

Semi-inclusive neutrino-nucleus reactions and deuteron target

Oscar Moreno

Grupo de Física Nuclear

Universidad Complutense de Madrid



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O. Moreno, T. W. Donnelly, J. W. Van Orden, W. P. Ford,
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SUMMARY

- Definition of semi-inclusive scattering
- Motivation
- Kinematics: lepton and transfer variables, the space of residual nucleus variables
- Dynamics: generalized Rosenbluth factors and hadronic response functions, tensor contraction and cross-sections
- Remarks on incident neutrino kinematics and nuclear dynamics
- Results with deuteron target
- Conclusions

DEFINITION

- In semi-inclusive charged current neutrino-nucleus reactions (anti)neutrinos interact with a nuclear target and the final charged lepton is detected in coincidence with a another particle.

$$X(\nu_\ell, \ell^- x)$$

$$X(\bar{\nu}_\ell, \ell^+ x)$$

DEFINITION

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$$X(\nu_\ell, \ell^- x)$$

$$X(\bar{\nu}_\ell, \ell^+ x)$$

- Case of most interest: nucleon emission

$${}^A_Z X(\nu_\ell, \ell^- p) {}^{A-1}_{Z-1} Y$$

$${}^A_Z X(\nu_\ell, \ell^- n) {}^{A-1}_{Z+1} Y$$

$${}^A_Z X(\bar{\nu}_\ell, \ell^+ n) {}^{A-1}_{Z-1} Y$$

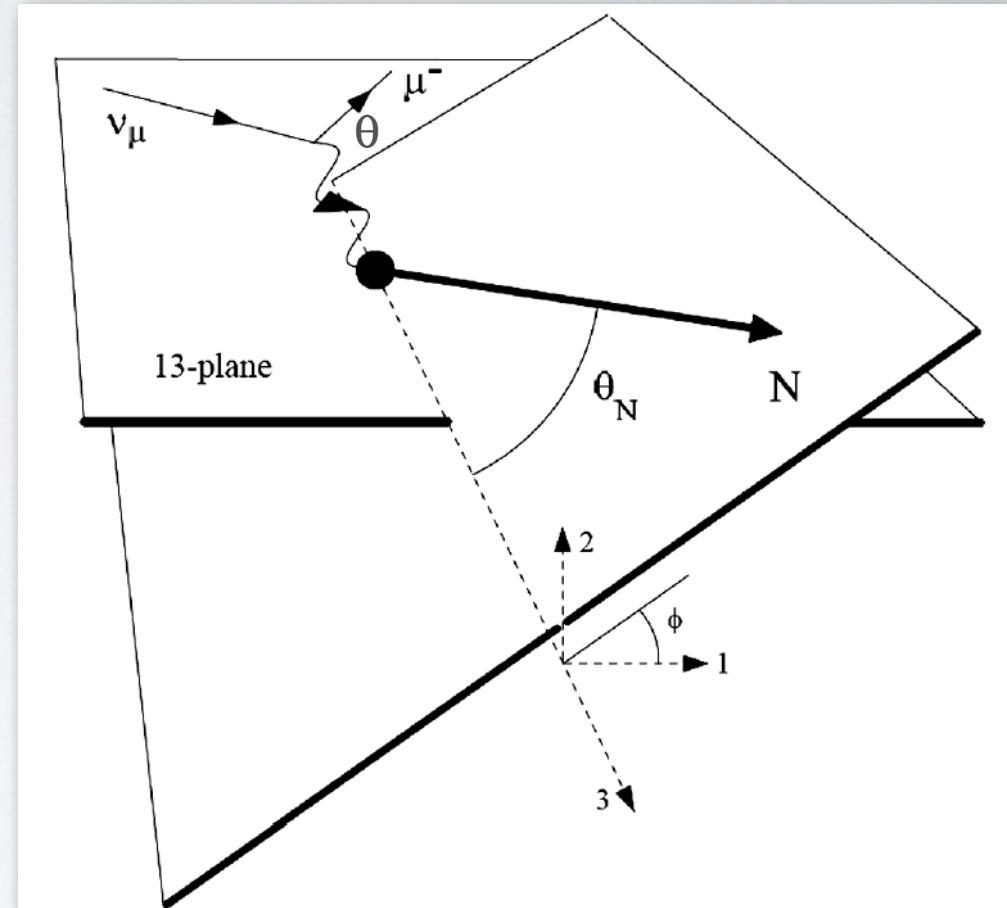
$${}^A_Z X(\bar{\nu}_\ell, \ell^+ p) {}^{A-1}_{Z-2} Y$$

MOTIVATION

- An increasing number of neutrino experiments allow for semi-inclusive measurements (ArgoNeuT, MicroBooNE: μ and p).
- Semi-inclusive measurements provide information on the incident neutrino kinematics.
- Hadronic structure studies.

KINEMATICS

- Leptonic variables: k, k', θ
- Exchange variables: q, ω
- Detected hadron variables: p_N, θ_N, ϕ



KINEMATICS

- Leptonic variables: k, k', θ
- Exchange variables: q, ω
- Detected hadron variables:

$\underline{p_N}, \theta_N, \phi$



$$\mathbf{p} \equiv -\mathbf{p}_{A-1} = \mathbf{p}_N - \mathbf{q}$$

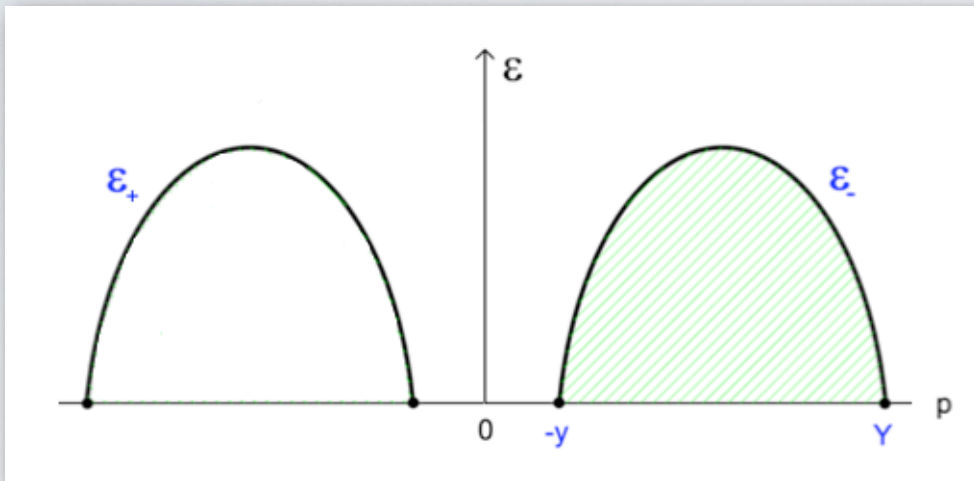
$$p = |\mathbf{p}_m|$$

$$\mathcal{E} \equiv E_{A-1} - E_{A-1}^0 = \omega - E_s + m_N - \sqrt{m_N^2 + p^2 + q^2 + 2pq \cos \theta_{pq}} - \sqrt{W_{A-1}^0{}^2 + p^2 + W_{A-1}^0}$$

