Fully relativistic treatment of pionic correlations and meson-exchange currents

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Contents

Quasielastic neutrino scattering

General formalism for (ν_μ, μ^-) reaction.

Energies of the incident neutrino and detected muon: $\epsilon = E_\nu$, $\epsilon' = m_\mu + T_\mu$, and their momenta are $\mathbf{k}, \mathbf{k}'$.
The four-momentum transfer is $k^\mu - k'^\mu = (\omega, \mathbf{q})$, with $Q^2 = q^2 - \omega^2 > 0$.
The lepton scattering angle is θ,

Double-differential cross section

$$\frac{d^2\sigma}{dT_\mu d\cos\theta}(E_\nu) = \left(\frac{M_W^2}{M_W^2 + Q^2}\right)^2 \frac{G^2\cos^2\theta_c}{4\pi}\frac{k'}{\epsilon} v_0 S_\pm \qquad (1)$$

$G = 1.166 \times 10^{-11} \quad \mathrm{MeV}^{-2} \sim 10^{-5}/\mathrm{m_p^2}$ is the Fermi constant,
θ_c is the Cabibbo angle, $\cos\theta_c = 0.975$,
kinematical factor $v_0 = (\epsilon + \epsilon')^2 - q^2$.

Nuclear structure function

A linear combination of the five nuclear response functions for neutrinos (+) and for antineutrinos (-)

$$S_{\pm} = V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T \pm 2V_{T'}R_{T'} \ , \quad (2)$$

V_K do not depend on the details of the nuclear target.

$$V_{CC} = 1 - \delta^2 \frac{Q^2}{v_0} \quad (3)$$

$$V_{CL} = \frac{\omega}{q} + \frac{\delta^2}{\rho'} \frac{Q^2}{v_0} \quad (4)$$

$$V_{LL} = \frac{\omega^2}{q^2} + \left(1 + \frac{2\omega}{q\rho'} + \rho\delta^2\right) \delta^2 \frac{Q^2}{v_0} \quad (5)$$

$$V_T = \frac{Q^2}{v_0} + \frac{\rho}{2} - \frac{\delta^2}{\rho'} \left(\frac{\omega}{q} + \frac{1}{2}\rho\rho'\delta^2\right) \frac{Q^2}{v_0} \quad (6)$$

$$V_{T'} = \frac{1}{\rho'} \left(1 - \frac{\omega\rho'}{q}\delta^2\right) \frac{Q^2}{v_0}. \quad (7)$$

$\delta = m'/\sqrt{Q^2}$, muon mass m', $\rho = Q^2/q^2$, and $\rho' = q/(\epsilon + \epsilon')$.

Response functions

We evaluate the five nuclear response functions R_K, $K = CC, CL, LL, T, T'$ (C=Coulomb, L=longitudinal, T=transverse).

$$
\begin{aligned}
R_{CC} &= W^{00} && (8)\\
R_{CL} &= -\frac{1}{2}\left(W^{03} + W^{30}\right) && (9)\\
R_{LL} &= W^{33} && (10)\\
R_T &= W^{11} + W^{22} && (11)\\
R_{T'} &= -\frac{i}{2}\left(W^{12} - W^{21}\right). && (12)
\end{aligned}
$$

Hadronic tensor $W^{\mu\nu}$

$$
W^{\mu\nu} = \sum_f \overline{\sum_i} \langle f|J^\mu(q,\omega)|i\rangle^* \langle f|J^\nu(q,\omega)|i\rangle
$$

Weak CC current operator $J^\mu(q,\omega)$.

Relativistic Fermi gas (RFG)

- Simplest approach that treats exactly relativity, gauge invariance and translational invariance.
- The nucleons are described by plane wave spinors
- Parameter: Fermi momentum k_F. (also separation energy E_s)
- Final states: excitations of the np-nh kind.
- The hadronic tensor is expanded as sum of one-particle one-hole (1p-1h) , two-particle two-hole (2p-2h), ...channels

$$W^{\mu\nu} = W^{\mu\nu}_{1p1h} + W^{\mu\nu}_{2p2h} + \cdots \tag{13}$$

1p-1h channel

The impulse approximation gives the well-known response functions of the RFG, proportional to a single-nucleon response function U_K times the scaling function $f(\psi)$

$$R_K = \frac{N\xi_F}{m_N \eta_F^3 \kappa} U_K f(\psi) \tag{14}$$

N is the neutron number, $\eta_F = k_F/m_N$, and $\xi_F = \sqrt{1+\eta_F^2} - 1$.

Scaling function of the RFG

$$f(\psi) = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2) \tag{15}$$

ψ is the scaling variable

$$\psi^2 = \frac{1}{\xi_F} \max\left\{ \kappa\sqrt{1+\frac{1}{\tau}} - \lambda - 1, \xi_F - 2\lambda \right\} \tag{16}$$

where $\lambda = \omega/(2m_N)$, $\kappa = q/(2m_N)$, and $\tau = \kappa^2 - \lambda^2$.

2p-2h channel

Final states: two particles $\mathbf{p}'_1$ and $\mathbf{p}'_2$ above the Fermi momentum, $p'_i > k_F$, and two holes $\mathbf{h}_1$ and $\mathbf{h}_2$ below the Fermi momentum, $h_i < k_F$.
Spin (isospin) indices: s'_i (t'_i) and s_i (t_i).

