Charge Densities from Electron Scattering

achievements for stable nuclei, challenges for unstable nuclei

Ingo Sick

Purpose of talk

discuss relation data $\leftrightarrow$ densities
address limitations
particularly with respect to low-L measurements

Start with historical perspective on extracting $\rho(r)$

1. Fifties

first low-q experiments
interpreted in terms of PWBA

$$\sigma(E,\theta) = \sigma_{Mott}\ F^2(q)$$

momentum transfer $q \sim 2\ E\ sin(\theta/2)$

σ_{Mott} = cross section for point-nucleus

$$F(q) = \int_0^\infty \rho(r)\ \frac{sin(qr)}{qr}\ r^2 dr$$

Form factor = Fourier-transform of density

Soon first calculations of σ via solution of Dirac equation

accounts for distortion of e-waves

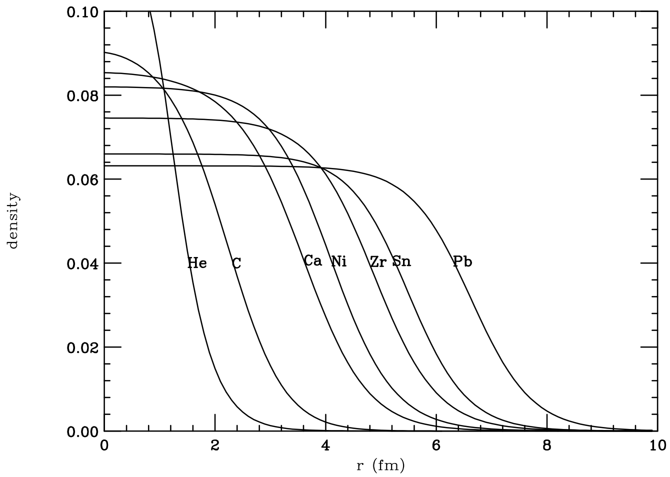
makes relation $\sigma \leftrightarrow \rho(r)$ less transparent

Determination of model densities

3PF $\rho(r) = (1 + w(r/c)^2)/(1 + e^{(r-c)/z})$

3PG $\rho(r) = (1 + w(r/c)^2)/(1 + e^{(r^2-c^2)/z^2})$

Establish general evolution of $\rho(r)$ with Z, A



Integral properties of interest

– half-density radius c

strongly correlated with rms-radius R

– surface thickness z

not too correlated with c

– central depression w

problematic, discuss below

Knowhow for determination of R

largely lost since

re-activated + improved with recent problems with proton R

new insights (see below)

2. Sixties

higher-q data (mainly from HEPL)

more complicated shapes of ρ

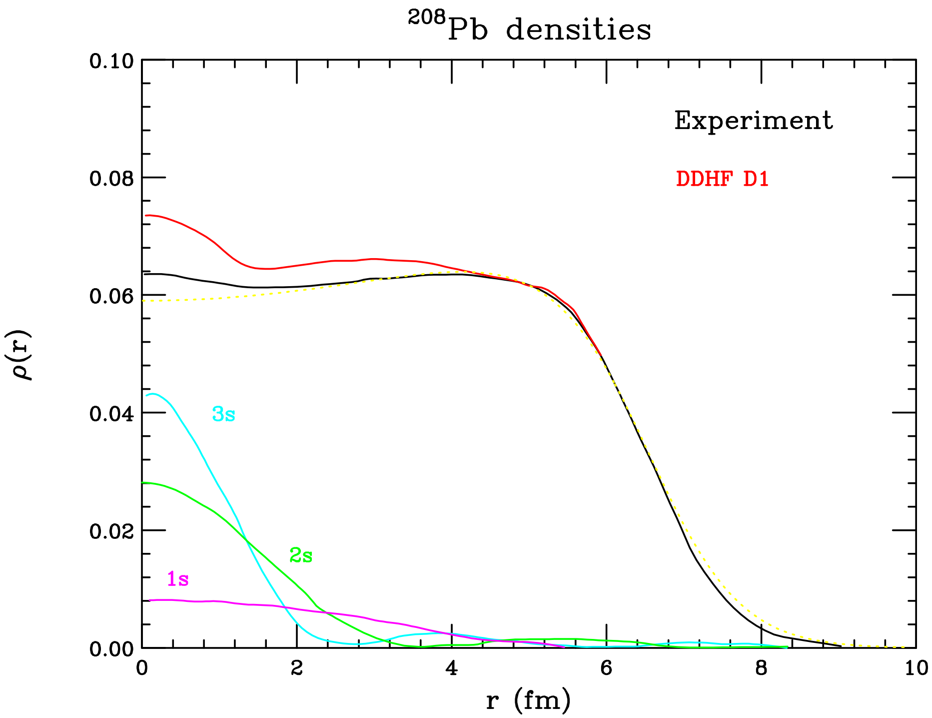
Fermi/Gauss + modifications

not always clear if given feature significant

or very model-dependent

example: w

Example: ^{208}Pb: central peak/depression [1]



3PG parameterization yields large $w \sim 0.33$
$\longrightarrow$ central depression?

SOG (model-independent density) does not really show it

3. Seventies

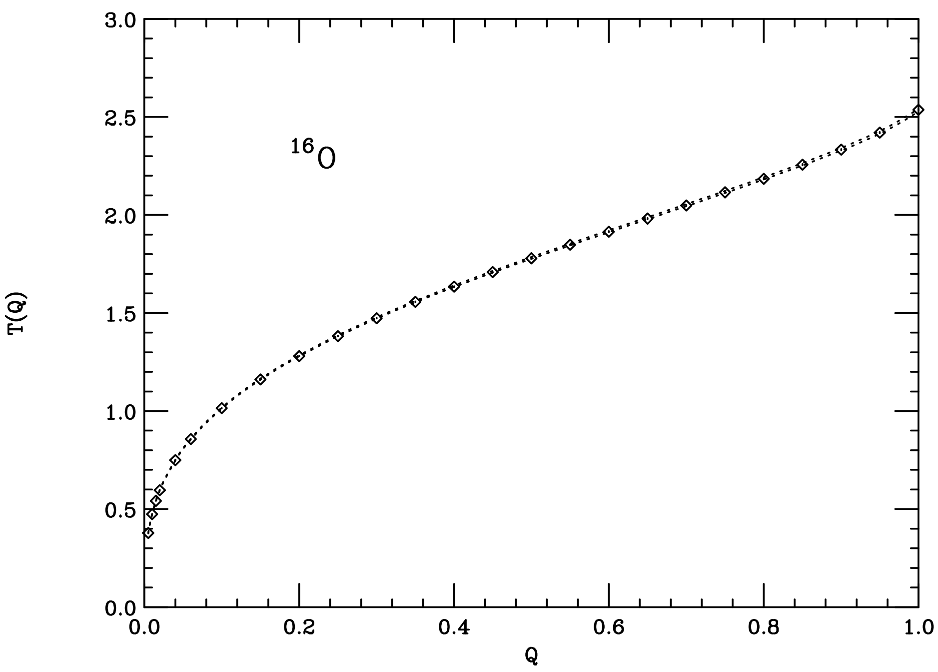
Extraction of model-independent information

possible only for *integral* quantities due to finite q_{max}

Partial moment function (F. Lenz [2])

