

# Relativistic Mean Field and Superscaling

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*Two-body current contributions in neutrino-nucleus scattering*

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# OUTLINE

- **Quasielastic  $(e, e')$  data & Scaling/Superscaling**
- **The Relativistic Impulse Approximation**
  - *The Relativistic Mean Field (RMF)*
  - *Analysis of electron scattering & Scaling behavior*
  - *Application to neutrino (CC and NC) processes*
  - *Comparison with the SuperScaling Approach (SuSA)*
  - *MiniBooNE, MINER $\nu$ A & NOMAD experiments*
- **Summary & Conclusions**

# Analysis of experimental cross sections

Experimental scaling function:

$$F(q, y) = \frac{[d\sigma/d\omega d\Omega']_{exp}}{\bar{\sigma}_{eN}(q, \omega; p = -y, \varepsilon = 0)}$$

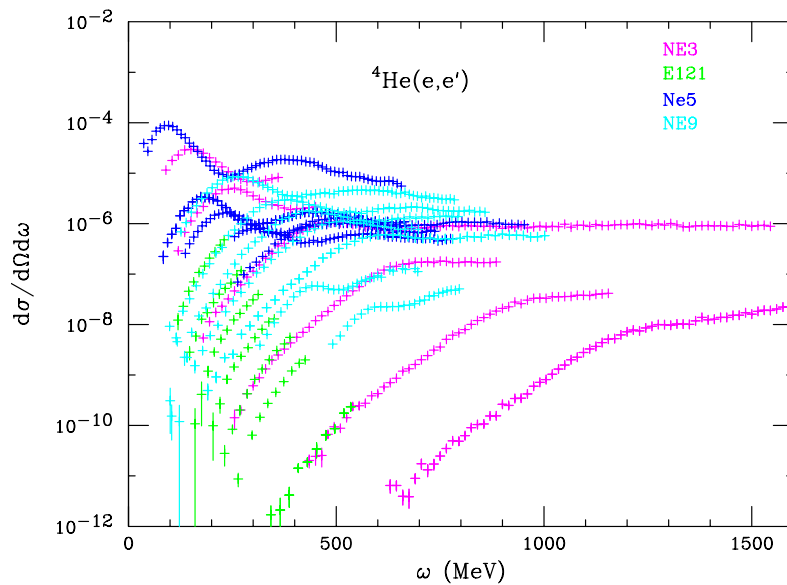
$$\bar{\sigma}_{eN}(q, \omega; p, \varepsilon) \equiv \frac{1}{2\pi} \int d\phi_N \frac{E_N}{q} [Z\sigma_{ep}(q, \omega; p, \varepsilon, \phi_N) + N\sigma_{en}(q, \omega; p, \varepsilon, \phi_N)]$$

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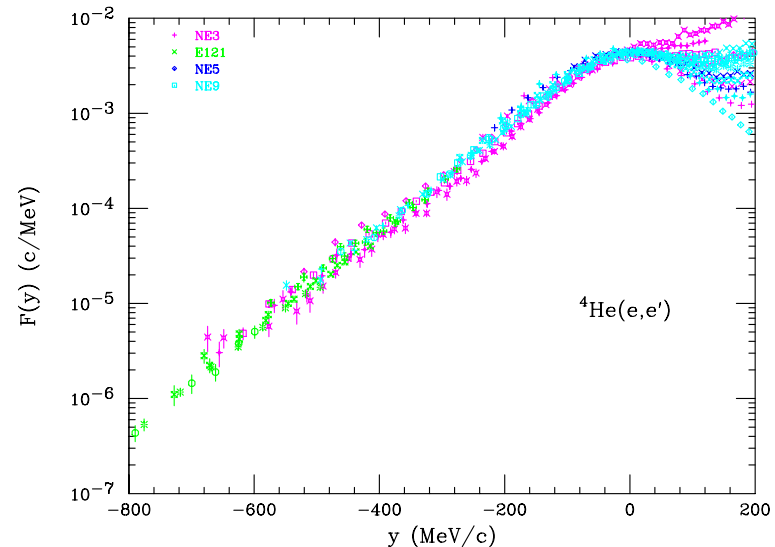
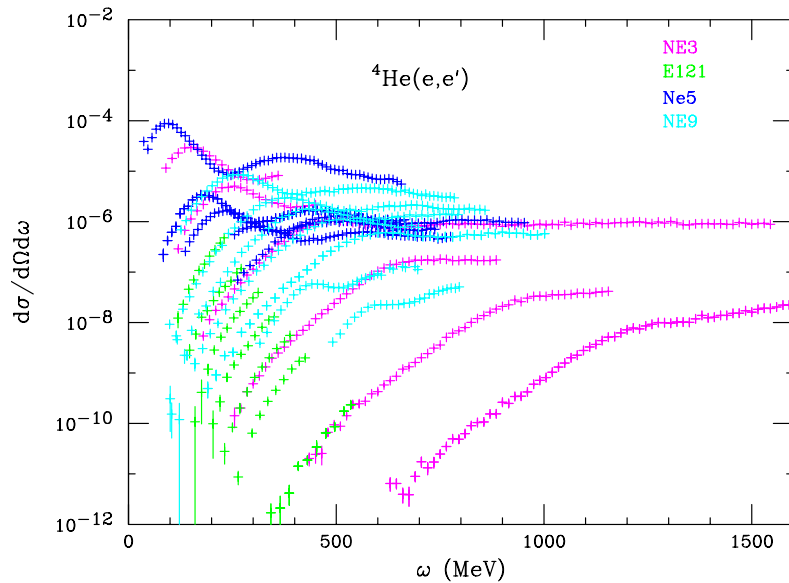


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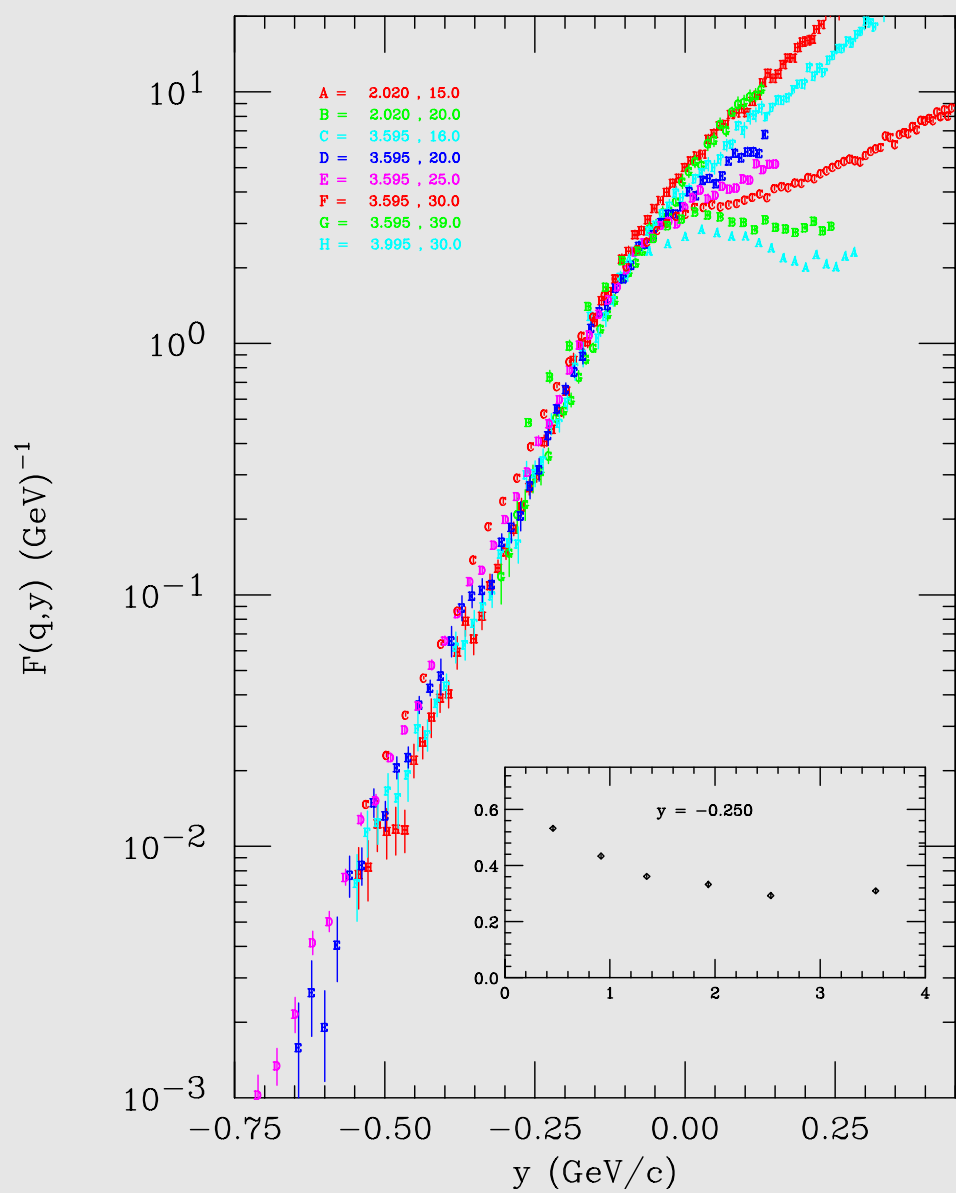
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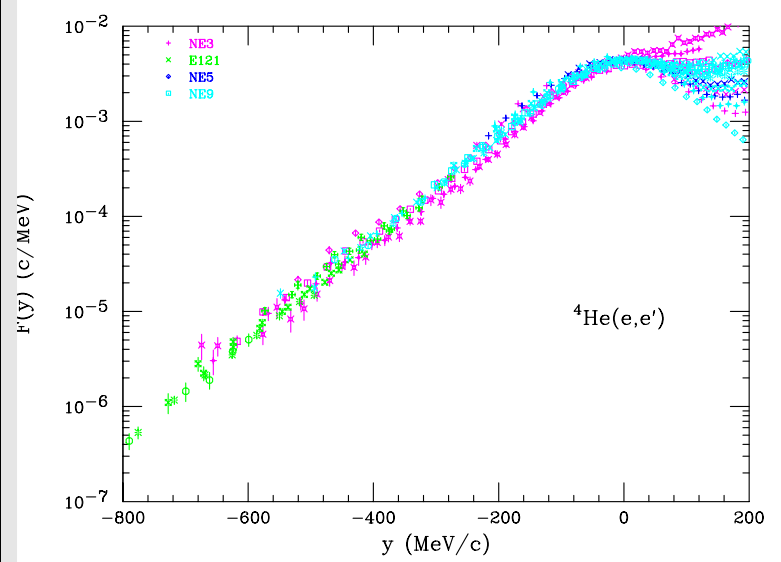
Scaling of the first kind:  $q \rightarrow \infty \implies F(q, y) \longrightarrow F(y) \equiv F(\infty, y)$

# Analysis of experimental cross sections



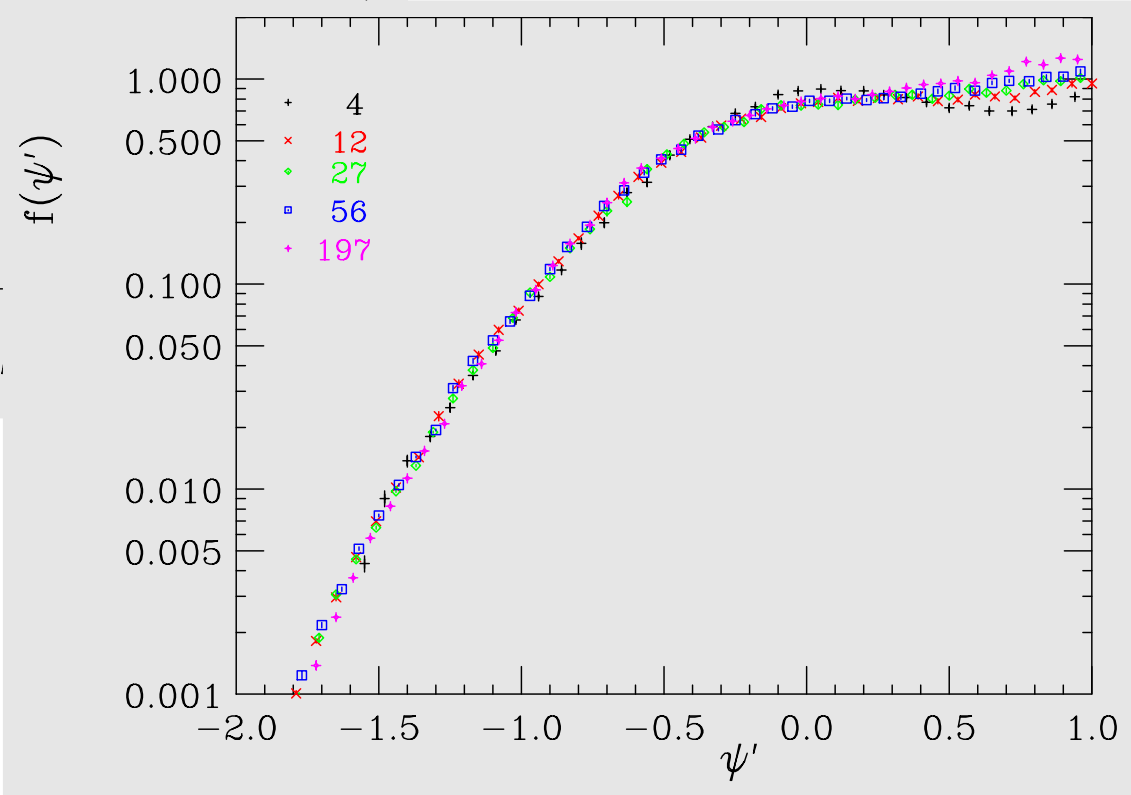
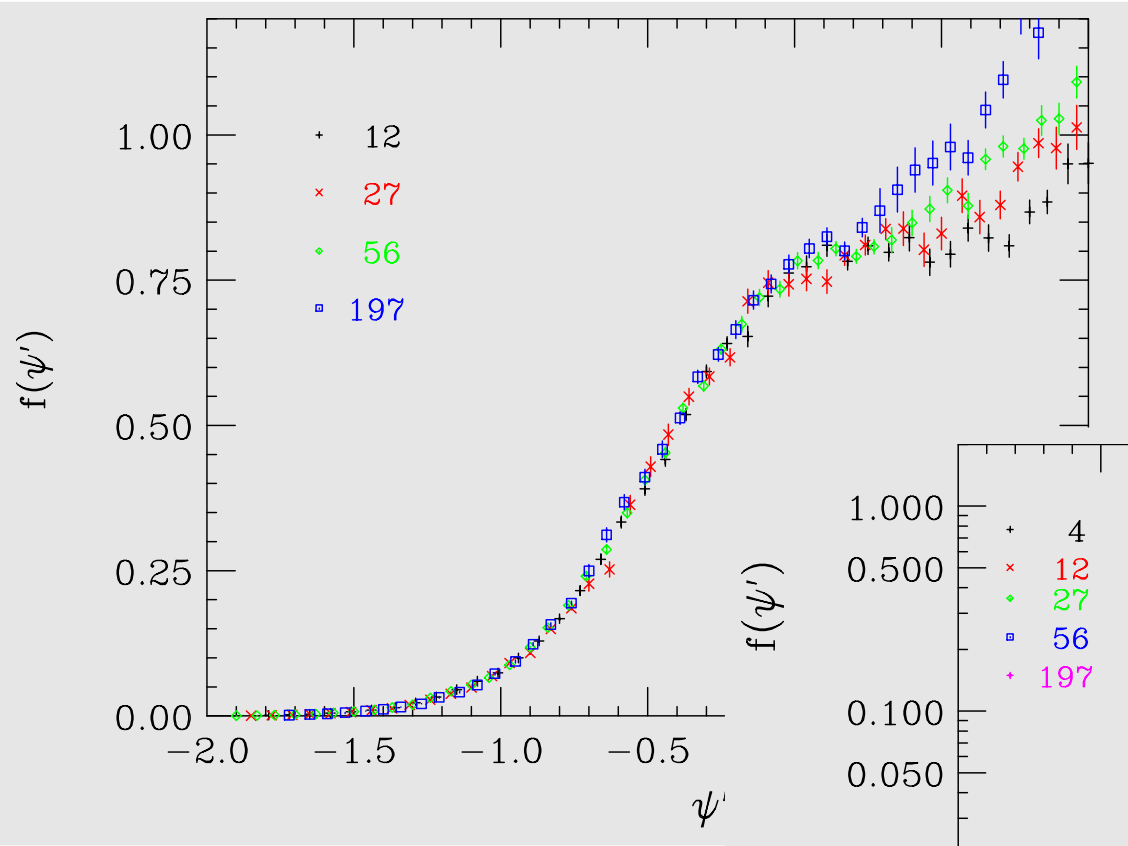
$$F(q, y) = \frac{[d\sigma/d\omega d\Omega']_{exp}}{\bar{\sigma}_{eN}(q, \omega; p = -y, \varepsilon = 0)}$$

$$q, \omega; p, \varepsilon, \phi_N) + N\sigma_{en}(q, \omega; p, \varepsilon, \phi_N)]$$

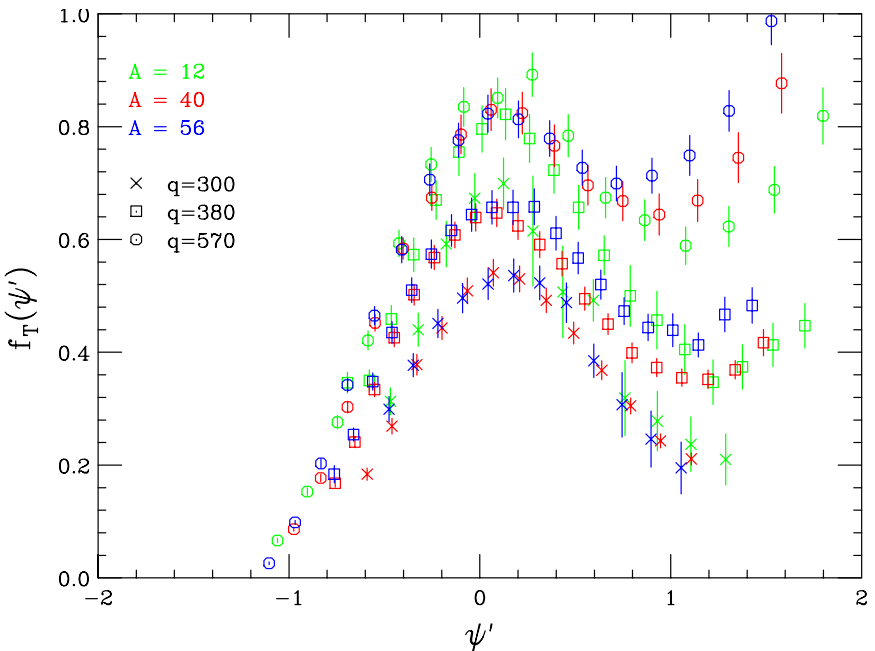
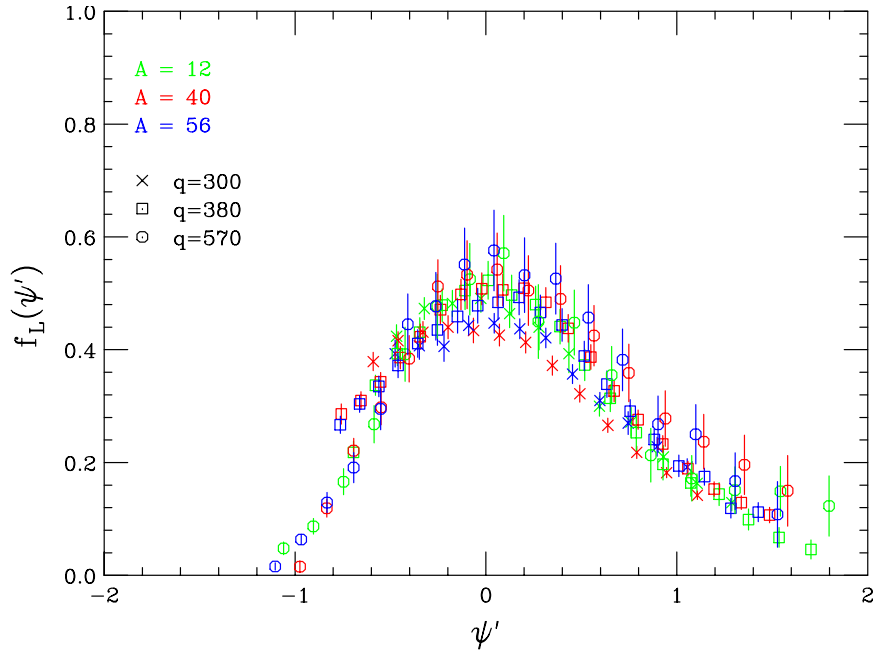