2p-2h hadronic tensor in the RFG model

$$\begin{aligned} W^{\mu\nu}_{2p-2h} &= \frac{V}{(2\pi)^9} \int d^3p'_1 d^3h_1 d^3h_2 \frac{M^4}{E_1 E_2 E'_1 E'_2} \Theta(p'_1, p'_2, h_1, h_2) \\ & \quad r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2) \delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \end{aligned} \quad (17)$$

where by momentum conservation, $\mathbf{p'_2} = \mathbf{h_1} + \mathbf{h_2} + \mathbf{q} - \mathbf{p'_1}$.
E_i and E'_i are the on-shell energies of the holes and particles,
The volume of the system is $V = 3\pi^2 Z/k_F^3$, for symmetric matter, $Z = N = A/2$.
Pauli blocking step functions:

$$\Theta(p'_1, p'_2, h_1, h_2) = \theta(p'_2 - k_F)\theta(p'_1 - k_F)\theta(k_F - h_1)\theta(k_F - h_2).$$

Elementary 2p-2h hadronic tensor

The non trivial part of the calculation is the function

$$r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2) = \frac{1}{4} \sum_{s_1 s_2 s'_1 s'_2} \sum_{t_1 t_2 t'_1 t'_2} j^\mu(1', 2', 1, 2)^*_A j^\nu(1', 2', 1, 2)_A.$$

- Two-body MEC antisymmetrized matrix element $j^\mu(1', 2', 1, 2)_A$
- The factor $1/4$ accounts for the antisymmetry of the 2p-2h wave function to avoid double counting.
- The sum over isospin combines all the possible charge channels in the final state, corresponding to emission of PP, NN, and PN pairs.
- In our formalism we separate the contributions of these charge states.
- This will allow us to apply the formalism to asymmetric nuclei $N \neq Z$.
- This will be of interest for neutrino experiments based on ^{40}Ar, ^{56}Fe or ^{208}Pb detectors.

Calculation of the 2p-2h responses

Calculation of the 9D integral:

- we use the rotational symmetry of the response functions around the **q** direction.
- This allows us to integrate over one of the azimuthal angles.
- We choose $\phi'_1 = 0$ and multiply the responses by a factor 2π.
- The energy delta function enables analytical integration over p'_1.
- The 2p-2h integral is then reduced to 7 dimensions.

The numerical integration method is described in: I. Ruiz Simo, C. Albertus, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T. W. Donnelly,

- PRD 90, 033012 (2014) (phase space in Lab system)
- PRD 90, 053010 (2014) (phase space in CM system)

2p-2h Phase space function

$$F(q,\omega) \equiv \int d^3p'_1 d^3h_1 d^3h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \delta(E'_1 + E'_2 - E_1 - E_2 - \omega)\Theta(p'_1, p'_2, h_1, h_2)\,. \quad (18)$$

with $\mathbf{p'_2} = \mathbf{h_1} + \mathbf{h_2} + \mathbf{q} - \mathbf{p'_1}$.

- Proportional to the hadronic tensor expected for $r^{\mu\nu} = 1$ (constant current matrix elements).
- The results will be modified here when including the effects of the two-body MEC current.
- All of the models of 2p-2h response functions should agree at the level of the 2p-2h phase-space integral $F(q,\omega)$
- Calculation of this function should be a good starting point to compare different nuclear models.

Calculation of the phase space function

Integration over p'_1 gives

$$F(q,\omega) = 2\pi \int d^3h_1 d^3h_2 d\theta'_1 \sin\theta'_1 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \tag{19}$$
$$\times \sum_{\alpha=\pm} \frac{{p'_1}^2}{\left|\frac{p'_1}{E'_1} - \frac{\mathbf{p}'_2 \cdot \hat{\mathbf{p}}'_1}{E'_2}\right|} \Theta(p'_1, p'_2, h_1, h_2)\Bigg|_{p'_1 = {p'_1}^{(\alpha)}},$$

the sum runs over the two solutions ${p'_1}^{(\pm)}$ of energy conservation
The integrand can be infinite for some angles but the divergence is integrable of the kind

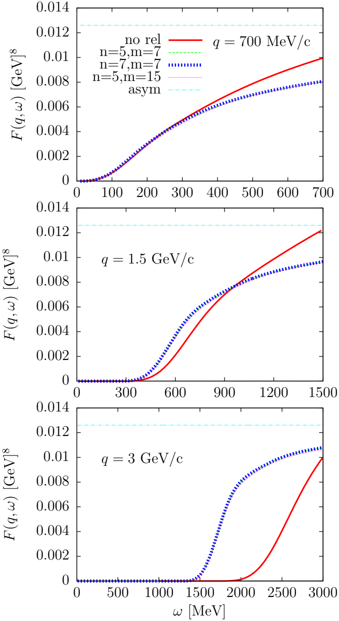
$$\int_0^\epsilon \frac{dx}{\sqrt{x}} = 2\sqrt{\epsilon}$$

In Ref. PRD 90, 033012 (2014) we made a systematic analisis of the singular points. We integrate numerically within each one of the allowed intervals, up to a distance ϵ to the singular point. The integral around the singular point is made using the semi-analytical method of Ruiz Simo et al., PRD 90, 033012

Phase space

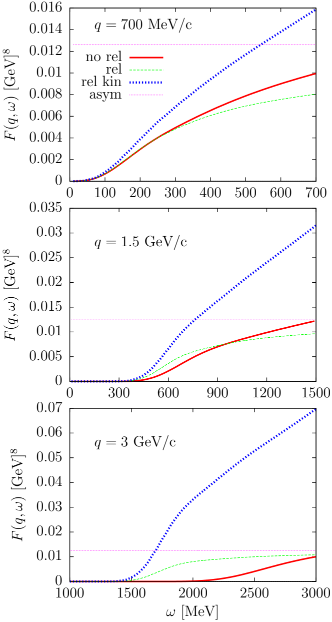
Total phase-space function for three values of the momentum transfer.
The number of integration points in each dimension in the hole variables is indicated by n.
The number of integration points over the emission angle θ'_1 is indicated as m.
We also show the non-relativistic, exact result and the relativistic asymptotic value.

$$F(q,\omega) \stackrel{\omega\to\infty}{\longrightarrow} 4\pi \left(\frac{4}{3}\pi k_F^3\right)^2 \frac{m_N^2}{2}.$$



Phase space

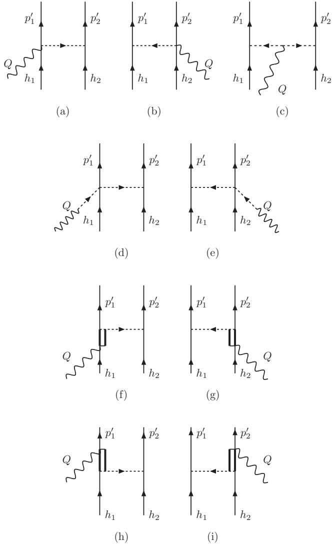
Effect of implementing relativistic kinematics in a non-relativistic calculation of $F(q, \omega)$.
Solid lines: non-relativistic result.
Thick dotted lines: relativistic kinematics only without the relativistic factors m_N/E.
Thin dashed lines: fully relativistic result.



