2p-2h excitations in neutrino scattering: angular distribution and frozen approximation

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in collaboration with

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Outline

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- Angular distribution: CM vs LAB frame
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Introduction

- The analysis of current and future neutrino oscillation experiments requires having under control the nuclear effects, inherent to any ν-nucleus scattering event, in order to reduce systematic errors to the level of a few percent.
- The topic of 2-body current contributions to neutrino-nucleus scattering has attracted much interest from both the theoretical and experimental sides since the publication of the double differential CCQE cross section measurement by MiniBooNE[1] collaboration.
- From the theoretical side it is important to understand the origin of the quantitative discrepancies between different calculations, taking into account the involved approximations in each of them and checking their accuracy or theoretical justification.
- A very important objective for this kind of calculations is being fast enough to be incorporated in the Monte Carlo codes used by the experimental collaborations. For this purpose, reasonable approximations will be necessary in order to reduce their computational time, but still yielding an accurate enough result. In this context, we will propose the frozen nucleon approximation.
- We also want to show the equivalence between the isotropic two-nucleon angular distribution in the center-of-mass (CM) frame and in the LAB one[2], in order to understand properly some difficulties that arise in this last frame of reference.

[1] A. A. Aguilar-Arevalo *et al.* **Phys. Rev. D 81** (2010) 092005.

[2] I. Ruiz Simo *et al.* **Phys. Rev. D 90** (2014) no.3, 033012; I. R. Simo *et al.* **Phys. Rev. D 90** (2014) no.5, 053010.

2p-2h phase space in the RFG

The hadron tensor for the 2p-2h channel is given by:

$$W^{\mu\nu}_{2p2h} = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{m_N^4}{E_1E_2E'_1E'_2} r^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2)$$
$$\delta(E'_1+E'_2-E_1-E_2-\omega)\Theta(p'_1,p'_2,h_1,h_2)$$
$$\delta^3(\mathbf{p}'_1+\mathbf{p}'_2-\mathbf{h}_1-\mathbf{h}_2-\mathbf{q}) \quad (1)$$

where we have defined the product of step functions,

$$\Theta(p'_1,p'_2,h_1,h_2) \equiv \theta(p'_1-k_F)\theta(p'_2-k_F)\theta(k_F-h_1)\theta(k_F-h_2) \quad (2)$$

and $r^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2)$ is the elementary "hadron" tensor for the transition of a nucleon pair with given initial $(\mathbf{h}_1,\mathbf{h}_2)$ and final $(\mathbf{p}'_1,\mathbf{p}'_2)$ momenta, summed up over spin and isospin.

$$r^{\mu\nu} = \frac{1}{4}\sum_{\sigma,\tau} j^{\mu*}(\mathbf{p}'_i,s'_i,t'_i;\mathbf{h}_i,s_i,t_i)\, j^{\nu}(\mathbf{p}'_i,s'_i,t'_i;\mathbf{h}_i,s_i,t_i) \quad (3)$$

where $j^{\nu}(\mathbf{p}'_i,s'_i,t'_i;\mathbf{h}_i,s_i,t_i)$ is the electroweak current matrix element between plane wave nucleon states.

Frozen nucleon approximation

The frozen nucleon approximation is just a particular case of the mean-value theorem in several variables.

- Mean-value theorem

$$\boxed{\int_a^b f(x)dx = f(c)(b-a)} \qquad \text{with} \quad c \in [a,b] \tag{4}$$

$$\boxed{\int_{\mathcal{V}} f(\mathbf{r})d^n\mathbf{r} = f(\mathbf{c})\int_{\mathcal{V}} d^n\mathbf{r} = f(\mathbf{c})\mathcal{V}} \qquad \text{with} \quad \mathbf{c} \in \mathcal{V} \tag{5}$$

In our case, we would have:

$$\begin{aligned} W^{\mu\nu}_{\text{frozen}}(\omega,q) &= \int d^3h_1 d^3h_2 d^3p_1' \; f^{\mu\nu}(\mathbf{h}_1,\mathbf{h}_2,\mathbf{p}_1') = \\ &= \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3p_1' \; f^{\mu\nu}(\langle\mathbf{h}_1\rangle,\langle\mathbf{h}_2\rangle,\mathbf{p}_1') \end{aligned} \tag{6}$$

where $(\langle\mathbf{h}_1\rangle,\langle\mathbf{h}_2\rangle)$ are two unknown hole momenta inside the Fermi sphere. For high enough transferred momentum $q >> k_F > h_i$ the nucleons can be regarded at rest and then we can take $(\langle\mathbf{h}_1\rangle,\langle\mathbf{h}_2\rangle) = (\vec{0},\vec{0})$.

