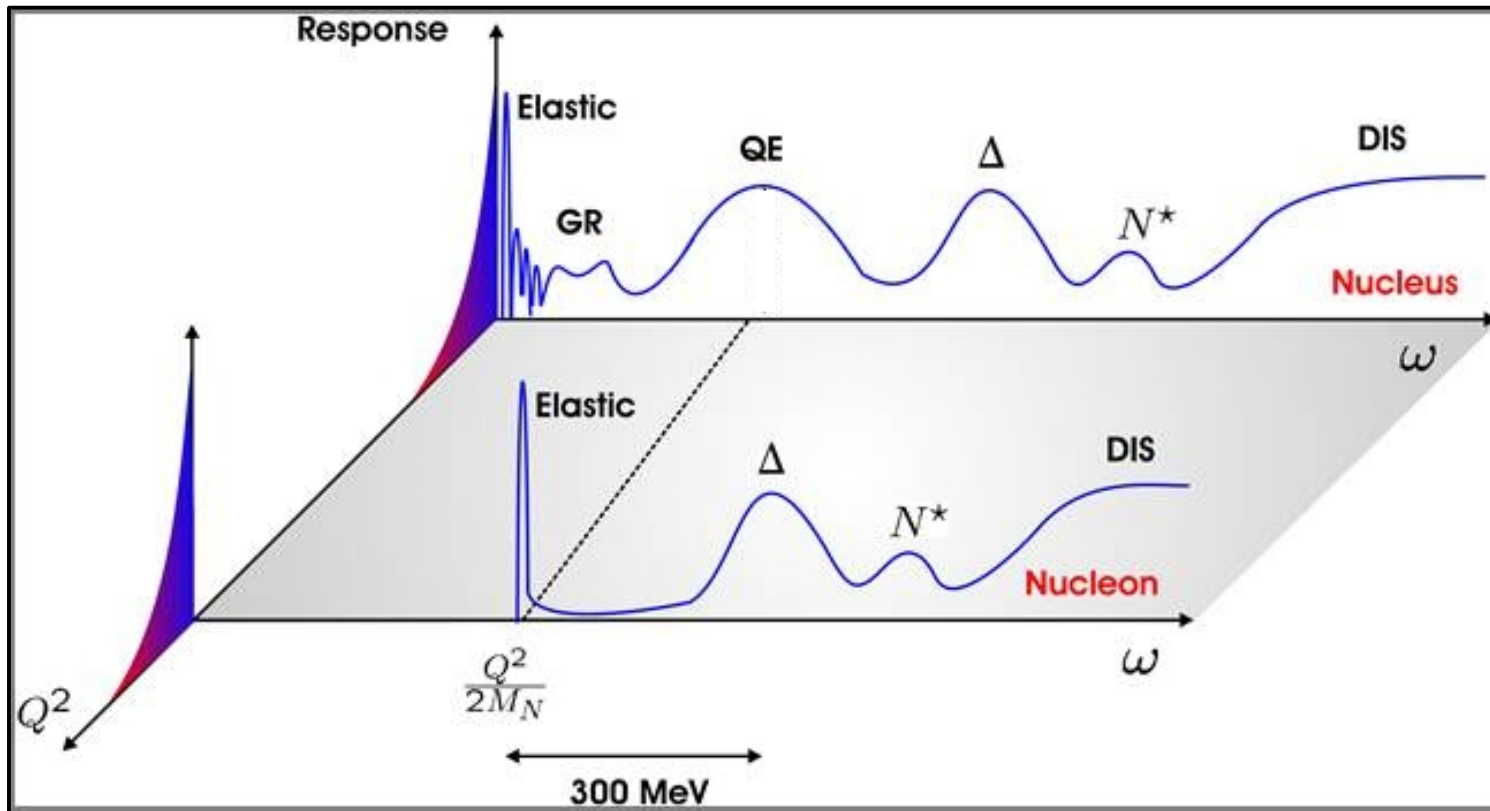
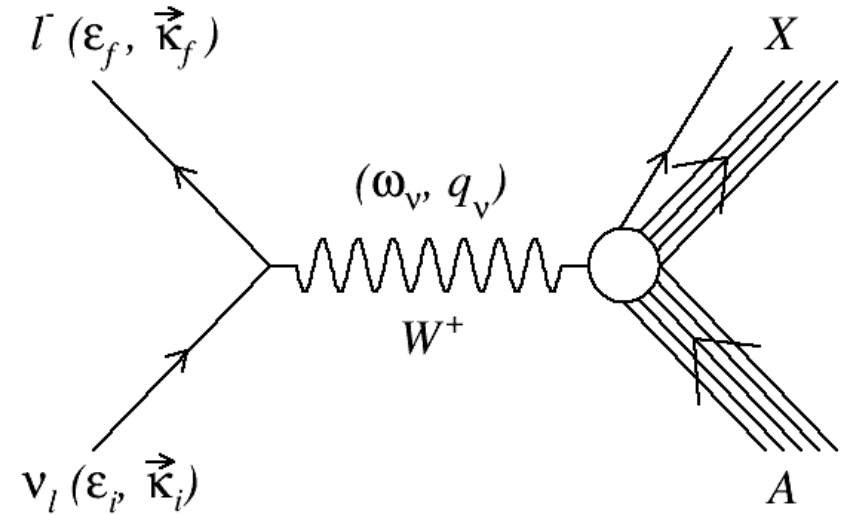
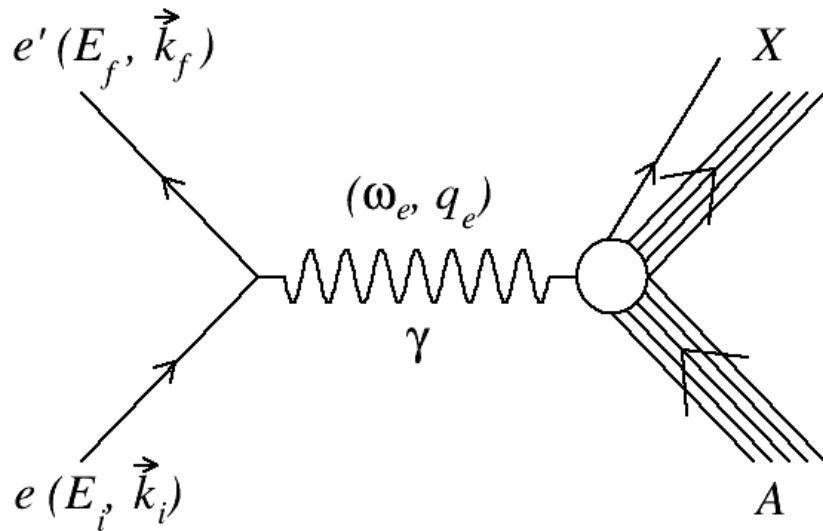


# Low-energy excitations to quasielastic scattering



# Quasielastic scattering



$$\left( \frac{d^2\sigma}{d\omega_e d\Omega} \right)_e = \frac{\alpha^2}{Q^4} \left( \frac{2}{2J_i + 1} \right) \frac{1}{k_f E_i} \times \zeta^2(Z', E_f, q_e) \left[ \sum_{J=0}^{\infty} \sigma_{L,e}^J + \sum_{J=1}^{\infty} \sigma_{T,e}^J \right]$$

$$\left( \frac{d^2\sigma}{d\omega_\nu d\Omega} \right)_\nu = \frac{G_F^2 \cos^2 \theta_c}{(4\pi)^2} \left( \frac{2}{2J_i + 1} \right) \epsilon_f \kappa_f \times \zeta^2(Z', \epsilon_f, q_\nu) \left[ \sum_{J=0}^{\infty} \sigma_{CL,\nu}^J + \sum_{J=1}^{\infty} \sigma_{T,\nu}^J \right]$$

$$\sigma_{L,e} = v_e^L R_e^L$$

$$\sigma_{T,e} = v_e^T R_e^T$$

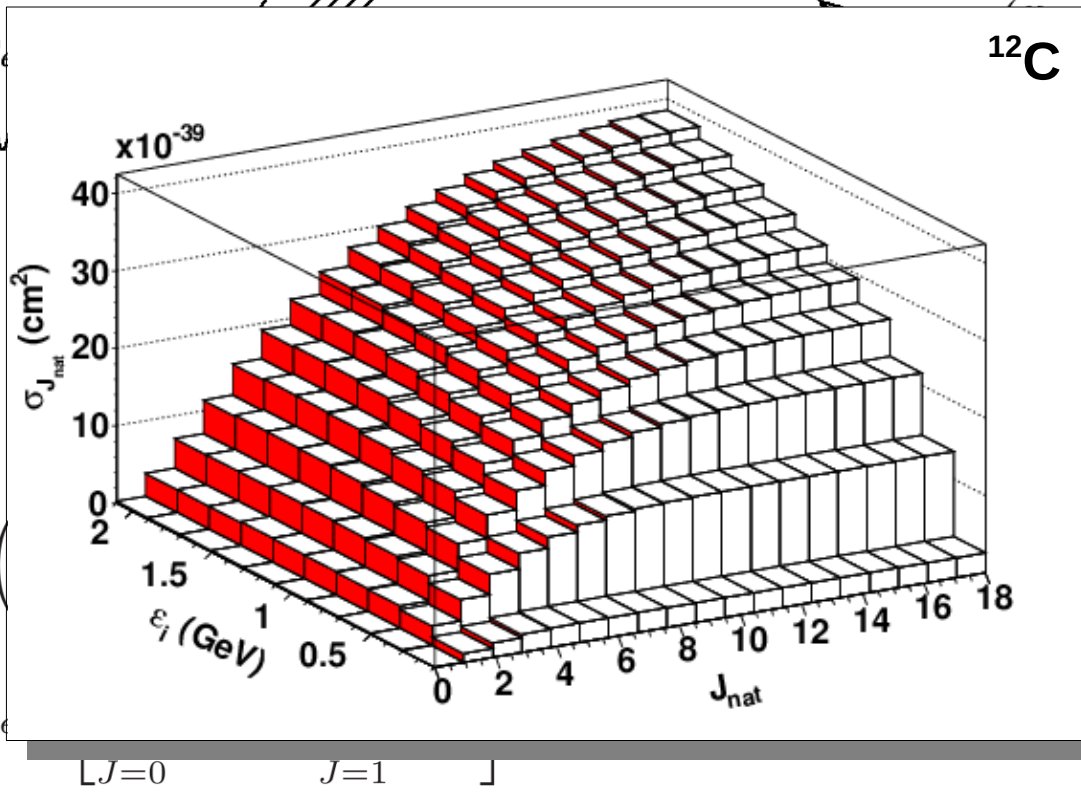
$$\sigma_{CL,\nu} = [v_\nu^M R_\nu^M + v_\nu^L R_\nu^L + 2 v_\nu^{ML} R_\nu^{ML}]$$

$$\sigma_{T,\nu} = [v_\nu^T R_\nu^T \pm 2 v_\nu^{TT} R_\nu^{TT}]$$

# Quasielastic scattering



$$\left( \frac{d^2\sigma}{d\omega_e d\Omega} \right)_e = \frac{\alpha^2}{Q^4} \left( \frac{d^2\sigma}{d\omega_\nu d\Omega} \right)_\nu \times \zeta^2(Z', E_f, q_e)$$



$$\left( \frac{d^2\sigma}{d\omega_\nu d\Omega} \right)_\nu = \frac{2\theta_c}{(2J_i + 1)} \varepsilon_f \kappa_f \left[ \sum_{J=0}^{\infty} \sigma_{CL,\nu}^J + \sum_{J=1}^{\infty} \sigma_{T,\nu}^J \right]$$

$$\sigma_{L,e} = v_e^L R_e^L$$

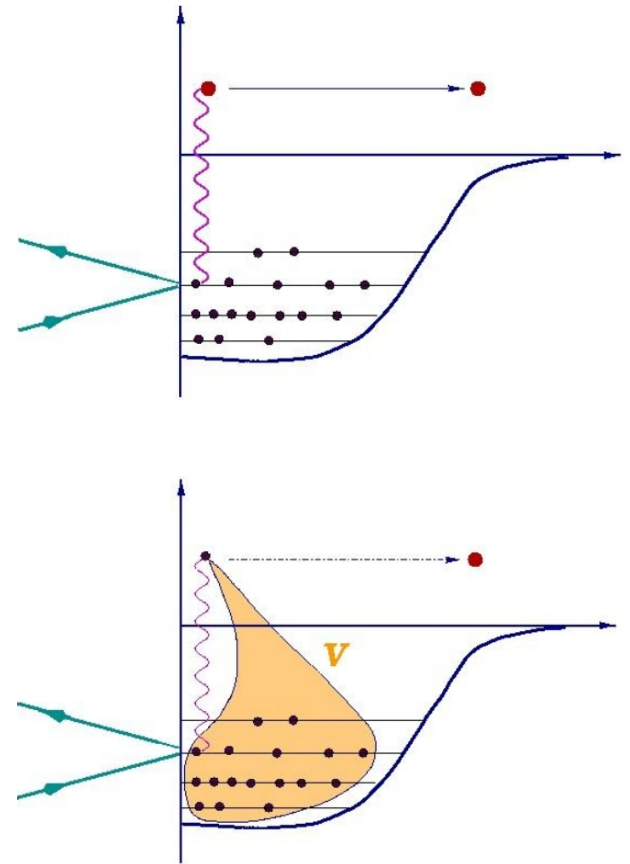
$$\sigma_{T,e} = v_e^T R_e^T$$

$$\sigma_{CL,\nu} = [v_\nu^M R_\nu^M + v_\nu^L R_\nu^L + 2 v_\nu^{ML} R_\nu^{ML}]$$

