Is there room (or how much room is there) for 2p 2h contributions to neutrino-nucleus cross-sections according to MiniBoone data?

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Nuclei are complex interacting systems

Please forgive me for introducing some ***kindergarten*** physics:

$$\mathcal{H}\ \boldsymbol{\psi}(1,\ldots,A)= [\Sigma\ \mathbf{p}_i/2M + V(1,\ldots.,A)]\ \boldsymbol{\psi}(1,\ldots,A)$$

This problem can be *exactly* rewritten using the following:

$$V(1,\ldots.,A)=\mathcal{V}(1)+\ldots+\mathcal{V}(A)+[V(1,\ldots,A)-\mathcal{V}(1)-\ldots\mathcal{V}(A)]$$

Thus:

$$\mathcal{H}\ \boldsymbol{\psi}(1,\ldots,A)= [\Sigma[\mathbf{p}_i/2M + \mathcal{V}(i)]\ + \mathcal{C}(1,\ldots,A)\ ]\boldsymbol{\psi}(1,\ldots,A)$$

Where $\mathcal{C}(1,\ldots,A)$ represents the *explicit* correlation content for this extraction of the central part. There are as many extractions or representatios of the mean field or central part as we want.

Choice of central part

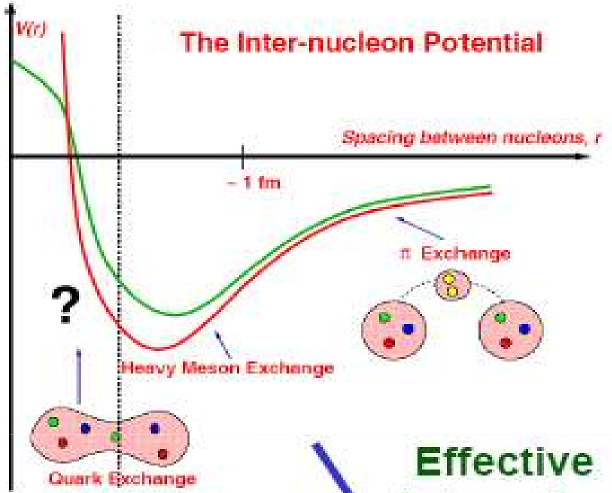
For the central part one may pick an harmonic oscillator. This is easy to solve, but it may not yield the smallest *explicit* correlation term.

We may deal with the problem in perturbation theory, taking the mean field term as the 'large' term, and the explicit correlations as the perturbation. In such case, the first order solution of the problem will be given by a (possible antisuymmetrized) product of single-particle states:

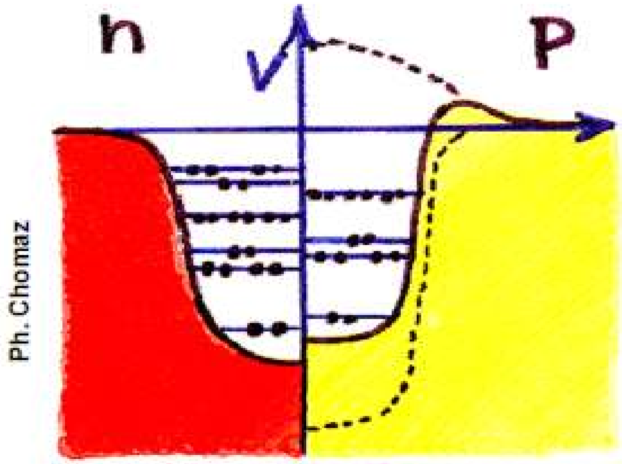
$$\boldsymbol{\psi}(1,\ldots,A) = \varphi_1(1)\ldots\varphi_A(A)$$

Different separations of the central part would yield the same solutions, it sumed up on all orders of perturbation theory (if convergence is reached, **which for most separations is NOT**). But to a given order, differences will arise.

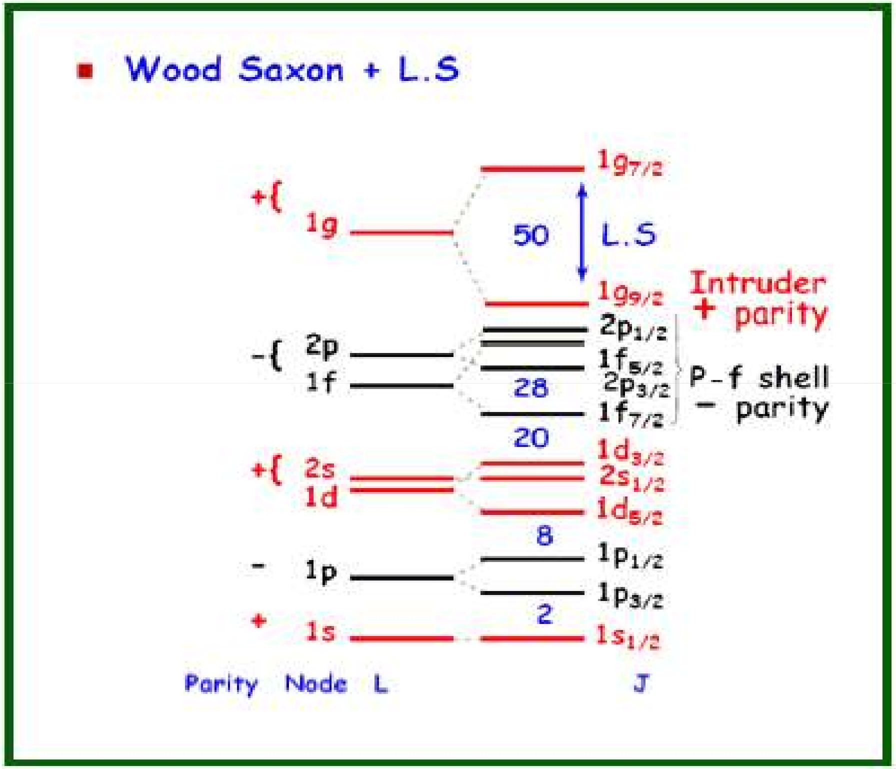
Mean Field Model of Nuclei



Effective interaction



Nucleons in a self-consistent mean-field



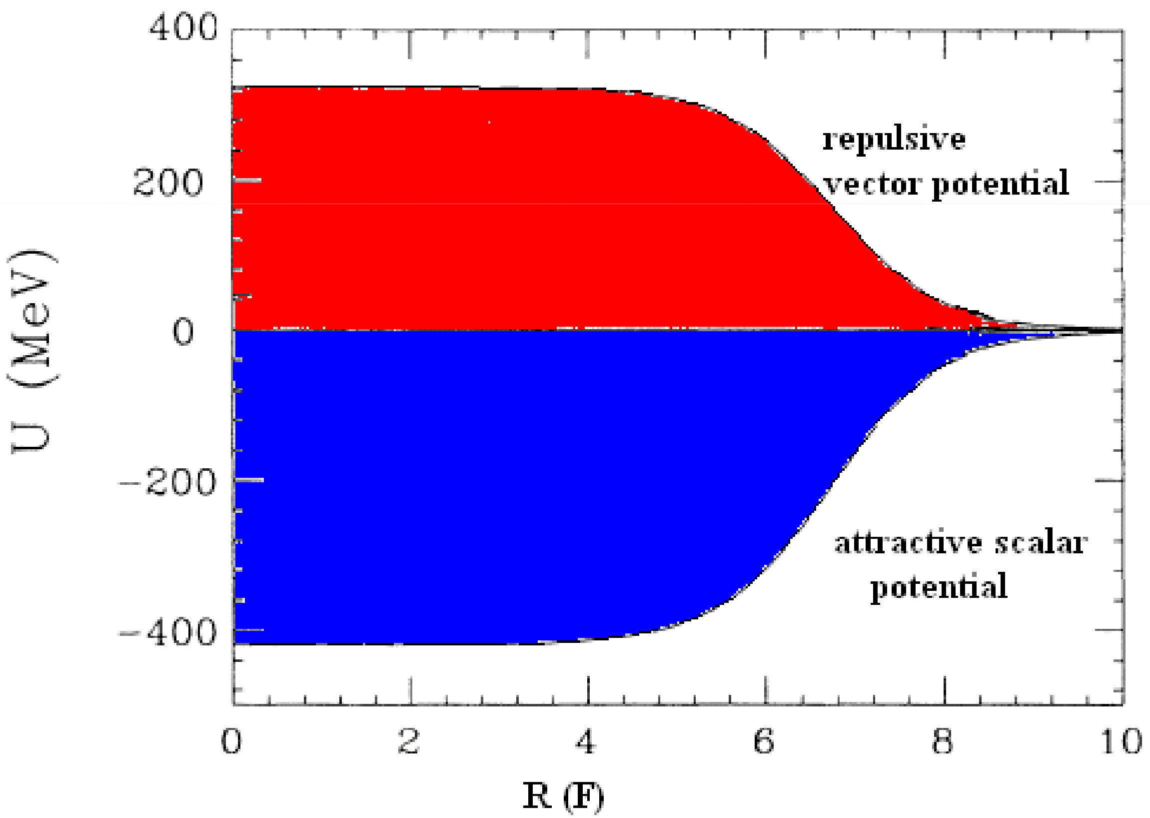
- fermion system at low energies
- suppression of collisions by Pauli exclusion
- independent particle motion
- shell structure
- mean field approximation

Self Consistent Mean Field

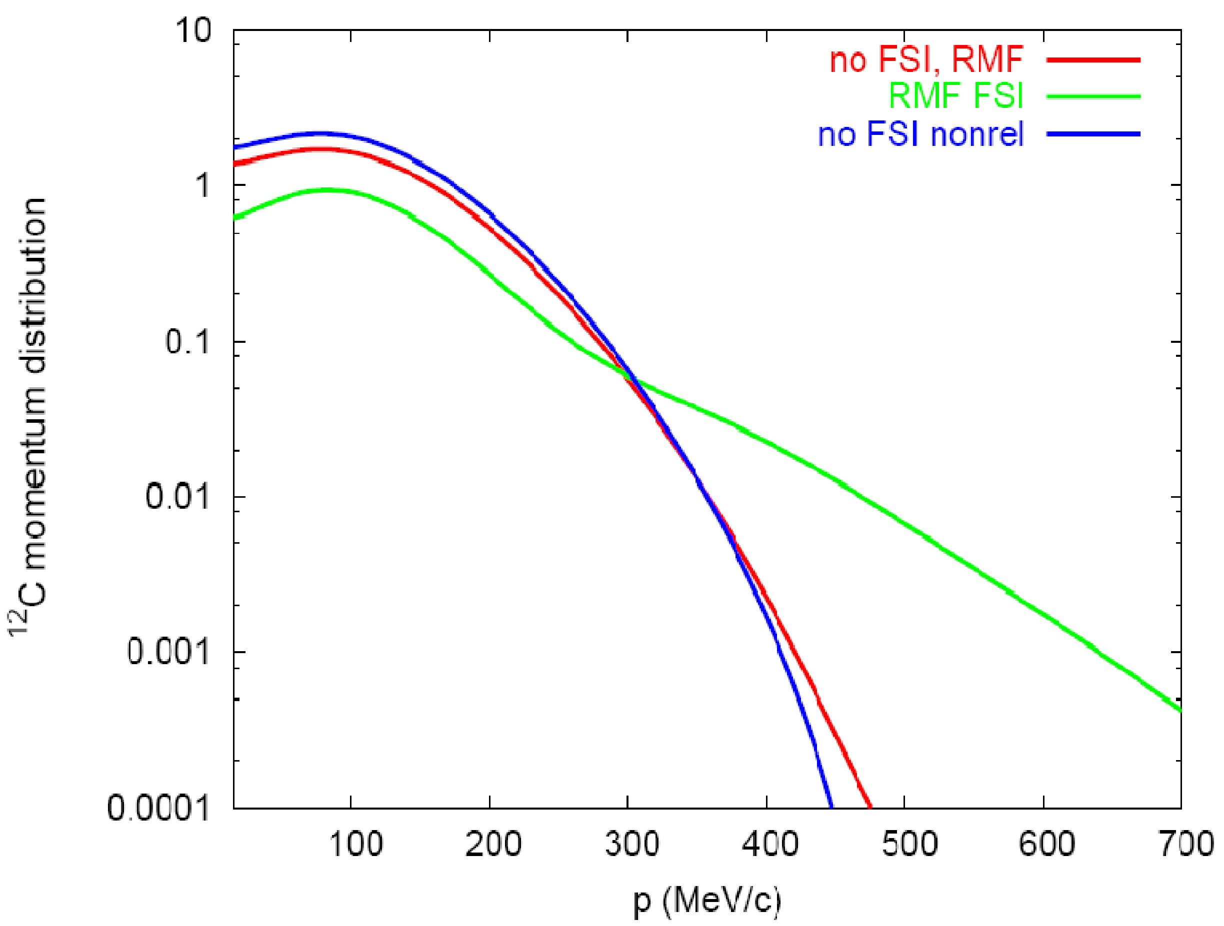
The problem can also be addressed in a variational fashion. We look for the solution written in terms of the single-particle wave function which minimizes the energy. This yields the self-consistence (Hartree or Hartree-Fock) equations, potentials and wave functions. These would be the ones minimizing explicit correlations.

And further, this can be done in a *non-relativistic* or in *a relativistic* way.