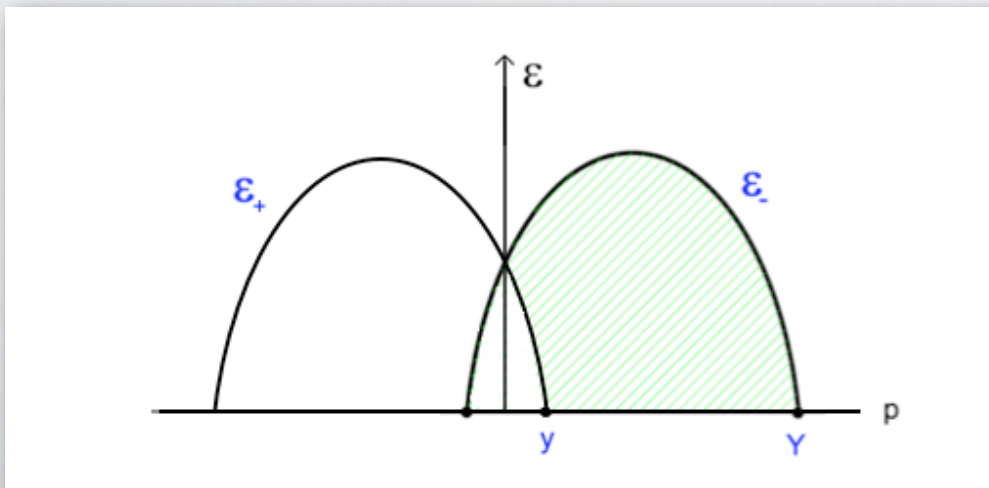
$$\mathcal{E} \approx E_m - E_s$$

KINEMATICS

Kinematically allowed region in the (ϵ, p) plane for fixed q and ω



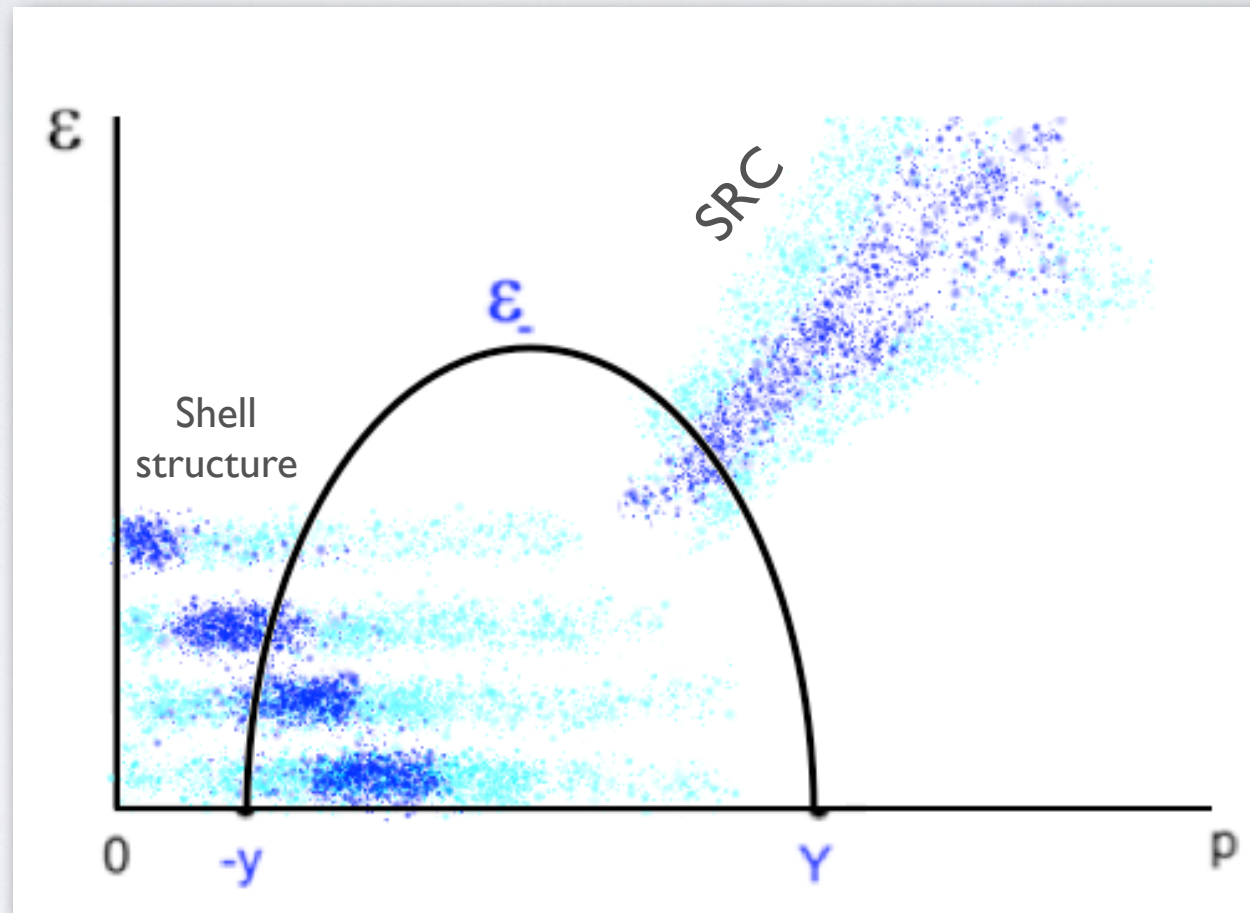
For $\omega < \omega_{QE}$ ($\Rightarrow y < 0, x > 1$)



For $\omega > \omega_{QE}$ ($\Rightarrow y > 0, x < 1$)

KINEMATICS

Probability distribution of the residual system in the (\mathcal{E}, p) plane

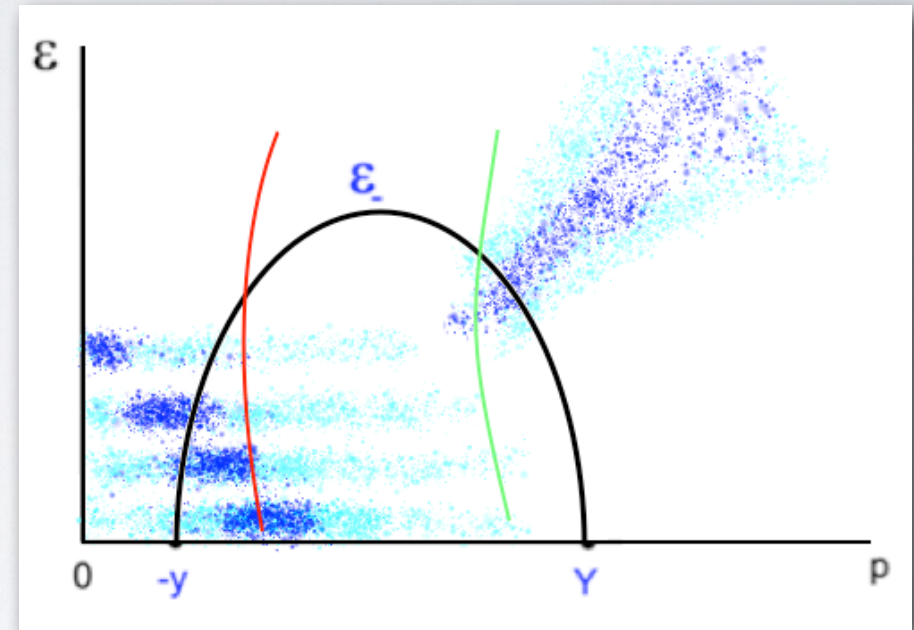


(for fixed q and ω d, for $\omega < \omega_{QE}$)

KINEMATICS

In practice, one performs an integration of the cross-section over the incoming neutrino energy weighted with the neutrino flux.

For a given set of values of the semi-inclusive variables $(k', \theta, p_N, \theta_N, \phi)$, a range of incident neutrino energy corresponds to a curve in the (ϵ, p) -plane.



The nuclear target modelling is more demanding than in the inclusive case: (ϵ, p) -dependence needed. The possibility of kinematic discrimination would allow for the calibration of nuclear models.

DYNAMICS

Leptonic and hadronic tensors:

$$d\sigma \sim \eta_{\mu\nu} W^{\mu\nu} \sim \eta_{\mu\nu}^s W_s^{\mu\nu} + \chi \eta_{\mu\nu}^a W_a^{\mu\nu}$$

$$\eta_{\mu\nu} \equiv 2mm' \overline{\sum_{if}} j_{\mu}^* j_{\nu}$$

$$W^{\mu\nu} \equiv \overline{\sum_{if}} J_{fi}^{\mu*}(\mathbf{q}) J_{fi}^{\nu}(\mathbf{q})$$

DYNAMICS

Leptonic and hadronic tensors contraction:

$$\begin{aligned} \eta_{\mu\nu}^s W_s^{\mu\nu} = v_0 \{ & [\hat{V}_{CC} W^{CC} + \hat{V}_{CL} W^{CL} + \hat{V}_{LL} W^{LL} \\ & + \hat{V}_T W^T + \hat{V}_{TT} W^{TT} + \hat{V}_{TC} W^{TC} + \hat{V}_{TL} W^{TL}] \\ & + [\hat{V}_{\underline{TT}} W^{\underline{TT}} + \hat{V}_{\underline{TC}} W^{\underline{TC}} + \hat{V}_{\underline{TL}} W^{\underline{TL}}] \} \end{aligned}$$

$$\begin{aligned} \eta_{\mu\nu}^a W_a^{\mu\nu} = v_0 \{ & [\hat{V}_{T'} W^{T'} + \hat{V}_{TC'} W^{TC'} + \hat{V}_{TL'} W^{TL'}] \\ & + [\hat{V}_{\underline{CL}'} W^{\underline{CL}'} + \hat{V}_{\underline{TC}'} W^{\underline{TC}'} + \hat{V}_{\underline{TL}'} W^{\underline{TL}'}] \} \end{aligned}$$

DYNAMICS

Generalized Rosenbluth factors from the leptonic tensor:

$$\hat{V}_{CC}$$

$$\hat{V}_{CL}$$

$$\hat{V}_{LL}$$

$$\hat{V}_T$$

$$\hat{V}_{TT}$$

$$\hat{V}_{TC}$$

$$\hat{V}_{TL}$$

$$\hat{V}_{T'}$$

$$\hat{V}_{TC'}$$

$$\hat{V}_{TL'}$$

DYNAMICS

Generalized Rosenbluth factors from the leptonic tensor:

$$\hat{V}_{CC} = \frac{1}{2} \{ (a_V^2 + a_A^2) - [a_V^2 (\delta - \delta')^2 + a_A^2 (\delta + \delta')^2] \tan^2 \tilde{\theta} / 2 \}$$

$$\delta \equiv \frac{m}{\sqrt{|Q^2|}}$$

$$\delta' \equiv \frac{m'}{\sqrt{|Q^2|}}$$

DYNAMICS

Response functions from the hadronic tensor:

$$\begin{array}{ccccccc}
 W^{CC} & W^{CL} & W^{LL} & W^T & W^{TT} & W^{TC} & W^{TL} \\
 & & & & & & \\
 & & W^{T'} & W^{TC'} & W^{TL'} & &
 \end{array}$$

DYNAMICS

Response functions from the hadronic tensor:

For inclusive cross-sections:



