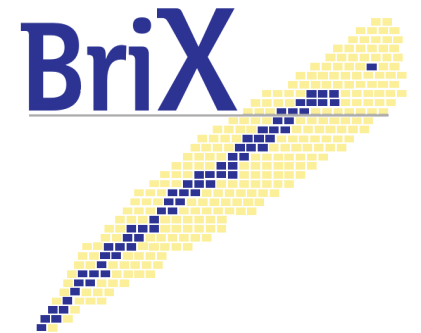


From quasielastic to pion production

Pion production within the Relativistic Mean Field model



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Department of Physics and Astronomy,
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*Workshop on
“Two-body current contributions in neutrino-nucleus scattering”,
CEA-Saclay, France, April 18-22, 2016*

Outline

I Introduction

II Quasielastic scattering

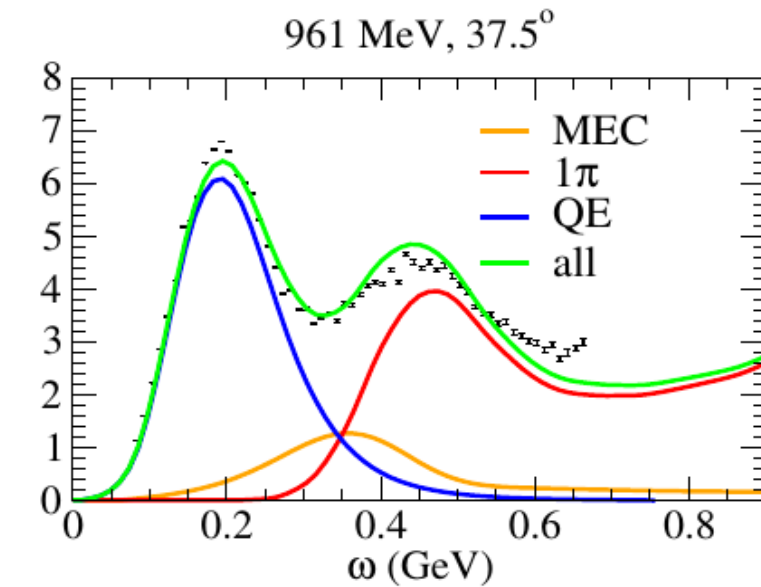
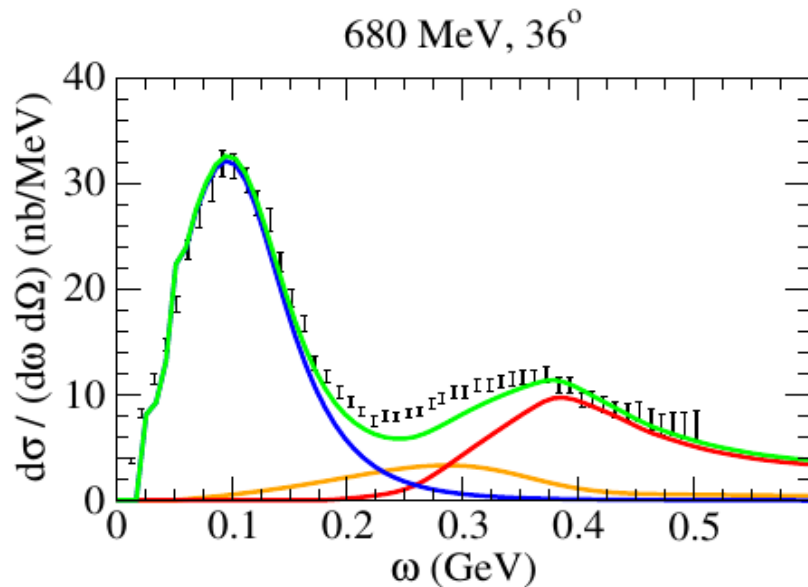
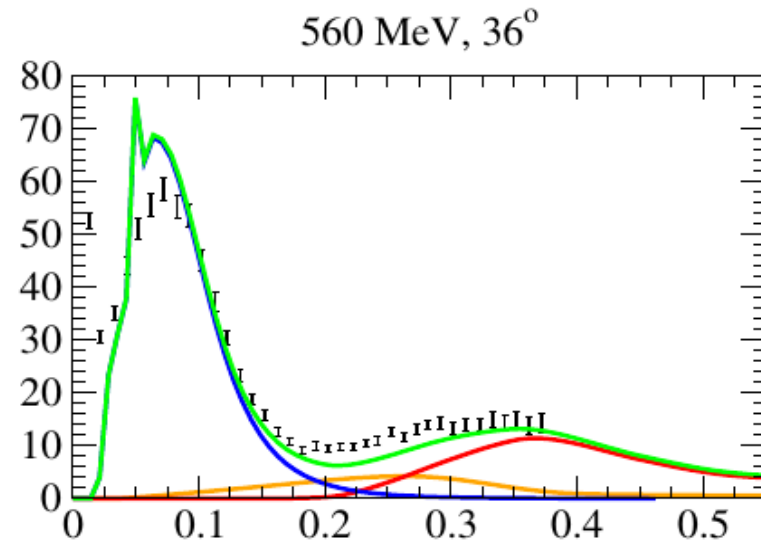
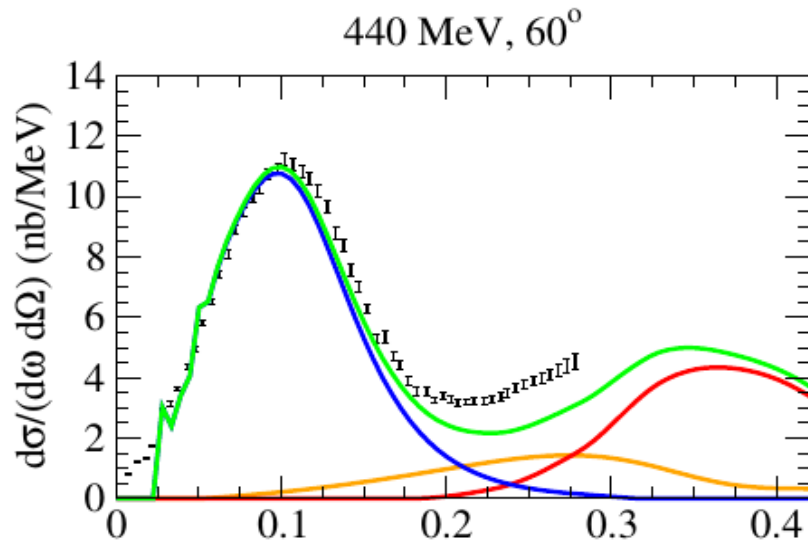
III Pion-production on nucleons

IV Pion-production on nuclei

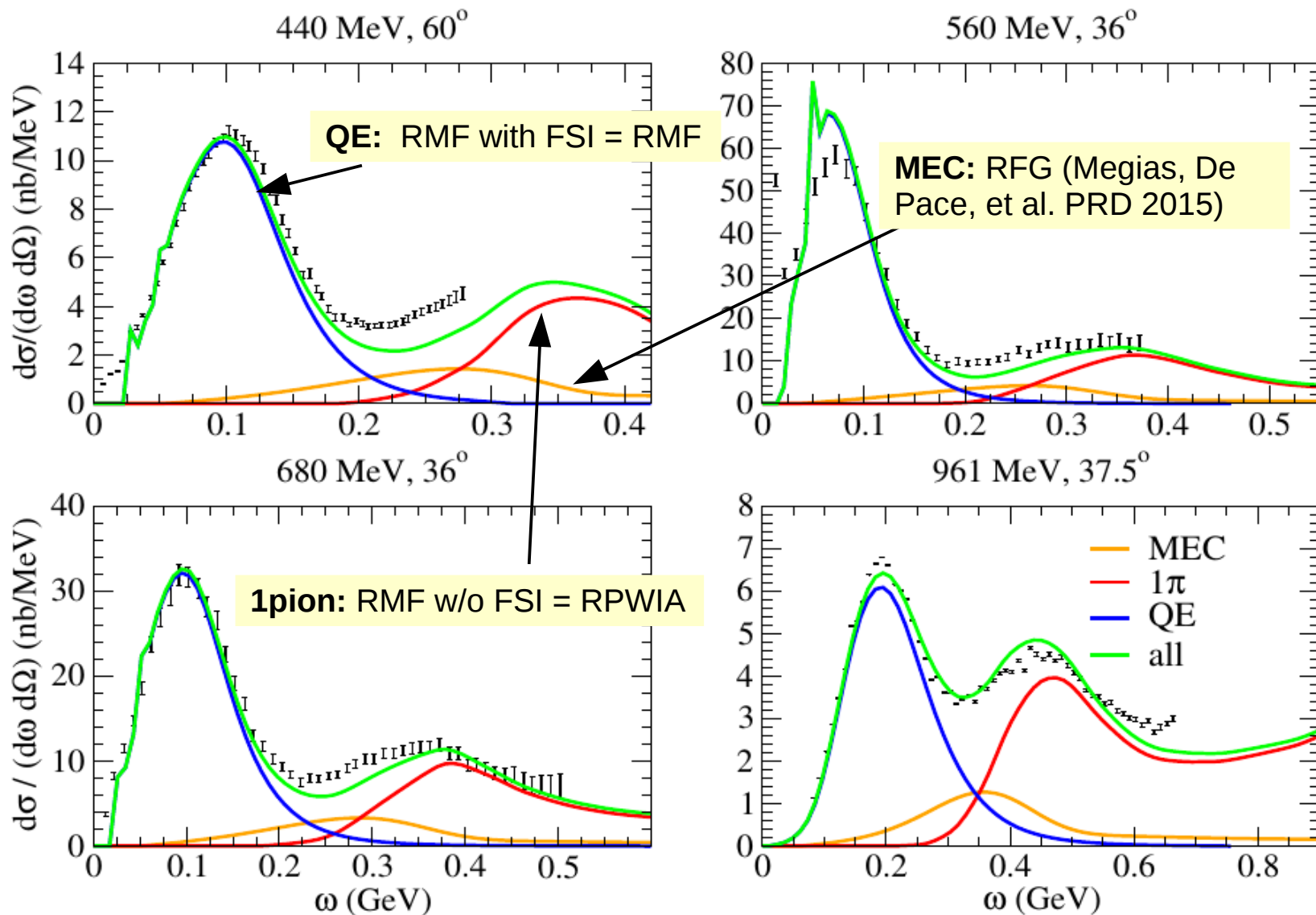
V Results

VI Conclusions

Introduction: what we know from (e,e')

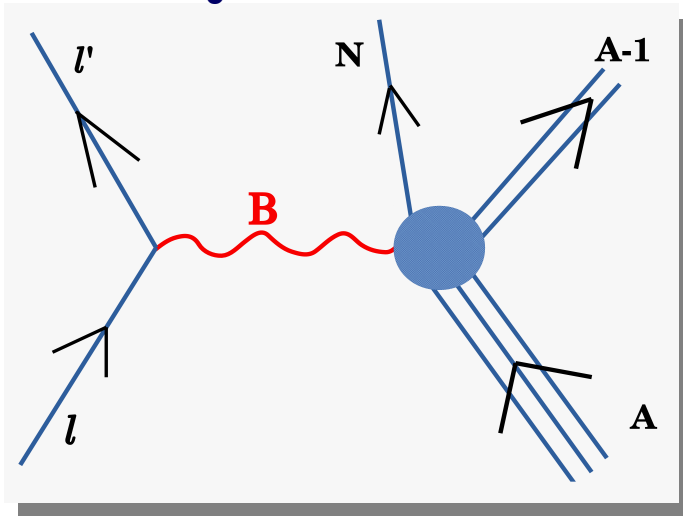


Introduction: what we know from (e,e')



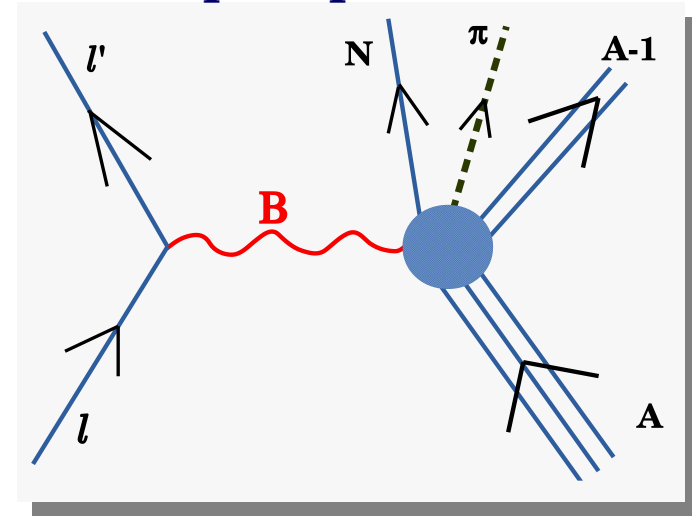
Introduction: cross sections

Quasielastic



$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_N} = \frac{m_i m_f}{(2\pi)^5} \frac{M_B M_N p_N}{M_A f_{rec}} \frac{k_f}{\varepsilon_i} \sum_{fi} |\mathcal{M}_{fi}|^2$$

One-pion production



$$\frac{d^8\sigma}{d\varepsilon_f d\Omega_f dE_\pi d\Omega_\pi d\Omega_N} = \frac{m_i m_f}{(2\pi)^8} \frac{M_N p_N k_\pi}{E_N f_{rec}} \frac{k_f}{\varepsilon_i} \sum_{fi} |\mathcal{M}_{fi}|^2$$

$$\mathcal{M}_{fi} = j_{lep}^\mu S_{\mu\nu} J_{had}^\nu$$

Quasielastic scattering

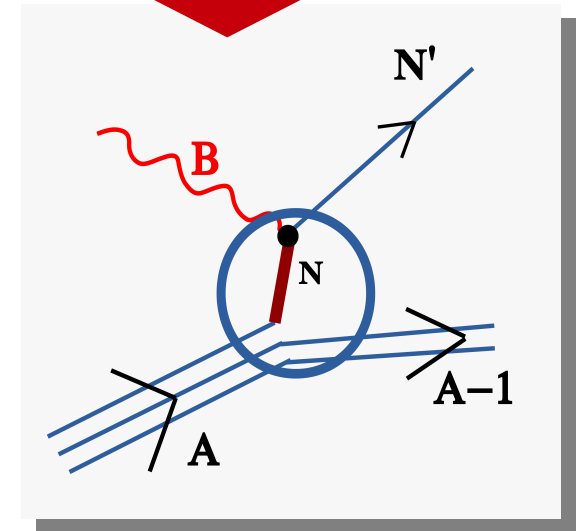
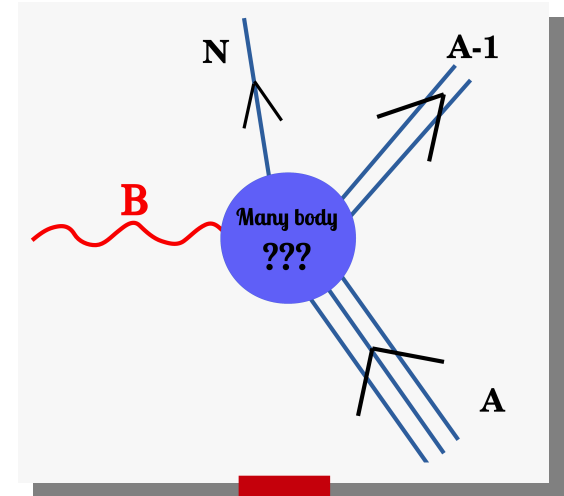
Impulse approximation

$$J_{had}^{\mu} = \langle N, A - 1 | \hat{O}_{many-body}^{\mu} | A \rangle$$

**Relativistic
Impulse
Approximation**

$$J_{had}^{\mu} = \sum_i^A \int d\mathbf{r} \bar{\Psi}_F(\mathbf{r}) \hat{O}_{one-body}^{\mu} \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

where
$$\hat{O}_{one-body}^{\mu} = F_1 \gamma^{\mu} + i \frac{F_2}{2M_N} \sigma^{\mu\alpha} Q_{\alpha}$$



Impulse approximation

$$J_{had}^{\mu} = \langle N, A - 1 | \hat{O}_{many-body}^{\mu} | A \rangle$$

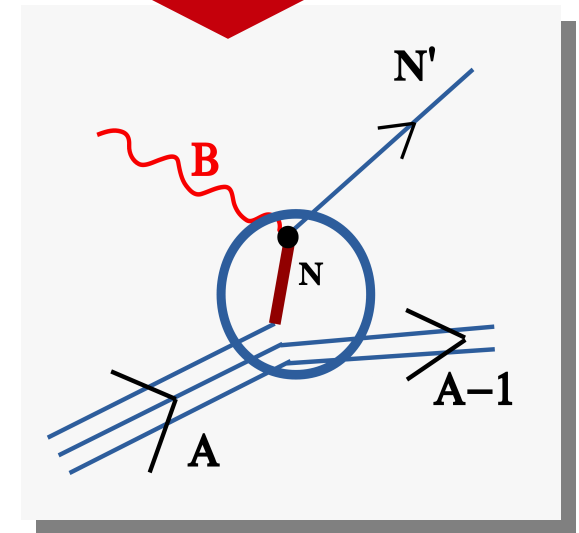
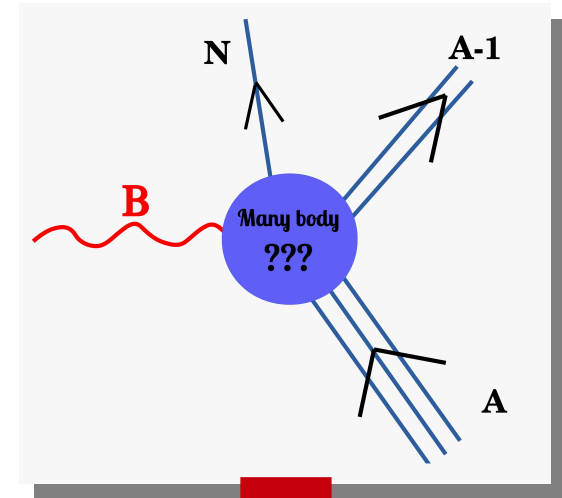
**Relativistic
Impulse
Approximation**

**Relativistic
mean-field
model**

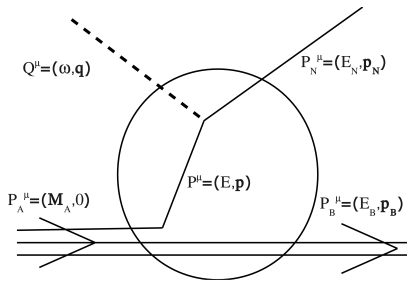
$$J_{had}^{\mu} = \sum_i^A \int dr \bar{\Psi}_F(\mathbf{r}) \hat{O}_{one-body}^{\mu} \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$

where

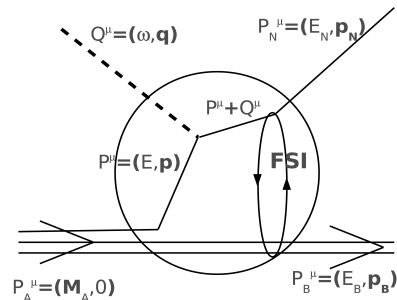
$$\hat{O}_{one-body}^{\mu} = F_1 \gamma^{\mu} + i \frac{F_2}{2M_N} \sigma^{\mu\alpha} Q_{\alpha}$$



RMF: quasielastic results



RPWIA: Scattered nucleon wf is described as a Dirac plane wave.