$$\sigma_{T,\nu} = [v_\nu^T R_\nu^T \pm 2 v_\nu^{TT} R_\nu^{TT}]$$

# Formalism: HF-CRPA approach

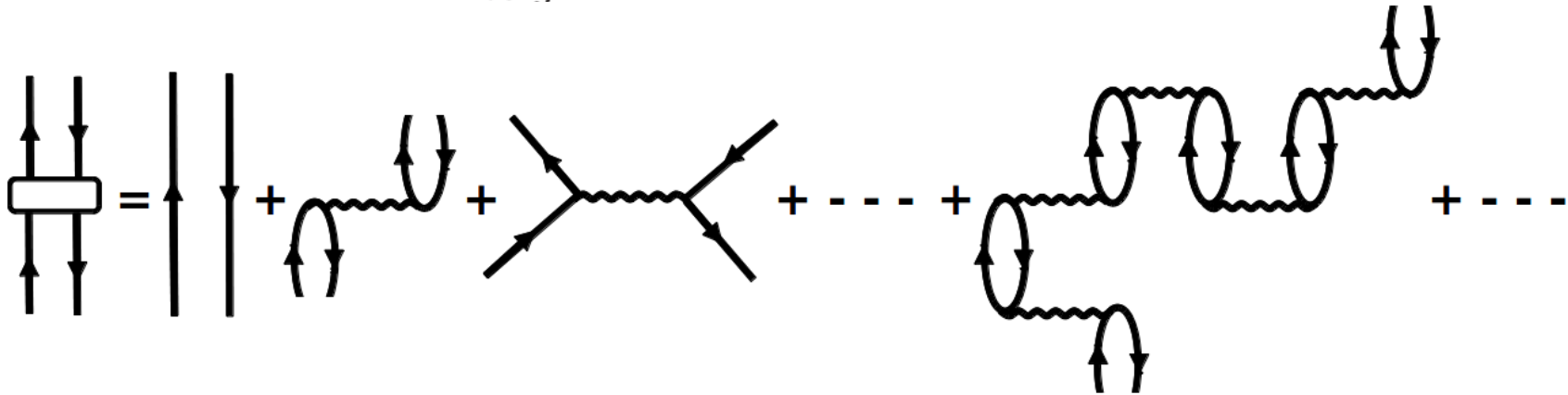
- We start by describing the nucleus with a Hartree-Fock (HF) approximation. The mean-field (MF) potential is obtained by solving the HF equations and using a Skyrme (SkE2) two-body interaction.
- Once we have bound and continuum single-nucleon wave functions, we introduce long-range correlations between the nucleons through a continuum Random Phase Approximation (CRPA).
- RPA equations are solved using a Green's function approach.



## Formalism: HF-CRPA approach

- The Skyrme (SkE2) nucleon-nucleon interaction, which was used in the HF calculations, is also used to perform CRPA calculations. That makes our approach **self-consistent**.

$$\Pi^{(RPA)}(x_1, x_2; E_x) = \Pi^{(0)}(x_1, x_2; E_x) + \frac{1}{\hbar} \int dx dx' \Pi^{(0)}(x_1, x; E_x) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; E_x)$$



- The effects of final state interactions (FSI) of the ejected nucleon with the residual nucleus, the distortion of the ejected nucleon waves, are taken into account.

# Formalism: Folding procedure

- A limitation of RPA formalism at lower energies:
  - energy position of the giant resonances is generally well predicted
  - width is underestimated
  - height is overestimated

CRPA: Without Folding, CRPA: With Folding

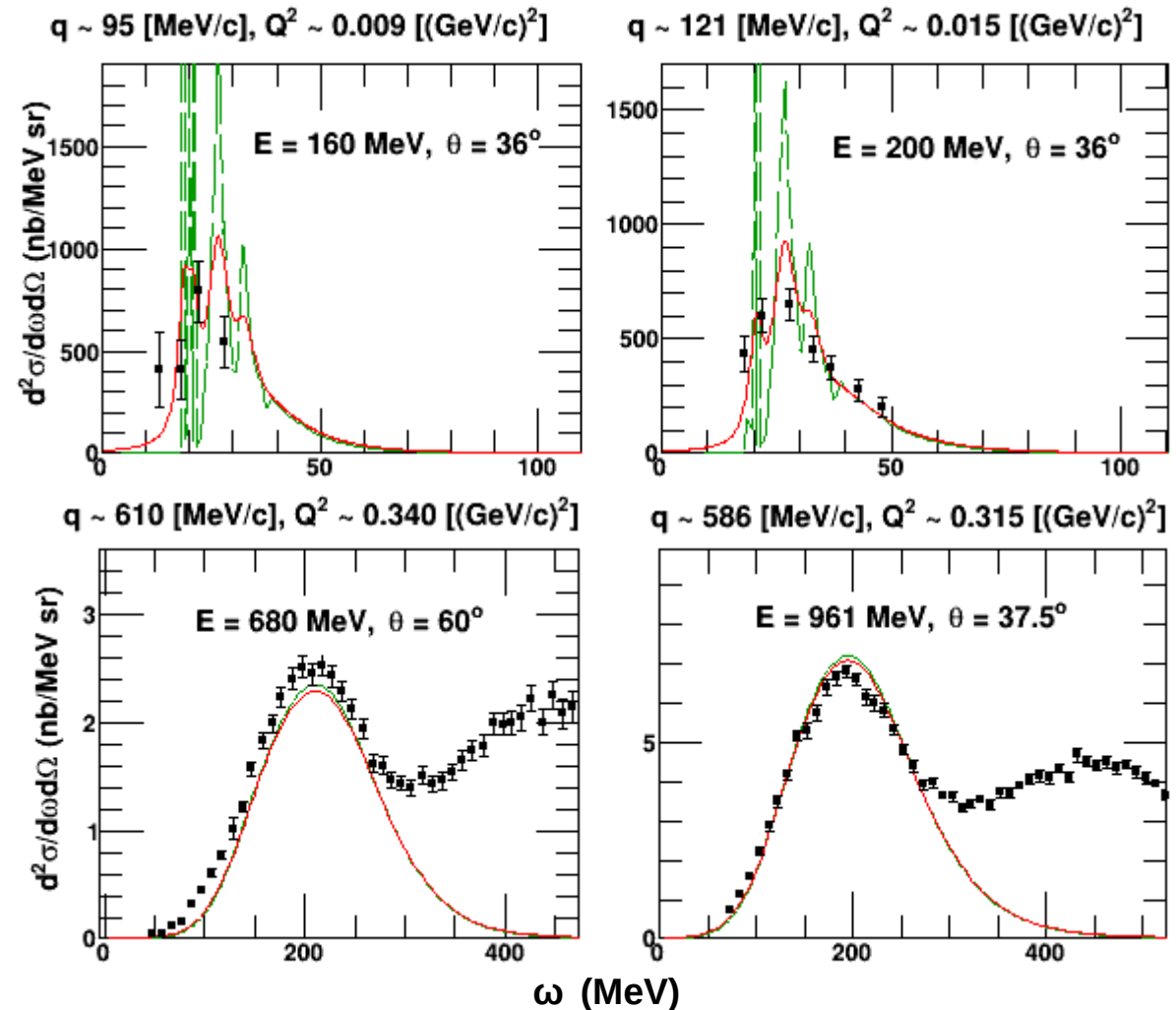


Folding

$$R'(q, \omega') = \int_{-\infty}^{\infty} d\omega R(q, \omega) L(\omega, \omega')$$

$$L(\omega, \omega') = \frac{1}{2\pi} \left[ \frac{\Gamma}{(\omega - \omega')^2 + (\Gamma/2)^2} \right]$$

$$\Gamma = 3 \text{ MeV}$$



# Formalism: Relativistic corrections

- Non-relativistic model  $\rightarrow$  higher energies ( $\sim$ GeV)  $\rightarrow$  problem!



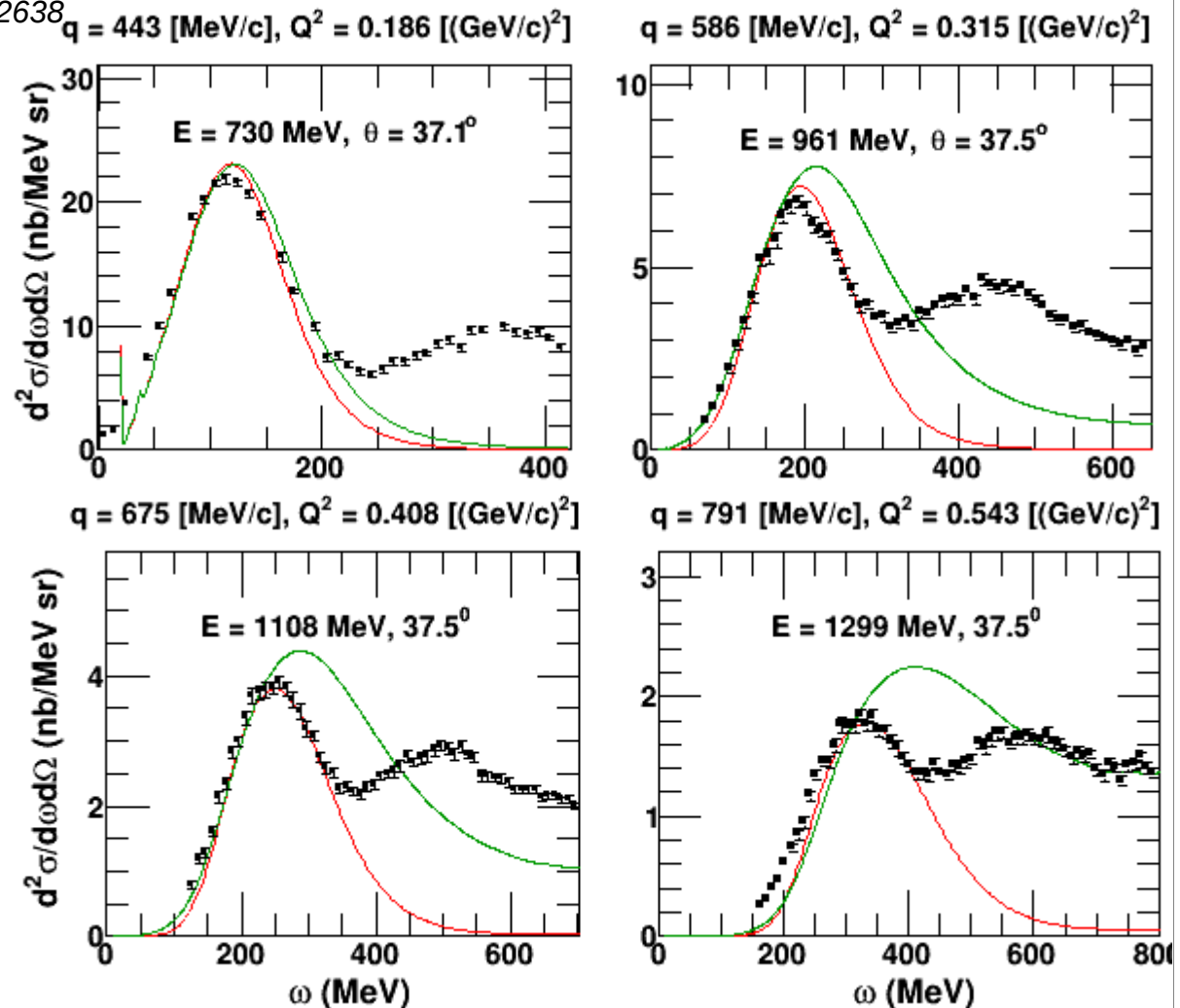
## 'Semi-relativistic' modeling

(S. Jeschonnek and T. W. Donnelly, PRC57, 2638, (1998))

CRPA: Non-relativized, CRPA: Relativized

Kinematic effects:

- $\lambda \rightarrow \lambda(1+\lambda)$ ,  $\lambda = \omega/2M_N$
- QE peak:  $\omega = q^2/2M_N \Rightarrow Q^2/2M_N$