ϕ dependence

DYNAMICS

General differential cross-section:

$$d\sigma_\chi = \frac{G^2 \cos^2 \theta_c}{2(2\pi)^5} \frac{m_N W_{A-1} v_0}{k \varepsilon' E_N E_{A-1}} \mathcal{F}_\chi^2 d^3 \mathbf{k}' d^3 \mathbf{p}_N d^3 \mathbf{p}_{A-1} \delta^4(K + P_A - K' - P_{A-1} - P_N).$$

Integration over the unobserved residual system variables:

$$\frac{d\sigma_\chi}{dk' d\Omega_{\mathbf{k}'} d\Omega_{\mathbf{p}_N}} = \frac{G^2 \cos^2 \theta_c}{2(2\pi)^5} \frac{m_N W_{A-1}}{M_A^0} \frac{p_N k'^2 v_0}{k \varepsilon' F_{\text{rec}}} \mathcal{F}_\chi^2,$$

REMARKS

Determination of the incident neutrino momentum

- From momentum conservation:

$$k = k' \cos \theta \pm \left\{ \left[p_N \cos \theta_N \pm (p^2 - p_N^2 \sin^2 \theta_N)^{1/2} \right]^2 - k'^2 \sin^2 \theta \right\}^{1/2}$$

REMARKS

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- From energy conservation:

$$\epsilon = \epsilon' + E_N - M_A^0 + E_{A-1} \Rightarrow$$

$$k = \left\{ \left[(k'^2 + m'^2)^{1/2} + (p_N^2 + m_N^2)^{1/2} - M_A^0 + \epsilon + (p^2 + W_{A-1}^{(0)2})^{1/2} \right]^2 - m^2 \right\}^{1/2}$$

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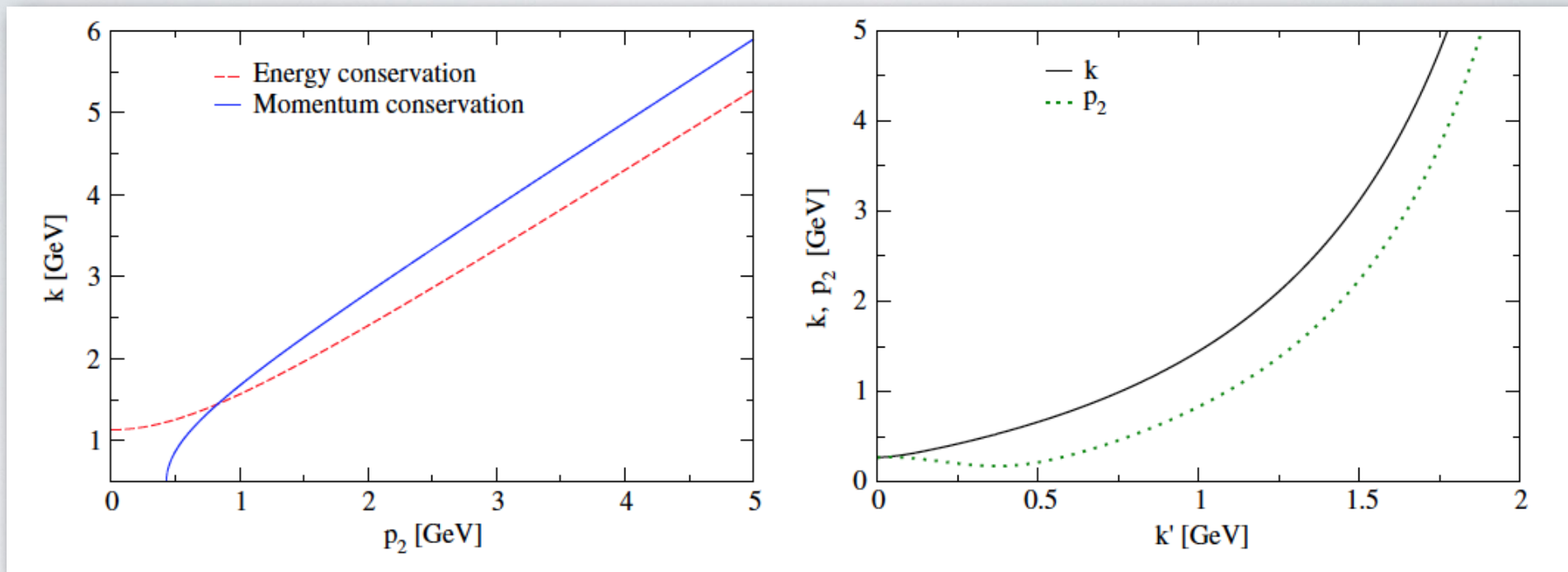
$$k = \left\{ \left[(k'^2 + m'^2)^{1/2} + (p_N^2 + m_N^2)^{1/2} - M_A^0 + \epsilon + (p^2 + W_{A-1}^{(0)2})^{1/2} \right]^2 - m^2 \right\}^{1/2}$$

For the particular case of deuteron target:

$$W_{A-1}^{(0)} = m_p, \quad \mathcal{E} = 0$$

RESULTS WITH DEUTERON TARGET

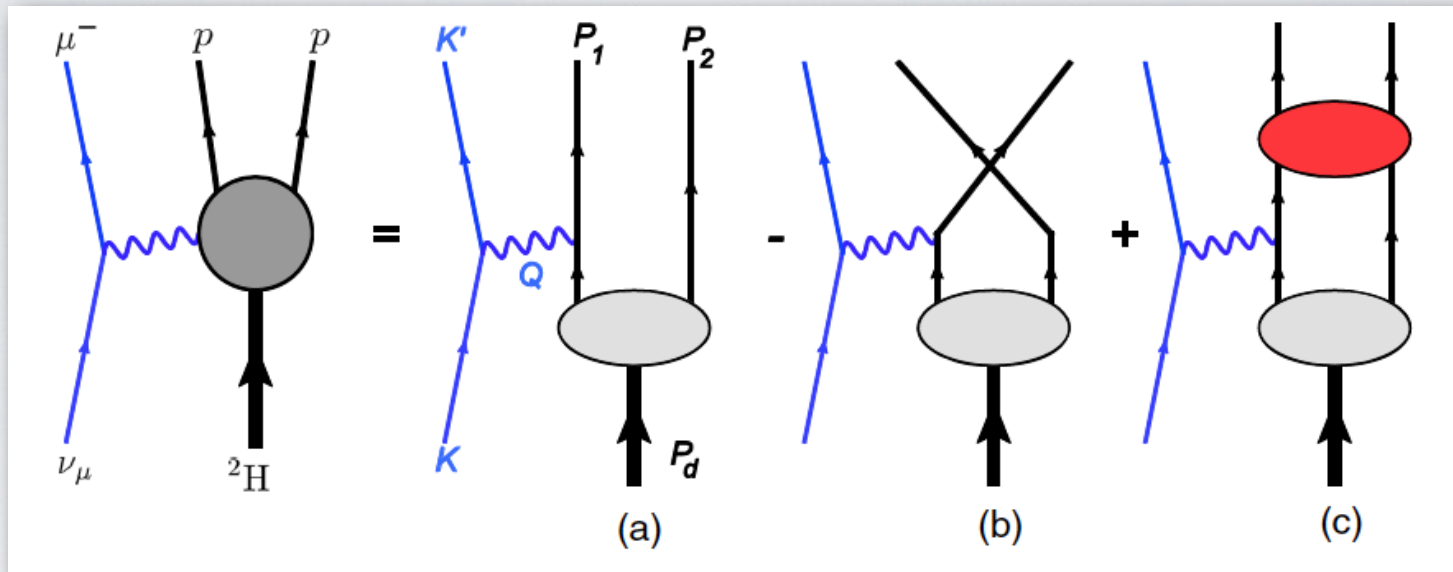
Determination of the incident neutrino momentum (graphical example):



$$k' = 1 \text{ GeV}, \theta = 60^\circ, p_1 = 0.5 \text{ GeV}, \theta_1 = 20^\circ$$