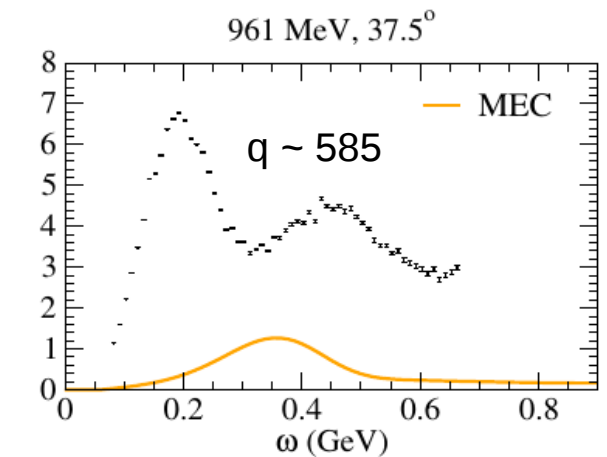
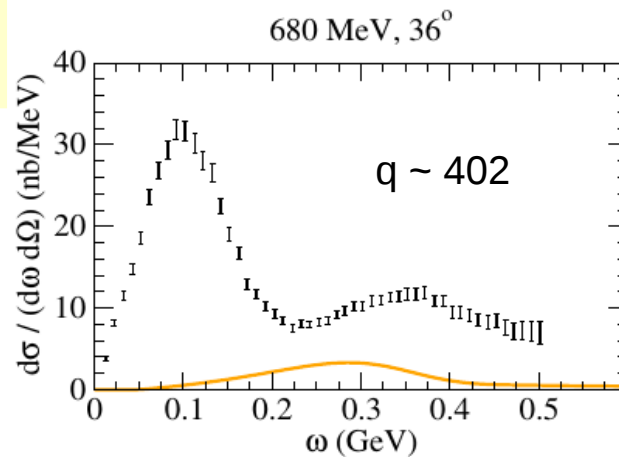
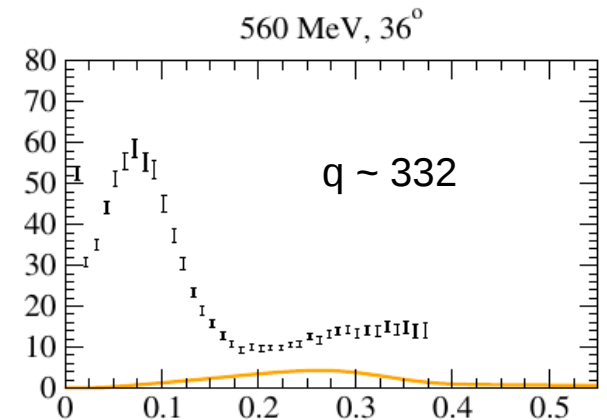
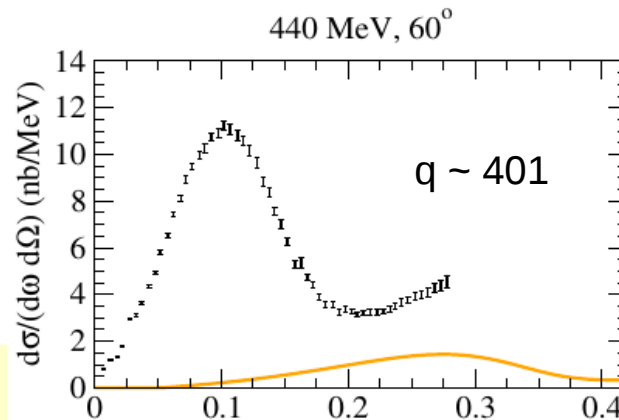


RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

$$[-i\boldsymbol{\alpha} \cdot \nabla + V(r) + \beta(M + S(r))]\Psi_i(\mathbf{r}) = E_i\Psi_i(\mathbf{r})$$

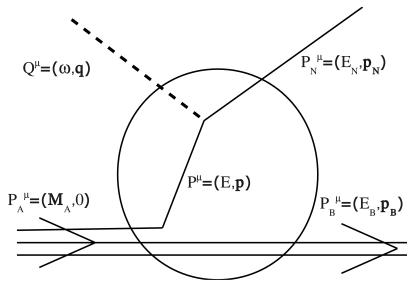
$$J_{had}^\mu = \sum_i^A \int d\mathbf{r} \bar{\Psi}_F(\mathbf{r}) \hat{O}_{one-body}^\mu \Psi_B(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}}$$

Intermediate energies (typical QE regime)

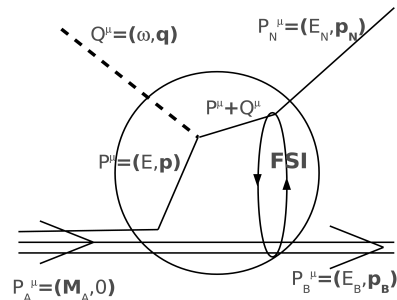


MEC: (Megias, ..., De Pace, et al. PRD 2015)

RMF: quasielastic results



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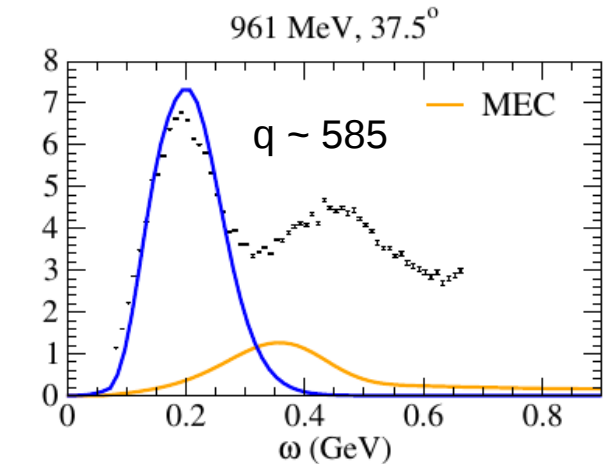
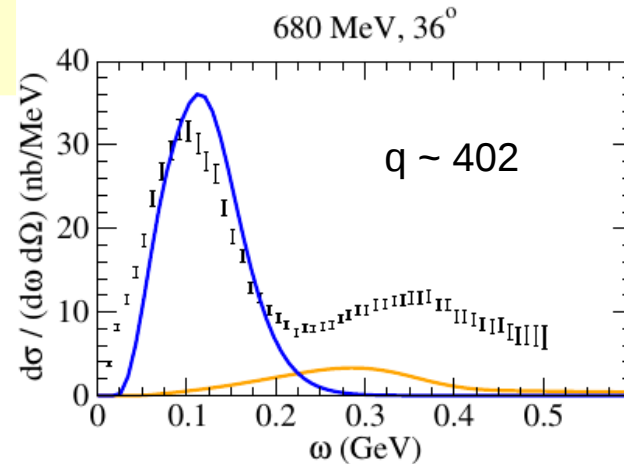
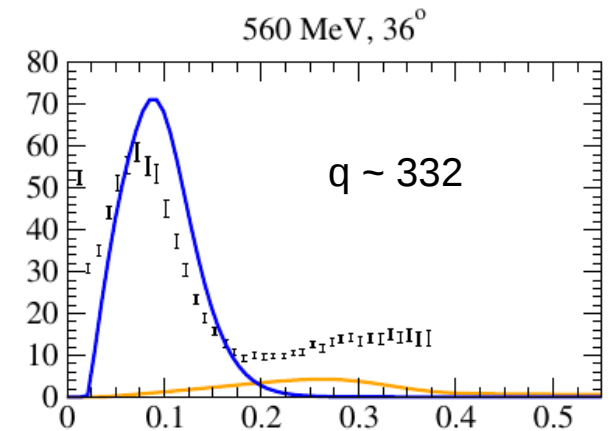
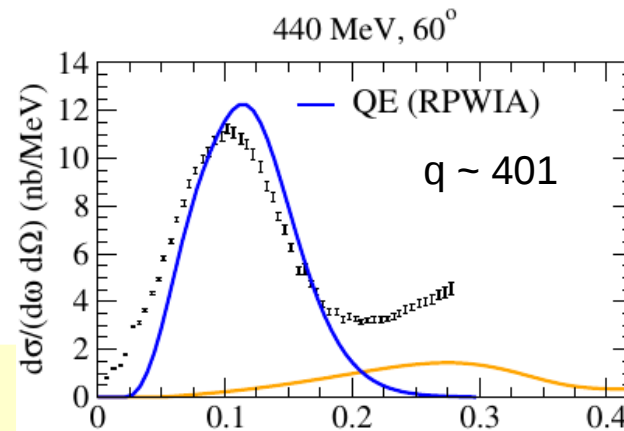


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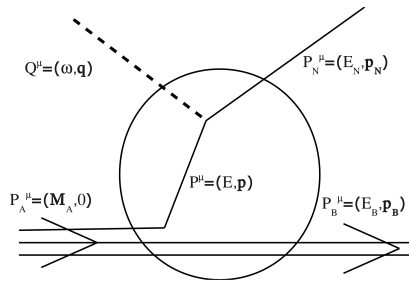
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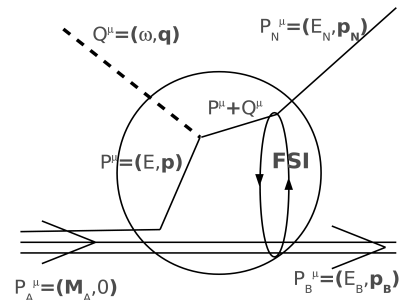
Intermediate energies (typical QE regime)



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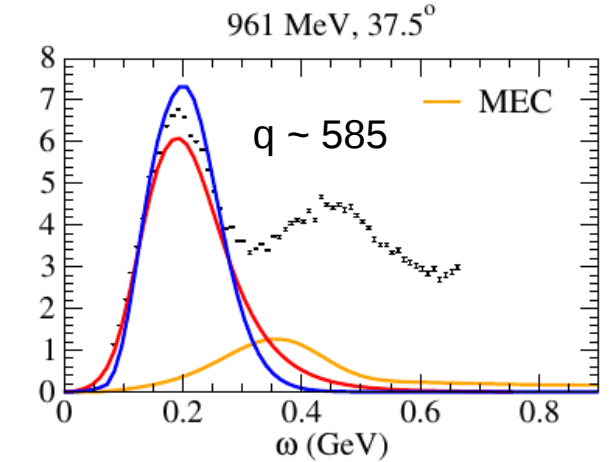
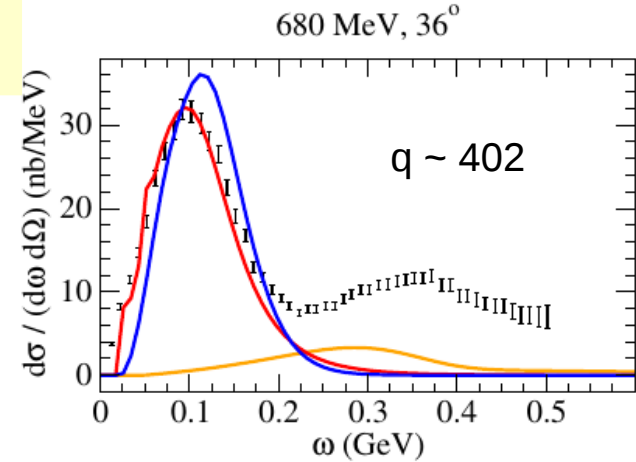
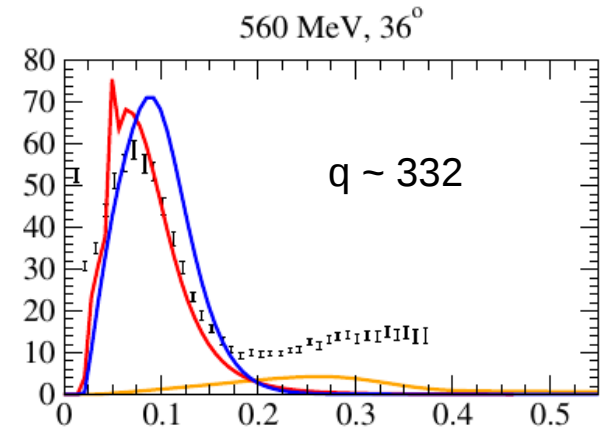
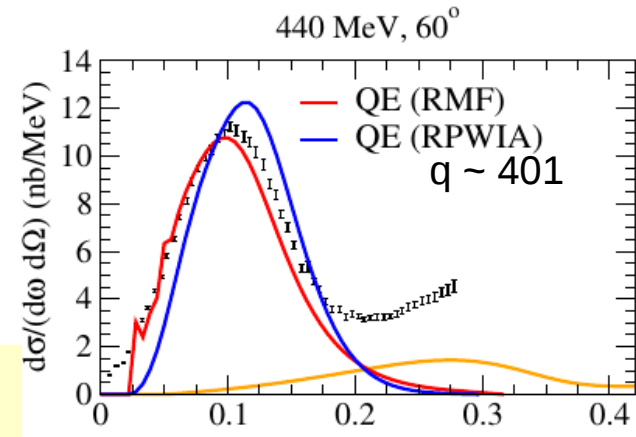


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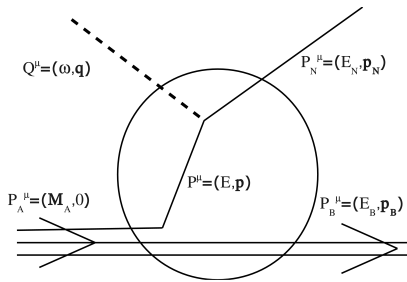
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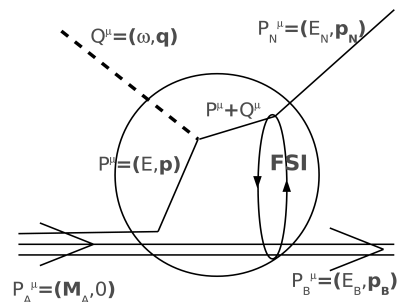
Intermediate energies (typical QE regime)



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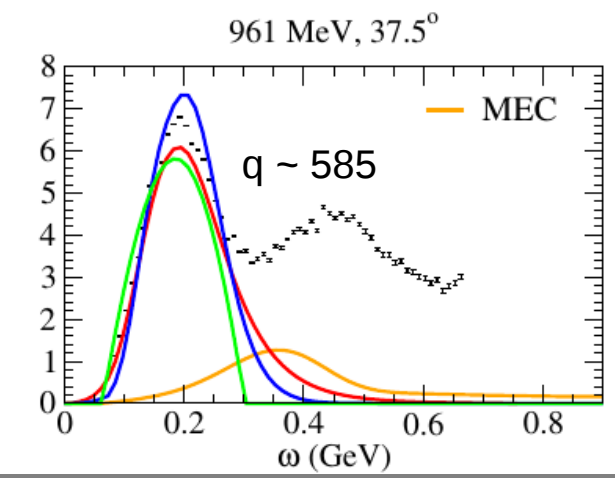
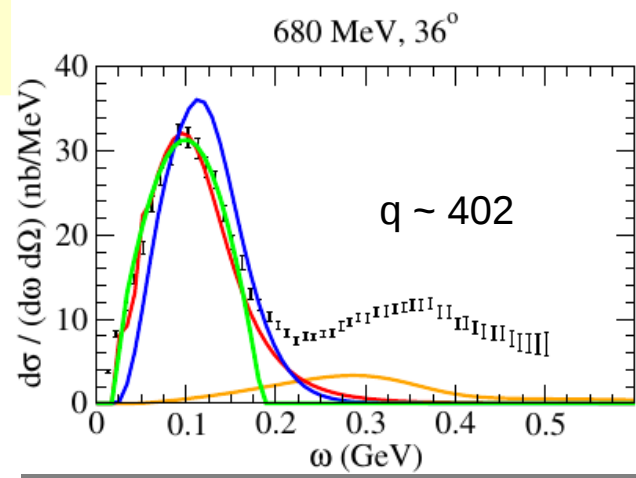
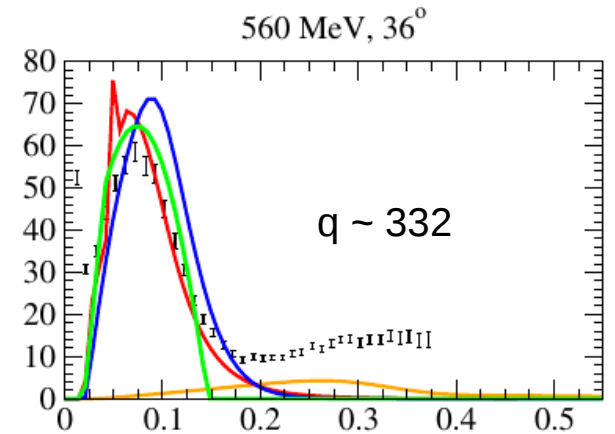
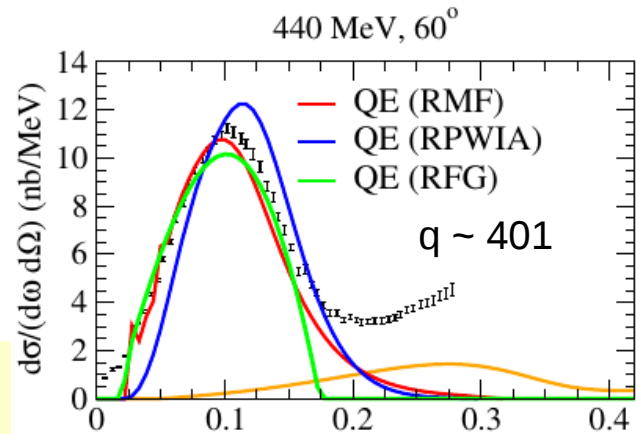


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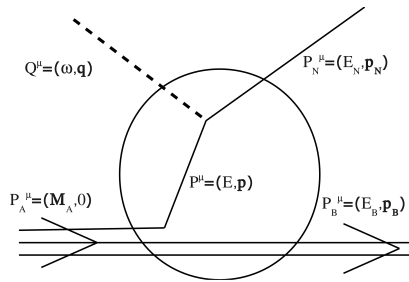
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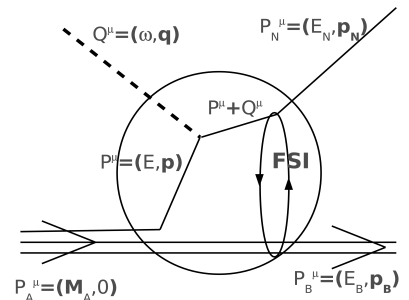
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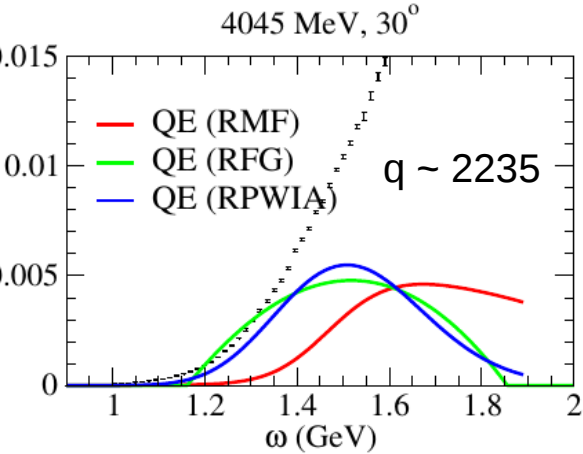
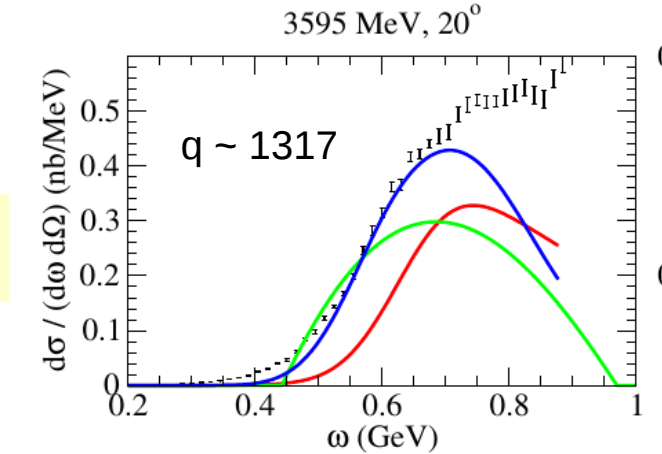
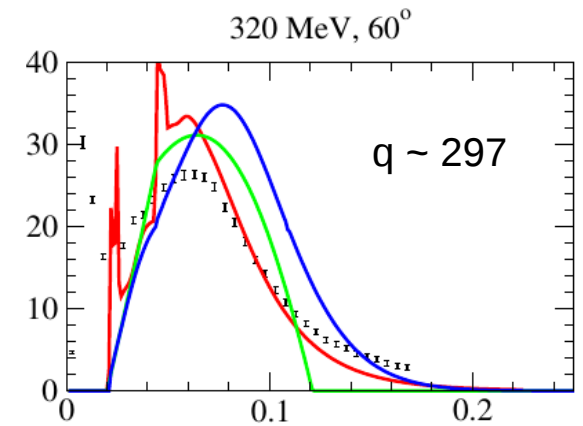
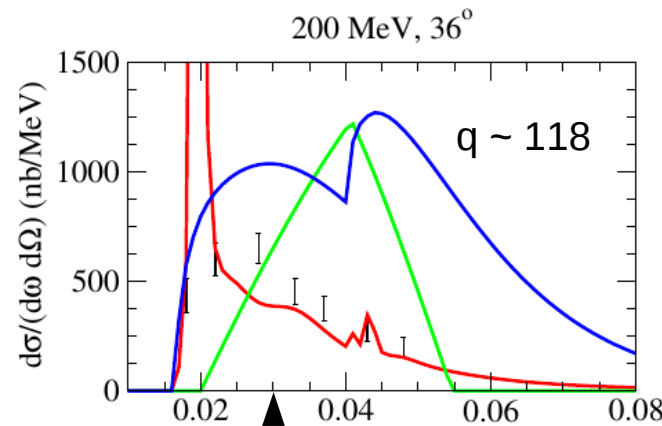
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Low energies

High energies



RMF: *transverse > longitudinal*

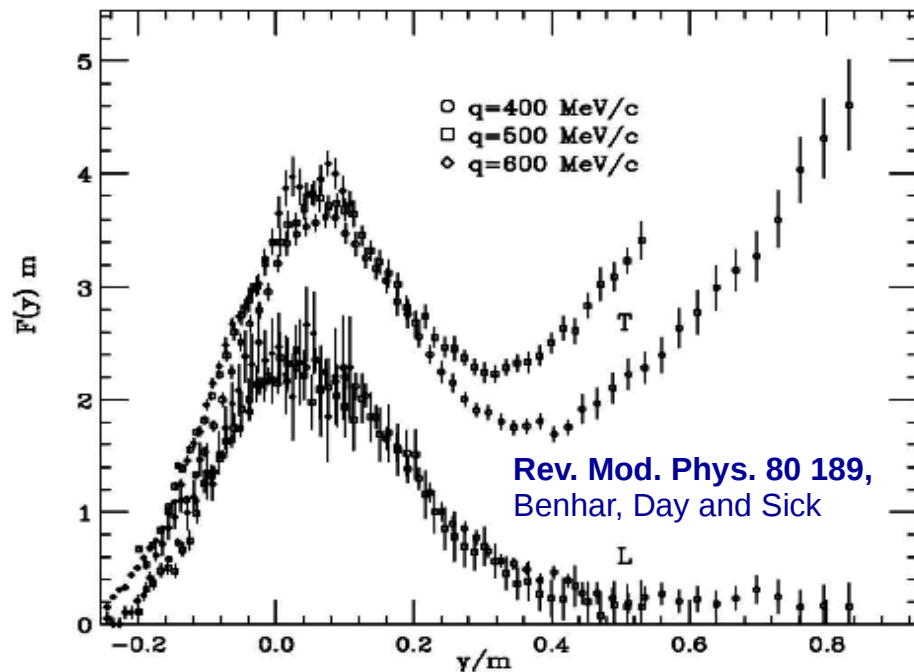
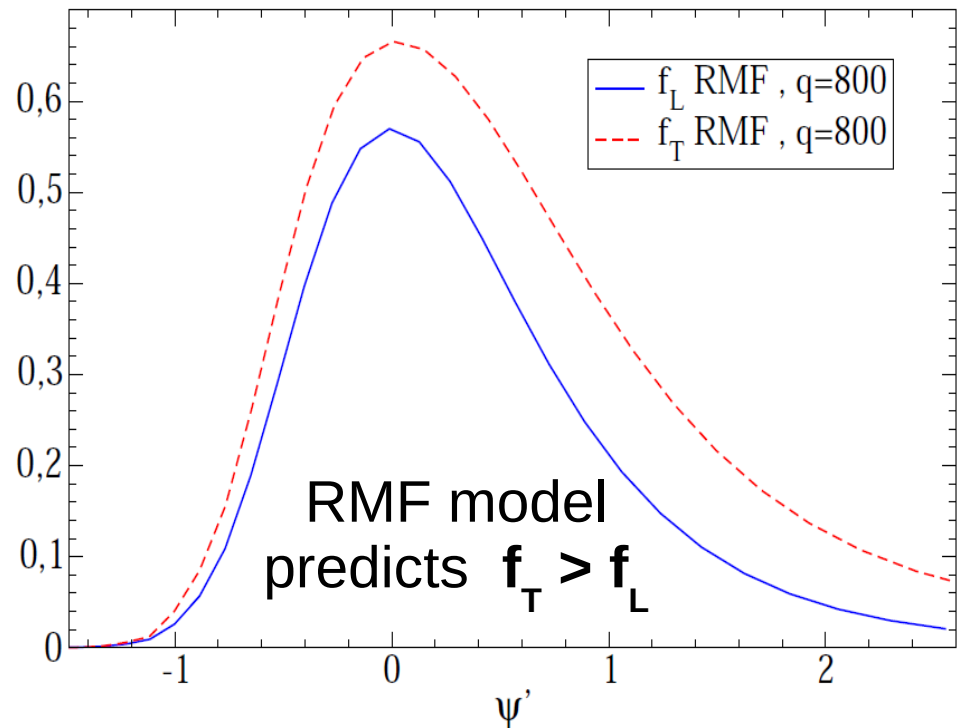


FIG. 30. Longitudinal (lower data set) and transverse responses of ^{12}C (Finn *et al.*, 1984), plotted in terms of the scaling function $F(y)$.



