Short-range correlations in neutrino-nucleus scattering

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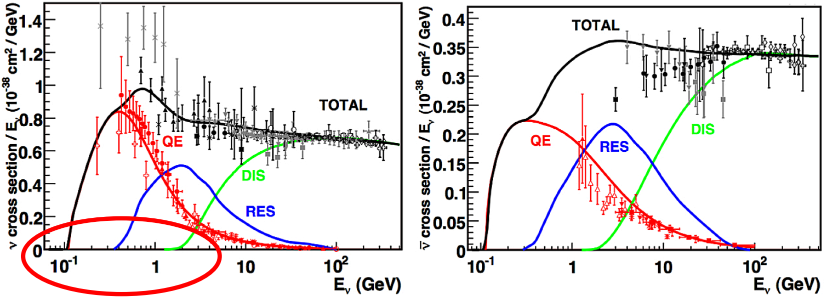
Ref: J.A. Formaggio, *et al.*
Rev.Mod.Phys. 84 (2012) 1307

Motivation



- QE - Quasi-elastic scattering: nucleon stays intact
  $$\nu_\mu + n \to \mu^- + p$$
- RES - Resonance production: nucleon is excited
  $$\nu_\mu + n \to \mu^- + \Delta^+$$
  $$\hookrightarrow p + \pi$$
- DIS - Deep inelastic scattering: nucleon breaks up
  $$\nu_\mu + n \to \mu^- + X$$

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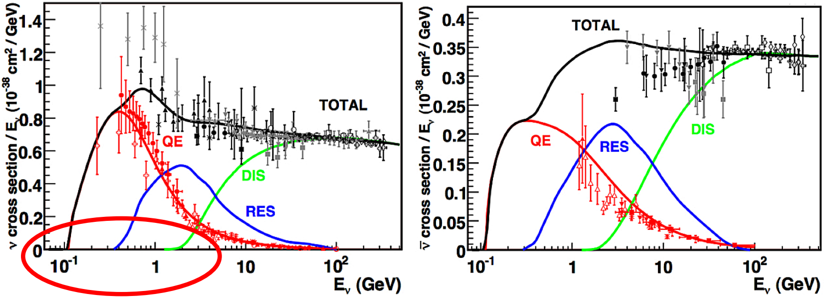
Motivation



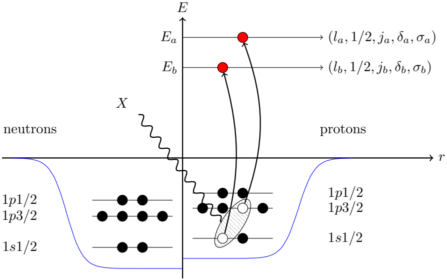
Multinucleon effects

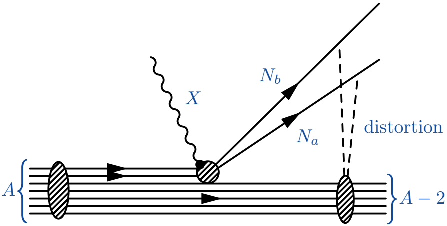
Dip region: multinucleon effects necessary to explain data

$$\nu_\mu + A \to \mu^- + p + (A-1)^* \quad 1N \text{ knockout or } 1p1h$$

$$\left.\begin{array}{l} \nu_\mu + A \to \mu^- + p + p + (A-2)^* \\ \nu_\mu + A \to \mu^- + p + n + (A-2)^* \end{array}\right\} 2N \text{ knockout or } 2p2h$$

Nuclear model

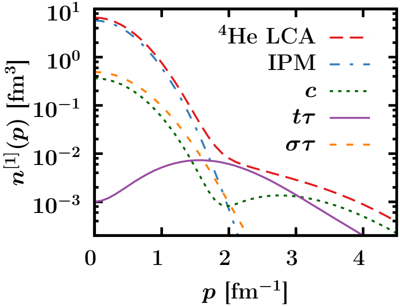


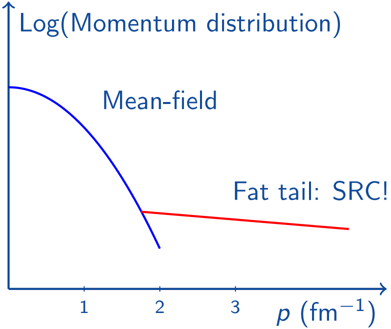


- Ground state nucleus is a shell model
  - Calculated with a Hartree-Fock approximation using an effective Skyrme NN force (SkE2)
  - Accounts for binding energies and nuclear structure
  - Pauli-blocking effects included inherently
- Continuum wave functions are calculated using the same NN potential
  - Orthogonality is preserved between initial and final states
  - Distortion effects of the residual nucleus on the ejected nucleons are incorporated

