Electron Scattering off Nuclei With Neutron and Proton Excess

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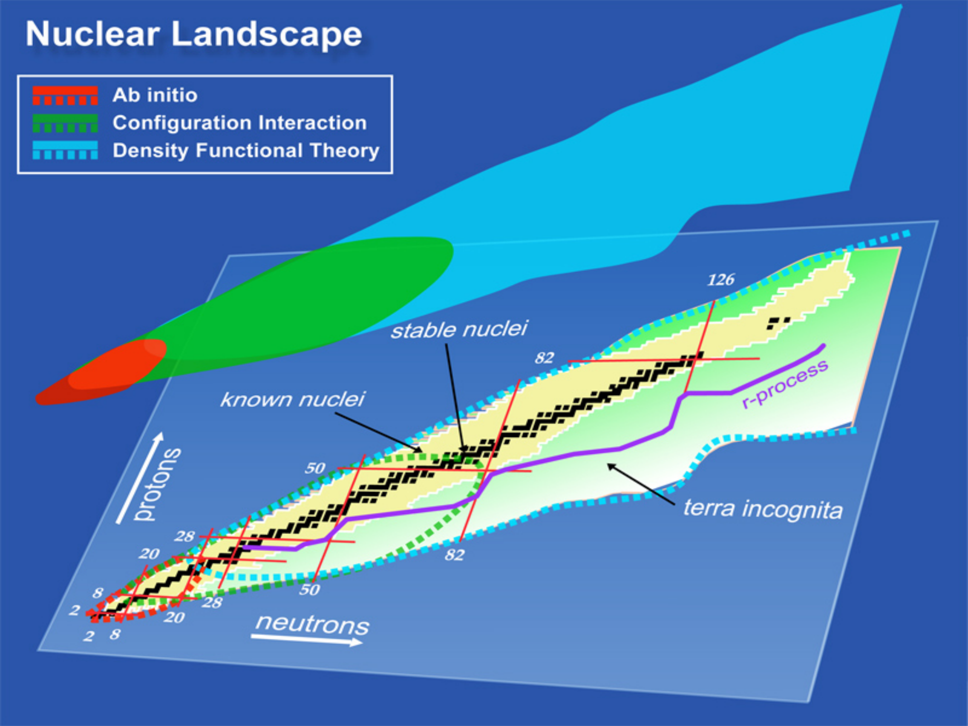
Nuclear Landscape
Ab initio
Configuration Interaction
Density Functional Theory
stable nuclei
known nuclei
126
82
50
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82
50
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8
2
r-process
terra incognita
protons
neutrons

PURPOSE

We study the evolution of the scattering observables for electron scattering on isotopic and isotonic chains

Isotopes

$Z = 8 : \quad {}^{14}\mathrm{O} - {}^{28}\mathrm{O}$

$Z = 20 : \quad {}^{36}\mathrm{Ca} - {}^{56}\mathrm{Ca}$

Phys. Rev. C **87**, 054620 (2013)

Isotones

$N = 14 : \quad {}^{40}\mathrm{Mg} - {}^{56}\mathrm{Ni}$

$N = 20 : \quad {}^{28}\mathrm{O} - {}^{46}\mathrm{Fe}$

$N = 28 : \quad {}^{22}\mathrm{O} - {}^{34}\mathrm{Ca}$

Phys. Rev. C **89**, 034604 (2014)

WHY ELECTRON SCATTERING?

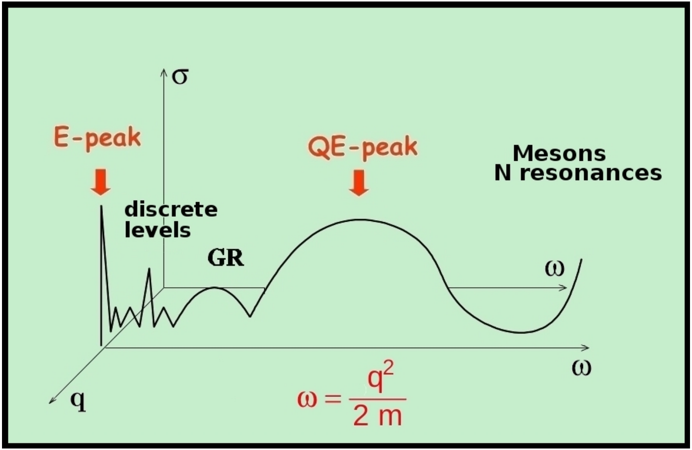
The key element for understanding the structure and dynamics of hadronic matter is its response to an external probe

⇓

The electromagnetic probe is the best reliable tool to investigate the nuclear response

1. Validity of the Born approximation
2. Explore the whole target volume
3. The nuclear response can be mapped as a function of its excitation energy

THE NUCLEAR RESPONSE



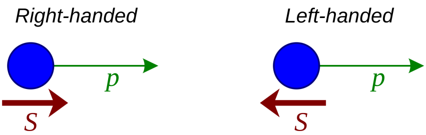
Elastic scattering: global properties
Quasi-elastic scattering: single-particle properties

STUDIED PROCESSES

1. Elastic scattering

a) Unpolarized incident electrons

b) Longitudinally polarized incident electrons



2. Inclusive quasi-elastic scattering (e, e')

The **width** of the QE peak can give a direct measurement of the average momentum of nucleons in nuclei

