

- (S=0, T=1): Hellemans, Pastore, Duguet, Bennaceur, Davesne, Meyer, Bender, Heenen, PRC88, 064323 (2013)
- S=1: Pastore, Tarpanov, Davesne, Navarro, PRC 92, 024305 (2015)
- 1. A functional is stable if the lowest critical density at which a pole occurs in nuclear matter calculations is larger than the central density of ⁴⁰Ca (~1.2 ρ_{sat})
- 2. One has also to verify that this pole represents a distinct global minimum in the (ρ_c,q) plane

Critical densities for the most commonly used Gogny forces



- D1, D1S, and D1M satisfy the stability criteria in all the channels hence they are free of spurious finite-size instabilities
- D1N should be treated with some caution
- Calculation at 1st order in the CF expansion can be considered enough for instabilities studies

Critical densities for the Gogny forces with tensor terms



- In the S =1 channels differences between the results at 1st and 2nd order in the CF expansion appear
- The stability criteria are satisfied in all the (S,M,T) channels by all the Gogny forces including tensor terms of type D1MT* and D1ST*
- At least at 1st and 2nd order in the CF expansion, the GT2 force is unstable in all the S =1 channels.

I)Charge exchange excitations and β decay in QRPA with the Gogny force $${\rm Summary}$$

- Generalization of QRPA approach to charge exchange sector
- For spherical and deformed nuclei GT (and IAR) results in good agreement with data
- Satisfactory agreement with experimental half-lives justifies the additional study on the exotic neutron-rich N = 82, 126 and 184 isotonic chains (r-process)

Perspectives

- First forbidden transitions
- Weak magnetism

II) RPA nuclear matter response functions with Gogny

Summary

- Calculations performed by truncating the CF expansion at 2nd order, the highest one so far reached in the context of finite-range forces
- The responses calculated with different parametrizations show important differences
- D1, D1S, D1M are stable; D1N should be treated with some caution
- First fully-antisymmetrized calculation for Gogny forces including tensor terms
- D1ST* and D1MT* are stable; GT2 is unstable

Perspectives

- Insert this tool in the fitting procedure to construct new Gogny type forces
- Compare these results with the ones obtained in (C)RPA for finite nuclei at finite q
- Neutrino-nucleus cross sections and neutrino mean free path in neutron matter