# Formalism: Regularization of the residual interaction

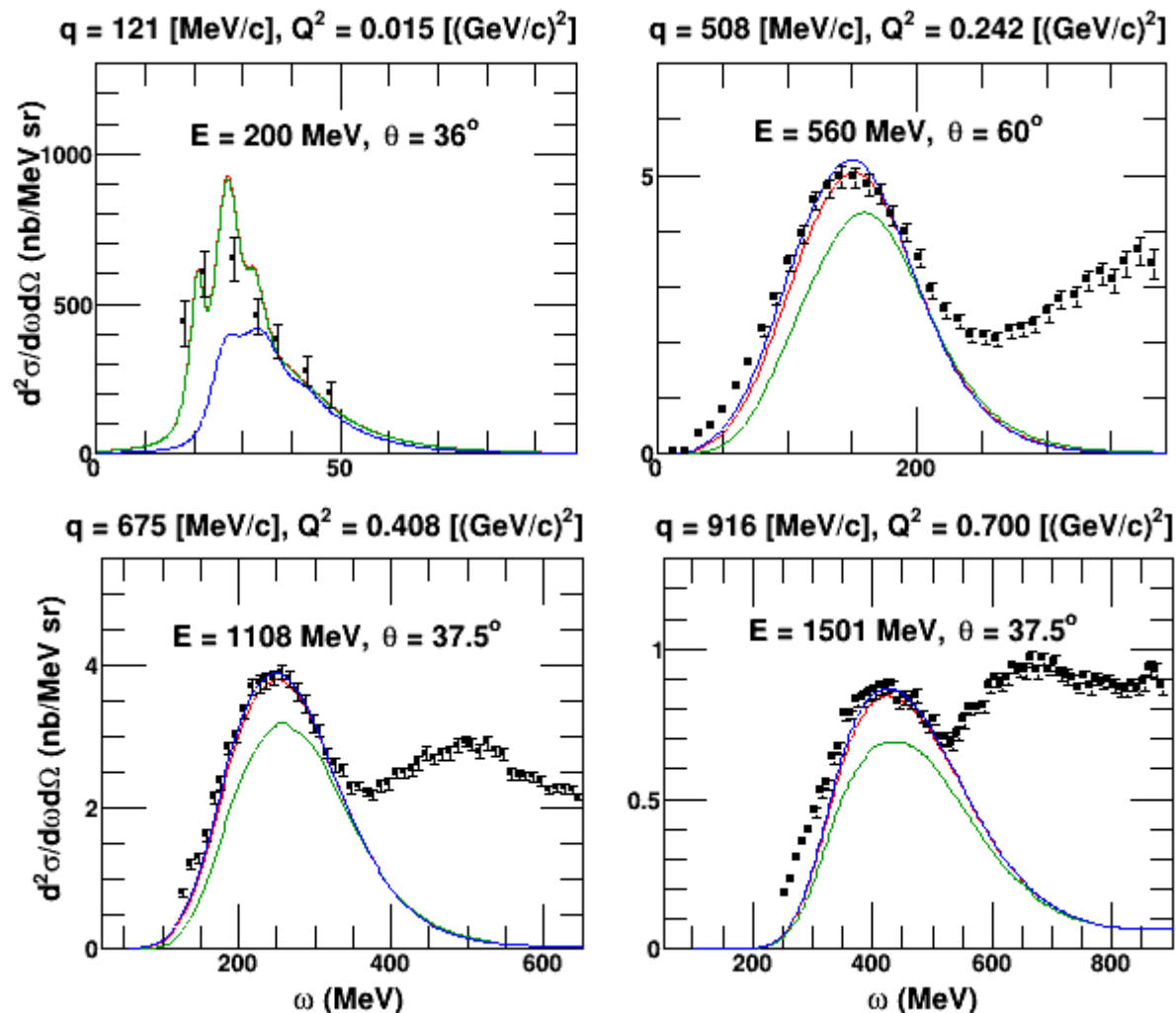
- SkE2 interaction → optimized against ground-state and low-excitation energy properties
  - at higher  $Q^2$ , unrealistically strong!



$$V(Q^2) = V(Q^2 = 0) \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

With  $\Lambda = 455$  MeV, optimized with a  $\chi^2$  fitting of theory-experiment comparison from low  $\omega$  up to the QE peak, over broad set of available data on  $A(e, e')$  scattering.

CRPA: without dipole, CRPA: with dipole, HF





# Formalism: Coulomb correction for outgoing lepton

- Effect of coulomb potential of the nucleus on the charged lepton:

→ Low energies: Fermi function  $F(Z', E) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}$   $\eta \sim \mp Z' \alpha$

→ High energies:

CRPA: without MEMA, CRPA: with MEMA

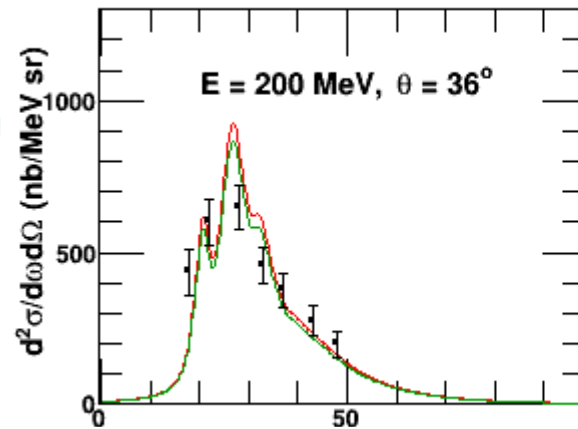
Modified effective momentum approximation

(J. Engel, PRC57,2004 (1998))

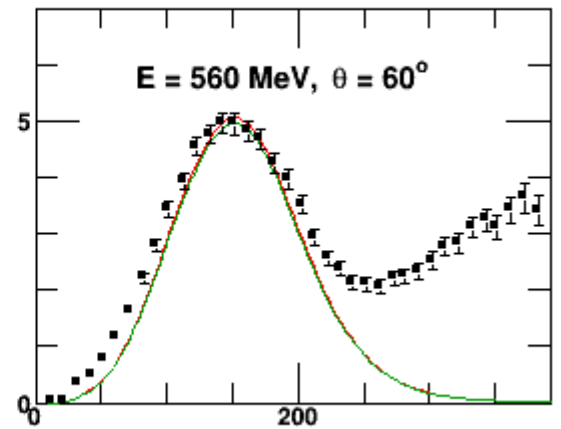
$$q_{\text{eff}} = q + 1.5 \left( \frac{Z' \alpha \hbar c}{R} \right)$$

$$\zeta(Z', E, q) = \sqrt{\frac{q_{\text{eff}} E_{\text{eff}}}{q E}}$$

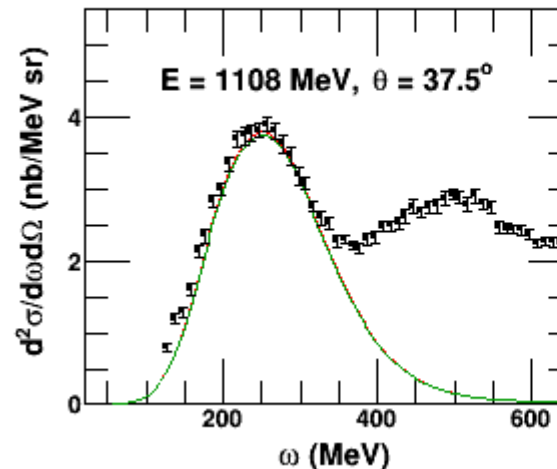
q = 121 [MeV/c], Q<sup>2</sup> = 0.015 [(GeV/c)<sup>2</sup>]



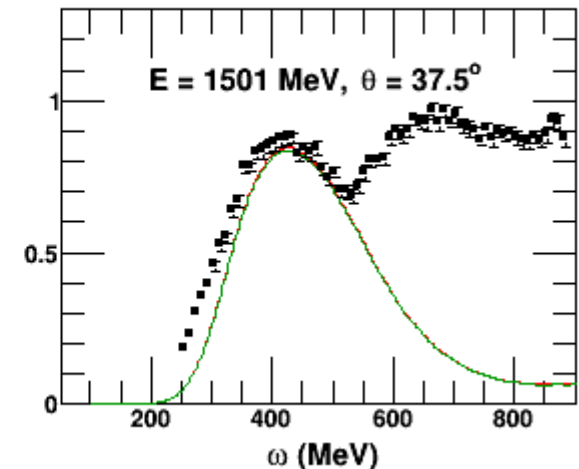
q = 508 [MeV/c], Q<sup>2</sup> = 0.242 [(GeV/c)<sup>2</sup>]



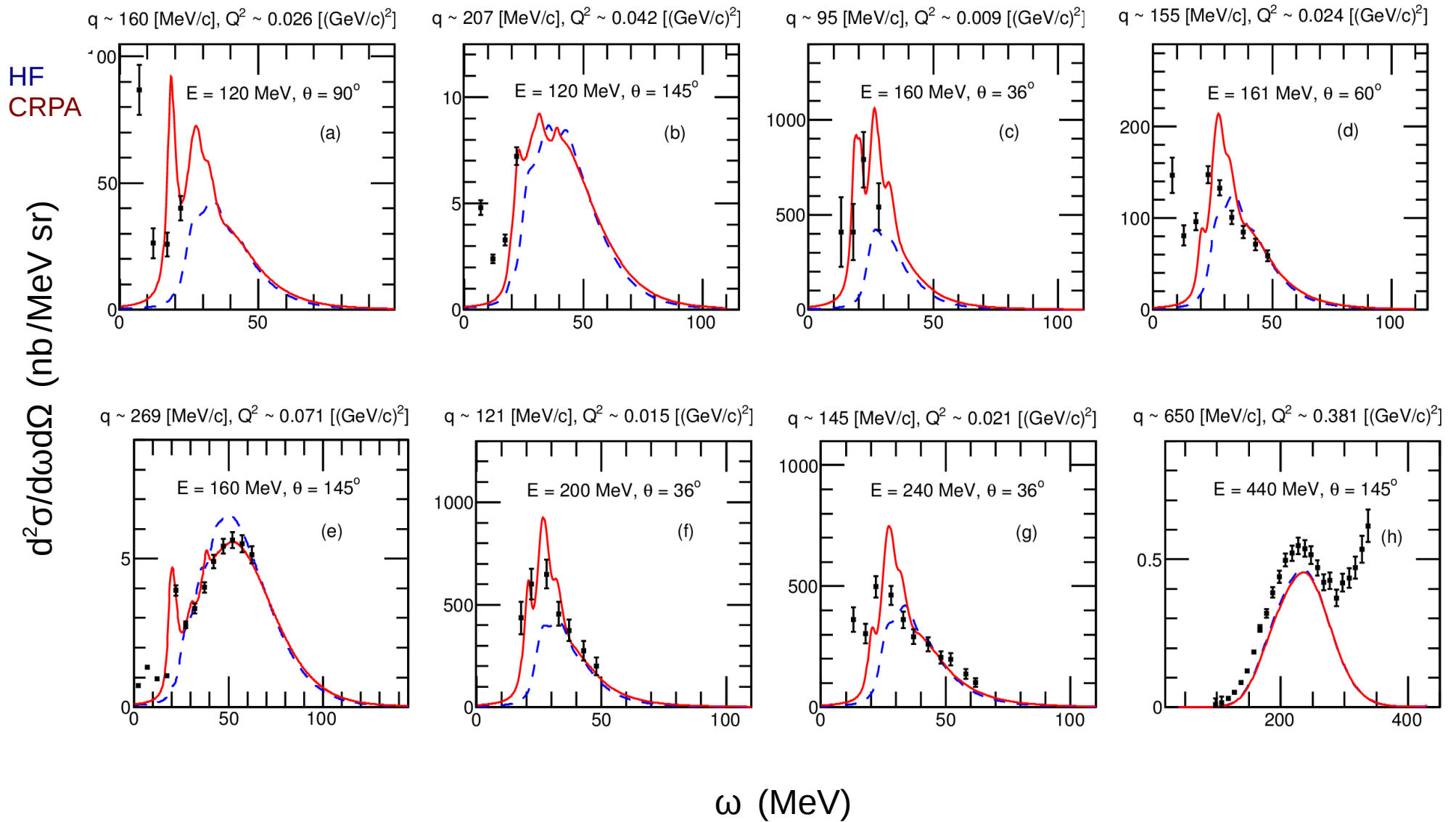
q = 675 [MeV/c], Q<sup>2</sup> = 0.408 [(GeV/c)<sup>2</sup>]



q = 916 [MeV/c], Q<sup>2</sup> = 0.700 [(GeV/c)<sup>2</sup>]