OUTLINE

1. Covariant Density Functional Theory
2. Elastic Scattering
3. Quasi-Elastic Scattering

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1. Covariant Density Functional Theory

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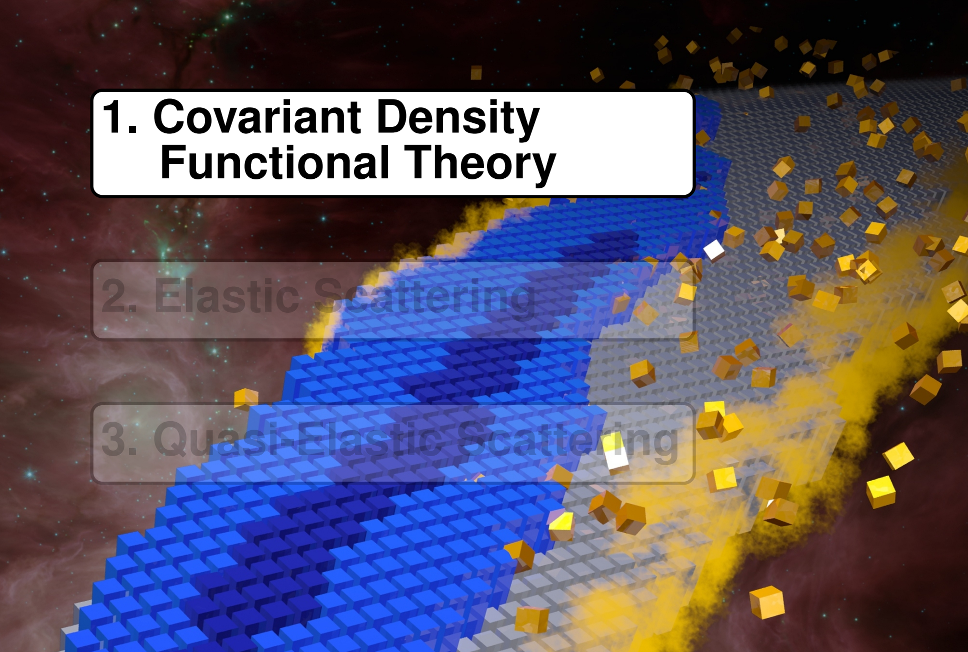
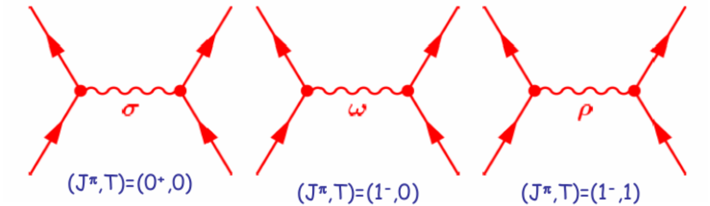


COVARIANT DENSITY FUNCTIONAL THEORY

Quantum Hadrodynamics (QHD)

Effective theory of nuclear structure

The nucleus is described as a system of Dirac nucleons coupled to the exchange mesons and the electromagnetic field through an effective Lagrangian



$$S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r}) \qquad V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau}\vec{\rho}(\mathbf{r}) + eA(\mathbf{r})$$

Sigma-meson: attractive scalar field

Omega-meson: short-range repulsive

Rho-meson: isovector field

COVARIANT DENSITY FUNCTIONAL THEORY

Relativistic Hartree-Bogoliubov model

$$\begin{pmatrix} \hat{h} - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h} + m + \lambda \end{pmatrix} \begin{pmatrix} U(r) \\ V(r) \end{pmatrix} = E \begin{pmatrix} U(r) \\ V(r) \end{pmatrix}$$

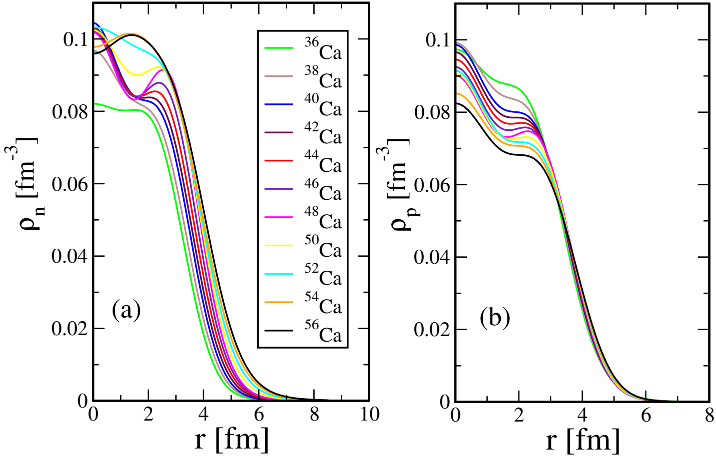
Consistent treatment of mean-field and pairing interactions