Nucleon angular distribution in LAB

If we write the complete expression for $f^{\mu\nu}(\vec{0},\vec{0},\mathbf{p}'_1)$, we get:

$$
\begin{aligned}
W^{\mu\nu}_{\rm frozen}(\omega,q) &= \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3p'_1\,\delta(E'_1+E'_2-\omega-2m_N)\,\Theta(p'_1,p'_2,0,0) \\
&\times \frac{m_N^2}{E'_1E'_2}\,r^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\vec{0},\vec{0}) \qquad (7)
\end{aligned}
$$

where now $\mathbf{p}'_2=\mathbf{q}-\mathbf{p}'_1$.

The Dirac delta function on energies allows to perform analytically the integral over p'_1 and then we have reduced the 7D integration problem to 1D integration over the polar angle θ'_1 if we can demonstrate that this approximation is good enough.

$$
W^{\mu\nu}_{\rm frozen}(\omega,q) = \left(\frac{4}{3}\pi k_F^3\right)^2 2\pi\int_0^\pi d\theta'_1\,\Phi^{\mu\nu}(\theta'_1) \qquad (8)
$$

where the emission angle distribution is:

$$
\begin{aligned}
\Phi^{\mu\nu}(\theta'_1) &= \sin\theta'_1 \int dp'_1\,{p'_1}^2\,\delta(E'_1+E'_2-\omega-2m_N)\,\Theta(p'_1,p'_2,0,0) \\
&\times \frac{m_N^2}{E'_1E'_2}\,r^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\vec{0},\vec{0}) \qquad (9)
\end{aligned}
$$

Nucleon angular distribution in the two-nucleon CM frame

In general, $\Phi^{\mu\nu}(\theta_1')$ will have two contributions which come from solving the Dirac delta function for p_1', which in general has two different momenta $(p_1')_\pm$ that fulfill the condition of vanishing the argument of the δ-function. We will denote them by $\Phi_\pm^{\mu\nu}(\theta_1')$ and we can write:

$$\Phi^{\mu\nu}(\theta_1') = \Phi_+^{\mu\nu}(\theta_1') + \Phi_-^{\mu\nu}(\theta_1') \tag{10}$$

In Monte Carlo event generators the angular distribution is obtained from an isotropic (as pure phase space considerations require) two-nucleon distribution in the CM frame of them, and then transformed back to the LAB frame. Here we want to show that the LAB distribution is recovered with this procedure as well.

Nucleon angular distribution in the two-nucleon CM frame

First we fix in the LAB frame the kinematics of the leptons, which completely determines (ω, q), and that of the two initial nucleons in the Fermi gas $(\mathbf{h}_1, \mathbf{h}_2)$, which for simplicity we will take at rest (frozen nucleon approximation) $h_1 = h_2 = 0$. Therefore, the total final momentum in LAB is $\mathbf{p}' = \mathbf{q} = \mathbf{p}'_1 + \mathbf{p}'_2$.
Now we have to perform a Lorentz boost along the direction of $\mathbf{q}$ (Z-axis) that keep the two final nucleons in the condition of their momenta sum up to zero, i.e, $(\mathbf{p}'_1 + \mathbf{p}'_2)_{\rm cm} = 0$.
The total final energy in the CM frame can be determined from invariance of the squared final four-momentum. And this final energy in the CM frame is shared equally between both nucleons because of their equal masses.

$$(E')_{\rm cm} = \sqrt{E'^2 - \mathbf{p}'^2} \quad \text{where} \quad (E', \mathbf{p}') = (\omega + 2m_N, \mathbf{q}) \tag{11}$$

$$(E'_1)_{\rm cm} = (E'_2)_{\rm cm} = \frac{1}{2}(E')_{\rm cm} \tag{12}$$

Nucleon angular distribution in the two-nucleon CM frame

By carrying out the Lorentz boost from the CM frame back to the LAB one, we can relate the kinematic variables (emission angles and momenta) in the LAB frame with their counterparts in the CM frame:

$$p'_1 = \sqrt{\gamma^2 \left[(E'_1)_{\rm cm} + v\,(p'_1)_{\rm cm} \cos\left(\theta'_1\right)_{\rm cm}\right]^2 - m_N^2} \qquad (13)$$

