A unified framework for short-range correlations in nuclear structure and reactions

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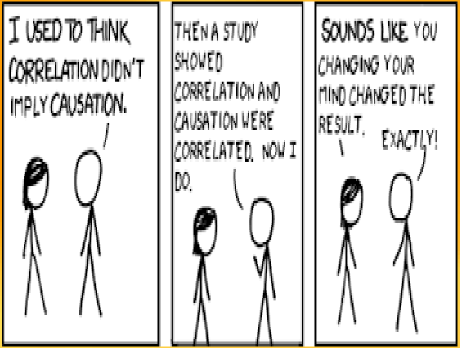
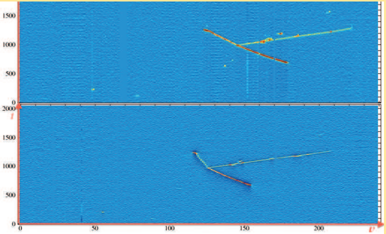
Talking about nuclear correlations



- Whole is different from the sum of the "parts"
- "Parts" can be effective degrees of freedom
- In nuclei: "Parts" are quasi-nucleons moving in a mean-field potential *(scheme dependent)*

- Momentum correlations: $P^{(2)}(\vec{p}_1, \vec{p}_2) \neq P^{(1)}(\vec{p}_1)\, P^{(1)}(\vec{p}_2)$
- Spatial correlations: $P^{(2)}(\vec{r}_1, \vec{r}_2) \neq P^{(1)}(\vec{r}_1)\, P^{(1)}(\vec{r}_2)$
  1. short-range: $P^{(2)}(\vec{r}_1, \vec{r}_2) \neq 0$ for $|\vec{r}_1 - \vec{r}_2| \approx R_N$ (nucleon radius)
  2. long-range: $P^{(2)}(\vec{r}_1, \vec{r}_2) \neq 0$ for $|\vec{r}_1 - \vec{r}_2| \approx R_A$ (nuclear radius)

Research goals: comprehensive picture of SRC



"hammer events" in $(\nu_\mu, \mu^- pp)$ (arXiv:1405.4261)

- Learn about SRC physics (nuclear structure AND reactions) in a unified framework
- Develop an approximate flexible method for computing nuclear momentum distributions
- Study the mass and isospin dependence of SRC
- Provide a unified framework to establish connections with measurable quantities that are sensitive to SRC
  1. Inclusive $A(e, e')$ at $x_B \gtrsim 1.5$
  2. Two-nucleon knockout: $A(e, e'pN)$, $A(\nu_\mu, \mu^- pp)$

Nuclear correlation operators (I)

- Shift complexity from wave functions to operators

$$\mid \Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\widehat{\mathcal{G}} \mid \Phi\rangle \qquad \text{with,} \qquad \mathcal{N} \equiv \langle\Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi\rangle$$

$\mid \Phi\rangle$ is an IPM single Slater determinant

- Nuclear correlation operator $\widehat{\mathcal{G}}$

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left(\prod_{i<j=1}^{A} \left[1 + \hat{l}\,(i,j) \right] \right) \,,$$

- Major source of correlations: central (Jastrow), tensor and spin-isospin

$$\hat{l}\,(i,j) = -g_c(r_{ij}) + f_{\sigma\tau}(r_{ij})\vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j + f_{t\tau}(r_{ij})\widehat{S}_{ij}\vec{\tau}_i \cdot \vec{\tau}_j \;.$$

Nuclear correlation operators (II)

- Expectation values between correlated states Ψ can be turned into expectation values between uncorrelated states Φ

$$\langle \Psi \mid \widehat{\Omega} \mid \Psi \rangle = \frac{1}{\mathcal{N}} \langle \Phi \mid \widehat{\Omega}^{\text{eff}} \mid \Phi \rangle$$

- "Conservation Law of Misery": $\widehat{\Omega}^{\text{eff}}$ is an A-body operator

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^{\dagger}\, \widehat{\Omega}\, \widehat{\mathcal{G}} = \left(\sum_{i<j=1}^{A} \left[1 - \hat{l}(i,j)\right] \right)^{\dagger} \widehat{\Omega} \left(\sum_{k<l=1}^{A} \left[1 - \hat{l}(k,l)\right] \right)$$

- Truncation procedure for short-distance phenomena:

K. Wilson's OPE: $\Psi^{\dagger}(\vec{R} - \frac{\vec{r}}{2})\Psi(\vec{R} + \frac{\vec{r}}{2}) \approx \sum_{n} c_n(\vec{r}) O_n(\vec{R}) \quad (\mid \vec{r} \mid \approx 0)$

Low-order correlation operator approximation (LCA)

- LCA: N-body operators receive SRC-induced $(N+1)$-body corrections

Norm $\mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}} \mid \Phi \rangle$: aggregated SRC effect