# Comparison with electron scattering data: $^{12}\text{C}$ ( $e, e'$ )

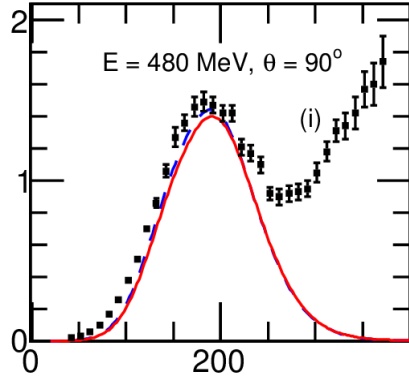


# Comparison with electron scattering data: $^{12}\text{C}$ ( $e, e'$ )

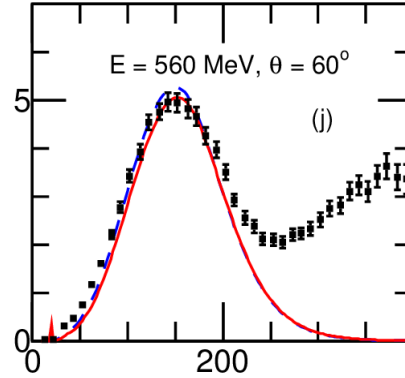
HF  
CRPA

$d^2\sigma/d\omega d\Omega$  (nb/MeV sr)

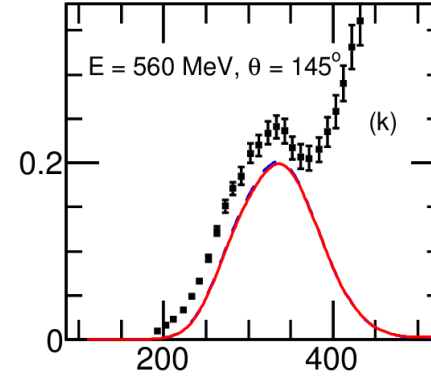
$q \sim 576$  [MeV/c],  $Q^2 \sim 0.305$  [(GeV/c) $^2$ ]



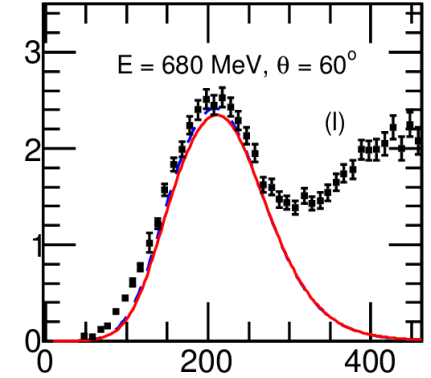
$q \sim 508$  [MeV/c],  $Q^2 \sim 0.242$  [(GeV/c) $^2$ ]



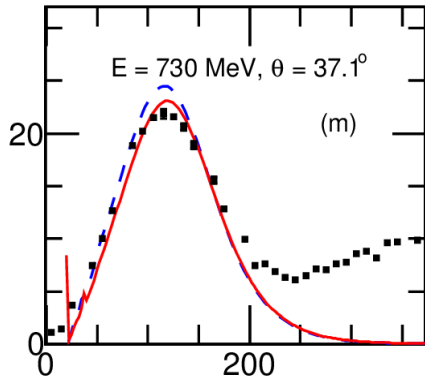
$q \sim 795$  [MeV/c],  $Q^2 \sim 0.548$  [(GeV/c) $^2$ ]



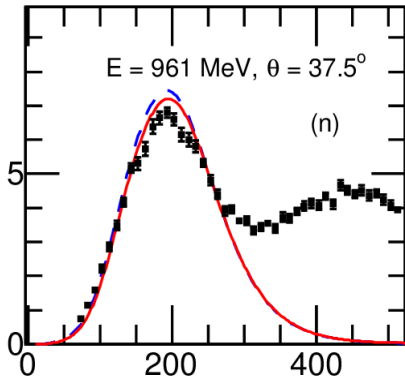
$q \sim 610$  [MeV/c],  $Q^2 \sim 0.340$  [(GeV/c) $^2$ ]



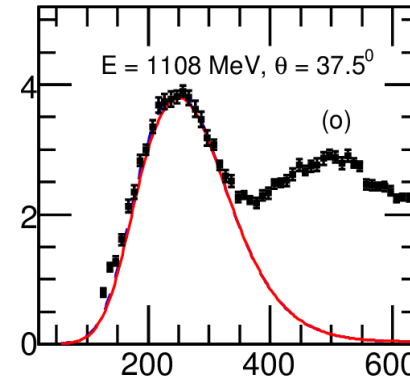
$q \sim 443$  [MeV/c],  $Q^2 \sim 0.186$  [(GeV/c) $^2$ ]



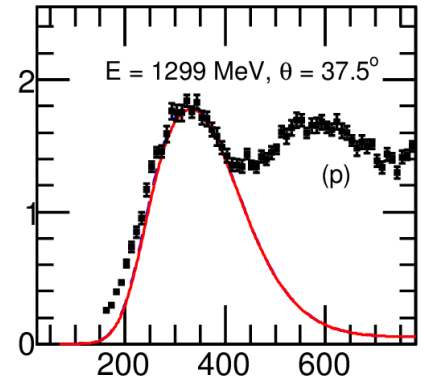
$q \sim 586$  [MeV/c],  $Q^2 \sim 0.315$  [(GeV/c) $^2$ ]



$q \sim 675$  [MeV/c],  $Q^2 \sim 0.408$  [(GeV/c) $^2$ ]



$q \sim 791$  [MeV/c],  $Q^2 \sim 0.543$  [(GeV/c) $^2$ ]



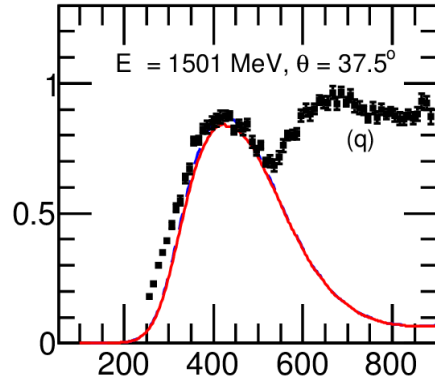
$\omega$  (MeV)

# Comparison with electron scattering data: $^{12}\text{C}$ (e,e')

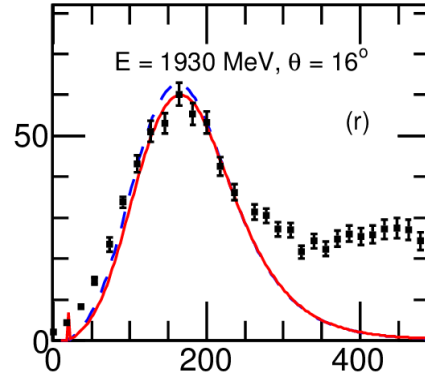
HF  
CRPA

$d^2\sigma/d\omega d\Omega$  (nb/MeV sr)

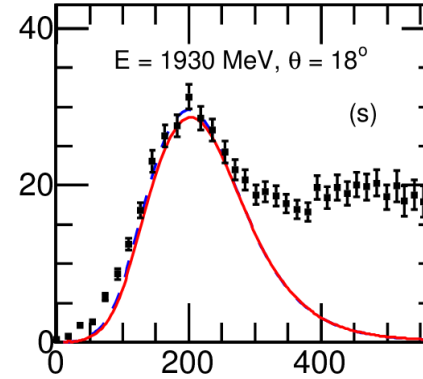
$q \sim 916$  [MeV/c],  $Q^2 \sim 0.700$  [(GeV/c) $^2$ ]



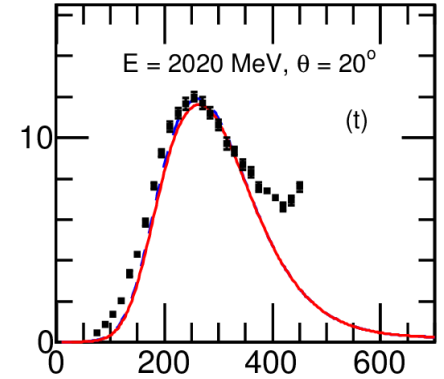
$q \sim 536$  [MeV/c],  $Q^2 \sim 0.267$  [(GeV/c) $^2$ ]



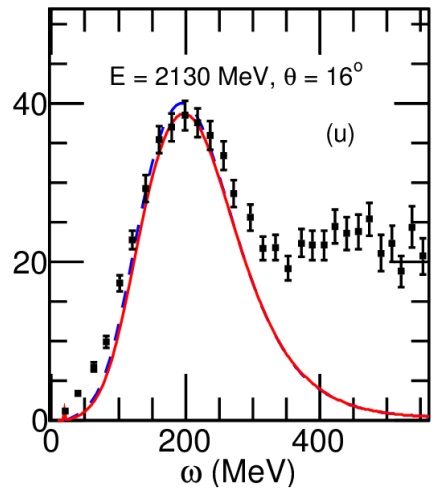
$q \sim 601$  [MeV/c],  $Q^2 \sim 0.331$  [(GeV/c) $^2$ ]



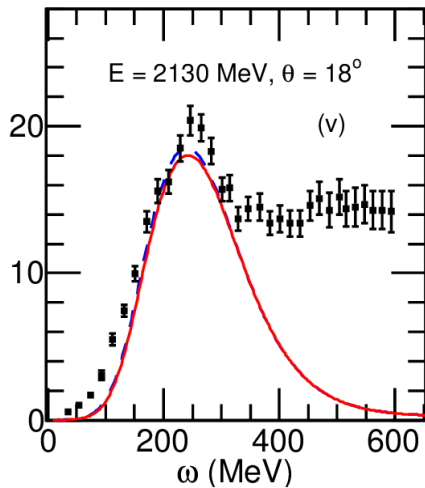
$q \sim 700$  [MeV/c],  $Q^2 \sim 0.436$  [(GeV/c) $^2$ ]



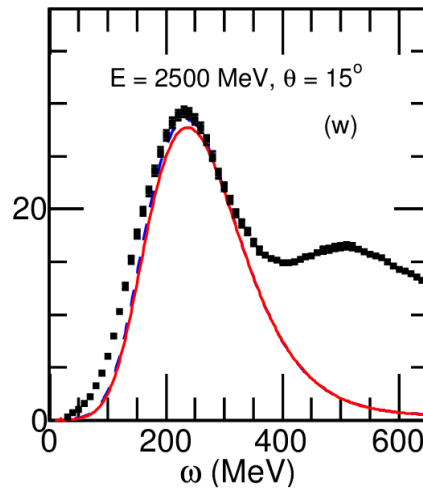
$q \sim 594$  [MeV/c],  $Q^2 \sim 0.323$  [(GeV/c) $^2$ ]



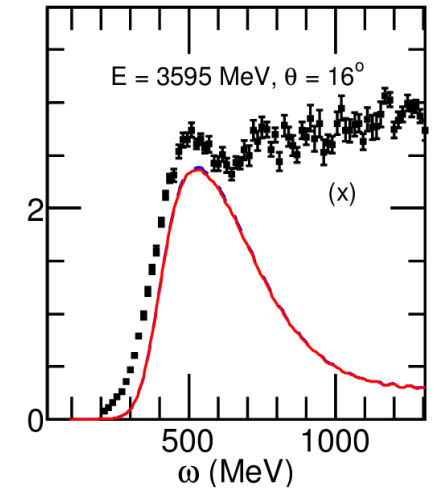
$q \sim 667$  [MeV/c],  $Q^2 \sim 0.399$  [(GeV/c) $^2$ ]



$q \sim 658$  [MeV/c],  $Q^2 \sim 0.391$  [(GeV/c) $^2$ ]