The origin of that effect lies in the distortion of the lower components of the bound and scattered nucleon wave functions (mainly, the latter).

Therefore, this effect does not occur in other models based on non-relativistic or semi-relativistic approaches.

The analysis of data seems to support that $f_T > f_L$.

RMF for QE: Summary

Both the distortion of initial and final nucleon wave functions have a huge impact in the cross sections

Positive:

- Long tails corresponding to high momentum of the outgoing nucleon and an asymmetric shape: good agreement with data.
- Excellent behavior at intermediate transfer momentum ($300 < q < 900$ MeV).
- Relatively good behavior at low q (much better than Fermi gas models!!).
- Prediction of a transverse enhancement ($f_L < f_T$).

Negative:

- One would expect that the behavior of RMF for increasing momentum transfer ($q > 1000$ MeV) tends to RPWIA one, but it does not happen...

RMF for QE: Summary

Both the distortion of initial and final nucleon wave functions have a huge impact in the cross sections

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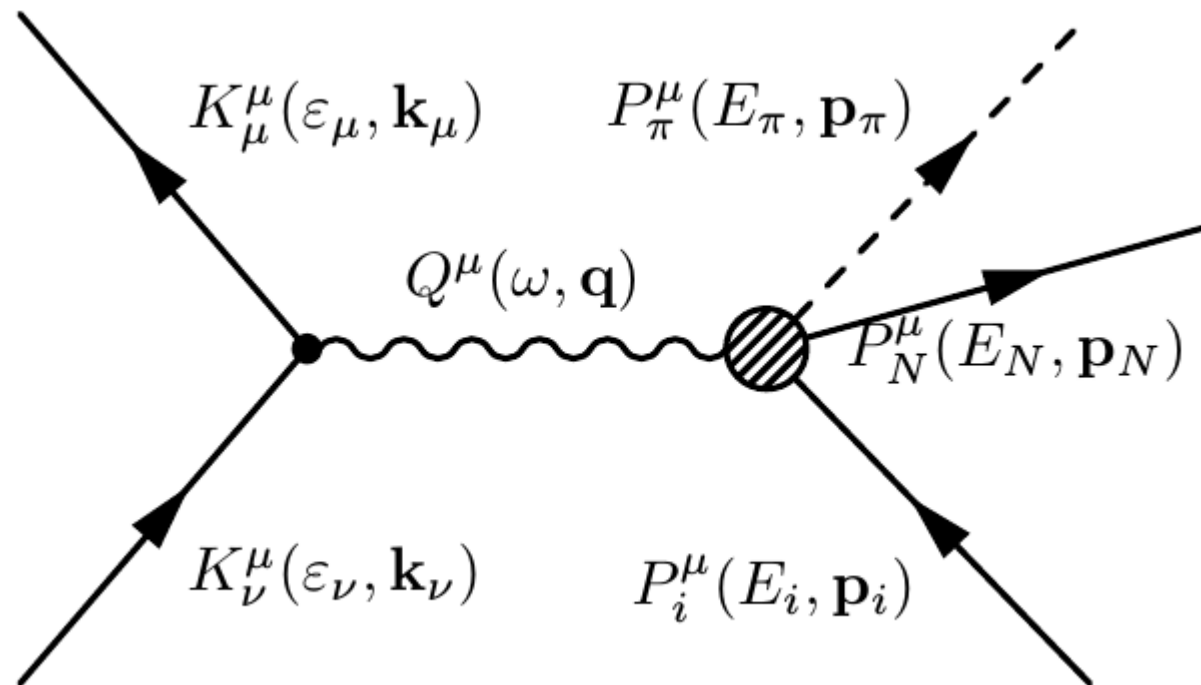
- Long tails correspond to high energy of the outgoing nucleon and an asymmetric shape: q
- Excellent behavior in the region $(300 < q < 900)$
- Relatively good agreement with gas models!!).
- Prediction of

Negative:

- One would expect that the RMF of increasing momentum transfer ($q > 1000 \text{ MeV}$) tends to RPA one, but it does not happen...

We want this model in the pion-production region!!!

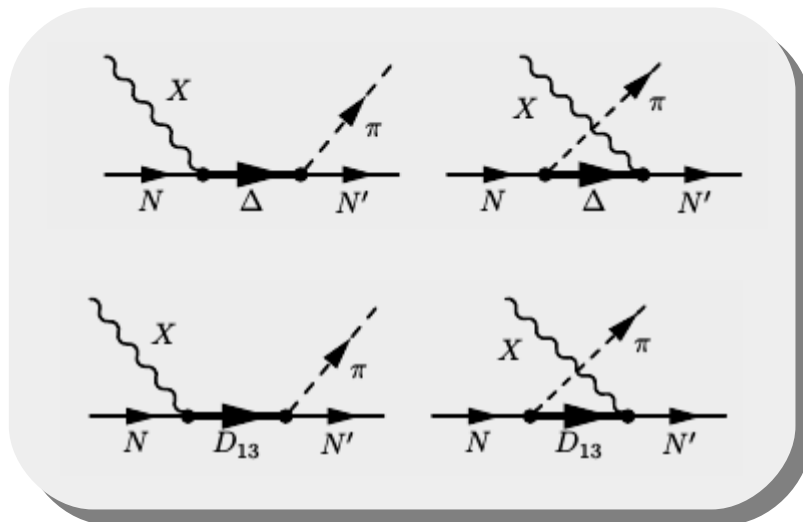
Electroweak one-pion production on nucleons



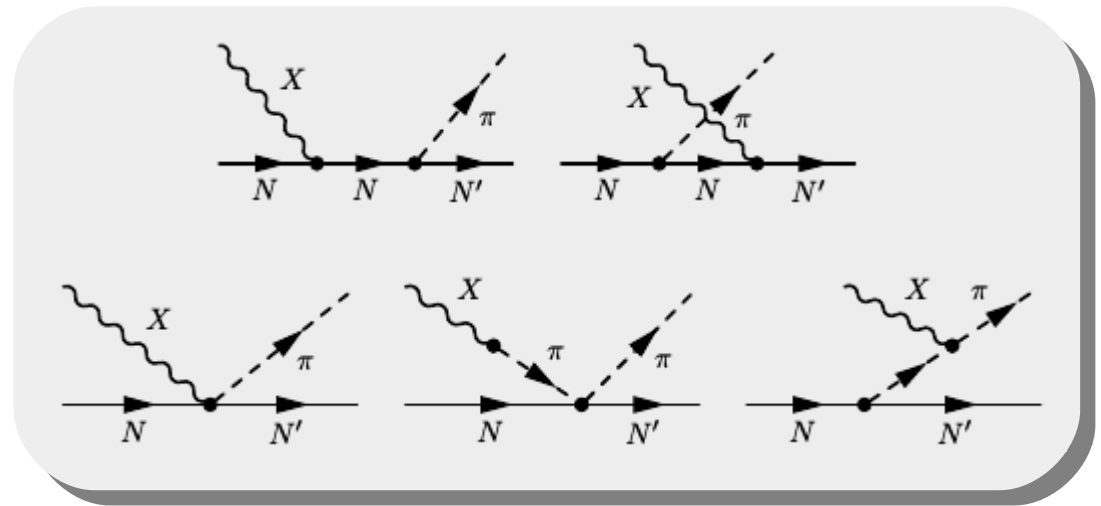
One-pion production on nucleons

We use the same one-pion-production mechanisms as in **Valencia model** (PRD 76 (2007) 033005, PRD 87 (2013) 113009)

Resonant contributions:

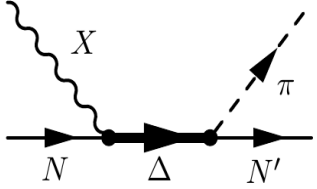


Contributions from the effective pion-nucleon Lagrangian of ChPT (non-resonant contributions):



$$J^\nu = \langle J_{\Delta P}^\nu \rangle + \langle J_{C\Delta P}^\nu \rangle + \langle J_{CT,V}^\nu \rangle + \langle J_{CT,A}^\nu \rangle + \langle J_{NP}^\nu \rangle + \langle J_{CNP}^\nu \rangle + \langle J_{PF}^\nu \rangle + \langle J_{PP}^\nu \rangle$$

Delta resonance



$$J^\mu = \bar{u}(\mathbf{p}_f, s_f) \Gamma_{\Delta\pi N}^\alpha S_{\Delta, \alpha\beta} \Gamma_{WN\Delta}^{\beta\mu} u(\mathbf{p}_i, s_i)$$

Nucleon-Delta transition vertex:

$$\Gamma_{WN\Delta}^{\beta\mu} = \left[\frac{C_3^V(Q^2)}{M} (g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu) + \frac{C_4^V(Q^2)}{M_N^2} (g^{\beta\mu} Q \cdot K_\Delta - Q^\beta K_\Delta^\mu) + \frac{C_5^V(Q^2)}{M_N^2} (g^{\beta\mu} Q \cdot P_i - Q^\beta P_i^\mu) + C_6^V(Q^2) g^{\beta\mu} \right] \gamma^5$$

$$+ \frac{C_3^A(Q^2)}{M} (g^{\beta\mu} \not{Q} - Q^\beta \gamma^\mu) + \frac{C_4^A(Q^2)}{M_N^2} (g^{\beta\mu} Q \cdot K_\Delta - Q^\beta K_\Delta^\mu) + C_5^A(Q^2) g^{\beta\mu} + \frac{C_6^A(Q^2)}{M_N^2} Q^\beta Q^\mu$$

Delta propagator:

$$S_{\Delta, \alpha\beta} = \frac{-(K_\Delta + M_\Delta)}{K_\Delta^2 - M_\Delta^2 + iM_\Delta \Gamma_{\text{width}}} \left(g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{2}{3M_\Delta^2} K_{\Delta, \alpha} K_{\Delta, \beta} - \frac{2}{3M_\Delta} (\gamma_\alpha K_{\Delta, \beta} - K_{\Delta, \alpha} \gamma_\beta) \right)$$

with the energy dependent Delta width: $\Gamma_{\text{width}}(W) = \frac{1}{12\pi} \frac{(f_{\pi N\Delta})^2}{m_\pi^2 W} (\rho_{\pi, cm})^3 (M + E_{N, cm})$

1) Traditional

Delta decay:

$$\Gamma_{\Delta\pi N}^\alpha = \frac{f_{\pi N\Delta}}{m_\pi} K_\pi^\alpha$$

2) Pascalutsa (it only couples to the physical spin-3/2 degrees of freedom of the Delta)

$$\Gamma_{\Delta\pi N}^\alpha = \frac{f_{\pi N\Delta}}{m_\pi M_N} \epsilon^{\alpha\rho\sigma\tau} K_{\pi, \rho} \gamma_\sigma \gamma_5 k_{\Delta, \tau}$$

Non-resonant contributions

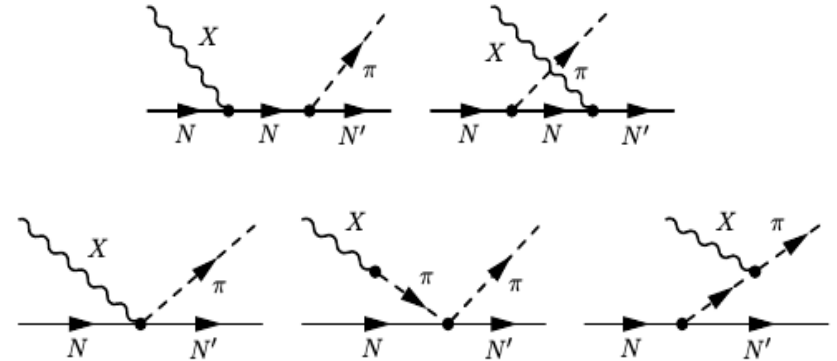
Chiral Perturbation Theory applied to the pion-nucleon system gives the following effective Lagrangian at lowest order in $1/f_\pi$

$$\mathcal{L}^{eff} = \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi\pi NN}$$

$$+ \mathcal{L}_{\gamma NN} + \mathcal{L}_{\gamma\pi\pi} + \mathcal{L}_{\gamma\pi NN}$$

$$+ \mathcal{L}_{WNN} + \mathcal{L}_{W\pi} + \mathcal{L}_{W\pi\pi} + \mathcal{L}_{W\pi NN}^V + \mathcal{L}_{W\pi NN}^A$$

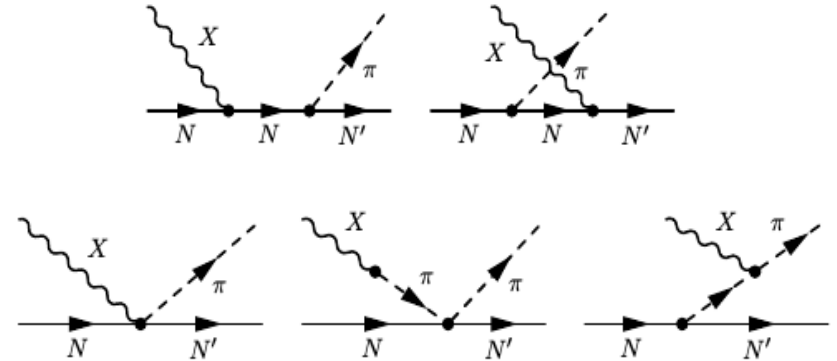
$$+ \mathcal{L}_{ZNN} + \mathcal{L}_{Z\pi} + \mathcal{L}_{Z\pi\pi} + \mathcal{L}_{Z\pi NN}^V + \mathcal{L}_{Z\pi NN}^A.$$



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$$\begin{aligned} \mathcal{L}^{eff} = & \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi\pi NN} \\ & + \mathcal{L}_{\gamma NN} + \mathcal{L}_{\gamma\pi\pi} + \mathcal{L}_{\gamma\pi NN} \\ & + \mathcal{L}_{WNN} + \mathcal{L}_{W\pi} + \mathcal{L}_{W\pi\pi} + \mathcal{L}_{W\pi NN}^V + \mathcal{L}_{W\pi NN}^A \\ & + \mathcal{L}_{ZNN} + \mathcal{L}_{Z\pi} + \mathcal{L}_{Z\pi\pi} + \mathcal{L}_{Z\pi NN}^V + \mathcal{L}_{Z\pi NN}^A. \end{aligned}$$

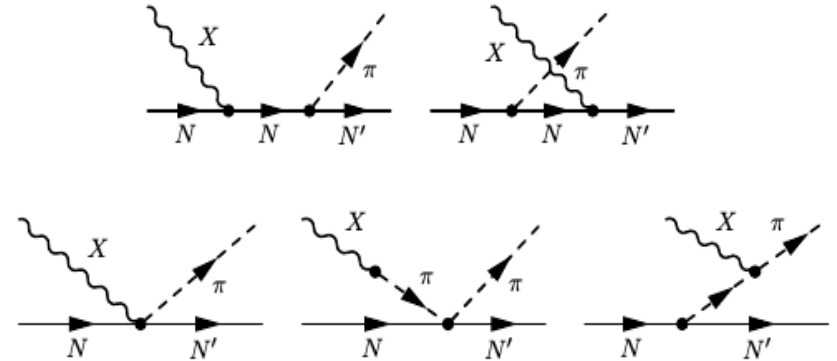


$$\begin{aligned} \mathcal{L}_{\pi NN} = & -\frac{g_A}{2f_\pi} \left[\bar{\psi}_p (\gamma^\mu \gamma^5 \partial_\mu \pi_0) \psi_p - \bar{\psi}_n (\gamma^\mu \gamma^5 \partial_\mu \pi_0) \psi_n \right. \\ & \left. + \sqrt{2} \bar{\psi}_p (\gamma^\mu \gamma^5 \partial_\mu \pi_+) \psi_n - \sqrt{2} \bar{\psi}_n (\gamma^\mu \gamma^5 \partial_\mu \pi_-) \psi_p \right], \end{aligned}$$

Non-resonant contributions

Chiral Perturbation Theory applied to the pion-nucleon system gives the following effective Lagrangian

$$\begin{aligned} \mathcal{L}^{eff} &= \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi\pi NN} \\ &+ \mathcal{L}_{\gamma NN} + \mathcal{L}_{\gamma\pi\pi} + \mathcal{L}_{\gamma\pi NN} \\ &+ \mathcal{L}_{WNN} + \mathcal{L}_{W\pi} + \mathcal{L}_{W\pi\pi} + \mathcal{L}_{W\pi NN}^V + \mathcal{L}_{W\pi NN}^A \\ &+ \mathcal{L}_{ZNN} + \mathcal{L}_{Z\pi} + \mathcal{L}_{Z\pi\pi} + \mathcal{L}_{Z\pi NN}^V + \mathcal{L}_{Z\pi NN}^A. \end{aligned}$$

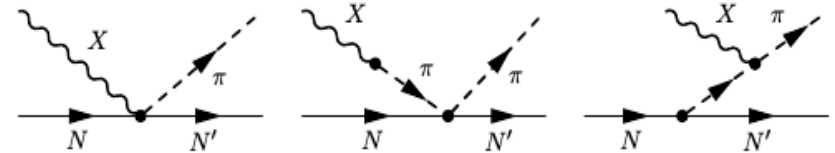
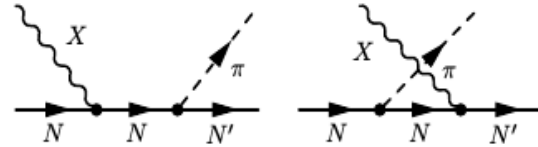


$$\begin{aligned} \mathcal{L}_{\pi NN} &= -\frac{g_A}{2f_\pi} \left[\bar{\psi}_p (\gamma^\mu \gamma^5 \partial_\mu \pi_0) \psi_p - \bar{\psi}_n (\gamma^\mu \gamma^5 \partial_\mu \pi_0) \psi_n \right. \\ &\quad \left. + \sqrt{2} \bar{\psi}_p (\gamma^\mu \gamma^5 \partial_\mu \pi_+) \psi_n - \sqrt{2} \bar{\psi}_n (\gamma^\mu \gamma^5 \partial_\mu \pi_-) \psi_p \right], \\ \mathcal{L}_{\pi\pi NN} &= -\frac{i}{4f_\pi^2} \left[\bar{\psi}_p \gamma^\mu (\pi_- \partial_\mu \pi_+ - \pi_+ \partial_\mu \pi_-) \psi_p - \bar{\psi}_n \gamma^\mu (\pi_- \partial_\mu \pi_+ - \pi_+ \partial_\mu \pi_-) \psi_n \right. \\ &\quad \left. + \sqrt{2} \bar{\psi}_p \gamma^\mu (\pi_+ \partial_\mu \pi_0 - \pi_0 \partial_\mu \pi_+) \psi_n + \sqrt{2} \bar{\psi}_n \gamma^\mu (\pi_0 \partial_\mu \pi_- - \pi_- \partial_\mu \pi_0) \psi_p \right], \end{aligned}$$

Non-resonant contributions

Chiral Perturbation Theory applied to the pion-nucleon system gives the following effective Lagrangian

$$\begin{aligned} \mathcal{L}^{eff} &= \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi\pi NN} \\ &+ \mathcal{L}_{\gamma NN} + \mathcal{L}_{\gamma\pi\pi} + \mathcal{L}_{\gamma\pi NN} \\ &+ \mathcal{L}_{WNN} + \mathcal{L}_{W\pi} + \mathcal{L}_{W\pi\pi} + \mathcal{L}_{W\pi NN}^V + \mathcal{L}_{W\pi NN}^A \\ &+ \mathcal{L}_{ZNN} + \mathcal{L}_{Z\pi} + \mathcal{L}_{Z\pi\pi} + \mathcal{L}_{Z\pi NN}^V + \mathcal{L}_{Z\pi NN}^A. \end{aligned}$$



$$\mathcal{L}_{\pi NN} = -\frac{g_A}{2f_\pi} \left[\bar{\psi}_p (\gamma^\mu \gamma^5 \partial_\mu \pi_0) \psi_p - \bar{\psi}_n (\gamma^\mu \gamma^5 \partial_\mu \pi_0) \psi_n + \sqrt{2} \bar{\psi}_p (\gamma^\mu \gamma^5 \partial_\mu \pi_+) \psi_n - \sqrt{2} \bar{\psi}_n (\gamma^\mu \gamma^5 \partial_\mu \pi_-) \psi_p \right],$$

$$\mathcal{L}_{\pi\pi NN} = -\frac{i}{4f_\pi^2} \left[\bar{\psi}_p \gamma^\mu (\pi_- \partial_\mu \pi_+ - \pi_+ \partial_\mu \pi_-) \psi_p - \bar{\psi}_n \gamma^\mu (\pi_- \partial_\mu \pi_+ - \pi_+ \partial_\mu \pi_-) \psi_n + \sqrt{2} \bar{\psi}_p \gamma^\mu (\pi_+ \partial_\mu \pi_0 - \pi_0 \partial_\mu \pi_+) \psi_n + \sqrt{2} \bar{\psi}_n \gamma^\mu (\pi_0 \partial_\mu \pi_- - \pi_- \partial_\mu \pi_0) \psi_p \right],$$

$$\mathcal{L}_{W\pi} = -\frac{g}{2} f_\pi \cos \theta_c (\partial^\mu \pi_- W_\mu^+ + \partial^\mu \pi_+ W_\mu^-),$$