$$\Rightarrow F(q, y) \longrightarrow F(y) \equiv F(\infty, y)$$

# What are QE ( $e, e'$ ) data showing us? *Cross sections*

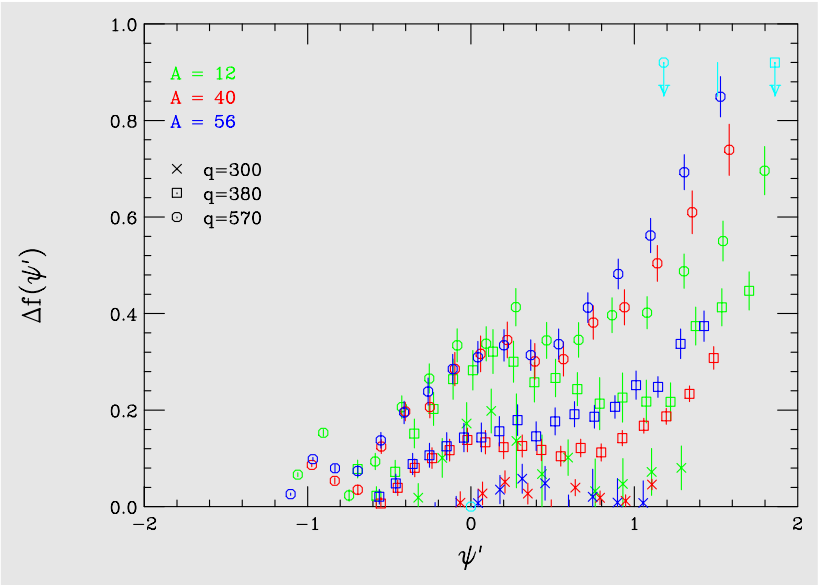
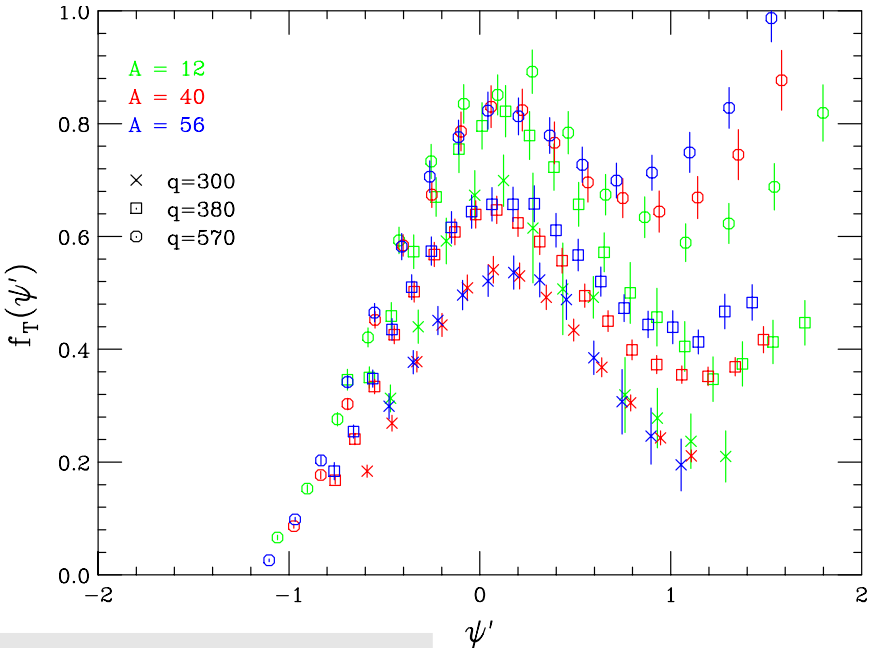
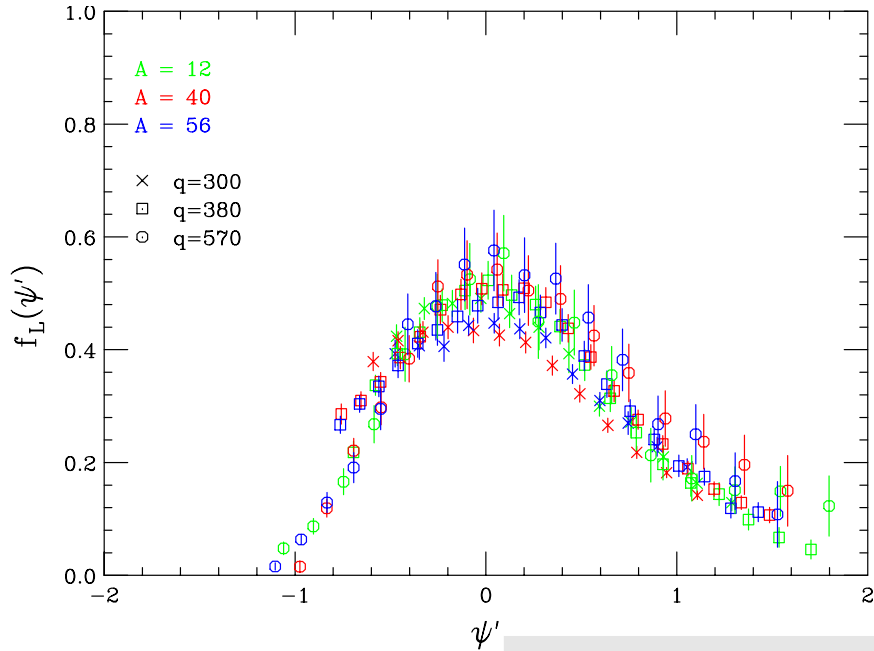


# What are QE ( $e, e'$ ) data showing us? $L/T$ contributions





# What are QE ( $e, e'$ ) data showing us? $L/T$ contributions



# The SuperScaling Approach (SuSA)

- *Scaling of the first kind below the QE peak ( $\psi \leq 0$ )*
- *Excellent scaling of the second kind in the same region*
- *Breaking of scaling above the QE peak ( $\psi > 0$ )  $\implies$  Effects beyond the IA  
(mainly located in the T channel)*
- **LONGITUDINAL RESPONSE SUPERSCALES**

# The SuperScaling Approach (SuSA)

- *Scaling of the first kind be*

PRC60 (1999) 065502

PRL82 (1999) 3212

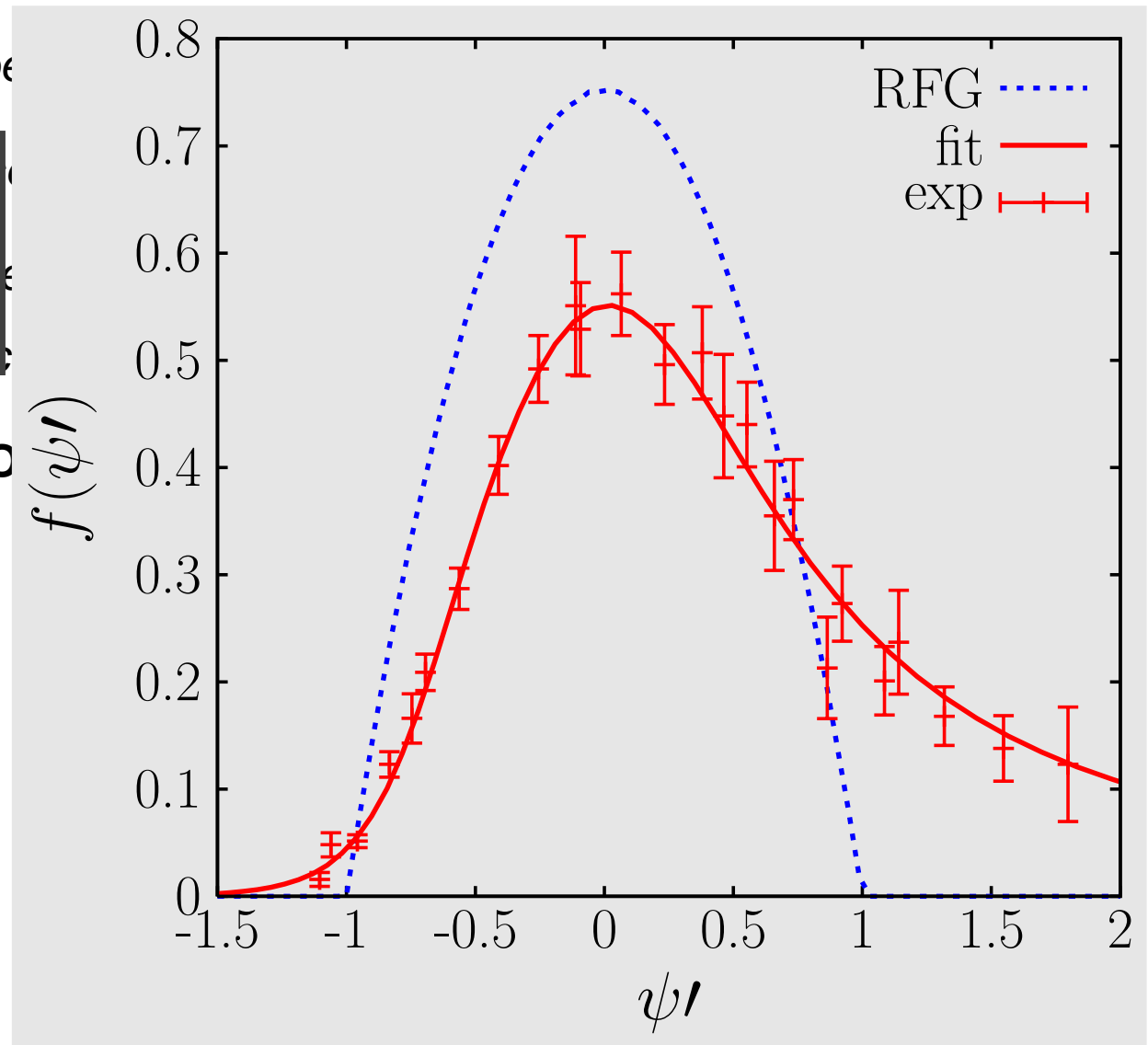
PRC65 (2002) 025502

- **LONGITUDINAL RESPO**

*Experimental superscaling function: asymmetric shape with a long tail extended to positive  $\psi$ -values*



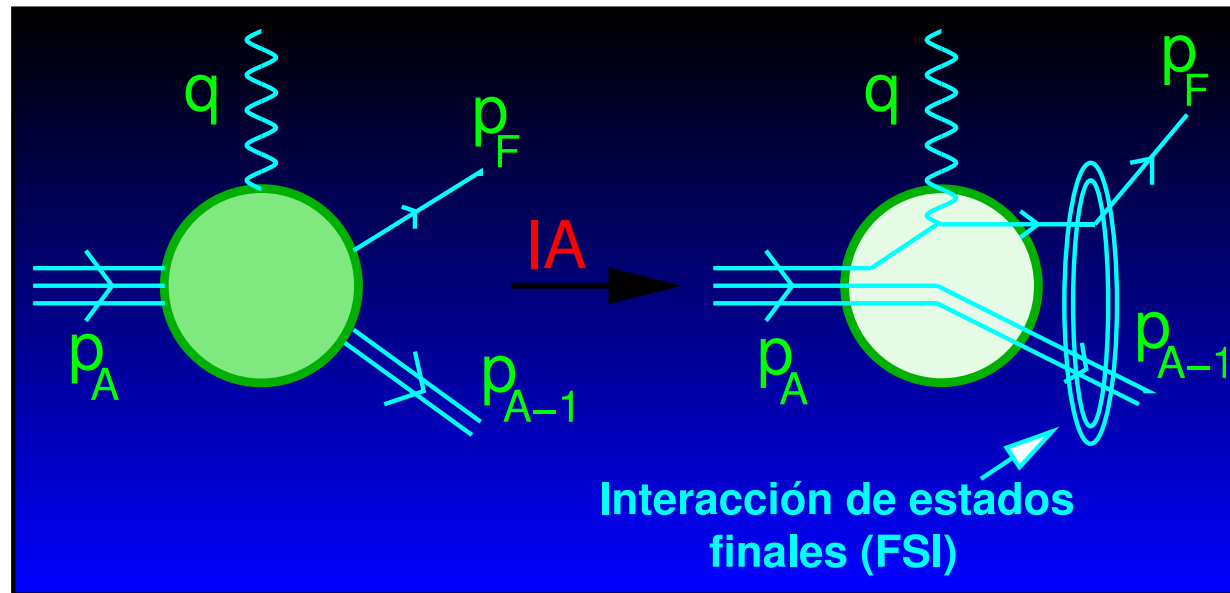
**strong constraints to models**



# THE MODEL: RELATIVISTIC IMPULSE APPROXIMATION

APPLICATION TO  $(e, e')$  PROCESSES

# Relativistic Impulse Approximation (RIA)



**Nuclear Current  $\implies$  One-body operator**

$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p} \bar{\Psi}_F(\vec{p} + \vec{q}) \hat{J}_N^\mu \Psi_B(\vec{p})$$

Scattering off a nucleus  $\implies$  incoherent sum of single-nucleon scattering processes

# Ingredients in RIA: nucleon w.f. & current operator

*Solutions of Dirac equation with phenomenological relativistic potentials*

- $\Psi_B$ : Bound nucleon w.f.  $\implies$  **Relativistic Mean Field (RMF)**
- $\Psi_F$ : Ejected nucleon w.f.  $\implies$  **Final State Interactions (FSI)**

**RMF  $\Leftrightarrow$  rROP  $\Leftrightarrow$  RPWIA  $\Leftrightarrow$  RGFA**

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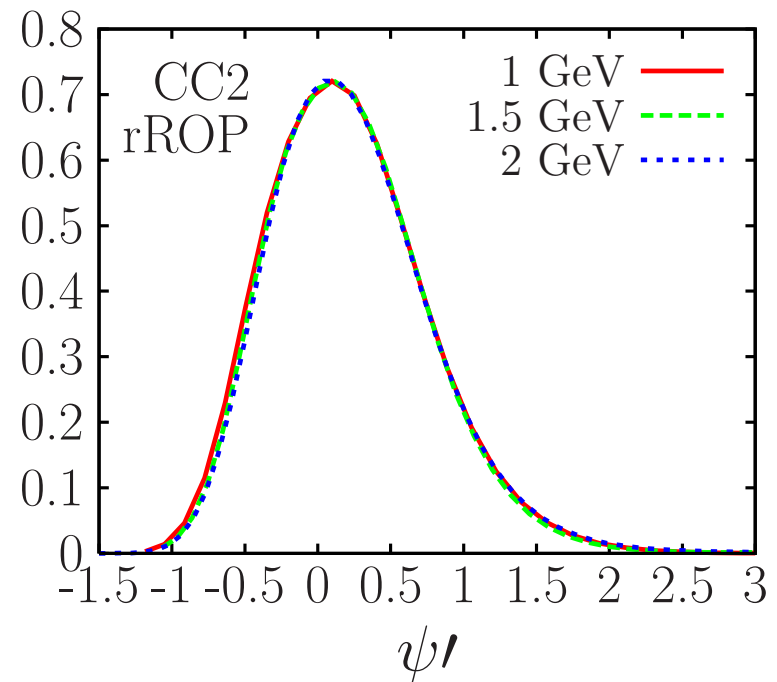
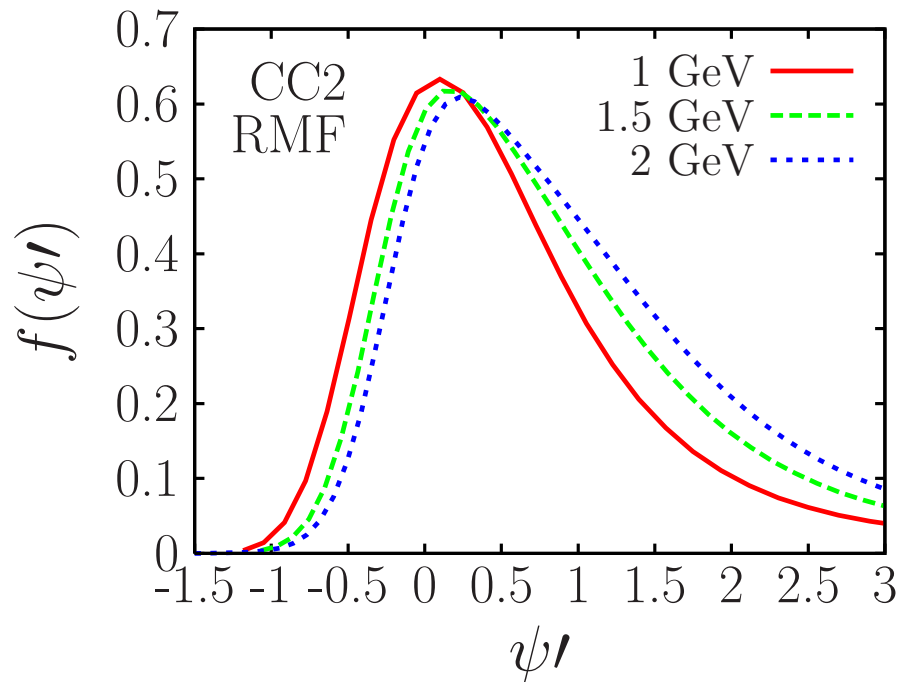
- **Electromagnetic current:  $(e, e')$**

$$\hat{J}_{cc1}^\mu = (F_1 + F_2)\gamma^\mu - \frac{F_2}{2m_N}(\bar{P} + P_N)^\mu$$

$$\hat{J}_{cc2}^\mu = F_1\gamma^\mu + \frac{iF_2}{2m_N}\sigma^{\mu\nu}Q_\nu$$

**Off-shell & Gauge ambiguities  $(Q_\mu J^\mu \neq 0)$**

# Scaling of the first kind in $^{12}C(e, e')$



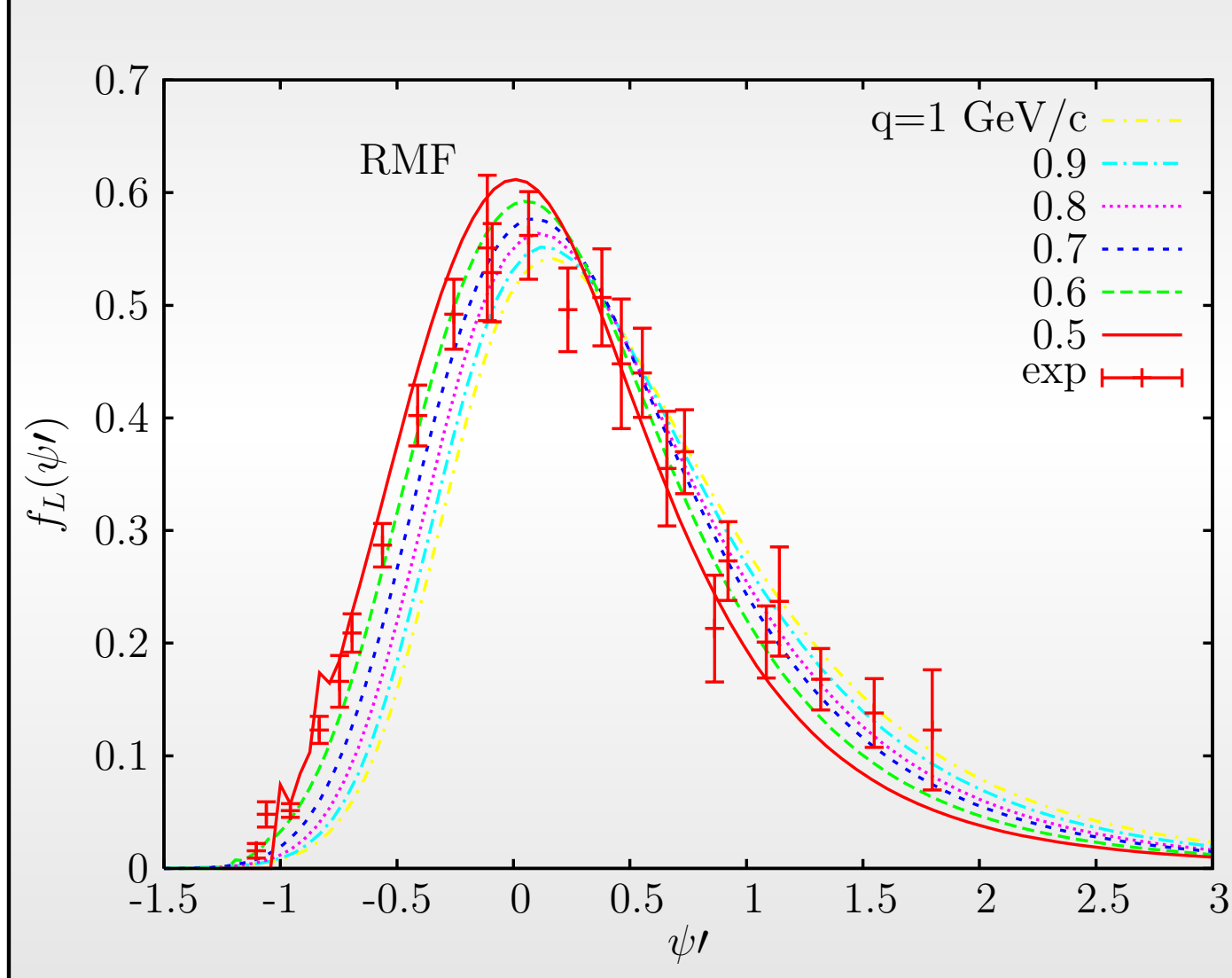
**RMF**: shift in  $\psi' < 0$  and breakdown of scaling at roughly  $\sim 25 - 30\%$  for  $\psi' > 0$  (compatible with data).