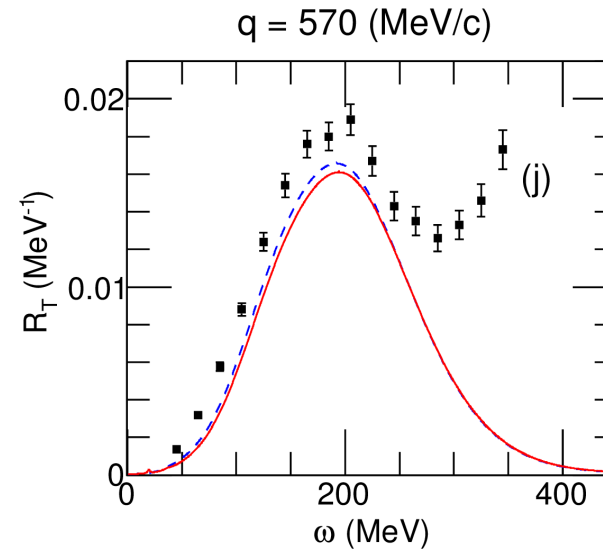
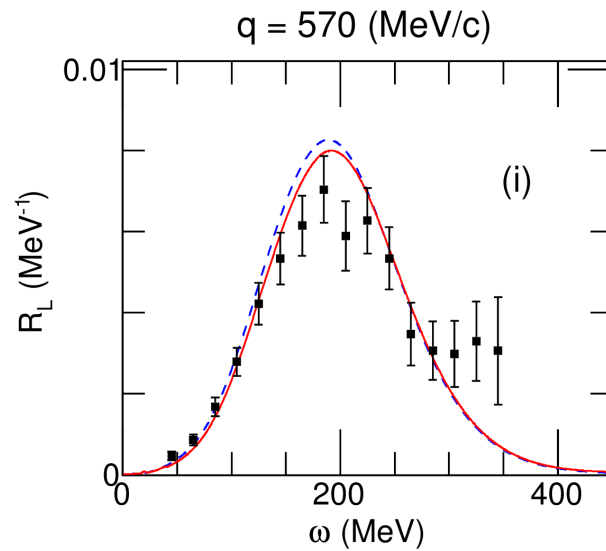
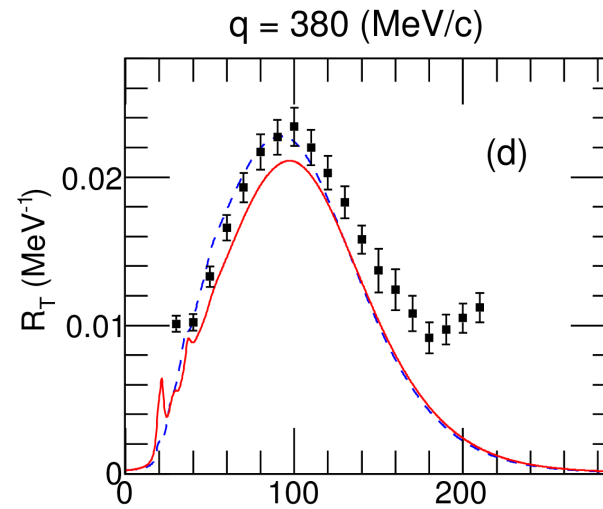
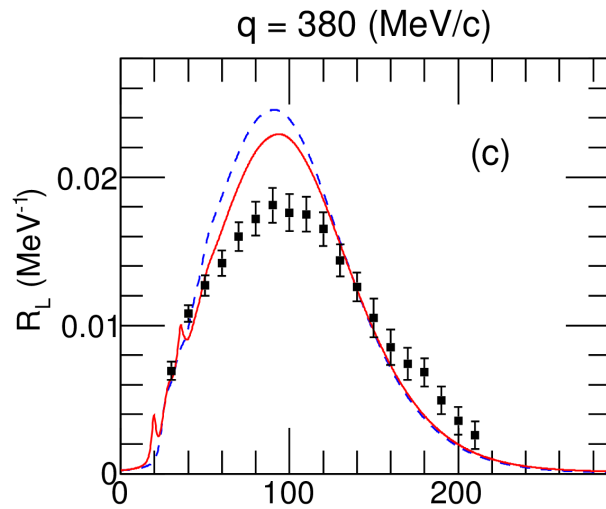
$q \sim 1043$  [MeV/c],  $Q^2 \sim 0.872$  [(GeV/c) $^2$ ]



$\omega$  (MeV)

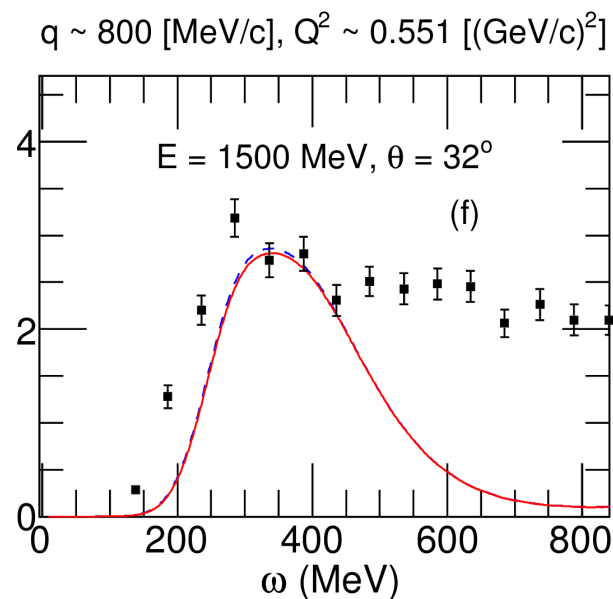
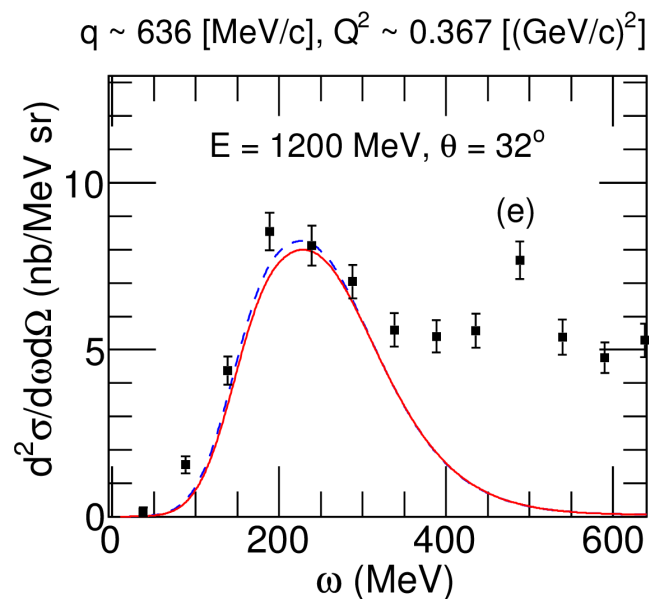
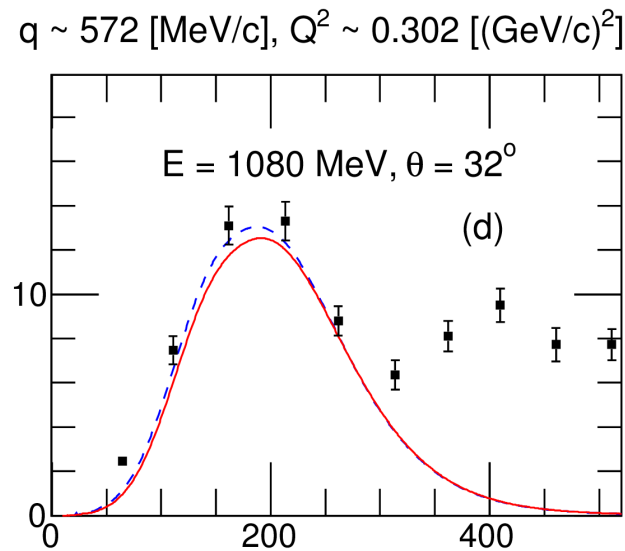
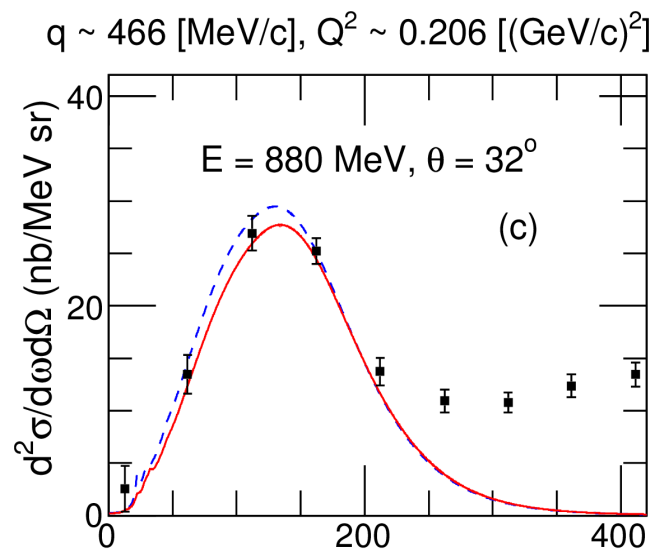
# Longitudinal and Transverse structure for $^{12}\text{C}$ ( $e, e'$ )

HF  
CRPA



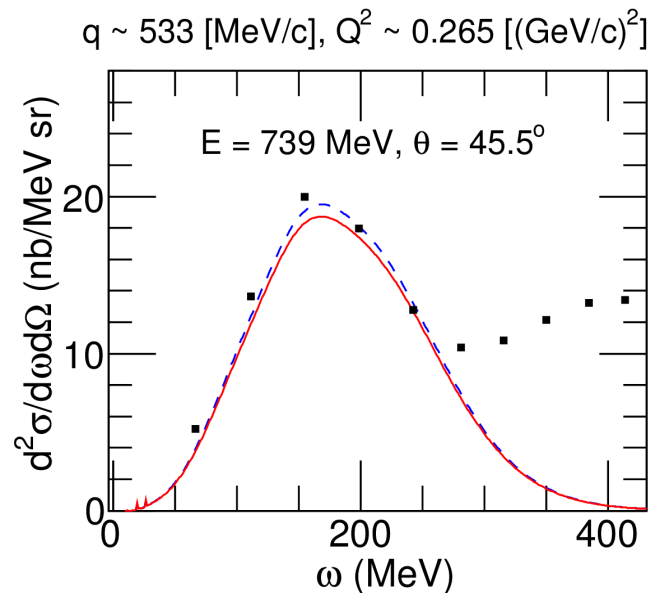
# Comparison with electron scattering data: $^{16}\text{O}$ (e,e')

HF  
CRPA



# Comparison with electron scattering data: $^{26}\text{Ca} (e, e')$

HF  
CRPA

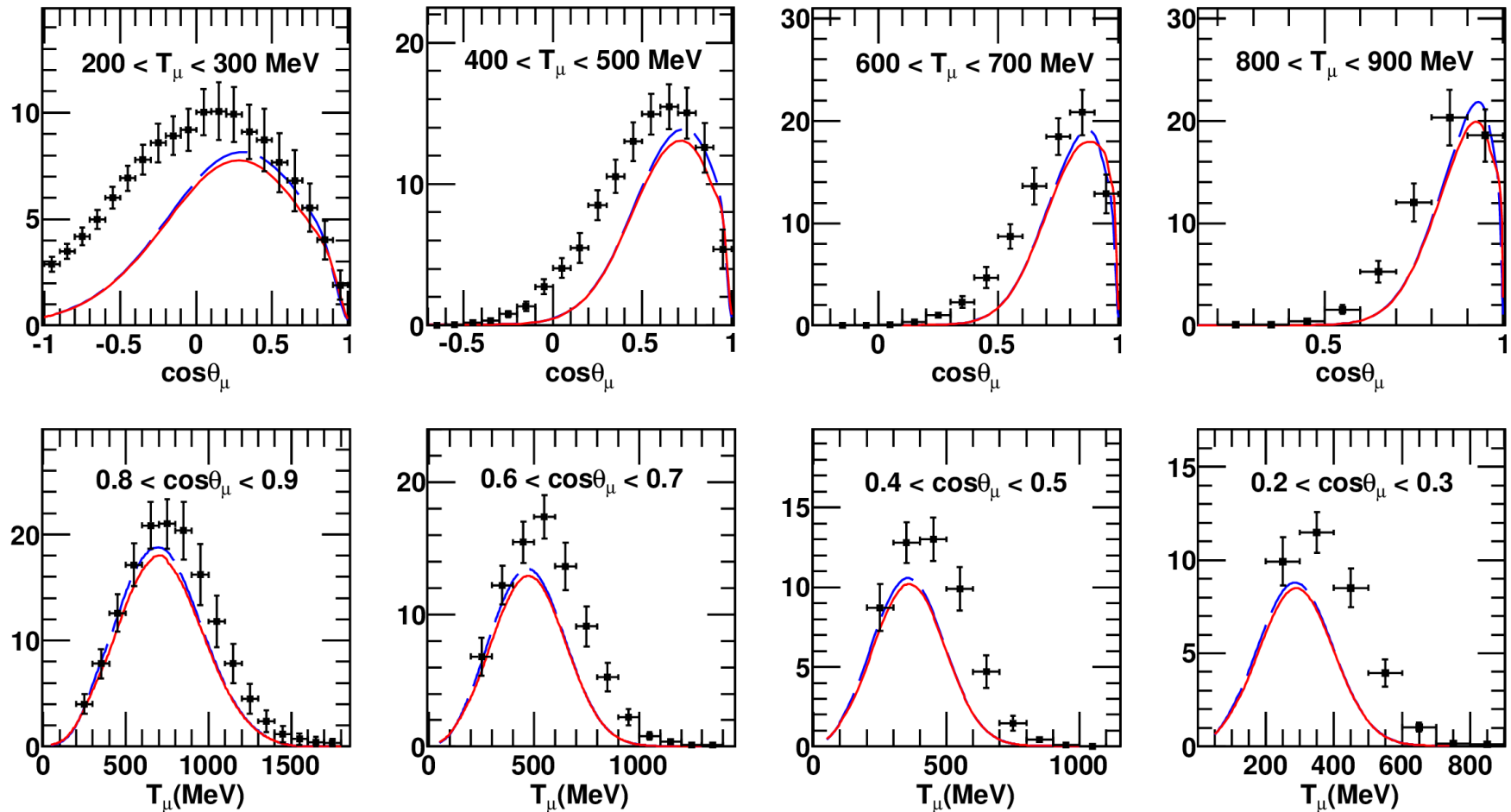


- Good overall agreement with electron scattering data on variety of nuclear target ( $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{26}\text{Ca}$ ) from low-energy excitations to the QE region, validates the reliability of our approach.