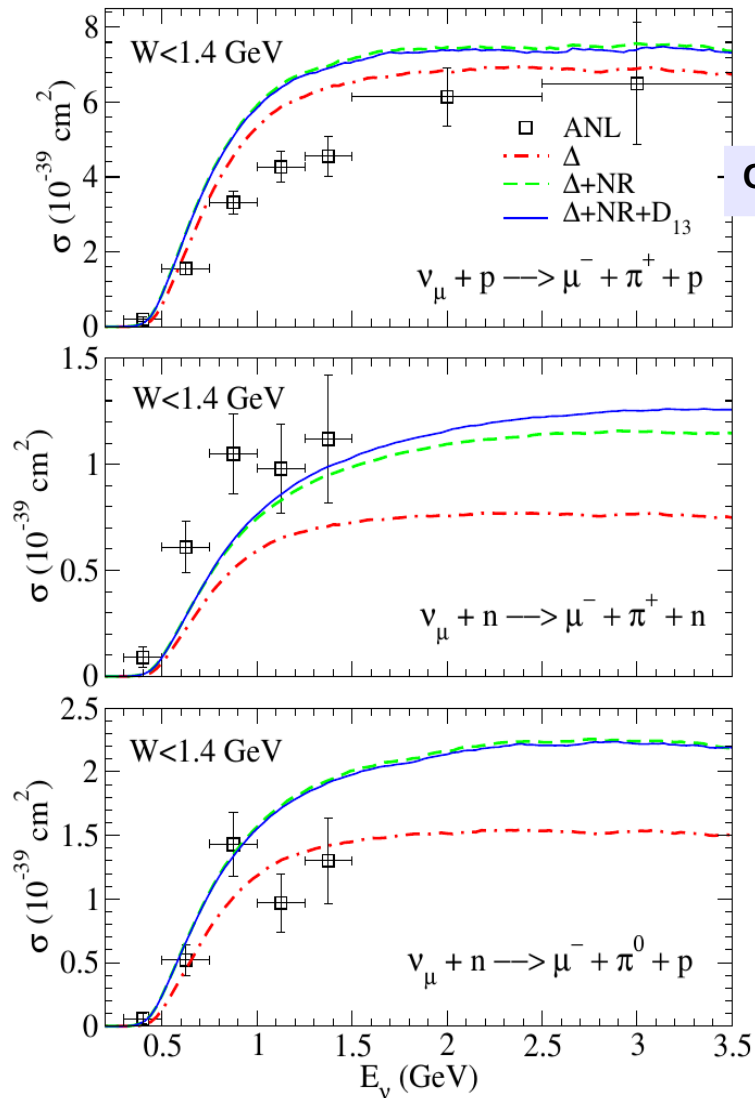
$$\begin{aligned} \mathcal{L}_{WNN} &= \frac{-g \cos \theta_c}{2\sqrt{2}} \left[\bar{\psi}_p \gamma^\mu (1 - g_A \gamma^5) \psi_n W_\mu^+ + \bar{\psi}_n \gamma^\mu (1 - g_A \gamma^5) \psi_p W_\mu^- \right] \\ &\rightarrow \frac{-g \cos \theta_c}{2\sqrt{2}} \left[\bar{\psi}_p (\hat{\Gamma}_V^\mu - \hat{\Gamma}_A^\mu) \psi_n W_\mu^+ + \bar{\psi}_n (\hat{\Gamma}_V^\mu - \hat{\Gamma}_A^\mu) \psi_p W_\mu^- \right], \end{aligned}$$

$$\mathcal{L}_{W\pi\pi} = i \frac{g}{2} \cos \theta_c \left[(\pi_- \partial^\mu \pi_0 - \partial^\mu \pi_- \pi_0) W_\mu^+ + (\pi_+ \partial^\mu \pi_0 - \partial^\mu \pi_+ \pi_0) W_\mu^- \right]$$

.... etc. (see, for instance, S. Scherer and M. R. Schindler, Springer 2012) 23

Checking our implementation

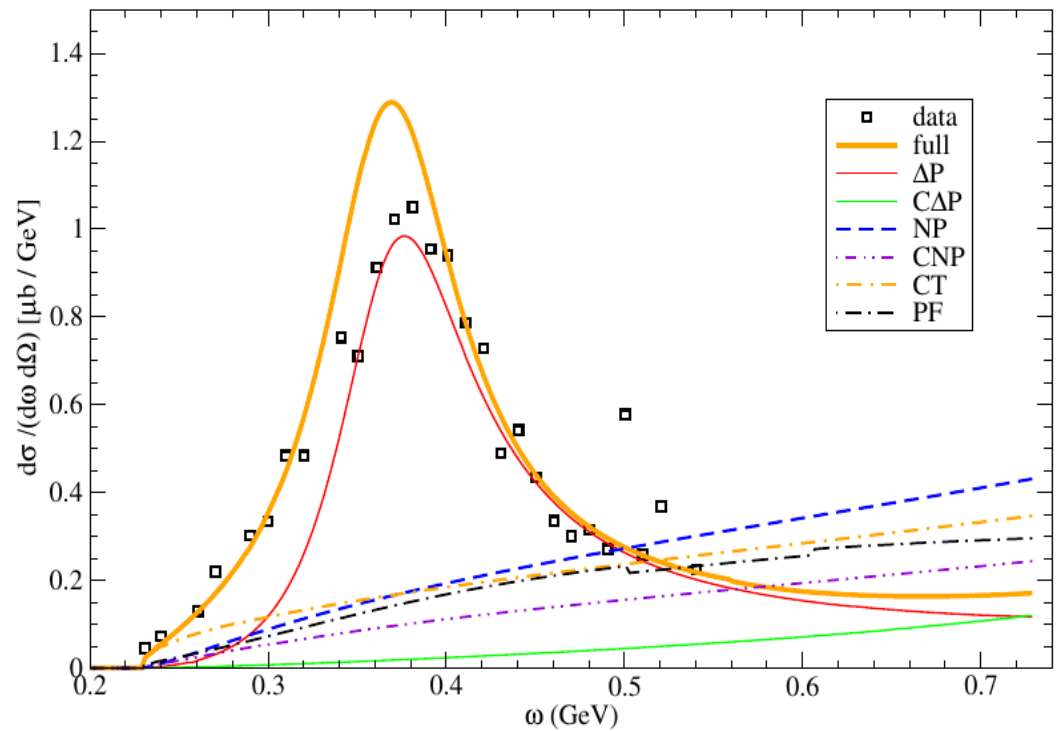
CC-neutrino induced pion production



$$C_5^A(0) = 1.2; M_A = 1.05 \text{ GeV}$$

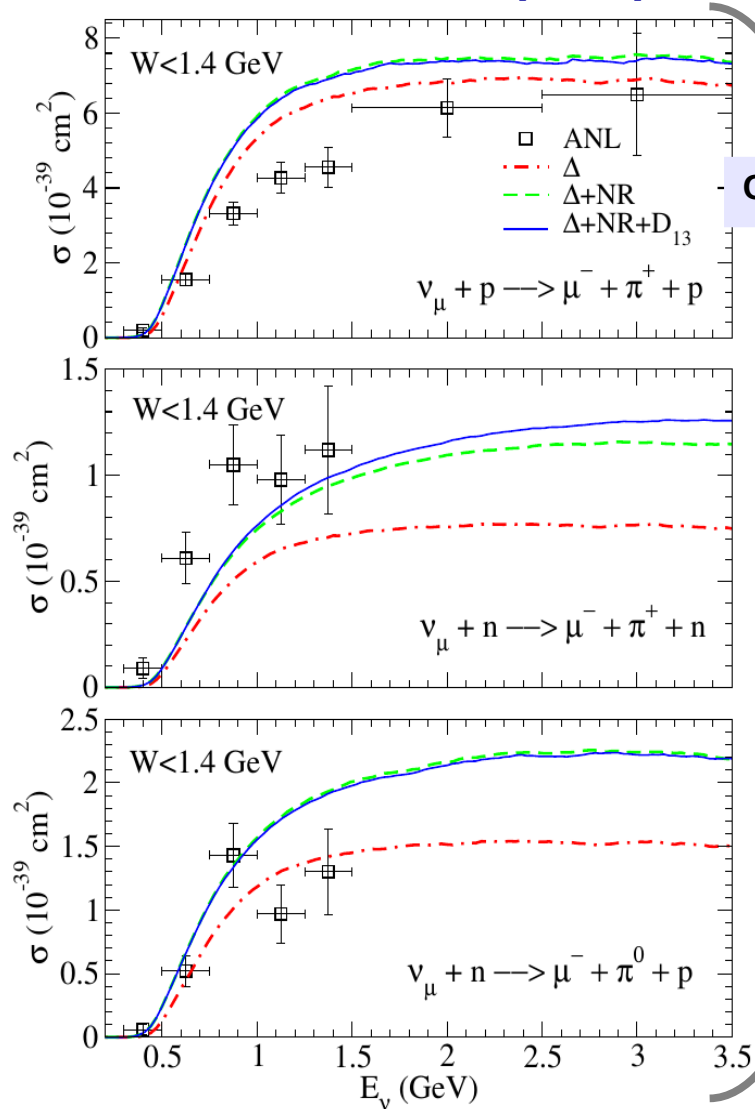
Electron induced pion production

$$E = 730 \text{ MeV}, \theta_e = 37.1^\circ$$



Checking our implementation

CC-neutrino induced pion production

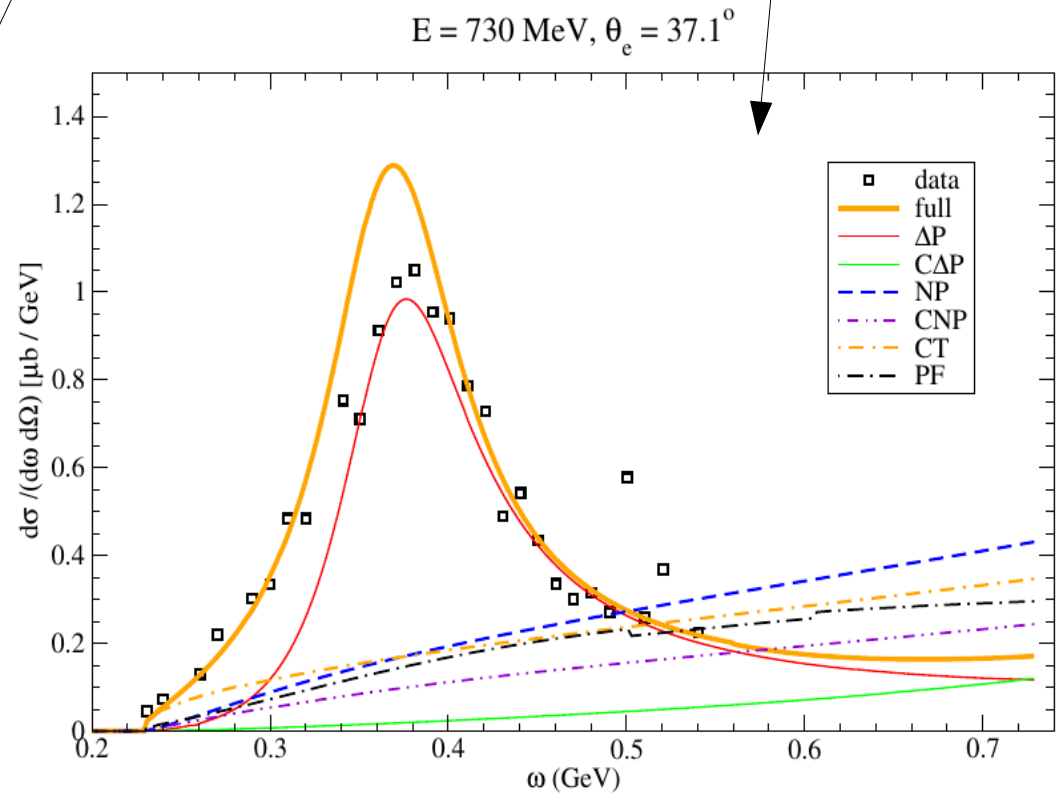


$C_5(0) = 1.2; M_A = 1.05$ GeV

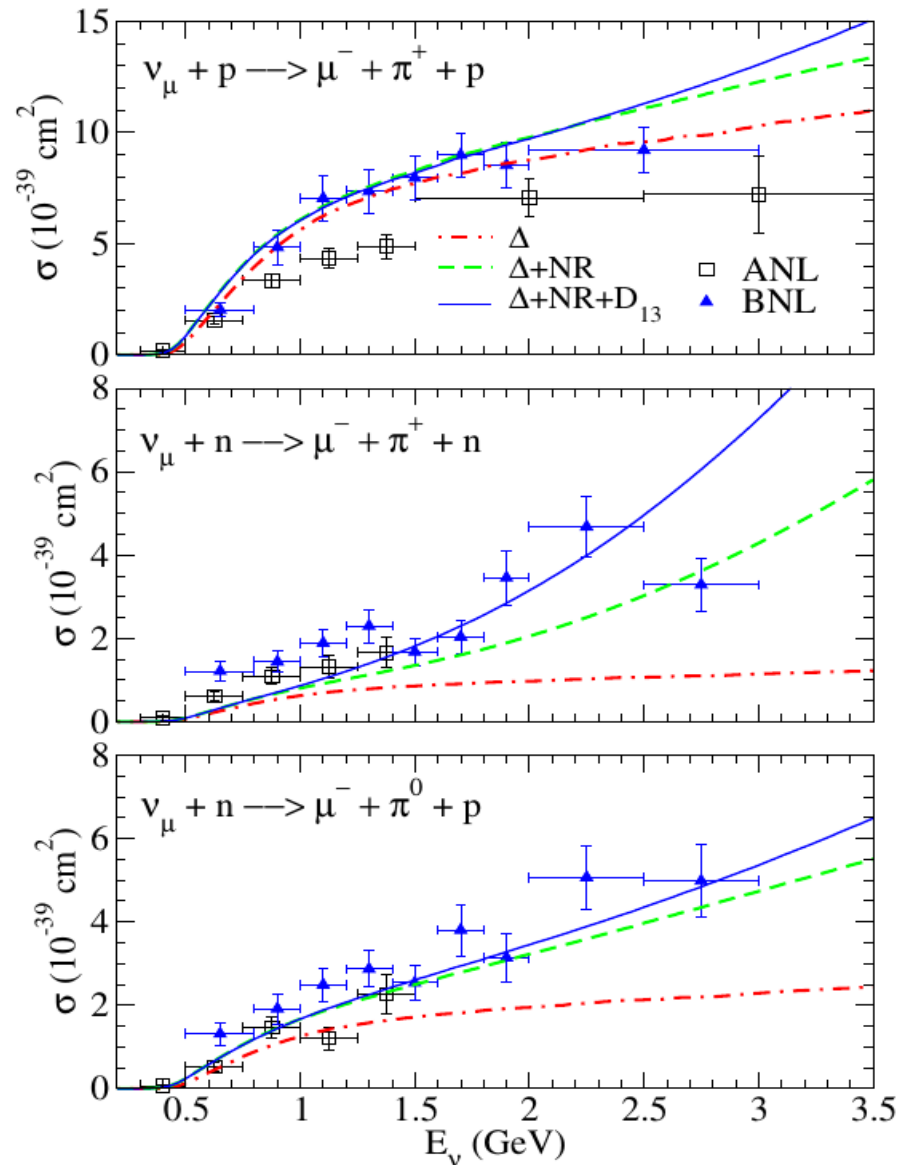
Same results as in HNV
(PRD 76, 033005 (2007))

Same results as in
J. Zmuda's PhD Thesis

Electron induced pion production



Checking our implementation

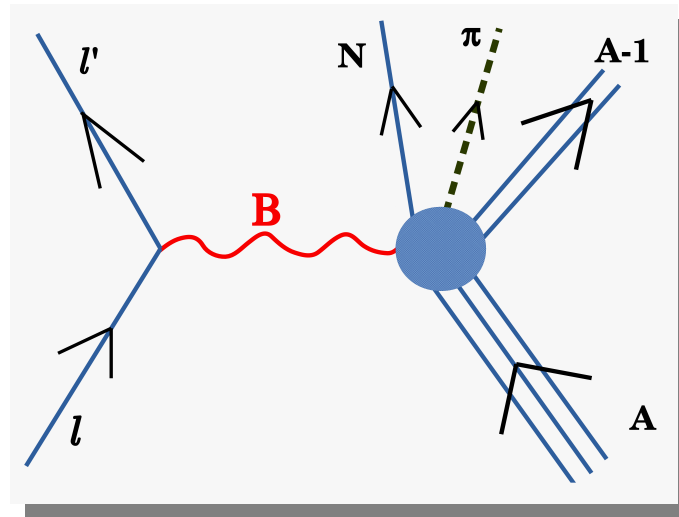


Fit to BNL data:

$$C_5^A(0) = 1.2$$

$$M_A = 1.05 \text{ GeV}$$

Electroweak one-pion production on nuclei

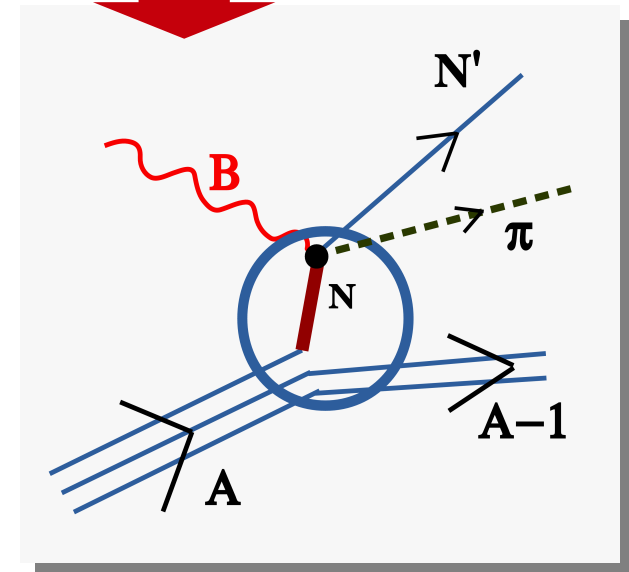
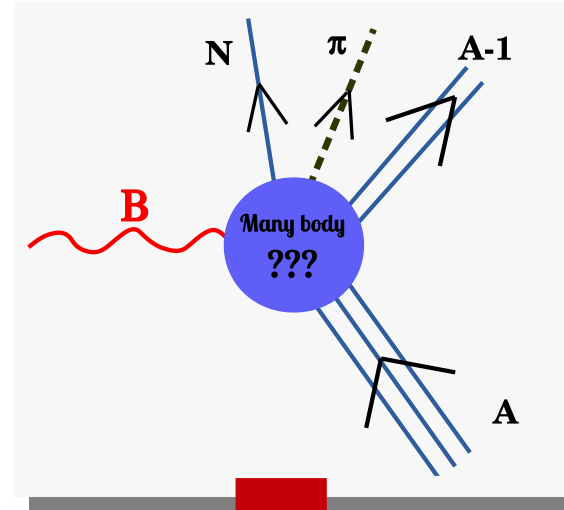


Impulse approximation

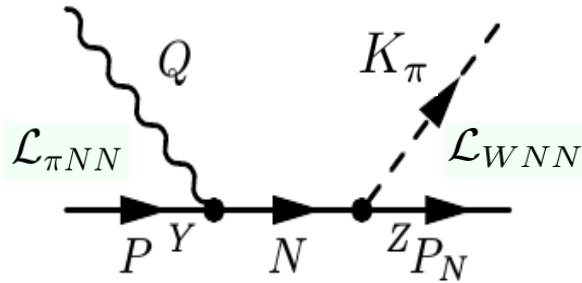
$$J_{had}^{\mu} = \langle N, \pi, A - 1 | \hat{O}_{many-body}^{\mu} | A \rangle$$

**Relativistic
Impulse
Approximation**

$$J_{had}^{\mu} = \sum_i^A \int d\mathbf{r} \bar{\Psi}_F(\mathbf{r}) \phi^*(\mathbf{r}) \hat{O}_{one-body}^{\mu}(\mathbf{r}) \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$



Hadronic Current (example, Nucleon pole)



$$J_{NP}^\nu(Q, P_N, K_\pi) = i \frac{g}{2\sqrt{2}} \frac{-ig_A \cos \theta_c}{\sqrt{2} f_\pi} \int d^4 Y \int d^4 Z \int \frac{d^4 K}{(2\pi)^4} e^{-iQ \cdot Y} \times \bar{\psi}_n(Z) (-i \not{\partial} \phi^*(Z)) \frac{K + M}{K^2 - M^2} e^{iK \cdot (Y-Z)} \gamma^\mu (1 - g_A \gamma^5) \psi_n(Y)$$

After some algebra and considering the initial and final states as states with well defined energy obtain:

$$J_{NP}^\nu(Q, P_N, K_\pi) = (2\pi) \delta(E_N + E_\pi - \omega - E) i \frac{g}{2\sqrt{2}} \frac{-ig_A \cos \theta_c}{\sqrt{2} f_\pi} \times \int dz \bar{\psi}_n(\mathbf{z}) \int \frac{d\mathbf{p}}{(2\pi)^{3/2}} e^{i(\mathbf{p}+\mathbf{q}) \cdot \mathbf{z}} (E_\pi \gamma^0 + i\boldsymbol{\gamma} \cdot \nabla) \phi^*(\mathbf{z}) \frac{K + M}{K^2 - M^2} \psi_n(\mathbf{p})$$

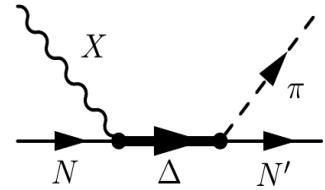
with $K^\mu = (E_N + E_\pi, \mathbf{q} + \mathbf{p})$.