- LCA expansion of the norm $\mathcal{N}$

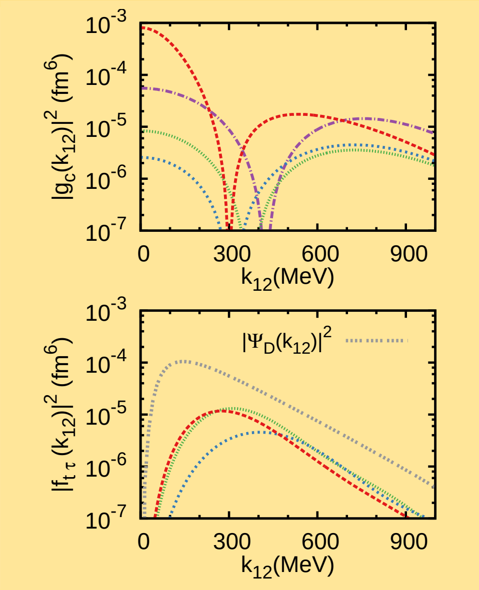
$$\mathcal{N} = 1 + \frac{2}{A} \sum_{\alpha<\beta} {}_{\text{nas}}\langle \alpha\beta \mid \hat{l}^{\dagger}(1,2) + \hat{l}^{\dagger}(1,2)\hat{l}(1,2) + \hat{l}(1,2) \mid \alpha\beta \rangle_{\text{nas}}.$$

  1. $\mid \alpha\beta\rangle_{\text{nas}}$: normalized and anti-symmetrized two-nucleon IPM-state
  2. $\sum_{\alpha<\beta}$ extends over all IPM states $\mid \alpha\rangle \equiv \mid n_\alpha l_\alpha j_\alpha m_{j_\alpha} t_\alpha \rangle$,
- $(\mathcal{N} - 1)$: measure for aggregated effect of SRC in the ground state
- Aggregated quantitative effect of SRC in A relative to ^{2}H

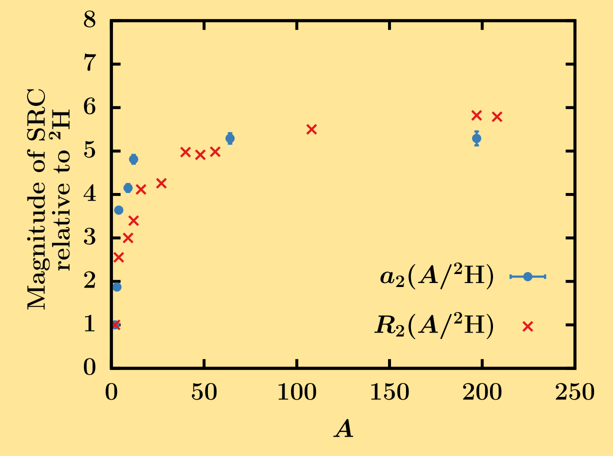
$$R_2(A/^2\text{H}) = \frac{\mathcal{N}(A) - 1}{\mathcal{N}(^2\text{H}) - 1} = \frac{\text{measure for SRC effect in } A}{\text{measure for SRC effect in } ^2\text{H}}.$$

- Input to the calculations for $R_2(A/^2\text{H})$:
  1. HO IPM states with $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$
  2. A-independent universal correlation functions $[g_c(r), f_{t\tau}(r), f_{\sigma\tau}(r)]$

Central, tensor, spin-isospin correlation function

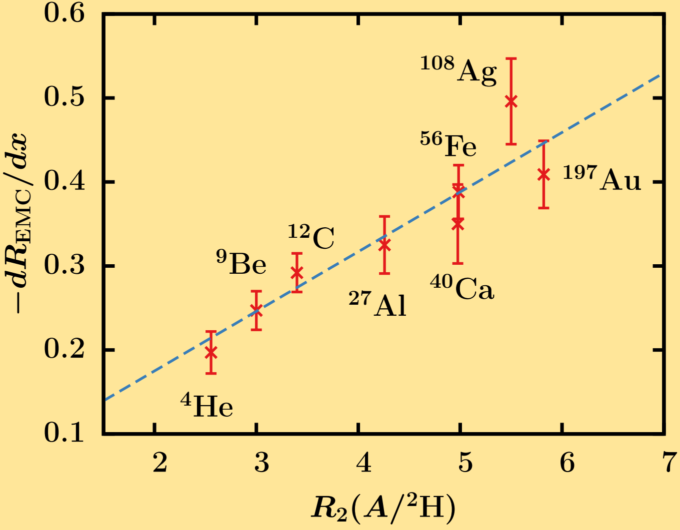


- the $g_C(k_{12})$ looks like the correlation function of a monoatomic classical liquid (reflects finite-size effects)
- the $g_c(k_{12})$ are ill constrained
- $|f_{t\tau}(k_{12})|^2$ is well constrained! (*D*-state deuteron wave function)
- $|f_{t\tau}(k_{12})|^2 \sim |\Psi_D(k_{12})|^2$
- very high relative pair momenta: central correlations
- moderate relative pair momenta: tensor correlations



1. $A \lesssim 40$: strong mass dependence in SRC effect
2. $A > 40$: soft mass dependence
3. SRC effect saturates for A large *(for large A aggregated SRC effect per nucleon is about $5\times$ larger than in ^2H)*