Meson exchange currents

- We start from the weak pion production model of E. Hernandez, J.M. Nieves and M. Valverde, PRD 76 (2007) 033005,
- We take the pion-production amplitudes from the nucleon and we couple a second nucleon line to the emitted pion.
- The MEC operator is written as the sum of four contributions, seagull (a,b), pion-in-flight (c), pion-pole (d,e), and Δ pole (f–i).

$$j^{\mu}_{\rm MEC} = j^{\mu}_{\rm sea} + j^{\mu}_{\pi} + j^{\mu}_{\rm pole} + j^{\mu}_{\Delta}$$



Seagull current

The MEC operators can be decomposed as a sum of vector (V) and axial-vector (A) currents.
The vector operators contribute also to electron scattering and are constrained by electromagnetic probes,
The axial operators only appear in weak processes like neutrino scattering.

$$j^{\mu}_{\rm sea} = (I_V)_{\pm} \, J^{\mu}_{\rm sea},$$

$(I_V)_{\pm} = (I_V)_x \pm i(I_V)_y$ is the $\pm$-component of the operator

$$\boldsymbol{I}_V = i\,[\boldsymbol{\tau}(1) \times \boldsymbol{\tau}(2)] \tag{20}$$

$J^{\mu}_{\rm sea}$ is the isospin-independent seagull current,

$$J^{\mu}_{\rm sea} = (J^{\mu}_{\rm sea})_V + (J^{\mu}_{\rm sea})_A\,. \tag{21}$$

Seagull current

$$
\begin{aligned}
(J^{\mu}_{\text{sea}})_V &= \frac{f^2_{\pi NN}}{m^2_\pi} F^V_1(Q^2) V^{(s'_1,s_1)}_{\pi NN}(\mathbf{p}'_1,\mathbf{h}_1)\bar{u}_{s'_2}(\mathbf{p}'_2)\gamma_5\gamma^\mu u_{s_2}(\mathbf{h}_2) - (1 \leftrightarrow 2) \\
(J^{\mu}_{sea})_A &= \frac{f^2_{\pi NN}}{m^2_\pi} \frac{1}{g_A} V^{(s'_1,s_1)}_{\pi NN}(\mathbf{p}'_1,\mathbf{h}_1) F_\rho\left(k^2_{22}\right) \bar{u}_{s'_2}(\mathbf{p}'_2)\,\gamma^\mu\, u_{s_2}(\mathbf{h}_2) - (1 \leftrightarrow 2)
\end{aligned}
$$

With

$$
k_{ij} = p'_i - h_j, \quad i,j = 1,2.
$$

and

$$
V^{(s'_i,s_j)}_{\pi NN}(\mathbf{p}'_i,\mathbf{h}_j) = \frac{\bar{u}_{s'_i}(\mathbf{p}'_i)\,\gamma_5\ \not{k}_{ij}\, u_{s_j}(\mathbf{h}_j)}{k^2_{ij} - m^2_\pi}
$$

accounts for the propagation and absorption of the exchanged pion

- The electromagnetic current operator can be obtained by keeping only the V current and taking the z component of the isospin operator

$$
(I_V)_\pm \to (I_V)_z = i\,[\boldsymbol{\tau}(1) \times \boldsymbol{\tau}(2)]_z \tag{22}
$$

Pion-in-flight current

The weak pion-in-flight current, has vanishing axial part:

$$
\begin{aligned}
j^\mu_\pi &= (I_V)_\pm \, J^\mu_\pi & (23)\\
J^\mu_\pi &= (J^\mu_\pi)_V & (24)\\
(J^\mu_\pi)_V &= \frac{f^2_{\pi NN}}{m^2_\pi} F^V_1(Q^2) V^{(s'_1,s_1)}_{\pi NN}(\mathbf{p}'_1, \mathbf{h}_1) & \\
&\times \; V^{(s'_2,s_2)}_{\pi NN}(\mathbf{p}'_2, \mathbf{h}_2)\,(k^\mu_{11} - k^\mu_{22}) & (25)\\
(J^\mu_\pi)_A &= 0. & (26)
\end{aligned}
$$

Agrees with the pion-in-flight electromagnetic MEC taking the z component of the isospin operator

Pion pole current

Has only the axial component and therefore it is absent in the electromagnetic case.

$$
\begin{aligned}
j^{\mu}_{\rm pole} &= (I_V)_{\pm}\, J^{\mu}_{\rm pole} && (27)\\
J^{\mu}_{\rm pole} &= \left(J^{\mu}_{\rm pole}\right)_A && (28)\\
\left(J^{\mu}_{\rm pole}\right)_V &= 0 && (29)\\
\left(J^{\mu}_{\rm pole}\right)_A &= \frac{f^2_{\pi NN}}{m^2_\pi}\frac{1}{g_A} F_\rho\left(k^2_{11}\right) V^{(s'_2,s_2)}_{\pi NN}(\mathbf{p}'_2,\mathbf{h}_2) \\
&\times\quad Q^\mu\, \frac{\bar{u}_{s'_1}(\mathbf{p}'_1)\, \not{Q}\, u_{s_1}(\mathbf{h}_1)}{Q^2-m^2_\pi} - (1 \leftrightarrow 2) && (30)
\end{aligned}
$$

Is proportional to Q^μ, and only contributes to the longitudinal and time components of the hadronic tensor.