Density-dependent coupling constants

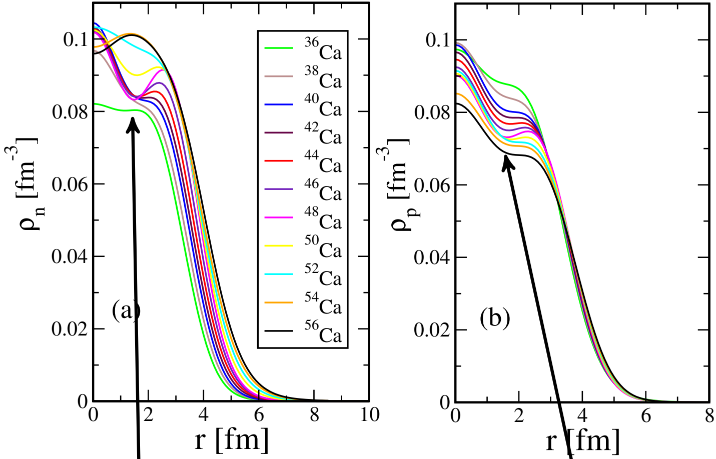
$$g_i(\rho) = g_i(\rho_{\text{sat}})\, f_i(x) \qquad f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \qquad i = \sigma, \omega$$

$$g_\rho(\rho) = g_\rho(\rho_{\text{sat}})\, f_\rho(x) \qquad f_\rho(x) = \exp[-a_\rho(x - 1)] \qquad x = \frac{\rho}{\rho_{\text{sat}}}$$

COVARIANT DENSITY FUNCTIONAL THEORY



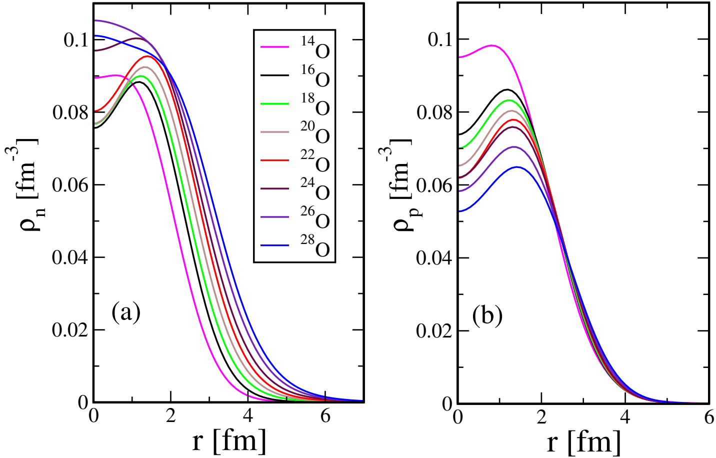
COVARIANT DENSITY FUNCTIONAL THEORY



Pronounced shell effects of neutron density profiles in the nuclear interior

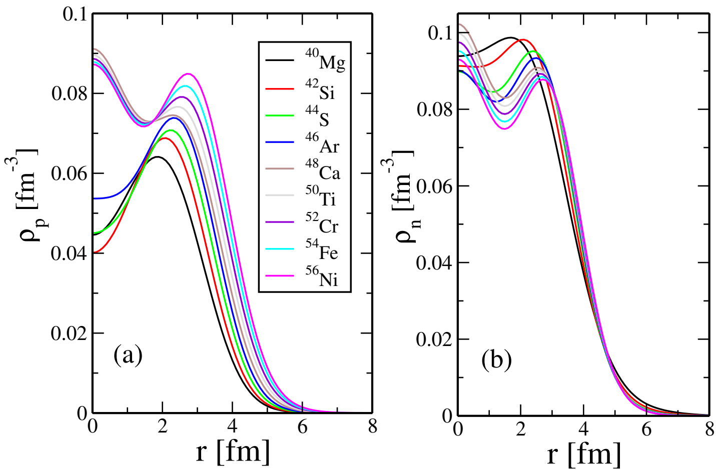
Decrease of proton densities in the nuclear interior