Within the (self-consistent) Relativistic Mean Field, they appear strong mean field potentials, meaning that stronger (implicit) correlations can be represented

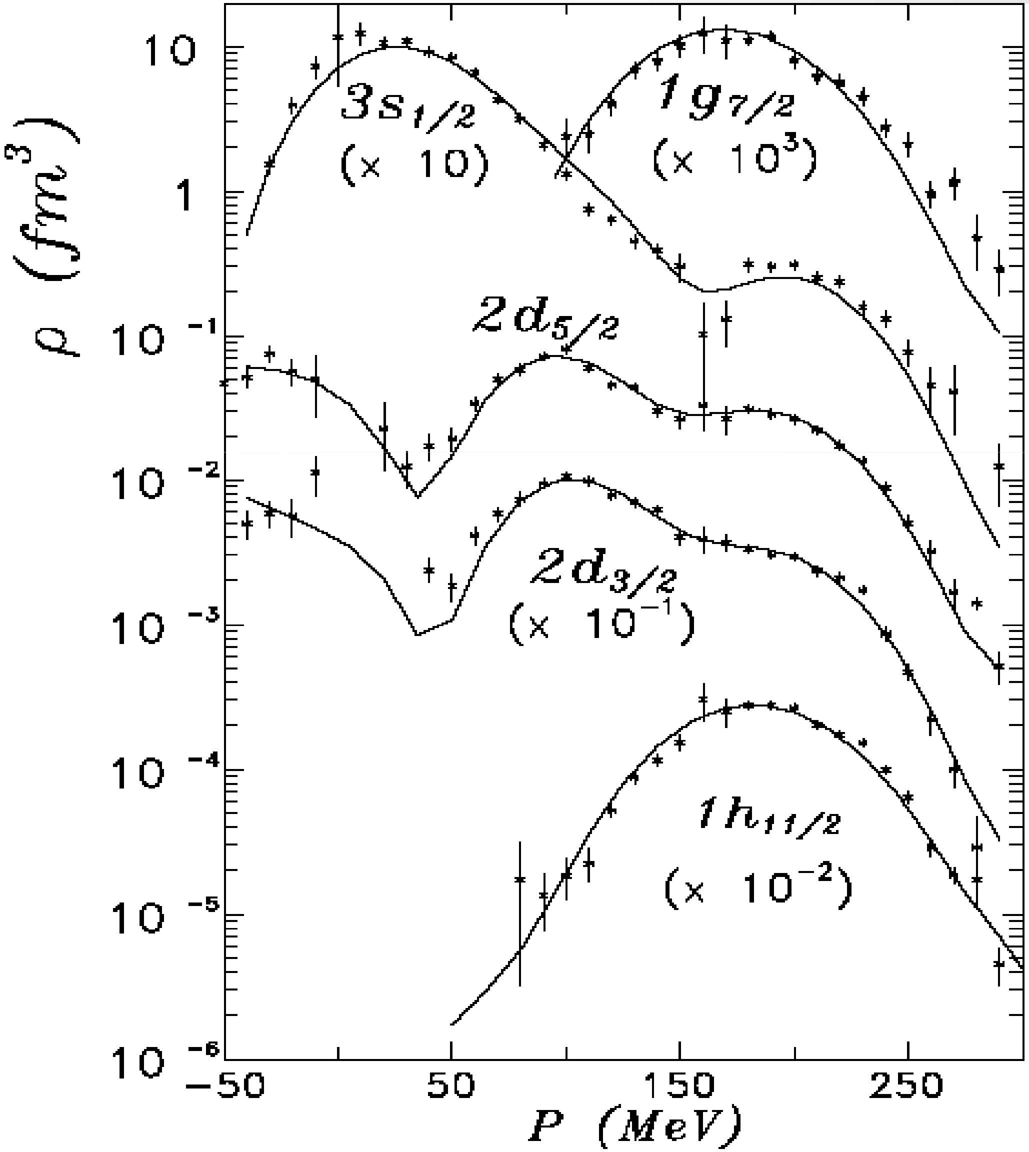


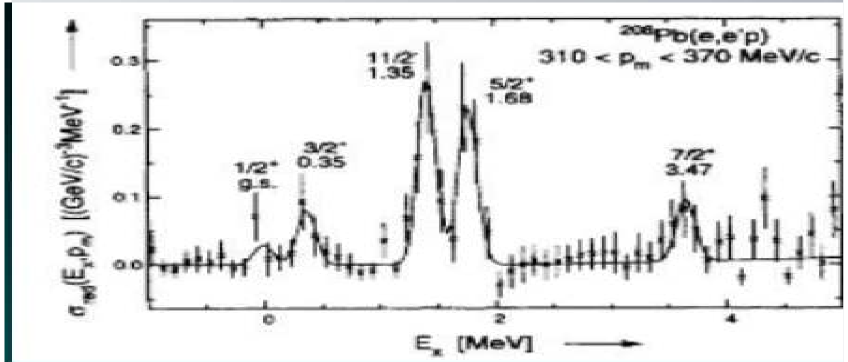
- Strong (hundreds of MeV's) repulsive vector and attractive scalar potentials are obtained with the Dirac treatment
- The small (tens of MeV) binding energy arises as a result of cancellations and is just the 'tip' of the iceberg



- As an example, here we show the distorted momentum distribution computed within RMF for the initial and final states
- The additional strength beyond p=300 MeV/c is due, in the RMF, to the strong potentials in the final state (FSI effects) which enhance the effective contribution of the nucleon at high momentum
- In this representation, the high momentum tail originates in the 'central' part. In another representation, it will originate in the explicit correlations term

The RMF yields good agreement with exclusive (e,e’p) data



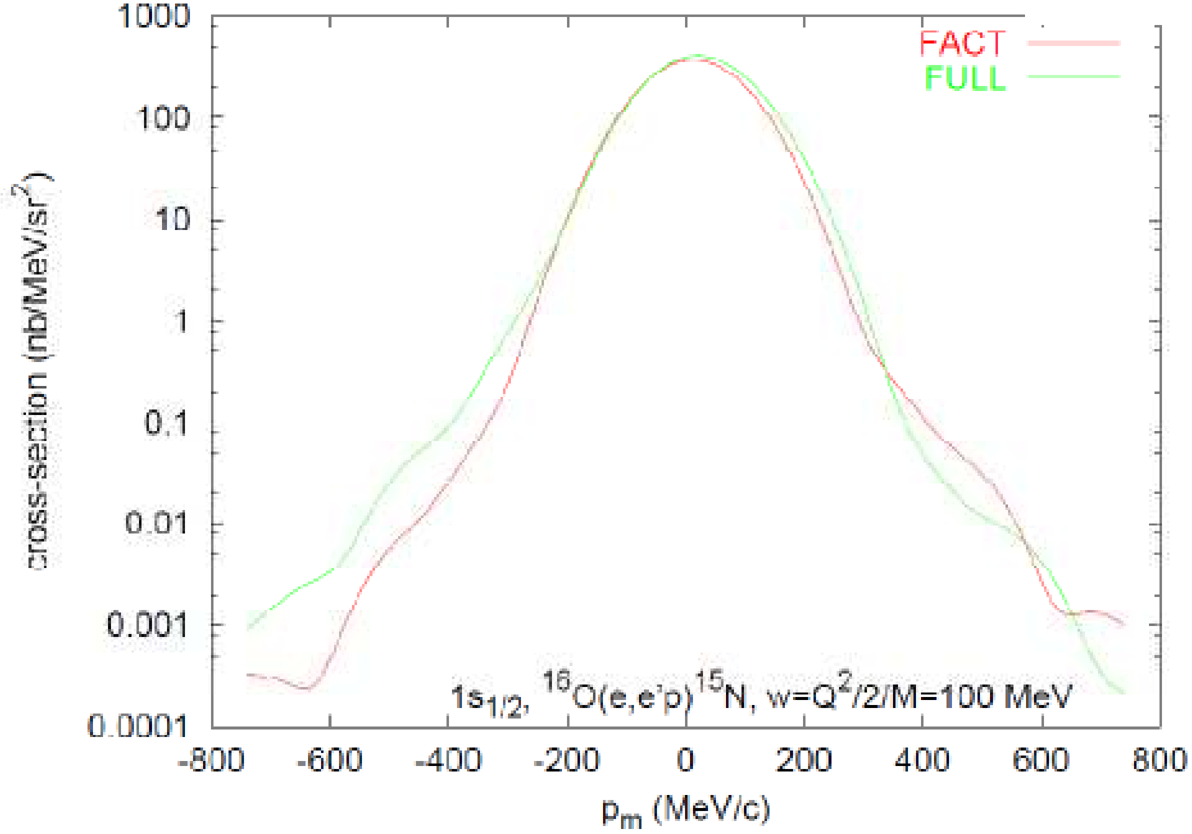


Reasonably good agreement with data under exclusive kinematics

	$3s_{1/2}$	$2d_{3/2}$	$1h_{11/2}$	$2d_{5/2}$	$1g_{7/2}$
Non rel. (Ref. [41])	50 %	53 %	42 %	44 %	19 %
Non rel. (Ref. [42])	55 %	57 %	58 %	54 %	26 %
Rel. (Refs. [40, 6])	70 %	72 %	64 %	60 %	30 %

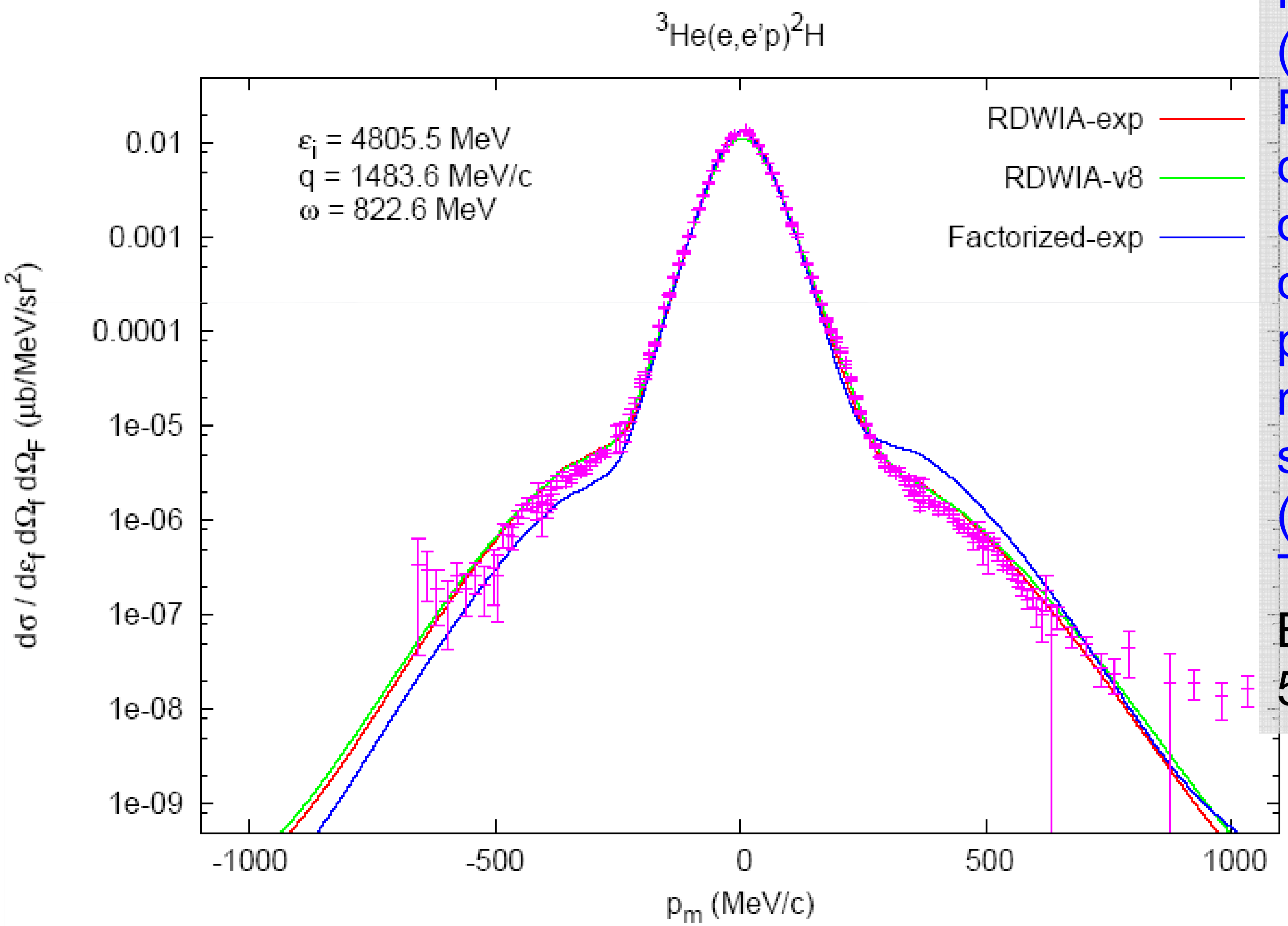
The one-boson exchange approximation allows us to decouple the direct dependence on the *energy* and *scattering angle* of the probe *via* the Mott cross-section for electrons or the equivalent expressions for neutrinos. This is the foundation for factorization

$$\frac{d^5\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{ep} S(E_m, \vec{p}_m) \qquad \rho^{exp}(\mathbf{p}_m) = \frac{\left(\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F}\right)^{exp}}{E_F p_F f_{rec} \sigma_{ep}} .$$



Factorization approach

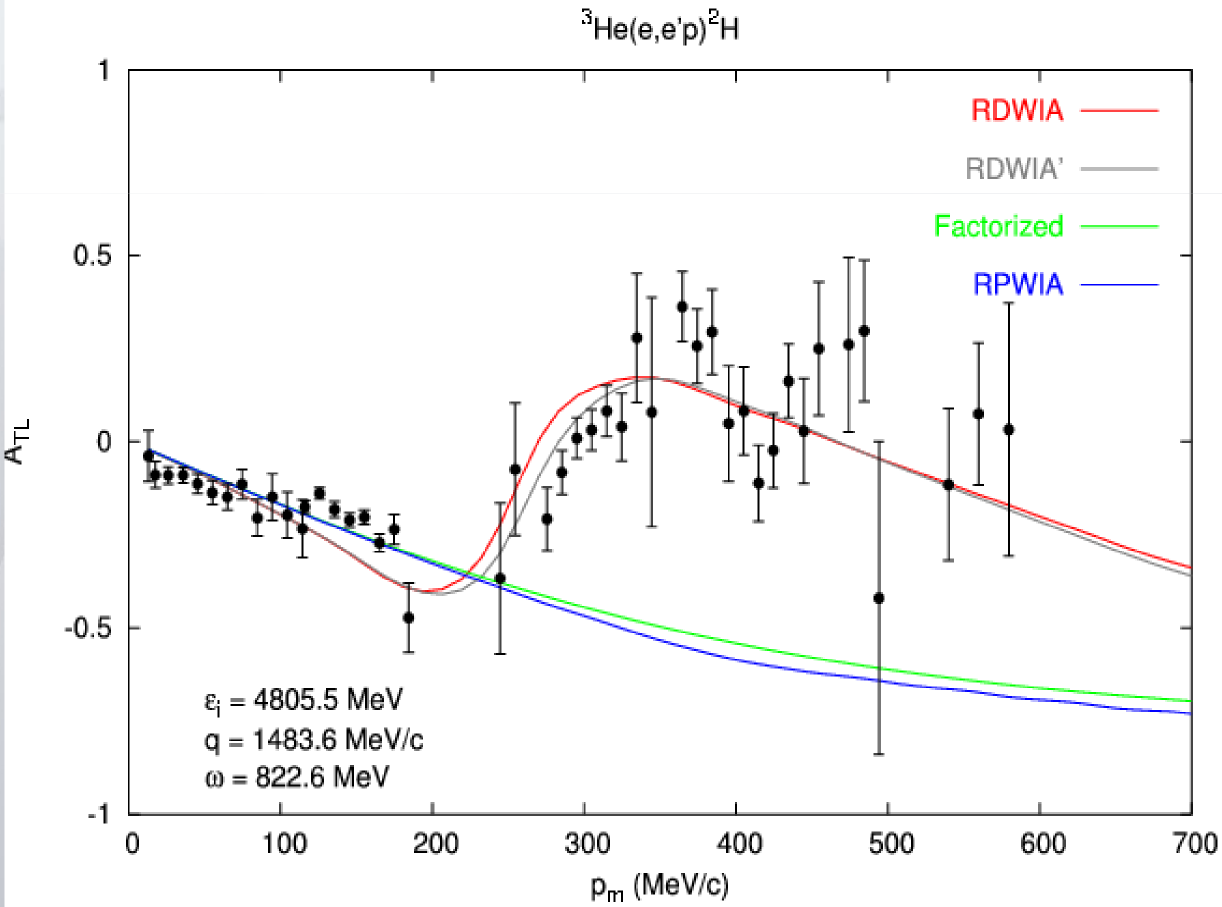
Breakdown of factorization will be seen at **demanding** kinematics (q-ω constant, high momentum)