Magnitude of EMC effect versus $R_2(A/^2\mathrm{H})$



LCA can predict magnitude of EMC effect for any $A(N, Z) \geq 4$

Single-nucleon momentum distribution $n^{[1]}(p)$

- Probability to find a nucleon with momentum p

$$n^{[1]}(p) = \int \frac{d^2\Omega_p}{(2\pi)^3} \int d^3\vec{r}_1 \, d^3\vec{r}_1{}' \, d^{3(A-1)}\{\vec{r}_{2-A}\} e^{-i\vec{p}\cdot(\vec{r}_1{}'-\vec{r}_1)} \times \Psi^*(\vec{r}_1, \vec{r}_{2-A}) \Psi(\vec{r}_1{}', \vec{r}_{2-A}).$$

- Corresponding single-nucleon operator $\hat{n}_p$

$$\hat{n}_p = \frac{1}{A} \sum_{i=1}^{A} \int \frac{d^2\Omega_p}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{r}_i{}'-\vec{r}_i)} = \sum_{i=1}^{A} \hat{n}_p^{[1]}(i).$$

- Effective correlated operator $\hat{n}_p^{\mathrm{LCA}}$
  (SRC-induced corrections to IPM $\hat{n}_p$ are of two-body type)
- Normalization property $\int dp \, p^2 n^{[1]}(p) = 1$ can be preserved by evaluating $\mathcal{N}$ in LCA

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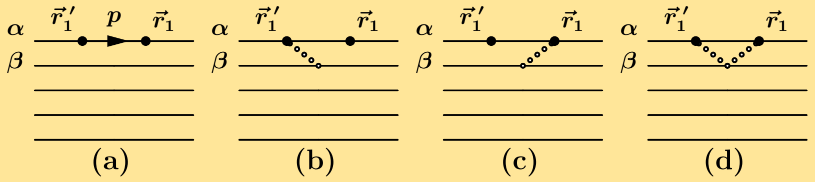
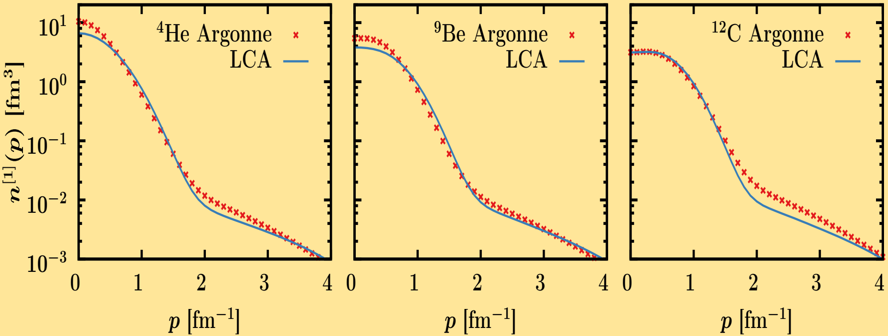


(a): IPM contribution
(b)-(d): SRC contributions

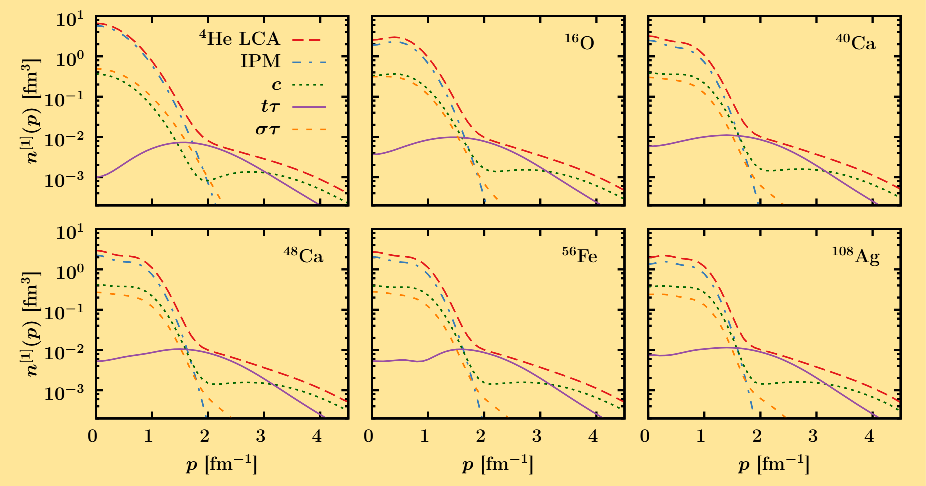
$n^{[1]}(p)$ for light nuclei: LCA (Ghent) vs QMC (Argonne)



QMC: PRC89(2014)024305 LCA: JPG42(2015)055104

1. $p \lesssim p_F = 1.25$ fm^{-1}: $n^{[1]}(p)$ is "Gaussian" (IPM PART)
2. $p \gtrsim p_F$: $n^{[1]}(p)$ has an "exponential" fat tail (CORRELATED PART)
3. fat tail in QMC and LCA are in reasonable agreement

Major source of correlated strength in $n^{[1]}(p)$?