COVARIANT DENSITY FUNCTIONAL THEORY



Similar results are found for the oxygen chain

COVARIANT DENSITY FUNCTIONAL THEORY

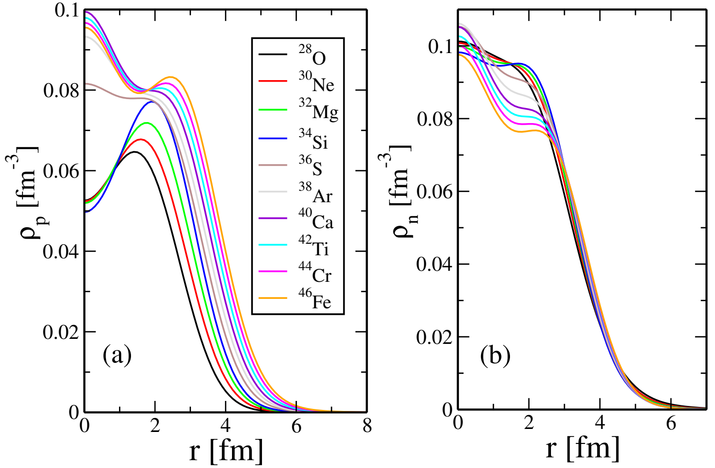


Proton-deficient nuclei ^{40}Mg - ^{46}Ar

Stable nuclei ^{48}Ca - ^{54}Fe

Proton-rich nucleus ^{56}Ni

COVARIANT DENSITY FUNCTIONAL THEORY



Proton-deficient nuclei

^{28}O - ^{34}Si

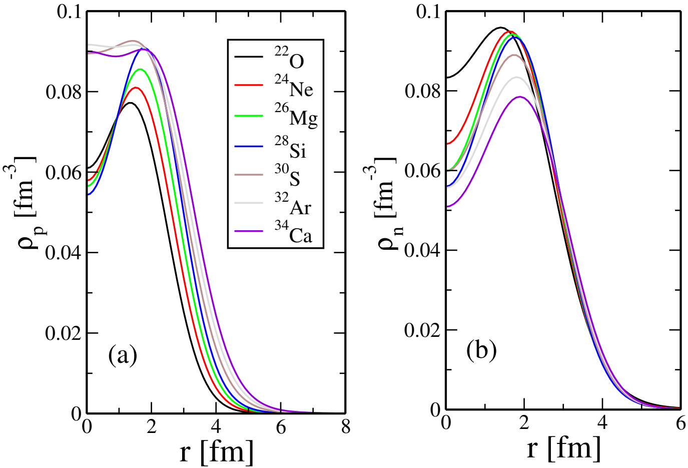
Stable nuclei

^{36}S - ^{40}Ca

Proton-rich nuclei

^{42}Ti - ^{46}Fe

COVARIANT DENSITY FUNCTIONAL THEORY



Proton-deficient nuclei ^{22}O - ^{26}Mg

Stable nucleus ^{28}Si

Proton-rich nuclei ^{30}S - ^{34}Ca

OUTLINE

1. Covariant Density Functional Theory

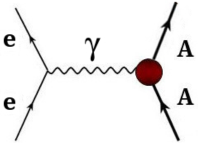
2. Elastic Scattering

3. Quasi-Elastic Scattering

ELASTIC ELECTRON SCATTERING

PLANE-WAVE BORN APPROXIMATION (PWBA)

One-photon exchange



The effects of the nuclear Coulomb field on incoming and outgoing electrons are neglected

Differential cross section

$$\frac{d\sigma}{d\Omega} = \sigma_{\mathrm{M}} \left| F_p(q) \right|^2$$

Relativistic Mott cross section

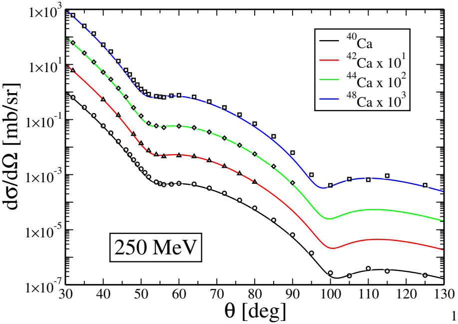
$$\sigma_{\mathrm{M}}(\theta) = \left(\frac{Ze^2}{2E} \right)^2 \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}$$

Charge form factor

$$F_p(q) = \int d^3r \, j_0(qr) \, \boxed{\rho_{ch}(r)}$$

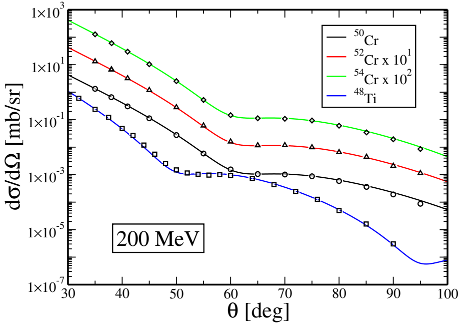
Nuclear charge density distribution

ELASTIC ELECTRON SCATTERING

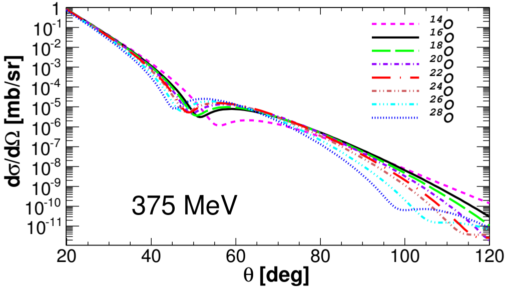


Same behavior as PWBA when Coulomb distortion is included

Distorted-Wave Born Approximation (DWBA)

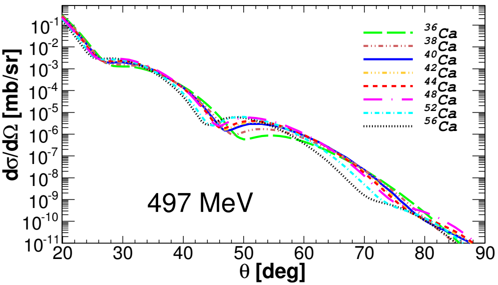


ELASTIC ELECTRON SCATTERING

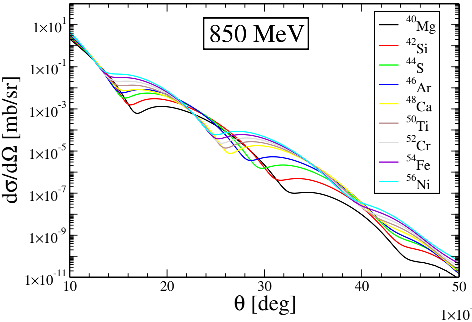


Theoretical results for isotopic chains

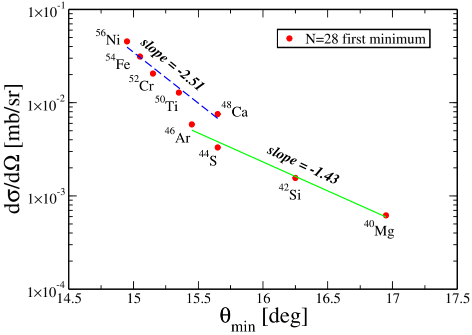
Shift of the minima toward smaller angles