Scaling of the first kind: excellent in **rROP** approach (and **RPWIA**)

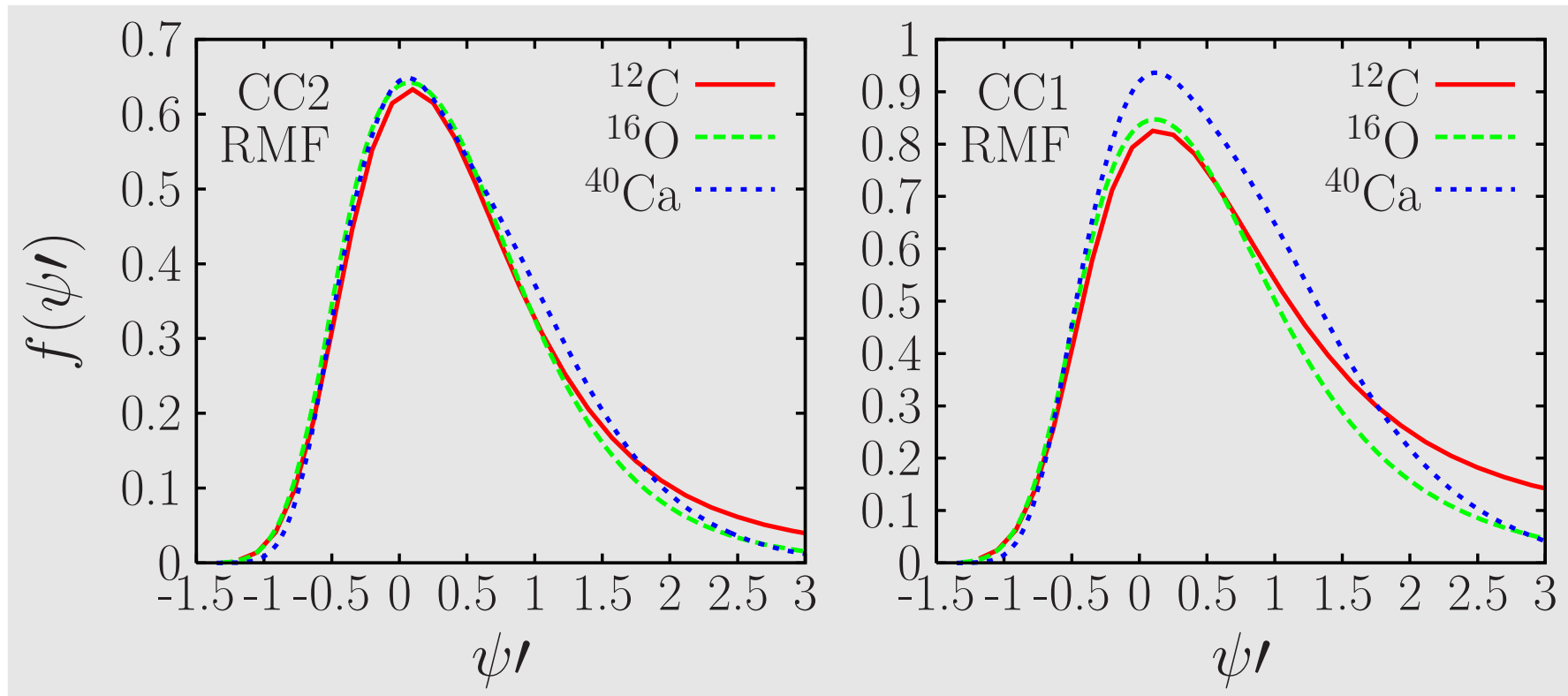


# How Scaling of the 1<sup>er</sup> kind behaves (RMF)

Scaling of first kind. Results for  $^{12}\text{C}(e, e')$  and  $\epsilon_e = 1 \text{ GeV}$ . CC2 current prescription

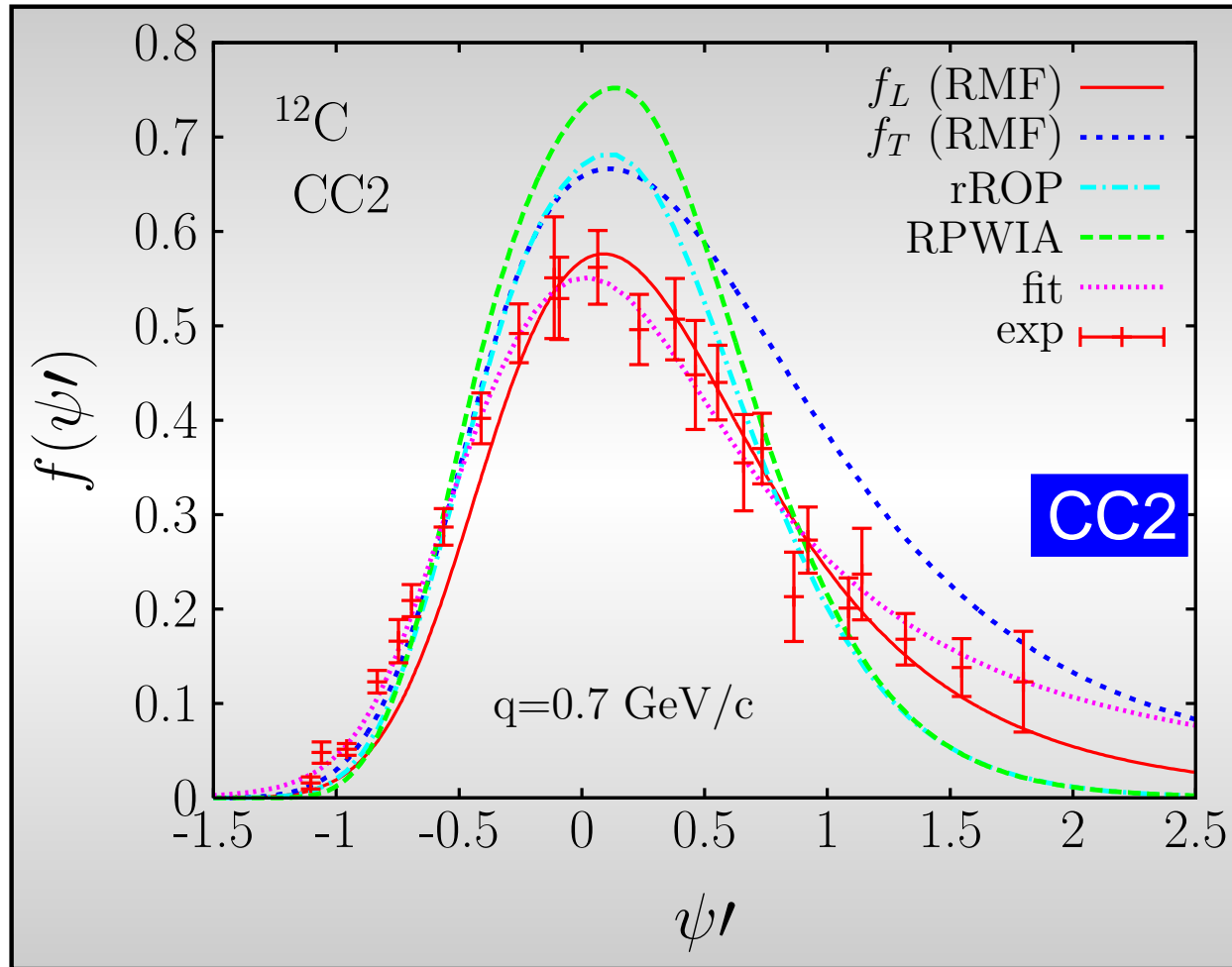


# Scaling of the second kind in RIA



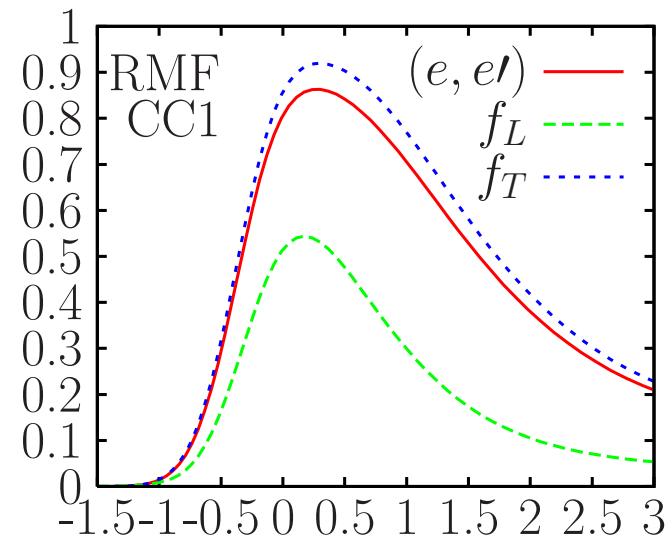
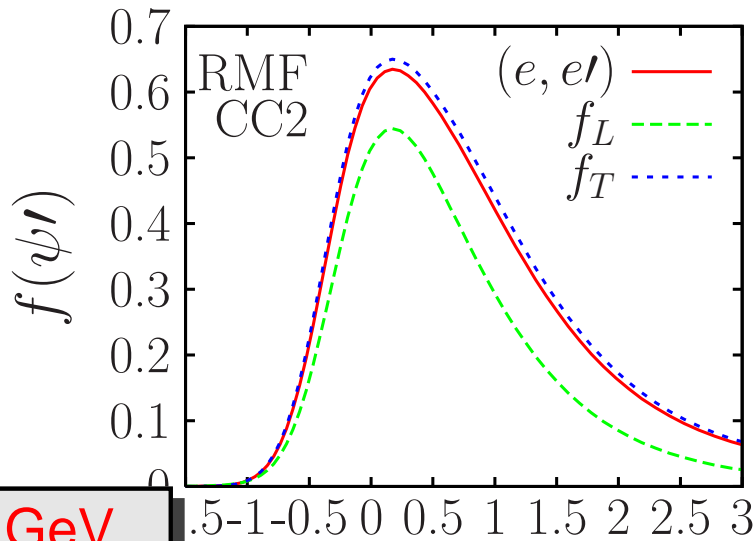
**Scaling of  $2^a$  kind: excellent with the CC2 current operator**

# RMF: Comparison with $(e, e')$ data

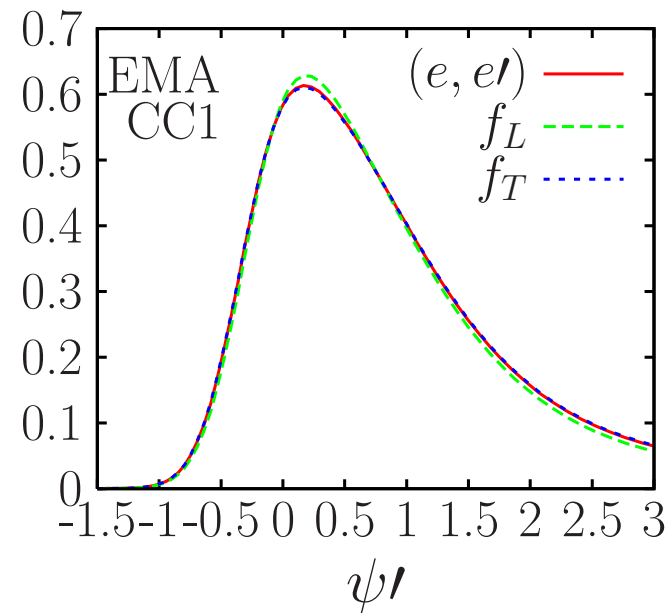
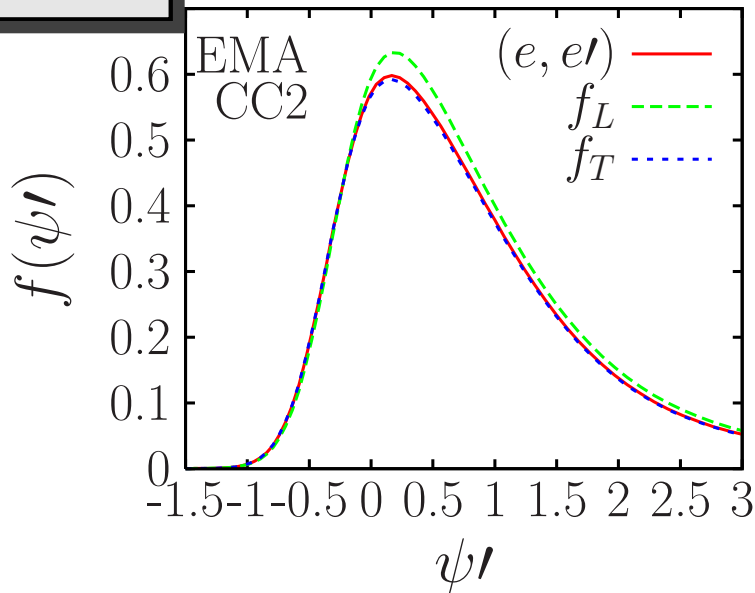


Only the description of FSI provided by RMF leads to an asymmetric function  $f(\psi')$  in accordance with the behavior shown by data. Moreover,  $f_T > f_L$

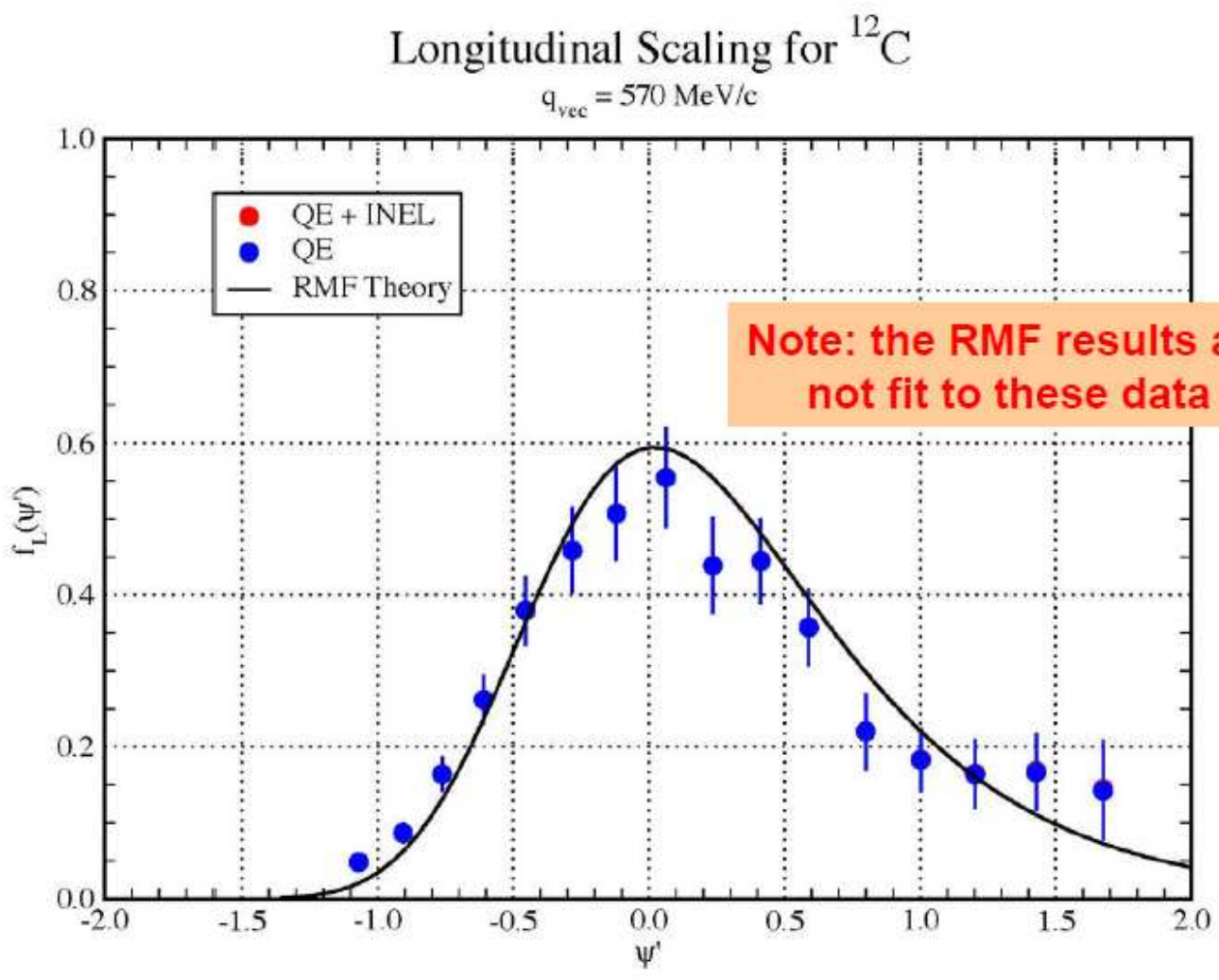
# Scaling of the 0<sup>th</sup> kind in RMF



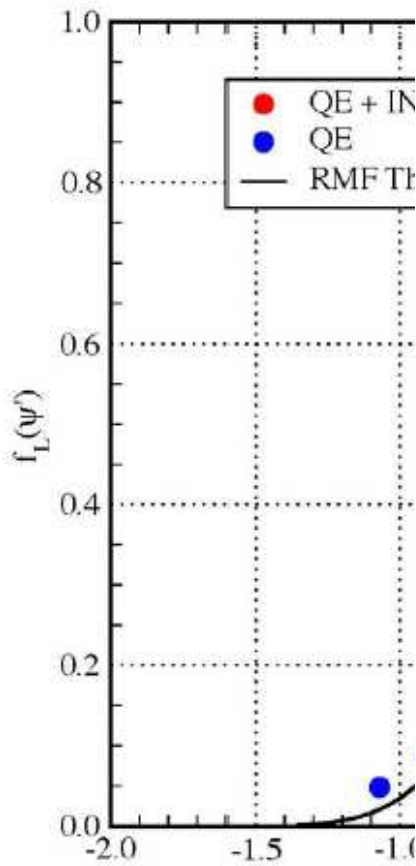
$\varepsilon_e = 1 \text{ GeV}$   
 $q = 1 \text{ GeV}/c$



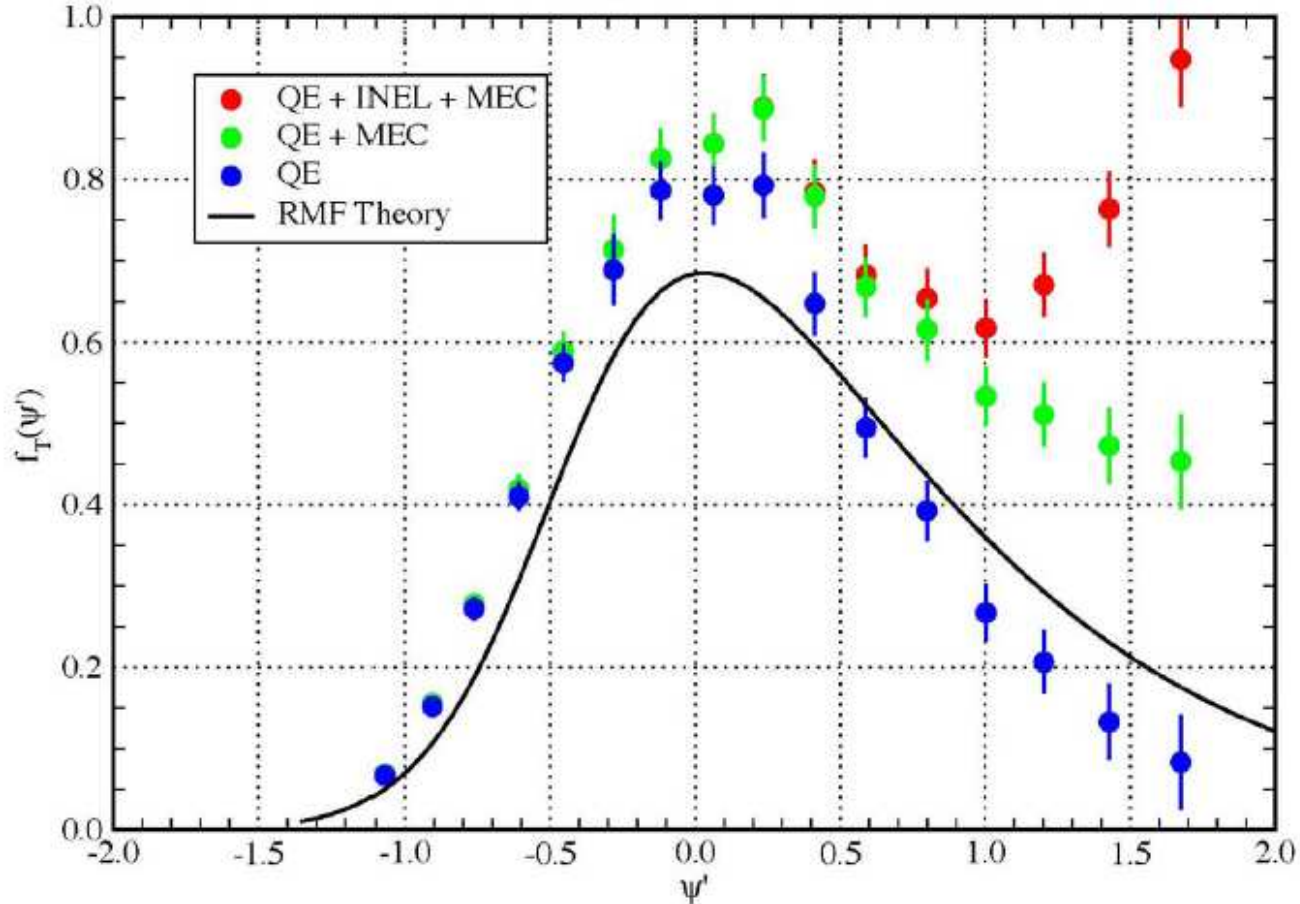
# Scaling in QE $L/T$ -channels



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Transverse Scaling for  $^{12}\text{C}$   
 $q_{\text{vec}} = 570 \text{ MeV}/c$



# INTERACTION OF NEUTRINOS with NUCLEI

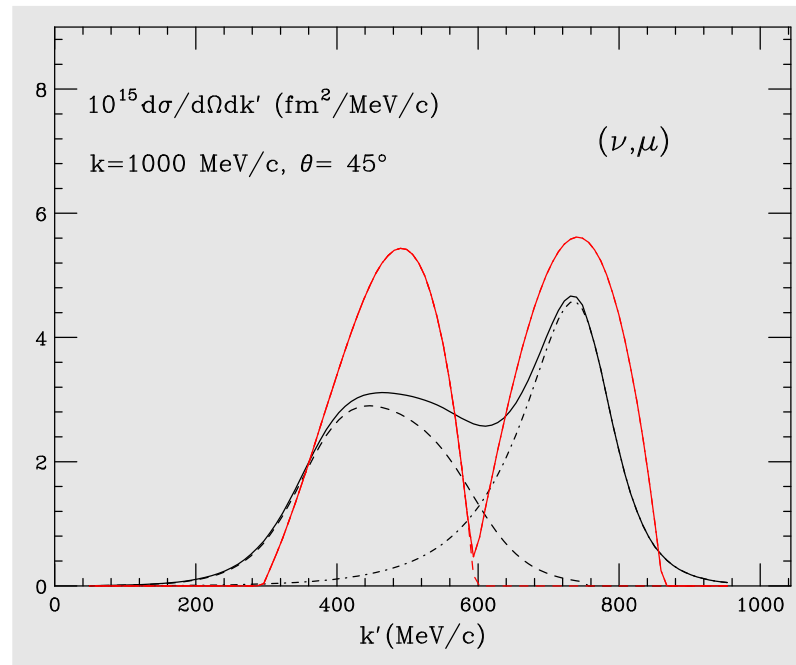
# Scaling applied to $(\nu, \mu)$ processes

## SuSA (“SuperScaling Analysis”)

- **Hyphotesis:** *the universal character of the function  $f_{exp}(\psi')$  extracted from the analysis of  $L(e, e')$  data  $\implies$  it can be applied to CC  $(\nu_\mu, \mu)$  processes.*

Prediction of “realistic”  $(\nu, \mu)$  cross sections

[MiniBoone, MINER $\nu$ A, NOMAD, T2K]





# RMF applied to $(\nu, \mu)$ & Scaling

## PROCEDURE

- Evaluate the inclusive  $(\nu, \mu)$  cross section with a specific RIA model and divide it by the corresponding single-nucleon cross section [weighted by the appropriate proton ( $Z$ ) and neutron ( $N$ ) numbers]  $\implies$  **THEORETICAL SCALING FUNCTION**
- Does the theoretical RIA scaling function satisfy scaling properties?
  - Scaling of the first kind:  $f(q, \psi) \xrightarrow{q \rightarrow \infty} f(\psi)$
  - Scaling of the second kind:  $f(\psi)$  – independent on the nucleus

# RMF applied to $(\nu, \mu)$ & Scaling

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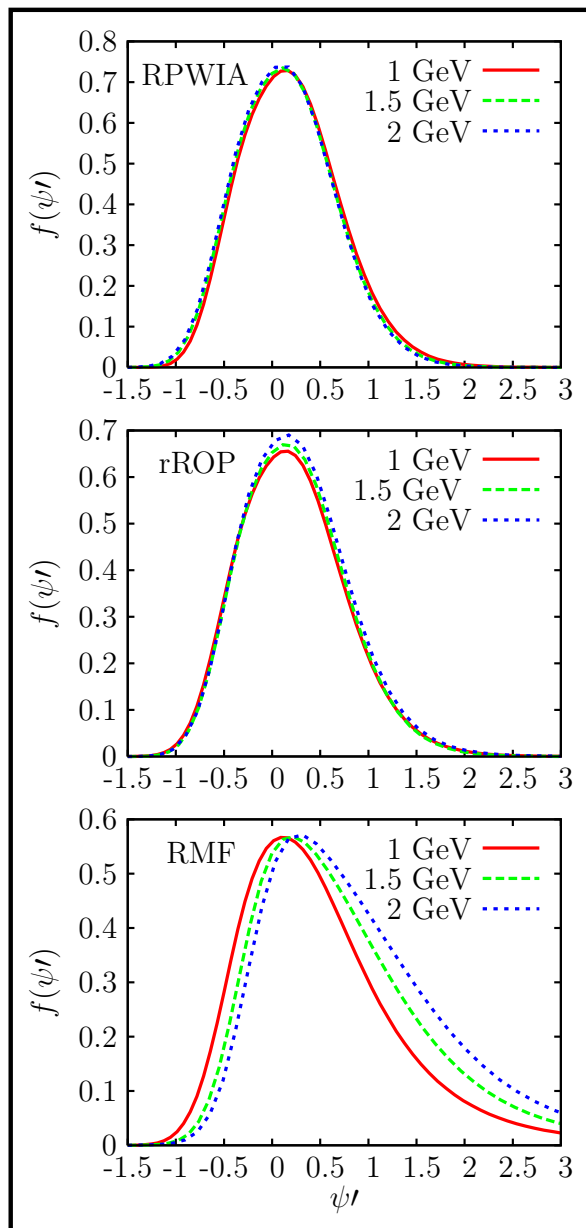
$(\nu, \mu)$  cross section with a specific RIA model and divide it by the nucleon cross section [weighted by the appropriate proton ( $Z$ ) and neutron ( $N$ ) cross sections]

→ **THEORETICAL SCALING FUNCTION**

Do scaling functions satisfy scaling properties?