$T(Q) = \int_0^Q r(Q')dQ'$ with Q=fraction of charge out to radius r

$T(Q)/Q$ = linear moment averaged over charge Q



Strictly model-independent, welldefined δT, not intuitive, never caught on

Less model-dependent densities

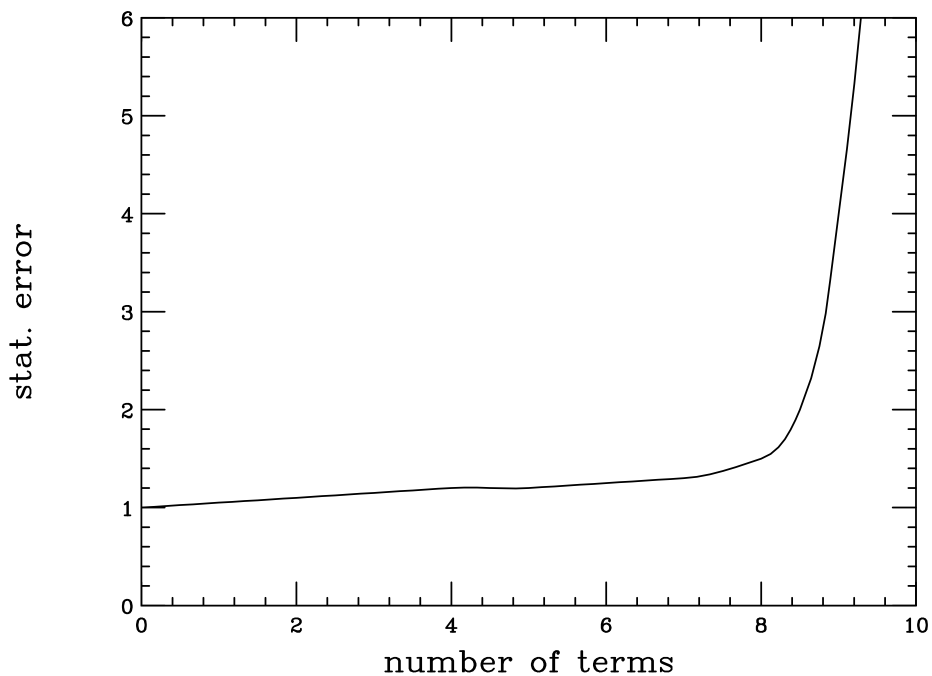
attempts to overcome model dependence

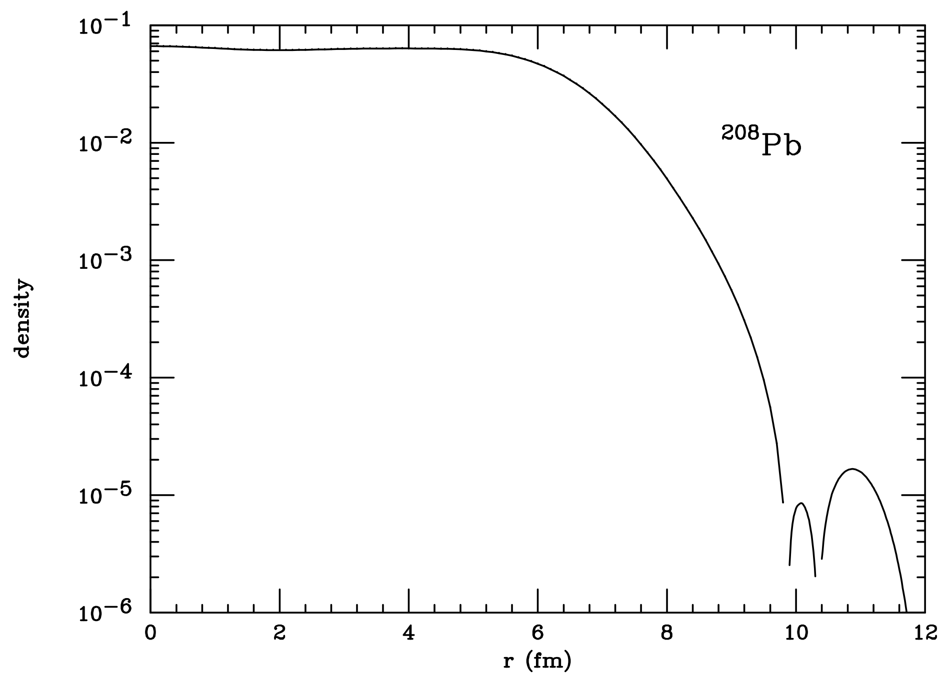
practical once $q_{max} \sim 3fm^{-1}$

expansion of $\rho(r)$ in complete basis

typically Fourier-Bessel

but: $\delta\rho(r)$ arbitrary (function of N) unless constrain $F(q > q_{max})$





to get significant $\delta\rho(r)$ must introduce physics constraint

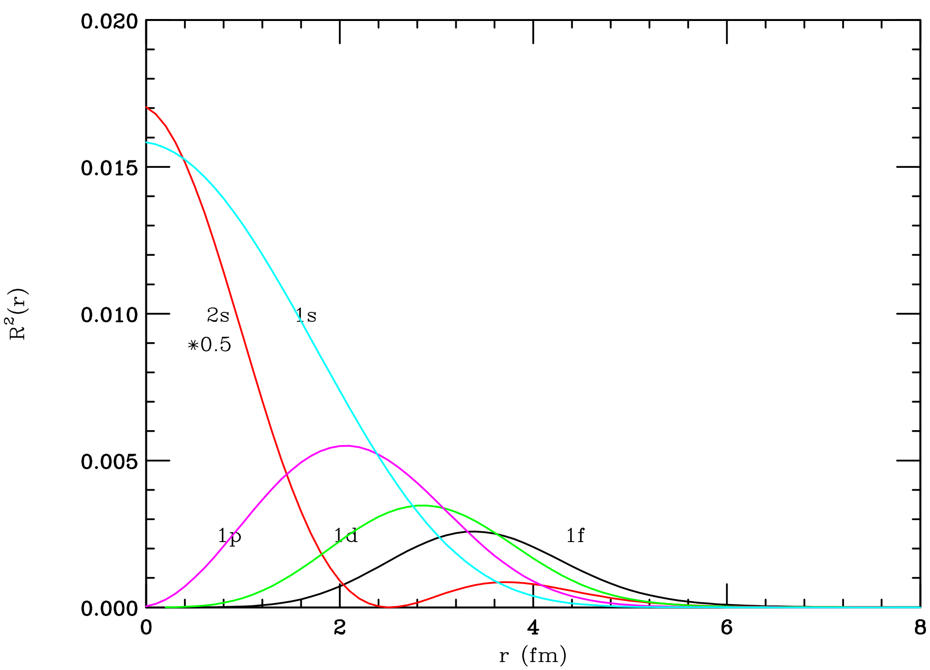
FB misbehaves at $r \sim r_{max}$, detrimental for determination of R (see below)

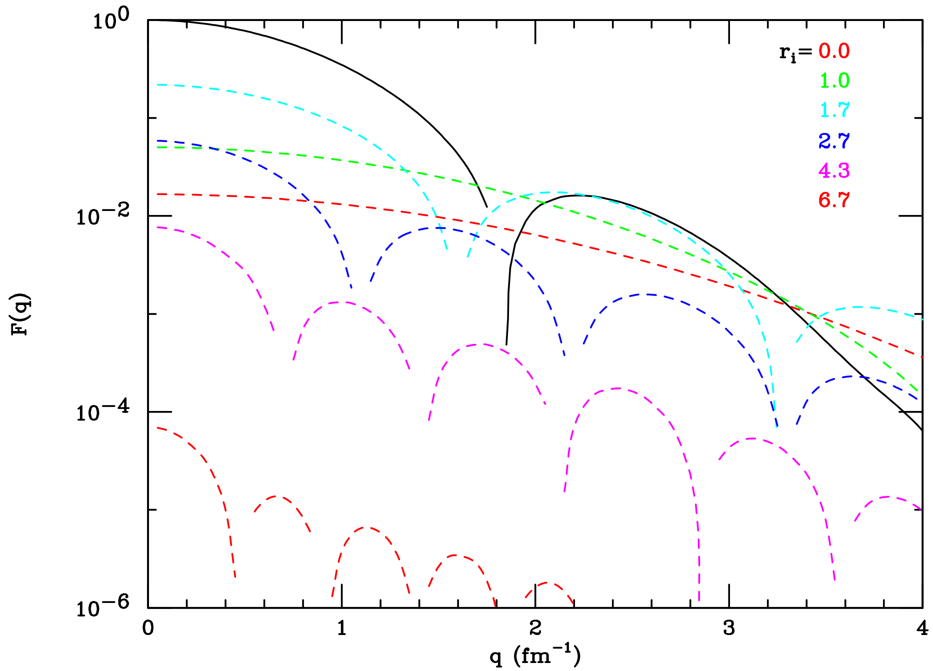
Physics constraint: minimal width Γ of peaks in ρ [3]

smoothly varying $R()$, peaks of minimal width Γ
property of solutions of Schrödinger equation: minimal curvature, $\langle E \rangle$