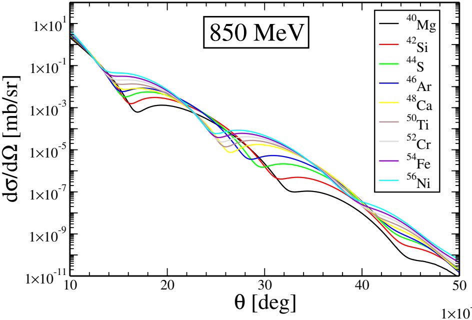
ELASTIC ELECTRON SCATTERING



Theoretical results
for N=28 chain

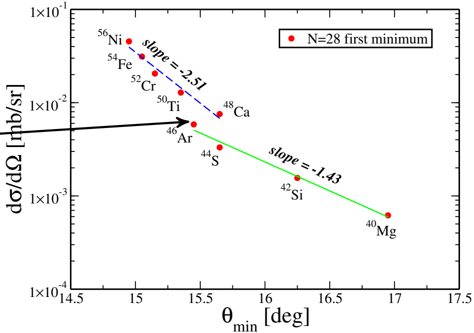


ELASTIC ELECTRON SCATTERING

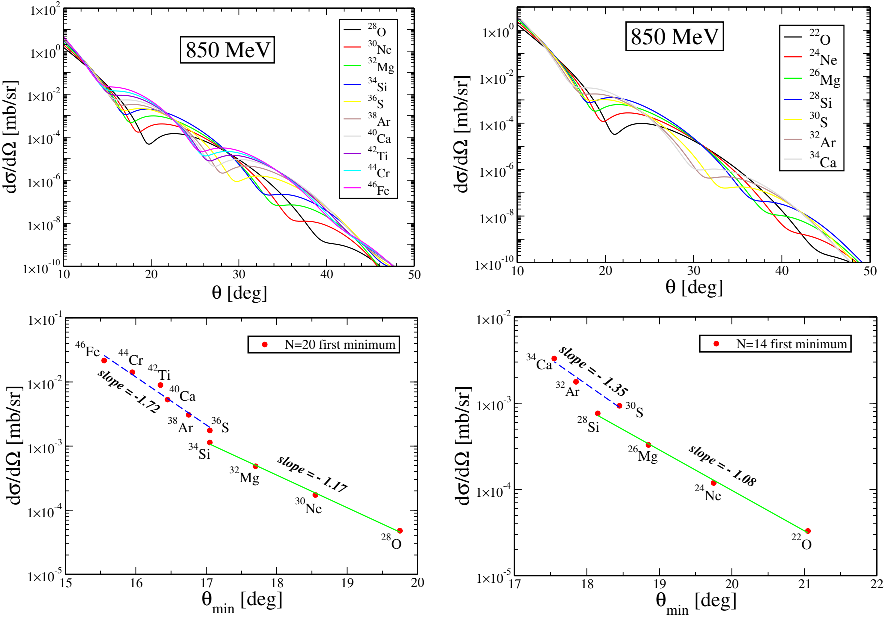


Theoretical results
for N=28 chain

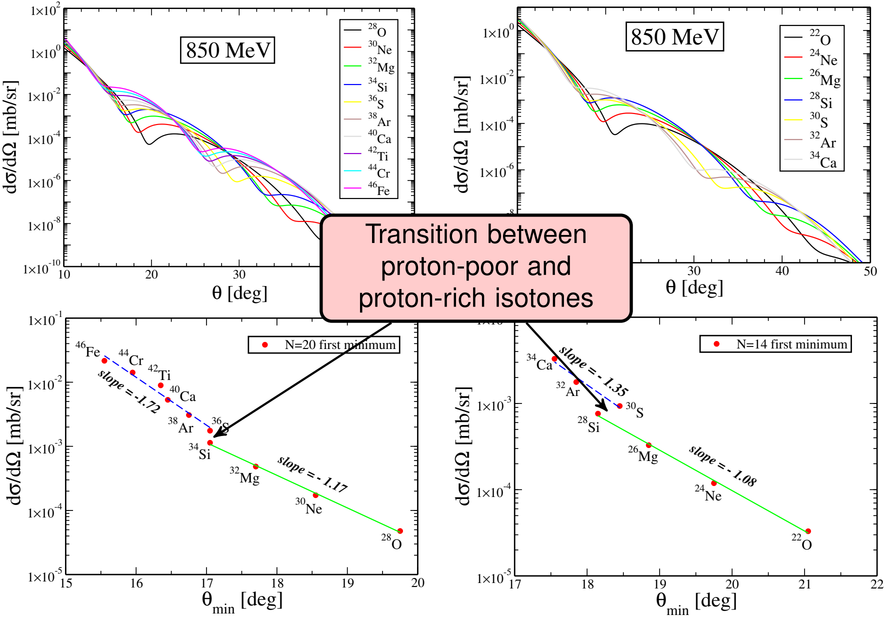
Transition between proton-poor and proton-rich isotones



ELASTIC ELECTRON SCATTERING

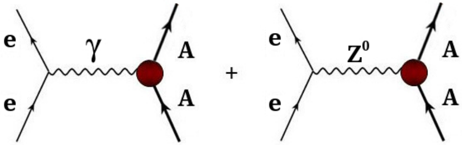


ELASTIC ELECTRON SCATTERING



PARITY-VIOLATING ELASTIC SCATTERING

One-photon + Z^0-boson exchange



Elastic scattering between **longitudinally polarized** incoming electrons and a target nucleus