Short-range correlations

Fat tails in the single-nucleon momentum distribution cannot be explained using an IPM (see talk J. Ryckebusch)



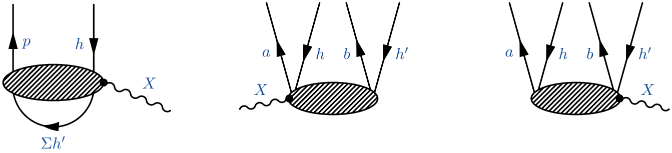


Ref: J. Ryckebusch, *et al.*, J. Phys. G: Nucl. Part. Phys. 42 055104 (2015)

Short-range correlations

Fat tails in the single-nucleon momentum distribution cannot be explained using an IPM (see talk J. Ryckebusch)

- Nucleons occur in pairs with high relative momenta and low center-of-mass momenta (SRC pairs)
  - tensor correlations dominate at intermediate relative pair momenta
  - central correlations dominate at high relative pair momenta
- A signature of SRC is back-to-back $2N$ knockout
- SRC also have an effect on $1N$ knockout



Ref: J. Ryckebusch, *et al.*, Nucl.Phys. A624, 581 (1997)
S. Janssen, *et al.*, Nucl.Phys. A672, 285 (2000)
(electron-scattering model with SRC + MEC)

Short-range correlations

Correlated wave functions $|\Psi\rangle$ are constructed by acting with a many-body correlation operator $\widehat{\mathcal{G}}$ on the uncorrelated Hartree-Fock wave functions $|\Phi\rangle$

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\widehat{\mathcal{G}}|\Phi\rangle, \qquad \text{with} \qquad \mathcal{N} = \langle\Phi|\widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}}|\Phi\rangle$$

The central (c), tensor ($t\tau$) and spin-isospin ($\sigma\tau$) correlations are responsible for majority of the strength

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}}\left(\prod_{i<j}^{A}\left[1+\widehat{l}(i,j)\right]\right)$$

with $\widehat{\mathcal{S}}$ the symmetrization operator and

$$\widehat{l}(i,j) = -g_c(r_{ij}) + f_{t\tau}(r_{ij})\widehat{S}_{ij}\left(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\right) + f_{\sigma\tau}(r_{ij})\left(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)\left(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\right).$$

$g_c(r_{ij})$, $f_{t\tau}(r_{ij})$ and $f_{\sigma\tau}(r_{ij})$ are the respective correlation functions

Short-range correlations

Transition matrix elements between correlated states $|\Psi\rangle$ can be written as matrix between uncorrelated states $|\Phi\rangle$, with an effective transition operator

$$\langle \Psi_f | \widehat{J}_\mu^{\text{nucl}} | \Psi_i \rangle = \frac{1}{\sqrt{\mathcal{N}_i \mathcal{N}_f}} \langle \Phi_f | \widehat{J}_\mu^{\text{eff}} | \Phi_i \rangle,$$

with

$$\widehat{J}_\mu^{\text{eff}} = \widehat{\mathcal{G}}^\dagger \widehat{J}_\mu^{\text{nucl}} \widehat{\mathcal{G}} = \left(\prod_{j<k}^{A} \left[1 + \widehat{l}(j,k) \right] \right)^\dagger \widehat{J}_\mu^{\text{nucl}} \left(\prod_{l<m}^{A} \left[1 + \widehat{l}(l,m) \right] \right).$$