Parameterize ρ as sum of *many* gaussians SOG with width Γ, placed at radii r_i

Example: ^{12}C

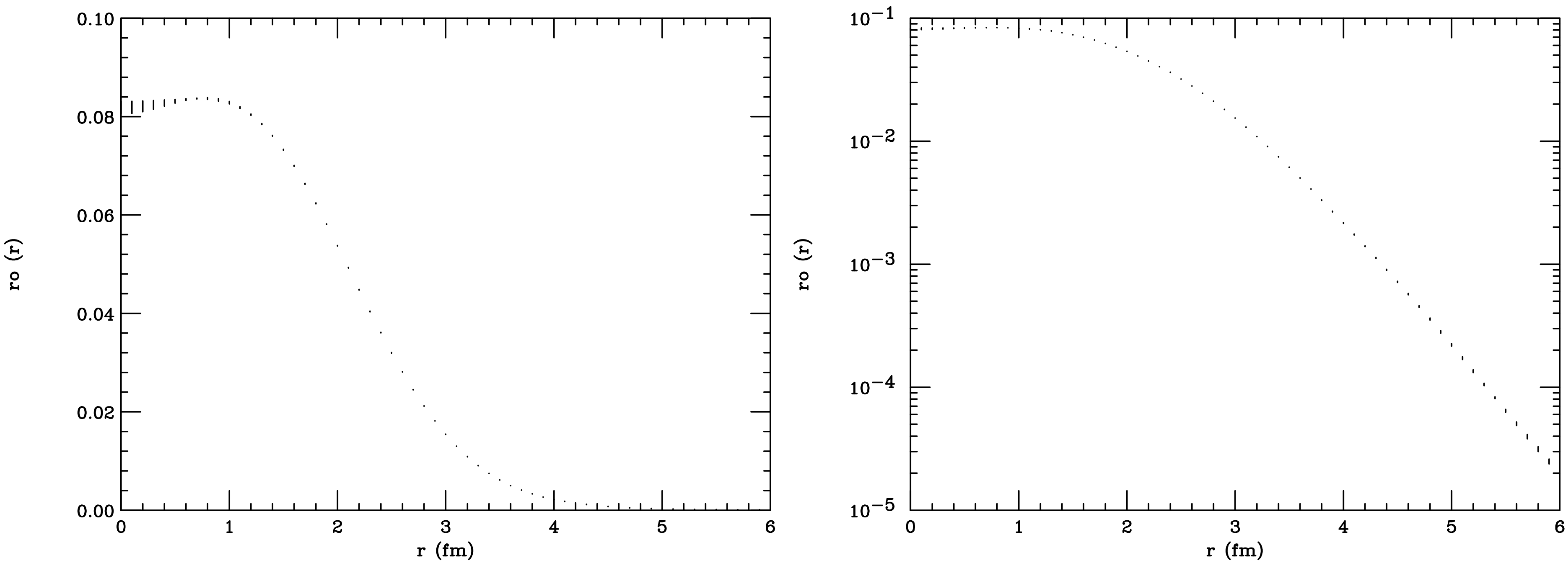




With physics constraint get significant $\delta\rho(r)$

note: large $r \leftrightarrow$ high-frequency oscillations in q

Get density with significant error bar: ^{12}C



Note (for later)

$\delta\rho$ in tail small (accurate extensive data)

displays typical exponential behavior

$$R(r) \sim e^{-\kappa r}/r$$

4. Eighties: most transparent extraction of ρ: DFT [4]

form factor $F(q)$ = Fourier transform of ρ(r)

density ρ(r) = Fourier transform of $F(q)$

$$\rho(r) = ... \int_0^\infty F(q) \frac{sin(qr)}{qr} q^2 dq$$

Can exploit? Potential difficulties:

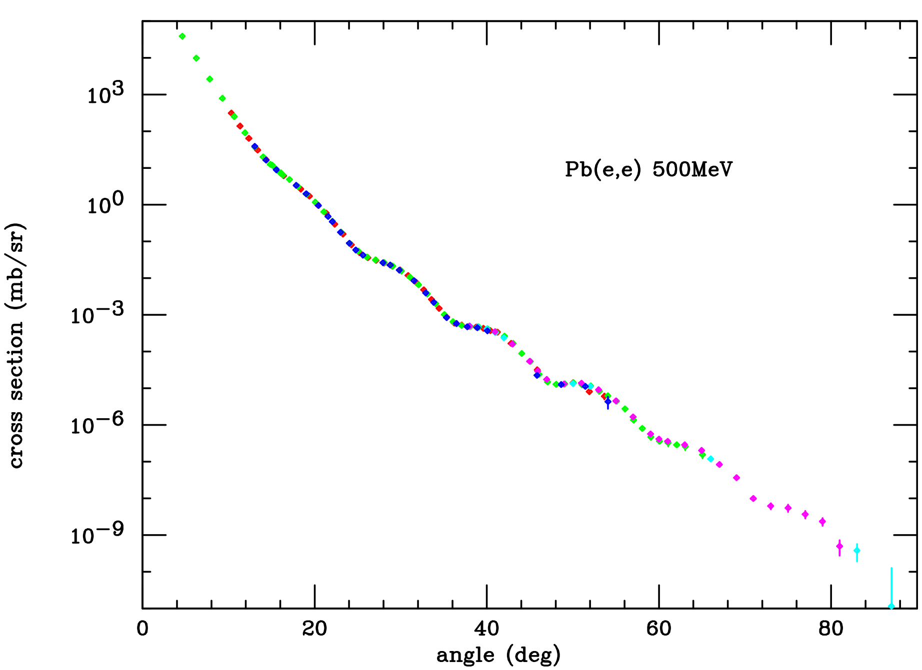
- $F(q)$ not available, only cross sections
  Coulomb distortion!
- q_{max} not infinite

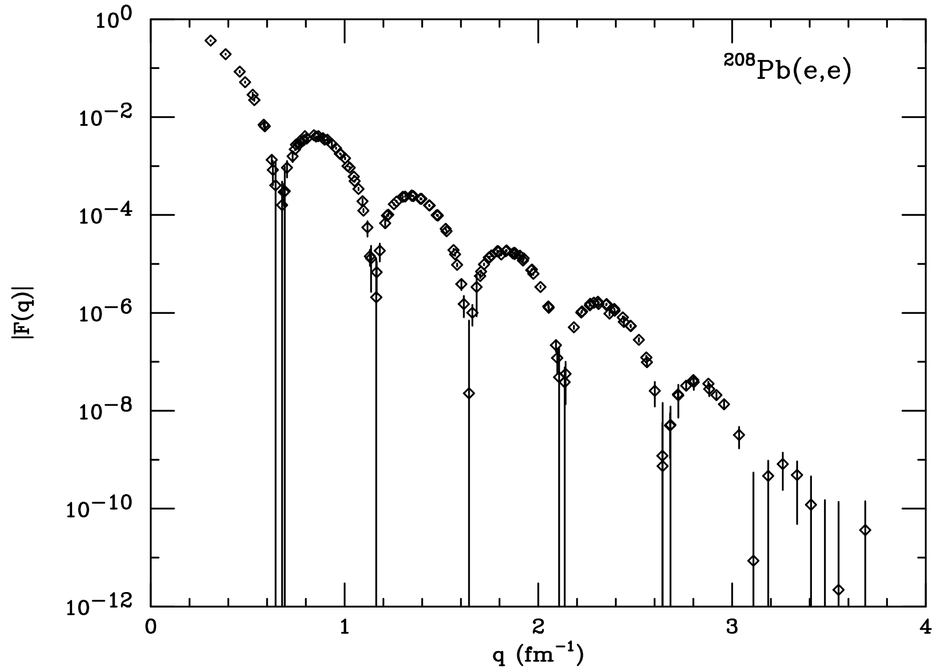
Advantage of electrons

Coulomb distortion relatively small
only function of Z (known)
can remove to get $F(q) \pm \delta F(q)$ via DW code

$$F^2_{exp} = F^2_{model} \frac{\sigma_{exp}}{\sigma^{DW}_{model}}, \qquad \delta F^2_{exp} = \delta\sigma_{exp}/\sigma_{point}$$