Dirac equation

$$[\boldsymbol{\alpha}\cdot\mathbf{p}+U_{\pm}(r)]\Psi_{\pm} = E\Psi_{\pm}$$

Total potential

$$U(r)_{\pm} = V(r)\pm\gamma_5\, A(r)$$

Axial potential

$$A(r) = \frac{G_F}{2\sqrt{2}}\,\rho_W(r)$$

Weak charge density

$$\rho_W(r) = \int \mathrm{d}\boldsymbol{r}'\, G_E\left(|\mathbf{r}-\mathbf{r}'|\right)\times\left[-\rho_n(r') + (1-4\sin^2\Theta_W)\rho_p(r')\right]$$

Computation of two different cross sections

$\frac{d\sigma_-}{d\Omega}$: Obtained with left-handed electrons

$\frac{d\sigma_+}{d\Omega}$: Obtained with right-handed electrons

PARITY-VIOLATING ELASTIC SCATTERING

Parity-violating asymmetry

$$A_{pv} = \frac{\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}}{\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}}$$

Free from uncertainties of strong interactions

Powerful tool to measure the neutron density distribution inside nuclei

Parity-violating asymmetry in Born approximation

$$A_{pv} = \frac{G_F\, Q^2}{4\sqrt{2}\,\pi\alpha}\left[4\sin^2\Theta_W - 1 + \frac{F_n(q)}{F_p(q)}\right]$$

Form factors

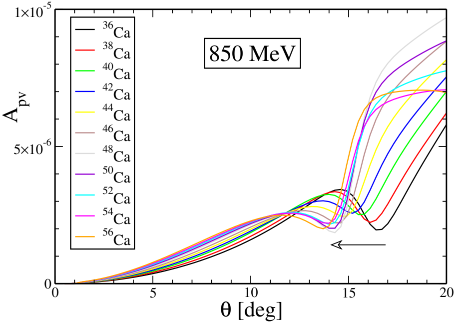
$$F_n(q) = \int \mathrm{d}^3 r\; j_0(qr)\; \rho_n(r)$$

$$F_p(q) = \int \mathrm{d}^3 r\; j_0(qr)\; \rho_p(r)$$

Neutron density distribution inside the nucleus

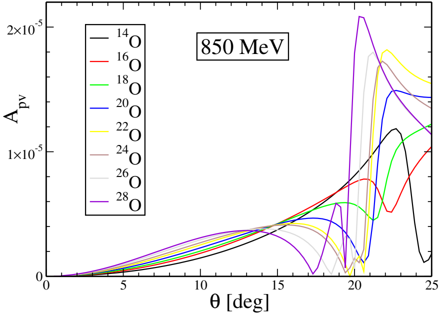
Proton density distribution

PARITY-VIOLATING ELASTIC SCATTERING



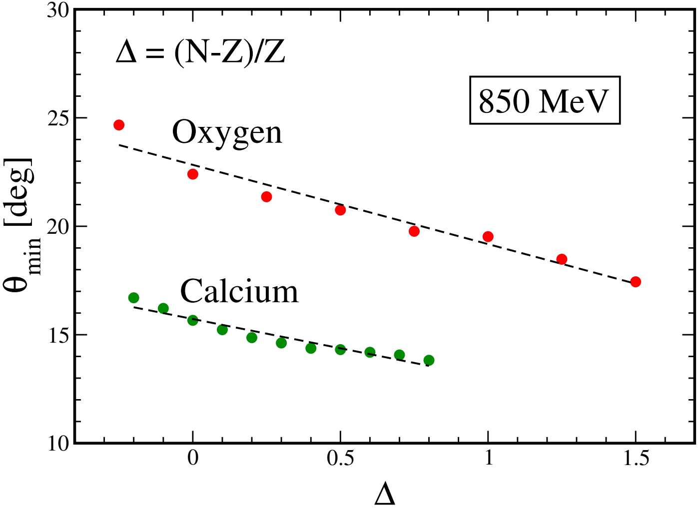
Coulomb distortion effects are included

Shift of the minima toward smaller angles



PARITY-VIOLATING ELASTIC SCATTERING

Linear correlation between θ_{min} of A_{pv}
and the neutron excess Δ



OUTLINE

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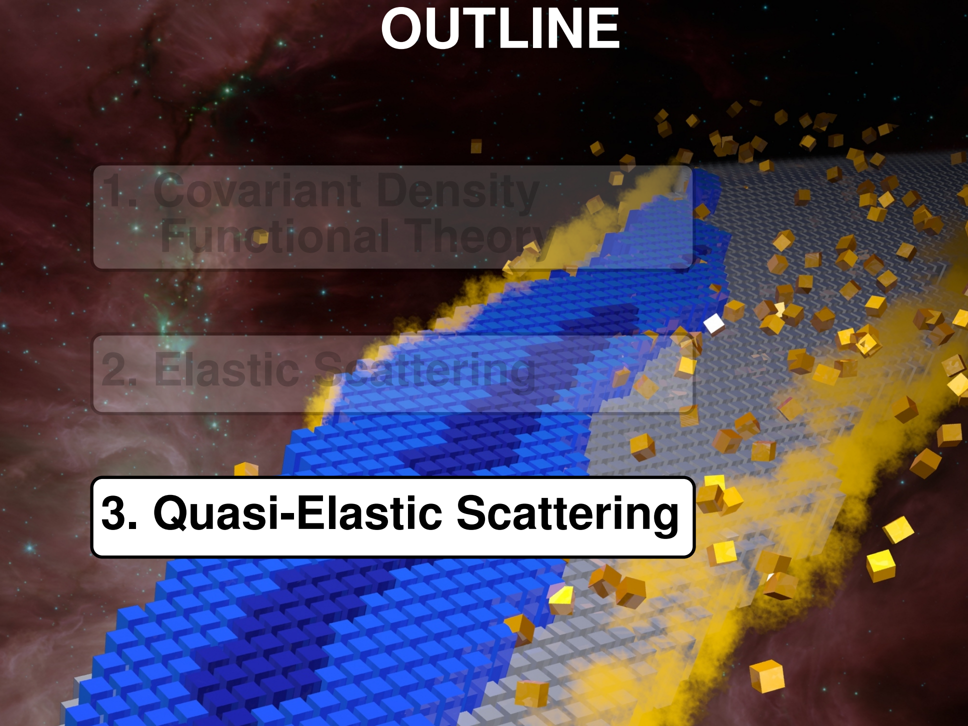
3. Quasi-Elastic Scattering