In the IA, the many-body nuclear current can be written as a sum of one-body operators

$$\widehat{J}_\lambda^{\text{eff}} = \left(\prod_{j<k}^{A} \left[1 + \widehat{l}(j,k) \right] \right)^\dagger \sum_{i=1}^{A} \widehat{J}_\lambda^{[1]}(i) \left(\prod_{l<m}^{A} \left[1 + \widehat{l}(l,m) \right] \right).$$

Short-range correlations

Use the fact that SRC is a short-range phenomenon

- Terms linear in the correlation operator are retained
- A-body operator $\rightarrow$ 2-body operator

$$\widehat{J}_\lambda^{\text{eff}} \approx \underbrace{\sum_{i=1}^{A} \widehat{J}_\lambda^{[1]}(i)}_{\text{one-body (IA)}} + \underbrace{\sum_{i<j}^{A} \widehat{J}_\lambda^{[1],\text{in}}(i,j), + \left[\sum_{i<j}^{A} \widehat{J}_\lambda^{[1],\text{in}}(i,j)\right]^\dagger}_{\text{two-body (SRC)}}$$

where

$$\widehat{J}_\lambda^{[1],\text{in}}(i,j) = \left[\widehat{J}_\lambda^{[1]}(i) + \widehat{J}_\lambda^{[1]}(j)\right]\widehat{l}(i,j)$$

- Effective nuclear current is the sum of a one-body (IA) and two-body (SRC) current

Short-range correlations

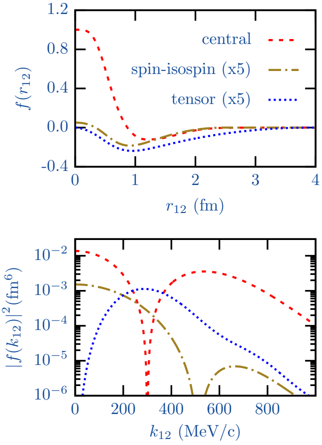


Figure: Correlation functions

- The correlations have a short range: $f(r_{ij}) \to 0$ at $r_{ij} > 3$ fm
- Tensor correlation function dominates for intermediate relative momenta $200 - 400$ MeV/c
- Central correlation function dominates at high relative momenta
- Spin-isospin correlation function overall relatively small
- These correlation functions are input, obtained from literature

Ref: C. C. Gearhaert, PhD thesis, Washington University, (1994). (central)
S. C. Pieper, *et al.* Phys.Rev. C46, 1741 (1992). (tensor and spin-isospin)

One-nucleon knockout

Directly calculate the double differential cross section

$$\frac{d\sigma}{dE_{l'}d\Omega_{l'}} = 4\pi\sigma^X \zeta f_{rec}^{-1}\left[v_{CC}W_{CC} + v_{CL}W_{CL} + v_{LL}W_{LL} + v_T W_T - h v_{T'} W_{T'}\right],$$

with v and σ^X containing leptonic information, e.g.

$$\sigma^{\text{Mott}} = \left(\frac{\alpha\cos(\theta_{e'}/2)}{2E_e \sin^2(\theta_{e'}/2)}\right)^2, \qquad \sigma^W = \left(\frac{G_F \cos\theta_c E_\mu}{2\pi}\right)^2,$$

and the response functions containing the nuclear information

$$\begin{aligned} W_{CC} &= |\mathcal{J}_0|^2 \\ W_{CL} &= 2\,\mathrm{Re}\left(\mathcal{J}_0\mathcal{J}_3^\dagger\right) \\ W_{LL} &= |\mathcal{J}_3|^2 \\ W_T &= |\mathcal{J}_+|^2 + |\mathcal{J}_-|^2 \\ W_{T'} &= |\mathcal{J}_+|^2 - |\mathcal{J}_-|^2 \end{aligned}$$

$$\begin{aligned} \mathcal{J}_0 &= \langle \Psi_f |\widehat{J}_0(q)| \Psi_i\rangle \\ \mathcal{J}_+ &= \langle \Psi_f |\widehat{J}_+(q)| \Psi_i\rangle \\ \mathcal{J}_- &= \langle \Psi_f |\widehat{J}_-(q)| \Psi_i\rangle \\ \mathcal{J}_3 &= \langle \Psi_f |\widehat{J}_3(q)| \Psi_i\rangle \end{aligned}$$