Example: cross sections for Pb





experimental F(q) contain same information as σ

To deal with finite q_{max}: study $\int$ as function of upper integration limit

$$\rho(r, q_{max}) = ... \int_0^{q_{max}} F(q) \frac{sin(qr)}{qr} q^2 dq$$

$\rho(r, q_{max})$ = damped oscillatory function of q_{max}
extrema at every zero of $F(q)$, $sin(qr)$
in limit $q_{max} = \infty$ equals $\rho(r)$

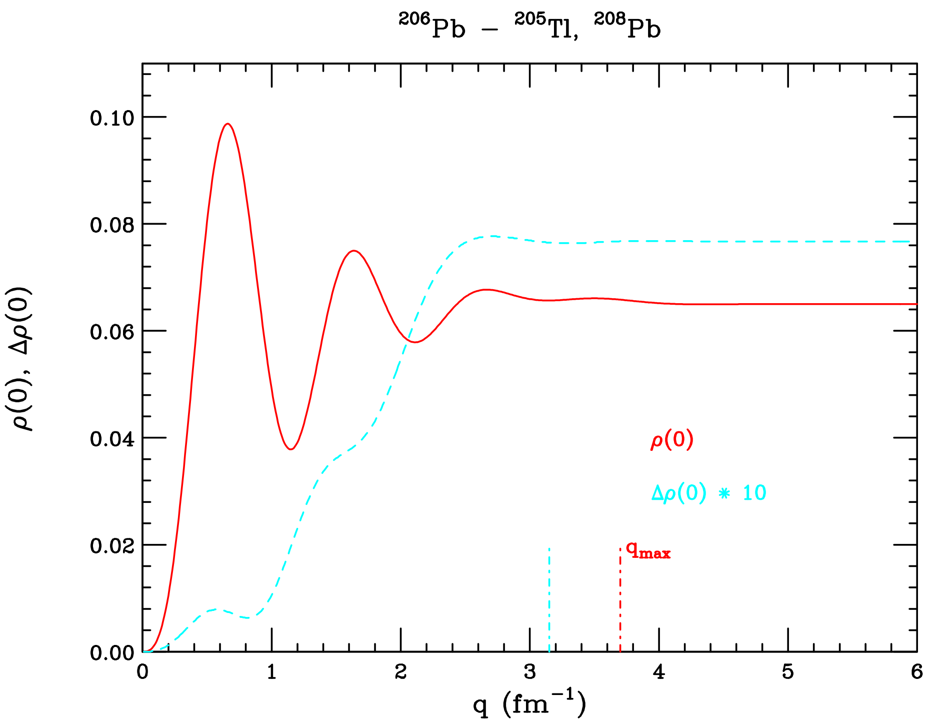
difference last minimum/maximum = uncertainty

$\rightarrow$ model-independent estimate of $\delta\rho$ $\rightarrow$ benchmark for $\delta\rho$

Illustration: $r = 0$ (most difficult case)

$$\rho(0, q_{max}) = ... \int_0^{q_{max}} F(q)\; q^2 dq$$

Example: Pb (red curve)



difference last mini/maxi <1%

Major assets of Direct Fourier Transform

bias-free determination of $\delta\rho$, transparent relation data - ρ, benchmark $\delta\rho$

$F(q)$ most useful to get insight, only in the end need solution Dirac eq.

Data of low q_{max} for exotic nuclei: DFT not possible

must rely on model densities
must aim at *integral* properties such as rms, c, z, ...

Discuss first: rms-radius $R = \langle r^2 \rangle^{1/2}$

the integral quantity of interest
most often quoted
can compare to radii from electronic/muonic atoms
needed to convert isotope shifts to radii
particularly relevant for unstable nuclei
discuss partly in PWBA (more transparent), in practice use DWBA

R strongly correlated with half-density radius c, discuss only R

Determination of R is deceptively easy:

$$F(q) = 1 - q^2\langle r^2 \rangle/6 + q^4\langle r^4 \rangle/120 - ...$$

with $F(q)$ obtained as in DFT

apparently model-independent way to get R
$\pm$ everybody uses it (at least conceptually)
but it does not work!!

Insights (in part not so recent [5])

1. Expansion of $F(q)$ in q^2 has very small convergence radius, $\sim 6/R^2$

2. $F(q)$ does not correspond to a density, Fourier transform diverges

3. Higher moments always contribute

if go to small enough q_{max}:

then finite size effect too small

measure only the "1" in $1 - q^2R^2/6$

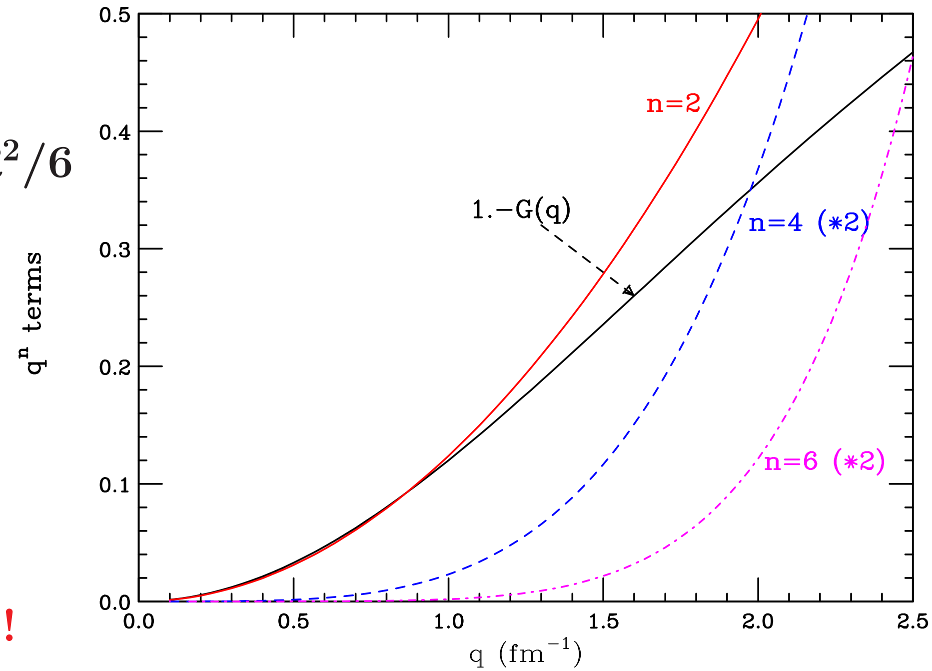
if q_{max} is larger for small δR:

then $\langle r^4 \rangle$ important

but $\delta\langle r^4 \rangle$ affects δR

get smaller $\delta\langle r^4 \rangle$ through higher q_{max}

get same problem with $\langle r^6 \rangle$, ...