1st kind:  $f(q, \psi) \xrightarrow{q \rightarrow \infty} f(\psi)$

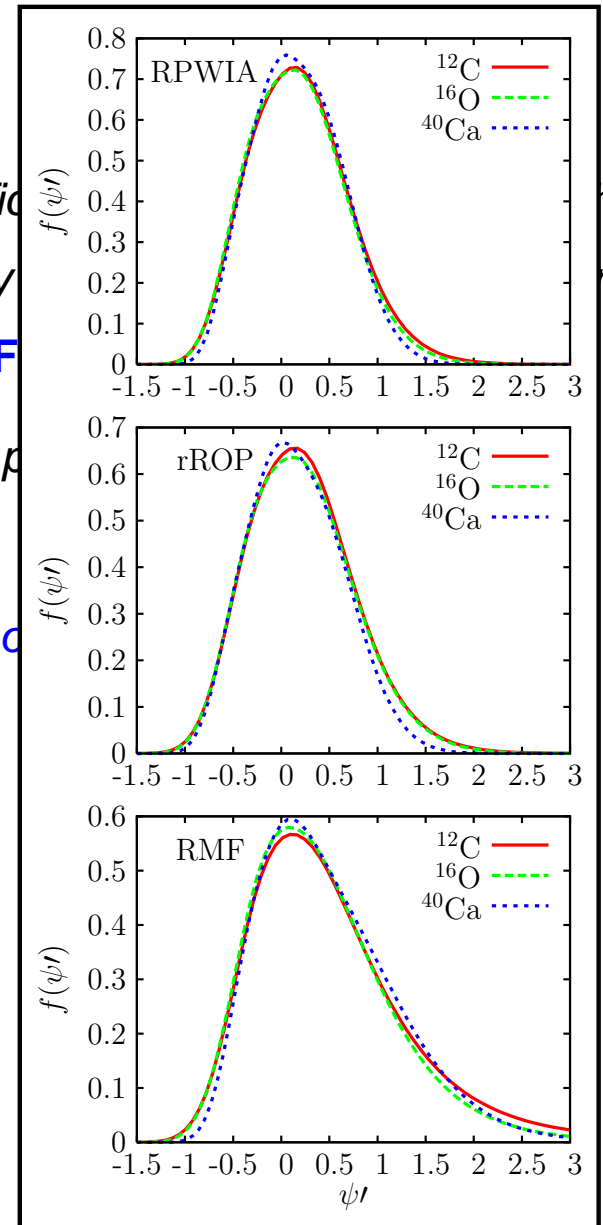
2nd kind:  $f(\psi)$  – independent on the nucleus



# RMF applied to $(\nu, \mu)$ & Scaling

## PROCEDURE

- Evaluate the inclusive  $(\nu, \mu)$  cross section with a specific model and the corresponding single-nucleon cross section [weighted by neutron ( $N$ ) numbers]  $\implies$  **THEORETICAL SCALING FUNCTION**
- Does the theoretical **RIA** scaling function satisfy scaling properties?
  - **Scaling of the first kind:**  $f(q, \psi) \xrightarrow{q \rightarrow \infty} f(\psi)$
  - **Scaling of the second kind:**  $f(\psi)$  – independent of  $q$



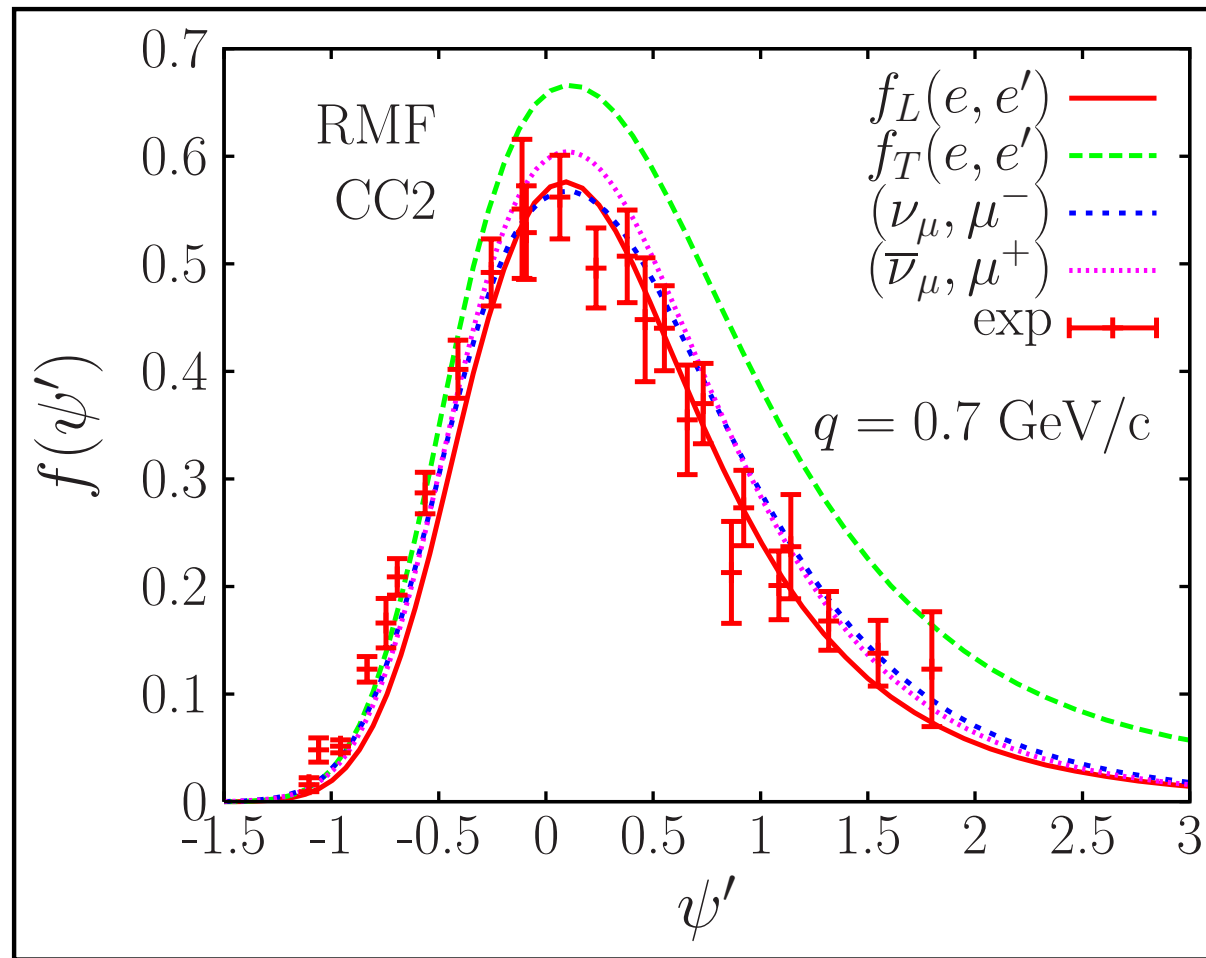
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  - Scaling of the second kind:  $f(\psi)$  – independent on the nucleus
- Is the function  $f(\psi)$  obtained from  $(\nu, \mu)$  cross sections evaluated within RIA consistent with the function  $f(\psi)$  obtained from  $(e, e')$  calculations (with the same model)?, and with  $f_{exp}(\psi)$ ?

Similar scaling function  $f(\psi)$  for  $(e, e')$  and  $(\nu, \mu)$  processes?

# $(e, e')$ vs $(\nu, \mu)$ . SuSA vs RMF

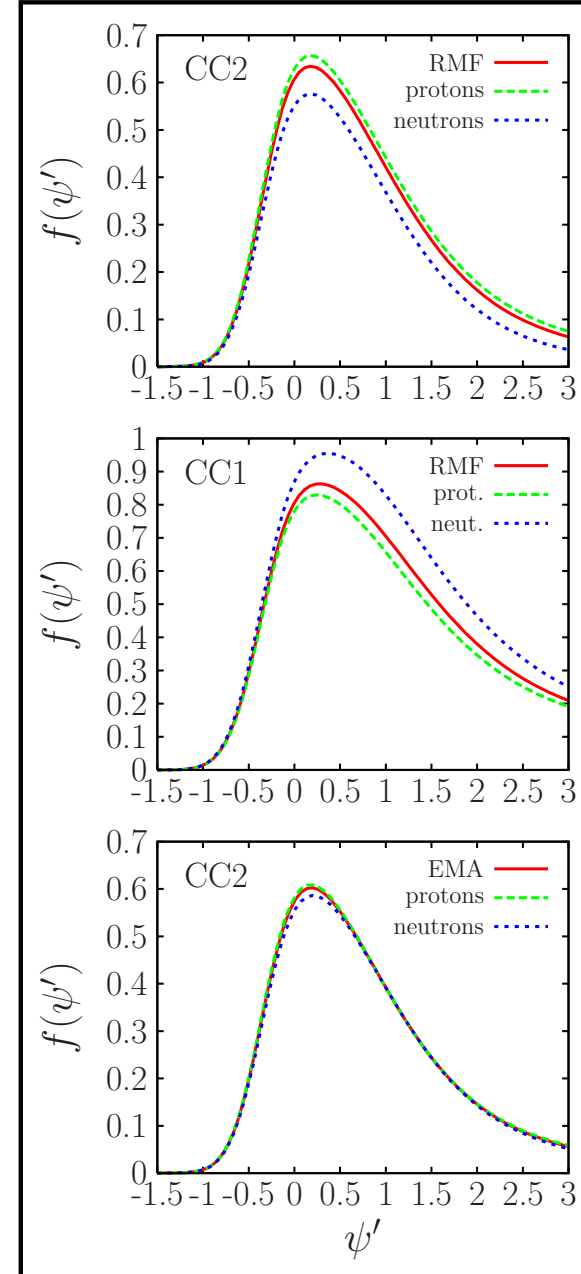


**Basic result:** the function  $f(\psi)$  evaluated for  $(\nu, \mu)$  processes agrees better with the contribution  $f_L(\psi)$  [corresponding to  $(e, e')$ ] than with  $f_T(\psi)$ .

# ISOSPIN: isoscalar vs isovector ( $3^{er}$ kind scaling)

Note that  $(\nu, \mu)$  are pure isovector, whereas  $(e, e')$  contains both isoscalar and isovector. Thus, SuSA applied to CC neutrino implies Scaling of the 3rd kind, i.e., isospin nature in the scaling functions is assumed to be universal.

Analysis of the separate  $VV$ ,  $AA$  and  $VA$  contributions in  $(\nu, \mu)$  reactions. SuSAv2 (Guillermo's talk)



## ELECTRON SCATTERING

- *Superscaling shows up in RIA, even with FSI:  $rROP$  &  $RMF$ .*
- *$RMF$ -FSI description leads to an asymmetric superscaling function which fits data.*
- *Some first-kind scaling violation with  $FSI$  switched on.*

*Contrary to most non-relativistic models,  $f_L(\psi) \neq f_T(\psi)$  within  $RMF$*

# BASIC CONCLUSIONS: $(e, e')$ & $(\nu_\mu, \mu)$

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## NEUTRINO SCATTERING

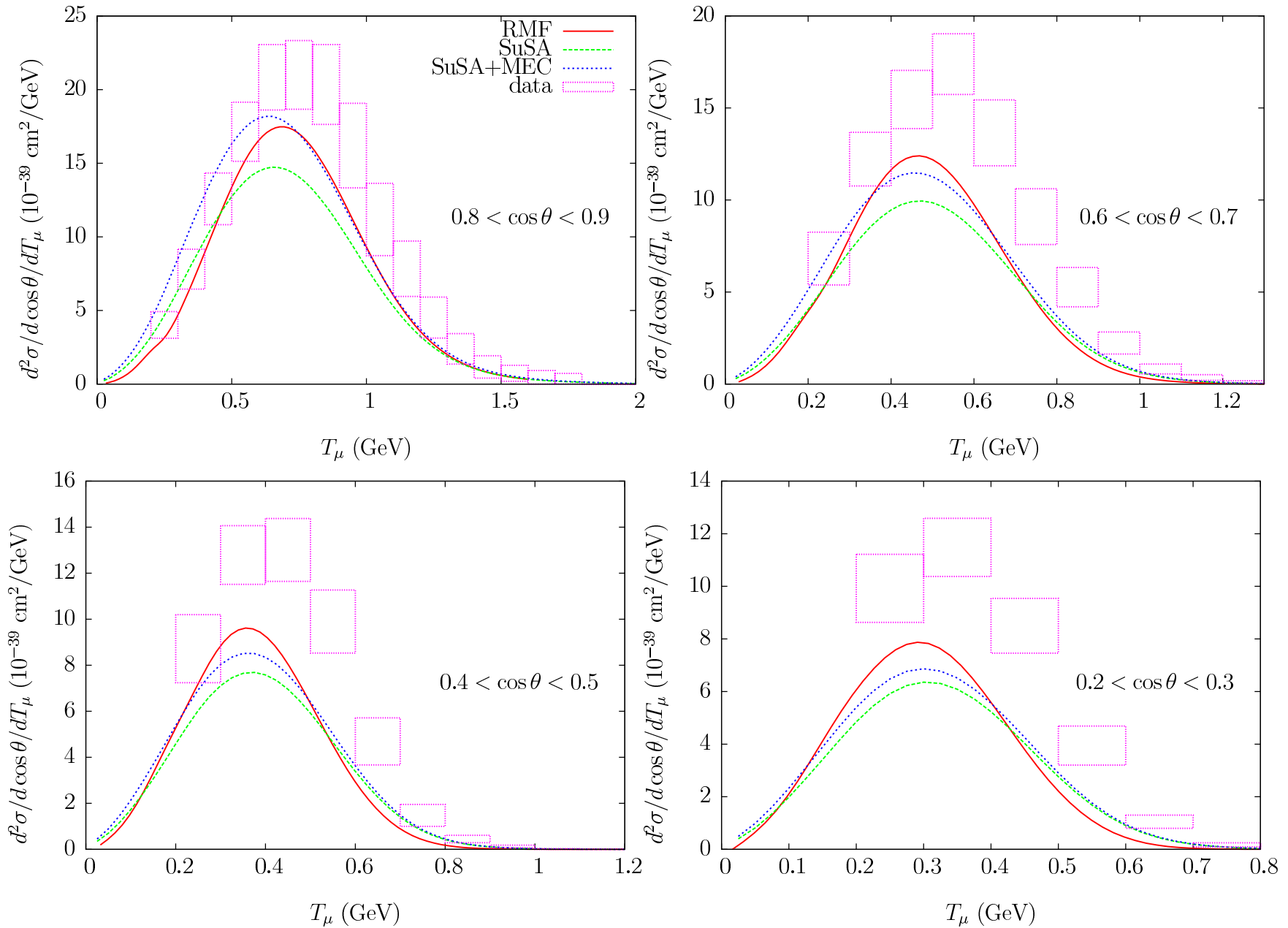
- *Superscaling fulfilled by  $RIA$  calculations.*
- *Scaling functions from QE  $(e, e')$  and  $(\nu, \mu)$  cross sections follow similar trends.*
- *Differences between  $(e, e')$  and  $(\nu, \mu)$   $RIA+RMF$  results are consistent with the isoscalar/isovector nucleon f.f. contributions in the two processes.*