$$\cos\theta'_1 = \frac{\gamma\left[v\,(E'_1)_{\rm cm} + (p'_1)_{\rm cm} \cos\left(\theta'_1\right)_{\rm cm}\right]}{\sqrt{\gamma^2 \left[(E'_1)_{\rm cm} + v\,(p'_1)_{\rm cm} \cos\left(\theta'_1\right)_{\rm cm}\right]^2 - m_N^2}} \qquad (14)$$

where $v = \frac{p'}{E'}$ and $\gamma = \frac{1}{\sqrt{1-v^2}}$ are the parameters of the Lorentz boost, and $(\theta'_1)_{\rm cm}$ is the emission angle of the first nucleon in the CM frame with respect to the direction of $\mathbf{q}$, and thus $(\theta'_1)_{\rm cm} \in [0, \pi]$

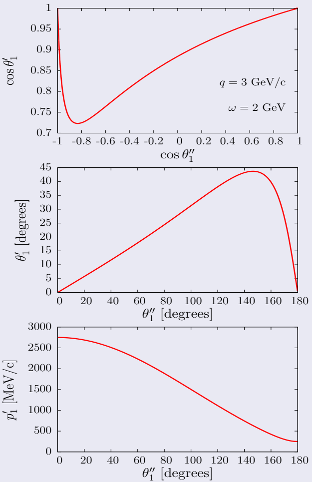


Figure: Correspondence between LAB variables (emission angle θ'_1 and momentum p'_1) and CM nucleon emission angle (called θ''_1 in the figure). The momentum and energy transfers are $q = 3$ GeV/c and $\omega = 2$ GeV.

- It can be seen in the figure that two different emission angles in the CM frame correspond to the same angle θ'_1 in the LAB frame, although with different momentum p'_1.
- Therefore we can say that for a given LAB angle, there are two different and possible values of p'_1 and these correspond to the two possible solutions of the energy conservation, which were previously called $(p'_1)_\pm$.
- Thus there are two branches which run over the same range of LAB angle, but correspond to different ranges of CM angle. And these two branches can be identified with the previously defined two contributions to the angular distribution $\Phi^{\mu\nu}_+(\theta'_1)$ and $\Phi^{\mu\nu}_-(\theta'_1)$.

Transformation of the angular distribution

Assuming an isotropic angular distribution in the CM frame, as two-particle phase space requires, it can be shown that the LAB angular distribution is retrieved after performing the Lorentz boost back to the LAB, and another easy interpretation of the divergence of the angular distribution is obtained.

Indeed, let me assume that the angular distribution in the CM frame is independent of the emission angle $(\theta_1')_{\rm cm}$ and, not considering Pauli blocking we can thus write:

$$n'_{\rm cm}\left[(\theta_1')_{\rm cm}\right] = C \tag{15}$$

The above function is such that $n'_{\rm cm}\left[(\theta_1')_{\rm cm}\right](d\Omega_1')_{\rm cm}$ gives the number of nucleons emitted within an elementary solid angle $(d\Omega_1')_{\rm cm}$ in the CM frame. And we impose this quantity to be equal to the number of nucleons emitted within an elementary solid angle $d\Omega_1'$ in the LAB frame, by virtue of conservation of probability. Therefore, we can write:

$$n'(\theta_1')d\Omega_1' = n'_{\rm cm}\left[(\theta_1')_{\rm cm}\right](d\Omega_1')_{\rm cm} \tag{16}$$

Transformation of the angular distribution

As the boost is performed along the **q** direction, the orthogonal directions are preserved and so the azimuthal angles, $d\phi'_1 = (d\phi'_1)_{\rm cm}$, and we only have to transform the polar angles between both frames:

$$n'(\theta'_1)\, d(\cos\theta'_1) = n'_{\rm cm}\left[(\theta'_1)_{\rm cm}\right] d(\cos\theta'_1)_{\rm cm} \tag{17}$$