1. $1.5 \lesssim p \lesssim 3$ fm^{-1} is dominated by tensor correlations
2. central correlations substantial at $p \gtrsim 3.5$ fm^{-1}

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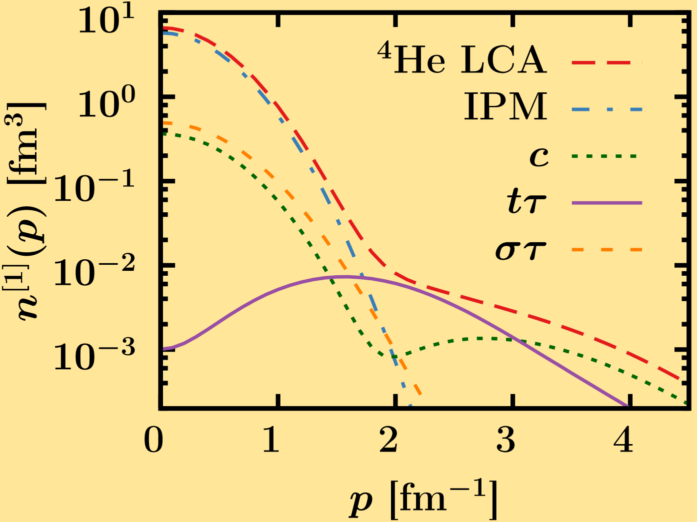

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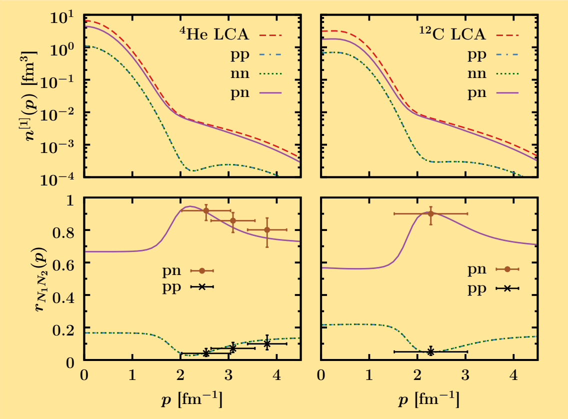
Isospin dependence of correlations: pp, nn and pn

$$n^{[1]}(p) \equiv n^{[1]}_{pp}(p) + n^{[1]}_{nn}(p) + n^{[1]}_{pn}(p) \qquad r_{N_1N_2}(p) \equiv n^{[1]}_{N_1N_2}(p)/n^{[1]}(p)$$

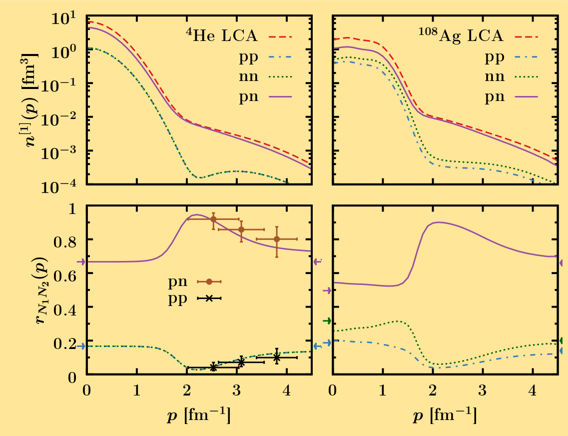


The fat tail is dominated by "pn" (momentum dependent)

- $r_{N_1N_2}(p)$: relative contribution of N_1N_2 pairs to $n^{[1]}(p)$ at p
- Naive IPM:
  $r_{pp} = \frac{Z(Z-1)}{A(A-1)}$,
  $r_{nn} = \frac{N(N-1)}{A(A-1)}$,
  $r_{pn} = \frac{2NZ}{A(A-1)}$.
- Data extracted from ^{4}He$(e,e'pp)/^4$He$(e,e'pn)$ (PRL 113, 022501) and $\frac{^{12}C(p,ppn)}{^{12}C(p,pp)}$ (Science 320, 1476) assuming that $r_{pp} \approx r_{nn}$

Isospin dependence of correlations: pp, nn and pn

$$n^{[1]}(p) \equiv n^{[1]}_{pp}(p) + n^{[1]}_{nn}(p) + n^{[1]}_{pn}(p) \qquad r_{N_1 N_2}(p) \equiv n^{[1]}_{N_1 N_2}(p)/n^{[1]}(p)$$



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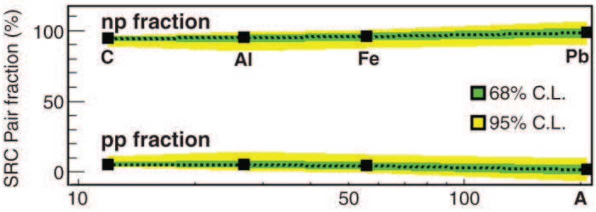
The fat tail is dominated by "pn" (momentum dependent)