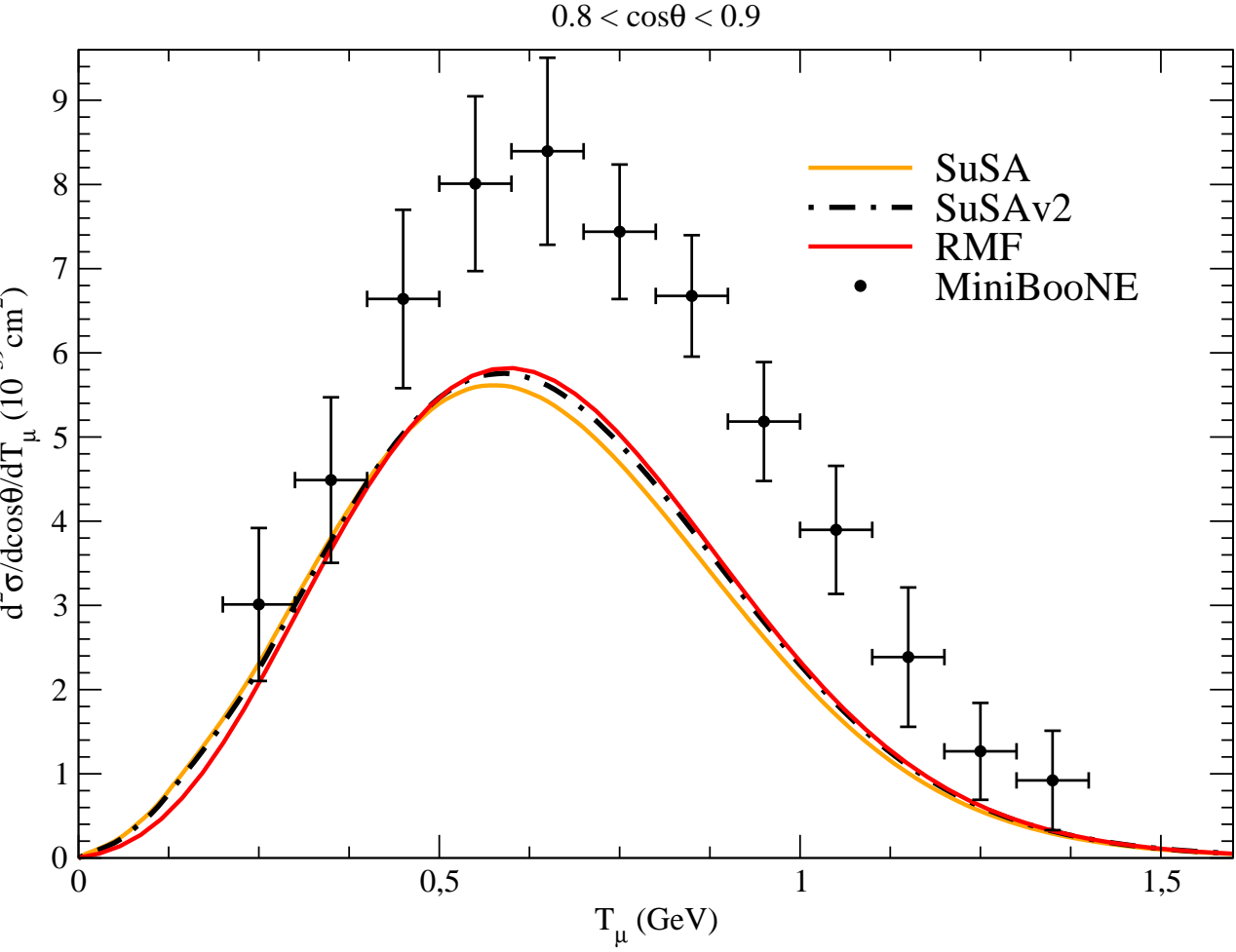
## **COMPARISON WITH DATA:**

**MiniBooNE, Miner $\nu$ A, NOMAD & T2K**

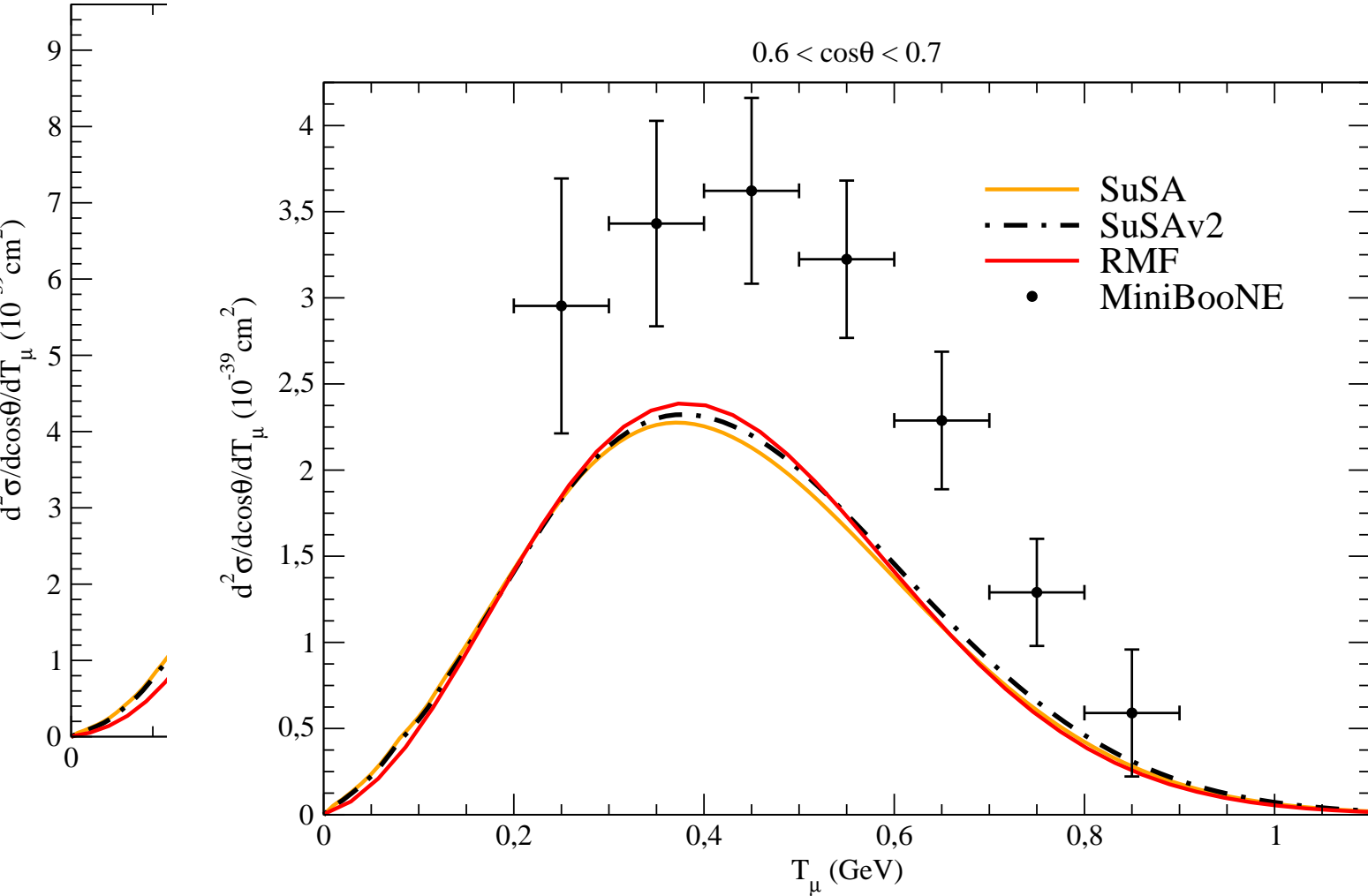
# Flux-averaged double-differential CCQE: SuSA & RMF



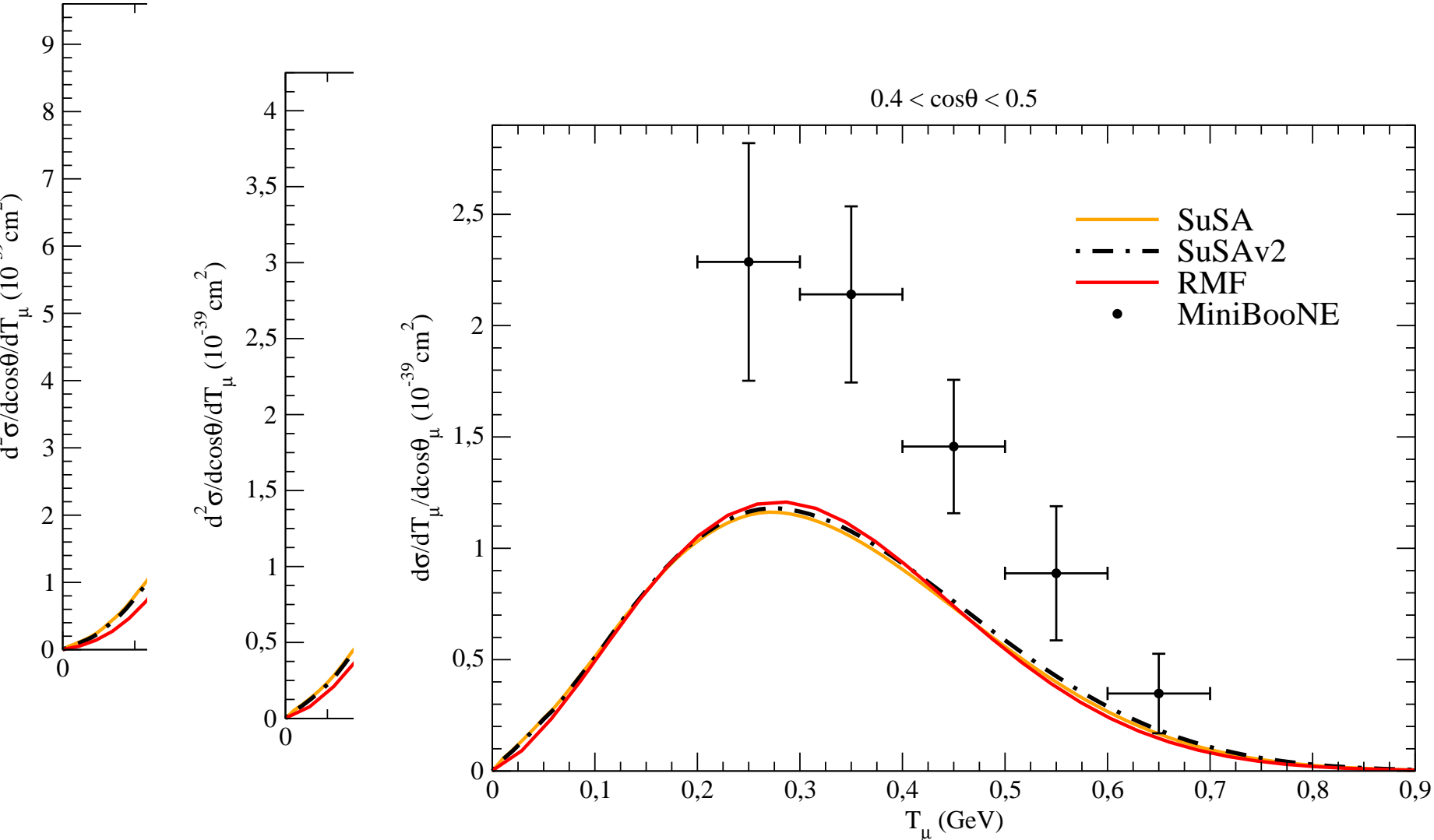
# Antineutrinos: strong enhancement of 2p2h effects?



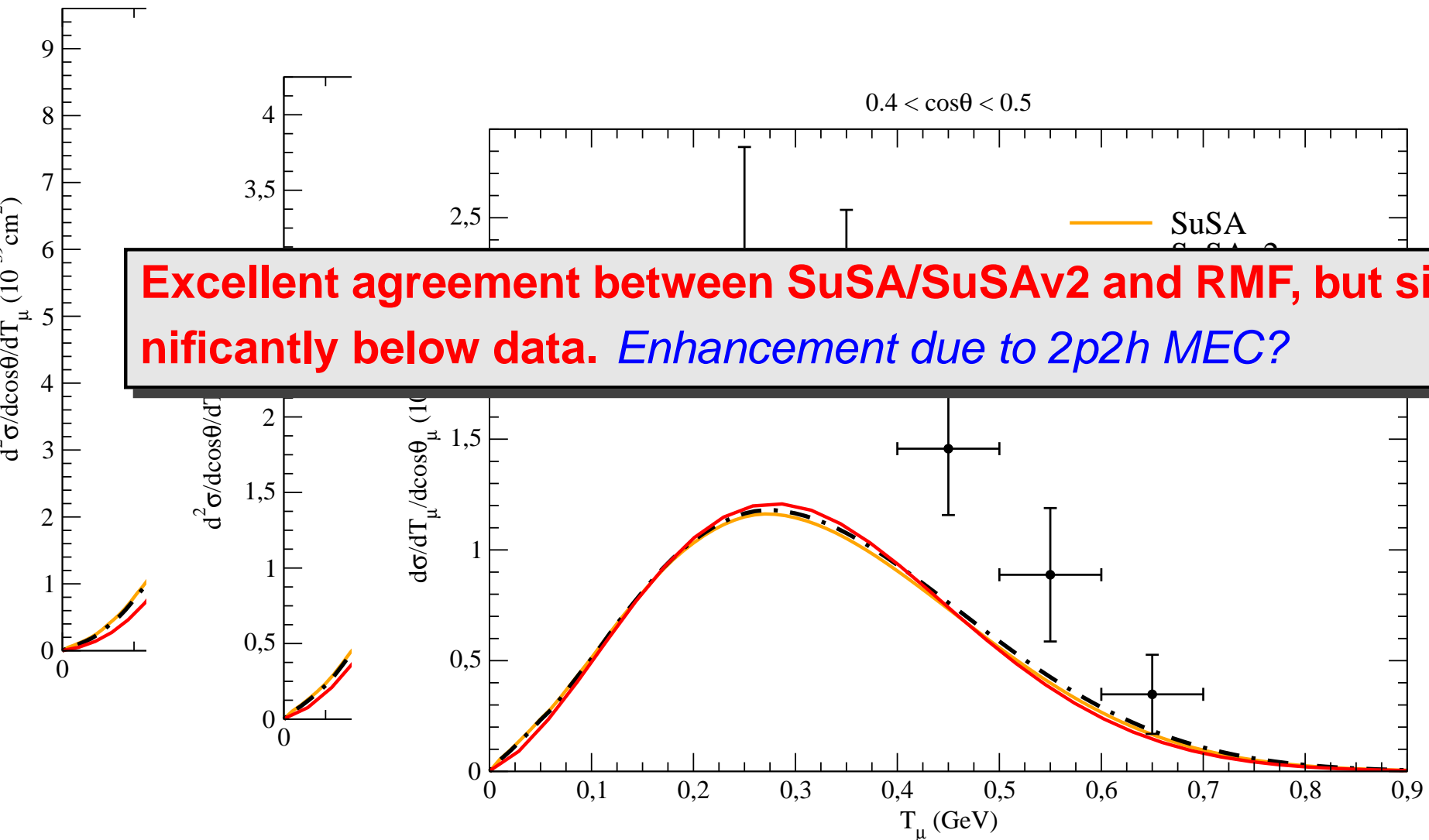
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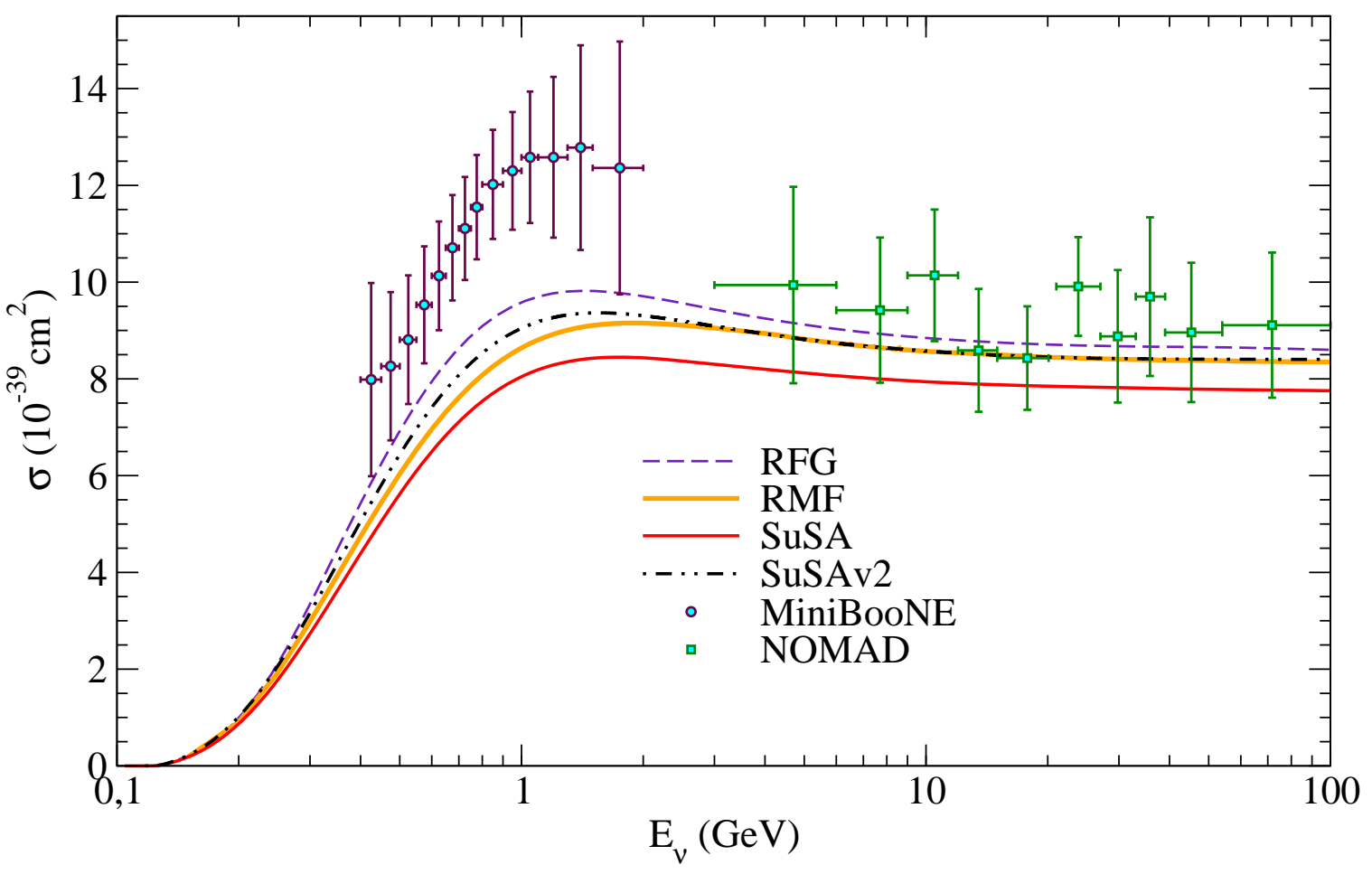
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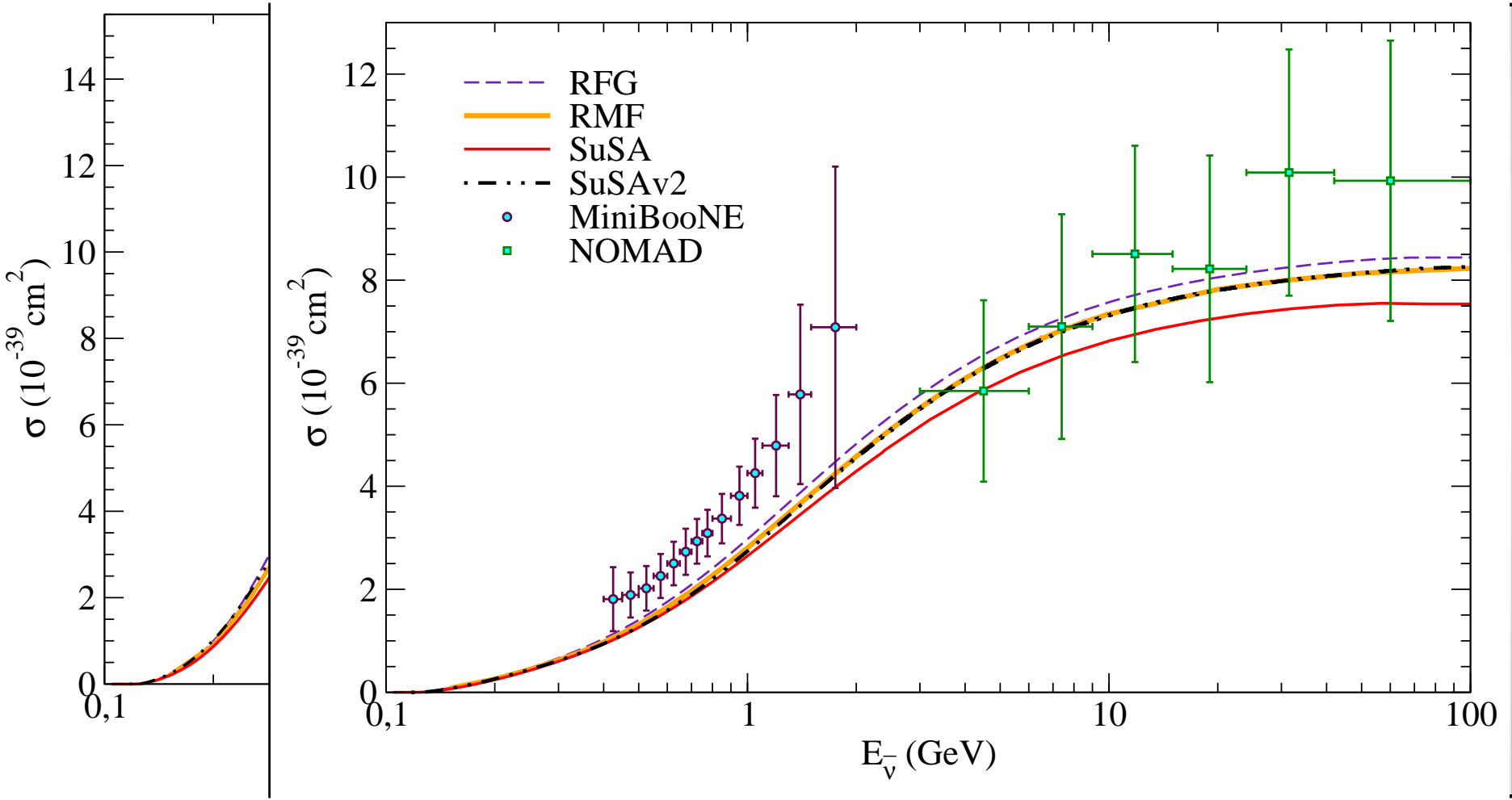
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# MiniBooNE & NOMAD: SuSA vs RMF