By differentiation of Eq. (14), we obtain

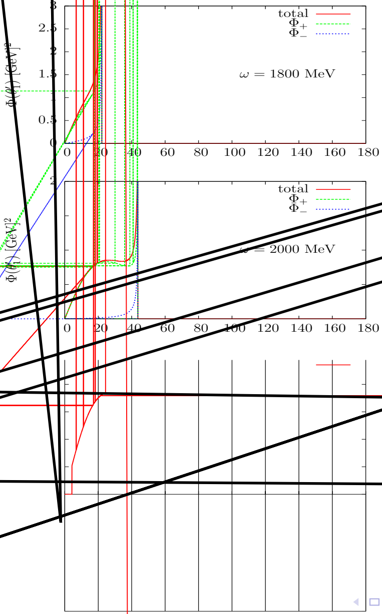
$$d(\cos\theta'_1) = \gamma\, (p'_1)_{\rm cm}\, \frac{p'_1 - v\, E'_1 \cos\theta'_1}{(p'_1)^2}\, d(\cos\theta'_1)_{\rm cm} \tag{18}$$

Thus resulting

$$n'(\theta'_1) = \frac{C}{\left|\frac{d(\cos\theta'_1)}{d(\cos\theta'_1)_{\rm cm}}\right|} = n'_+(\theta'_1) + n'_-(\theta'_1) \tag{19}$$

where in the last step of the above expression we have explicitly separated the two branches of the distribution, the one with negative derivative and the other one with positive derivative. The problem of the divergence in the LAB angular distribution occurs for the CM angle where $\frac{d(\cos\theta'_1)}{d(\cos\theta'_1)_{\rm cm}} = 0$.

Angular distribution



Model for Meson-exchange currents

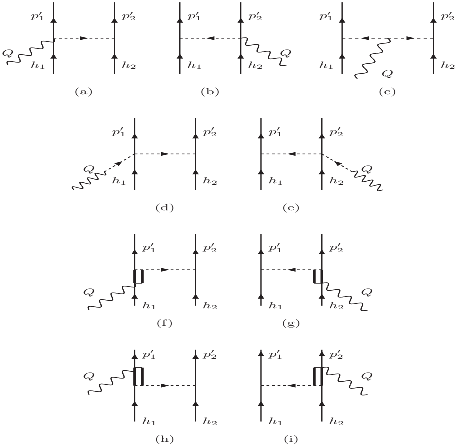


Figure: Feynman diagrams for the meson-exchange currents model: seagull (a) and (b), pion-in-flight (c), pion-pole (d) and (e), Δ-forward (f) and (g), and finally Δ-backward (h) and (i).

Angular distribution (preliminary results)

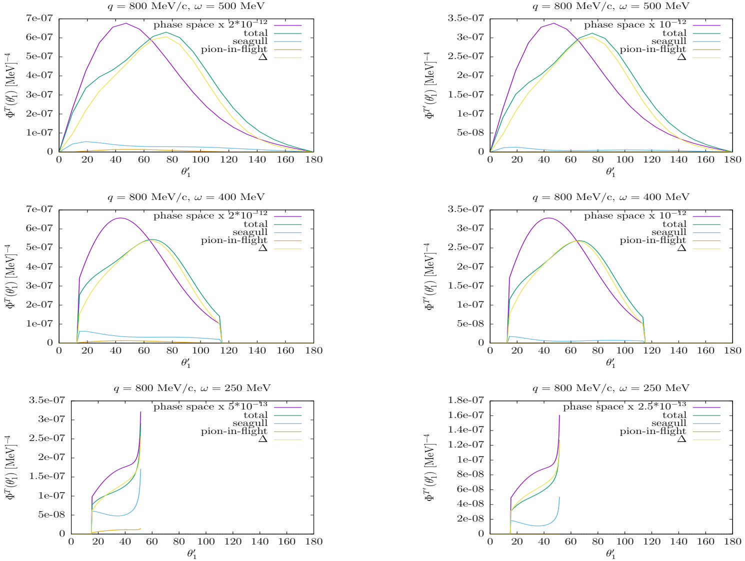


Figure: Angular distribution for the transverse T (left) and T′ (right) response functions in the "frozen nucleon approximation" for different values of the transferred energy ω, where the elementary contributions have been singled out.