SRC results - $1p1h$

The effective two-body operator affects the $1p1h$ cross section

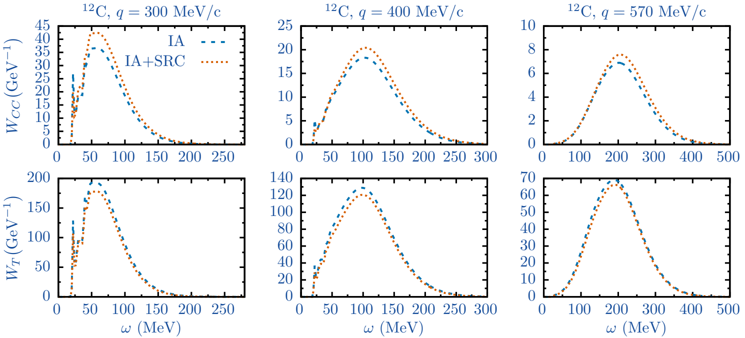


Figure: W_{CC} and W_T response functions for $1p1h$ $^{12}\mathrm{C}(\nu_\mu, \mu^-)$

- Small increase in longitudinal channel W_{CC}
- Small decrease in transverse channel W_T

Two-nucleon knockout

Start with the 8-fold exclusive differential cross section

$$\frac{d\sigma}{dE_{l'}d\Omega_{l'}dT_a d\Omega_a d\Omega_b} = \sigma^X \zeta f_{rec}^{-1}$$
$$\times \Big[v_{CC}W_{CC} + v_{CL}W_{CL} + v_{LL}W_{LL} + v_T W_T + v_{TT}W_{TT} + v_{TC}W_{TC}$$
$$+ v_{TL}W_{TL} - h(v_{T'}W_{T'} + v_{TC'}W_{TC'} + v_{TL'}W_{TL'})\Big],$$

The leptonic factors v and σ^X are independent of the number of knockout particles and five more response functions appear

$$W_{TT} = 2\,\mathrm{Re}\left(\mathcal{J}_+\mathcal{J}_-^\dagger\right)$$
$$W_{TC} = 2\,\mathrm{Re}\left(\mathcal{J}_0\left(\mathcal{J}_+ - \mathcal{J}_-\right)^\dagger\right)$$
$$W_{TL} = 2\,\mathrm{Re}\left(\mathcal{J}_3\left(\mathcal{J}_+ - \mathcal{J}_-\right)^\dagger\right)$$
$$W_{TC'} = 2\,\mathrm{Re}\left(\mathcal{J}_0\left(\mathcal{J}_+ + \mathcal{J}_-\right)^\dagger\right)$$
$$W_{TL'} = 2\,\mathrm{Re}\left(\mathcal{J}_3\left(\mathcal{J}_+ + \mathcal{J}_-\right)^\dagger\right)$$

$$\mathcal{J}_0 = \langle \Psi_f | \widehat{J_0}(q) | \Psi_i \rangle$$
$$\mathcal{J}_+ = \langle \Psi_f | \widehat{J_+}(q) | \Psi_i \rangle$$
$$\mathcal{J}_- = \langle \Psi_f | \widehat{J_-}(q) | \Psi_i \rangle$$
$$\mathcal{J}_3 = \langle \Psi_f | \widehat{J_3}(q) | \Psi_i \rangle$$

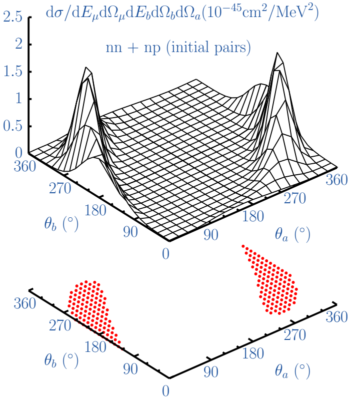
Adding $2p2h$ cross section to $1p1h$ double differential cross section
$\rightarrow$ integrate over outgoing nucleons $\int dT_a d\Omega_a d\Omega_b$