Imbalanced strongly interacting Fermi systems

Sciencexpress

Momentum sharing in imbalanced Fermi systems

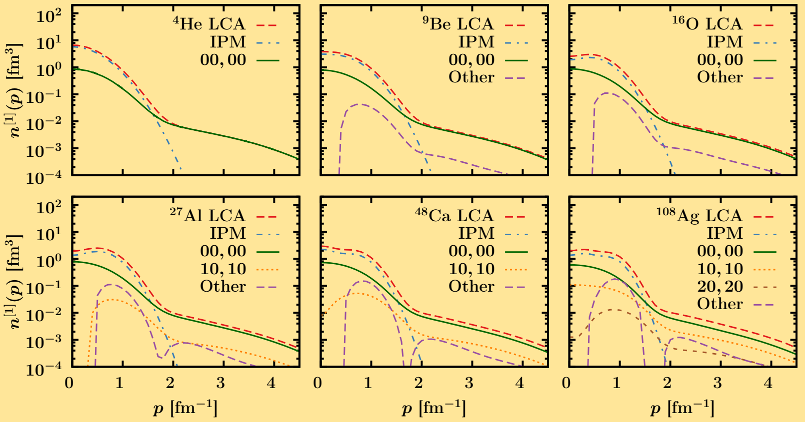
O. Hen,[1*] M. Sargsian,[2] L. B. Weinstein,[3] E. Piasetzky,[1] H. Hakobyan,[4,5] D. W. Higinbotham,[6] M



LCA predicts that ≈90% of correlated pairs is "pn", and ≈5% is"pp" (A independent)

Quantum numbers of SRC-susceptible IPM pairs?

$n^{[1],\mathrm{corr}}$ stems from correlation operators acting on IPM pairs.
What are relative quantum numbers (nl) of those IPM pairs?



$$\sum_{nl}\sum_{n'l'} n^{[1],\mathrm{corr}}_{nl,n'l'}(p) = n^{[1],\mathrm{corr}}(p)$$

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$n^{[1],\mathrm{corr}}$ stems from correlation operators acting on IPM pairs.
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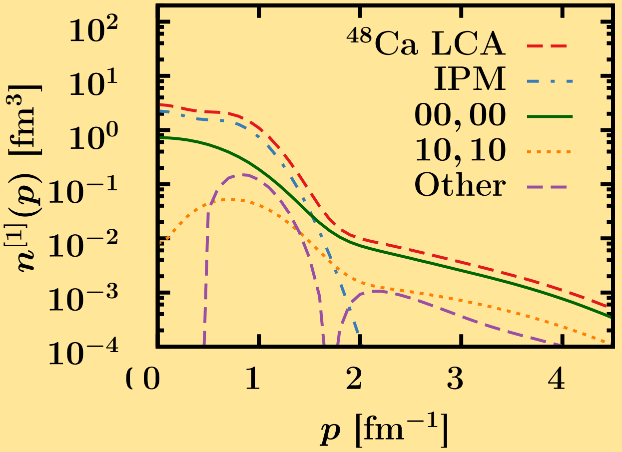


Major source of SRC: correlations acting on $(n = 0\ l = 0)$ IPM pairs

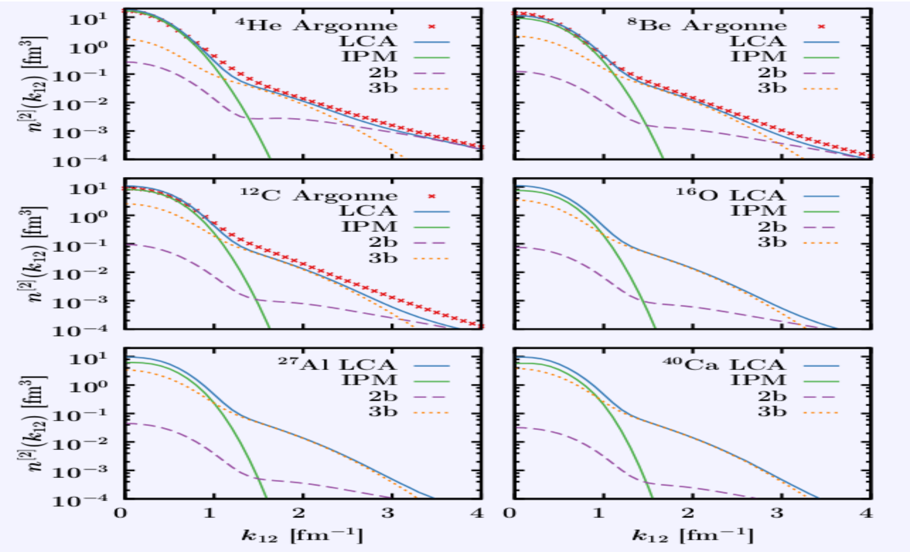
Two-nucleon momentum distribution (TNMD) $n^{[2]}\left(\vec{k}_{12}, \vec{P}_{12}\right)$

- Belongs to the class of four-point correlation functions (two tagged nucleons)
- Corresponding two-nucleon operator $\widehat{n}_{k_{12}P_{12}}$
- In LCA: effective correlated operator $\widehat{n}^{LCA}_{k_{12}P_{12}}$ (SRC-induced corrections are two-body ("2b") and three-body ("3b") operators)
- Relative TNMD: distribution of the relative momentum of the tagged pair

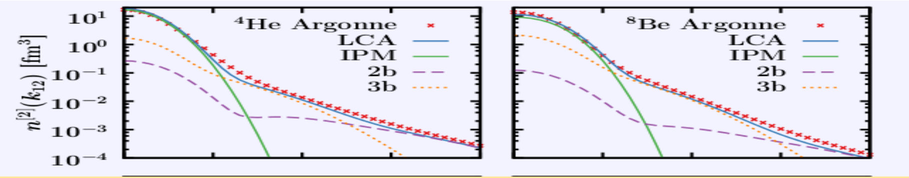
$$n^{[2]}(k_{12}) = \int d^3\vec{P}_{12} d^2\Omega_{k_{12}} n^{[2]}\left(\vec{k}_{12}, \vec{P}_{12}\right)$$