Approximation: To simplify, in the propagator we use asymptotic values for the momentum of the particles

$$\mathbf{k} = \mathbf{q} + \mathbf{p} \longrightarrow \mathbf{k} = \mathbf{p}_N + \mathbf{k}_\pi$$

$$J_{NP}^\nu(Q, P_N, K_\pi) = (2\pi) \delta(E_N + E_\pi - \omega - E) i \frac{g}{2\sqrt{2}} \frac{-ig_A \cos \theta_c}{\sqrt{2} f_\pi} \times \int dz \bar{\psi}_n(\mathbf{z}) e^{i\mathbf{q} \cdot \mathbf{z}} (E_\pi \gamma^0 + i\boldsymbol{\gamma} \cdot \nabla) \phi^*(\mathbf{z}) \frac{K + M}{K^2 - M^2} \psi_n(\mathbf{z})$$

Medium modifications of the Delta



The mass (M_Δ) and the width ($\Gamma_{\text{width}}^{\text{free}}$) are modified inside a nucleus. We use the *Oset and Salcedo* [*] formalism to implement these medium modifications (MM):

$$\Gamma_{\text{width}}^{\text{free}} \longrightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_\Delta), \quad M_\Delta^{\text{free}} \longrightarrow M_\Delta^{\text{in-medium}} = M_\Delta^{\text{free}} + \Re(\Sigma_\Delta).$$

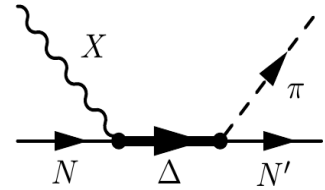
+ Γ_{Pauli} : some nucleons from Δ -decay are Pauli blocked (the Δ -decay width decreases).

+ The parametrization of $\Im(\Sigma_\Delta)$ and $\Re(\Sigma_\Delta)$ is given in terms of the nuclear density ρ :

$$\begin{aligned} -\Im(\Sigma_\Delta) &= C_{QE} (\rho/\rho_0)^\alpha + C_{A2} (\rho/\rho_0)^\beta + C_{A3} (\rho/\rho_0)^\gamma, \\ \Re(\Sigma_\Delta) &= 40 \text{ MeV} (\rho/\rho_0). \end{aligned}$$

References: [*] E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987).

Medium modifications of the Delta



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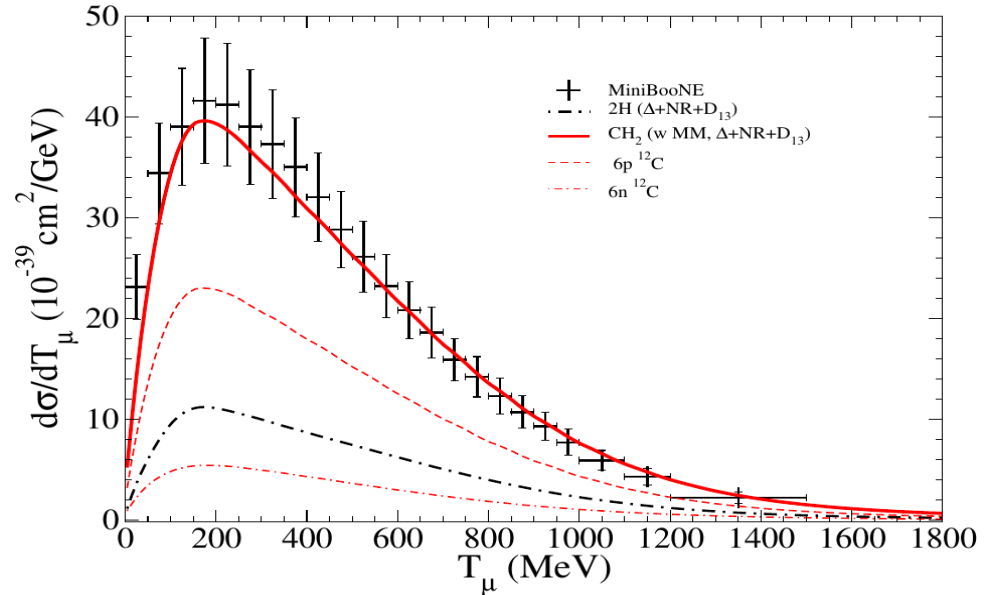
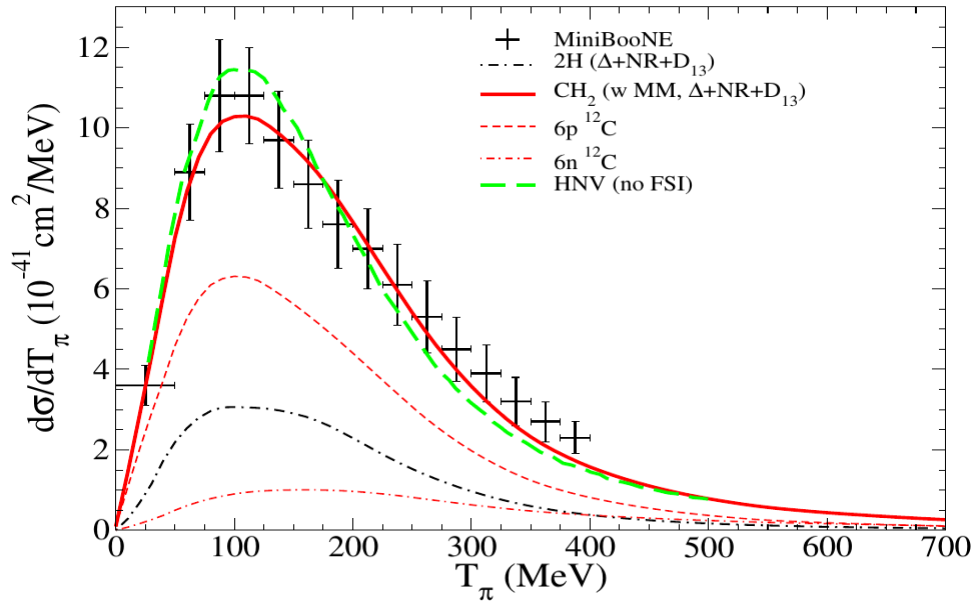
Now the operator explicitly depends on r

References: [*] E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 6

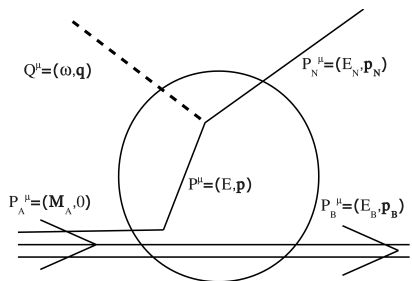
$$J_{had}^\mu = \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \bar{\psi}_p(\mathbf{r}) \phi^*(\mathbf{r}) \left[\Gamma_{\Delta\pi N}^\alpha S_{\Delta,\alpha\beta}(r) \Gamma_{WN\Delta}^{\beta\mu} \right] \psi_n(\mathbf{r})$$

Results

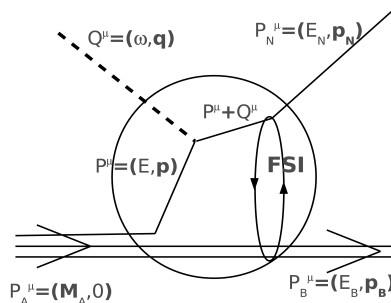
MiniBooNE Charged-Current $1\pi^+$



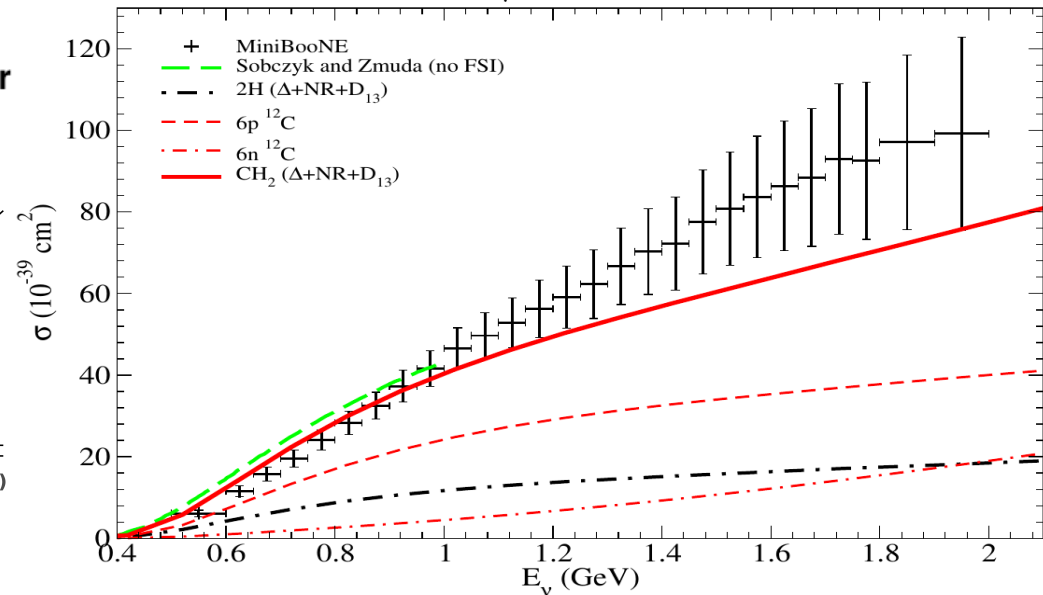
$$J_{had}^\mu = \sum_i^A \int dr \bar{\Psi}_F(\mathbf{r}) \phi^*(\mathbf{r}) \hat{O}_{one-body}^\mu(\mathbf{r}) \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$



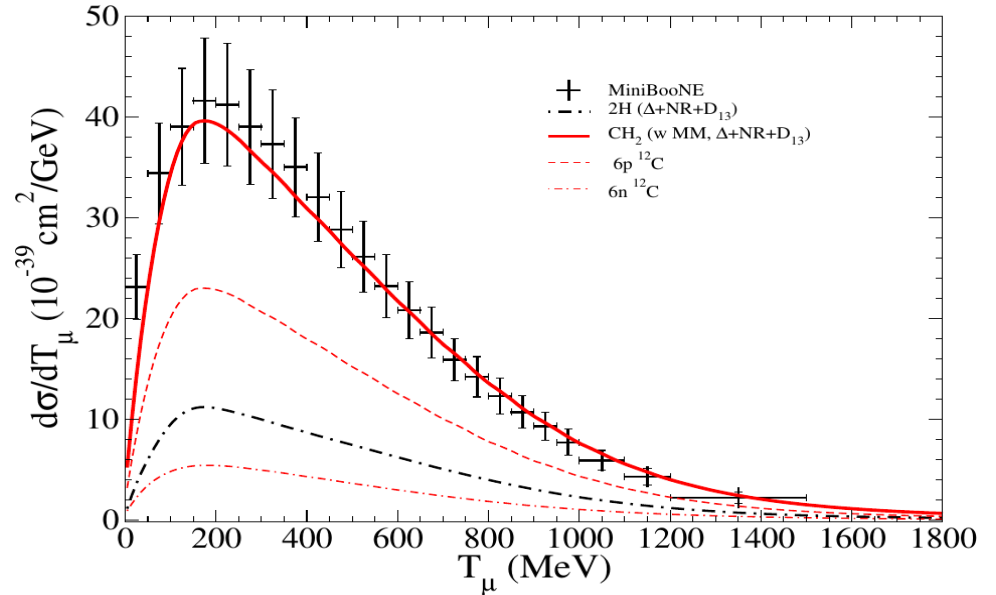
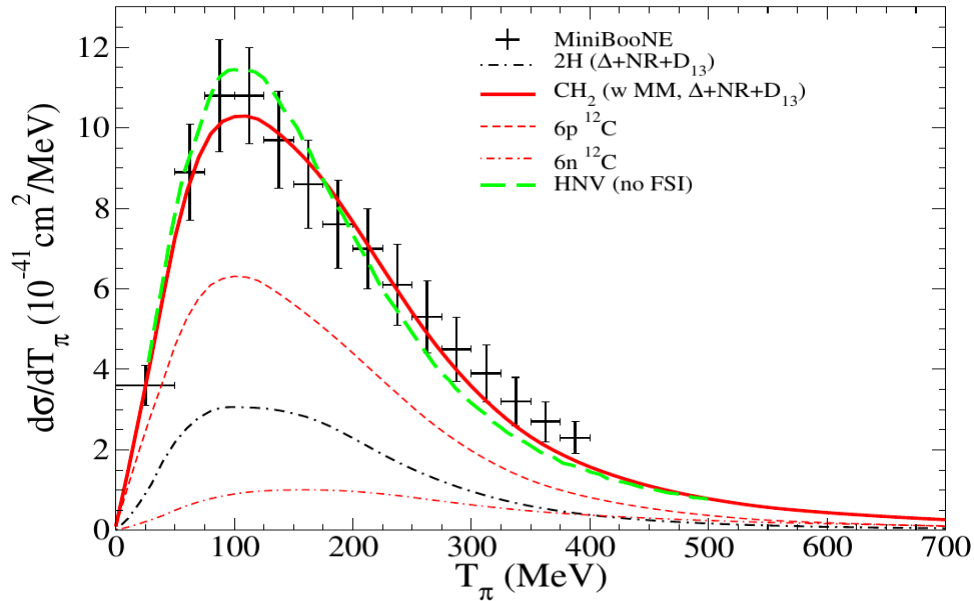
RPWIA



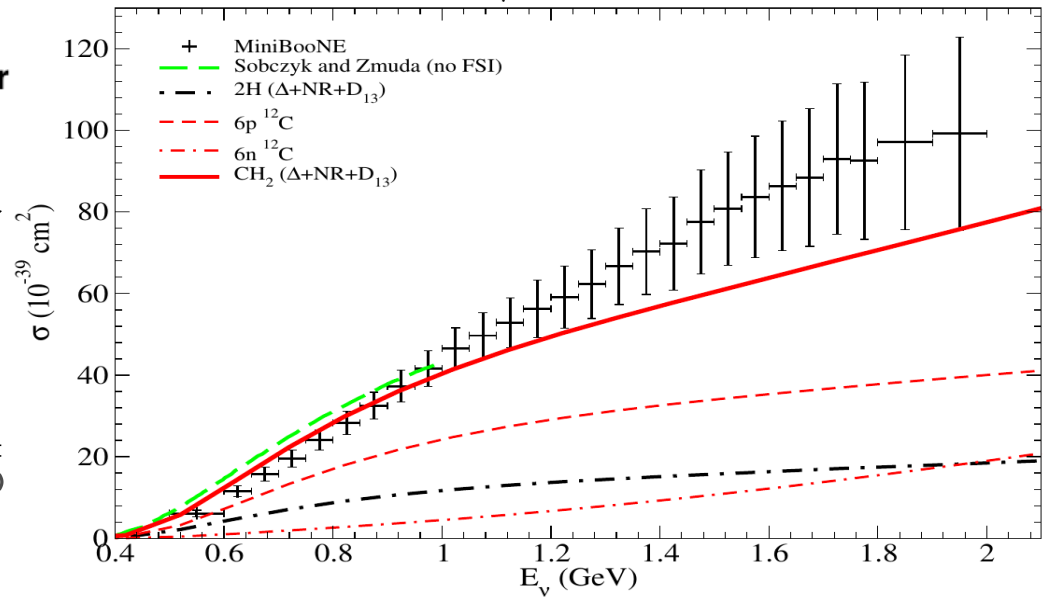
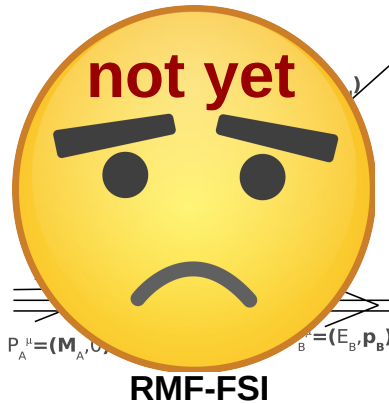
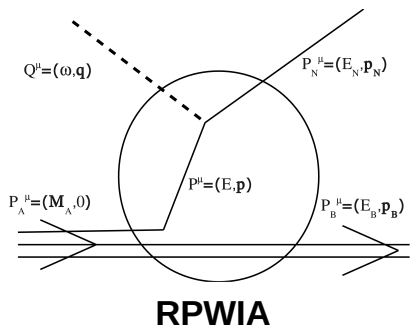
RMF-FSI



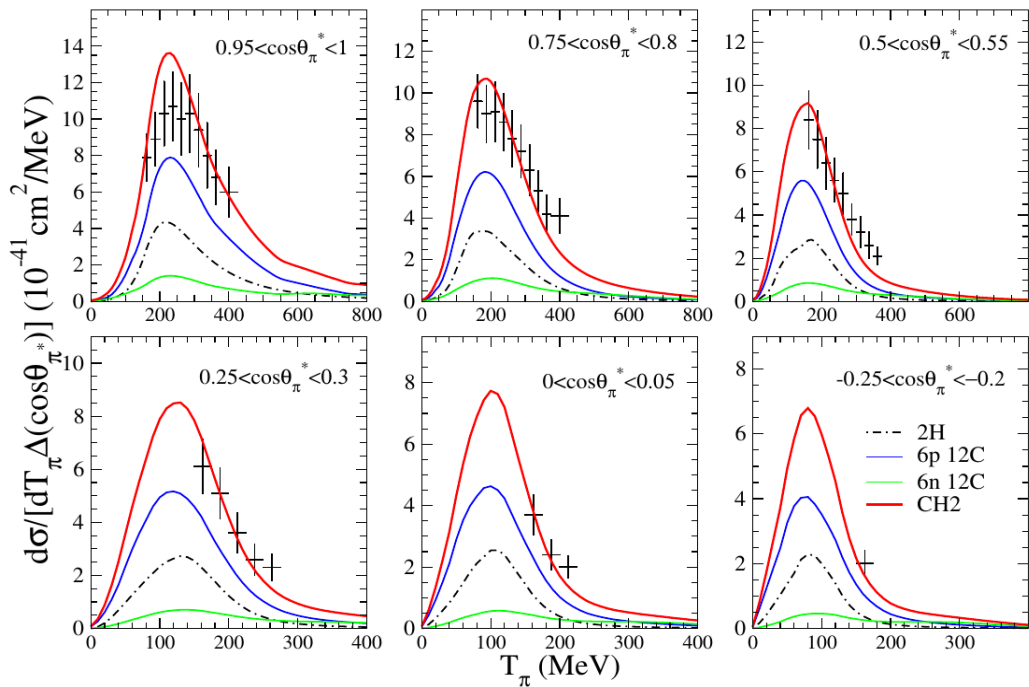
MiniBooNE Charged-Current $1\pi^+$



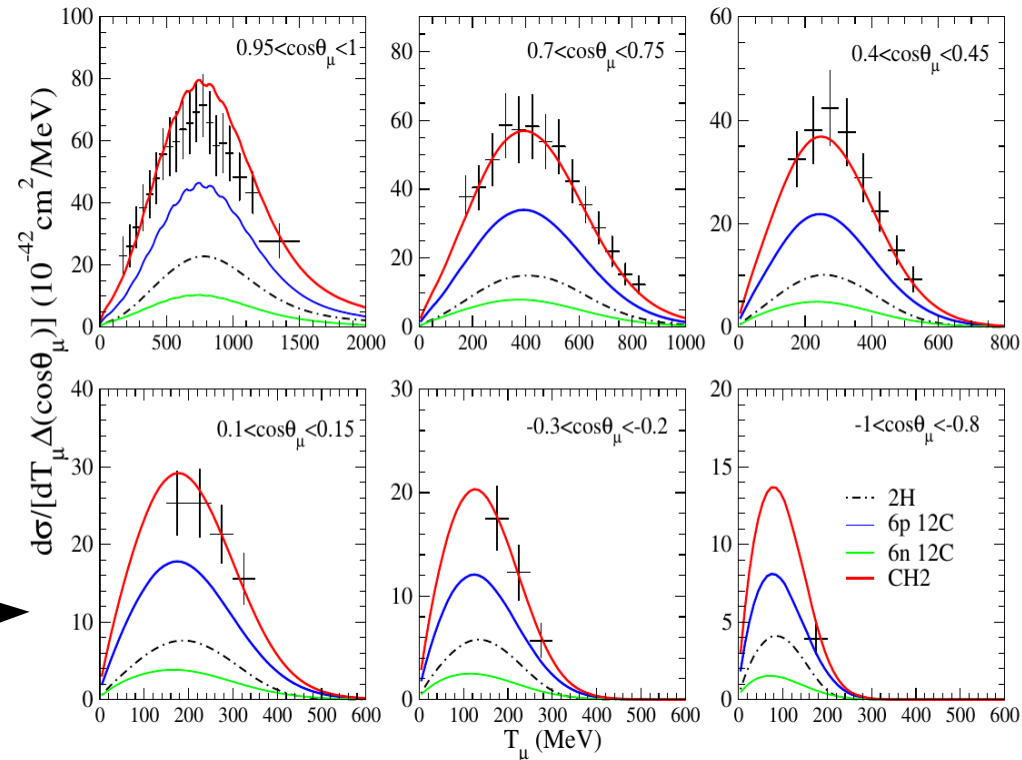
$$J_{had}^\mu = \sum_i^A \int dr \bar{\Psi}_F(\mathbf{r}) \phi^*(\mathbf{r}) \hat{O}_{one-body}^\mu(\mathbf{r}) \Psi_B(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}$$