SRC results - Exclusive $A(\nu_\mu, \mu^- N_a N_b)$

Exclusive

- 2 outgoing nucleons N_a and N_b observed in coincidence with the final lepton
- incoherent sum of pp$'$ and pn knockout
- $2N$ knockout from all possible shell combinations $(1s1/2)^2$, $(1s1/2)(1p3/2)$ and $(1p3/2)^2$

SRC results - Exclusive $A(\nu_\mu, \mu^- N_a N_b)$



$$\frac{d\sigma}{dE_\mu d\Omega_\mu dT_b d\Omega_b d\Omega_a}(\nu_\mu, \mu^- N_a N_b)$$

$$N_a = \mathrm{p}, N_b = \mathrm{p}', \mathrm{n}$$

- exclusive differential cross section shows clear back-to-back knockout signal: use this to calculate some of the integrals analytically

Figure: $E_{\nu_\mu} = 750$ MeV, $E_\mu = 550$ MeV, $\theta_\mu = 15^\circ$ and $T_p = 50$ MeV in lepton scattering plane $(\varphi_a, \varphi_b = 0^\circ)$ on ^{12}C.

- red area: $\boldsymbol{P}_{12} = \boldsymbol{p}_a + \boldsymbol{p}_b - \boldsymbol{q} < 300$ MeV/c
- $2N$ knockout cross section proportional to close proximity pairs (see talks J. Ryckebusch and C. Colle)

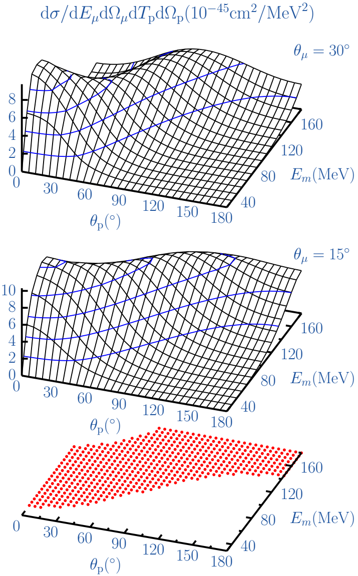
SRC results - Semi-exclusive $A(\nu_\mu, \mu^- N_a)$

Semi-exclusive (semi-inclusive?)

- 1 outgoing nucleon N_a observed in coincidence with final lepton
- $(A-1)^*$ excited above $2N$ emission threshold
- contribution of $2N$ knockout $A(l, l' N_a N_b)$ to semi-exclusive $A(l, l' N_a)$
- incoherent sum of pp′ and pn knockout (p is detected)
- $2N$ knockout from all possible shell combinations $(1s1/2)^2$, $(1s1/2)(1p3/2)$ and $(1p3/2)^2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_{l'}\mathrm{d}\Omega_{l'}\mathrm{d}T_p\mathrm{d}\Omega_p}(l, l'p) = \int \mathrm{d}\Omega_n \frac{\mathrm{d}\sigma}{\mathrm{d}E_{l'}\mathrm{d}\Omega_{l'}\mathrm{d}T_p\mathrm{d}\Omega_p\mathrm{d}\Omega_n}(l, l'pn) + \int \mathrm{d}\Omega_{p'} \frac{\mathrm{d}\sigma}{\mathrm{d}E_{l'}\mathrm{d}\Omega_{l'}\mathrm{d}T_p\mathrm{d}\Omega_p\mathrm{d}\Omega_{p'}}(l, l'pp')$$

SRC results - Semi-exclusive $A(\nu_\mu, \mu^- N_a)$



$$\int d\Omega_n \frac{d\sigma}{dE_{l'} d\Omega_{l'} dT_p d\Omega_p d\Omega_n}(l, l'pn)$$

We impose quasi-deuteron kinematics and replace $\boldsymbol{p}_b \to \boldsymbol{p}_b^{ave}$

$$\boldsymbol{p}_b^{ave} = \boldsymbol{q} - \boldsymbol{p}_a$$

This is equivalent with residual nucleus with zero recoil momentum ($f_{rec} = 1$)

Figure: $E_{\nu_\mu} = 750$ MeV, $E_\mu = 550$ MeV for in-plane kinematics ($\varphi_p = 0°$) on ^{12}C.