$\Delta(1232)$ current

The Δ-pole terms includes forward and backward Δ propagations. We start from the Δ-pole and the crossed-Δ-pole pion-production amplitudes of Hernandez et al.

$$j^\mu_\Delta = j^\mu_{\Delta,\mathrm{forw}} + j^\mu_{\Delta,\mathrm{back}} \tag{31}$$

$$\begin{aligned} j^\mu_{\Delta,\mathrm{forw}} &= -\frac{f^* f_{\pi NN}}{m_\pi^2}\sqrt{3}\left(U^{\mathrm{forw}}\right)_{t_1' t_2'; t_1 t_2} V^{(s_2',s_2)}_{\pi NN}(\mathbf{p}_2', \mathbf{h}_2) \\ &\times k^\alpha_{22}\, \bar{u}_{s_1'}(\mathbf{p}_1') G_{\alpha\beta}(h_1 + Q)\Gamma^{\beta\mu}(h_1, Q) u_{s_1}(\mathbf{h}_1) \\ &+ (1 \leftrightarrow 2) \end{aligned} \tag{32}$$

$$\begin{aligned} j^\mu_{\Delta,\mathrm{back}} &= -\frac{f^* f_{\pi NN}}{m_\pi^2}\sqrt{3}\left(U^{\mathrm{back}}\right)_{t_1' t_2'; t_1 t_2} V^{(s_2',s_2)}_{\pi NN}(\mathbf{p}_2', \mathbf{h}_2) \\ &\times k^\beta_{22}\, \bar{u}_{s_1'}(\mathbf{p}_1') \hat{\Gamma}^{\mu\alpha}(p_1', Q) G_{\alpha\beta}(p_1' - Q) u_{s_1}(\mathbf{h}_1) \\ &+ (1 \leftrightarrow 2). \end{aligned} \tag{33}$$

$f^* = 2.13$ is the $\pi N \Delta$ coupling constant.

Δ propagator

The Δ-propagator $G_{\alpha\beta}(P)$ is the Rarita-Schwinger propagator of a spin 3/2 particle

$$G_{\alpha\beta}(P) = \frac{P_{\alpha\beta}(P)}{P^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta(P^2)}, \tag{34}$$

where $P_{\alpha\beta}$ is the projector over spin-$\frac{3}{2}$,

$$P_{\alpha\beta}(P) = -(\not{P} + M_\Delta)\left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{P_\alpha P_\beta}{M_\Delta^2} + \frac{1}{3}\frac{P_\alpha\gamma_\beta - P_\beta\gamma_\alpha}{M_\Delta}\right]$$

M_Δ and Γ_Δ stand for the $\Delta(1232)$ mass and width.

$N \to \Delta$ transition vertex

Forward term $P_\Delta = P + Q$.

$$\Gamma^{\beta\mu}(P,Q) = \Gamma_V^{\beta\mu}(P,Q) + \Gamma_A^{\beta\mu}(P,Q) \tag{35}$$

$$\Gamma_V^{\beta\mu}(P,Q) = \left[\frac{C_3^V}{M} \left(g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu \right) + \frac{C_4^V}{M^2} \left(g^{\beta\mu} Q \cdot P_\Delta - Q^\beta P_\Delta^\mu \right) + \frac{C_5^V}{M^2} \left(g^{\beta\mu} Q \cdot P - Q^\beta P^\mu \right) + C_6^V g^{\beta\mu} \right] \gamma_5$$

$$\Gamma_A^{\beta\mu}(P,Q) = \frac{C_3^A}{M} \left(g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu \right) + \frac{C_4^A}{M^2} \left(g^{\beta\mu} Q \cdot P_\Delta - Q^\beta P_\Delta^\mu \right) + C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2} Q^\beta Q^\mu, \tag{36}$$

$C_i^{V,A}(Q^2)$ are the vector and axial-vector form factors.
In the backward term we use the $\Delta \to N$ transition vertex

$$\hat{\Gamma}^{\mu\alpha}(P',Q) = \gamma^0 \left[\Gamma^{\alpha\mu}(P',-Q) \right]^\dagger \gamma^0. \tag{37}$$

Isospin transition operators

The quantities $(U^{\rm forw})_{t'_1 t'_2;t_1 t_2}$ and $(U^{\rm back})_{t'_1 t'_2;t_1 t_2}$ are the matrix elements of the following forward and backward isospin operators

$$U^{\rm forw} = \left(T_i \left(T^\dagger\right)_{+1}\right) \otimes \tau_i \tag{38}$$

$$U^{\rm back} = \left(T_{+1}\, T_i^\dagger\right) \otimes \tau_i , \tag{39}$$

T_{+1} is the spherical component of the isovector transition operator $\frac{3}{2} \to \frac{1}{2}$, normalized as

$$\left\langle \frac{3}{2}, t_\Delta \middle| (T^\dagger)_\lambda \middle| \frac{1}{2}, t_N \right\rangle = C\left(\frac{1}{2}, 1, \frac{3}{2} \middle| t_N, \lambda, t_\Delta\right) \tag{40}$$

for $\lambda = \pm 1, 0$.

Using (in cartesian coordinates)

$$T_i\, T_j^\dagger = \frac{2}{3}\,\delta_{ij} - \frac{i}{3}\,\epsilon_{ijk}\,\tau_k . \tag{41}$$

We expand the $U^{\rm forw,back}$ operators as linear combination of the isospin matrices $\boldsymbol{\tau}(1), \boldsymbol{\tau}(2)$, and $\boldsymbol{I}_V$.