$$\Rightarrow k = 1.45 \text{ GeV}, p_2 = 0.83 \text{ GeV}$$

RESULTS WITH DEUTERON TARGET



Direct
term

Exchange
term

Final state
interactions

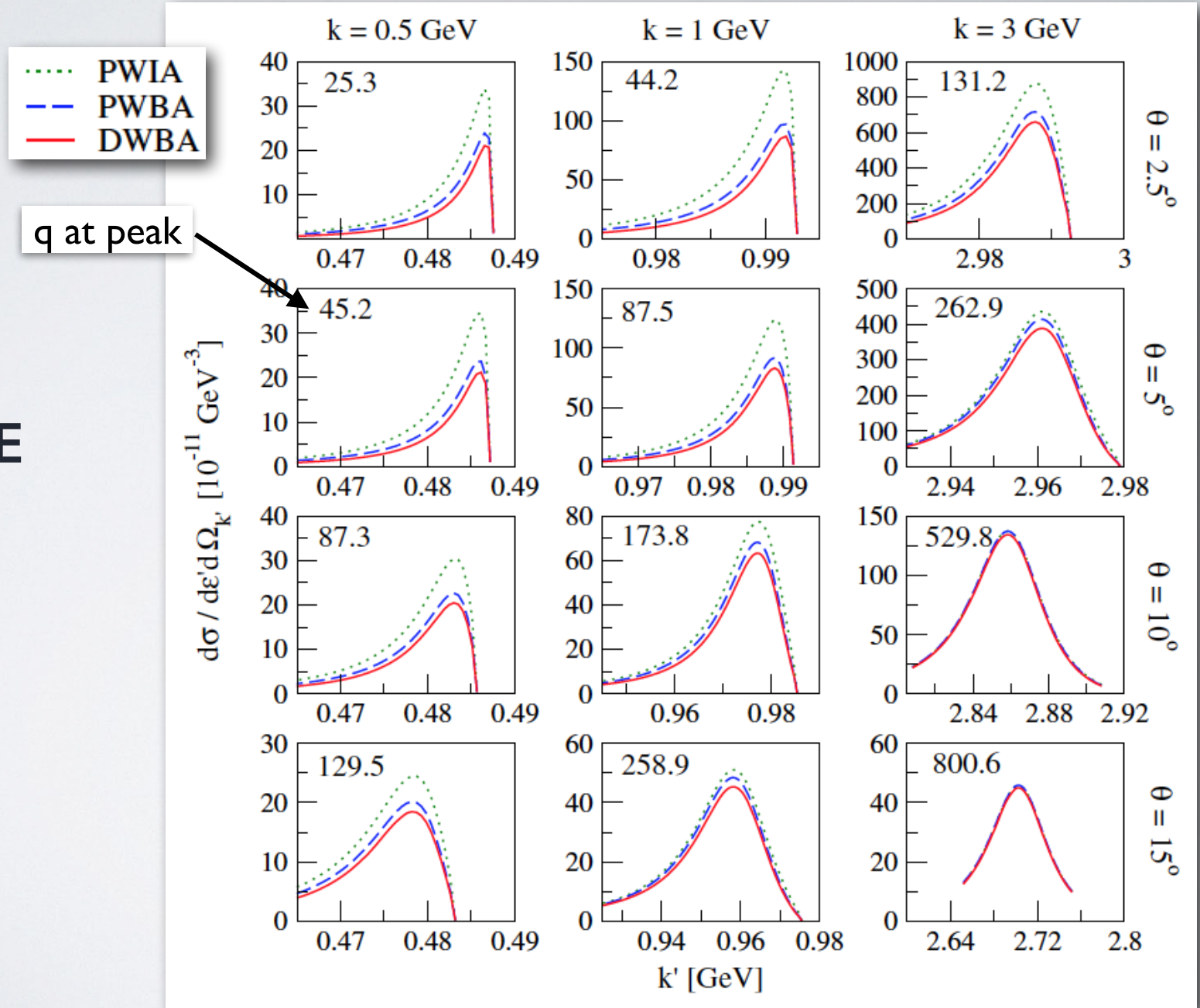
PWIA

PWBA

DWBA

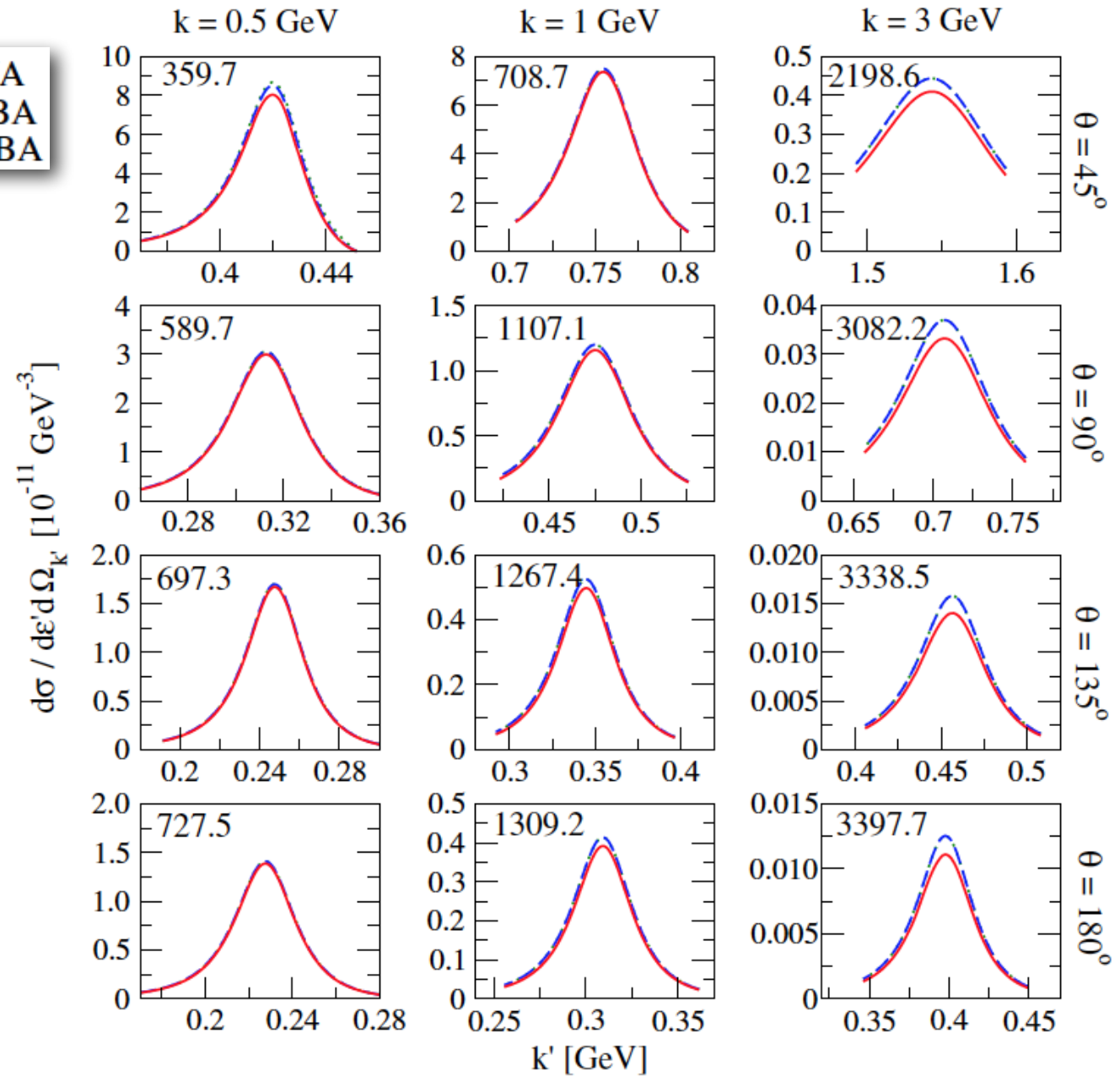
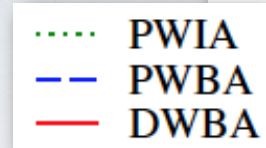
RESULTS WITH DEUTERON TARGET