Data: M.M. Ravchev, PRL 94 (2005) 192302
Full theoretical calculation of the overlap from Faddev calculations. No free parameters in these results, not even the spectroscopic factors (of the order of 0.65)
Theory from Few-Body Syst (2011) 50:359

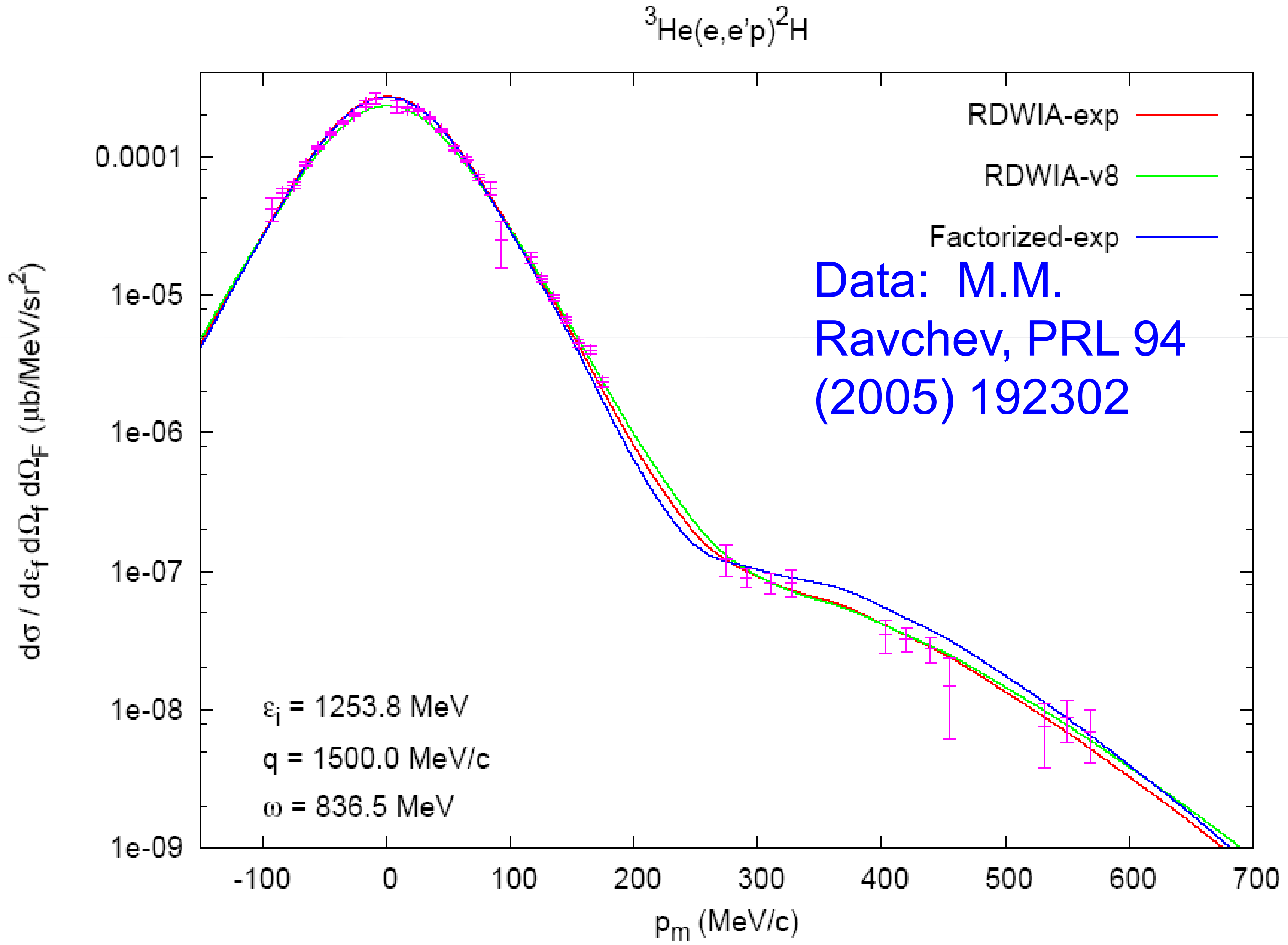
A_{TL} in ^{3}He, ^{4}He and ^{16}O

Asymmetry measured in (e,e'p) exclusive reactions. There are relativistic dynamical effects with a strong impact on A_{TL} which would be seen, particularly at moderate p_m.There is a noticeable difference in A_{TL} predictions for ^{3}He due to relativistic dynamics. This asymmetry is recovered with a relativistic potential in the FSI, within this approach. In other approaches it comes from MEC/beyond tree level diagrams.



M. Rvachev et al. PRL 94:12320,2005

Actually this experiment is interesting, light system, calculations based on ‘exact’ nuclear functions, no adjustable spectroscopic factor, FSI taken from a ***folding model optical potential, with no free parameters.***



Exclusive data at QE kinematics are well described within the RMF, with relativistic optical potentials (ROP) for the final state. But these are not suited for the inclusive case, for which we want to consider all possible final states. ROP include an imaginary term to implement the absorption, or flux lost into the unobserved (non exclusive) channels.

For the inclusive reaction, we can use the RMF with the same real potentials for the bound and the final nucleon. This fits very well the scaling response for the QE peak obtained from inclusive (e,e’) data at low-intermediate momentum transfer (say upto 1.5 GeV/c). Thus it can be considered that the RMF represents the nucleonic 1p1h contribution to the inclusive response at intermediate energy. As the energy increases, however, and because it lacks energy dependence, RMF is too strong and its prediction evaentually departs from the data.

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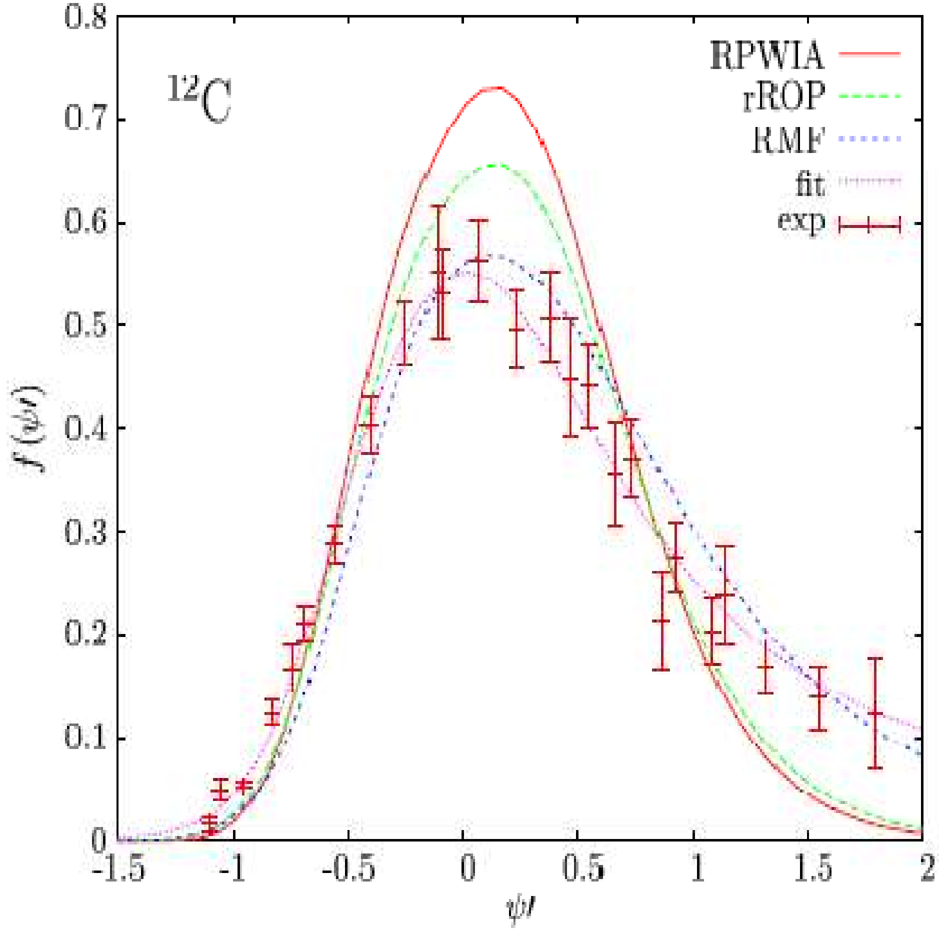
Comparison to inclusive data: Scaling analyses (J.A. Caballero et al., PRL 95 (2005) 252502)



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There is a formal tool which allows to build inclusive responses from the optical potentials fitted to the elastic nucleon scattering data.

Relativistic Green Function

(A. Meucci, F. Capuzzi, C. Giust, D. Pacati,
PRC67(2003)054601, NPA739 (2004)277)

- In this approach one can formally build the inclusive response from the optical potential fitted to reproduce nucleon elastic scattering data.
- It needs the optical potential, usually taken from phenomenological fits, in a large energy range if possible.

Relativistic Green Function

(A. Meucci, F. Capuzzi, C. Giust, D. Pacati, PRC67(2003)054601, NPA739 (2004)277)

$$W^{\mu\nu}(\omega,q) = \overline{\sum_i} \sum\!\!\!\!\!\!\int_f \langle \Psi_f \mid J^\mu(q) \mid \Psi_i\rangle \langle \Psi_i \mid J^{\nu\dagger}(q) \mid \Psi_f\rangle \times \delta(E_i + \omega - E_f),$$

$$W^{\mu\mu}(\omega,q) = -\frac{1}{\pi}\mathrm{Im}\langle \Psi_i \mid J^{\mu\dagger}(q) G(E_f) J^\mu(q) \mid \Psi_i\rangle$$

$$W^{\mu\mu}(\omega,q)) = \sum_n \Bigg[\mathrm{Re}\, T_n^{\mu\mu}(E_f-\varepsilon_n, E_f-\varepsilon_n) - \frac{1}{\pi}\mathcal{P}\int_M^\infty \mathrm{d}\mathcal{E}\frac{1}{E_f-\varepsilon_n-\mathcal{E}}\,\mathrm{Im}\, T_n^{\mu\mu}(\mathcal{E}, E_f-\varepsilon_n)\Bigg]$$

Relativistic Green Function

$$T_n^{\mu\mu}(\mathcal{E},E) = \lambda_n \langle \varphi_n \mid j^{\mu\dagger}(q)\sqrt{1-\mathcal{V}'(E)} \mid \tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle$$
$$\times \langle \chi_{\mathcal{E}}^{(-)}(E) \mid \sqrt{1-\mathcal{V}'(E)} j^{\mu}(q) \mid \varphi_n \rangle \,.$$

$$(\mathcal{E} - h^{\dagger}(E)) \mid \chi_{\mathcal{E}}^{(-)}(E)\rangle = 0$$

$$(\mathcal{E} - h(E)) \mid \tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle = 0$$

It is relatively simple to implement, it has the same ingredients than the RMF+ROP calculation, apart from an factor related to the derivative of the potentials on the energy, one has to take matrix elements between the bound state and the scattering state with the absorptive optical potential. In the RGF, there is also a matrix element with an scattering state computed with the complex conjugated optical potential, which compensates the absorption effects.