Isospin dependence of Δ current

$$\sqrt{3}\, U^{\rm forw} = \frac{1}{\sqrt{6}} \left[-2\, \tau_{+}(2) + (I_V)_{+} \right] \tag{42}$$

$$\sqrt{3}\, U^{\rm back} = \frac{1}{\sqrt{6}} \left[-2\, \tau_{+}(2) - (I_V)_{+} \right] \tag{43}$$

In the $(1 \leftrightarrow 2)$ terms the isospin operators are

$$\sqrt{3}\, U^{\rm forw} \quad \xrightarrow{(1\leftrightarrow 2)} \quad \frac{1}{\sqrt{6}} \left[-2\, \tau_{+}(1) - (I_V)_{+} \right] \tag{44}$$

$$\sqrt{3}\, U^{\rm back} \quad \xrightarrow{(1\leftrightarrow 2)} \quad \frac{1}{\sqrt{6}} \left[-2\, \tau_{+}(1) + (I_V)_{+} \right], \tag{45}$$

The Δ-current is the sum of three currents with specific isospin dependence

$$\begin{aligned} j^{\mu}_{\Delta} = {} & \tau_{+}(1)\; J^{\mu}_{\Delta 1}(1',2';\,1,2) + \tau_{+}(2)\; J^{\mu}_{\Delta 2}(1',2';\,1,2) \\ & + (I_V)_{+}\;\; J^{\mu}_{\Delta 3}(1',2';\,1,2) \end{aligned} \tag{46}$$

$J^{\mu}_{\Delta i}(1',2';\,1,2)$ depend only on spin and momenta.

Isospin dependence of the MEC

The total CC MEC for neutrino scattering can be written as

$$
\begin{aligned}
j^{\mu}_{\rm MEC} &= \tau_{+}(1)\, J^{\mu}_{1}(1'\,2';1\,2) + \tau_{+}(2)\, J^{\mu}_{2}(1'\,2';1\,2) \\
&+ (I_V)_{+}\; J^{\mu}_{3}(1'\,2';1\,2),
\end{aligned}
\tag{47}
$$

where

$$J^{\mu}_{1} = J^{\mu}_{\Delta 1} \tag{48}$$

$$J^{\mu}_{2} = J^{\mu}_{\Delta 2} \tag{49}$$

$$J^{\mu}_{3} = J^{\mu}_{\rm sea} + J^{\mu}_{\pi} + J^{\mu}_{\rm pole} + J^{\mu}_{\Delta 3}. \tag{50}$$

- This expression can be applied to antineutrinos by taking the $(-)$ component of the isospin operators.
- For electron scattering, one should take the z component of the isospin operators and keep only the V part of the current.
- The resulting electromagnetic MEC is in agreement with previous expressions
- Expression (47) will be useful to obtain the response functions for the separate charge channels PP, PN, NN

(ν_l, l^-) response functions

CC neutrino scattering can induce two possible 2p-2h transitions: $NP \to PP$ and $NN \to NP$.

Diagonal components of the hadronic tensor in the PP channel:

$$W^{\mu\mu}_{PP} = \frac{1}{2} \int\!\!\!\!\!\!\sum \left| \langle PP | j^{\mu}_{MEC}(1'2';12) - j^{\mu}_{MEC}(2'1';12) | NP \rangle \right|^2$$

where the $\int\!\!\!\!\!\sum$ symbol means integration over momenta and sum over nucleon spins

$$\int\!\!\!\!\!\!\sum f(1'2';12) \equiv \frac{V}{(2\pi)^9} \int d^3p'_1 d^3h_1 d^3h_2 \frac{M^4}{E_1 E_2 E'_1 E'_2} \sum_{s_1 s_2 s'_1 s'_2} f(1'2';12)$$
$$\Theta(p'_1, p'_2, h_1, h_2)\delta(E'_1 + E'_2 - E_1 - E_2 - \omega),$$

$f(1'2';12)$ is any function depending on the momenta and spins of the final 2p-2h states.

We have exchanged the momenta and spins of the final protons (fermions).

Using the isospin expansion of the MEC, taking the isospin matrix elements and changing variables $1' \leftrightarrow 2'$ in the final state

$$W^{\mu\mu}_{PP} = 4 \sum\!\!\!\!\!\!\int \left\{ \left| J^{\mu}_{PP}(1'2';12) \right|^2 - \mathrm{Re}\, J^{\mu}_{PP}(1'2';12)^* J^{\mu}_{PP}(2'1';12) \right\}.$$

"Effective current" for PP-emission with neutrinos

$$J^{\mu}_{PP} = J^{\mu}_{1} + J^{\mu}_{3}. \tag{51}$$

The first term is the "direct" contribution, and the second one is the exchange contribution —interference between the direct and exchange matrix elements.
The exchange contribution is usually neglected in the existing models of neutrino scattering.
For NP emission the exchanged particles are the two initial neutrons

$$W^{\mu\mu}_{NP} = 4 \sum\!\!\!\!\!\!\int \left\{ \left| J^{\mu}_{NP}(1'2';12) \right|^2 - \mathrm{Re}\, J^{\mu}_{NP}(1'2';12)^* J^{\mu}_{NP}(1'2';21) \right\},$$

Effective current for NP emission with neutrinos

$$J^{\mu}_{NP} = J^{\mu}_{2} + J^{\mu}_{3}. \tag{52}$$

Effective currents for 2p-2h charge channels

final state	ν	$\overline{\nu}$	e
PP	$J_1 + J_3$	$\times$	$J_1 + J_2$
NP	$J_2 + J_3$	$J_1 - J_3$	$-J_1 + J_2$ $2J_3$
NN	$\times$	$J_2 - J_3$	$-J_1 - J_2$