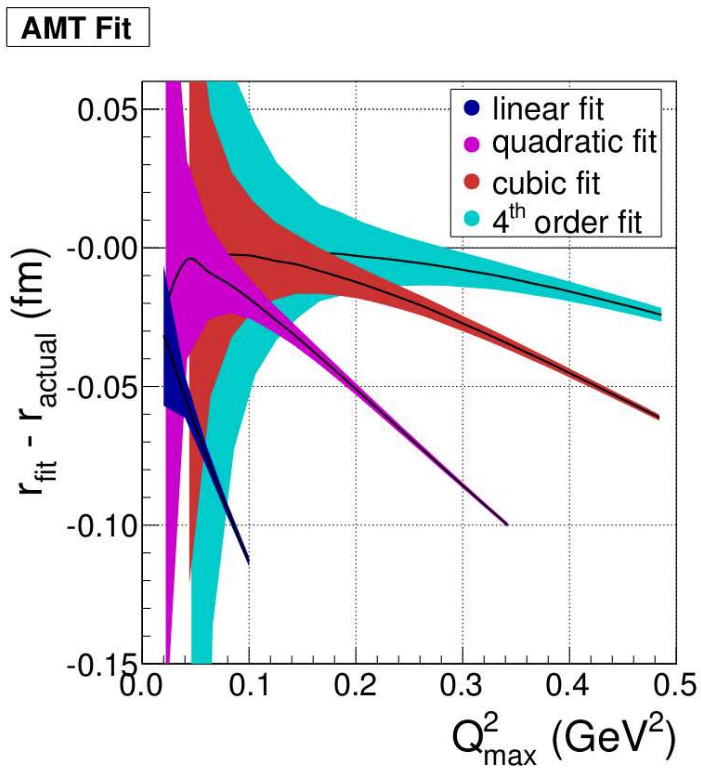


known since a decade for p, but ignored!

Problems with proton rms-radius have illustrated this point

Quantitative demonstration: Kraus *et al.* [6]

generate $F(q)$-data from known density, extract R from fit of $F(q)$
find systematic differences input-$R \leftrightarrow$ output-R



Expansion in powers of q^2 useless!

Consequence: need model densities/physics to (implicitly) fix higher moments

for resulting problems (model dependence) see below

Important consideration: which q's are important?

most often ignored

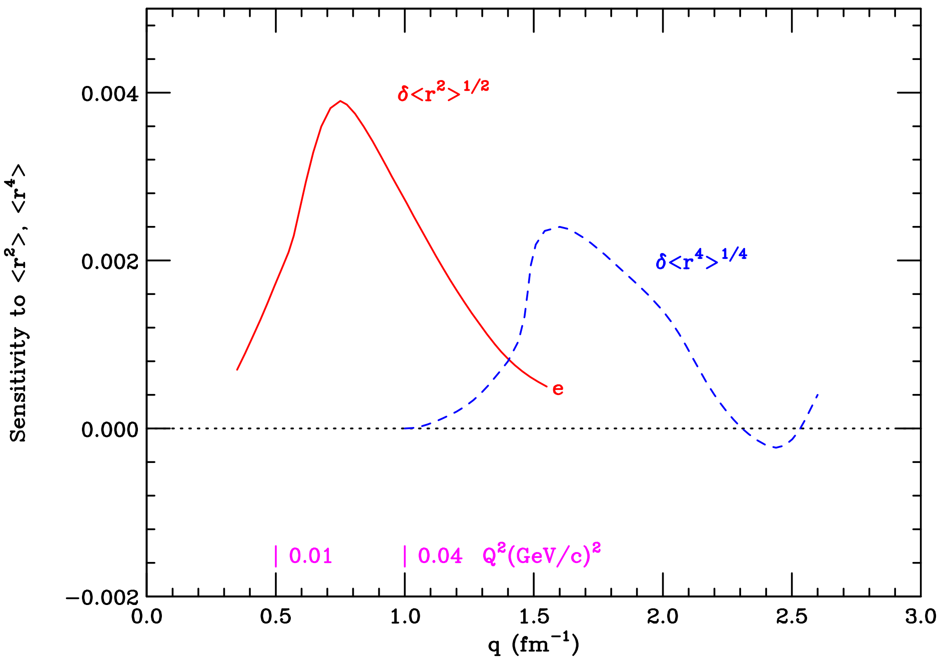
standard answer "small q" not good enough

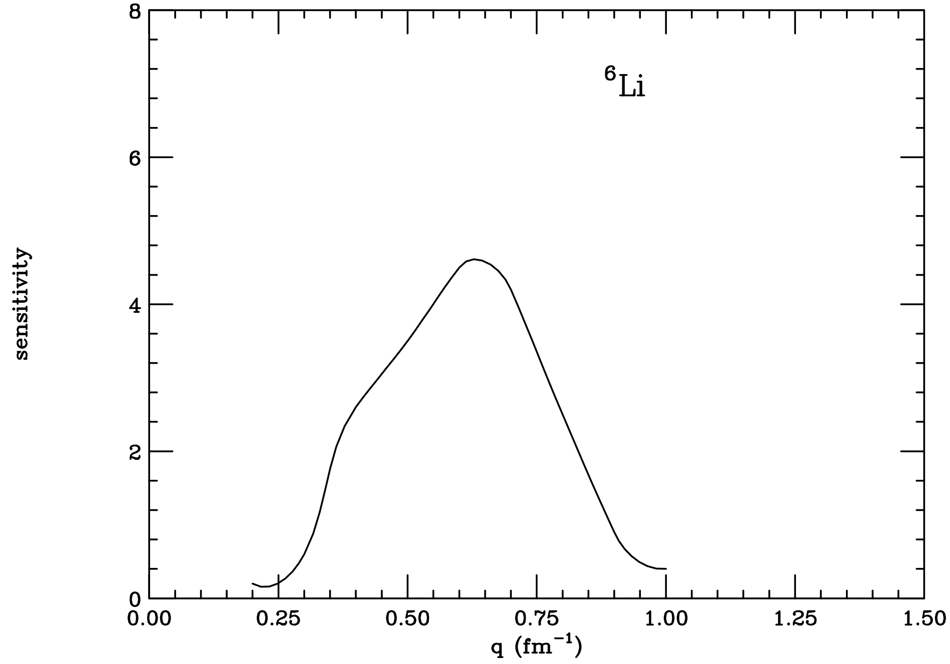
Quantitative study: notch test [7]

change data by (say) 1% in narrow region around q_0

refit data

plot changes of R as function of q_0



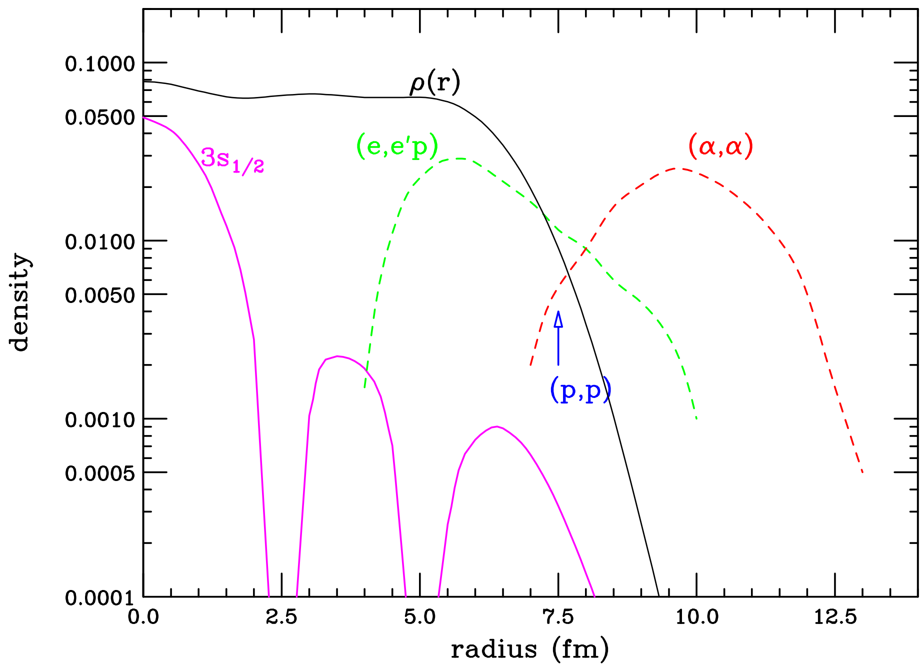


For nuclei heavier than proton:

maximum sensitivity to R at q where $F(q) \sim 0.7 - 0.8$

Important consideration: which r do matter?