# Comparison with MiniBooNE $\nu_\mu$ 'CCQE-like' data

HF  
CRPA

$\langle d^2\sigma/dT d\cos\theta_\mu \rangle$  ( $10^{-42} \text{ cm}^2 \text{ MeV}^{-1}$ )

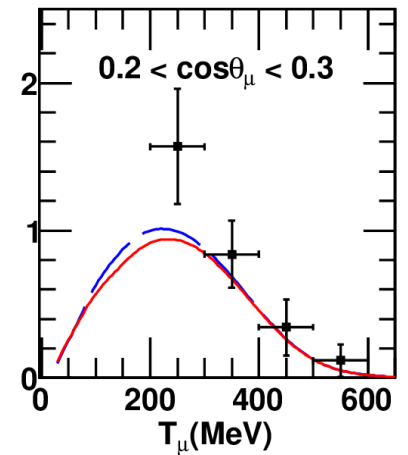
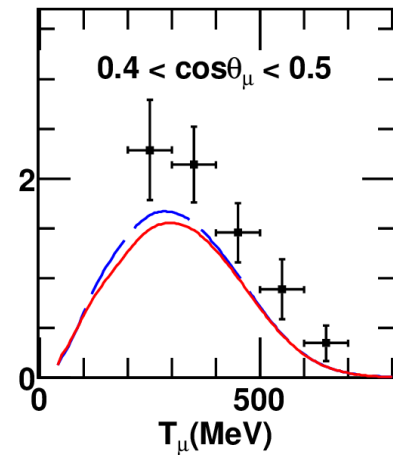
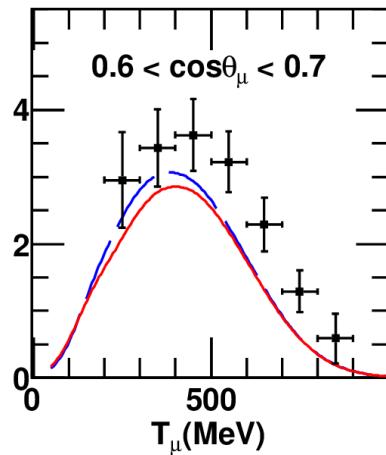
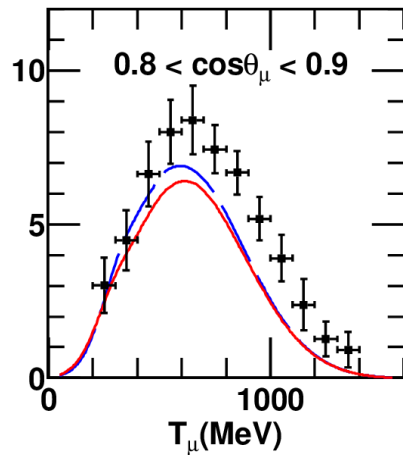
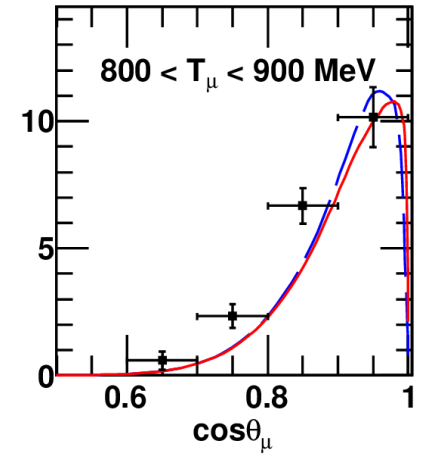
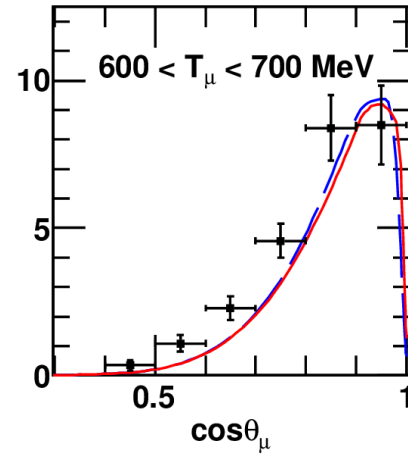
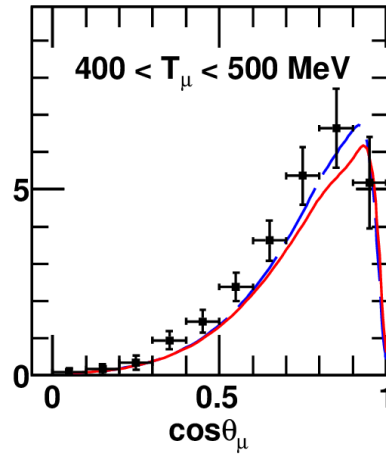
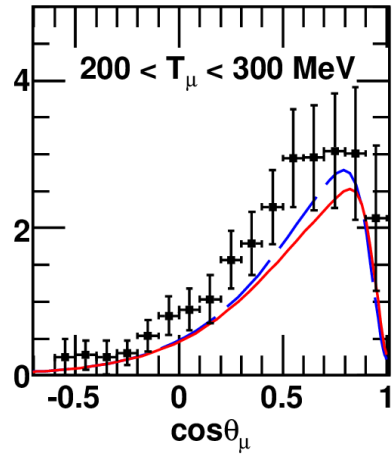




# Comparison with MiniBooNE $\bar{\nu}_\mu$ 'CCQE-like' data

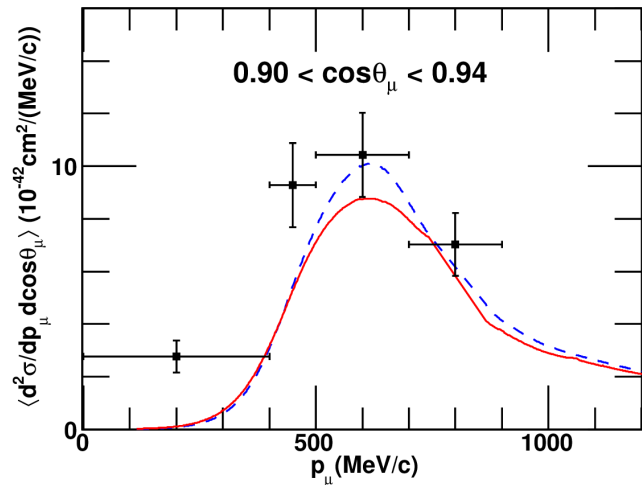
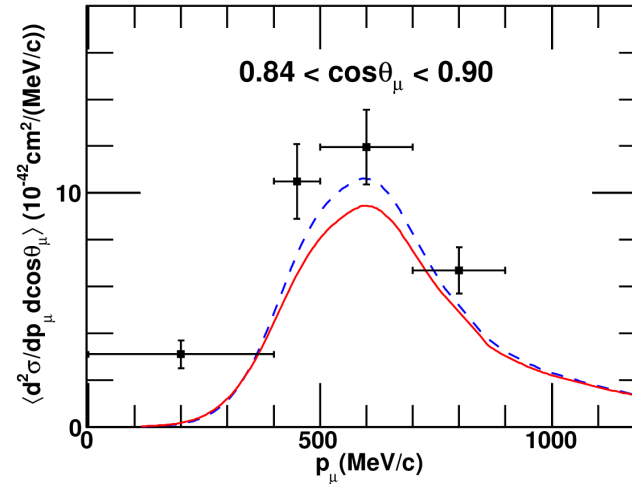
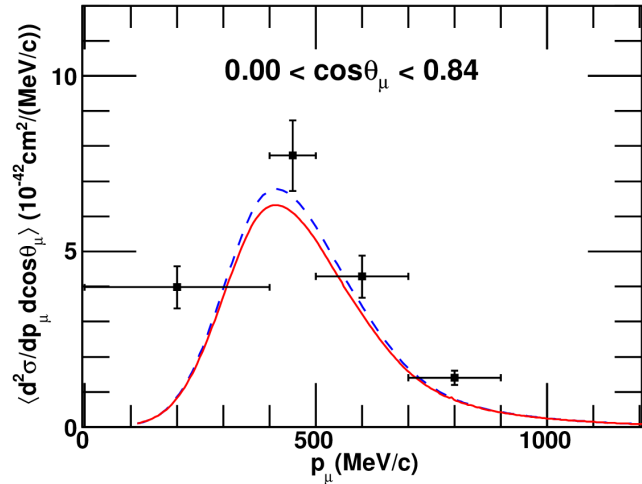
HF  
CRPA

$\langle d^2\sigma/dT_\mu d\cos\theta_\mu \rangle > (10^{-42} \text{ cm}^2 \text{ MeV}^{-1})$



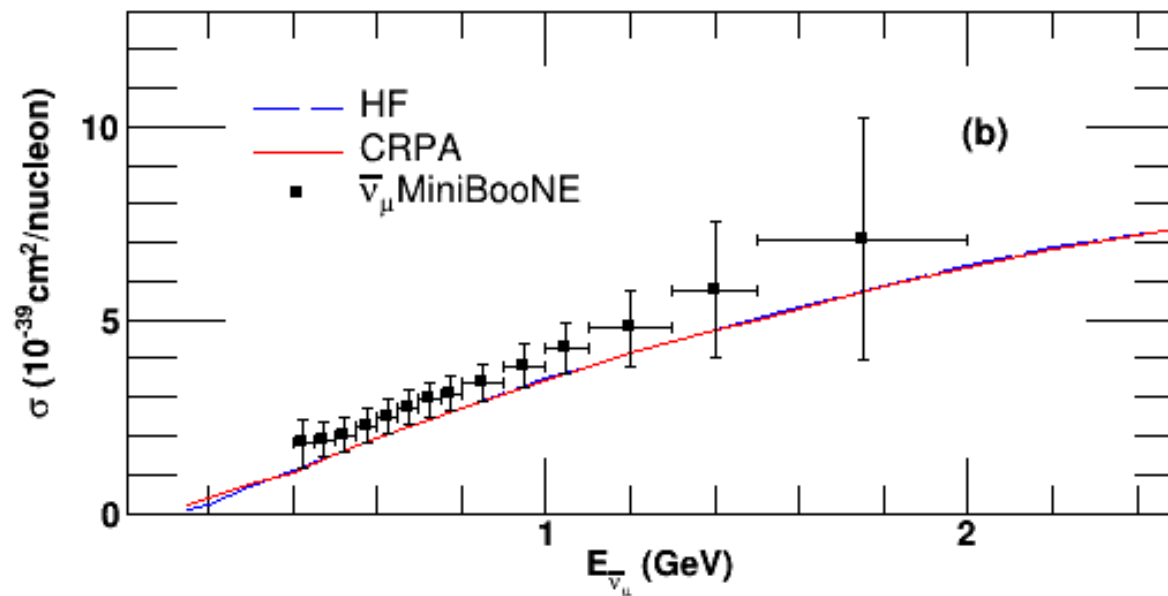
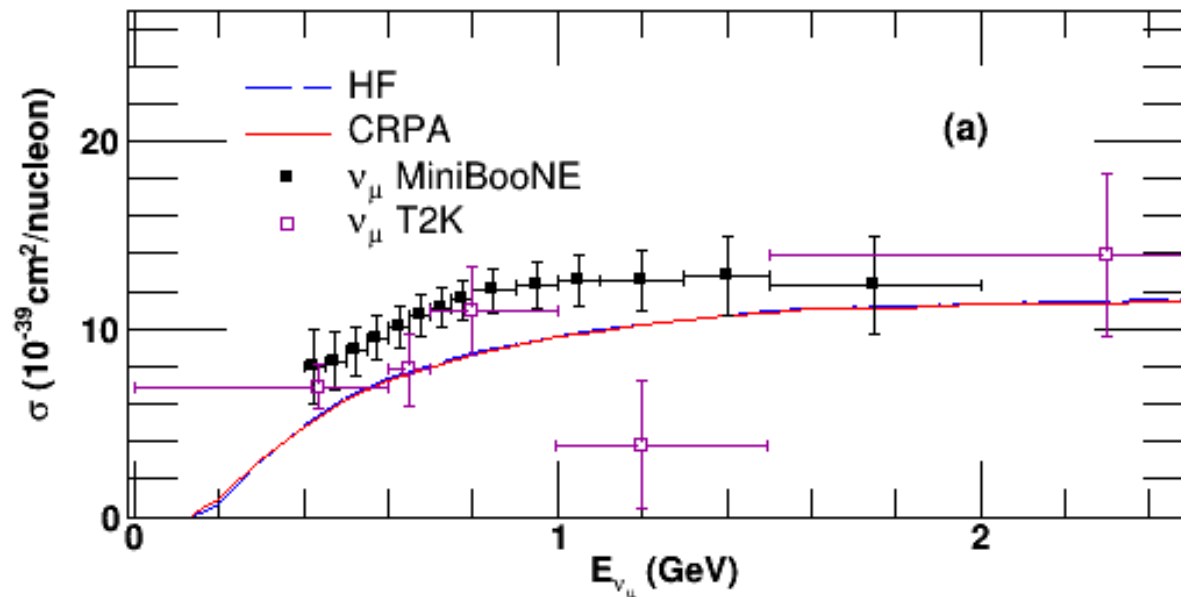
# Comparison with T2K $\nu_{\mu}$ inclusive QE data