Comparison with inclusive electron scattering data is OK

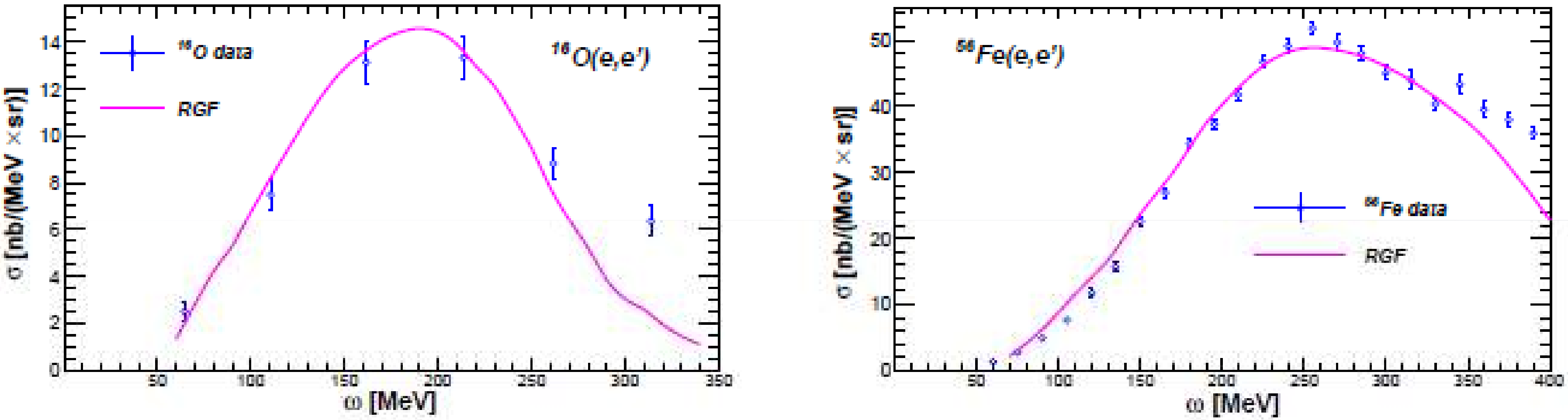
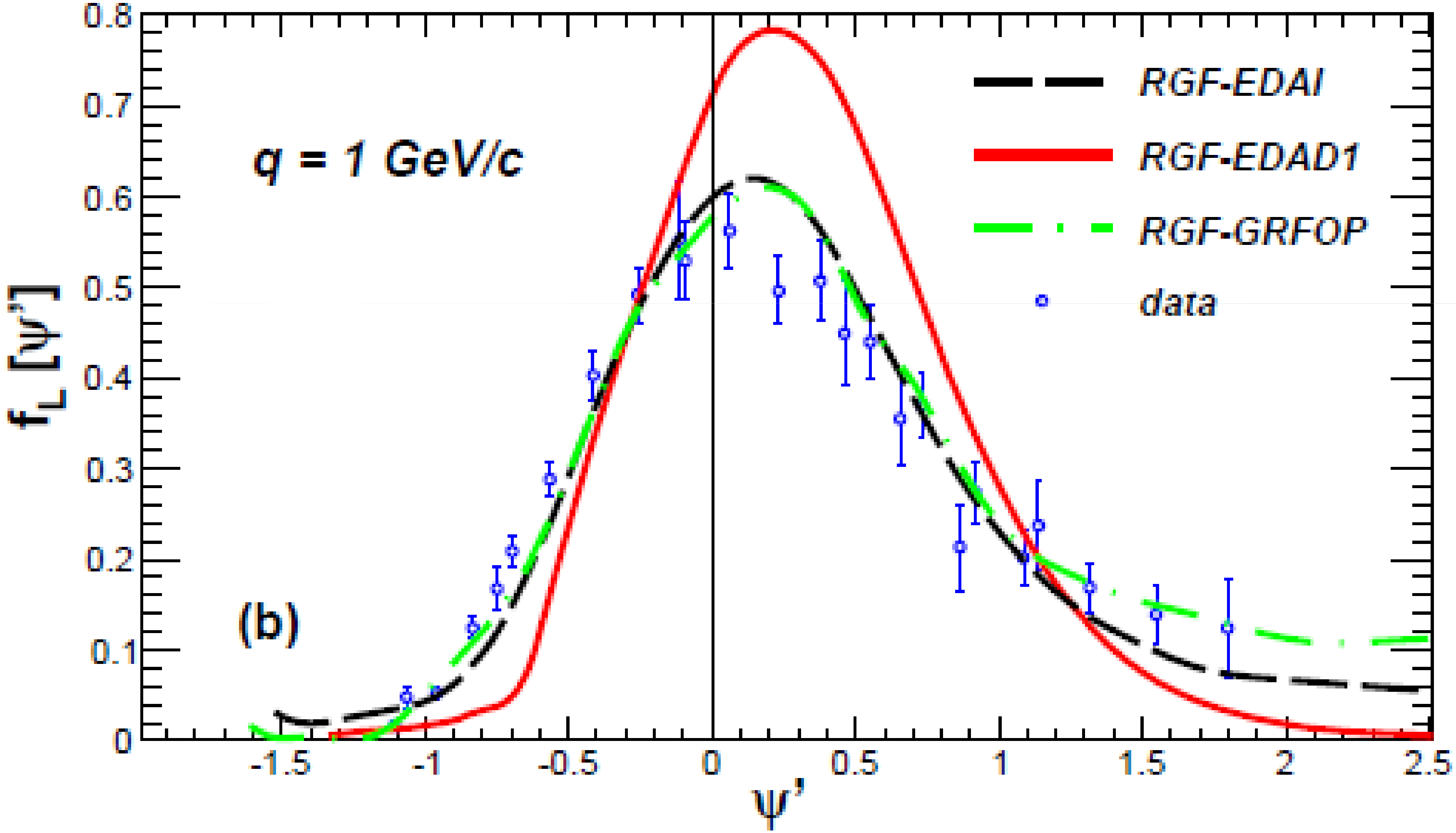
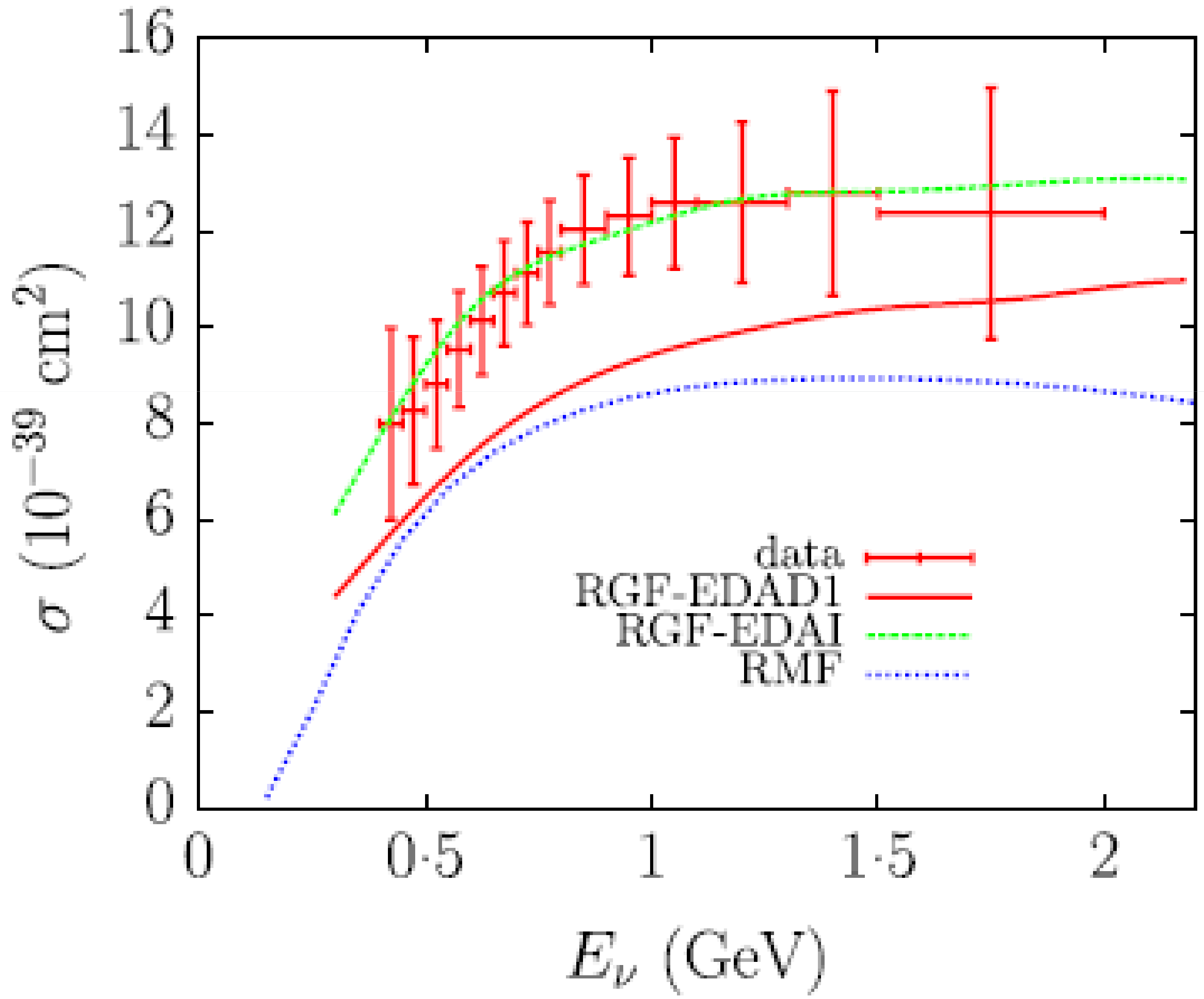


Figure 1. Differential cross sections of the reactions $^{16}\mathrm{O}(e,e')$ (beam energy $\varepsilon = 1080$ MeV and scattering angle $\vartheta = 32^{\circ}$) and $^{56}\mathrm{Fe}(e,e')$ ($\varepsilon = 2020$ MeV and $\vartheta = 20^{\circ}$) calculated in the RGF-DEM. Experimental data from [30] (^{16}O) and [31] (^{56}Fe).

RGF can be compared also with the electron scaling function



Comparison to MiniBoone CCQE neutrino-carbon data



GLOBAL OPTICAL POTENTIAL FOLDING APPROACH AND THE RELATIVISTIC GREEN FUNCTION APPROACH

M. Ivanov, J.R. Vignote, R. Álvarez Rodríguez, C. Giusti, A. Meucci and JM Udías

Goal: To obtain a more resticted geometry global potential, valid in a wide range of energies, covering (most of) all available data

Based on the relativistic folding model (after Horowitz, Love and Franey), the optical potential is obtained by folding the nuclear density with an effective NN interaction. Parameters of the NN interaction are taken from NN scattering data and/or fitted to some observables.

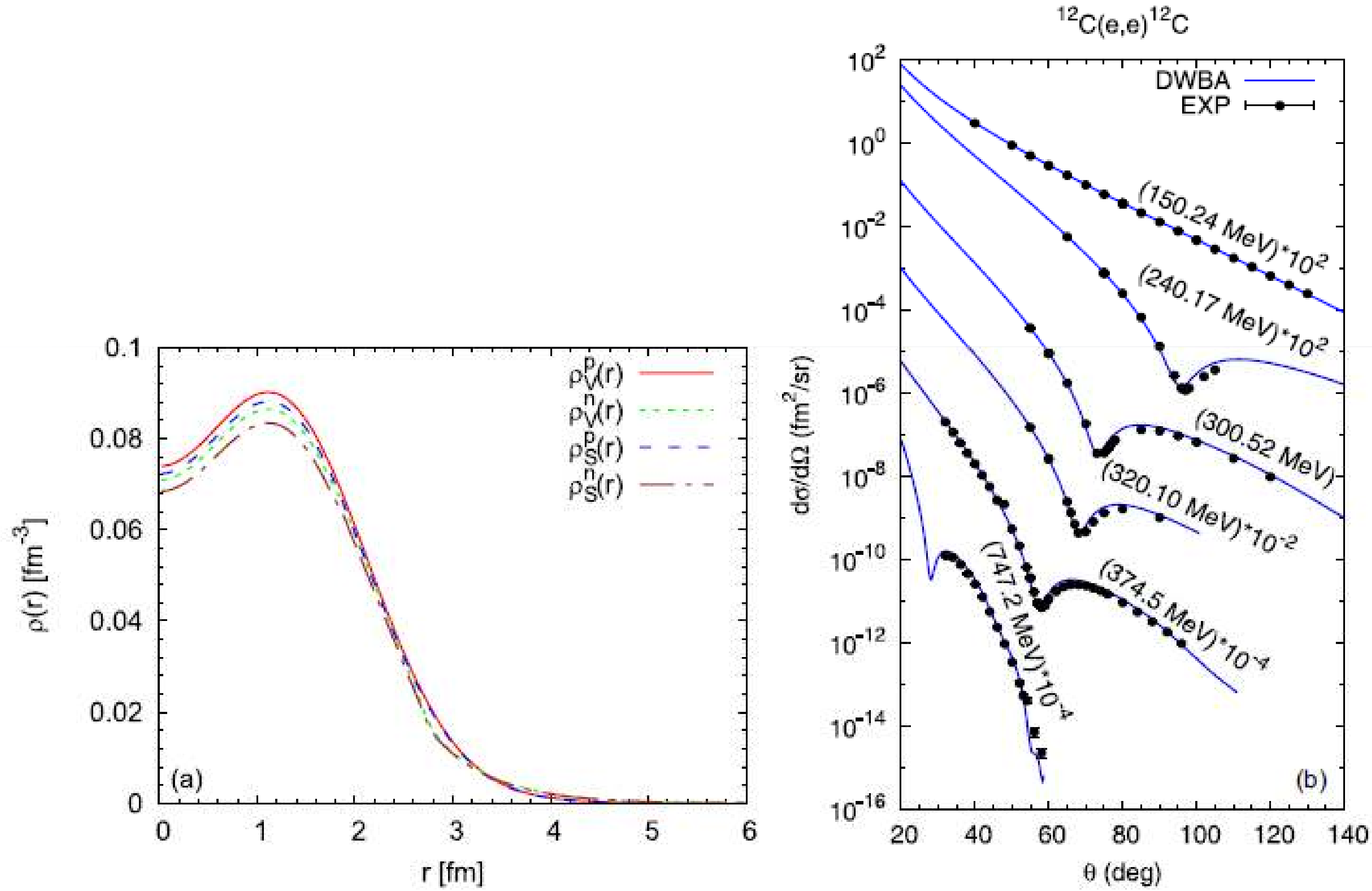
We first fix the nuclear density, for the proton vector, taking it from the experimental charge distribution. The scalar proton density will be given by the ratio vector/proton obtained from the RMF calculation with NLSH parameters. Neutron densities are taken from the experimental proton density scaled in the spatial direction in the same way as the RMF.

$$\tilde{\rho}^p_V(q) = \frac{\int d^3q e^{i\vec{q}\cdot\vec{r}} \rho_c(r)}{G(q)}$$

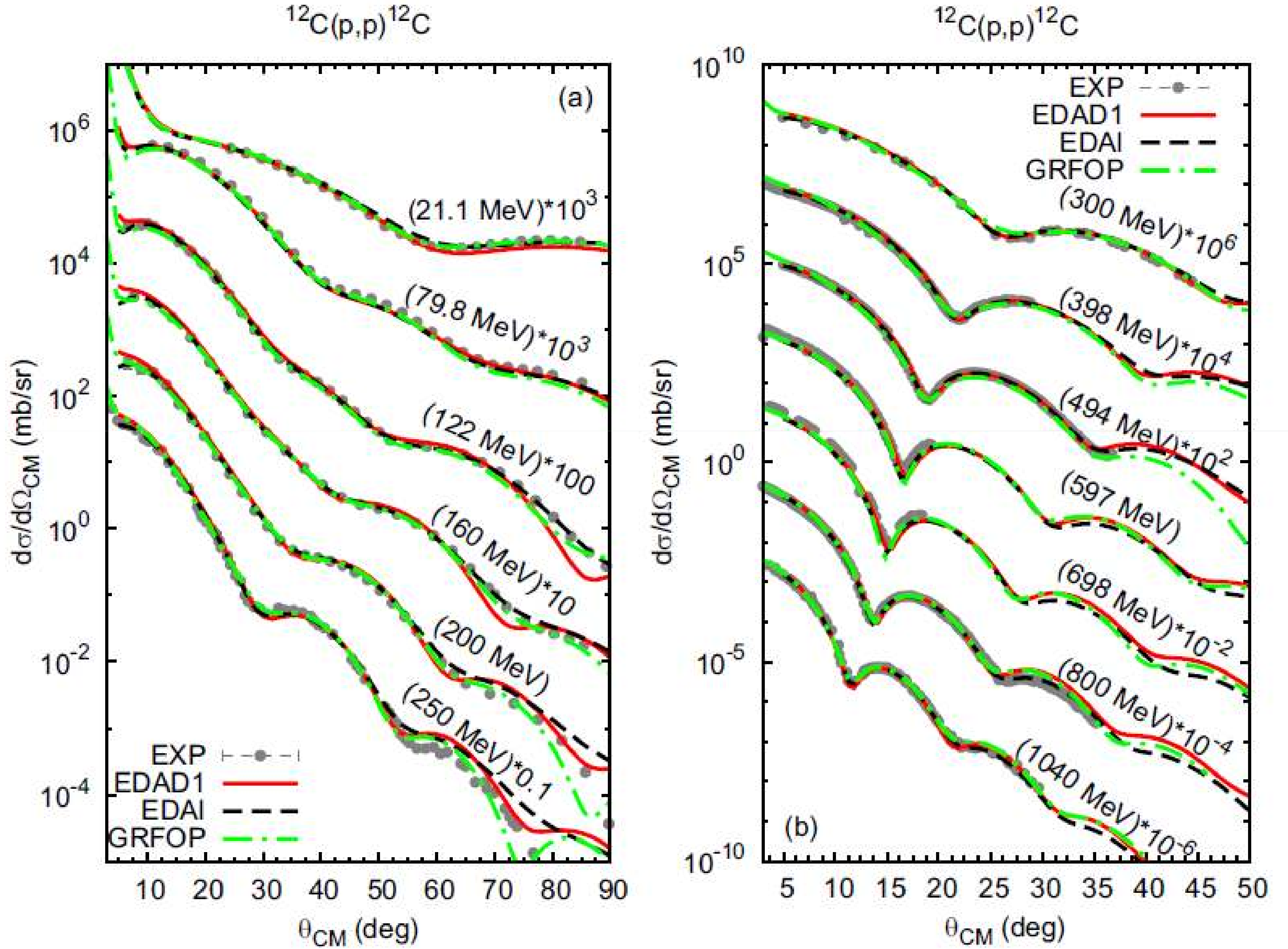
$$\rho_S(r) \simeq \frac{\int \rho_S^{\mathrm{NLSH}}(r) r^2 dr}{\int \rho_V^{\mathrm{NLSH}}(r) r^2 dr} \rho_V(r)$$