INCLUSIVE QUASI-ELASTIC SCATTERING

Inclusive differential cross section

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon\,\mathrm{d}\Omega}\right)_{QE} = \sigma_M \left[v_L R_L + v_T R_T\right]$$

Coefficients

$$v_L = \left(\frac{|Q^2|}{|\boldsymbol{q}|^2}\right)^2 \qquad v_T = \tan^2\frac{\theta}{2} - \frac{|Q^2|}{2|\boldsymbol{q}|^2} \qquad Q^2 = |\boldsymbol{q}|^2 - \omega^2$$

Longitudinal and transverse response functions

$$R_L(q,\omega) = W^{00}(q,\omega) \qquad R_T(q,\omega) = W^{11}(q,\omega) + W^{22}(q,\omega)$$

Hadron tensor

$$W^{\mu\mu}(q,\omega) = \overline{\sum_i} \sum\!\!\!\!\!\!\int_f \left|\langle\Psi_f|\hat{J}^\mu(\boldsymbol{q})|\Psi_i\rangle\right|^2 \delta(E_i + \omega - E_f)$$

INCLUSIVE QUASI-ELASTIC SCATTERING

Relativistic Green's function model

Equivalent expression for the hadron tensor

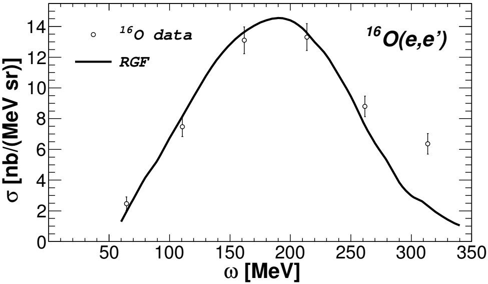
$$W^{\mu\mu}(q,\omega) = -\frac{1}{\pi}\mathrm{Im}\langle\Psi_i|J^{\mu\dagger}(\boldsymbol{q})G(E_f)J^{\mu}(\boldsymbol{q})|\Psi_i\rangle$$

Final expression for the hadron tensor

$$\begin{aligned} W^{\mu\mu}(q,\omega) = \sum_n \Bigg[& \mathrm{Re}\, T_n^{\mu\mu}(E_f - \varepsilon_n, E_f - \varepsilon_n) \\ & - \frac{1}{\pi}\mathcal{P}\int_M^\infty \mathrm{d}\mathcal{E}\, \frac{1}{E_f - \varepsilon_n - \mathcal{E}}\ \mathrm{Im}\, T_n^{\mu\mu}(\mathcal{E}, E_f - \varepsilon_n)\Bigg] \end{aligned}$$

$$\begin{aligned} T_n^{\mu\mu}(\mathcal{E}, E) = {} & \lambda_n \langle\varphi_n|j^{\mu\dagger}(\boldsymbol{q})\sqrt{1-\mathcal{V}'(E)}|\tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle \\ & \times \langle\chi_{\mathcal{E}}^{(-)}(E)|\sqrt{1-\mathcal{V}'(E)}\,j^{\mu}(\boldsymbol{q})|\varphi_n\rangle \end{aligned}$$