INCLUSIVE



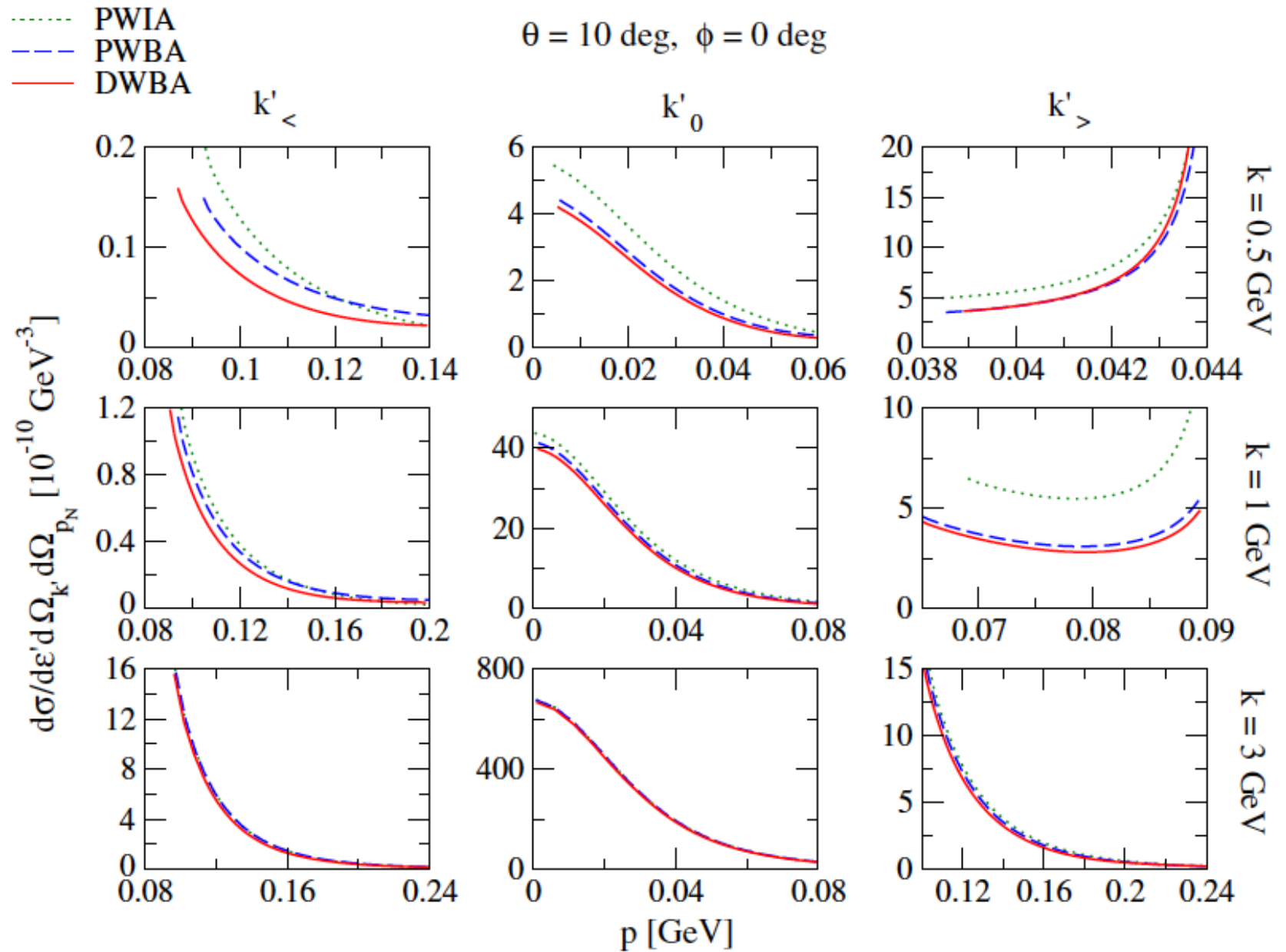
RESULTS WITH DEUTERON TARGET

INCLUSIVE



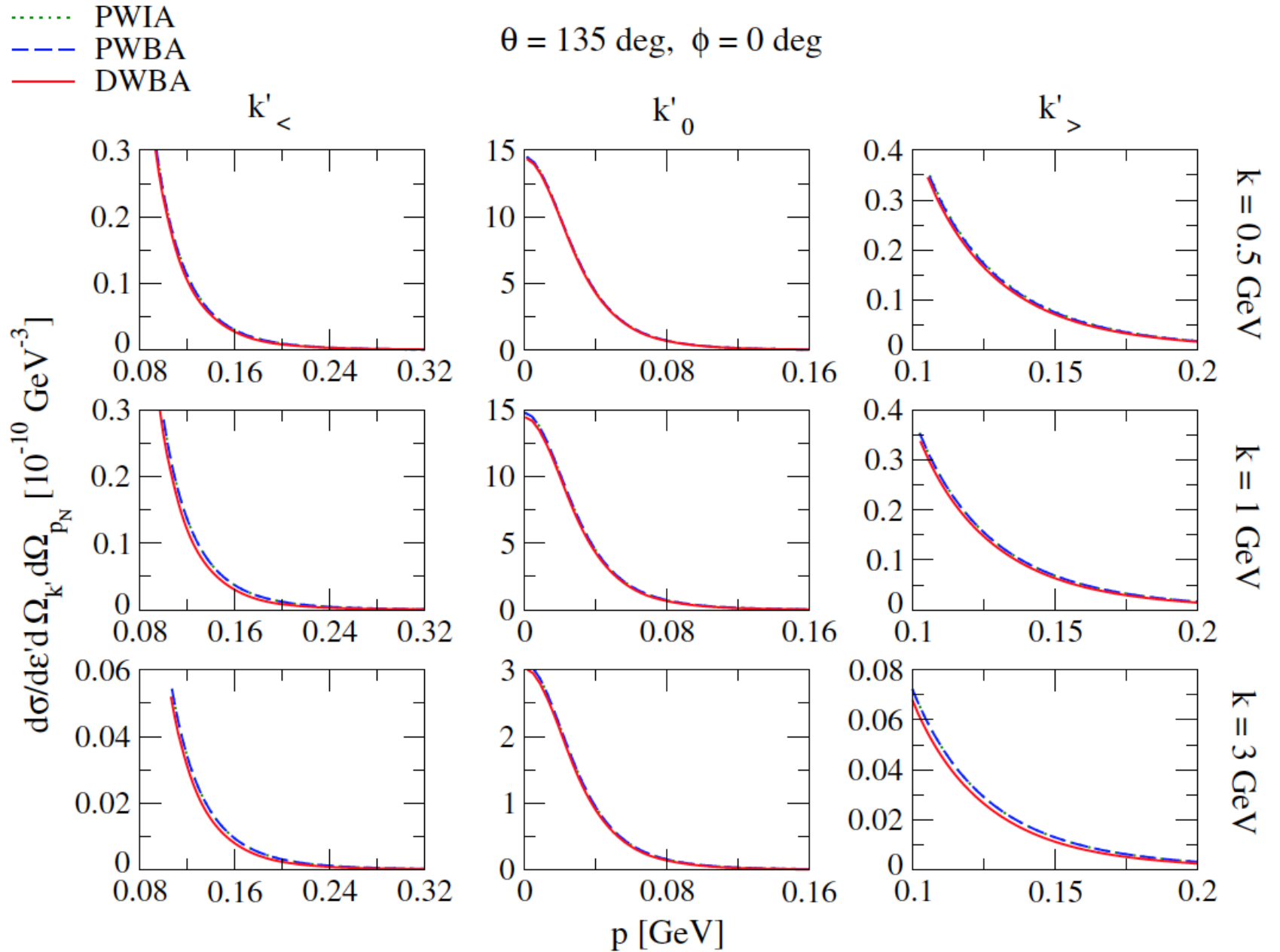
RESULTS WITH DEUTERON TARGET

SEMI
INCLUSIVE



RESULTS WITH DEUTERON TARGET

SEMI
INCLUSIVE

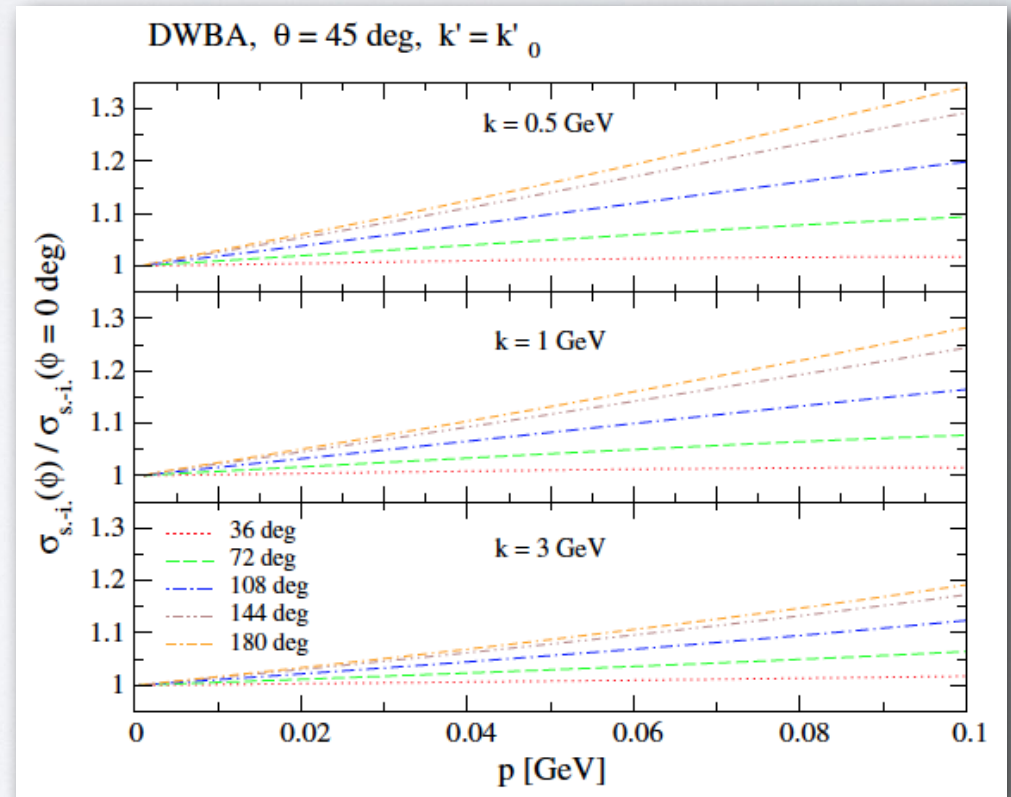
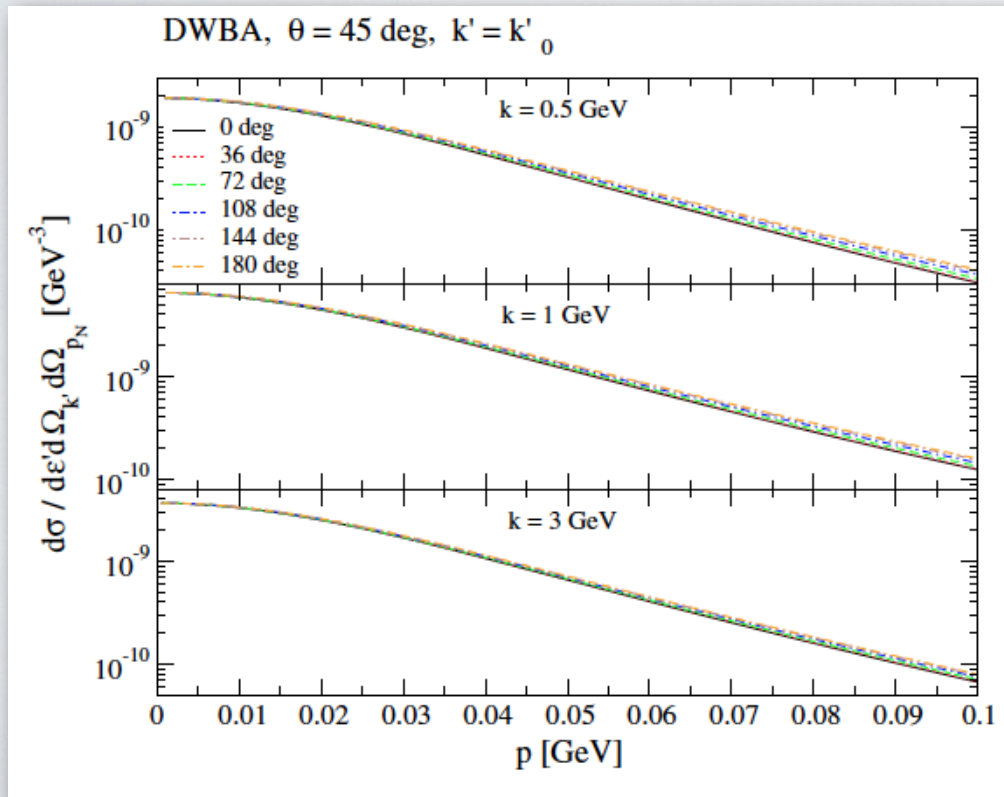