mainly relevant for comparison with *other* probes aiming at ρ's and R's
all too often ignored



(e,e) = only probe sensitive to *all* r

strongly interacting probes = strongly absorbed probes
only sensitive to very large r
can never determine an rms-radius!
but are regularly used to do so

Not everything simple with (e,e) either: R dominated by large r, $\int ..r^4\ dr$

Importance of large-r tail [8]

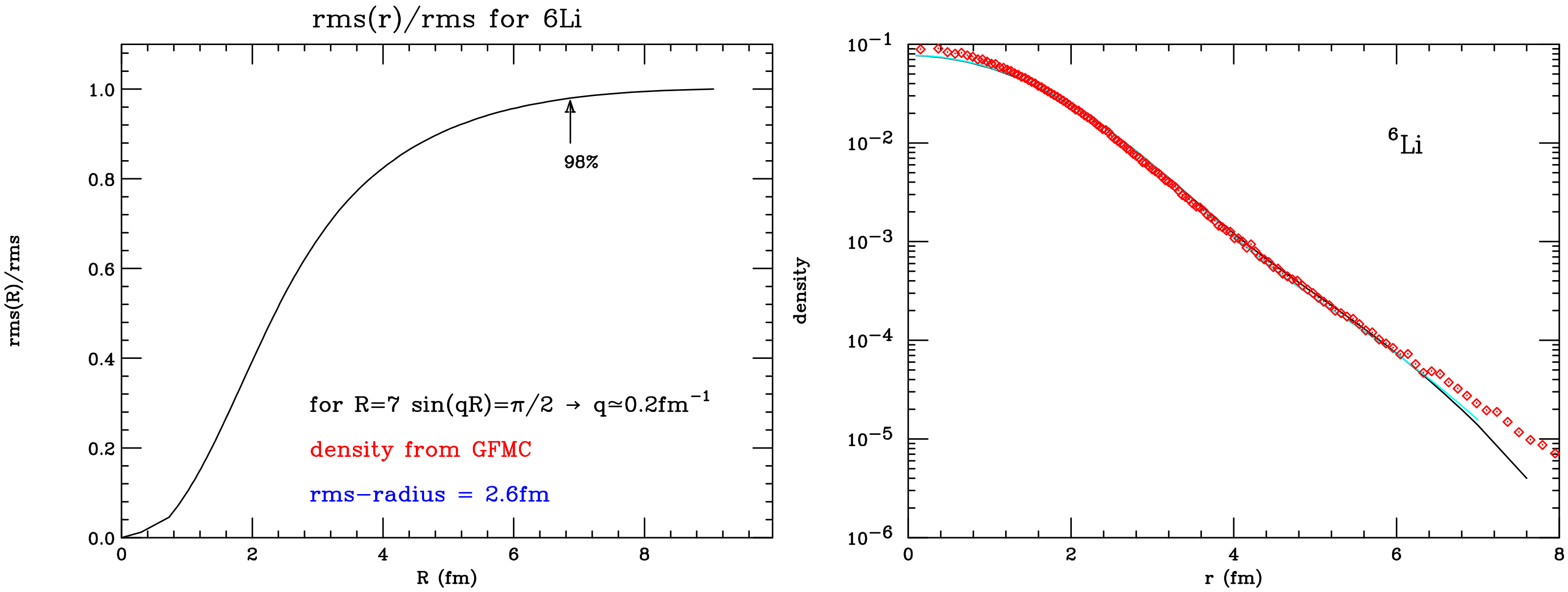
most often ignored in analyses using models
perhaps even worse in 'model-independent' analyses (see above)
parameterizations allow for too much flexibility of ρ at large r

Large-r particularly important for rms-radius and low SE!

Example: ^{6}Li

study $[\int_0^{r_{cut}} \rho(r)\ r^4 dr / \int_0^{\infty} \rho(r)\ r^4 dr]^{1/2}$ as function of cutoff r_{cut}

note: rms-radius of ^{6}Li $= 2.6 fm$

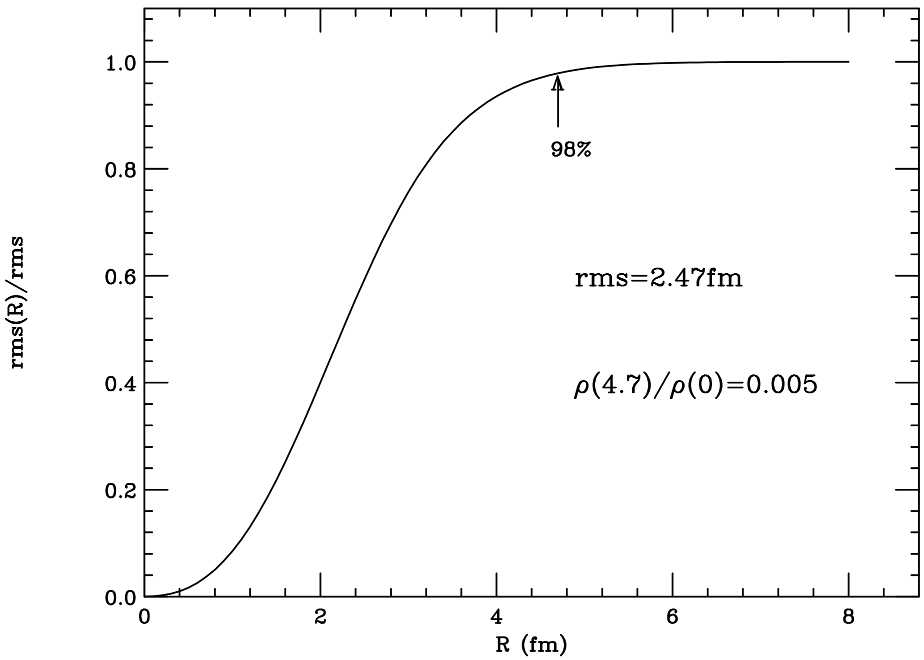


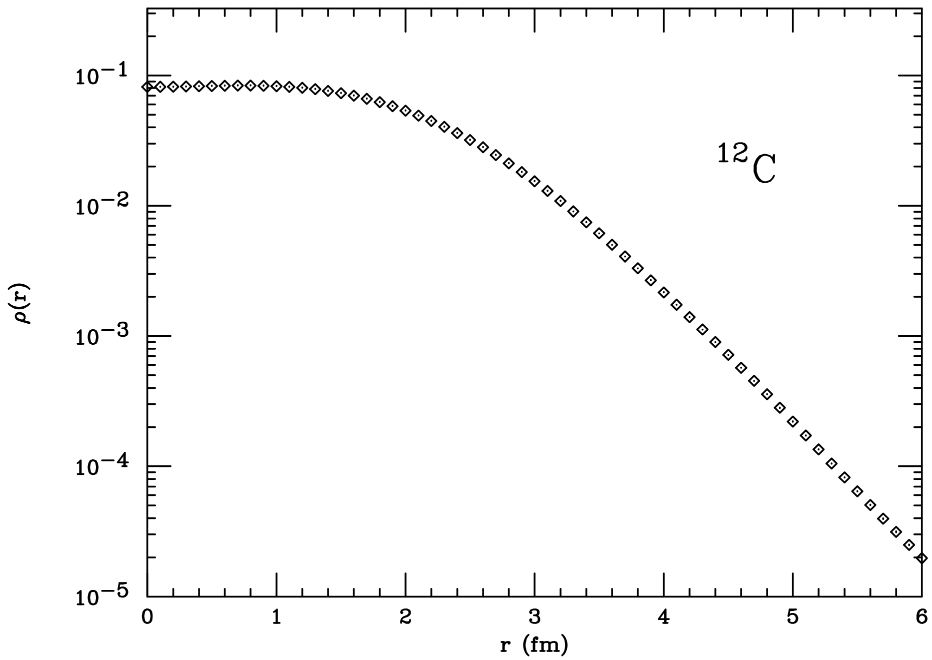
if want R to <2% must know ρ at $r > 7fm$ where $\rho/\rho(0) \sim 10^{-4}$!!