Non diagonal hadronic tensor components

$$W^{\mu\nu}_{PP} = 4 \sum\!\!\!\!\!\!\int \Big\{ J^{\mu}_{PP}(1'2';12)^* J^{\nu}_{PP}(1'2';12) - J^{\mu}_{PP}(1'2';12)^* J^{\nu}_{PP}(2'1';12)$$

$$W^{\mu\nu}_{NP} = 4 \sum\!\!\!\!\!\!\int \Big\{ J^{\mu}_{NP}(1'2';12)^* J^{\nu}_{NP}(1'2';12) - J^{\mu}_{NP}(1'2';12)^* J^{\nu}_{NP}(1'2';21$$

In the case of antineutrinos the allowed charge 2p-2h channels are $NP \rightarrow NN$ and $PP \rightarrow NP$. The formulae are similar with effective currents given in the table

(e, e') responses

- The three charge channels are opened.
- We take the matrix elements of the z-component of the MEC in isospin space.
- For NP emission with electrons two effective currents appear

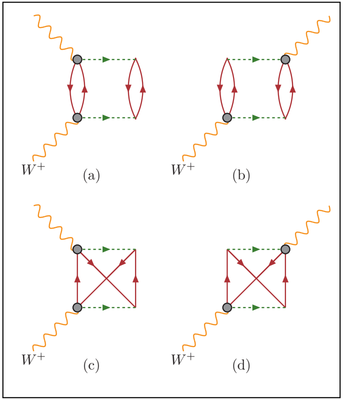
$$
\begin{aligned}
W^{\mu\mu}_{PP} &= \frac{1}{2}\sum\!\!\!\!\!\!\!\!\int \left\{ \left| J^{\mu}_{PP}(1'2';12) \right|^2 - \mathrm{Re}\, J^{\mu}_{PP}(1'2';12)^* J^{\mu}_{PP}(1'2';21) \right\} \\
W^{\mu\mu}_{NN} &= W^{\mu\mu}_{PP} \\
W^{\mu\mu}_{NP} &= \sum\!\!\!\!\!\!\!\!\int \left\{ \left| J^{\mu}_{NP1}(1'2';12) \right|^2 + \left| J^{\mu}_{NP2}(1'2';12) \right|^2 \right. \\
&\quad \left. + 2\mathrm{Re}\, J^{\mu}_{NP1}(1'2';12)^* J^{\mu}_{NP2}(2'1';12) \right\}
\end{aligned}
$$

The two effective currents for NP are

$$
\begin{aligned}
J^{\mu}_{NP1} &= -J^{\mu}_{1} + J^{\mu}_{2} \\
J^{\mu}_{NP2} &= 2J^{\mu}_{3}.
\end{aligned}
$$

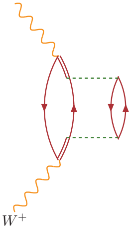
2p-2h polarization propagator

- Some contributions of 2p-2h to the virtual W^+ self-energy, or polarization propagator $\Pi^{\mu\nu}$.
- The response functions are related to the imaginary part of the polarization propagator, $\mathrm{Im}\,\Pi^{\mu\nu}$.
- The circle stands for the elementary model for $W^+N \to \pi N$ of Hernandez without the nucleon-pole diagrams.
- Diagrams (a,b) represent the direct contribution. Diagrams (c,d) are the exchange contributions.



Delta excitation

- Ambiguity in the separation between 2p-2h and Δ-peak contributions.
- Inside the nucleus the Δ can decay into two-nucleon emission without pions. $\longrightarrow$ 2p-2h channel ?
- This MEC diagram contains one self-energy insertion contributing to dressing the Δ propagator.
- Our prescription consists in considering only the real part of the Δ



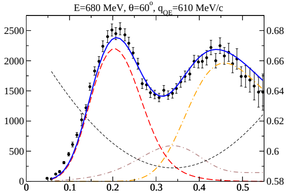
$$\frac{1}{P^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta(P^2)} \rightarrow$$

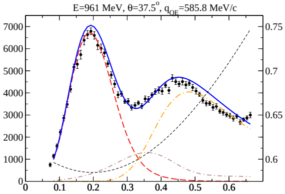
$$\mathrm{Re}\,\frac{1}{P^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta(P^2)}$$

$$= \frac{P^2 - M_\Delta^2}{(P^2 - M_\Delta^2)^2 + (M_\Delta\Gamma_\Delta(P^2))^2}.$$

Validation

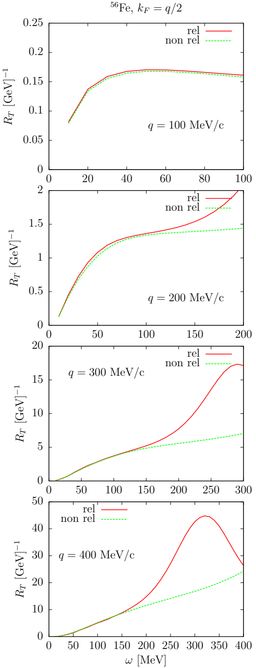
- The validation of the MEC contribution requires to compute the total (e, e') cross section including also both the quasi-elastic and inelastic
- Is possible to reproduce globally the experimental world-data for ^{12}C in the super-scaling approach using the MEC of De Pace et al. ?
- YES
- G.D. Megias, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly arXiv:1603.08396





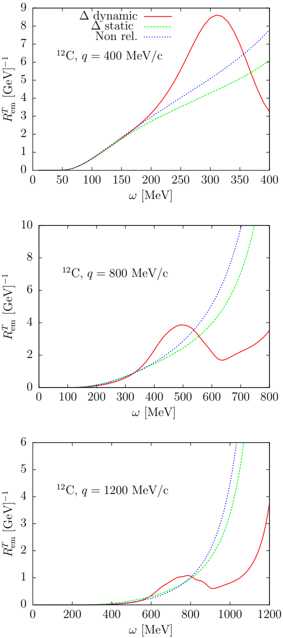
Non relativistic limit

Electromagnetic transverse response function for 2p-2h for low momentum q and $k_F = q/2$. We take $A = 56$.