HF  
CRPA

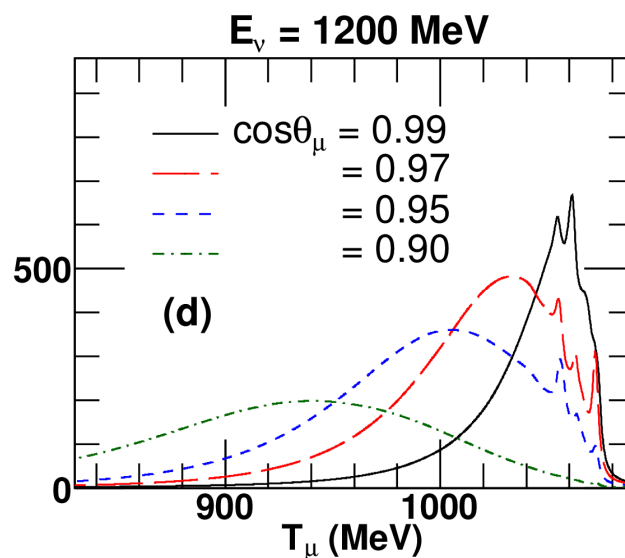
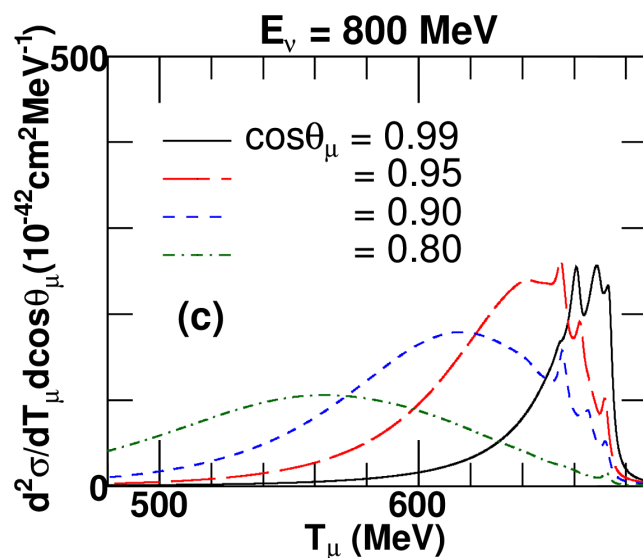
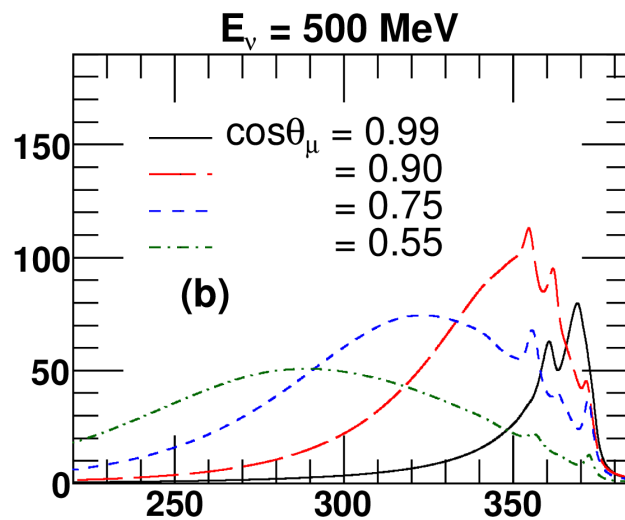
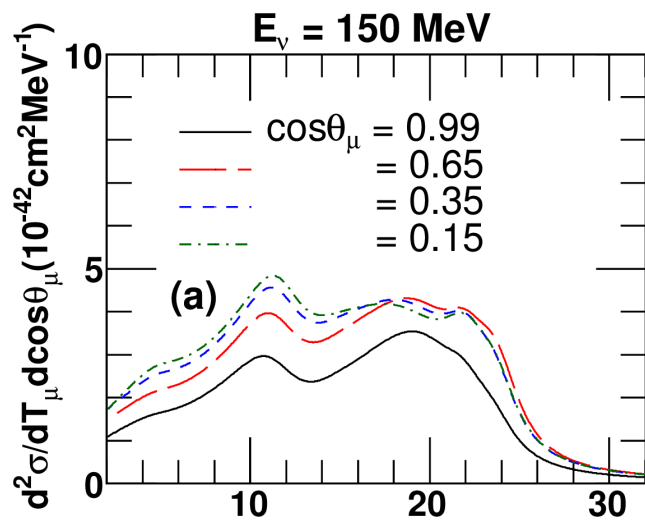


➤ Most forward bin  
( $0.94 < \cos\theta_{\mu} < 1.00$ )  
will be discussed later.

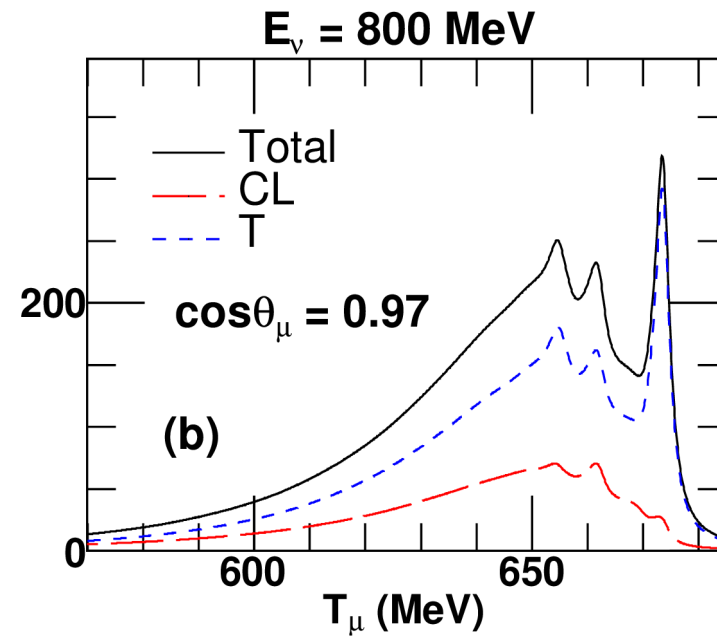
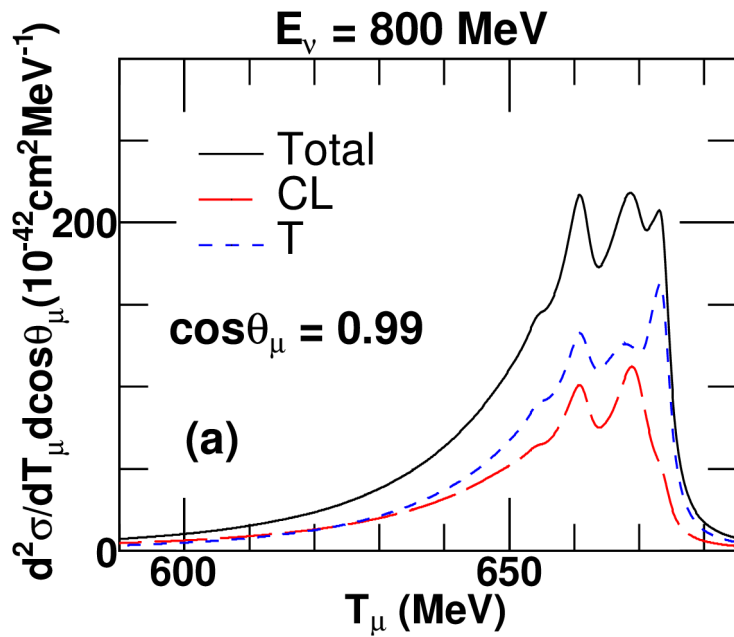
# Total cross section: comparison with MiniBooNE and T2K CCQE data



# Impact of low-energy excitations: Forward scattering (fixed energy)



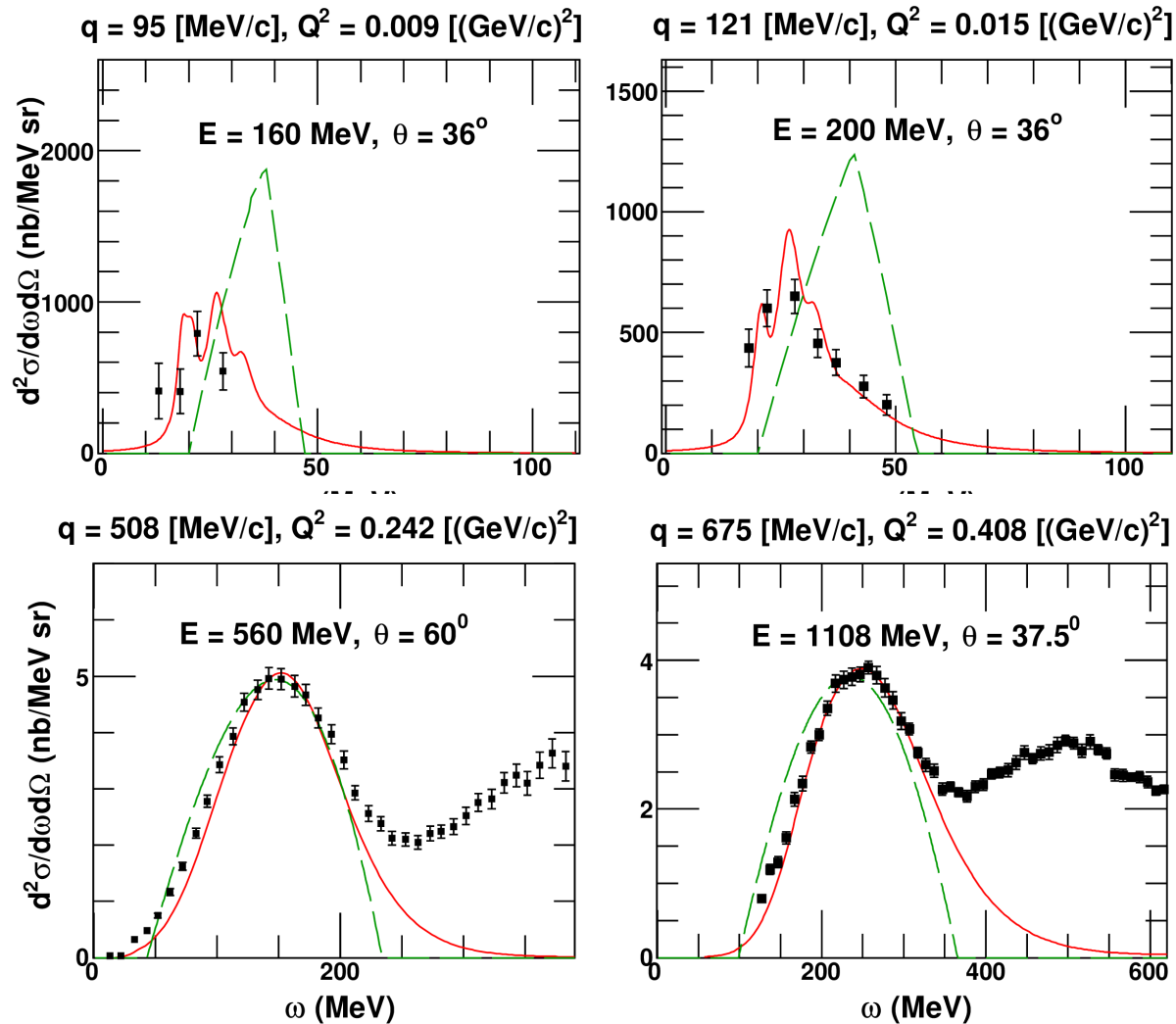
# Impact of low-energy excitations: Forward scattering (CL vs T)