Only a problem for ^{6}Li with long tail?

similar, though less pronounced for ^{12}C

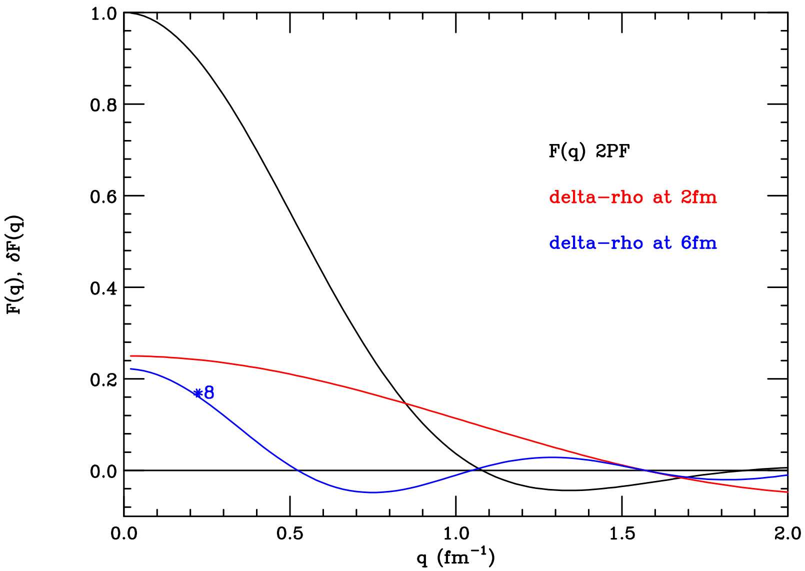
SE=16MeV!





Qualitative demonstration large $r \leftrightarrow$ low q

density = 2-parameter Fermi, 1/2-density radius $c = 4fm$
add local change of $\rho(2fm)$, or
add local change of $\rho(6fm)$, same rms-radius change



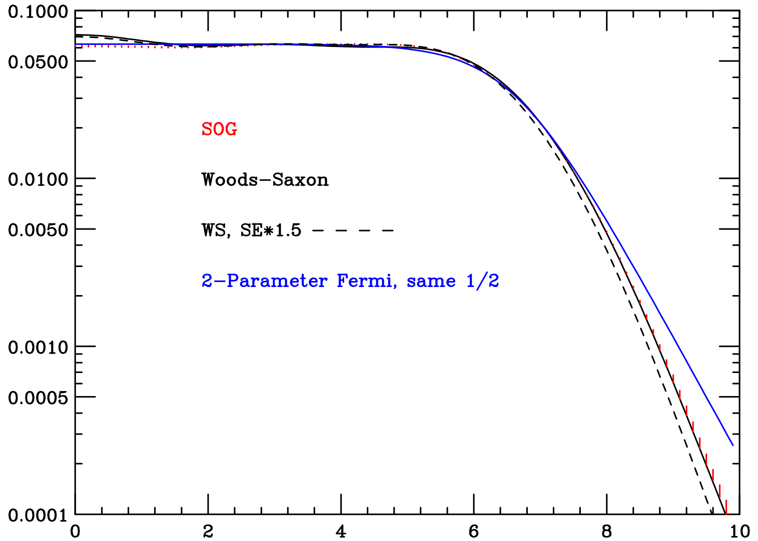
large r produce curvature of $F(q)$ at very low q
affects extrapolated slope $F(q=0)$ which yields rms-radius

Consequence: must be VERY careful about ρ(large r)!

parameterization $\longrightarrow$ implicit assumption on ρ(large r)

Example: compare tails for Pb

experimental SOG density
Woods-Saxon density with exp. separation energy
Woods-Saxon with increased separation energy
2-parameter Fermi with same 1/2-density radius



tail of 2PF leads to 2% larger rms-radius

Lesson for analysis using model-ρ's:

must constrain large-r behavior using physics knowledge

fall-off of $\rho(r)$ dominated by least-bound protons

What know on $\rho(r)$ at large r?

dominated by least-bound proton, radial wave function = Whittaker function

$$R(r) \sim W_{-\eta,l+1/2}(2\kappa r)/r \text{ with } \kappa^2 = 2\mu E/\hbar^2,\ \eta = (Z-1)e^2/\hbar\sqrt{\mu/2E}$$

depends only on removal energy

presumably available from accurate mass-measurements

easy to calculate

overall normalization (asymptotic normalization) generally not known $\rightarrow$shape

Shape automatically given if calculate $R(r)$ using Woods-Saxon potential

solution of Schrödinger equation for sensible potential

$\rightarrow$ physical shape of asymptotic $R(r)$

Note: Harmonic oscillator has *wrong* asymptotic shape

has been used often in past

has caused many problems (*e.g.* Darmstadt rms-radii)

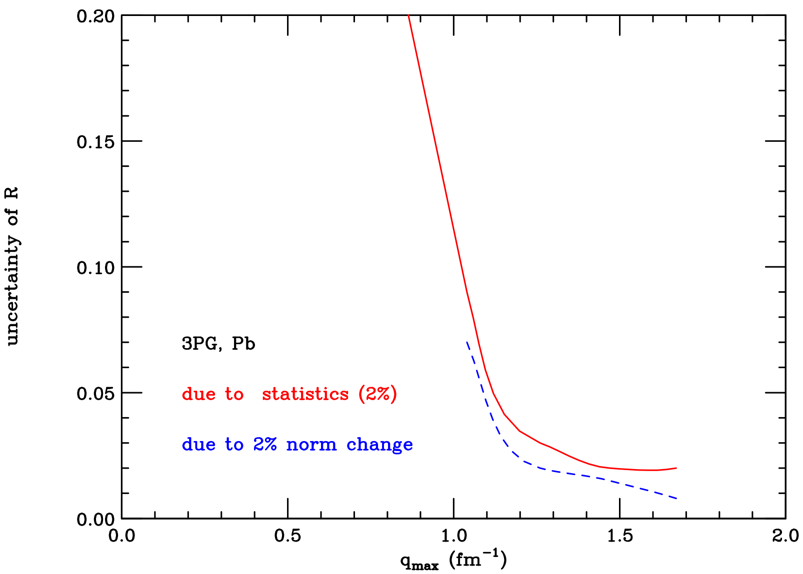
keep away from it

Important to use physics information on large-r behavior !