gfn UCM
Grupo de Física Nuclear

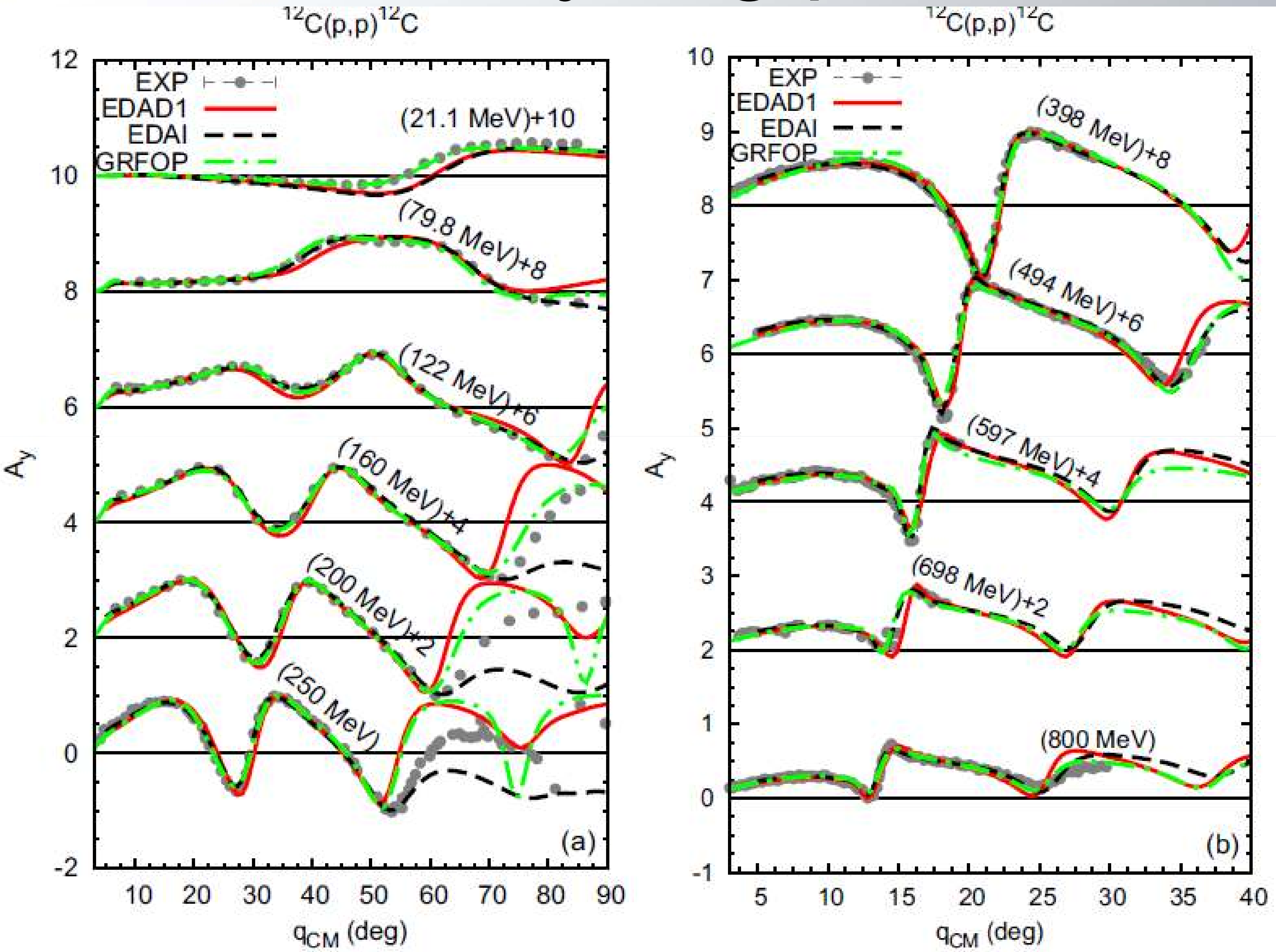
Nuclear charge densities are in good agreement with data



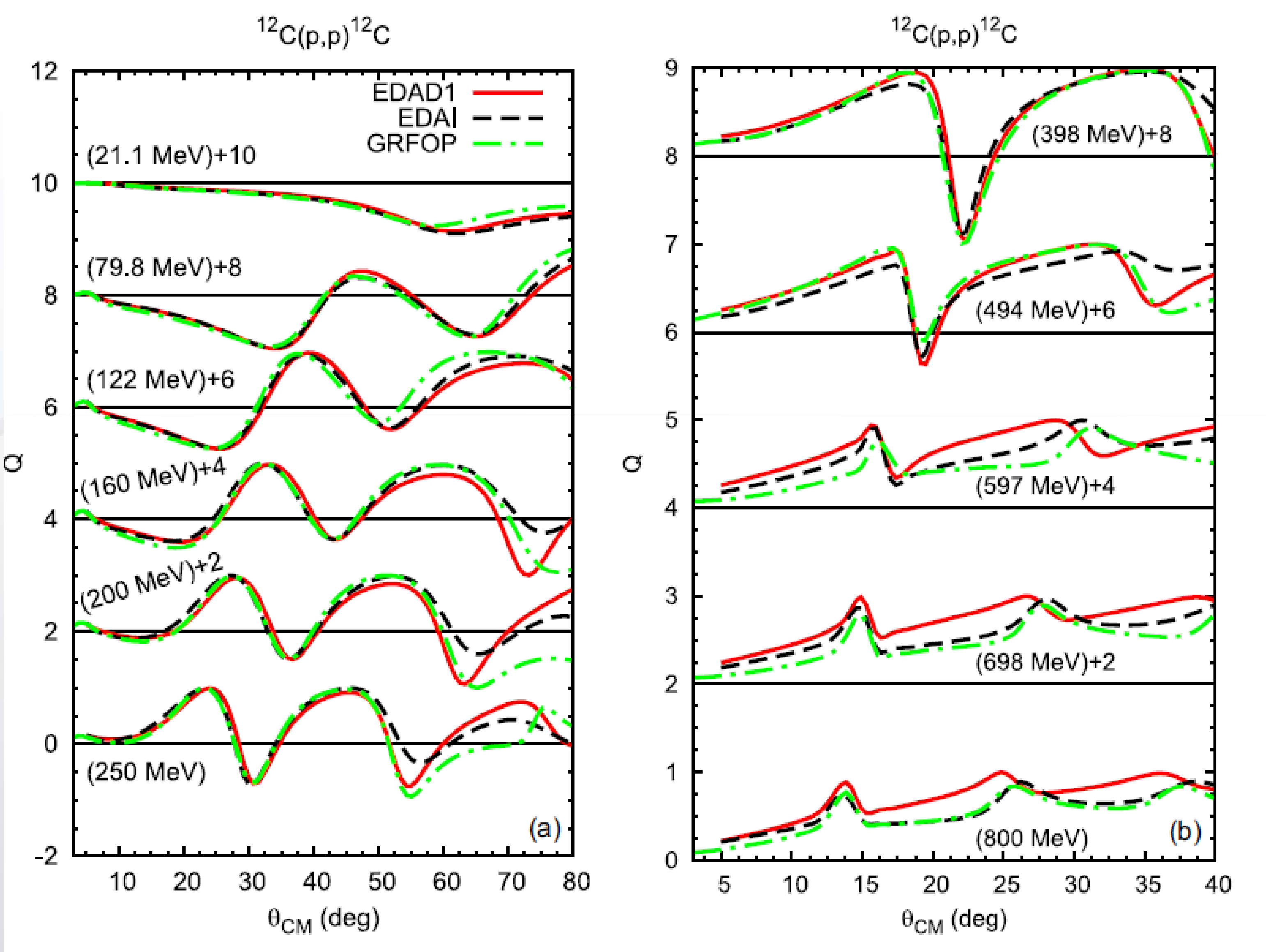
Fit of cross-sections



Fit of Analyzing power



Spin rotation functions



Fit results: summary

$$\chi^2_{\rm pdf} = \frac{1}{N - N_p} \sum_{j=1}^{N_S} \left[N_\sigma(j)\chi^2_\sigma(j) + N_{A_y}(j)\chi^2_{A_y}(j) \right]$$

$$\chi^2_{\rm pdf}({\rm EDAI}) = 2.2,\ \chi^2_{\rm pdf}({\rm GRFOP}) = 4.7,\ \chi^2_{\rm pdf}({\rm EDAD1}) = 5.6$$

Not such a good job as the A-specific purely phenomenological fit, but better than the A-dependent fit, with approximately the same number of parameters. None of the existing RIA folding parametrizations (RLF from Horowitz, MRW from Tjon, Maxwell high and low energy parametrizations and Hillhouse parameterizations cover the whole range of energy and/or fits the ^{12}C proton scattering data at the same level.