- **No direct connection between $n^{[2]}\left(\vec{k}_{12}, \vec{P}_{12}\right)$ and SRC dominated two-nucleon knockout cross sections**

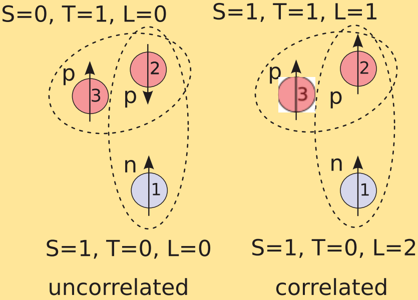
Relative TNMD: tail is dominated by “3-body” effects



Relative TNMD: tail is dominated by "3-body" effects

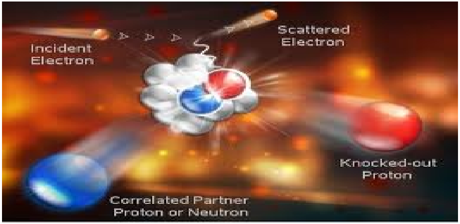


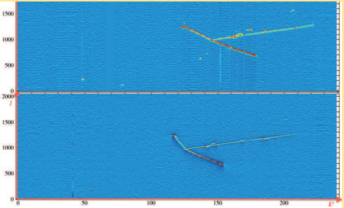
Correlations through the mediation of a third particle:



Feldmeier *et al.*, PRC 84 (2011), 054003

Exclusive two-nucleon knockout $A(e,e'NN),\ldots$



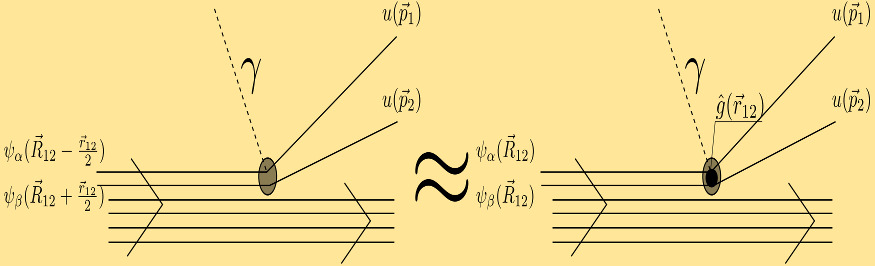


“hammer events” in $(\nu_\mu, \mu^- pp)$ (arXiv:1405.4261)

- The (virtual) photon-nucleon interaction is a one-body operator
- Two-nucleon knockout is the hallmark of SRC (one hits a nucleon and its correlated partner)
  1. $A(e,e'pN)$
  2. $A(\nu_\mu, \mu^- pp)$
  3. $A(p,pNN)$

Exclusive $A(e, e'NN)$ along the LCA lines

- SRC-prone IPM pairs: close-proximity ($n_{12} = 0, l_{12} = 0$) state
- The EXCLUSIVE $A(e, e'NN)$ cross sections can be factorized [PLB 383,1 (1996) and PRC 89, 024603 (2014)]



ZRA: Zero range approximation

Exclusive $A(e,e'NN)$ along the LCA lines

- SRC-prone IPM pairs: close-proximity ($n_{12}=0, l_{12}=0$) state
- The EXCLUSIVE $A(e,e'NN)$ cross sections can be factorized [PLB 383,1 (1996) and PRC 89, 024603 (2014)]

1. **$A(e,e'NN)$ cross section factorizes according to**

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e,e'NN) = K\sigma_{eNN}(k_+,k_-,q)F^{(D)}_{h_1,h_2}(P)$$

$F^{(D)}_{h_1,h_2}(P)$: FSI corrected conditional probability to find a dinucleon with c.m. momentum P in a relative ($n_{12}=0, l_{12}=0$) state

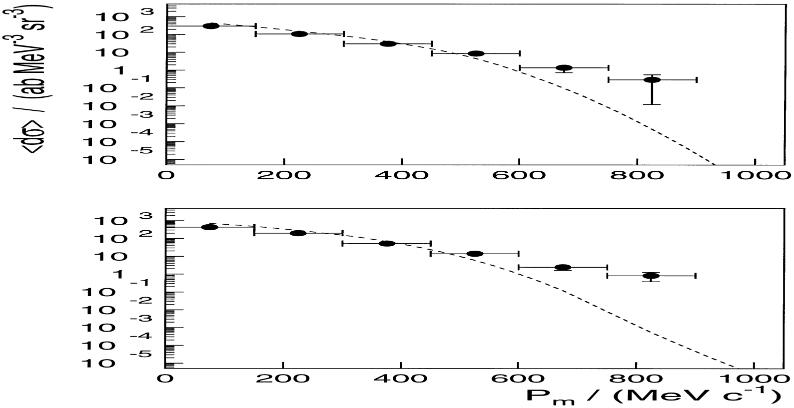
2. **A dependence of the $A(e,e'pp)$ cross sections is soft**