RESULTS WITH DEUTERON TARGET

SEMI-INCLUSIVE

ϕ DEPENDENCE

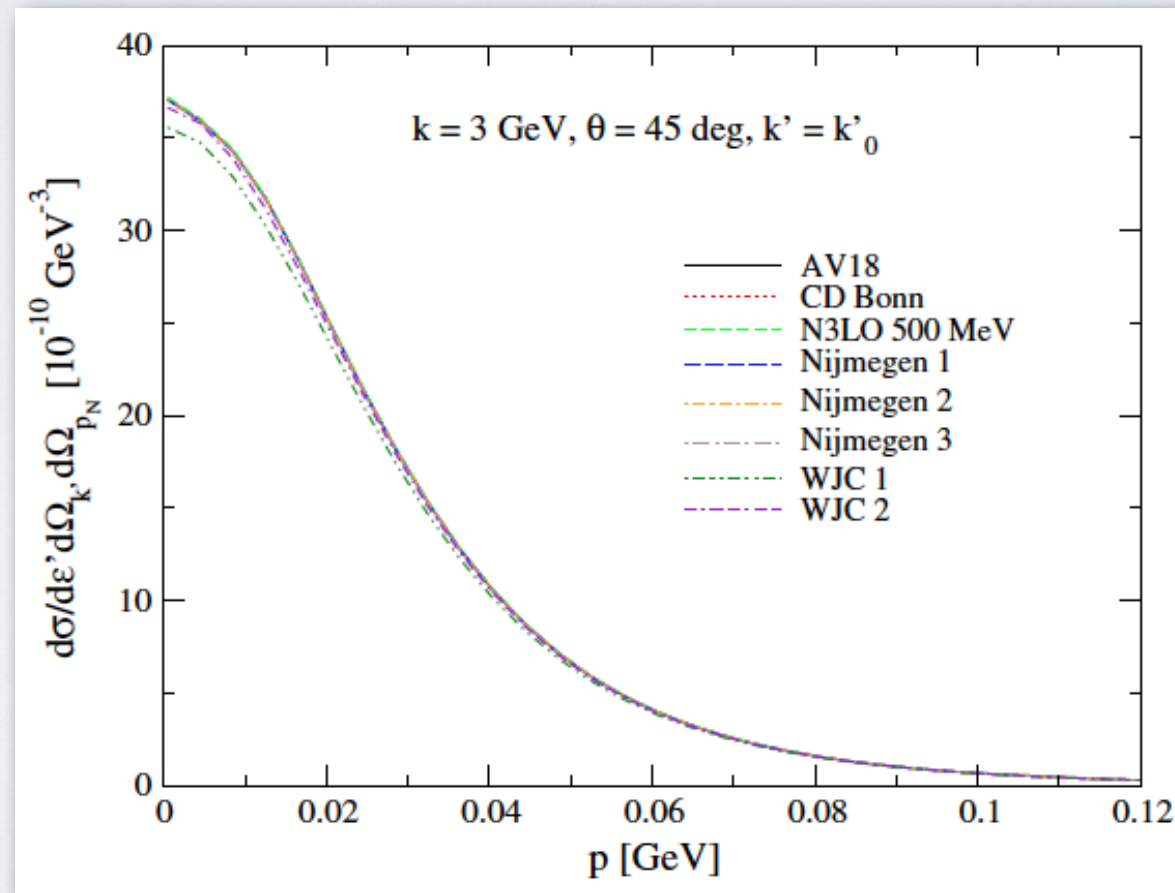
Ratios over result for $\phi=0$



RESULTS WITH DEUTERON TARGET

SEMI-INCLUSIVE

INITIAL STATE DEPENDENCE
(DIFFERENT DEUTERON WAVE FUNCTIONS)

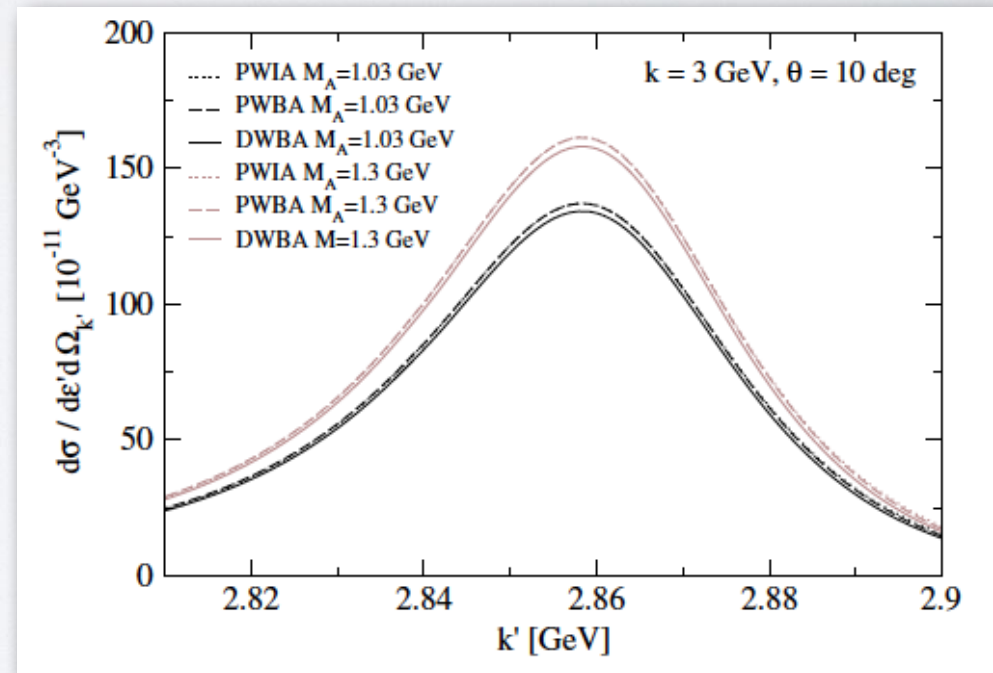
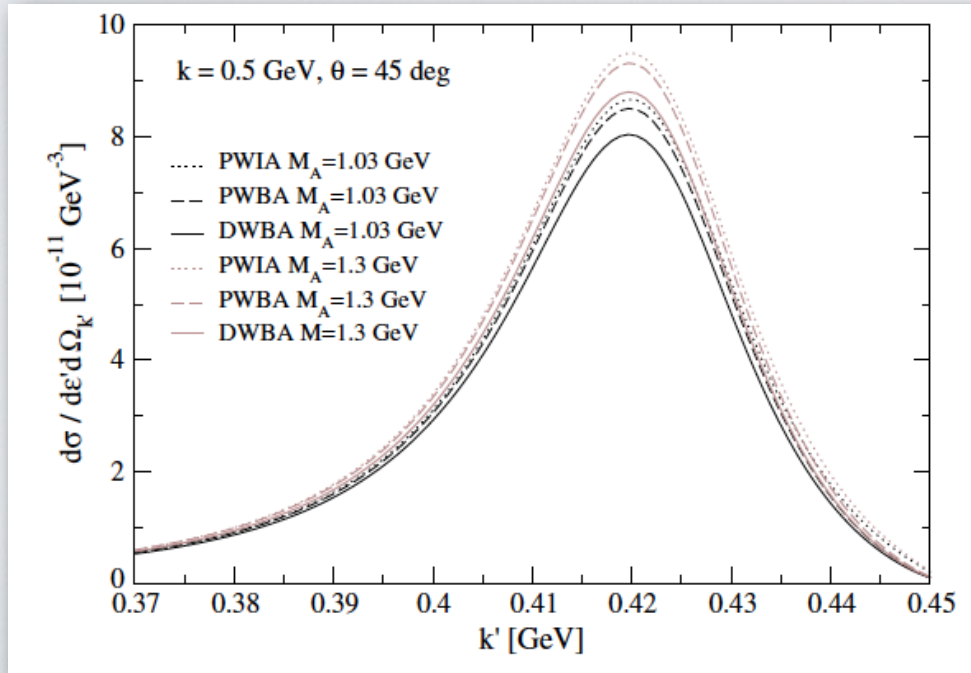