- red area: so-called ridge

  $E_m = \frac{A-2}{A-1}\frac{p_m^2}{2m_N} + S_{2N} + E_{A-2}^{hh'}$

SRC results - Inclusive $2p2h$ $A(\nu_\mu, \mu^-)$

Inclusive $2p2h$

- only final lepton is detected
- contribution of $2N$ knockout $A(l, l' N_a N_b)$ to $A(l, l')$
- incoherent sum of pp' and pn knockout
- $2N$ knockout from all possible shell combinations $(1s1/2)^2$, $(1s1/2)(1p3/2)$ and $(1p3/2)^2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_{l'}\mathrm{d}\Omega_{l'}}(l, l') = \int \mathrm{d}T_\mathrm{p}\mathrm{d}\Omega_\mathrm{p} \frac{\mathrm{d}\sigma}{\mathrm{d}E_{l'}\mathrm{d}\Omega_{l'}\mathrm{d}T_\mathrm{p}\mathrm{d}\Omega_\mathrm{p}}(l, l'p)$$

- $\int \mathrm{d}\Omega_p$ analytical integration
- $\int \mathrm{d}T_p$ numerical integration

SRC electron results - Inclusive $2p2h$

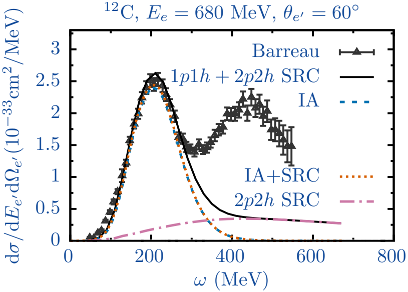


Figure: (e, e') scattering on ^{12}C

Strength of the $2p2h$ contribution

- tensor SRC dominates at small to intermediate ω
- central SRC dominates at large ω
- tensor dominated by pn pairs
- vector, axial and interference terms are equally important

- Inclusion of SRC in the $2p2h$ channel yields a broad background over the whole ω range

SRC results - Inclusive $2p2h$

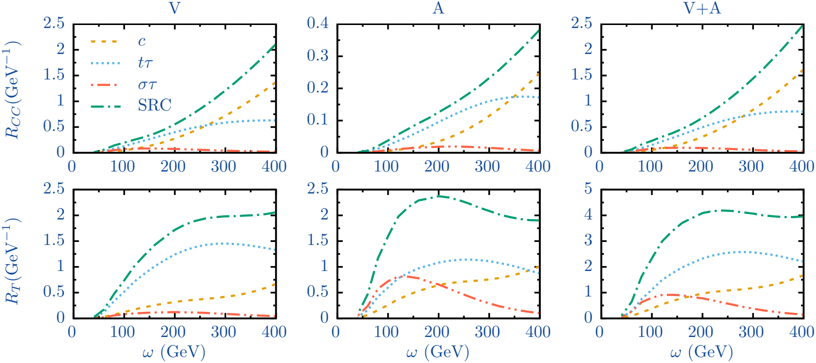


Figure: $2p2h$ SRC response functions R_{CC} and R_T for ^{12}C(ν_μ, μ^-) at $q = 400$ MeV/c

- The tensor part yields the biggest contribution for small ω transfers while the importance of the central part increases with ω. This is directly related to the correlation functions.
- Spin-isospin is relatively large for axial-transverse. Axial-transverse current and spin-isospin operator have a $\boldsymbol{\sigma} \cdot \boldsymbol{\tau}$ structure.

SRC results - Inclusive $2p2h$

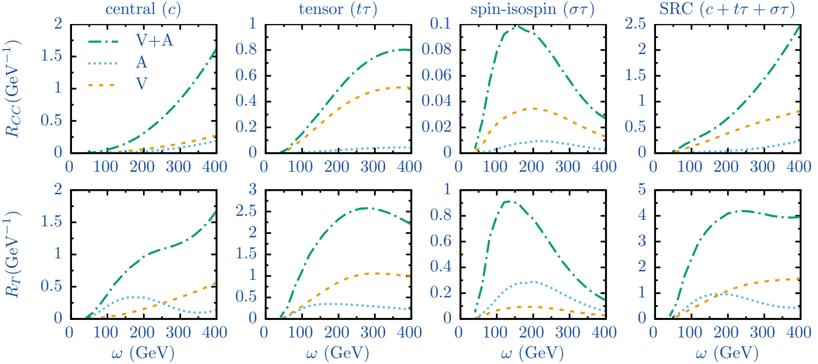


Figure: $2p2h$ SRC response functions R_{CC} and R_T for $^{12}\mathrm{C}(\nu_\mu, \mu^-)$ at $q = 400$ MeV/c

- Vector and axial-vector are both equally important.