MiniBooNE Charged-Current $1\pi^+$

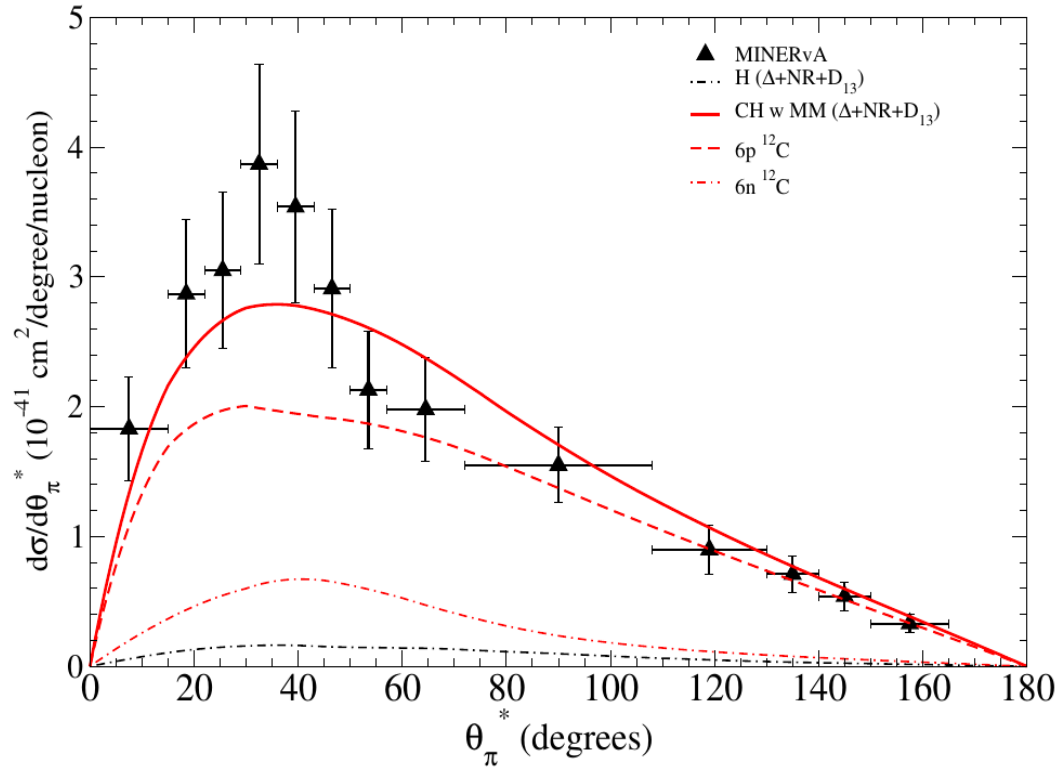


Double differential cross section
pion variables



Double differential cross section
muon variables

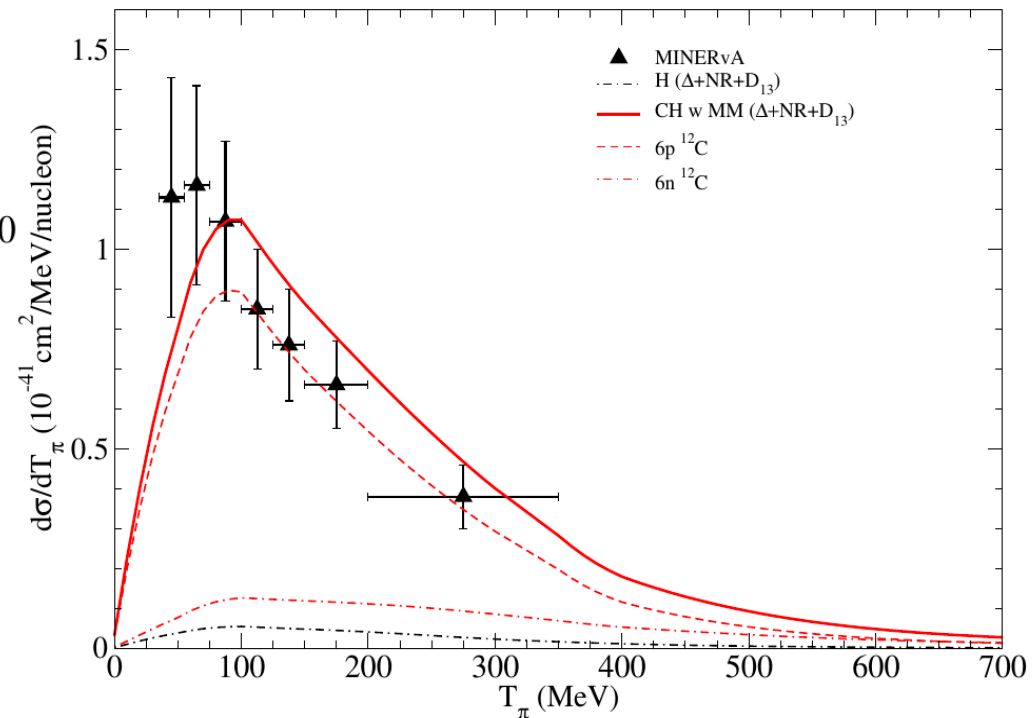
MINERvA Charged-Current $1\pi^+$



$W < 1400$ MeV

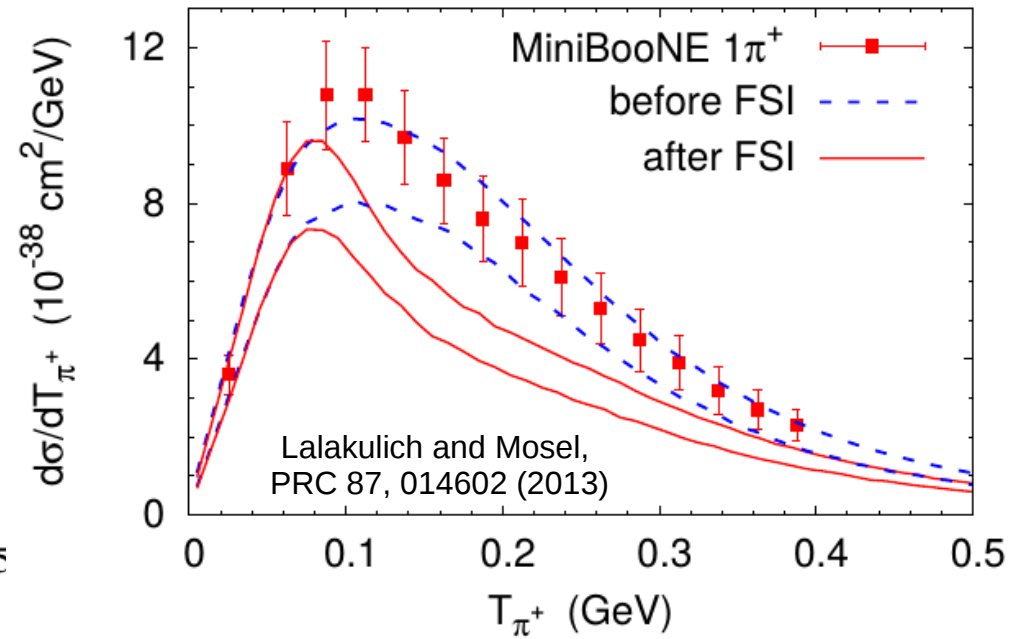
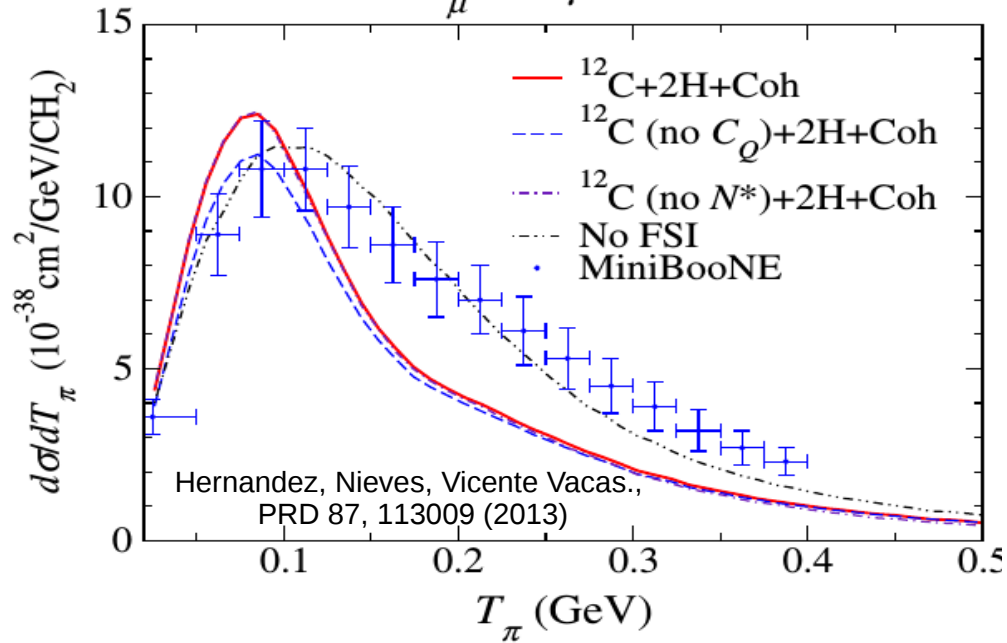
$$W^2 = M^2 + 2M\omega - Q^2$$

See U. Mosel, PRC 91, 065501 (2015) for discussion about W -cuts



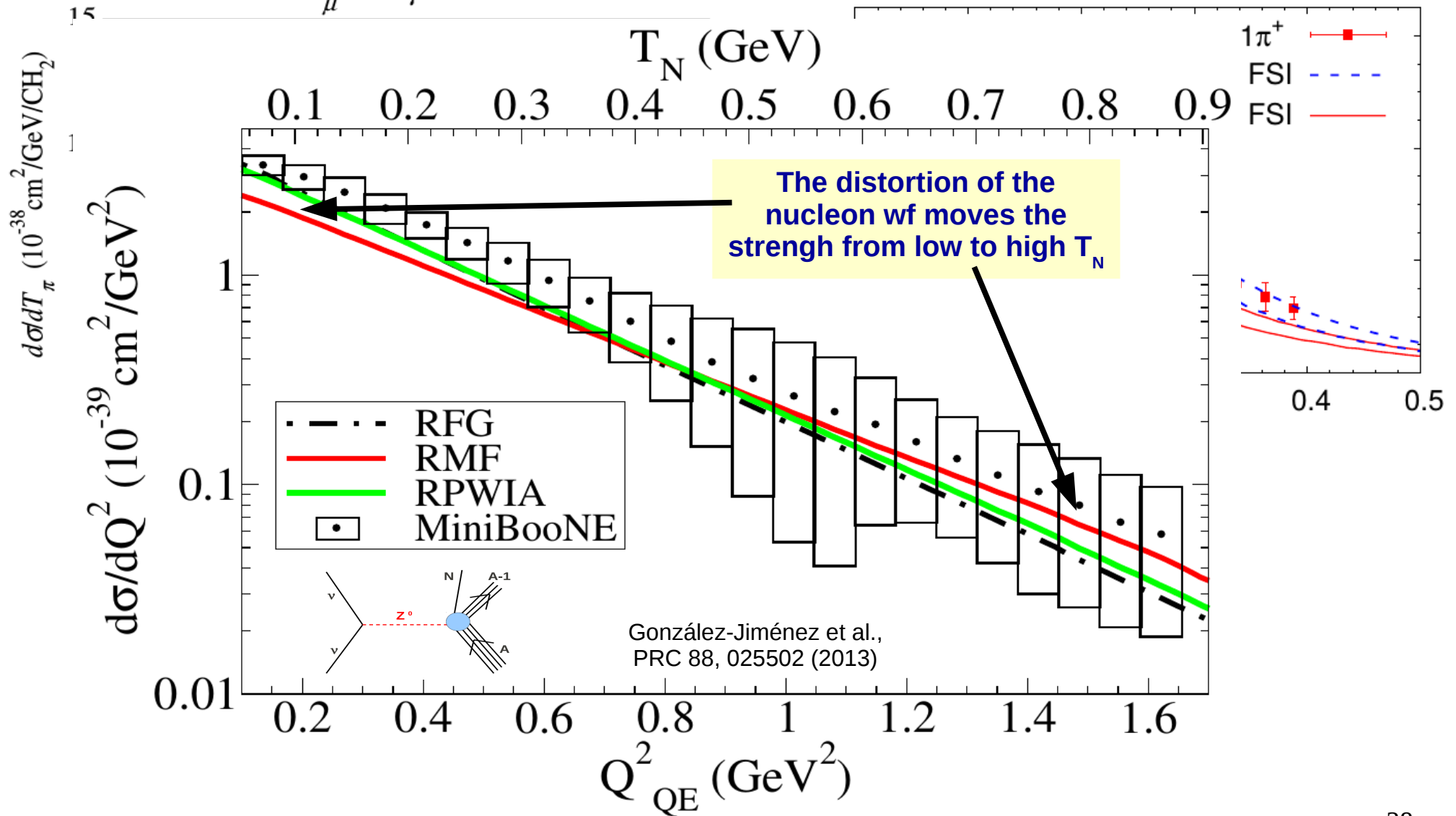
Maybe...

$$\nu_{\mu} N \rightarrow \mu^{-} \pi^{+} N'$$

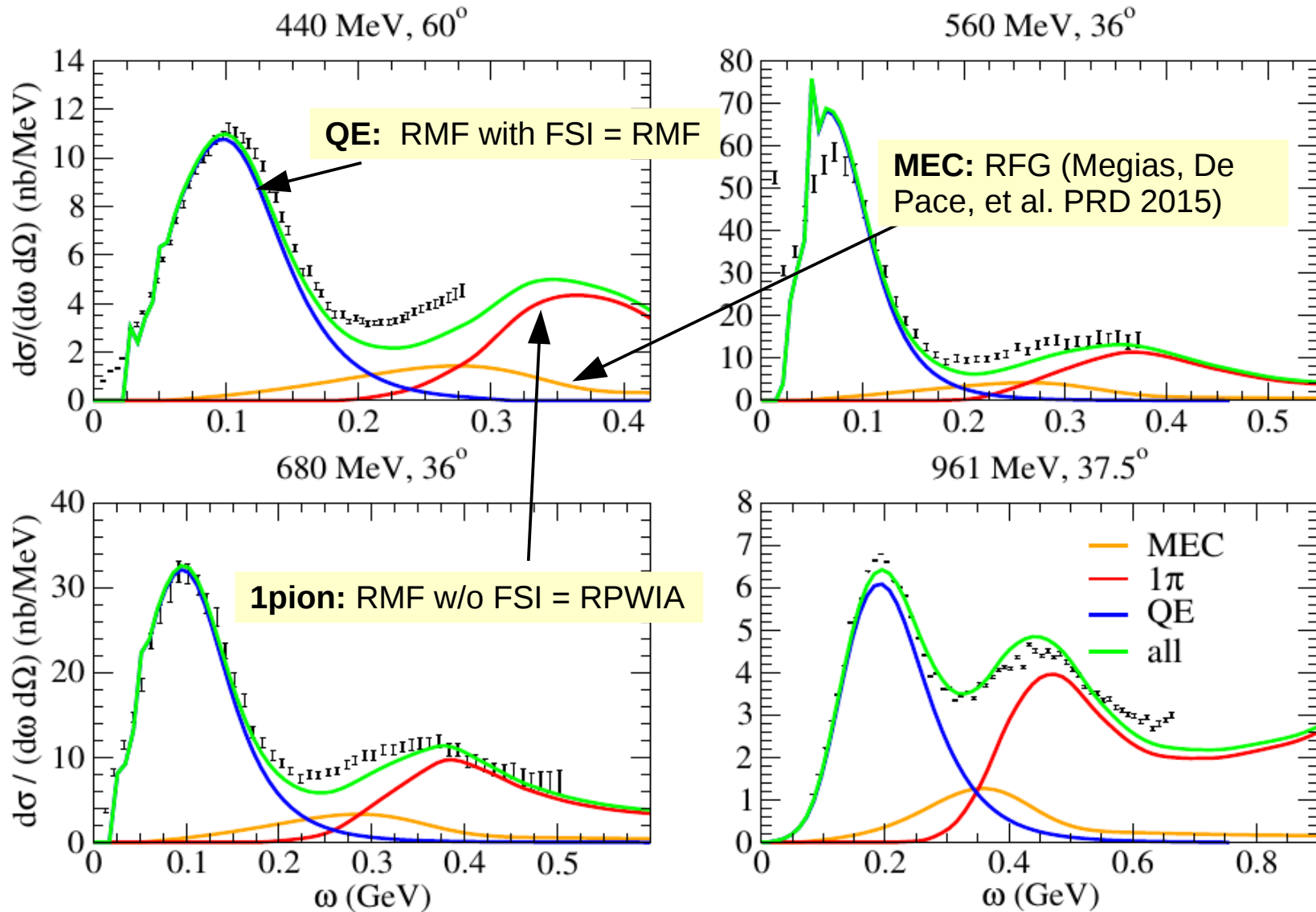


Maybe...

$$\nu_{\mu} N \rightarrow \mu^{-} \pi^{+} N'$$



Inclusive $^{12}\text{C}(e,e')$



Summary

- ✓ We described the quasielastic and one-pion production processes within a **relativistic mean-field model**.
 - ✗ Microscopic model, fully relativistic and we can make predictions for exclusive cross sections
- ✓ The agreement with inclusive (e,e') data as well as with Charged-Current $1\pi^+$ is quite good.
- ✓ Near future:
 - ✗ Incorporate the FSI for the outgoing nucleon and pion.
- ✓ Problems:
 - ✗ We sum amplitudes so we have interferences that, in some cases, we do not control.
 - ✗ We are still missing a lot of ingredients: coherent pion production, other resonances, ...

The end...

*Merci pour votre
attention*

Collaboration

Pion production (Ghent University)

Natalie Jachowicz

Tom Van Cuyck

Vishvas Pandey

Nils Van Dessel

Wim Cosyn

Jan Ryckebusch

Camille Colle

Quasielastic – Superscaling

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Universidad de Sevilla. Spain.

M. B. Barbaro. Università di Torino and
INFN. Italy.

J. M. Udías. Universidad Complutense
de Madrid. Spain.

M. V. Ivanov. Institute for Nuclear
Research and Nuclear Energy. Sofia,
Bulgaria.

A. Meucci and C. Giusti. Università
degli studi di Pavia.

T. W. Donnelly and O. Moreno.
Massachusetts Institute of Technology.
USA.

Backup slides

Relativistic mean-field model (I)

RMF model provides a microscopic description of the ground state of finite nuclei which is consistent with Quantum Mechanics, Special Relativity and symmetries of strong interaction.

The starting point is a Lorentz covariant Lagrangian density

$$\begin{aligned} \mathcal{L} = & \bar{\Psi} (i\gamma_\mu \partial^\mu - M) \Psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) \\ & - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \mathbf{R}_{\mu\nu} \mathbf{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & - g_\sigma \bar{\Psi} \sigma \Psi - g_\omega \bar{\Psi} \gamma_\mu \omega^\mu \Psi - g_\rho \bar{\Psi} \gamma_\mu \boldsymbol{\tau} \boldsymbol{\rho}^\mu \Psi - g_e \frac{1 + \tau_3}{2} \bar{\Psi} \gamma_\mu A^\mu \Psi . \end{aligned}$$

Extension of the original σ - ω Walecka model (Ann. Phys.83,491 (1974)).

where

$$\Omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu ,$$

$$\mathbf{R}^{\mu\nu} = \partial^\mu \boldsymbol{\rho}^\nu - \partial^\nu \boldsymbol{\rho}^\mu ,$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu .$$

$$U(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$$

Main approximations:

1) Mean-field approximation:

$$\omega_\mu \rightarrow \langle \omega_\mu \rangle \quad \sigma \rightarrow \langle \sigma \rangle \quad \rho_\mu \rightarrow \langle \rho_\mu \rangle$$

2) Static limit:

$$\partial^0 \omega_0 = \partial^0 \rho_0 = \partial^0 \sigma = 0 \quad \omega_\mu = \delta_{\mu 0} \omega_0 , \quad \rho_\mu = \delta_{\mu 0} \rho_0$$

3) Spherical symmetry for finite nuclei:

$$\omega_0 = \omega_0(r) \quad \rho_0 = \rho_0(r) \quad \sigma = \sigma(r)$$

Relativistic mean-field model (II)

Dirac equation for nucleons (eq. of motion for the barionic fields):

$$[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + V(r) + \beta(M + S(r))]\Psi_i(\mathbf{r}) = E_i\Psi_i(\mathbf{r})$$

where the scalar (S) and vector (V) potential are given by:

$$S(r) = g_\sigma\sigma(r),$$

$$V(r) = g_\omega\omega^0(r) + g_\rho\tau_3\rho_3^0(r) + e\frac{1 + \tau_3}{2}A^0(r)$$

Eqs. of motion for the mesons and the photon:

$$[-\nabla^2 + m_\sigma^2]\sigma(r) = -g_\sigma\rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r),$$

$$[-\nabla^2 + m_\omega^2]\omega^0(r) = -g_\omega\rho_B(r),$$

$$[-\nabla^2 + m_\rho^2]\rho_3^0(r) = -g_\rho\rho_\rho(r),$$

$$-\nabla^2 A^0 = e\rho_c,$$

Current densities

$$\rho_s(r) = \sum_i^A \bar{\Psi}_i(\mathbf{r})\Psi_i(\mathbf{r}),$$

$$\rho_B(r) = \sum_i^A \Psi_i^\dagger(\mathbf{r})\Psi_i(\mathbf{r}),$$

$$\rho_\rho(r) = \sum_i^A \Psi_i^\dagger(\mathbf{r})\tau_3\Psi_i(\mathbf{r})$$

$$\rho_c(r) = \sum_i^A \Psi_i^\dagger(\mathbf{r})\frac{1 + \tau_3}{2}\Psi_i(\mathbf{r})$$

Solution of the couple equations for the fields in a self-consistent way.

Relativistic mean-field model (III)

In general, the parameters are fit to reproduce some general properties of some closed shell spherical nuclei and nuclear matter.

Parameters for the NLSH model (fitted to the mean charge radius, binding energy and neutron radius of the ^{16}O , ^{40}Ca , ^{90}Zr , ^{116}Sr , ^{124}Sn and ^{208}Pb).