# Impact of low-energy excitations: CRPA vs RgFG

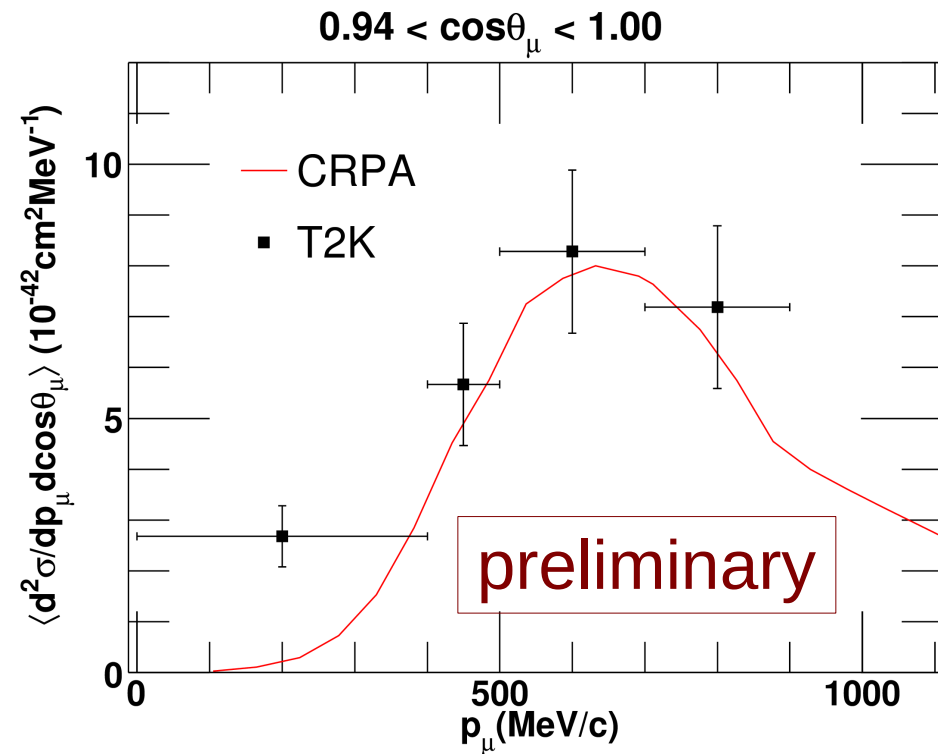
- Example from  $^{12}\text{C}(e,e')$

CRPA  
RgFG

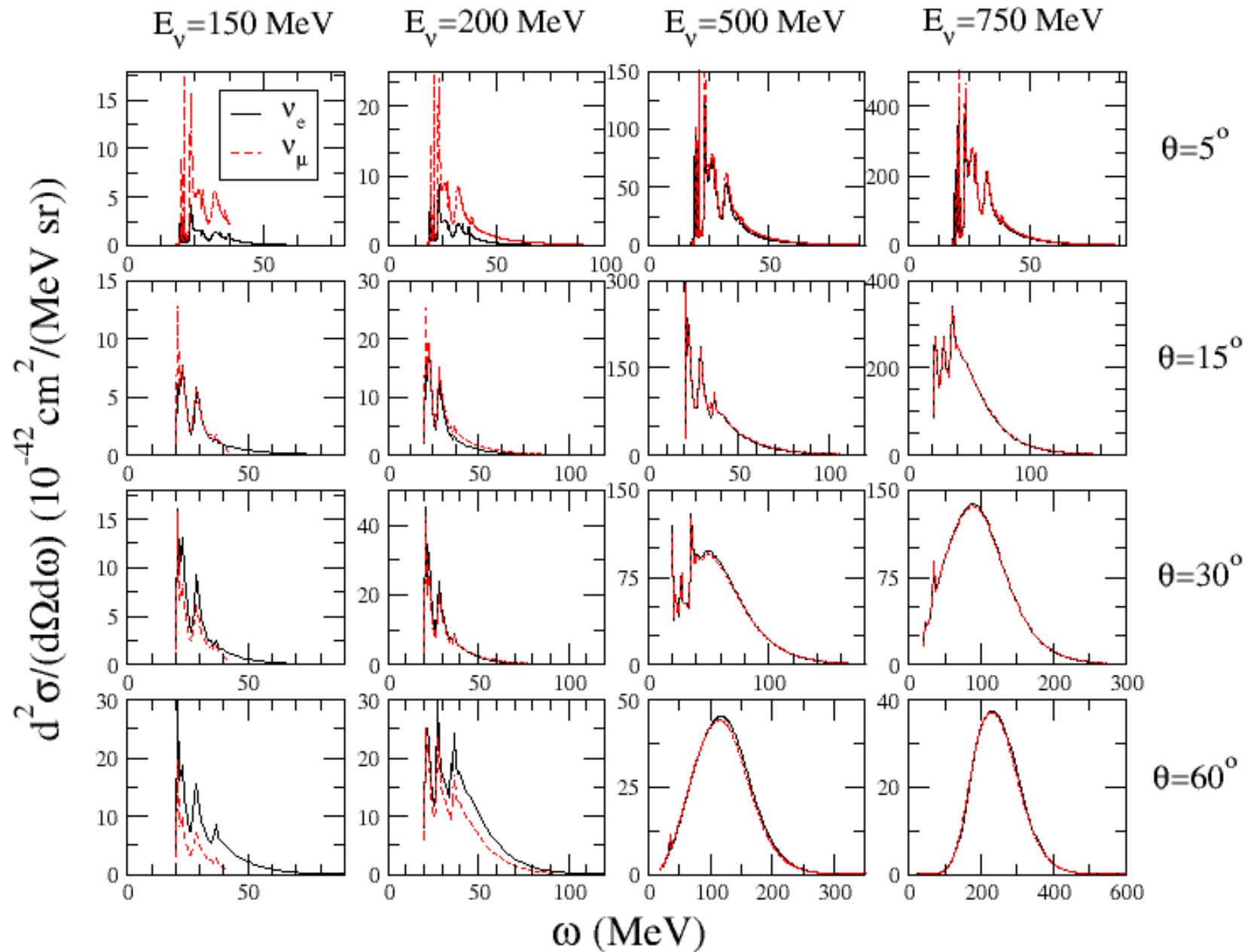


# Impact of low-energy excitations: Forward scattering (flux-folded)

- Significant contribution is expected from low-energy nuclear excitation ( $\omega < 50$  MeV).
- A quantitative analysis is underprogress.

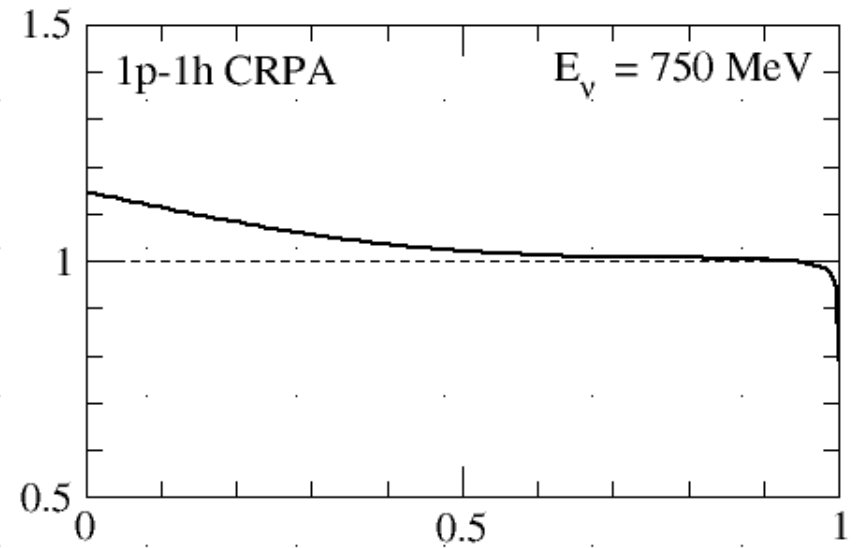
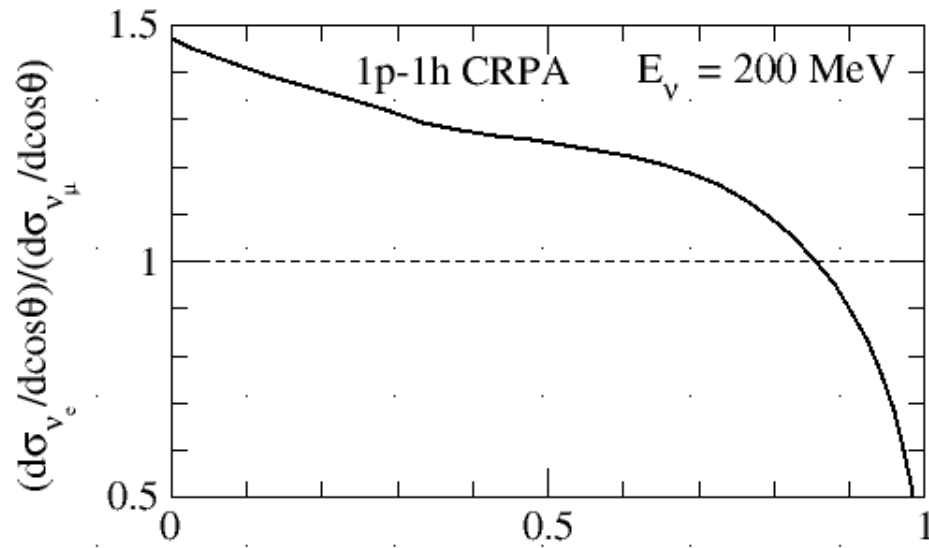


# Impact of low energies: $\nu_e$ vs $\nu_\mu$





# Impact of low energies: $\nu_e$ vs $\nu_\mu$



# Summary

- We present a CRPA approach for QE electron- and neutrino-nucleus scatterings. The model is validated against electron scattering data.
- We compare flux-folded neutrino-nucleus cross-section predictions with MiniBooNE and T2K experimental measurements. Our calculations successfully describe the gross feature of the measurements but underestimate the data. Missing strength can be associated to the processes beyond QE (np-nh, pion production, etc.), not present in our description.

## Impact of low energies

- We draw special attention to the contributions emerging from low-energy nuclear excitations ( $\omega < 50$  MeV), which remain inaccessible in the RFG-based Models, especially for forward scatterings.
- We compare electron-neutrino and muon-neutrino cross sections, relevant for  $\nu_{\mu} \rightarrow \nu_e$  oscillation studies, we observed some non-trivial behavior at low energies.

## References

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### *Electron-scattering data are from:*

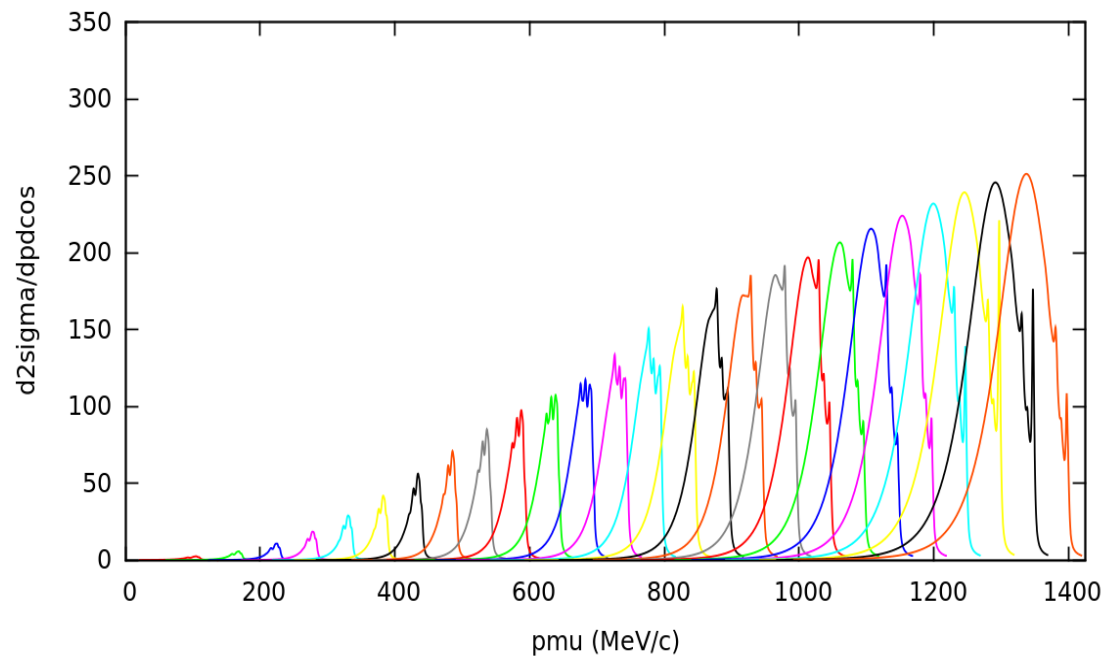
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Back-up

$\cos(\theta)=0.97$



$\cos(\theta)=0.97$