Comparison of potentials

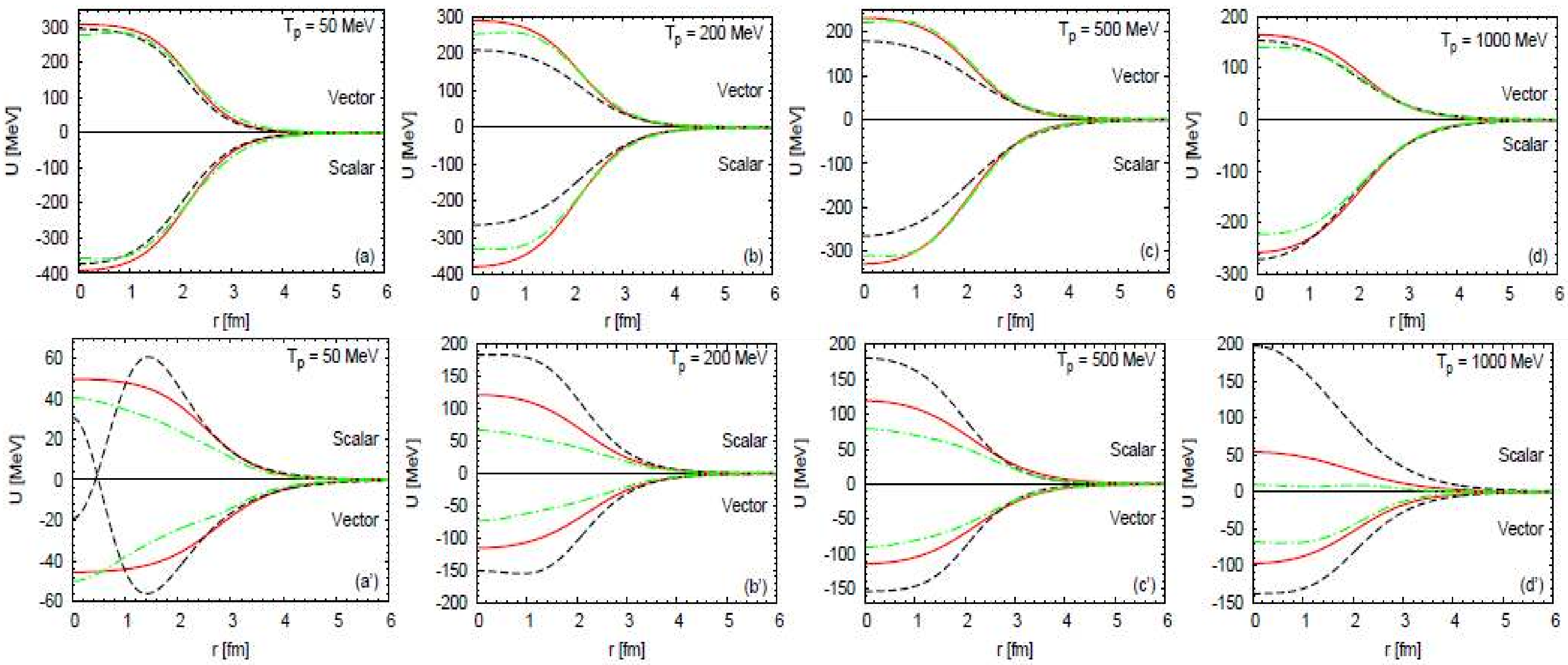
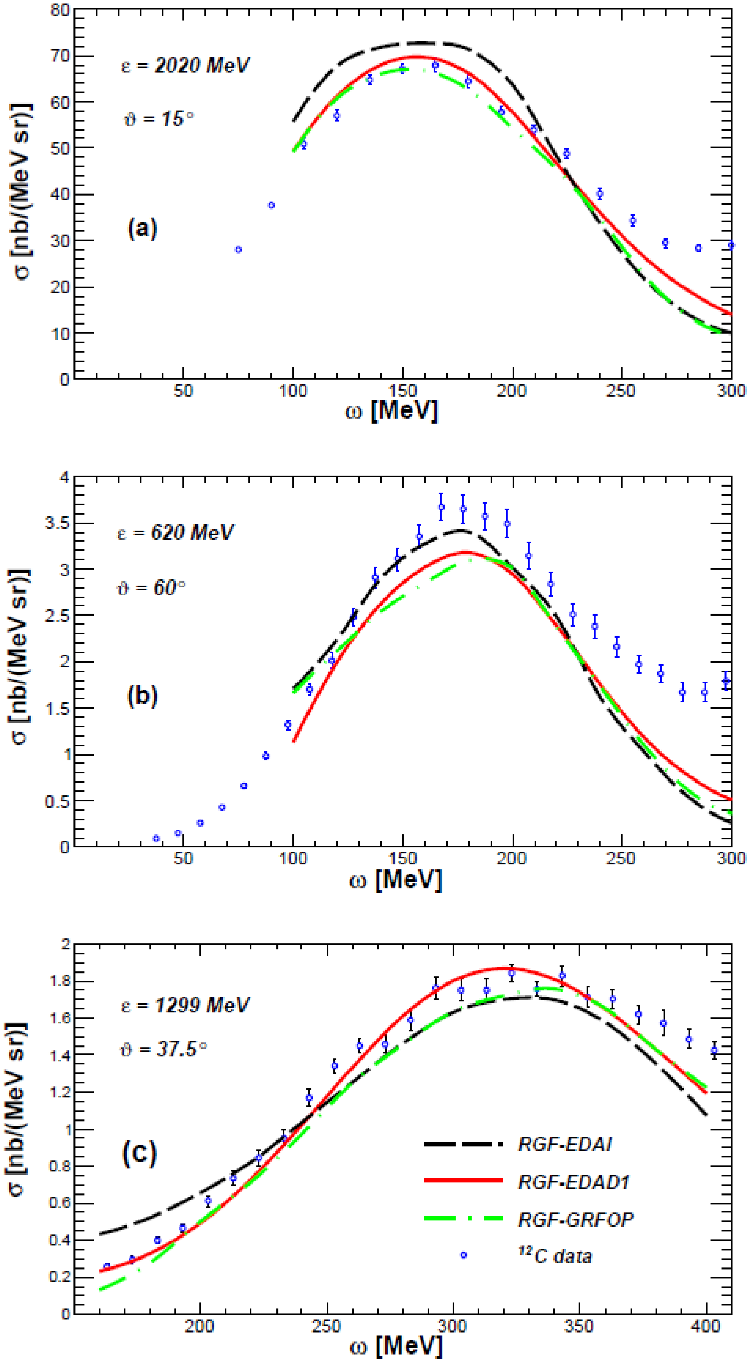
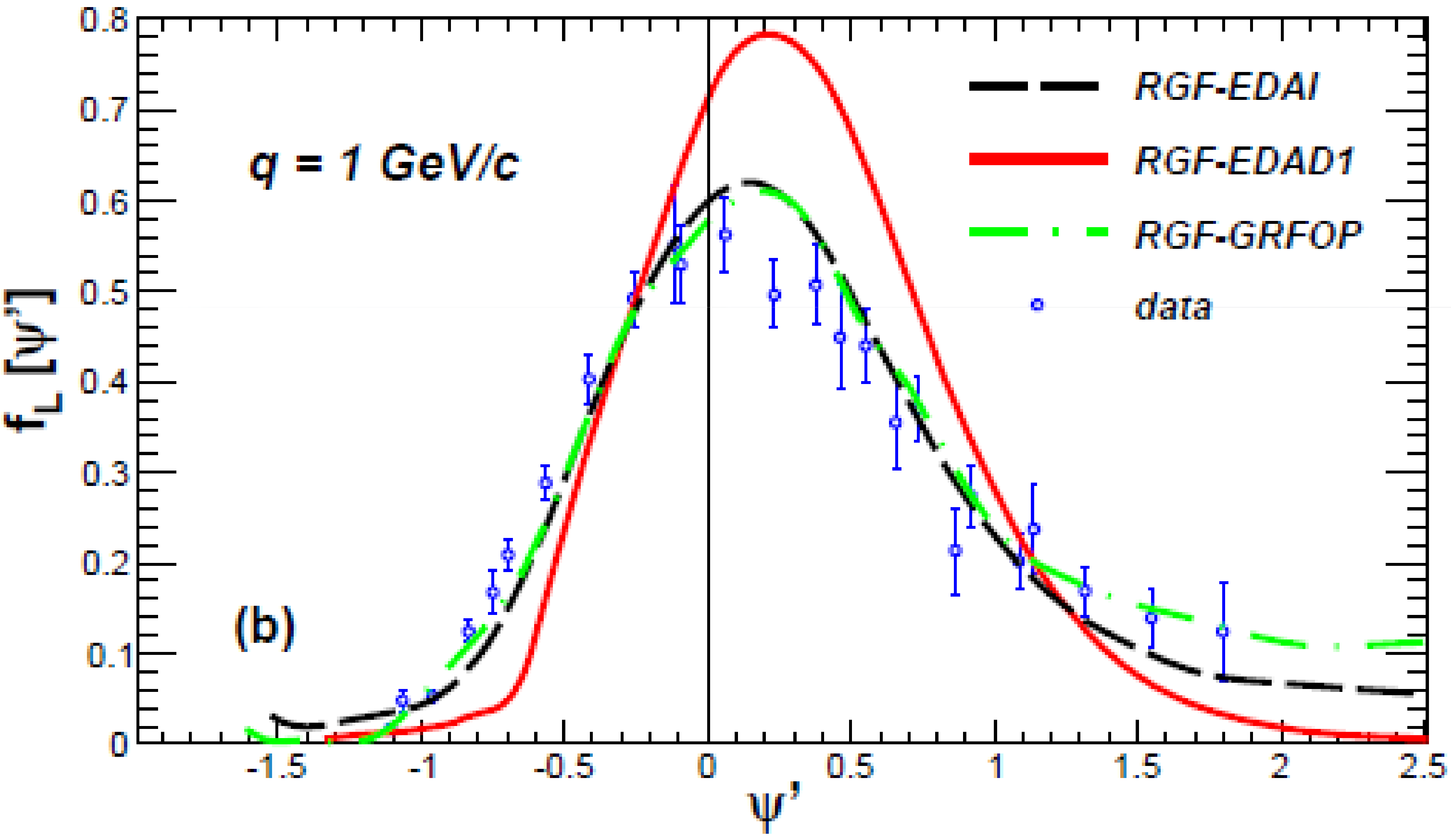


FIG. 5. (Color online) The real scalar and vector optical potentials for ^{12}C at T_p equal 50 MeV (a), 200 MeV (b)500 MeV (c), and 1000 MeV (d): EDAI (dashed lines), EDAD1 (solid lines), and GRFOP (dash-dotted lines). The corresponding imaginary parts are shown in (a′)–(d′) panels.

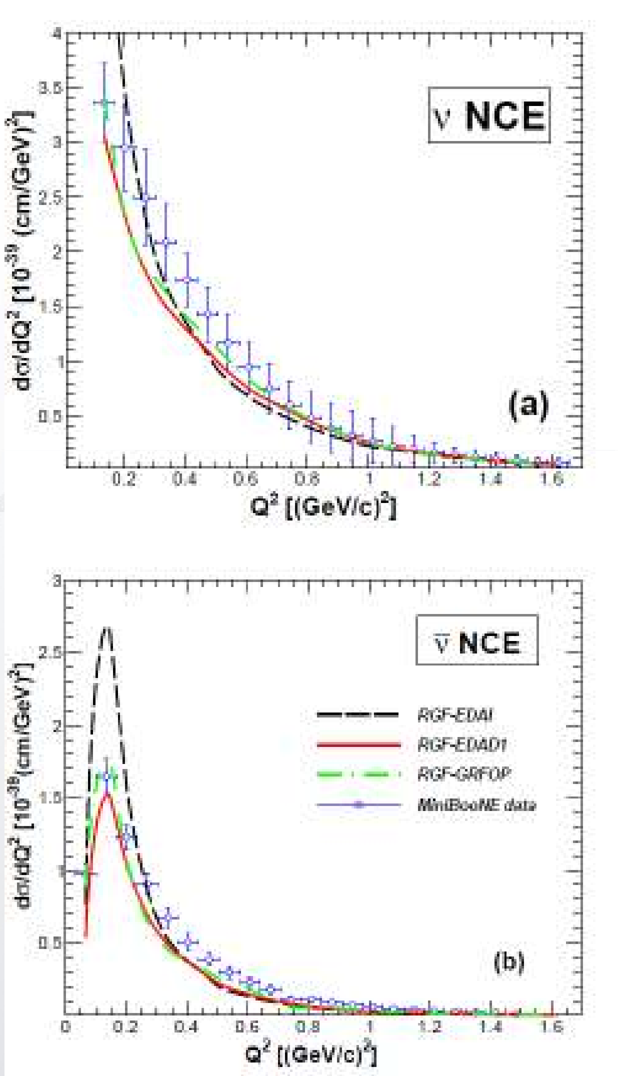
(e,e') results

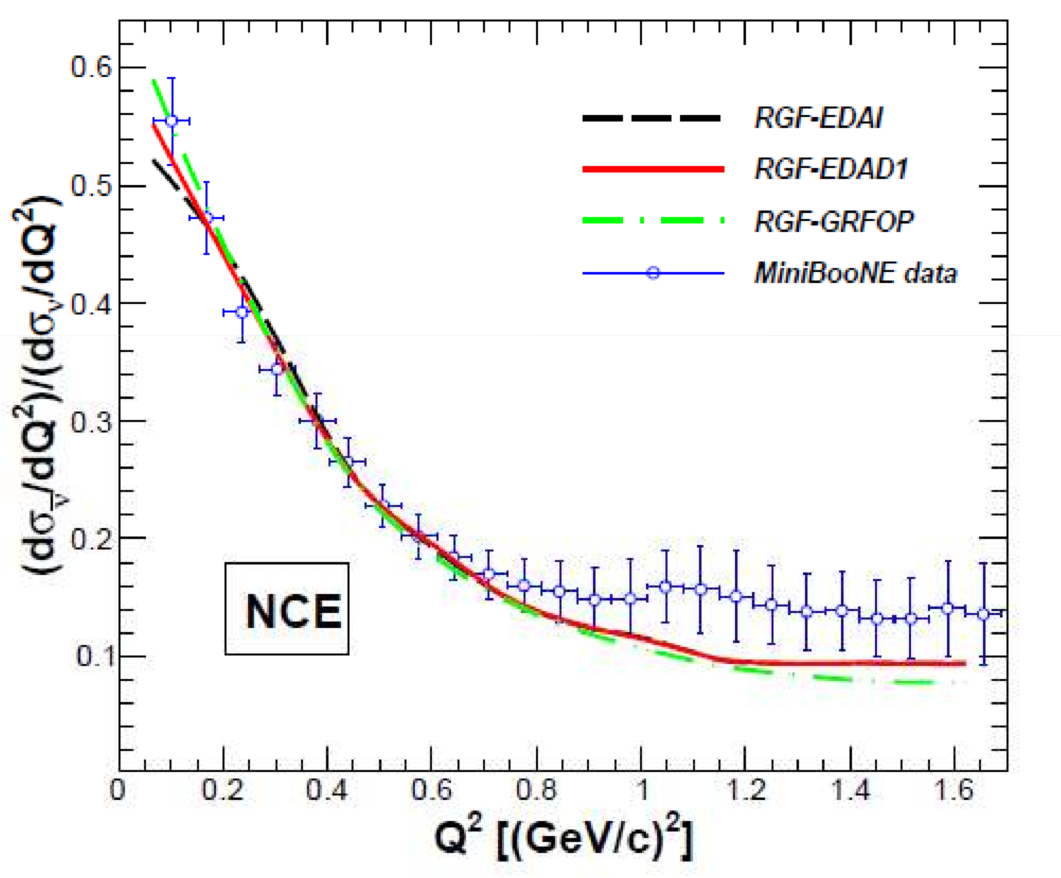

ε = 2020 MeV
ϑ = 15°
(a)
σ [nb/(MeV sr)]
ω [MeV]
ε = 620 MeV
ϑ = 60°
(b)
σ [nb/(MeV sr)]
ω [MeV]
ε = 1299 MeV
ϑ = 37.5°
(c)
σ [nb/(MeV sr)]
ω [MeV]
RGF-EDAI
RGF-EDAD1
RGF-GRFOP
12C data