RESULTS WITH DEUTERON TARGET

EFFECT OF AXIAL MASS

INCLUSIVE AXIAL MASS DEPENDENCE

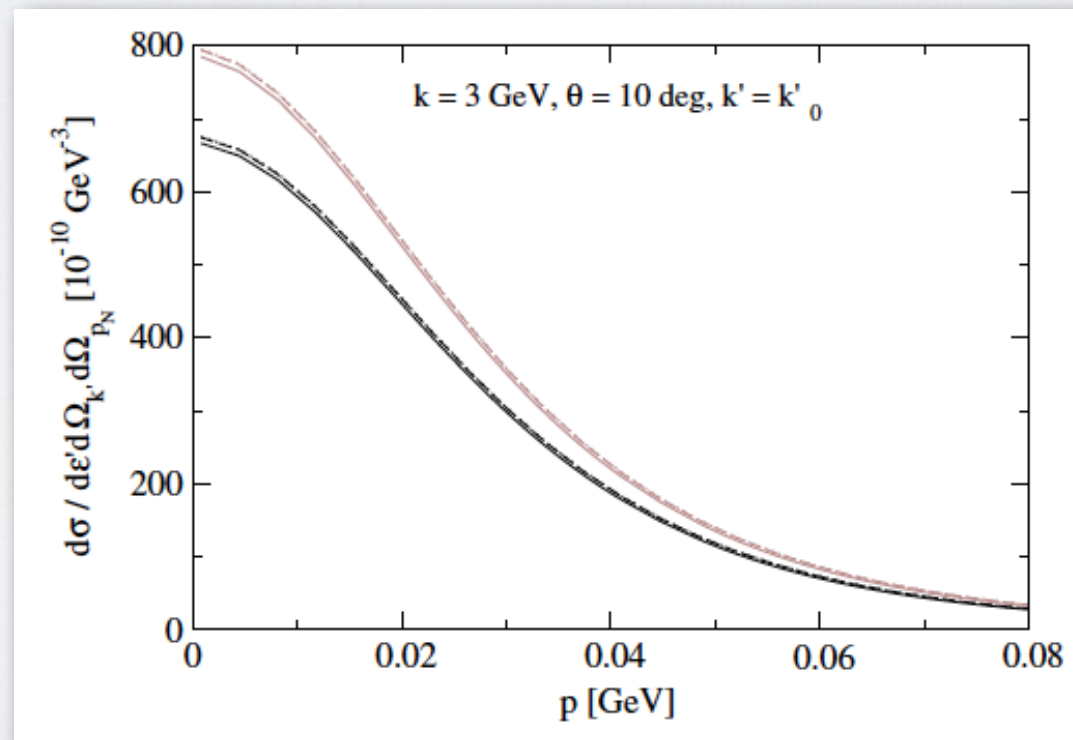
$$G_A(Q^2) = \frac{1.2695}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$



RESULTS WITH DEUTERON TARGET

EFFECT OF AXIAL MASS

SEMI-INCLUSIVE AXIAL MASS DEPENDENCE



CONCLUSIONS

GENERAL TARGET

- General expressions developed for any lepton masses, for both neutrinos and antineutrinos, for any hadronic target and products, and valid for weak charged- and neutral-current processes.
- Components of the leptonic and hadronic tensors are given in terms of charginelike, longitudinal and transverse projections of the electroweak current and organized into VV, AA and VA contributions.
- Transformation of hadronic variables to the (ϵ, p) variables, which are best suited to characterizing the nuclear dynamics.
- Translation of a neutrino energy range onto the (ϵ, p) -plane: kinematical discrimination of nuclear dynamics.

CONCLUSIONS

DEUTERON TARGET

- **Determination of the incident neutrino energy** using kinematics alone: a semi-inclusive measurement (pp or μp) below π production is actually exclusive.
- **Determination of the incident neutrino flux** from theoretical semi-inclusive cross-sections. In the range $500 \lesssim q \lesssim 1000$ MeV the PWIA/PWBA/DWBA calculations are indistinguishable.
- **Determination of the axial-mass** dependence of the nucleon isovector axial form factor G_A (for $q \gtrsim 150$ MeV).

OUTLOOK

DEUTERON TARGET

- Provide simple parametrizations of the DWBA semi-inclusive cross-section.
- Provide calculations for a heavy water target (D_2O).

Semi-inclusive neutrino-nucleus reactions and deuteron target

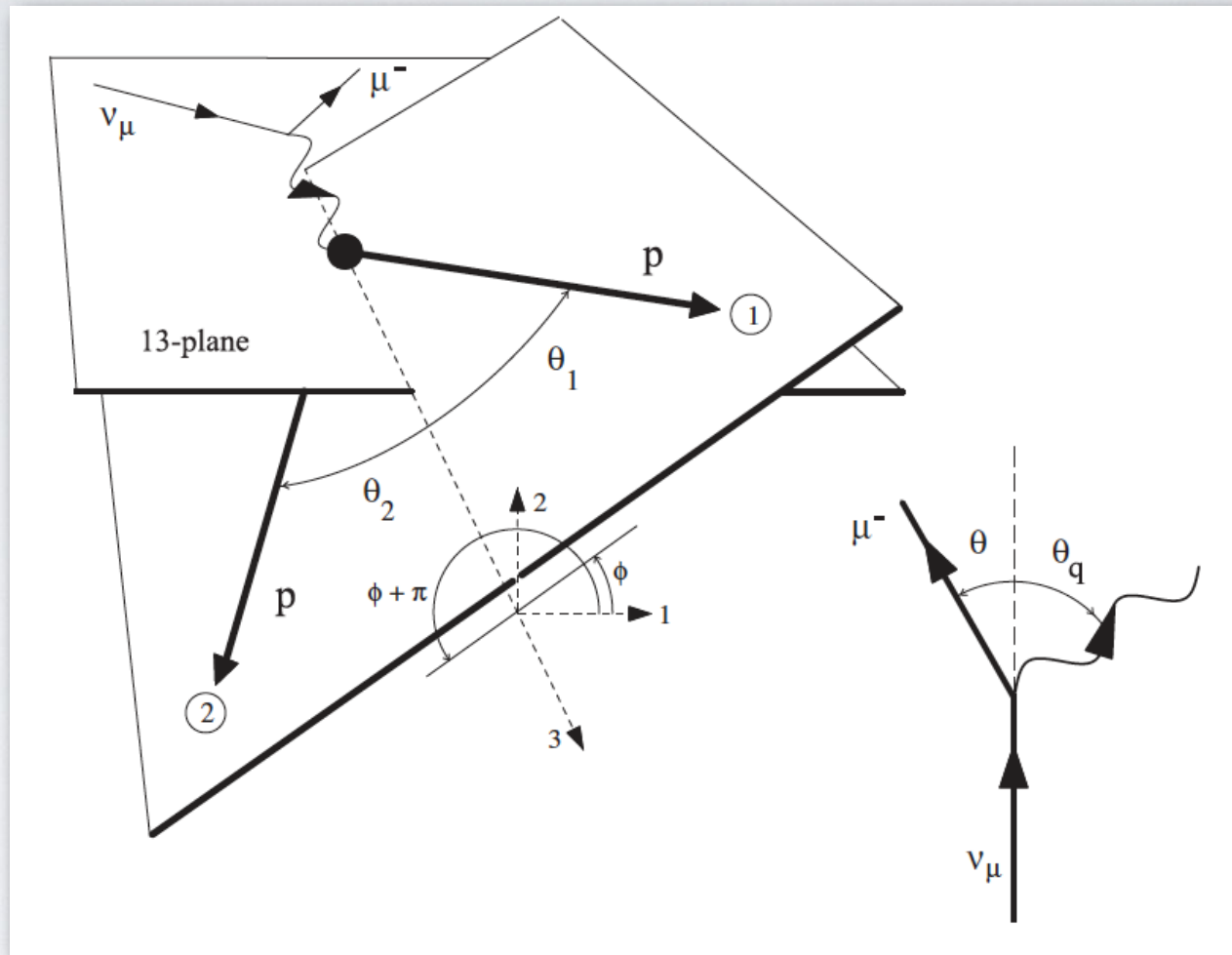
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EXTRA MATERIAL