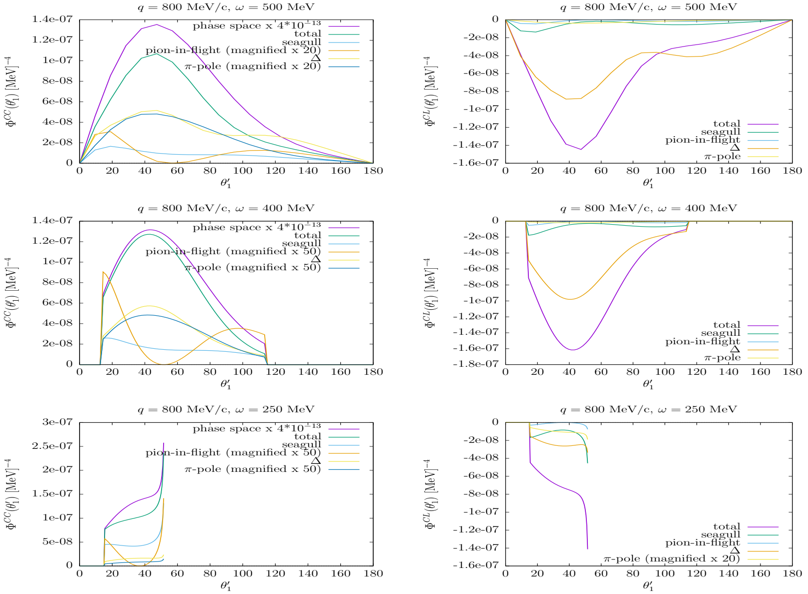


Figure: Angular distribution for the CC (left) and CL (right) response functions in the "frozen nucleon approximation" for different values of the transferred energy ω, where the elementary contributions have been singled out. These results are preliminary.

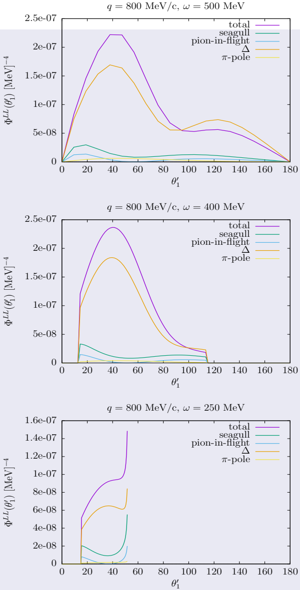


Figure: Angular distribution for the LL response function in the "frozen nucleon approximation" for different values of the transferred energy ω. The results are preliminary.

- Δ is the dominant contribution in all responses, except on CC and CL responses, where in addition a significant interference with seagull occurs.
- π-pole current is proportional to Q^μ and, therefore, its only non-vanishing components are the longitudinal (L) and time-like components, with exactly zero contribution to the transverse T and T' responses.
- Pion-in-flight current is purely of vector type, so in T' response gives exactly zero contribution because only the V-A interference contributes to it.
- Only in the transverse responses the peak of the angular distribution is a bit shifted towards larger emission angles with respect to the phase space alone result (with constant $r^{\mu\nu}$).
- In the CC and the CL responses, for the pion-in-flight contribution there are some emission angles in which the response is zero, but this is due to the frozen approximation $(\mathbf{h}_1, \mathbf{h}_2) = (\vec{0}, \vec{0})$ and also to the fact that the pion-in-flight current is proportional to the difference of momenta of the two exchanged pions, $\propto (k_1^\mu - k_2^\mu)$. And when $\mu = 0$ (as it is the case for the CC (00th component) or CL (03-component), there will be some angles for the nucleon emission where $k_1^0 = k_2^0$ and the response will vanish.

Frozen nucleon approximation for the integrated responses

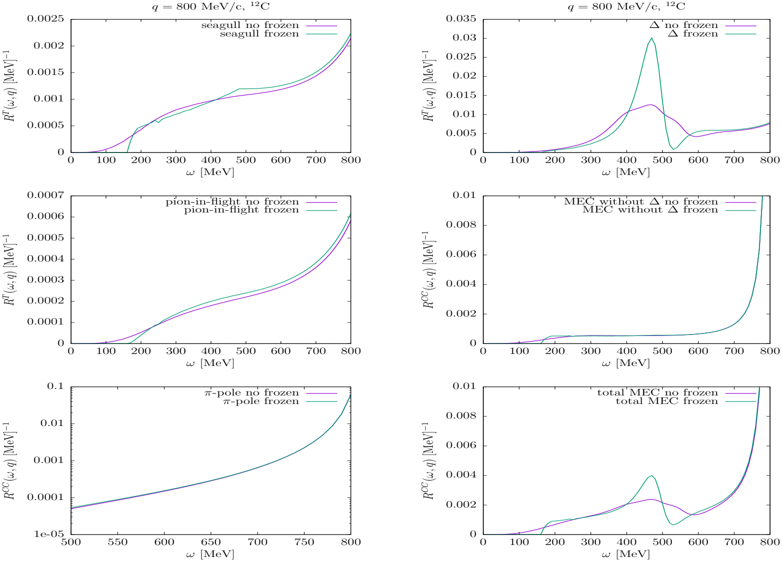


Figure: Comparison between frozen nucleon approximation (one-dimensional integration) and full integration (7D) for different weak response functions at $q = 800$ MeV/c in ^{12}C. The results are preliminary.