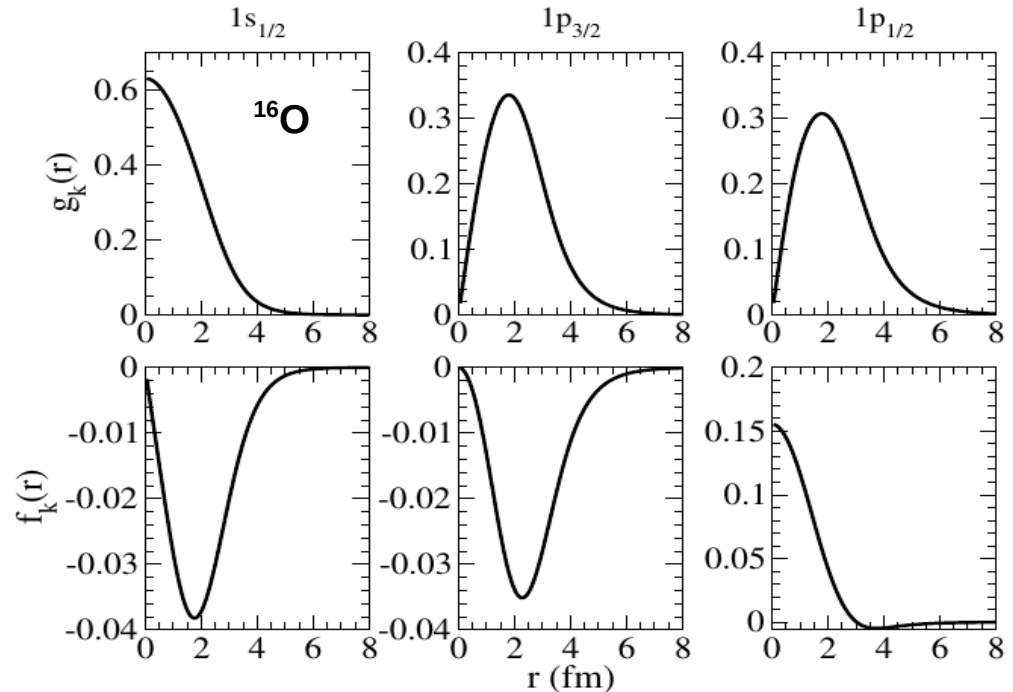
M_N	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2	g_3
939.0	526.059	783.0	763.0	10.444	12.945	4.3830	-6.9099	-15.8337

6 free parameters

$$[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + V(r) + \beta(M + S(r))]\Psi_i(\mathbf{r}) = E_i\Psi_i(\mathbf{r})$$

$$\Psi_k^{m_j}(\mathbf{r}) = \begin{pmatrix} g_k(r)\varphi_k^{m_j}(\Omega_r) \\ if_k(r)\varphi_{-k}^{m_j}(\Omega_r) \end{pmatrix},$$

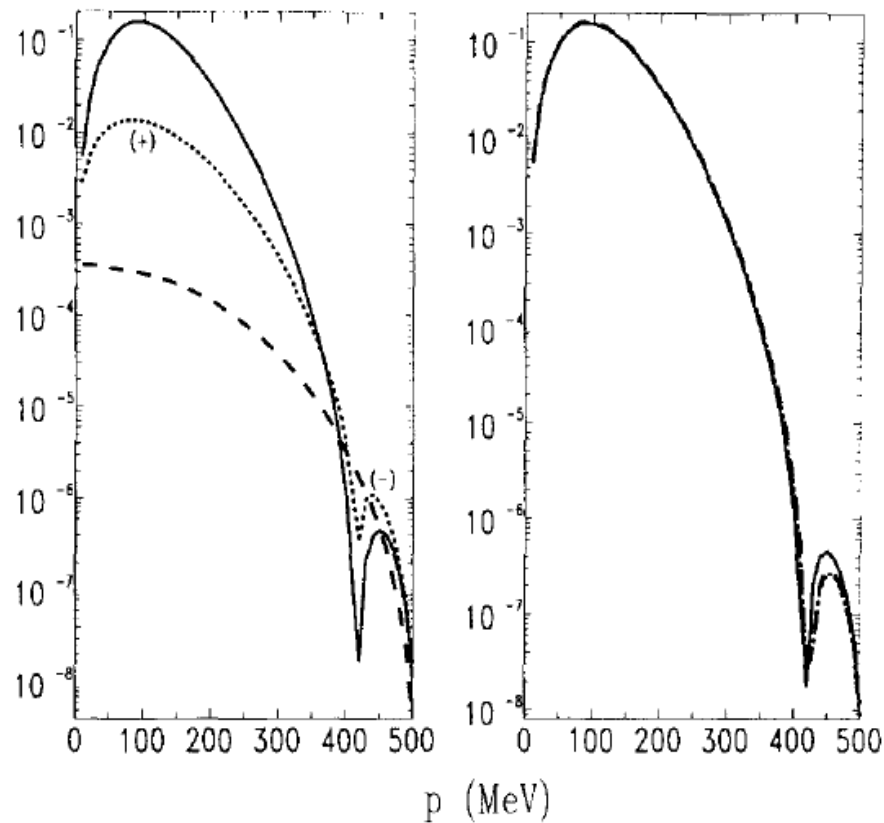
$$\varphi_k^{m_j}(\Omega_r) = \sum_{m_\ell s} \langle \ell m_\ell \frac{1}{2} s | j m_j \rangle Y_\ell^{m_\ell}(\Omega_r) \chi^s$$



Relativistic mean-field model

J.A. Caballero et al. / Nuclear Physics A 632 (1998) 323–362

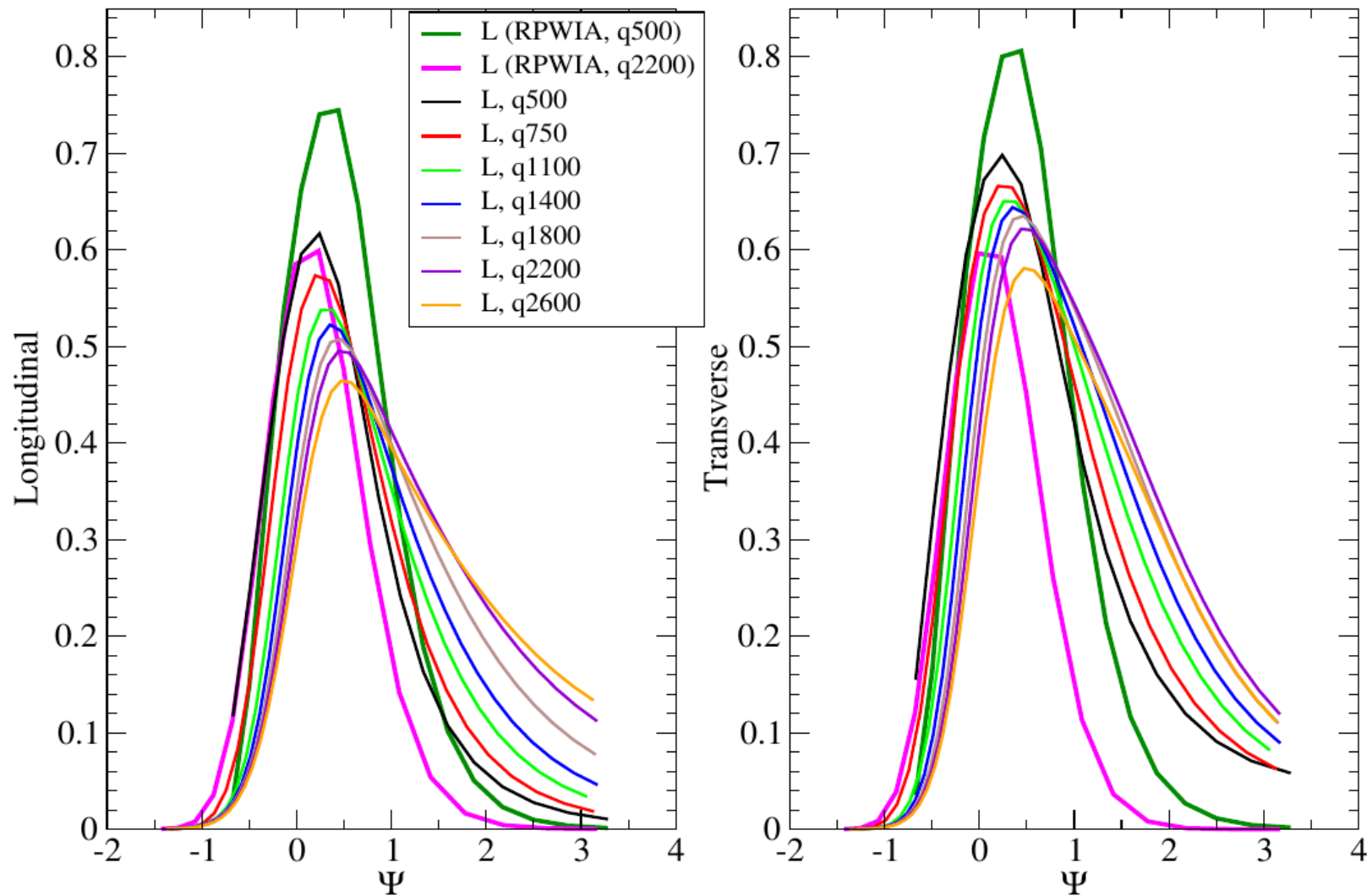
333



shell $1p_{1/2}$ in ^{16}O .

Fig. 1. Left panel: projection components of the momentum distribution (in units of fm^3): $N_{uu}(p)$ (solid), $N_{uv}(p)$ (dotted) and $N_{vv}(p)$ (dashed). Right panel: $N_{uu}(p)$ (solid), $N_{uu}^{(0)}(p)$ (dotted) and $N_{uu}^{n.r.}(p)$ (dashed) (see text for details).

Q dependence of RMF scaling functions



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2.2.1. *The static limit.* Most applications of the relativistic mean-field model are concerned with nuclear ground states, or more generally, stationary states. We can assume in all nuclear applications that the nucleon single-particle states do not mix isospin, i.e. they are pure proton or pure neutron states. As a consequence, only the third component of the isospin vectors is needed, i.e.

$$R_{\pm 1\mu} = 0 \quad \text{and} \quad \rho_{\pm 1\mu} = 0.$$

The mean-field equations are further greatly simplified due to stationarity. All time derivatives of densities and fields vanish, i.e.

$$\dot{\rho}_s = 0 \quad \dot{\Phi} = 0 \quad \dot{\rho}_\mu = 0 \quad \text{etc}$$

and all space-vector components of densities and fields vanish, i.e.

$$\rho_i = 0 \quad \rho_{0i} = 0 \quad \rho_i^{(\text{proton})} = 0 \quad V_i = 0 \quad R_{0i} = 0 \quad A_i = 0 \quad \text{for } i = 1, 2, 3.$$

Relativistic Mean Field Theory in Finite Nuclei

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In the static approximation we assume time-independence for the meson fields and a time-dependent phase $\exp(i\epsilon_i t)$ for the spinors ψ_i . Furthermore we restrict ourselves in this section to cases with time-reversal invariance and with good parity, as one has it for instance in the ground state of even-even nuclei. In this case the space-like components of all currents \vec{j} , \vec{j} , j_c and the pion field vanish and we are left with the stationary RMF equations:

Medium modifications of the Delta

Delta propagator:

$$S_{\Delta,\alpha\beta} = \frac{-(K_{\Delta} + M_{\Delta})}{K_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\text{width}}} \left(g_{\alpha\beta} - \frac{1}{3}\gamma_{\alpha}\gamma_{\beta} - \frac{2}{3M_{\Delta}^2}K_{\Delta,\alpha}K_{\Delta,\beta} - \frac{2}{3M_{\Delta}}(\gamma_{\alpha}K_{\Delta,\beta} - K_{\Delta,\alpha}\gamma_{\beta}) \right)$$

with the energy dependent Delta width:

$$\Gamma_{\text{width}}(W) = \frac{1}{12\pi} \frac{(f_{\pi N\Delta})^2}{m_{\pi}^2 W} (\rho_{\pi,cm})^3 (M + E_{N,cm})$$

$$\Gamma_{\text{width}}^{\text{free}} \longrightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_{\Delta}), \quad M_{\Delta}^{\text{free}} \longrightarrow M_{\Delta}^{\text{in-medium}} = M_{\Delta}^{\text{free}} + \Re(\Sigma_{\Delta}).$$

+ Γ_{Pauli} : some nucleons from Δ -decay are Pauli blocked (the Δ -decay width decreases).

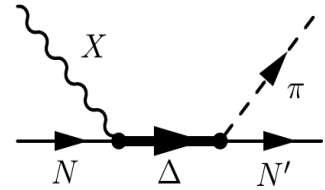
+ The parametrization of $\Im(\Sigma_{\Delta})$ and $\Re(\Sigma_{\Delta})$ is given in terms of the nuclear density ρ :

$$\begin{aligned} -\Im(\Sigma_{\Delta}) &= C_{QE} (\rho/\rho_0)^{\alpha} + C_{A2} (\rho/\rho_0)^{\beta} + C_{A3} (\rho/\rho_0)^{\gamma}, \\ \Re(\Sigma_{\Delta}) &= 40 \text{ MeV} (\rho/\rho_0). \end{aligned}$$

We modify the free $\Delta\pi N$ -decay constant ($f_{\Delta\pi N}$) to take into account the E -dependent medium modification of the Δ width:

$$f_{\Delta\pi N}^{\text{in-medium}}(W) = f_{\Delta\pi N} \sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE} (\rho/\rho_0)^{\alpha}}{\Gamma_{\text{width}}^{\text{free}}}}$$

Medium modifications of the Delta



$$-\Im(\Sigma_{\Delta}) = C_{QE} (\rho/\rho_0)^{\alpha} + C_{A2} (\rho/\rho_0)^{\beta} + C_{A3} (\rho/\rho_0)^{\gamma}$$

Each contribution corresponds to a different process:

- QE $\implies \Delta N \rightarrow \pi NN$ (still one pion in the final state)
- A2 $\implies \Delta N \rightarrow NN$ (no pions in the final state)
- A3 $\implies \Delta NN \rightarrow NNN$ (no pions in the final state)

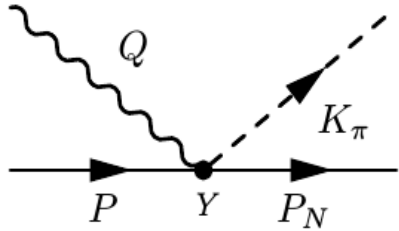
We modify the free Delta decay constant to take into account the E-dependent medium modification of the Delta-width

$$\Gamma_{\Delta\pi N}^{\alpha} = \frac{f_{\pi N\Delta} P_{\pi}^{\alpha}}{m_{\pi}}$$

$$f_{\Delta\pi N}^{\text{in-medium}}(W) = f_{\Delta\pi N} \sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE} (\rho/\rho_0)^{\alpha}}{\Gamma_{\text{width}}^{\text{free}}}}$$

References: [*] E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987).

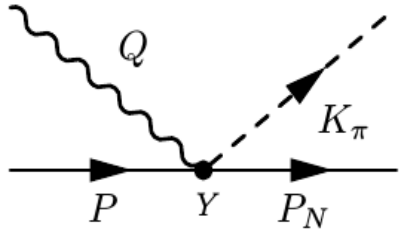
Hadronic Current (Contact Term)



$$\mathcal{L}_{W\pi NN}^V = i\frac{g}{4f_\pi}\cos\theta_c \left[\left(\sqrt{2}\bar{\psi}_p\gamma^\mu\pi_0\psi_n - \bar{\psi}_p\gamma^\mu\pi_-\psi_p + \bar{\psi}_n\gamma^\mu\pi_-\psi_n \right) W_\mu^+ \right. \\ \left. + \left(\bar{\psi}_p\gamma^\mu\pi_+\psi_p - \bar{\psi}_n\gamma^\mu\pi_+\psi_n - \sqrt{2}\bar{\psi}_n\gamma^\mu\pi_0\psi_p \right) W_\mu^- \right],$$

$$J_{CT,V}^\nu(Q) = \int d^4Y e^{-iQ\cdot Y} \left[\frac{ig}{2\sqrt{2}} \frac{i\cos\theta_c}{\sqrt{2}f_\pi} \bar{\psi}_p(Y)\phi^*(Y)\gamma^\nu\psi_n(Y) \right] \\ = (2\pi)\delta(E_N + E_\pi - \omega - E) \frac{ig}{2\sqrt{2}} \frac{i\cos\theta_c}{\sqrt{2}f_\pi} \\ \times \int d\mathbf{y} e^{i\mathbf{q}\cdot\mathbf{y}} \bar{\psi}_p(\mathbf{y})\phi^*(\mathbf{y})\gamma^\nu\psi_n(\mathbf{y}).$$

Hadronic Current (Contact Term)



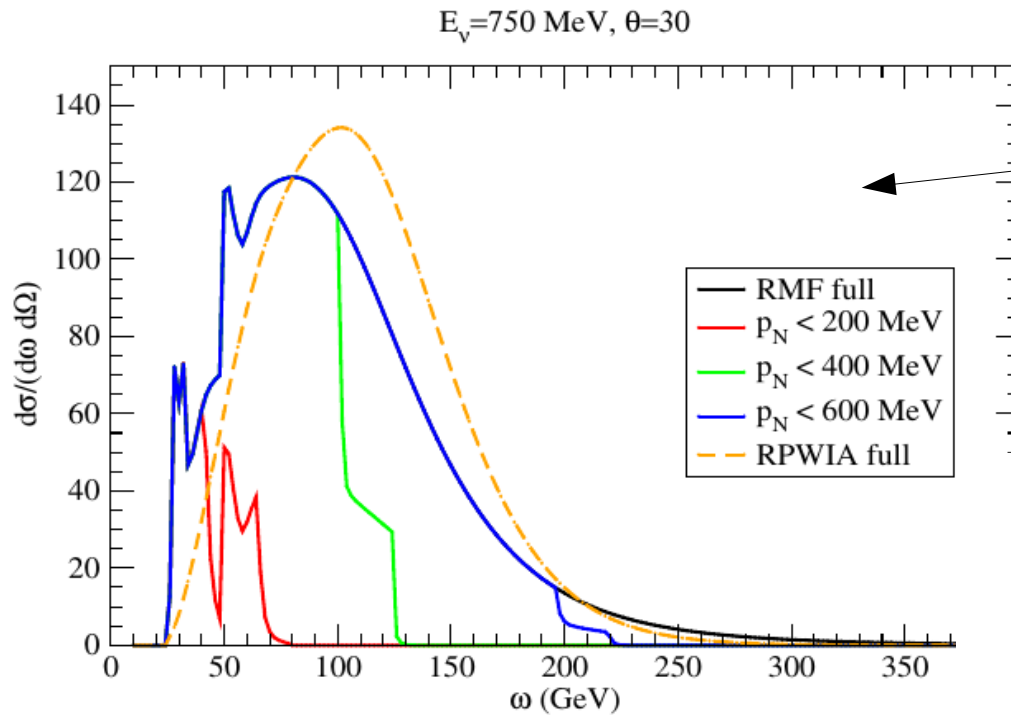
$$\mathcal{L}_{W\pi NN}^V = i \frac{g}{4f_\pi} \cos \theta_c \left[\left(\sqrt{2} \bar{\psi}_p \gamma^\mu \pi_0 \psi_n - \bar{\psi}_p \gamma^\mu \pi_- \psi_p + \bar{\psi}_n \gamma^\mu \pi_- \psi_n \right) W_\mu^+ \right. \\ \left. + \left(\bar{\psi}_p \gamma^\mu \pi_+ \psi_p - \bar{\psi}_n \gamma^\mu \pi_+ \psi_n - \sqrt{2} \bar{\psi}_n \gamma^\mu \pi_0 \psi_p \right) W_\mu^- \right],$$

$$J_{CT,V}^\nu(Q) = \int d^4 Y e^{-iQ \cdot Y} \left[\frac{ig}{2\sqrt{2}} \frac{i \cos \theta_c}{\sqrt{2} f_\pi} \bar{\psi}_p(Y) \phi^*(Y) \gamma^\nu \psi_n(Y) \right] \\ = (2\pi) \delta(E_N + E_\pi - \omega - E) \frac{ig}{2\sqrt{2}} \frac{i \cos \theta_c}{\sqrt{2} f}$$

$$\int d^3 y e^{i\mathbf{q} \cdot \mathbf{y}} \bar{\psi}_p(\mathbf{y}) \phi^*(\mathbf{y}) \gamma^\nu \psi_n(\mathbf{y}) \neq (2\pi)^3 \delta^3(\mathbf{p}_N + \mathbf{k}_\pi - \mathbf{q} - \mathbf{p}) \mathcal{N} \\ \times \bar{u}(\mathbf{p}_N, s_N) \gamma^\nu u(\mathbf{p}, s)$$

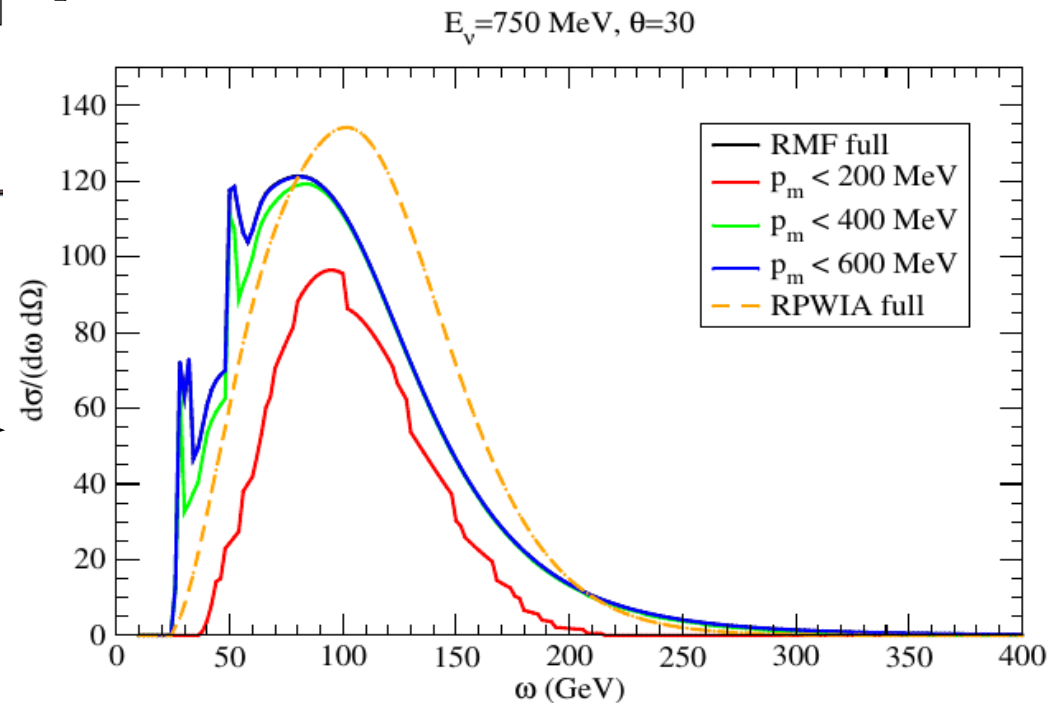
No free particles !!!