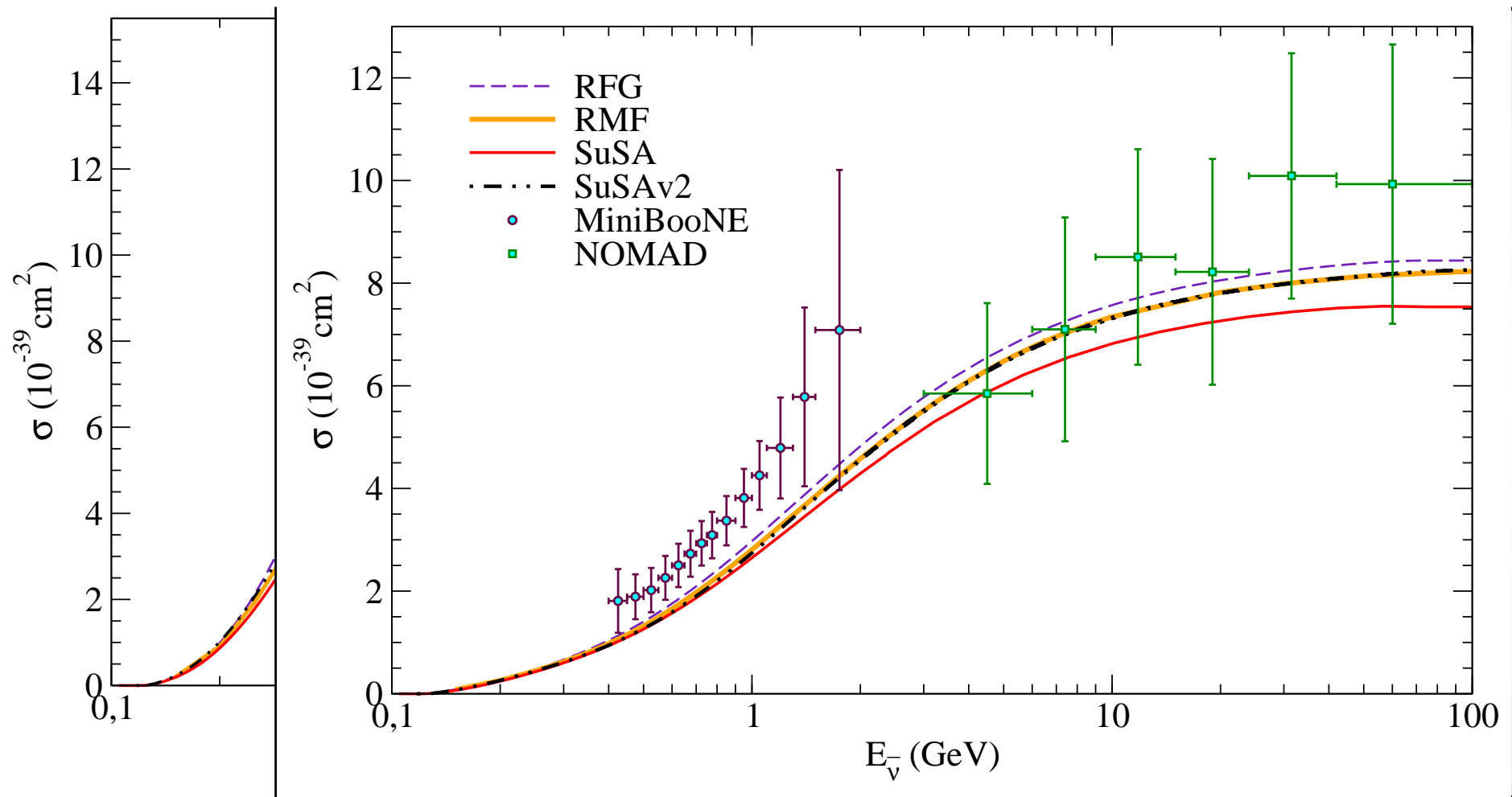
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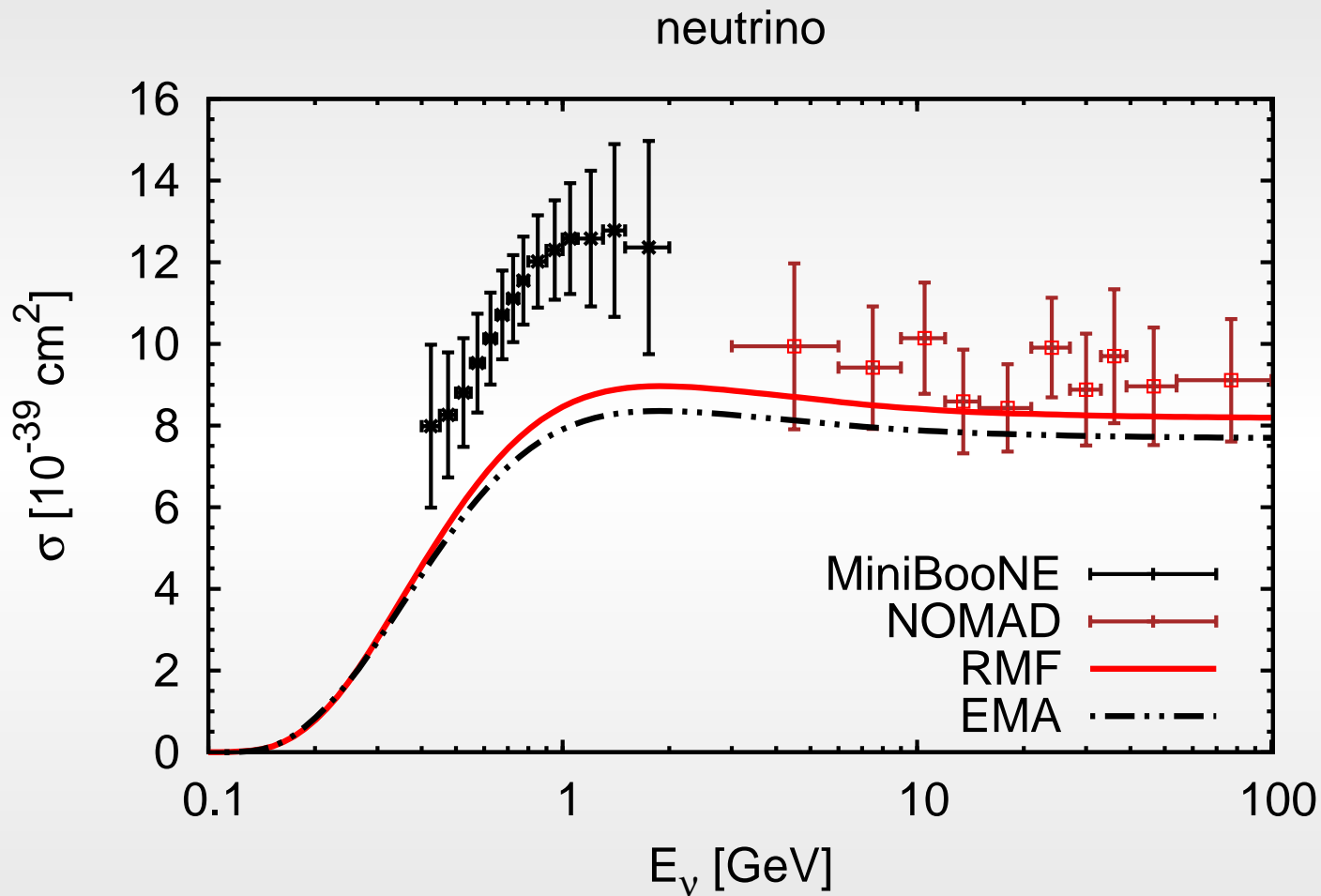


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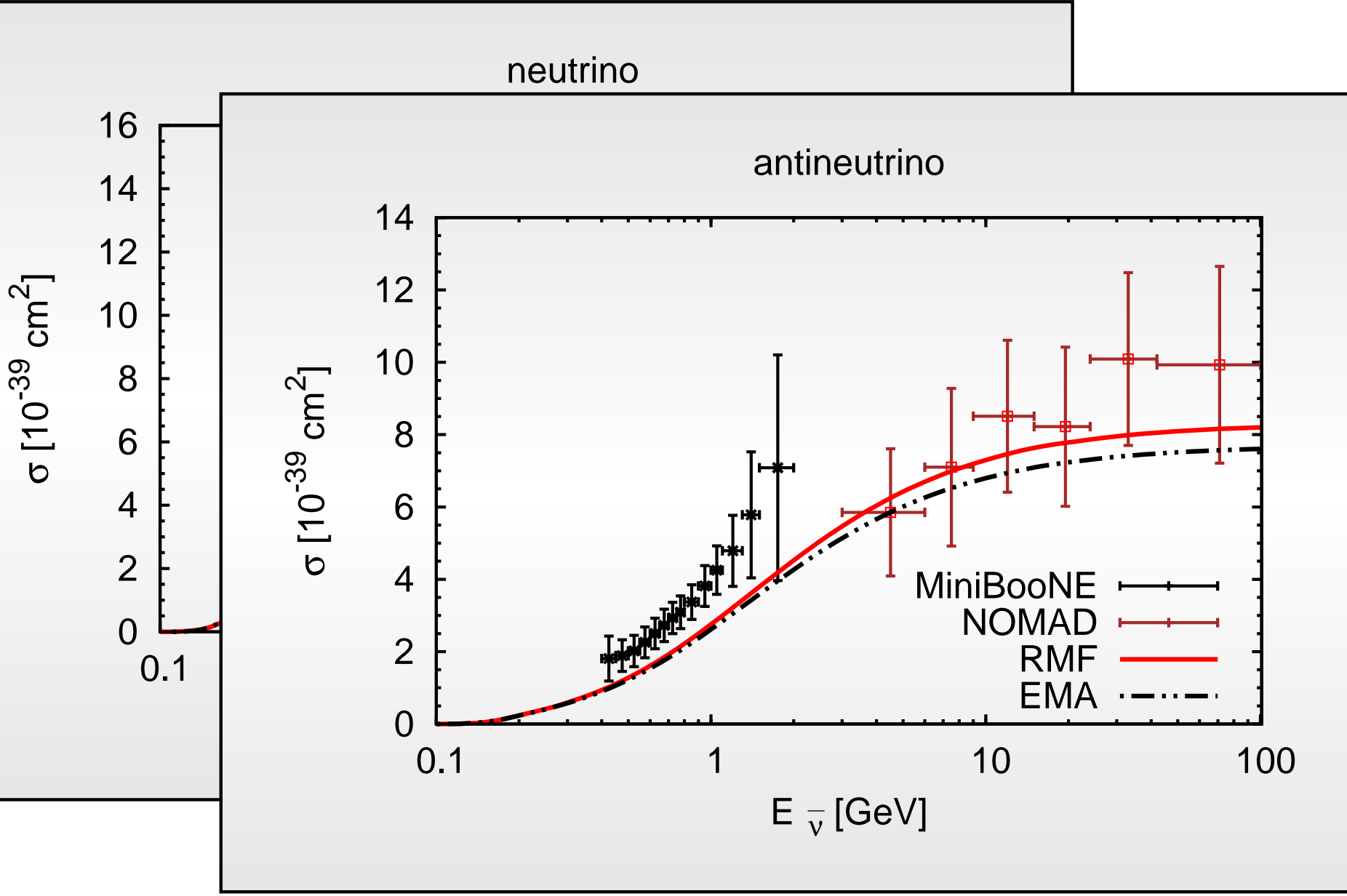
Good agreement between SuSA/SuSAv2 and RMF for all energies. The models reproduce NOMAD but underpredict MiniBooNE (standard value of the axial mass). [PLB 725 (2013) 170]



# NOMAD: spinor distortion in RMF

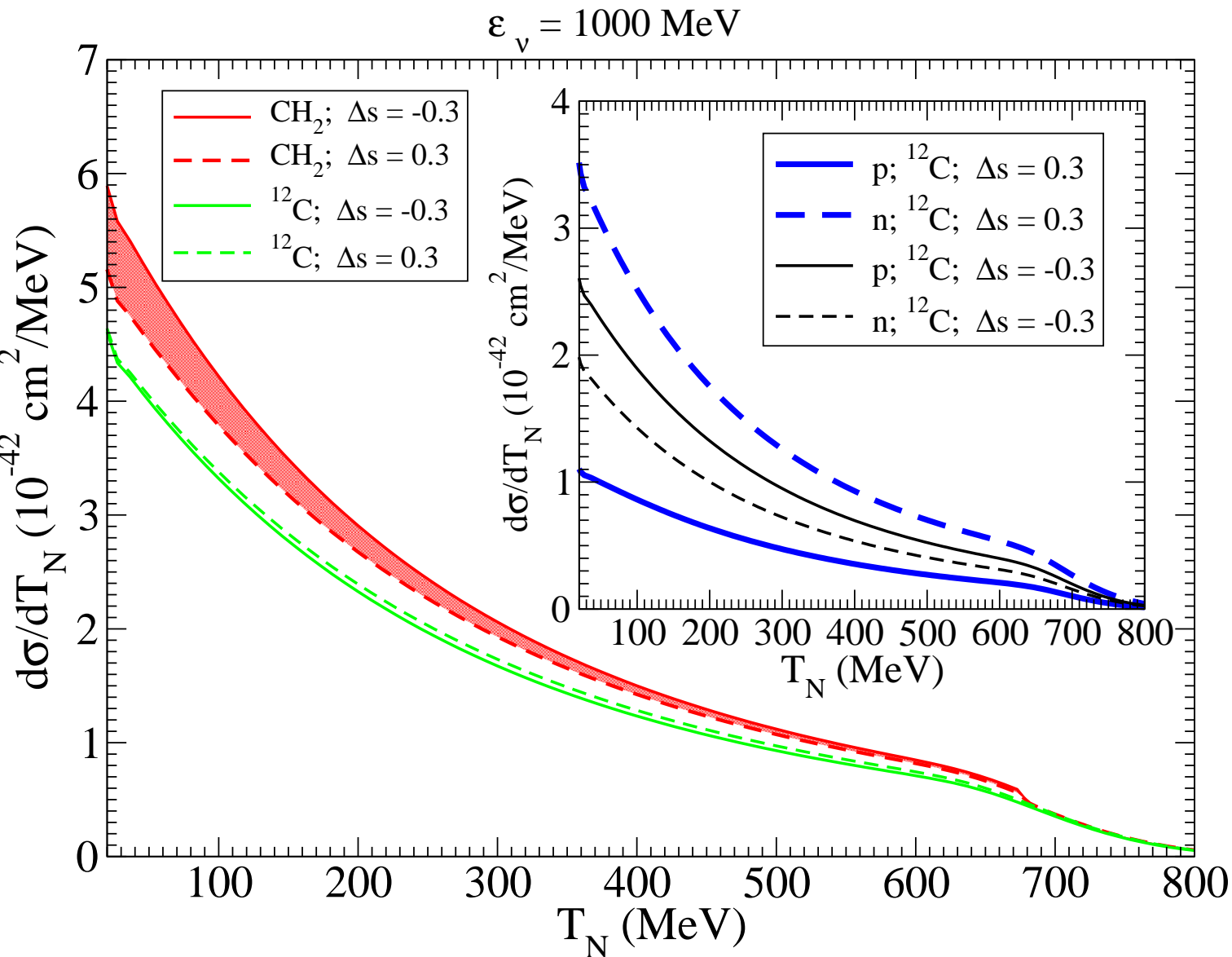


# NOMAD: spinor distortion in RMF

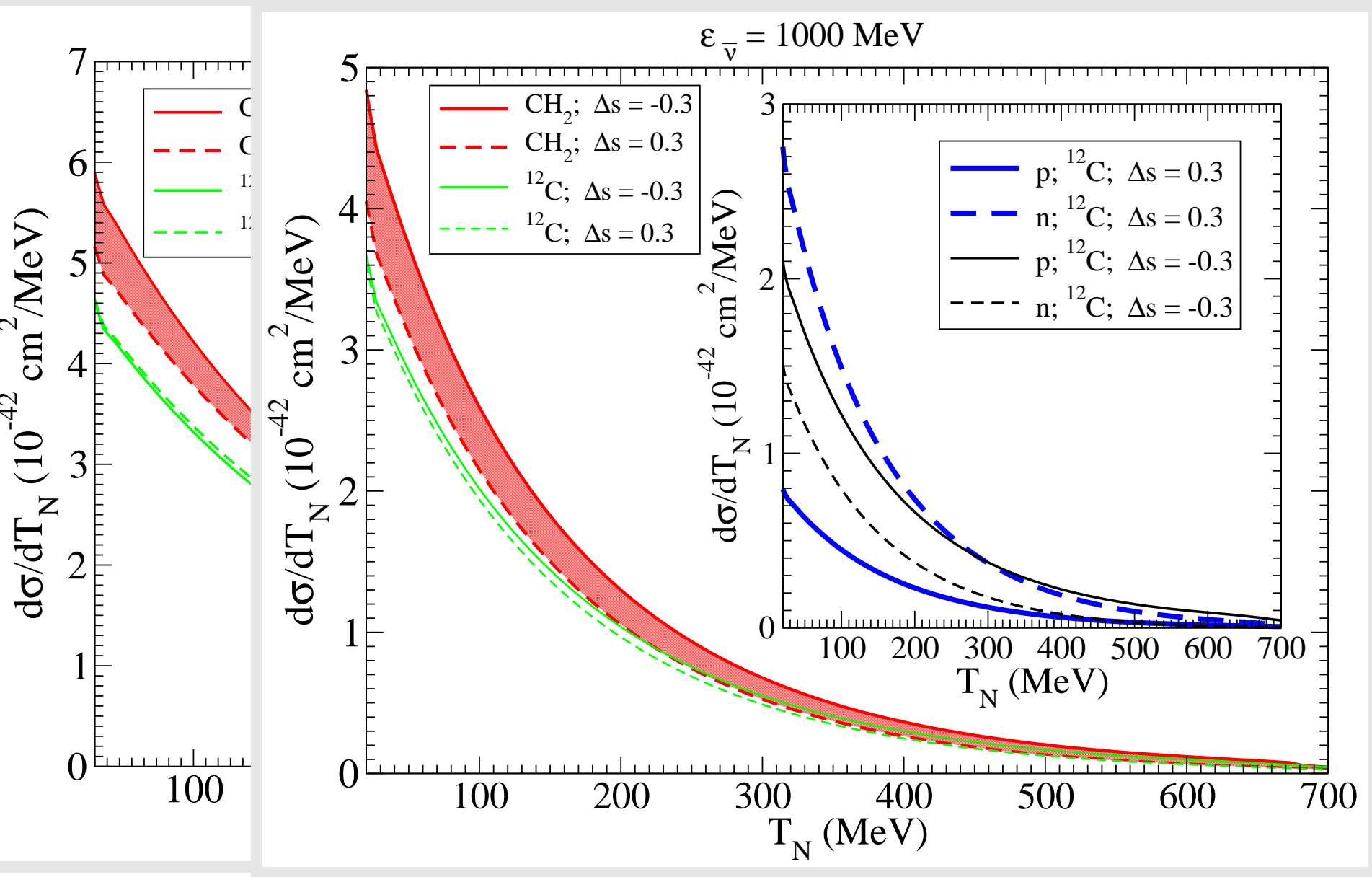


**APPLICATION TO NEUTRAL CURRENT  
NEUTRINO PROCESSES:  $(\nu, N)\nu'$**

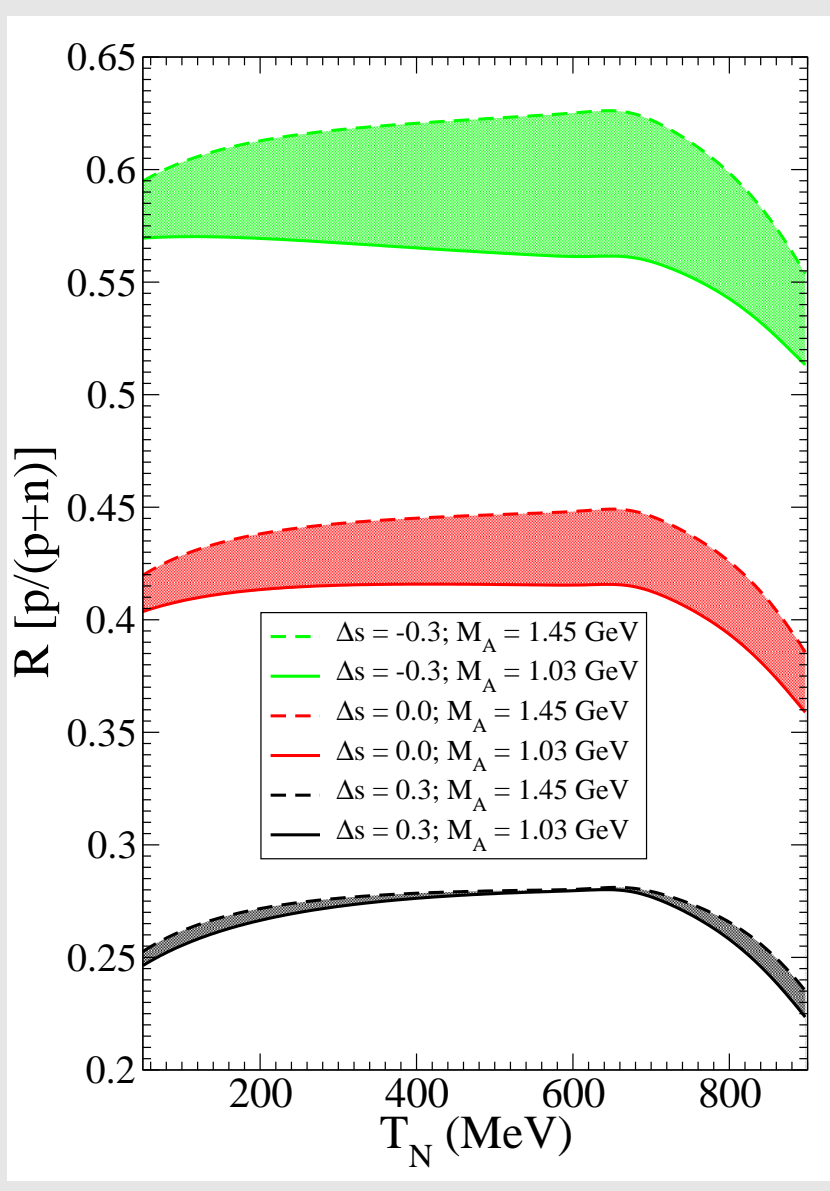
# NCQE & axial strangeness



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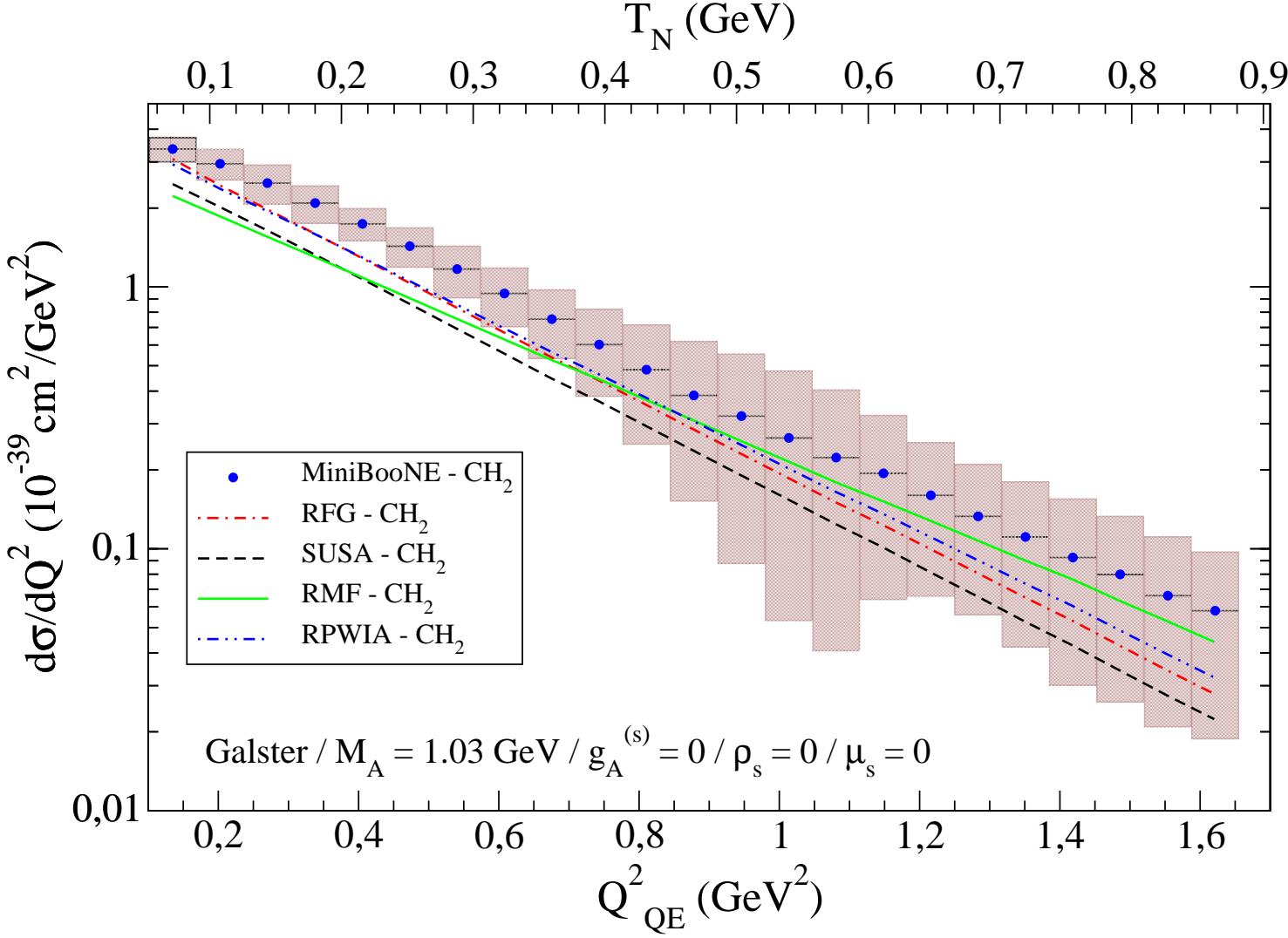