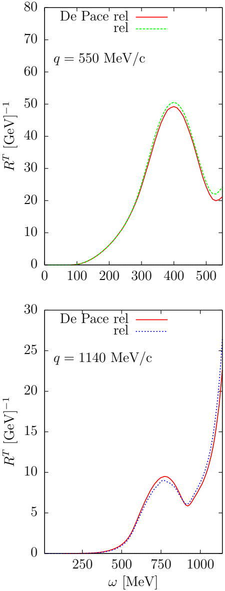
Relativistic effects

Electromagnetic 2p-2h transverse response function of ^{12}C from low to high momentum q
$k_F = 228$ MeV/c.
We show the total relativistic result and non relativistic, compared to the relativistic result with a constant Δ propagator.



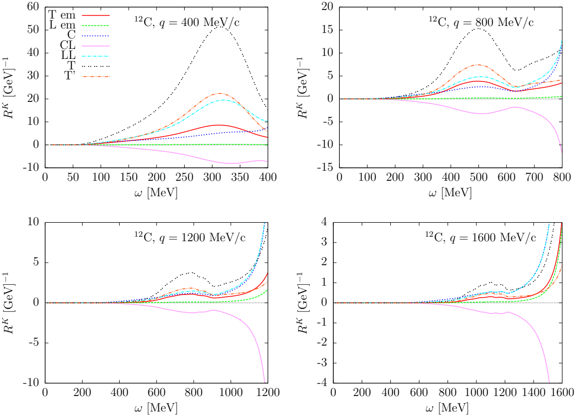
Comparison of models

Electromagnetic transverse response function for 2p-2h from ^{56}Fe for two values of q. Comparison with the model of ref. De Pace et al.



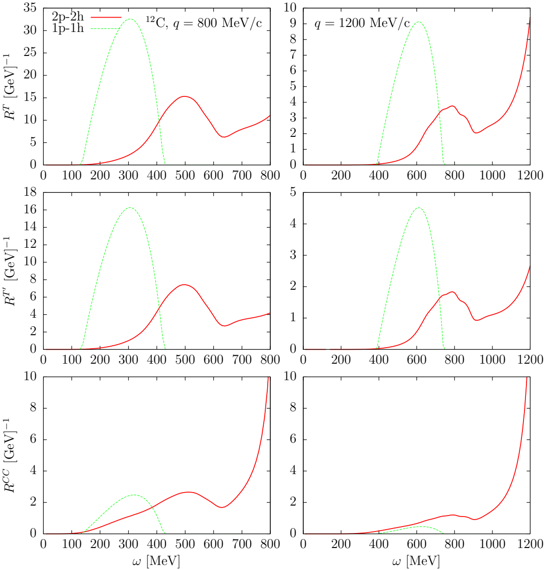
2p-2h response functions

Separate 2p-2h response functions of ^{12}C for four values of the momentum transfer. We show the L, T electromagnetic responses and the five weak responses for CC neutrino scattering.



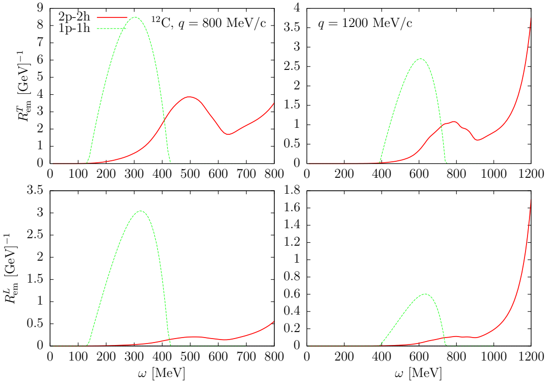
(ν_μ, μ)

Comparison between 1p-1h and 2p-2h response functions for CC neutrino scattering off ^{12}C for two values of the momentum transfer.



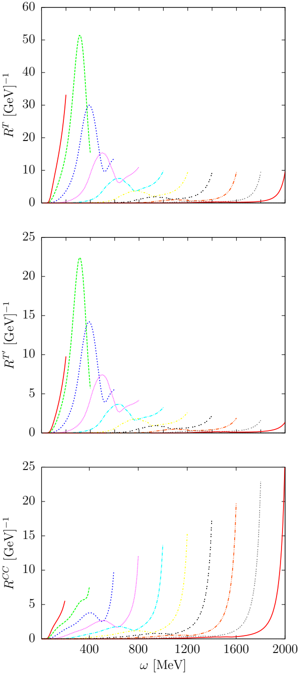
(e, e') Response functions

Comparison between 1p-1h and 2p-2h response functions for electron scattering off ^{12}C for two values of the momentum transfer.



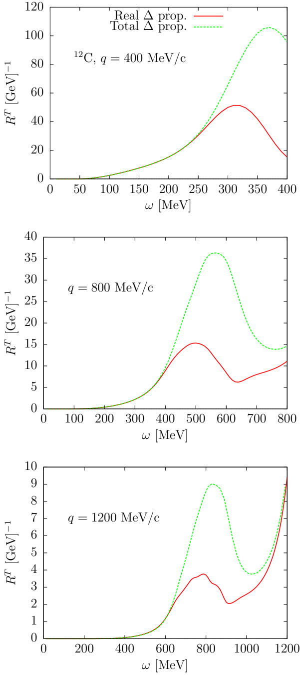
q-evolution of response functions

Evolution of the weak 2p-2h response functions from low to high values of q. Only the T, T' and CC responses are shown from left to right, for $q = 200$, 400, 600, 800, 1000, 1200, 1400, 1600, 1800 and 2000 MeV/c.