scaling function

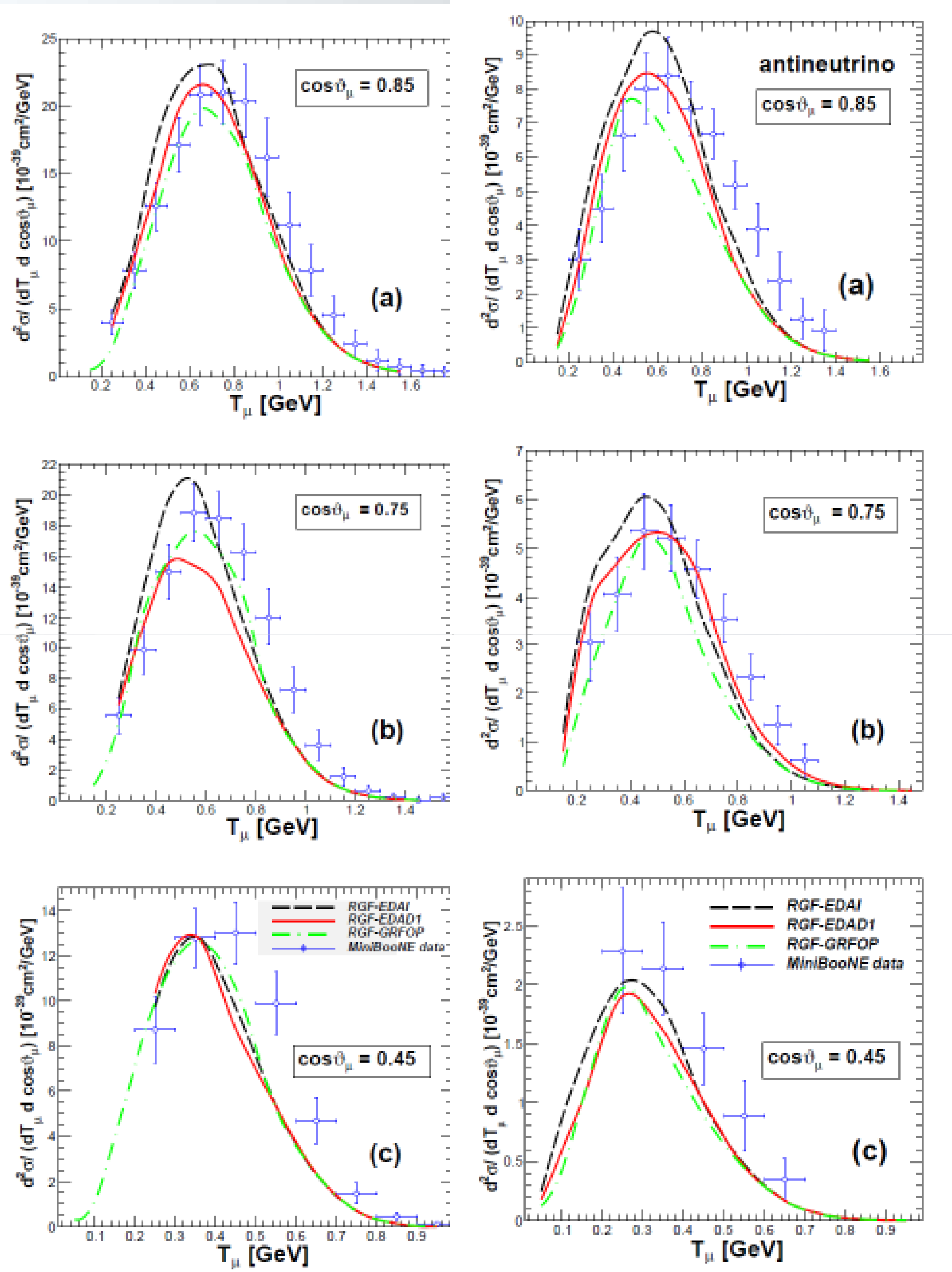


MiniBoone NCE data

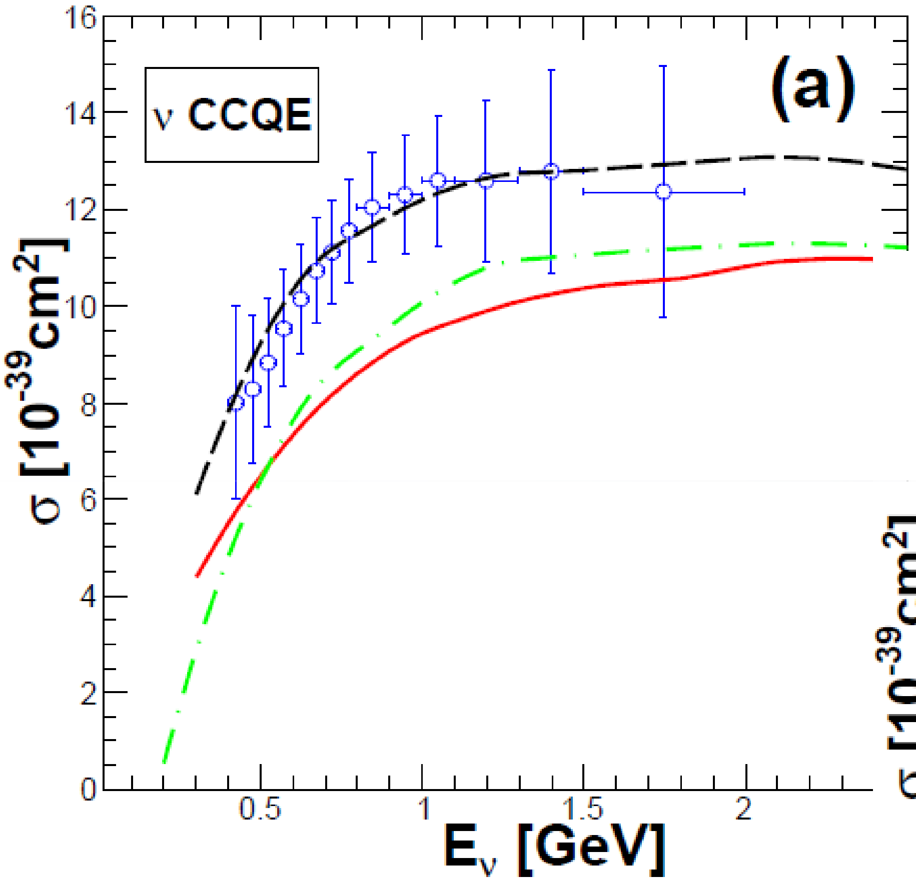


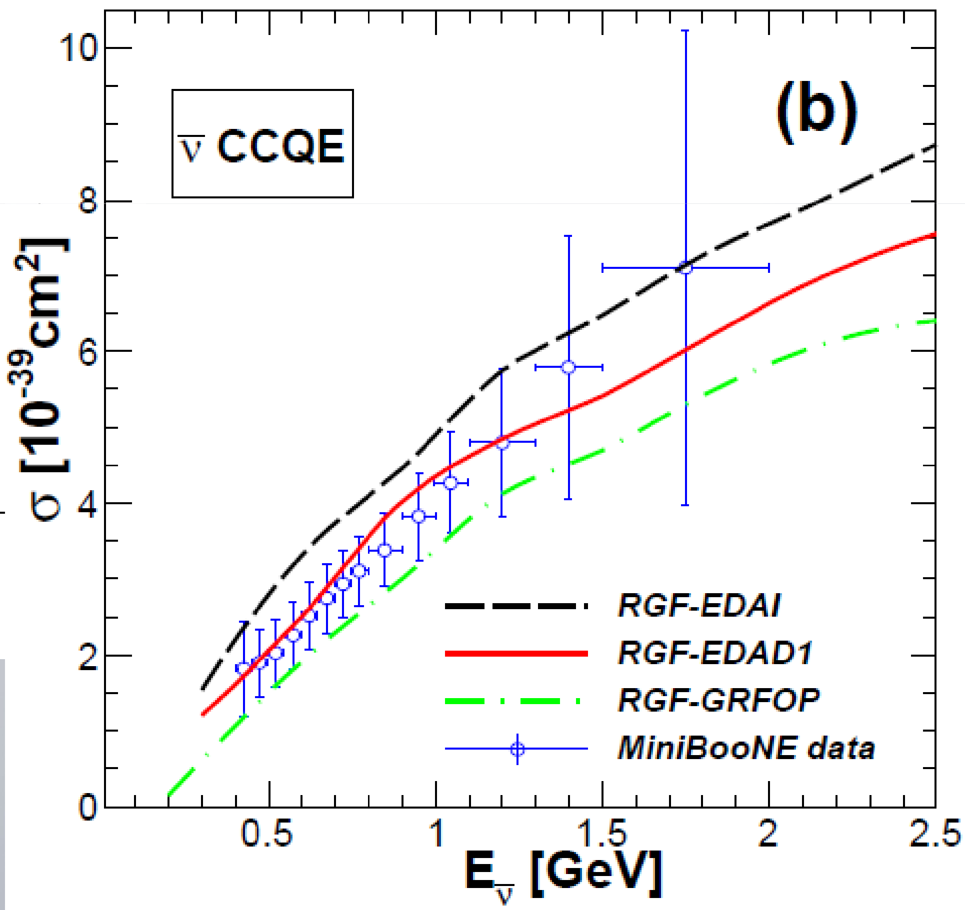


CC data


antineutrino
cosϑμ = 0.85
cosϑμ = 0.75
cosϑμ = 0.45
(a)
(b)
(c)
RGF-EDAI
RGF-EDAD1
RGF-GRFOP
MiniBooNE data
Tμ [GeV]
d²σ/ (dTμ d cosϑμ) [10⁻³⁹cm²/GeV]

CC Data (2)


(a)
ν CCQE
σ [10^-39 cm^2]
E_ν [GeV]
0
2
4
6
8
10
12
14
16
0.5
1
1.5
2


(b)
ν̄ CCQE
σ [10^-39 cm^2]
E_ν̄ [GeV]
RGF-EDAI
RGF-EDAD1
RGF-GRFOP
MiniBooNE data
0
2
4
6
8
10
0.5
1
1.5
2
2.5

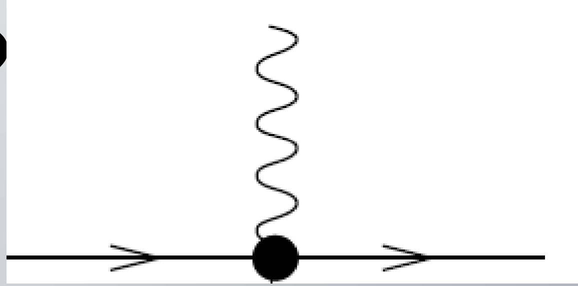
Summary

The RMF is successful in describing the universal inclusive scaling function representing the pure nucleonic response. But the potentials do not exhibit energy dependence in the potential, constituting a problem (too much FSI effect) for large nucleon energies.

Optical potentials do exhibit energy dependence and absorption. They reproduce well exclusive data, but cannot be applied as they are to the inclusive reaction. The RGF formalism can be employed for this purpose, but it depends on the phenomenological optical potentials. Available global optical potentials are not too constrained in shape and size, particularly for the imaginary term. We have built a new optical potential ^{12}C-GROP, with a large range in energy, more constrained than purely phenonemological fits. It is based on a RLF folding model. It reproduces well the scaling function and RMF predictions at low energies, while in principle exhibiting the adequate energy dependence.

RGF+GROP underpredict MiniBoone QE dta, which thus appear to include significant contributions from processes beyond 1p-1h excitations.

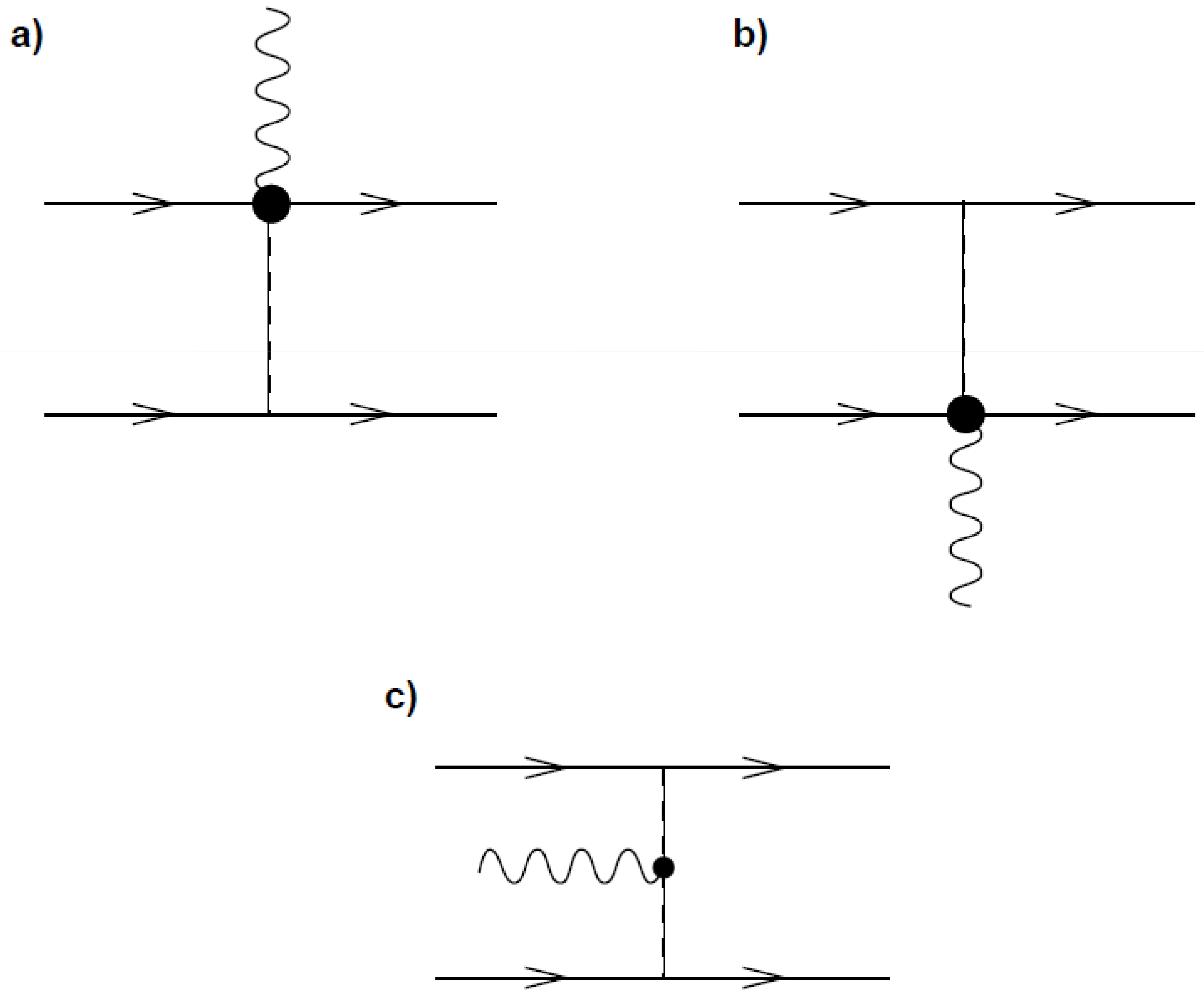
And what about the current operator?



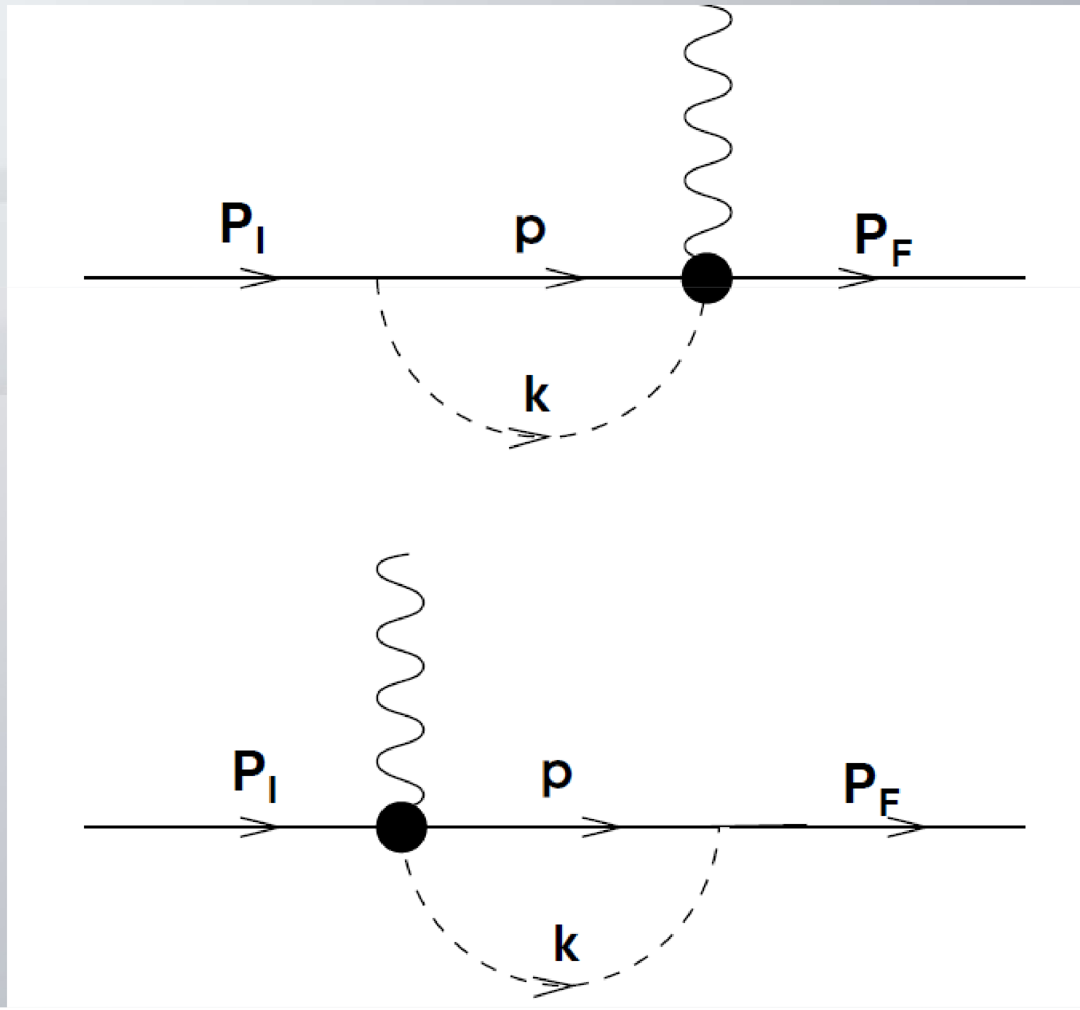
The single particle content of the model at tree level will couple the virtual boson. With the dressed (effective) nucleon. If using a free current operator, effects of truncation of the model into the mean field sector will show up as current no conservation, if only the one-body part of the current is considered.