SRC results - Inclusive $2p2h$

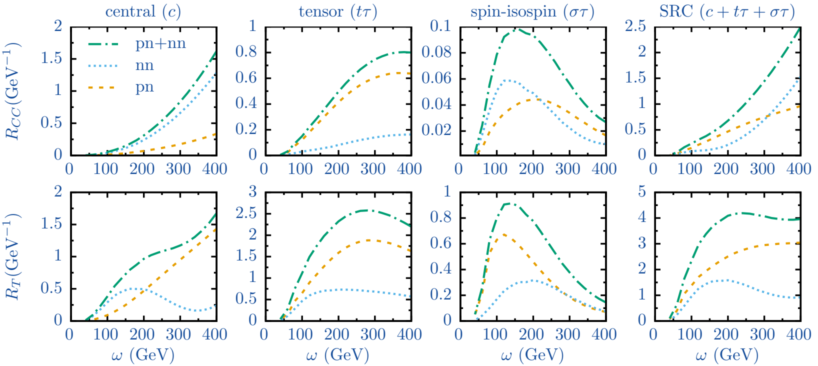
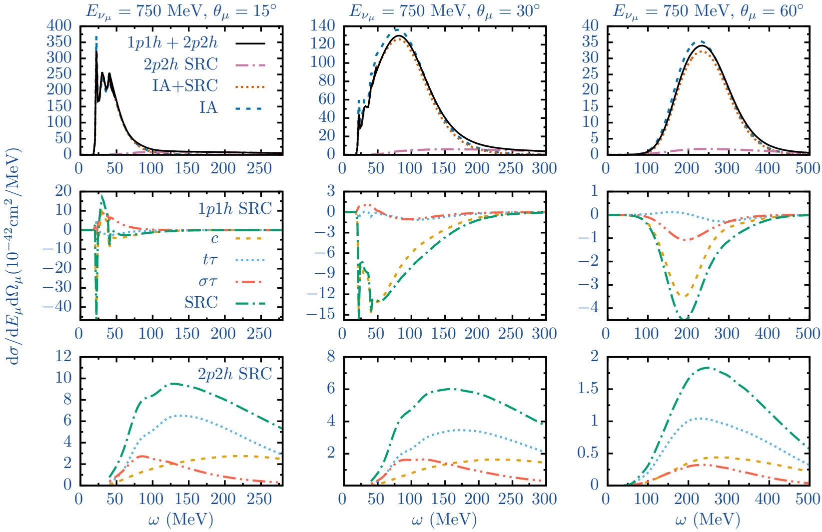


Figure: $2p2h$ SRC response functions R_{CC} and R_T for $^{12}\mathrm{C}(\nu_\mu, \mu^-)$ at $q = 400$ MeV/c

- Central-Coulomb (top left) does not distinguish between protons and neutrons $\rightarrow$ biggest contribution from nn pairs.
- Tensor correlations clearly dominated pn pairs.

SRC results - Inclusive 2p2h



- Inclusive 2p2h appears as a broad background to 1p1h

Meson-exchange currents

Extend the current model with MEC. First seagull and pion-in-flight currents

$$\widehat{J}_{\lambda}^{\text{eff}} = \underbrace{\sum_{i=1}^{A} \widehat{J}_{\lambda}^{[1]}(i)}_{\text{one-body (IA)}} + \underbrace{\sum_{i<j}^{A} \widehat{J}_{\lambda}^{[1],\text{in}}(i,j), + \left[\sum_{i<j}^{A} \widehat{J}_{\lambda}^{[1],\text{in}}(i,j)\right]^{\dagger}}_{\text{two-body (SRC)}}$$

$$+ \underbrace{\sum_{i<j}^{a} \widehat{J}_{\lambda}^{[2],\text{sea}}(i,j) + \sum_{i<j}^{a} \widehat{J}_{\lambda}^{[2],\text{pif}}(i,j)}_{\text{two-body (MEC)}}$$