(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e,e'pp)}{{}^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left({}^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{{}^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$

3. **C.m. width of SRC susceptible pairs is “large” (in p-space)**

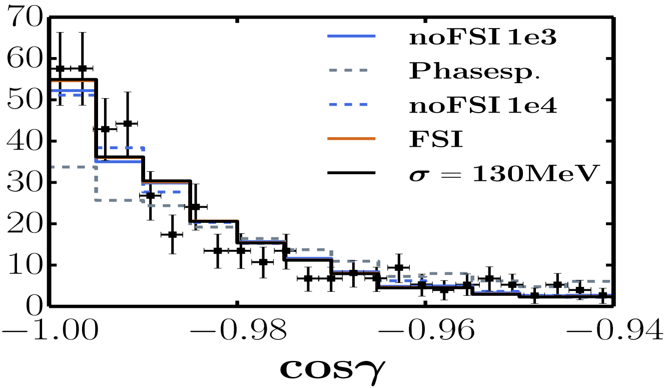
$^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)



For $P \lesssim 0.5$ GeV c.m. motion of correlated pairs in ^{12}C is mean-field like $\left(\exp \frac{-P^2}{2\sigma^2_{c.m}}\right)$! Data prove the proposed factorization in terms of $F^{(D)}_{h_1,h_2}(P)$.

$A(e, e'NN)$: Effect of the final-state interactions?

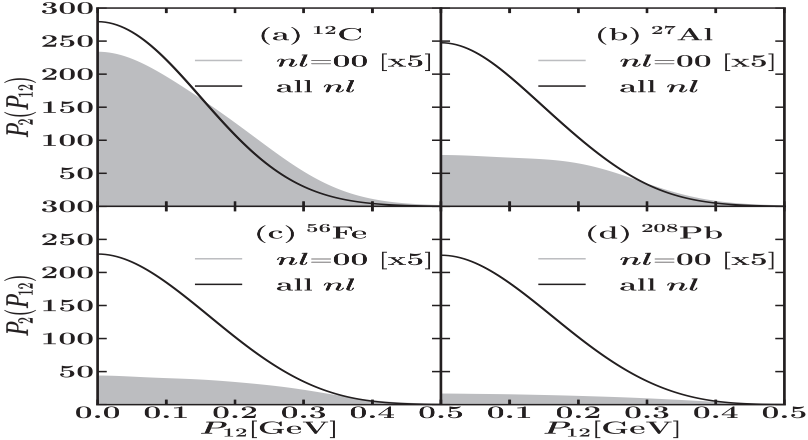
Opening-angle distribution of ^{4}He$(e, e'pp)$



1. **FSI (eikonal model) reduces the cross sections**
2. **FSI marginally affects the angular distributions**
   (FSI preserves factorization properties)

C.m. motion of correlated pp pairs

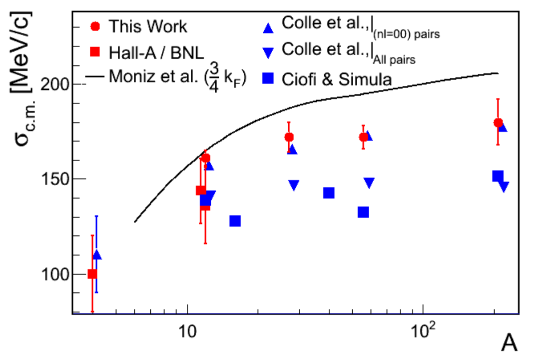
PHYSICAL REVIEW C **89**, 024603 (2014)



Width of c.m. distribution is a lever to discriminate between SRC-prone IPM pairs and the other IPM pairs

C.m. motion of correlated pp pairs

DATA IS PRELIMINARY! (COURTESY OF O. HEN AND E. PIASETZKY)



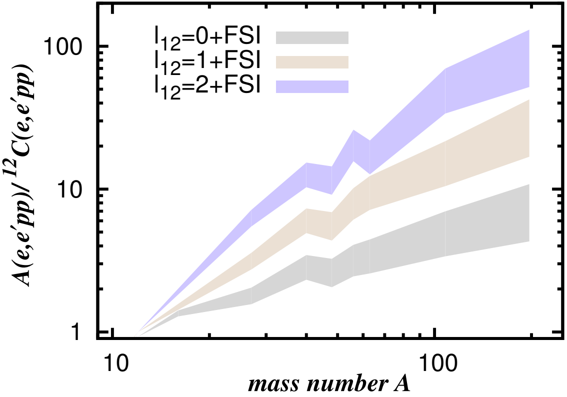
- Analysis of exclusive $A(e, e'pp)$ for ^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb by Data Mining Collaboration at Jefferson Lab
- Distribution of events against P is fairly Gaussian
- $\sigma_{c.m.}$: Gaussian widths from a fit to measured c.m. distributions

Mass dependence of the $A(e, e'pp)$ cross sections

PREDICTION: A dependence of $A(e, e'pp)$ c.s. is soft

(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e,e'pp)}{^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left(^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$