INCLUSIVE QUASI-ELASTIC SCATTERING



Oxygen kinematics

$E = 1080$ MeV

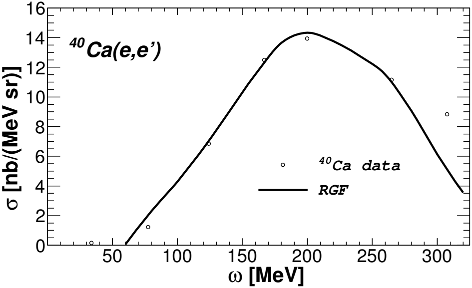
$\theta = 32°$

Calcium kinematics

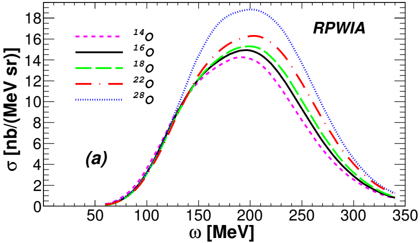
$E = 841$ MeV

$\theta = 45.5°$

Good agreement with experimental data in the region of the quasi-elastic peak

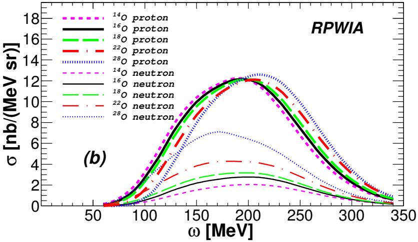


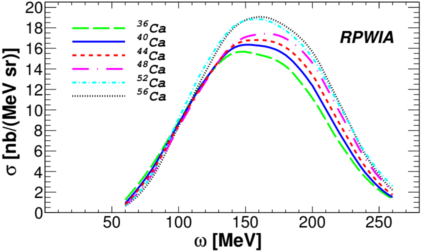
INCLUSIVE QUASI-ELASTIC SCATTERING



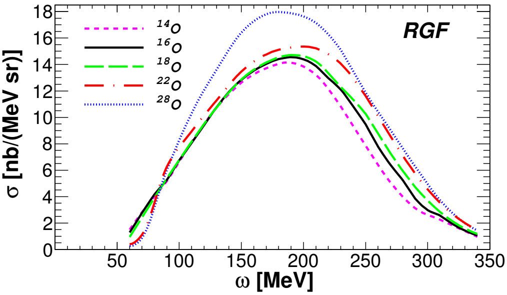
RPWIA

Differences between isotopes are entirely due to single-particle wave functions





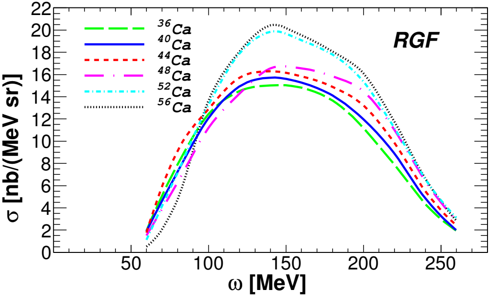
INCLUSIVE QUASI-ELASTIC SCATTERING



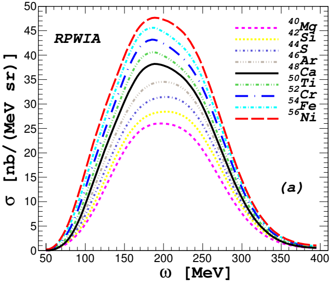
RGF

Differences in the low-energy transferred region and in the quasi-elastic peak

Importance of FSI

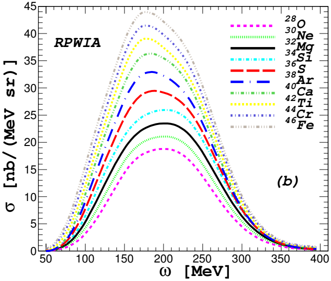


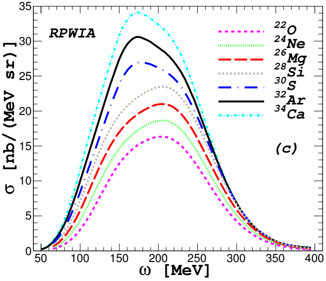
INCLUSIVE QUASI-ELASTIC SCATTERING



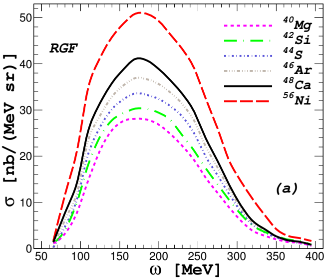
RPWIA

Results for isotonic chains



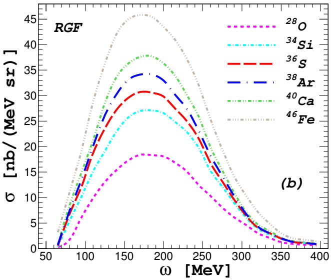


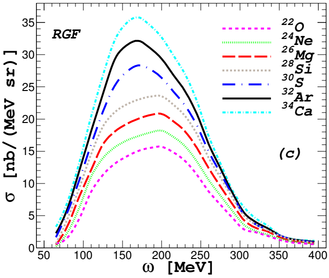
INCLUSIVE QUASI-ELASTIC SCATTERING



RGF

1. Visible distortion effects
2. Cross sections show a tail toward large values of the energy transferred





OUTLINE

1. Covariant Density Functional Theory

2. Elastic Scattering

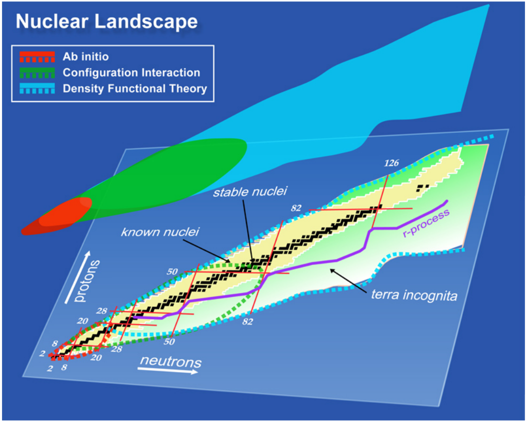
3. Quasi-Elastic Scattering

CONCLUSION

Electron scattering is the best reliable tool to investigate the nuclear properties

1. The interaction is well known and relatively weak
2. Free from most uncertainties of strong interaction
3. Explore the details of inner nuclear structures

Electron Scattering off Nuclei With Neutron and Proton Excess



1. Covarian Density Functional Theory

2. Elastic Scattering

3. QE Scattering