Generalized Rosenbluth factors (1/2)

$$\hat{V}_{CC} = \frac{1}{2} \{ (a_V^2 + a_A^2) - [a_V^2(\delta - \delta')^2 + a_A^2(\delta + \delta')^2] \tan^2 \tilde{\theta}/2 \},$$

$$\hat{V}_{CL} = -\frac{1}{2} (a_V^2 + a_A^2) \left[\nu - \frac{1}{\rho'} (\delta^2 - \delta'^2) \tan^2 \tilde{\theta}/2 \right],$$

$$\hat{V}_{LL} = \frac{1}{2} \left\{ (a_V^2 + a_A^2) \left[\nu^2 - \frac{1}{\rho'} (2\nu - \rho\rho'(\delta^2 - \delta'^2)) (\delta^2 - \delta'^2) \tan^2 \tilde{\theta}/2 \right] + [a_V^2(\delta - \delta')^2 + a_A^2(\delta + \delta')^2] \tan^2 \tilde{\theta}/2 \right\},$$

$$\hat{V}_T = \frac{1}{2} (a_V^2 + a_A^2) \left\{ \left[\frac{1}{2} \rho + \tan^2 \tilde{\theta}/2 \right] + \left(\frac{\nu}{\rho'} (\delta^2 - \delta'^2) - \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right) \tan^2 \tilde{\theta}/2 \right\} - (a_V^2 - a_A^2) \delta \delta' \tan^2 \tilde{\theta}/2,$$

$$\hat{V}_{TT} = \frac{1}{2} (a_V^2 + a_A^2) \left\{ -\frac{1}{2} \rho + \left[(\delta^2 + \delta'^2) - \frac{\nu}{\rho'} (\delta^2 - \delta'^2) + \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right] \tan^2 \tilde{\theta}/2 \right\},$$

$$\hat{V}_{TC} = -\frac{1}{2} (a_V^2 + a_A^2) \frac{1}{\rho'} \tan \tilde{\theta}/2 \times \left(\frac{1}{2} - \frac{1}{\rho} \left[(\delta^2 + \delta'^2) - \frac{\nu}{\rho'} (\delta^2 - \delta'^2) + \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right] \tan^2 \tilde{\theta}/2 \right)^{1/2},$$

Generalized Rosenbluth factors (2/2)

$$v_0 \equiv (\varepsilon + \varepsilon')^2 - q^2.$$

$$\nu \equiv \frac{\omega}{q},$$

$$\rho \equiv \frac{|Q^2|}{q^2} = 1 - \nu^2; \quad \rho' \equiv \frac{q}{\varepsilon + \varepsilon'},$$

$$\delta \equiv \frac{m}{\sqrt{|Q^2|}}; \quad \delta' \equiv \frac{m'}{\sqrt{|Q^2|}},$$

$$\tan^2 \tilde{\theta}/2 = \frac{|Q^2|}{v_0} = \frac{\rho \rho'^2}{1 - \rho'^2}.$$

$$\hat{V}_{TL} = -(\nu - \rho \rho' (\delta^2 - \delta'^2)) \hat{V}_{TC},$$

$$\hat{V}_{\underline{TT}} = 0,$$

$$\hat{V}_{\underline{TC}} = 0,$$

$$\hat{V}_{\underline{TL}} = 0,$$

$$\hat{V}_{T'} = a_V a_A \frac{1}{\rho'} (1 + \nu \rho' (\delta^2 - \delta'^2)) \tan^2 \tilde{\theta}/2,$$

$$\hat{V}_{TC'} = -a_V a_A \tan \tilde{\theta}/2$$

$$\times \left[\frac{1}{2} - \frac{1}{\rho} \left[(\delta^2 + \delta'^2) - \frac{\nu}{\rho'} (\delta^2 - \delta'^2) + \frac{1}{2} \rho (\delta^2 - \delta'^2)^2 \right] \tan^2 \tilde{\theta}/2 \right]^{1/2},$$

$$\hat{V}_{TL'} = -\nu \hat{V}_{TC'},$$

$$\hat{V}_{\underline{CL}'} = 0,$$

$$\hat{V}_{\underline{TC}'} = 0,$$

$$\hat{V}_{\underline{TL}'} = 0.$$

Generalized Rosenbluth factors

ERL for neutral current

$$v_{CC} = 1,$$

$$v_{CL} = -\nu,$$

$$v_{LL} = \nu^2,$$

$$v_T = \frac{1}{2}\rho + \tan^2\theta/2,$$

$$v_{TT} = -\frac{1}{2}\rho,$$

$$v_{TC} = -\frac{1}{\sqrt{2}\rho'}\tan\theta/2,$$

$$v_{TL} = -\nu v_{TC},$$

$$v_{T'} = \tan\theta/2\sqrt{\rho + \tan^2\theta/2},$$

$$v_{TC'} = -\frac{1}{\sqrt{2}}\tan\theta/2,$$

$$v_{TL'} = -\nu v_{TC'}.$$

Generalized Rosenbluth factors

ERL for charged current

$$V_{CC} = 1 - \delta'^2 \tan^2 \tilde{\theta}/2,$$

$$V_{CL} = -\nu - \frac{1}{\rho'} \delta'^2 \tan^2 \tilde{\theta}/2,$$

$$V_{LL} = \nu^2 + \left(1 + \frac{2\nu}{\rho'} + \rho \delta'^2\right) \delta'^2 \tan^2 \tilde{\theta}/2,$$

$$V_T = \frac{1}{2} \rho + \left(1 - \frac{\nu}{\rho'} \delta'^2 - \frac{1}{2} \rho \delta'^4\right) \tan^2 \tilde{\theta}/2,$$

$$V_{TT} = -\frac{1}{2} \rho + \left[1 + \frac{\nu}{\rho'} + \frac{1}{2} \rho \delta'^2\right] \delta'^2 \tan^2 \tilde{\theta}/2,$$

$$V_{TC} = -\frac{1}{\rho'} \tan \tilde{\theta}/2 \sqrt{-\frac{1}{\rho} V_{TT}},$$

$$V_{TL} = -(\nu + \rho \rho' \delta'^2) V_{TC},$$

$$V_{T'} = \left(-\frac{1}{\rho'} + \nu \delta'^2\right) \tan^2 \tilde{\theta}/2,$$

$$V_{TC'} = \tan \tilde{\theta}/2 \left\{ \frac{1}{2} - \frac{1}{\rho} \left[1 + \frac{\nu}{\rho'} + \frac{1}{2} \rho \delta'^2\right] \delta'^2 \tan^2 \tilde{\theta}/2 \right\}^{1/2}$$

$$V_{TL'} = -\nu V_{TC'}.$$

A BRIEF DETOUR TO A DIFFERENT PROCESS: ELASTIC NEUTRINO SCATTERING

- From semi-inclusive to inclusive
- From (mainly) charged-current to neutral-current
- From quasi-elastic to elastic
- From detection of ejected particles to detection of target recoil

In elastic neutrino scattering the target nucleus remains in its ground state. In general, the main contribution is the **coherent scattering**, proportional to N^2 and valid when $q \lesssim 1/R \approx 160 A^{-1/3}$ MeV; it is the only contribution for even-even targets.

A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

- Relationship between coherent electron-nucleus and coherent neutrino-nucleus cross-sections in PWBA:

$$\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} = \left[\frac{(a_V^\nu)^2 + (a_A^\nu)^2}{2 (a_A^e)^2} \right] \mathcal{A}_{(e,e)}^2 \left(\frac{d\sigma}{d\Omega}\right)_{(e,e)}$$

where the parity-violating elastic electron scattering asymmetry is defined as

$$\mathcal{A}_{(e,e)} = \frac{\left(\frac{d\sigma}{d\Omega}\right)^{h=+1} - \left(\frac{d\sigma}{d\Omega}\right)^{h=-1}}{\left(\frac{d\sigma}{d\Omega}\right)^{h=+1} + \left(\frac{d\sigma}{d\Omega}\right)^{h=-1}}$$

A BRIEF DETOUR TO A DIFFERENT PROCESS:

ELASTIC NEUTRINO SCATTERING

$$\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} = \left[\frac{(a_V^\nu)^2 + (a_A^\nu)^2}{2(a_A^e)^2}\right] \mathcal{A}_{(e,e)}^2 \left(\frac{d\sigma}{d\Omega}\right)_{(e,e)}$$

- Deviations from this prediction:
 - Coulomb distortion.
 - Effect of higher order corrections.
 - Different coupling of Z^0 to neutrinos and charged leptons.
 - Other effects affecting differently neutrinos and charged leptons.

A BRIEF DETOUR TO A DIFFERENT PROCESS: ELASTIC NEUTRINO SCATTERING

- Relationship between relative uncertainties:

$$\mathcal{E}\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} \approx 2 \mathcal{E}_{\mathcal{A}(e,e)}$$

to which the PV experiment statistical contribution is:

$$\mathcal{E}^{stat.}\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} \approx 2 \chi_{PV}^{-\frac{1}{2}} \mathcal{F}_{PV}^{-\frac{1}{2}}$$

A BRIEF DETOUR TO A DIFFERENT PROCESS: ELASTIC NEUTRINO SCATTERING

- For measurements at different kinematic conditions:

$$\left(\frac{d\sigma(k_\nu, \theta_\nu)}{d\Omega} \right)_{(\nu, \nu)} = \mathcal{K}(k_\nu, k_e, \theta_e) \mathcal{A}_{(e, e)}^2(\hat{k}_e, \hat{\theta}_e) \left(\frac{d\sigma(k_e, \theta_e)}{d\Omega} \right)_{(e, e)}$$

$$\mathcal{K} = \frac{k_e^2 (k_\nu - \omega_e)^2 [2 k_\nu^2 - \omega_e (2 k_\nu + M_A)]}{k_\nu^2 (k_e - \omega_e)^2 [2 k_e^2 - \omega_e (2 k_e + M_A)]}$$

A BRIEF DETOUR TO A
DIFFERENT PROCESS:
ELASTIC NEUTRINO
SCATTERING