# P/N RATIO: axial strangeness vs axial mass



*Correlation between axial strangeness & axial mass: for  $\Delta s$ -positive (grey band) NCQE is almost insensitive to  $M_A$ , whereas for  $\Delta s$ -negative (green) uncertainty due to  $M_A$  is  $\sim 10-12\%$ .*

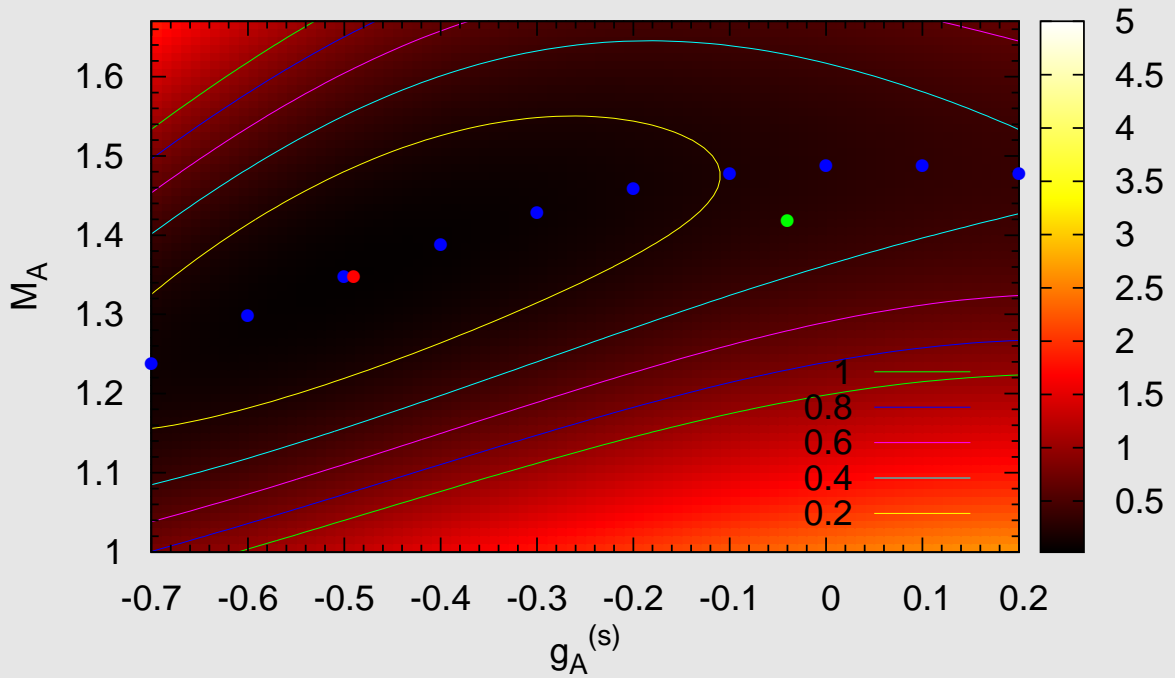
# Comparison with MiniBooNE data



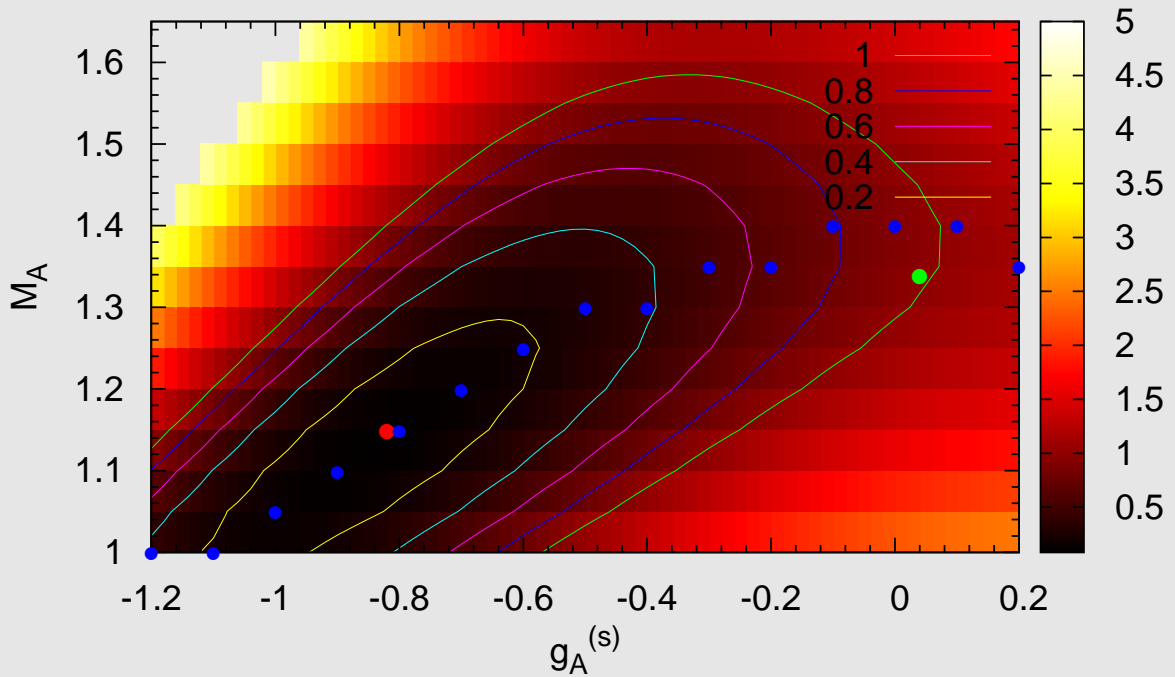


$\chi^2 / 21$  DOF; true CS with corr. coef.

SUSA;  $\chi^2_{\min}/\text{DOF}=0.88/21$ ;  $M_A=1.35\text{GeV}$ ;  $g_A^{(s)}=-0.49$



RMF;  $\chi^2_{\min}/\text{DOF}=1.85/21$ ;  $M_A=1.15\text{GeV}$ ;  $g_A^{(s)}=0.82$



# SUMMARY

- *The RIA/RMF describes in a reasonable way QE  $(e, e')$  data, satisfying scaling behavior and providing an asymmetric superscaling  $L$  function in accordance with data.*
- *Contrary to most NR/SR models (likewise RFG), RMF violates scaling of zeroth order, i.e.,  $f_T > f_L$ . This seems to be consistent with  $(e, e')$  data analysis.*
- *RMF applied to neutrino scattering also satisfies scaling/superscaling properties.*
- *RMF provides results in excellent agreement with SuSA/SuSAv2 approaches.*
- *Significant discrepancy with MiniBooNE data: **Important enhancement of 2p-2h effects***
- *RMF results (likewise SuSA/SuSAv2) in accordance with NOMAD and MinerVA data?*
- *Correlation between axial strangeness and axial mass in NC processes. Some discrepancy with MiniBooNE data: **role of 2p-2h?, strangeness? ...***

# COLLABORATION

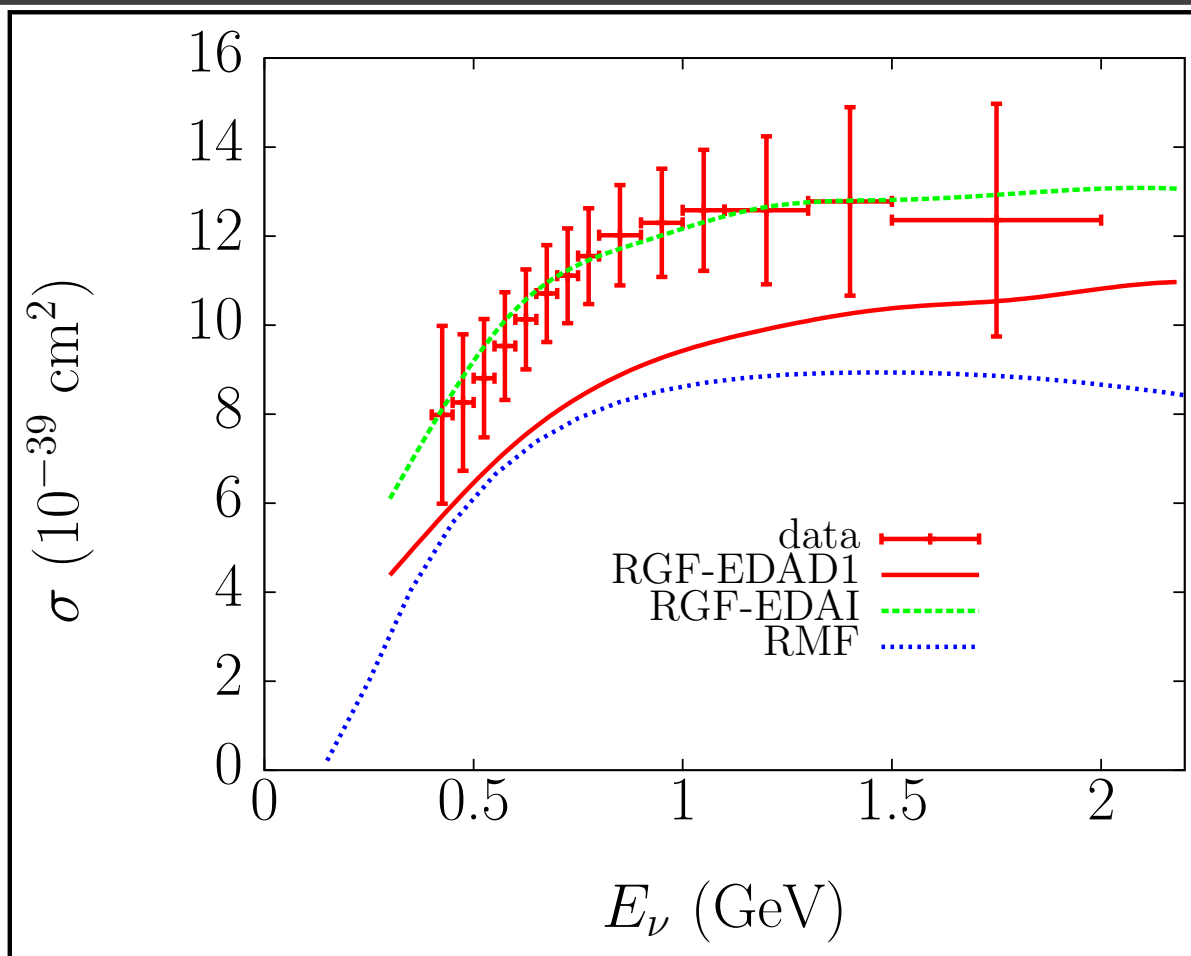
- **R. González-Jiménez, G. Megias** *Universidad de Sevilla*
- **J.E. Amaro**, *Universidad de Granada*
- **M.B. Barbaro, A. Molinari** *Università di Torino*
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**Juan Antonio Caballero (20/04/13)**

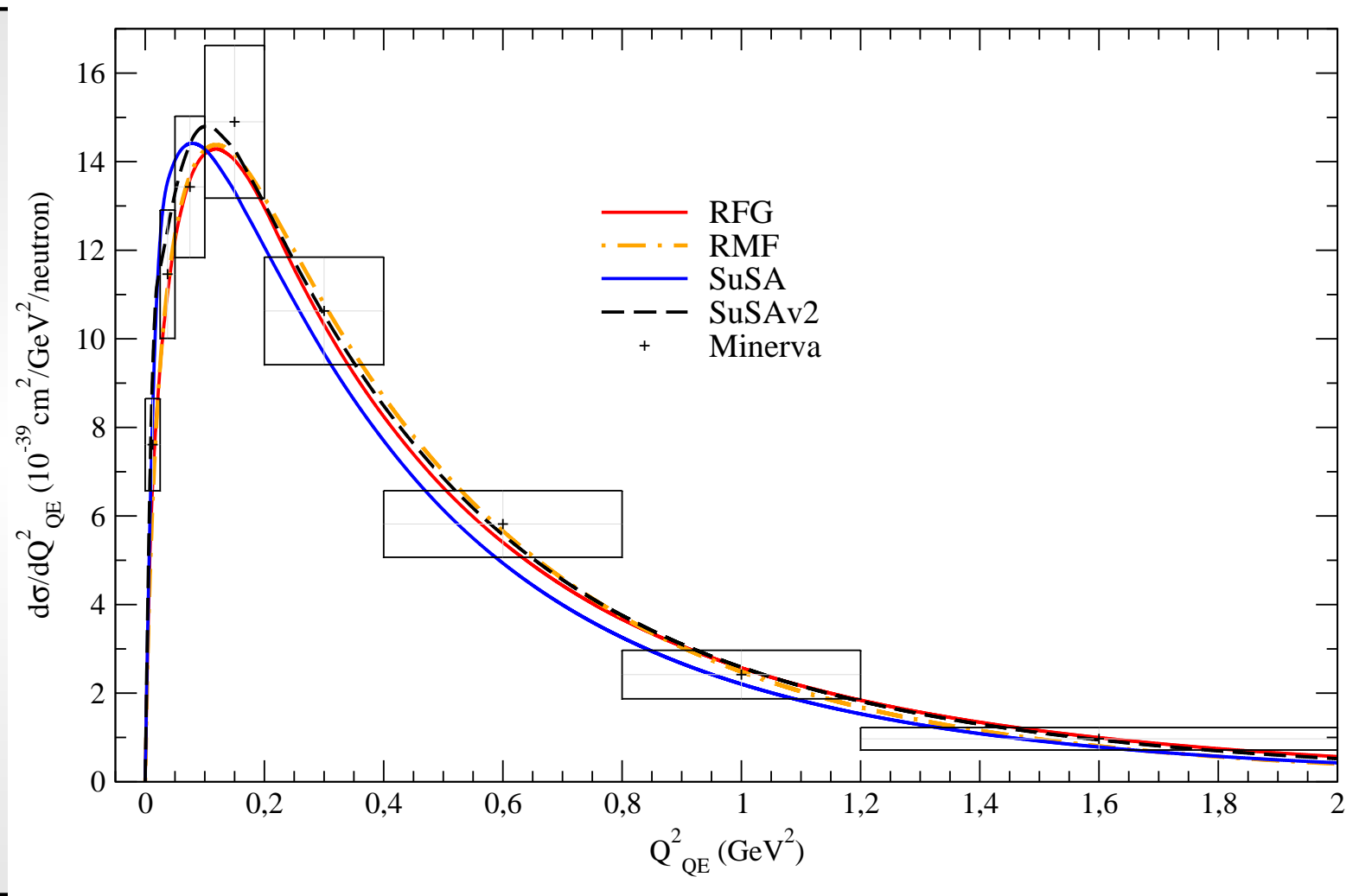
# Other Relativistic Approaches (Pavia Group)

Relativistic Green Function Approach (RGF): *FSI treated consistently in the exclusive & inclusive channels (same relativistic complex optical potential used in both cases, but flux conserved in the latter)*

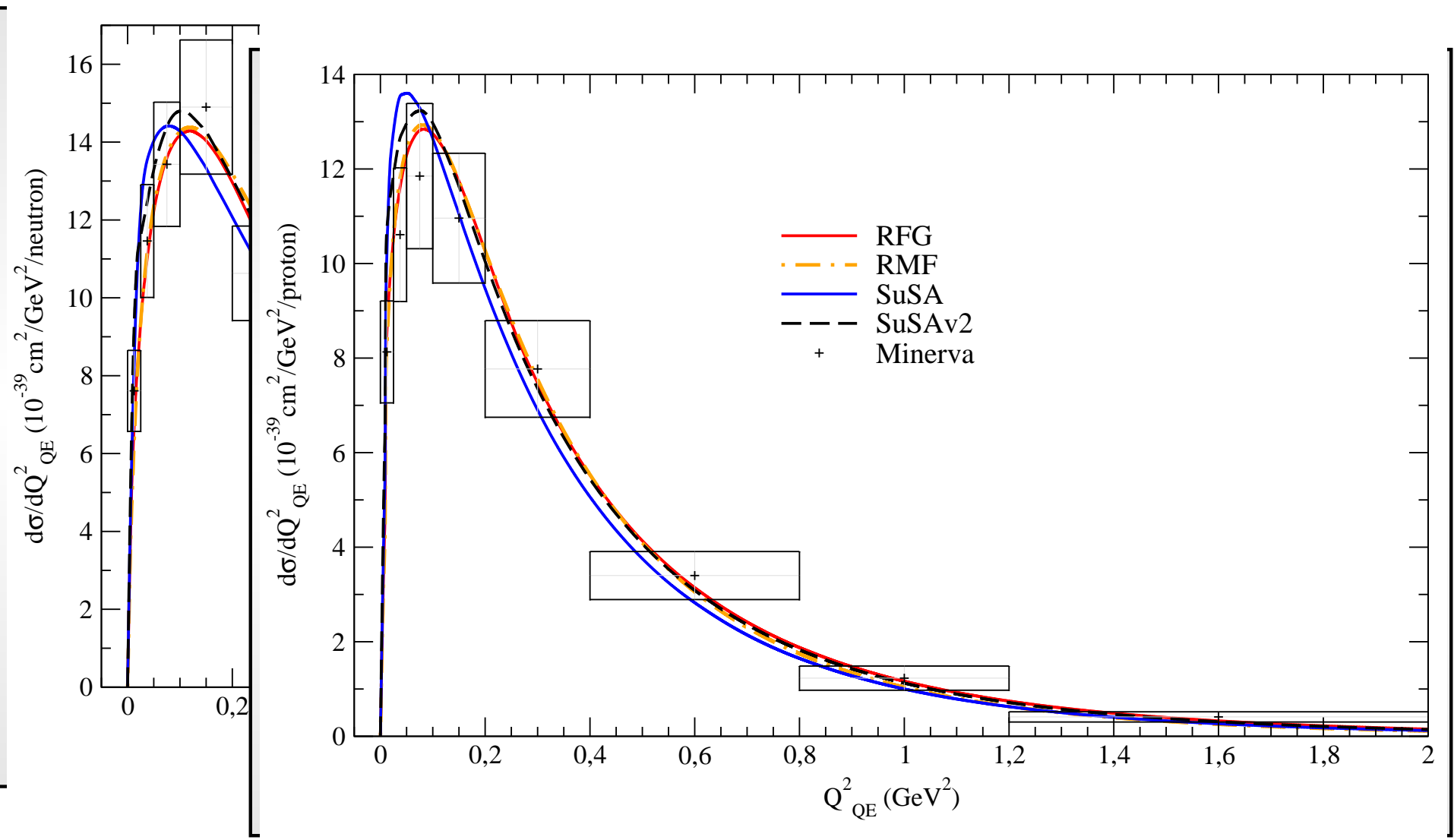
Phys. Rev. Lett. 107,  
172501 (2011)



# MINER $\nu$ A: SuSA vs RMF



# MINER $\nu$ A: SuSA vs RMF

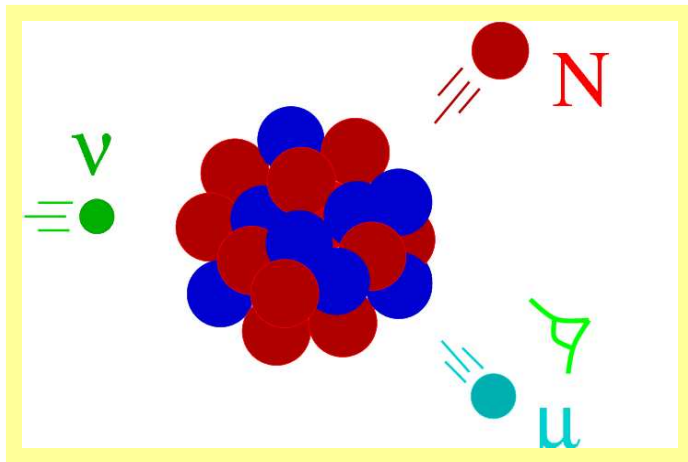


# NC vs CC neutrino-nucleus QE scattering

The dominant processes in the QE region are assumed to be:

## Charged-Current

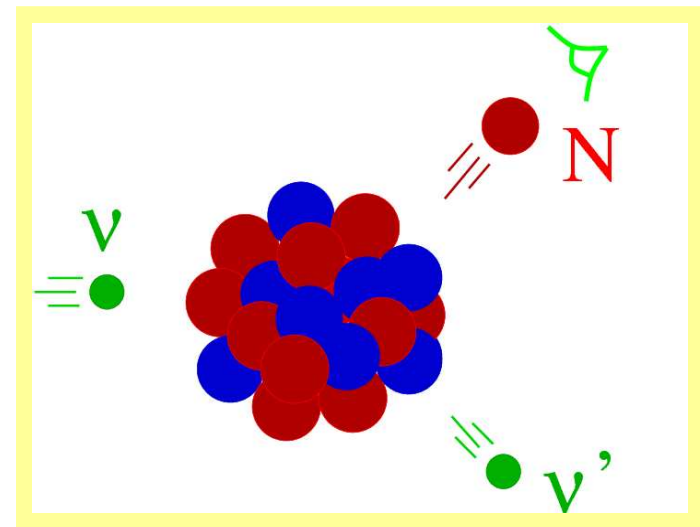
$$\nu + A \Rightarrow \mu + N + B$$



*Outgoing lepton detected, fixed  $Q^2$   
as in  $(e, e')$ : t-channel kinematics*