QE: cuts on the nucleon momentum



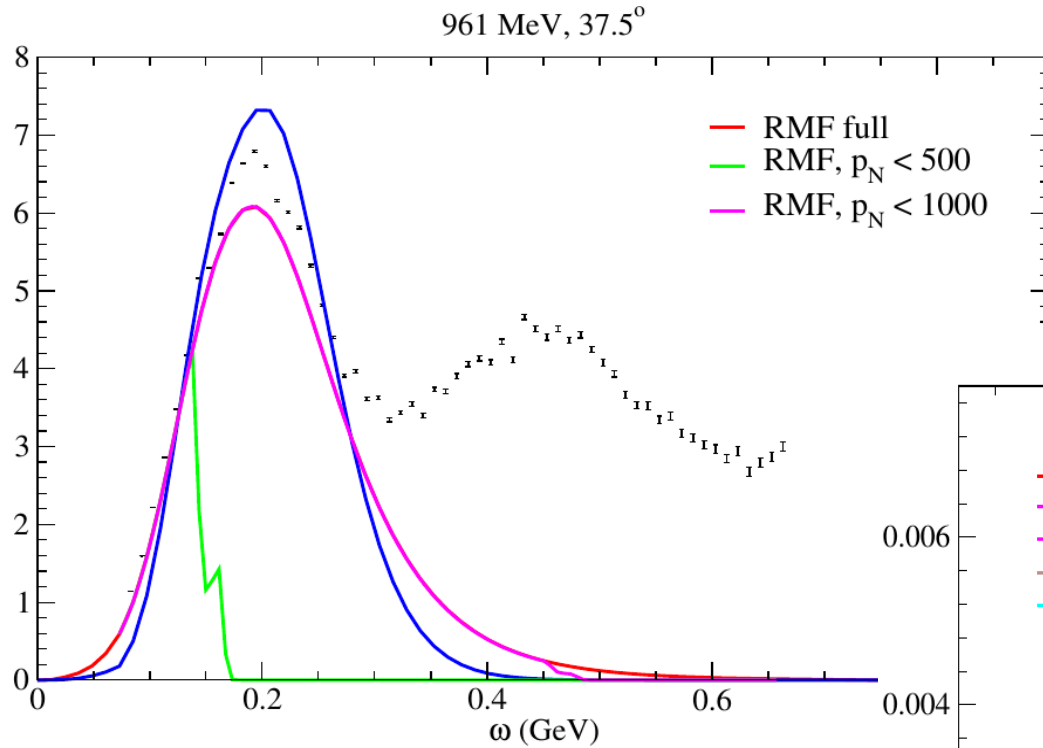
p_N is the momentum of the **outgoing nucleon**

p_m is the momentum of the **bound nucleon**

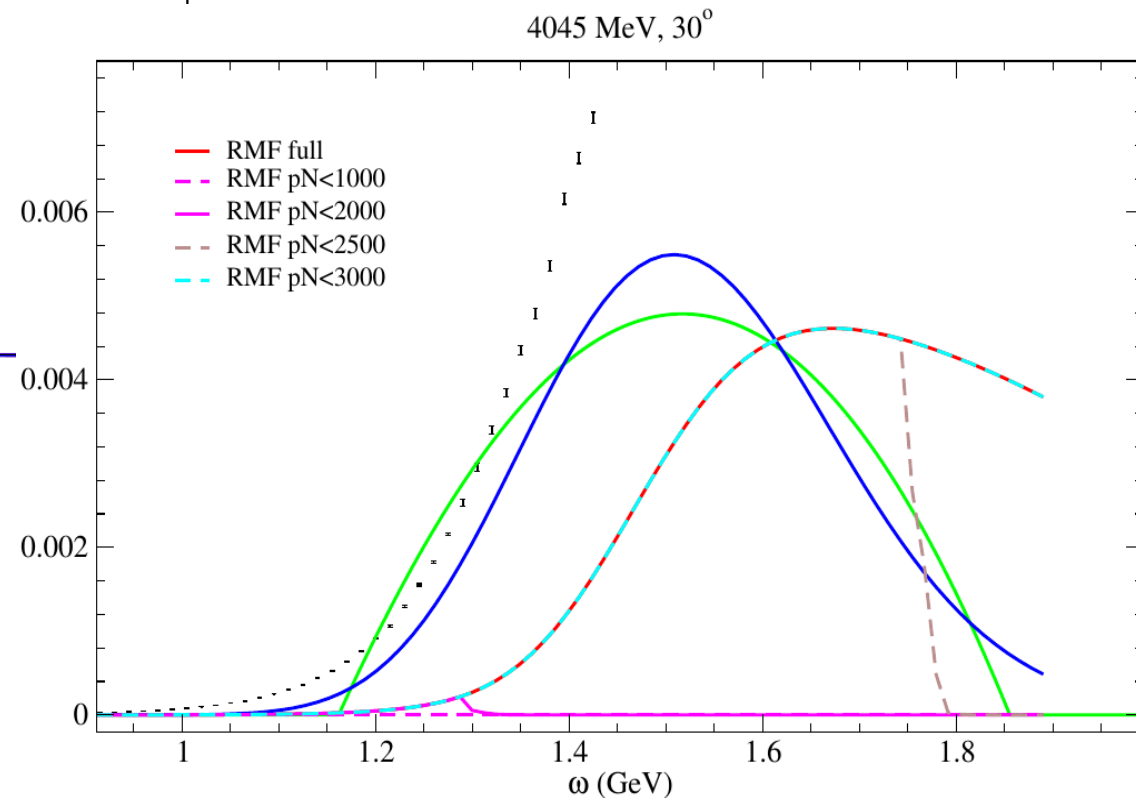


CCQE neutrino-12C scattering cross sections

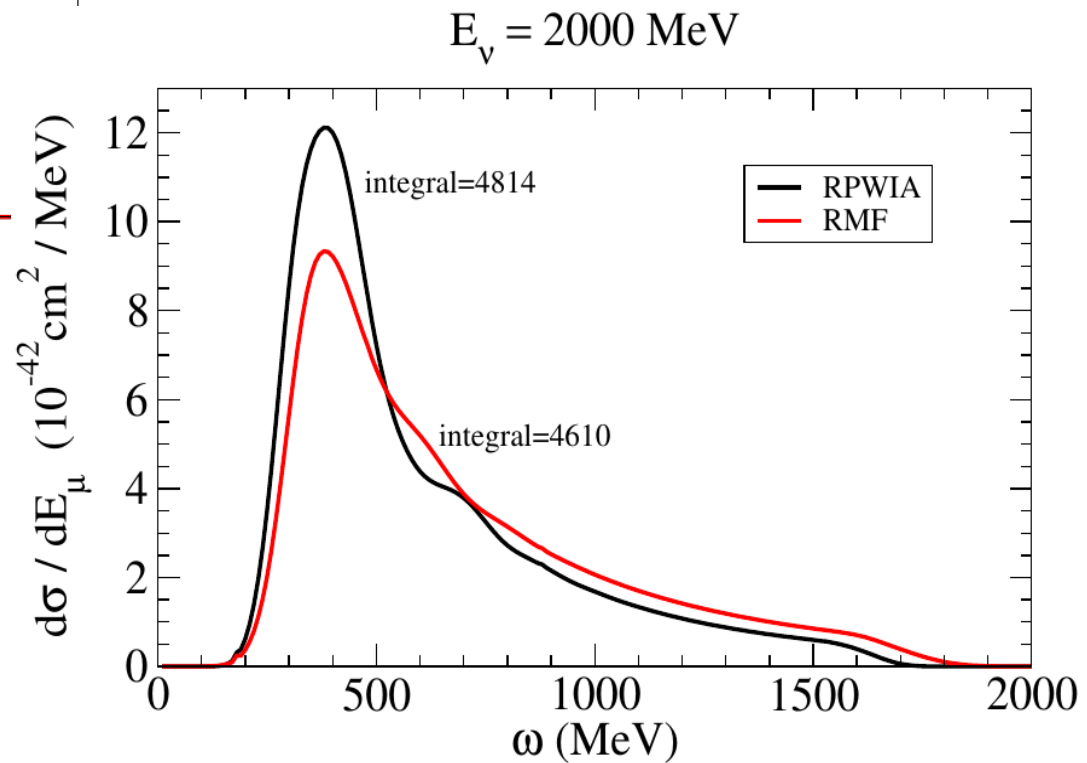
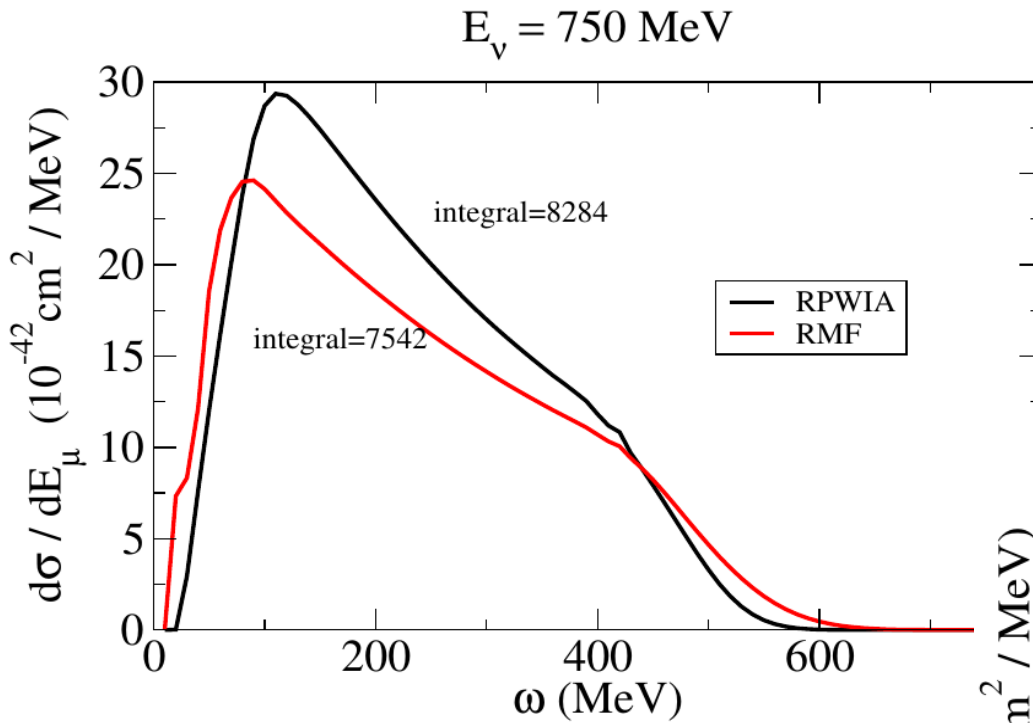
QE: cuts on the nucleon momentum



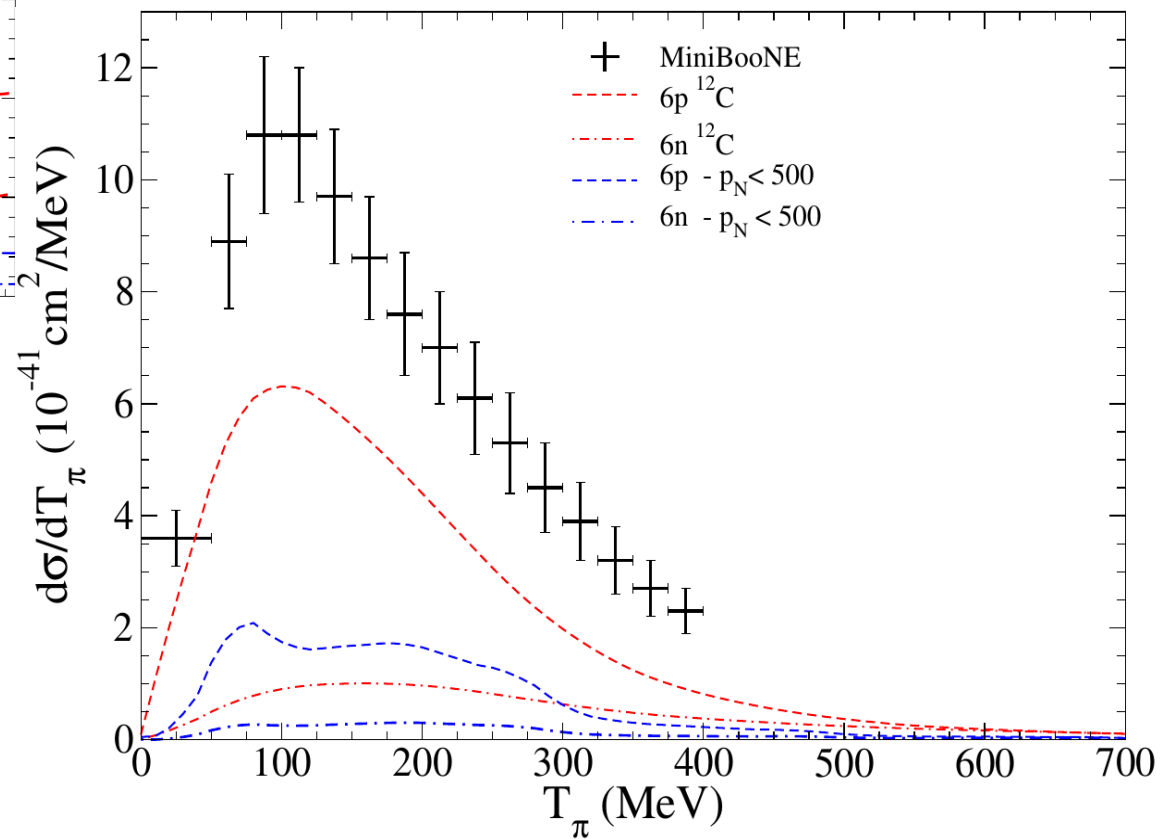
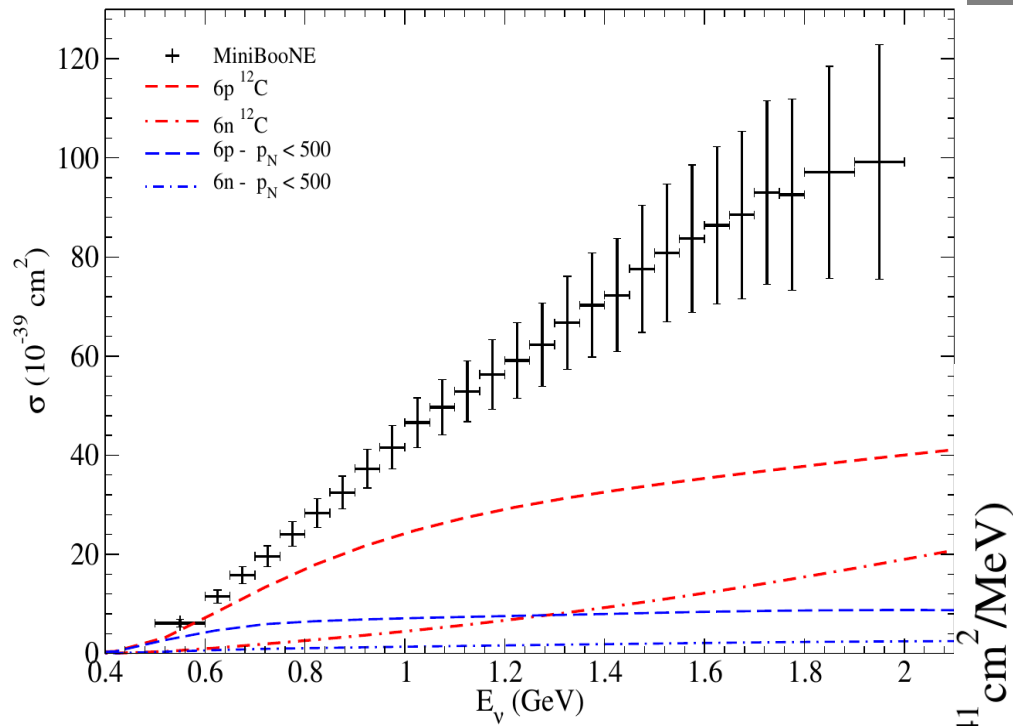
(e,e')



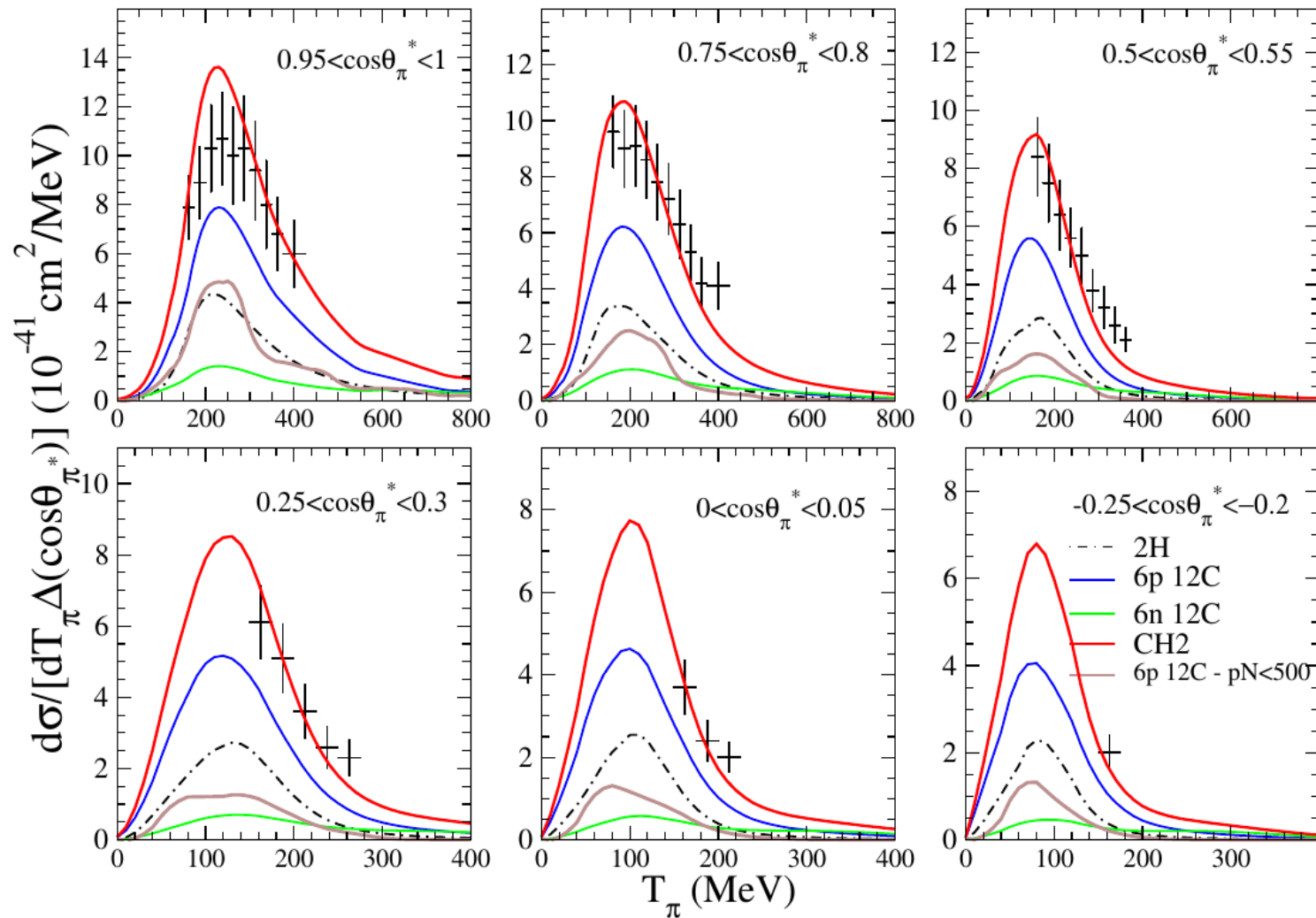
QE: RMF vs RPWIA (CCQE neutrino-¹²C)



Pion production (cuts on p_N)



Pion production (cuts on p_N)



Interferences

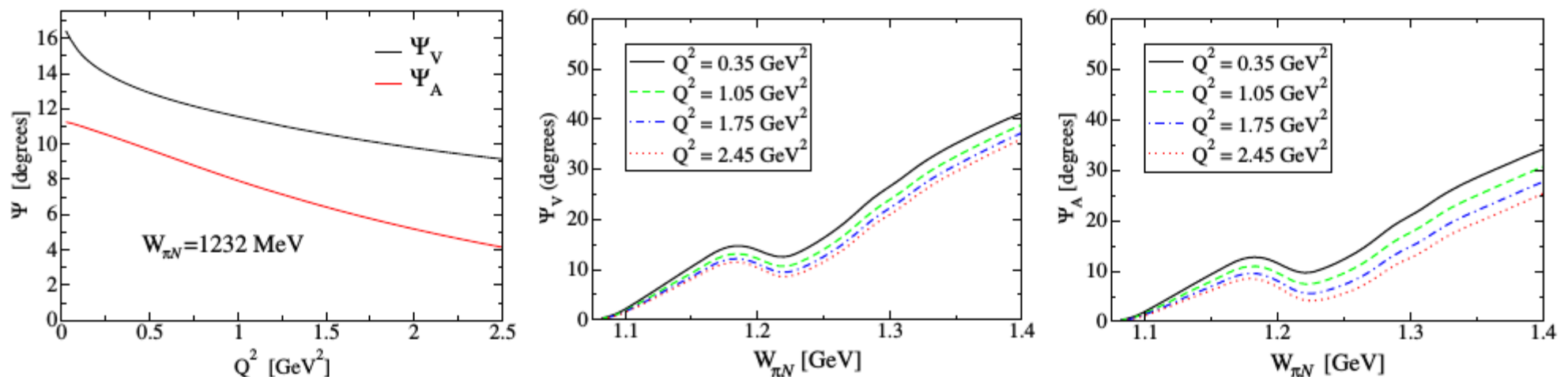
$$J^\nu = \langle J_{\Delta P}^\nu \rangle + \langle J_{C\Delta P}^\nu \rangle + \langle J_{CT,V}^\nu \rangle + \langle J_{CT,A}^\nu \rangle + \langle J_{NP}^\nu \rangle + \langle J_{CNP}^\nu \rangle + \langle J_{PF}^\nu \rangle + \langle J_{PP}^\nu \rangle$$

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Watson's theorem and the $N\Delta(1232)$ axial transition

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We present a new determination of the $N\Delta$ axial form factors from neutrino induced pion production data. For this purpose, the model of Hernandez *et al.* [Phys. Rev. D 76, 033005 (2007)] is improved by partially restoring unitarity. This is accomplished by imposing Watson's theorem on the dominant vector and axial multipoles. As a consequence, a larger $C_5^A(0)$, in good agreement with the prediction from the off-diagonal Goldberger-Treiman relation, is now obtained.



Other results