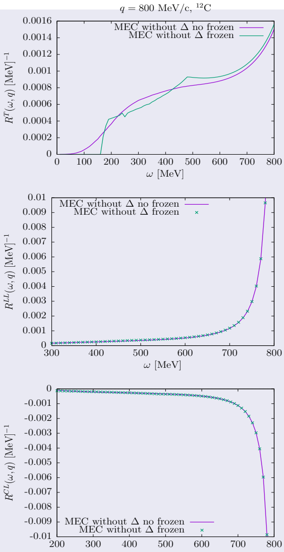


Figure: Comparison between frozen nucleon approximation and full integration for different weak response functions (T, LL and CL from top to bottom) at $q = 800$ MeV/c in ^{12}C. These results are still preliminary.

- The frozen nucleon approximation (with only one integration over θ_1' is an excellent approximation to the 7D integral except when the Δ current is considered, either alone or with the rest of contributions. This is because of the Δ-pole propagator, which can be placed on-shell for the Δ-resonance at some ω value for a given q, when $(h_1 + Q)^2 = M_\Delta^2$ or $(h_2 + Q)^2 = M_\Delta^2$.
- For other ω values, the on-shellness condition for the Δ is reached at other values of $\mathbf{h}_1$ or $\mathbf{h}_2$, which will be picked when performing the full integration and these points will be one contribution among thousands of others. Meanwhile, in the frozen approximation all the points $(\mathbf{h}_1, \mathbf{h}_2)$ are assigned the same weight or contribution to the full integral, and if it is the case that for that ω value, the point $(\mathbf{h}_1, \mathbf{h}_2) = (\vec{0}, \vec{0})$ hits the pole, then the only thing what prevents the Δ propagator from being completely divergent is the Δ width.
- Therefore, the net effect of the full integration around the Δ peak is a kind of smearing of the sharp Δ peak obtained with the frozen nucleon approximation.
- One of our next objectives is trying to put an effective width to the Δ propagator (maybe a parameterized one) that despite having no physical interpretation, we expect it to be useful to get accurate enough results to keep using the frozen approximation because of its simplicity and its extremely short time of computation.

Separate isospin channels for electron scattering (preliminary)

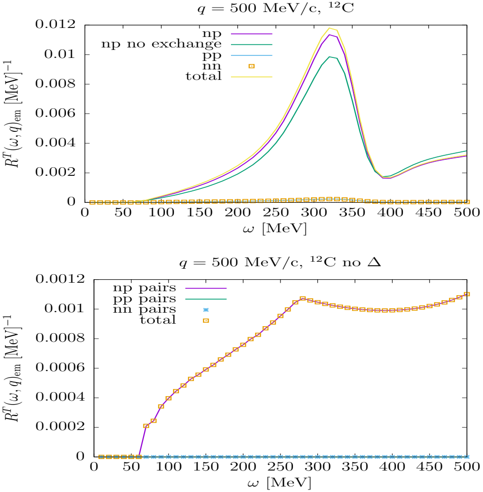


Figure: Separate isospin transverse response functions for (e, e') scattering in the frozen approximation scheme.

Conclusions and future plans

- We have improved our previous study of the 2p-2h phase-space by including now the square of the current matrix elements, but still keeping our final goal of finding a way to obtain accurate enough results without calculating the 7D integral.
- The frozen nucleon approximation (1D integral) seems to be a quite promising approach to achieve it, except when the Δ contribution is taken into account. We want to extend the approach in order to get an unified treatment of all the contributions within this approximation.
- We have obtained the correspondence between the CM angular distribution and the LAB one and we have understood in other way the origin of the divergence in the LAB angular distribution.
- We have given preliminary results for the angular distribution in the frozen approximation where the effects of different contributions have been isolated.
- We have also given preliminary results to test the validity of the frozen approximation for the integrated responses.
- We can give responses for separate isospin channels with neutrinos, antineutrinos and electrons.
- One of our next steps will be to extend the present model to give results for nuclei with $N \neq Z$, to be used to calculate nuclei such as ^{40}Ar, ^{56}Fe or ^{208}Pb.
- ...