- Effective current includes SRCs and MECs in a uniform way
- Interference between IA, SRC and MEC inherently included

Meson-exchange currents

Vector seagull and pion-in-flight currents

$$\widehat{\boldsymbol{J}}_V^{[2],\mathrm{sea,nr}}(\boldsymbol{q}) = -i\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\vec{\boldsymbol{\tau}}_1 \times \vec{\boldsymbol{\tau}}_2)_\pm F_1^V(Q^2)\left(\frac{\boldsymbol{\sigma}_1\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2)}{\boldsymbol{q}_2^2+m_\pi^2} - \frac{\boldsymbol{\sigma}_2\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1)}{\boldsymbol{q}_1^2+m_\pi^2}\right)$$

$$\widehat{\boldsymbol{J}}_V^{[2],\mathrm{pif,nr}}(\boldsymbol{q}) = i\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\vec{\boldsymbol{\tau}}_1 \times \vec{\boldsymbol{\tau}}_2)_\pm F_1^V(Q^2)\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1)\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2)}{(\boldsymbol{q}_1^2+m_\pi^2)\,(\boldsymbol{q}_2^2+m_\pi^2)}(\boldsymbol{q}_1-\boldsymbol{q}_2)$$

with $\pm$ corresponding with the incoming $W^\pm$

Ref: J. Ryckebusch, *et al.*, Nucl.Phys. A624, 581 (1997)
S. Janssen, *et al.*, Nucl.Phys. A672, 285 (2000)
(electron-scattering model with SRC + MEC)

Axial seagull currents ?

$$\widehat{\rho}_A^{[2],\mathrm{sea,nr}}(\boldsymbol{q}) = \frac{i}{g_A}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\vec{\boldsymbol{\tau}}_1 \times \vec{\boldsymbol{\tau}}_2)_\pm \left(F_\pi(Q_1^2)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2}{\boldsymbol{q}_2^2+m_\pi^2} - F_\pi(Q_2^2)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1}{\boldsymbol{q}_1^2+m_\pi^2}\right)$$

Summary and outlook

Summary SRC

- Started from a model for exclusive calculations which was tested against electron scattering data
- Calculated contribution of SRC to double differential QE cross section

Outlook

- Extending the model with meson-exchange currents in a consistent approach
  - Vector MEC model exists for electron scattering
  - Axial MEC are *challenging*

References

2p2h e-scattering calculations including SRC and MEC

- J. Ryckebusch, *et al.*, Nucl.Phys. A568, 828 (1994)
- J. Ryckebusch, *et al.*, Nucl.Phys. A624, 581 (1997)
- S. Janssen, *et al.*, Nucl.Phys. A672, 285 (2000)

Momentum distributions with SRC

- J. Ryckebusch, *et al.*, J.Phys.G: Nucl.Part.Phys. 42 055104 (2015)

CRPA calculations for ν-interactions

- N. Jachowicz, *et al.*, Phys.Rev. C65, 025501 (2002)
- V. Pandey, *et al.*, Phys.Rev. C89, 024601 (2014)
- V. Pandey, *et al.*, Phys.Rev. C92, 024606 (2015)

Relativistic corrections

Relativistic kinematic corrections are implemented by the following simple substitution for ω (computed nonrelativistically) in the computation of the response functions W_i

$$W_i(\omega, q) \rightarrow W_i\left(\omega\left(1 + \frac{\omega}{2m_N}\right), q\right),$$

with m_N the nucleon mass. This can be interpreted as a shift of the QE peak from its nonrelativistic position to the relativistic position

$$\omega = \frac{q^2}{2m_N} \rightarrow \omega = \frac{Q^2}{2m_N}$$

Ref: W. Alberico, *et al.* Nucl.Phys. A512, 541 (1990)
S. Jeschonnek, T.W. Donnelly, Phys.Rev. C57, 2438 (1998)
J. E. Amaro, *et al.*, Phys.Rev. C71, 065501 (2005)