The RMF when using the free current operator coupled to (effective) nucleons at tree level (thus only explicit one-body terms) already produces current-conserving matrix elements.

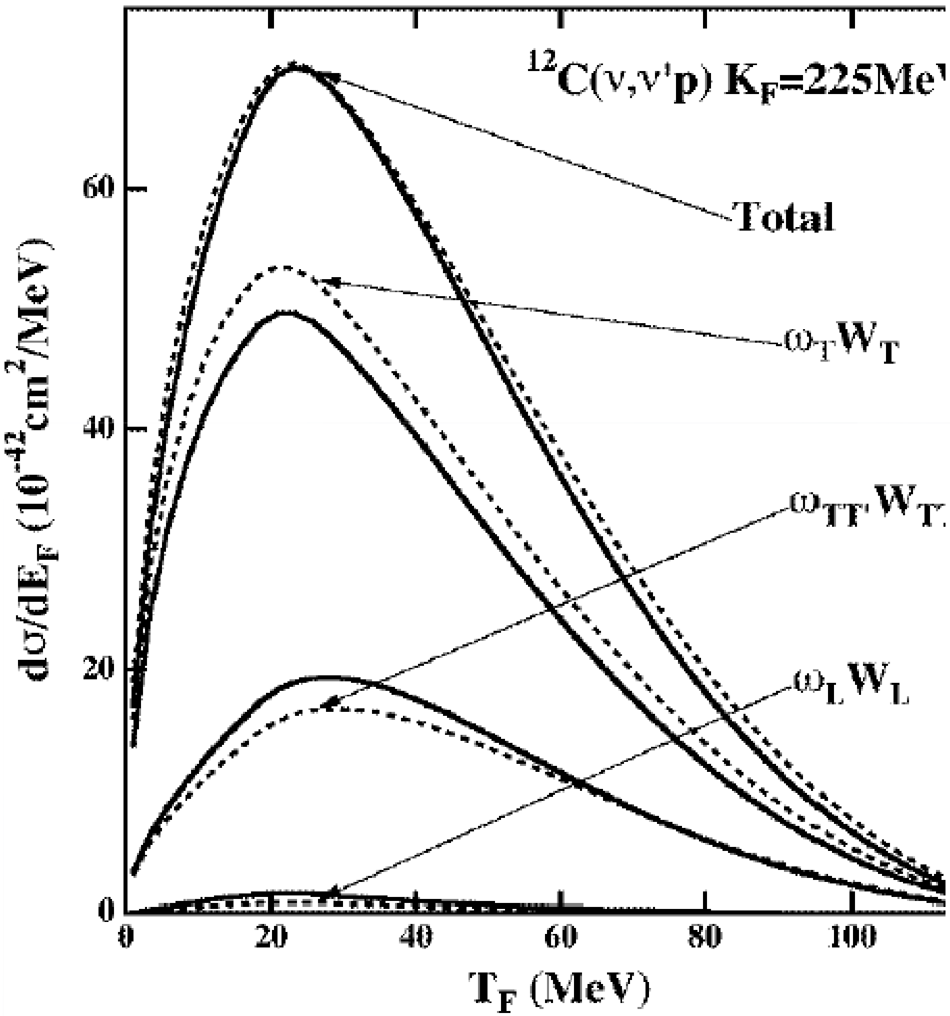
But they certainly are two-body contributions

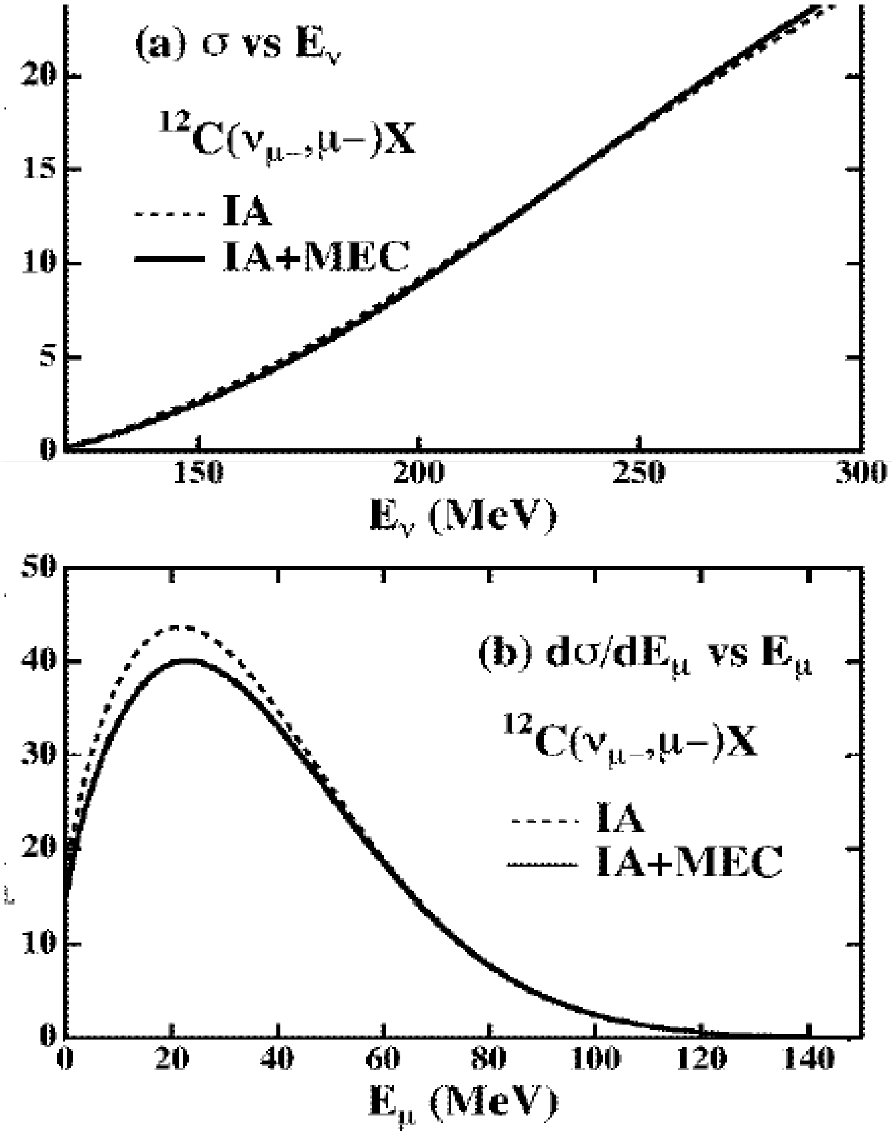


Besides 2p-2h, they also yield 1p-1h contributions



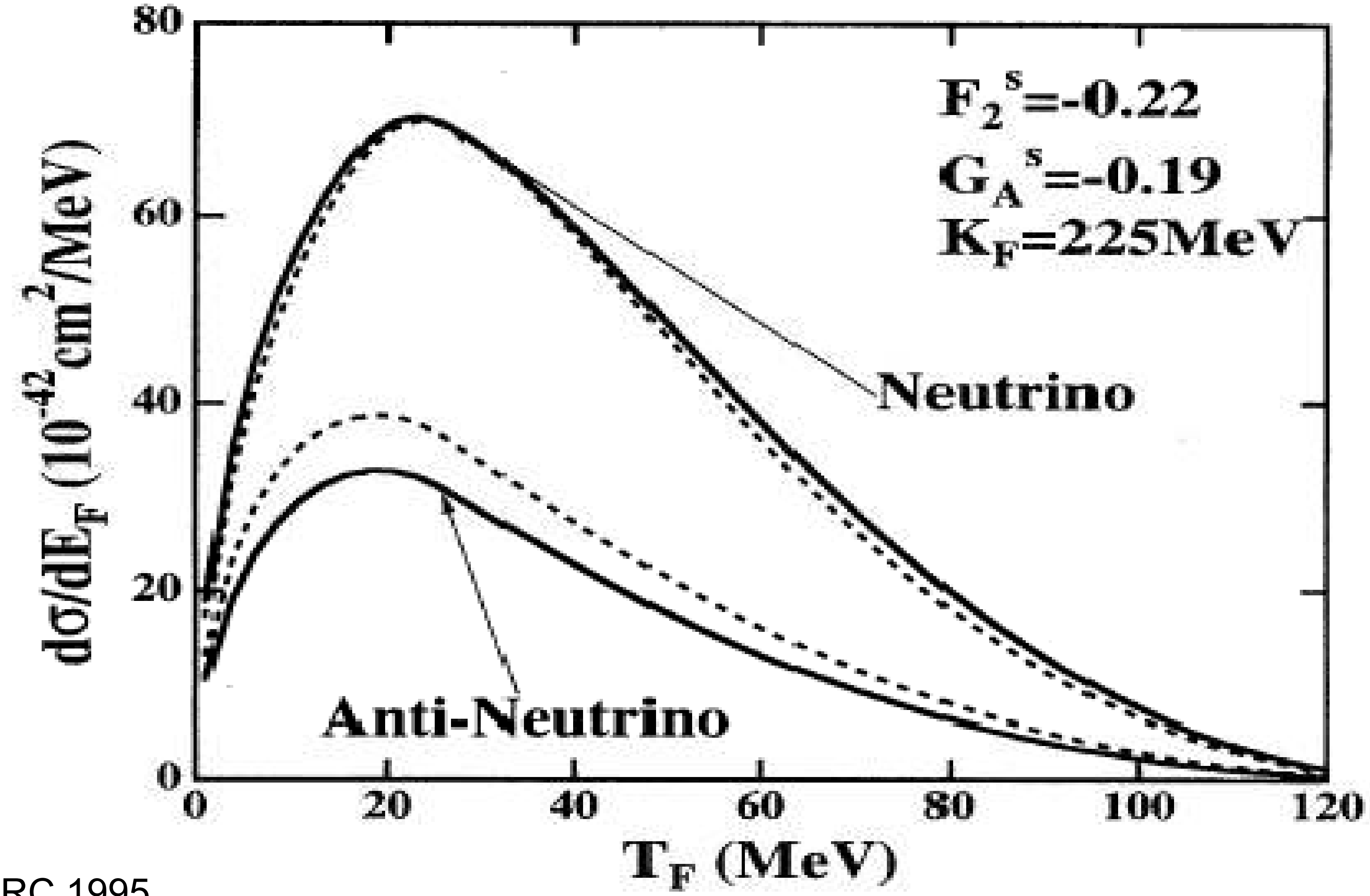
• Due to cancelling effects, it is a small effect for neutrino-nucleus scattering





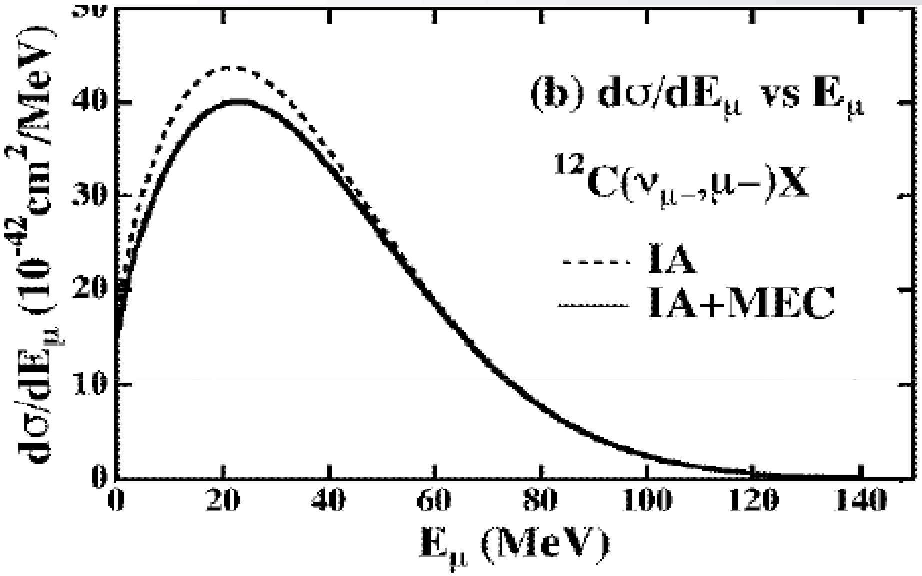
PRL74(1995)3399, PRC52(1995)3399

But they may be important for anti-neutrino as cancellation for neutrinos becomes constructive sum of effects for antineutrinos



PRC 1995

RMF and 1p1h MEC effects are very similar



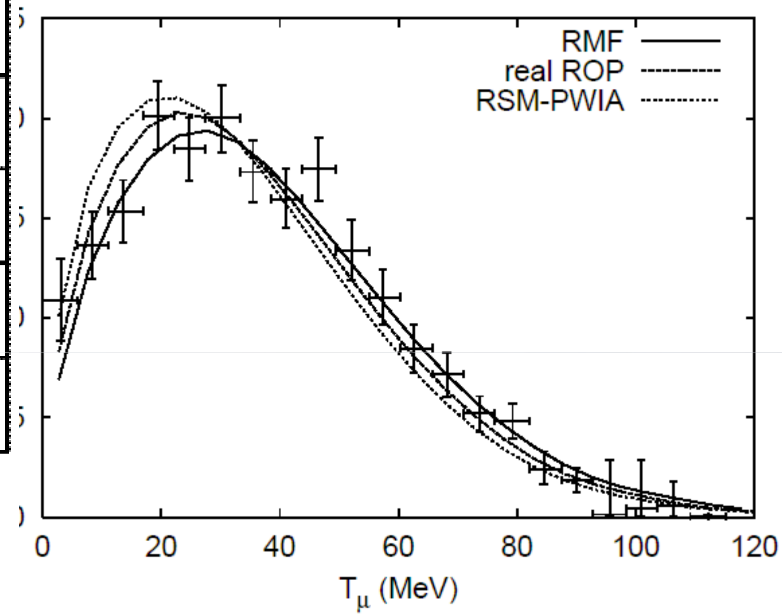


FIG. 5: Observed distribution of muon kinetic energies T_μ compared with the flux–averaged predictions of our RSM, in PWIA (dotted line) and including FSI within the RMF (solid) and purely real ROP (dashed) frameworks. The theoretical distributions have been normalized to give the same integrated values as the experimental points, and have been folded in energy with a bin size of 5 MeV, the same employed for the experimental data. Data are from Albert et al. [12].