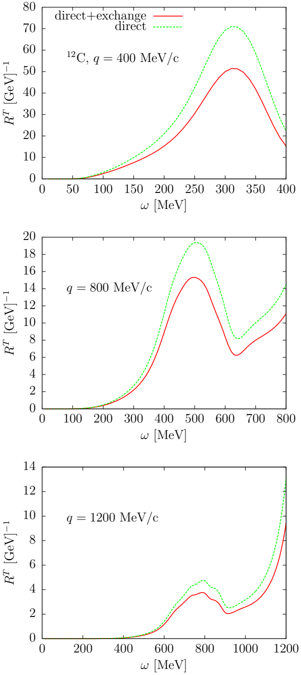
Δ propagator

Comparison of the T response function for two models of Δ current: using the total propagator or using only the real part.

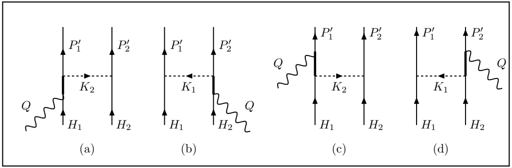


Direct-Exchange contribution

The T response function computed including only the direct contribution compared to the direct+exchange contribution.



Correlations



The OPE correlation current

$$
\begin{aligned}
j^{\mu}_{cor}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2) \;\; = \;\; & \frac{f^2}{m_\pi^2} \overline{u}(\mathbf{p}'_1)\tau_a\gamma_5 \; \not{K}_1 u(\mathbf{p}_1) \frac{1}{K_1^2 - m_\pi^2} \\
& \times \overline{u}(\mathbf{p}'_2) \left[\tau_a\gamma_5 \; \not{K}_1 S_F(P_2 + Q)\Gamma^{\mu}(Q) \right. \\
& \left. + \Gamma^{\mu}(Q) S_F(P'_2 - Q)\tau_a\gamma_5 \; \not{K}_1 \right] u(\mathbf{p}_2) \\
& + (1 \leftrightarrow 2) \, ,
\end{aligned} \tag{53}
$$

$S_F(P)$ is the Feynman propagator for the nucleon

$$
S_F(P) = \frac{\not{P} + m}{P^2 - m^2 + i\epsilon} \tag{54}
$$

$\Gamma^{\mu}(Q)$ is the electromagnetic or weak nucleon vertex,

$$
\Gamma^{\mu}_{\rm em}(Q) = F_1 \gamma^{\mu} + \frac{i}{2m} F_2 \sigma^{\mu\nu} Q_{\nu} \, . \tag{55}
$$

Divergence of the correlation responses

- The response functions computed using the correlation current in are divergent in the Fermi gas.
- As example for diagram (a) the current operator can be written as
$$j^\mu = \frac{I^\mu}{E_1 + \omega - E_{\mathbf{h}_1+\mathbf{q}} + i\epsilon}, \tag{56}$$
with a pole for
$$E_{\mathbf{h}_1+\mathbf{q}} = E_1 + \omega \tag{57}$$
in the limit $\epsilon \to 0$.
- The pole is double when taking the square of the current
- The 2p-2h correlation responses diverge in the Fermi gas because the probability of the first nucleon to eject a second nucleon is proporcional to the interaction time T that is infinite.

A regularization of correlation responses

- In a finite nucleus T must be finite bacuse the nucleon can leave the nucleus before interaction with a second nucleon.
- The current can be regularized by taking a finite value for ϵ.

$$\frac{T}{2} = \frac{1}{\epsilon}$$

- For exiting ^{12}C, the nucleon crosses a distance $R \sim 2$ fm in a time $T \sim R/c$.

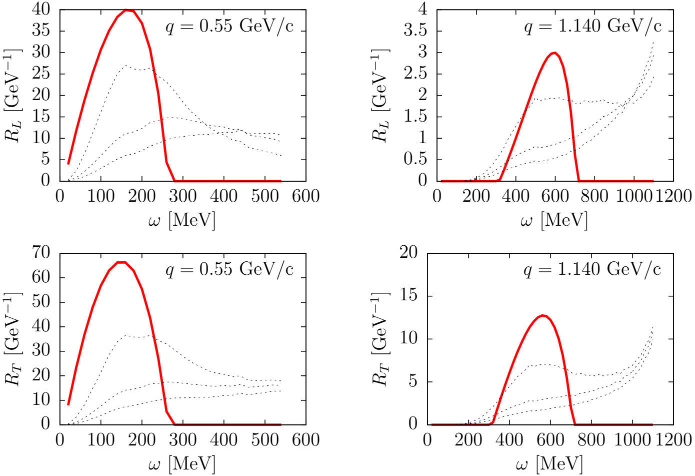
$$\epsilon \simeq \frac{2\hbar}{T} \simeq \frac{2\hbar c}{R} \simeq \frac{400}{2}\mathrm{MeV} \simeq 200\,\mathrm{MeV}\,.$$

- This value is very different from the nucleon width $\Gamma \sim 10$ MeV for inelastic interaction.
- In practice the value of ϵ can be taken as a parameter to be fitted to data.

2p-2h correlations

Amaro, Maieron, Barbaro, Caballero, Donnelly, PRC 82 (2010) 044601

Results for three values of $\epsilon = 100, 200, 300$ MeV.



Conclusions

- We have extended the MEC model of De Pace to the weak sector by adding the axial MEC operators
- This model of MEC has been validated within the SuSA-v2 approach by fitting the (e, e') data
- Our model can be applied to compute neutrino cross sections
- We can compute the separate 2p-2h charge channels, asymmetric matter and angular distributions
- We are preparing and will publish a fortran library of routines (NuMEC) to compute the elementary 2p-2h response functions

Thank you

Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table : Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

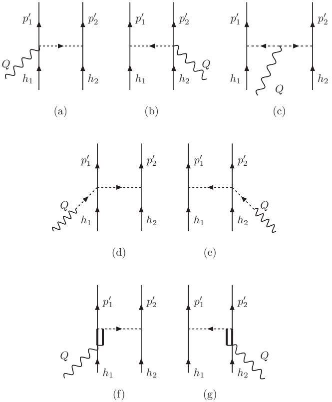
Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}%
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.



Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End