## Neutral-Current

$$\nu + A \Rightarrow \nu' + N + B$$

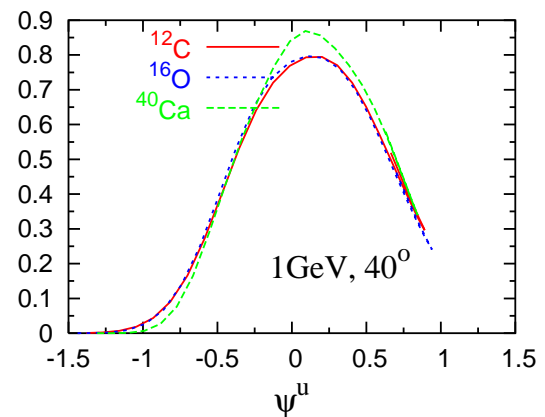
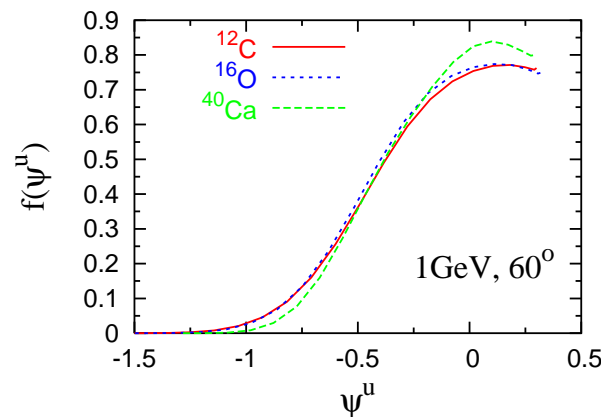
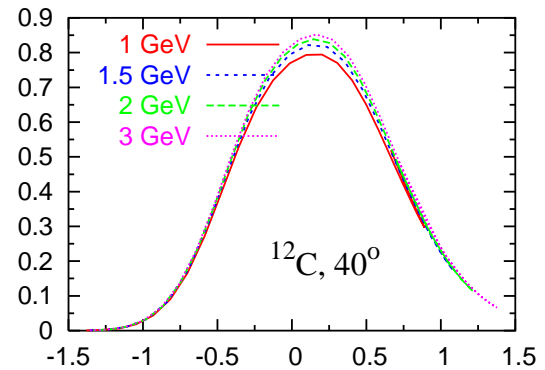
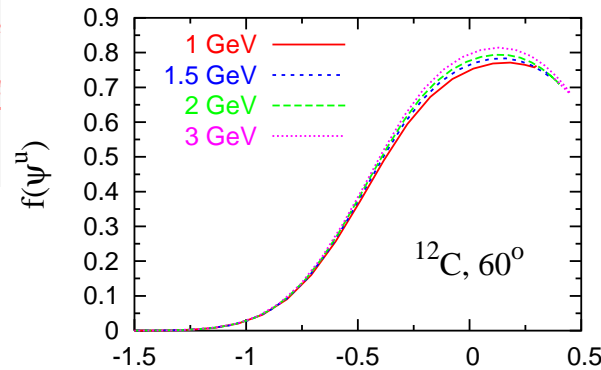


*Only the nucleon is detected,  $Q^2$  is not  
fixed: u-channel kinematics*

**Very different kinematics in both processes. Do they reveal different sensitivity to the nuclear dynamics underlying scaling?**

# Superscaling for NC in the RIA

$$f(\psi^{(u)}) = k_F \frac{\left[ \frac{d\sigma}{d\Omega_N dp_N} \right]_{NC}^{RLA}}{\bar{\sigma}_{sn}^{(u)}}$$



Superscaling also occurs at high degree within RPWIA for NC

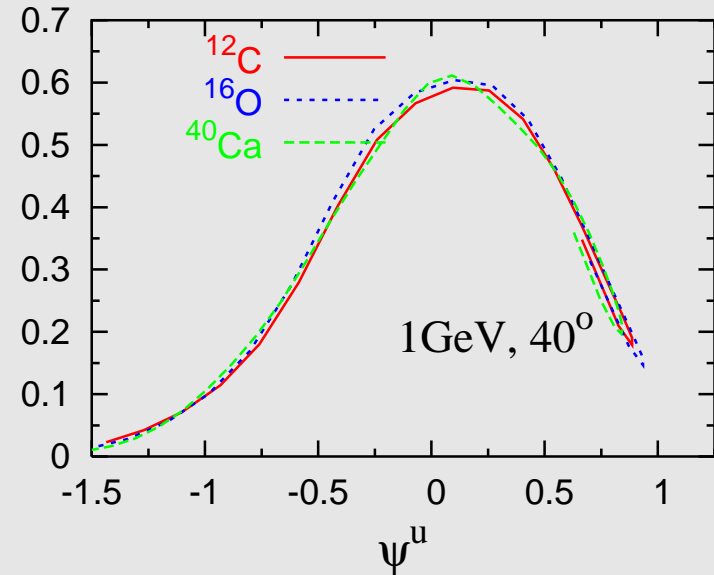
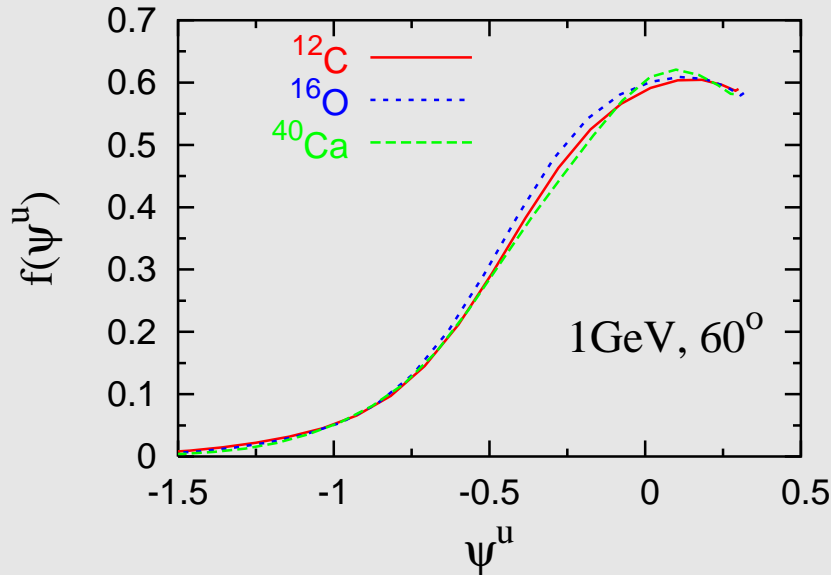
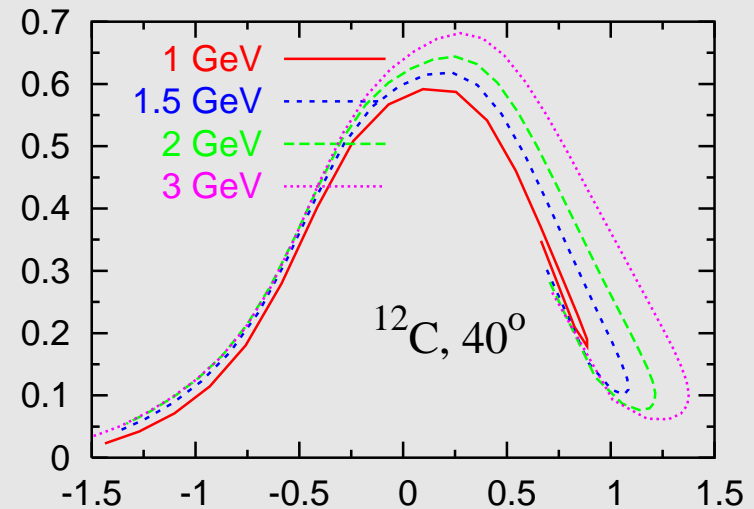
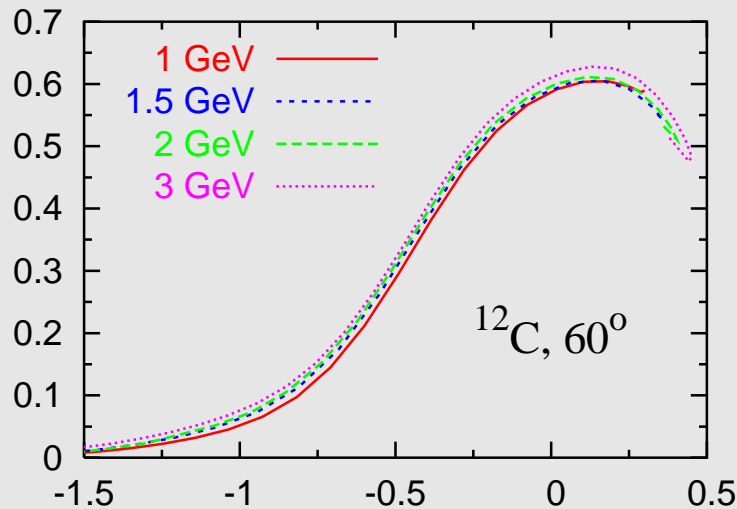
What happens when FSI are incorporated?



# Superscaling for NC in the RIA

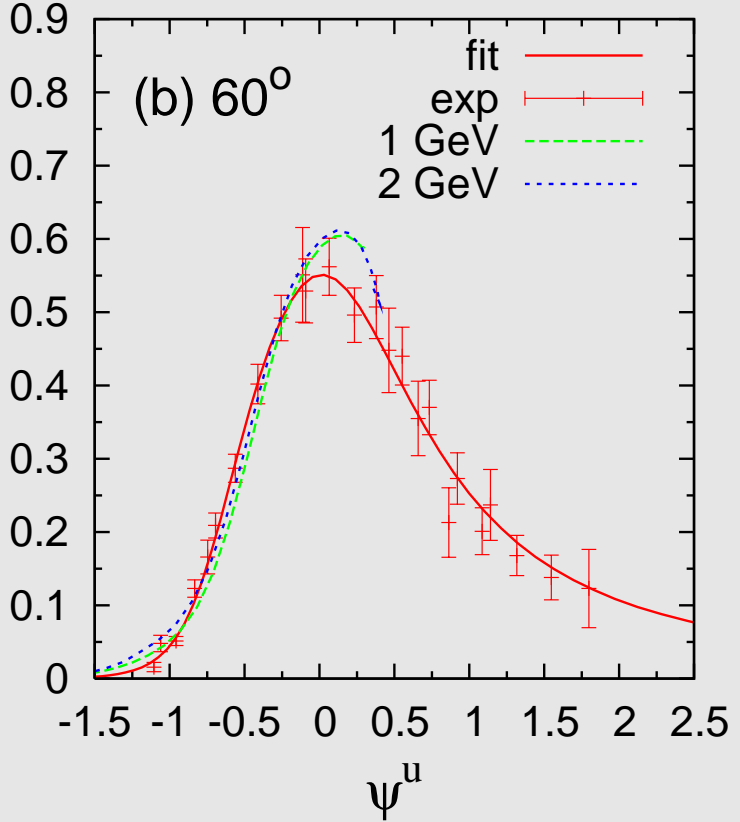
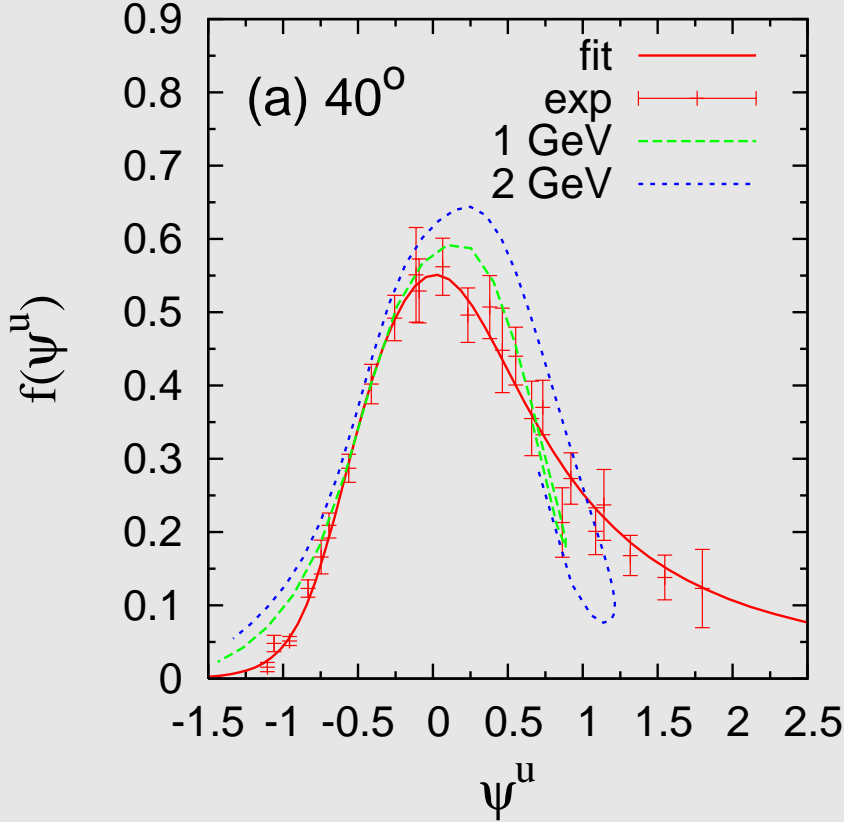
$f(\psi^u)$

$f(\psi^u)$

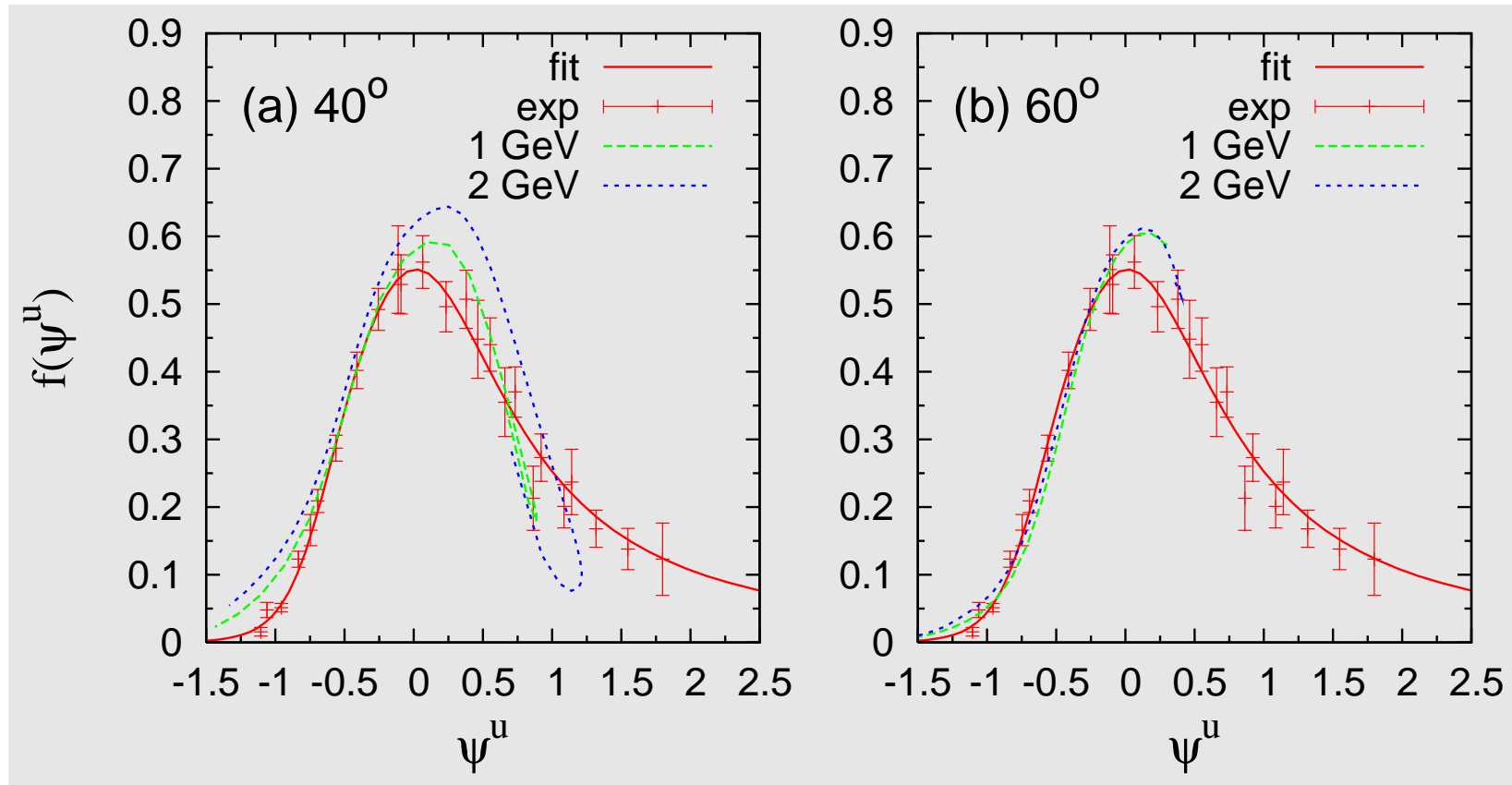


**What happens when FSI are incorporated?**

# Comparison with $f_{exp}(\psi)$ [from $(e, e')$ ]

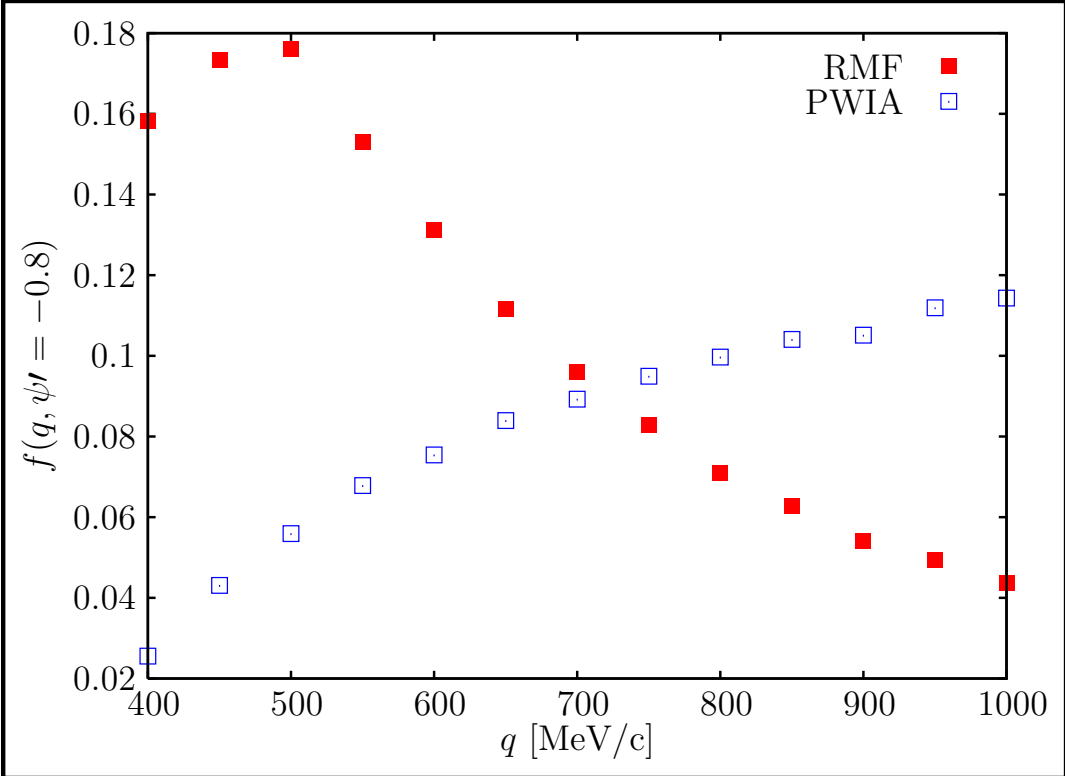
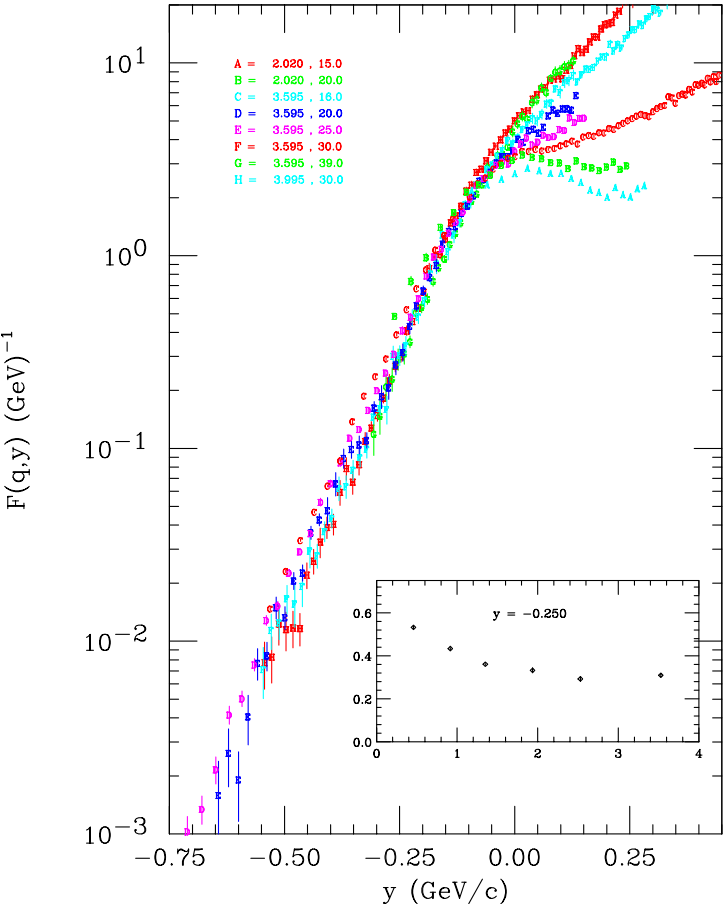


# Comparison with $f_{exp}(\psi)$ [from $(e, e')$ ]



- FSI break scaling of first kind for small scattering angles. On the contrary, for large values ( $\geq 60^\circ$ ), which give the largest contribution to the integrated cross section, scaling of first kind is highly fulfilled.
- Comparison with  $f_{exp}(\psi)$  suggests that a similar scaling function results valid for electrons (longitudinal), CC and NC neutrino reactions.

# Approach to scaling for fixed $\psi$ -values



Approach to scaling from above