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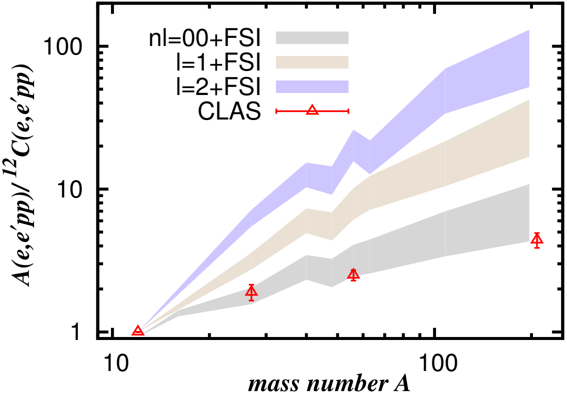
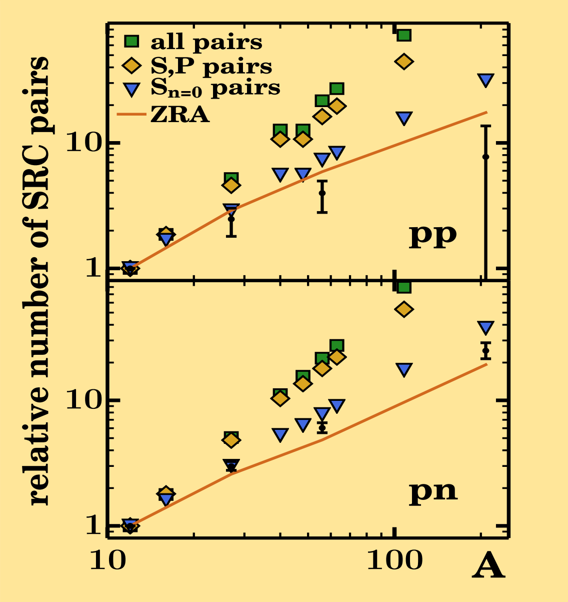


DATA COMPATIBLE WITH ABSORPTION ON $(n_{12} = 0, l_{12} = 0)$ PAIRS

arXiv:1503.06050

A dependence of number of pp and pn SRC pairs



- Analysis of $A(e, e'pp)$ and $A(e, e'p)$ (A=^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb) in "SRC" kinematics (Data Mining Collaboration @JLAB)
- FSI corrections applied to the data
- Reaction-model calculations in the large phase space: importance sampling
- Relative number of SRC pp-pairs and pn-pairs

CONCLUSIONS (I)

Stylized features of nuclear SRC: The mass and isospin dependence of the magnitude of the 2N and 3N correlations can be captured by some general principles

- LCA: efficient and realistic way of computing the SRC contributions to nuclear momentum distributions (NMD)
  1. Magnitude of EMC effect and $A(e,e')/D(e,e')$ scaling factor ($x_B \gtrsim 1.5$) can be predicted in LCA
  2. $A \leq 12$: LCA predictions for fat tails are in line with those of QMC
  3. LCA predictions for $\langle T_N \rangle$ and radii are "realistic" (consistency checks)
  4. Natural explanation for the universal behavior of the NMD tails
- Number of SRC-prone pairs in a nucleus $A(N,Z)$ is proportional with the number of pairs in a relative $(n_{12}=0, l_{12}=0)$ state

CONCLUSIONS (II)

- Insights from study of SRC contribution to NMD has implications for exclusive $A(e, e'NN)$:
  1. Scaling behavior of cross section ($\sim F(P)$) (CONFIRMED!)
  2. Very soft mass dependence of cross section (CONFIRMED!)
  3. Peculiar c.m. width of the SRC-susceptible pairs (CONFIRMED!)
- Aggregated effect of SRC: “universal” correlation operators acting on close-proximity pairs in a nodeless relative S state
- Generally applicable techniques for quantifying SRC: two-body effects in neutrino reactions (T. Van Cuyck’s talk), role of SRC in exotic forms of hadronic matter, ...
- SRC induced spatio-temporal fluctuations are measurable, are significant and are quantifiable (scales are set)


THANK YOU!

Selected publications

- J. Ryckebusch, M. Vanhalst, W. Cosyn
  "Stylized features of single-nucleon momentum distributions"
  arXiv:1405.3814 and Journal of Physics G **42** (2015) 055104.
- C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, L.B. Weinstein
  "Extracting the Mass Dependence and Quantum Numbers of Short-Range Correlated Pairs from $A(e,e'p)$ and $A(e,e'pp)$ Scattering"
  arXiv:1503.06050 and Physical Review C **92** (2015), 024604.
- C. Colle, W. Cosyn, J. Ryckebusch, M. Vanhalst
  "Factorization of electroinduced two-nucleon knockout reactions"
  arXiv:1311.1980 and Physical Review C **89** (2014), 024603.
- Maarten Vanhalst, Jan Ryckebusch, Wim Cosyn
  "Quantifying short-range correlations in nuclei"
  arXiv:1206.5151 and Physical Review C **86** (2012), 044619.
- Maarten Vanhalst, Wim Cosyn, Jan Ryckebusch
  "Counting the amount of correlated pairs in a nucleus"
  arXiv:1105.1038 and Physical Review C **84** (2011), 031302(R).