Should have been done in past analyses of data for stable nuclei

Essential insight for determination of R: role of normalization

generate cross sections for Pb using 3PG, $\pm 2\%$
in realistic q-range
analyze as function of q_{max} using 3PG
study effect of normalization change



Absolute normalization of cross sections very important

must make great effort to determine

Replace absolute normalization by floating of data?

traditional: people float data, as absolute norm difficult to determine
main purpose: get good-looking χ^2

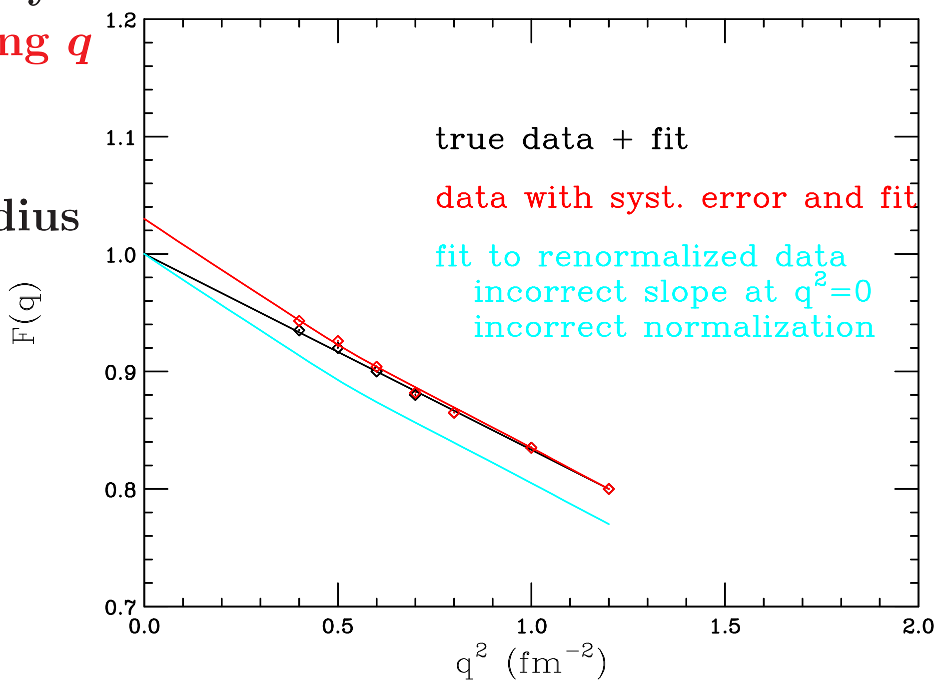
Problems:

1. Ignores $\sim$ 50% of effort of experimentalists

often great effort made to get ***absolute*** normalization, dont want to throw away

2. When floating *extrapolate* via fit to $q = 0$

become very sensitive to q-dependent systematic errors
errors clearly increasing with decreasing q
otherwise data taken at lower q
extrapolation increases effect
affects in particular extracted rms-radius



$\rightarrow$ **High premium on *absolute* $\boldsymbol{\sigma}$'s**

Alternative look at accuracy needed

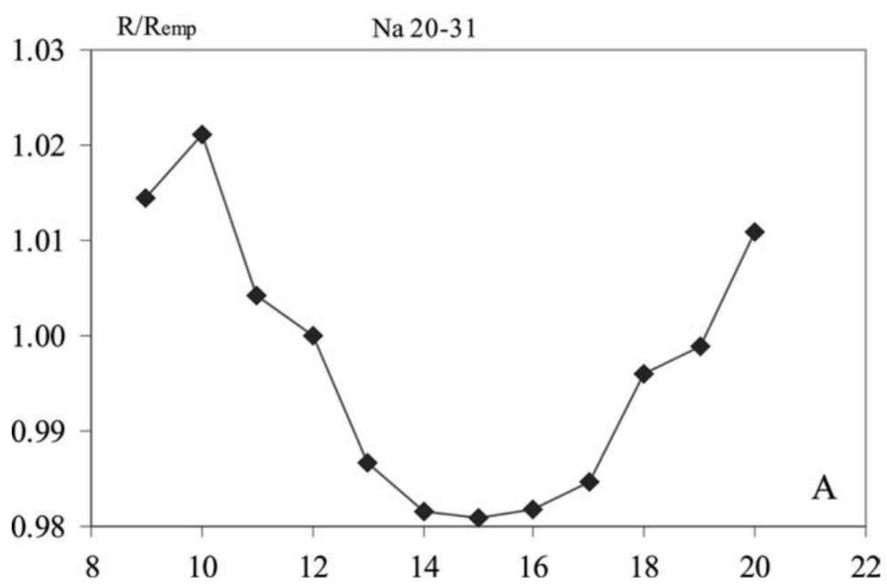
can estimate from sensitivity-curves discussed above
no need for detailed quantitative study

maximum sensitivity to R at q where $F(q) \sim 0.75$

$\rightarrow$ finite-size effect in $\sigma \sim 50\%$

If measure σ to $\pm 1\%$

measure R^2 to $\sim 2\%$
measure R to 1%
then can see evolution of R of interest



Absolute σ's to $\pm 1\%$ for stored ions??

already difficult enough for stable isotopes
already there dominating experimental error
presumably not possible

For stored ions

measure cross section *ratios* to stable isotope
$\pm$straightforward as these much more abundant
modest increase of time

Requirement

accurate (relative) luminosity monitor
same response for different isotopes
should be feasible
– elastic scattering at very low q (recoils near $90°$)
– or exploiting atomic electrons (?), nuclear Bremsstrahlung

But remember

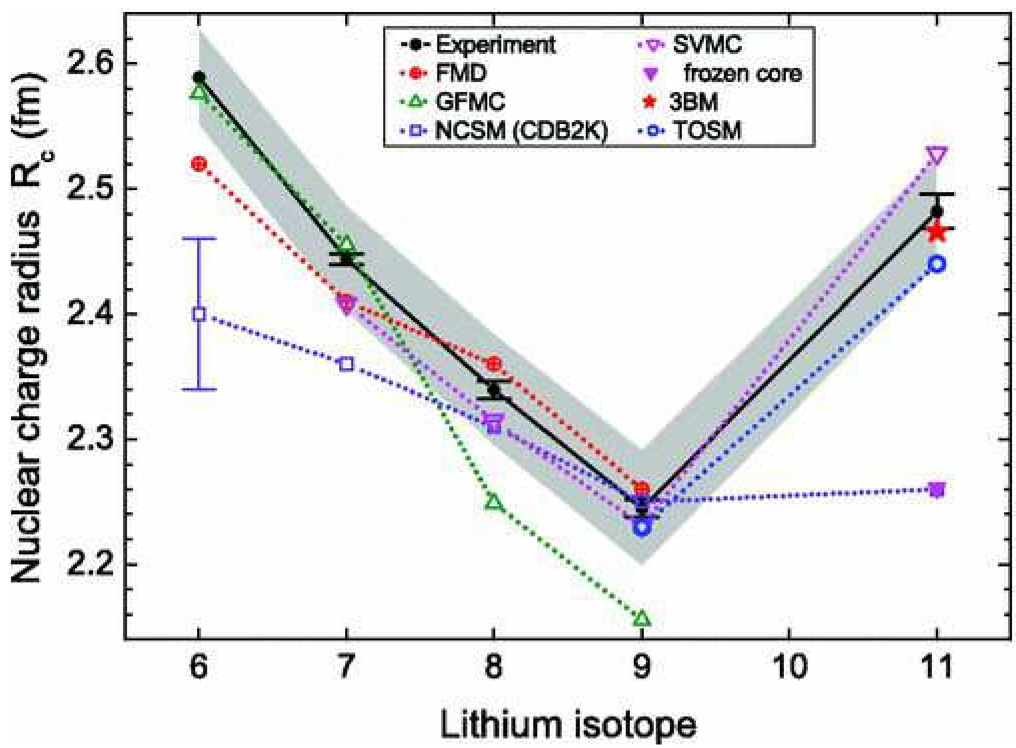
results only as good as result for reference nucleus
(re-)analyze reference nucleus data respecting above insights

Conditio sine qua non: precise *ratio*-measurements!

Special difficulty for unstable isotopes: halo, for small SE

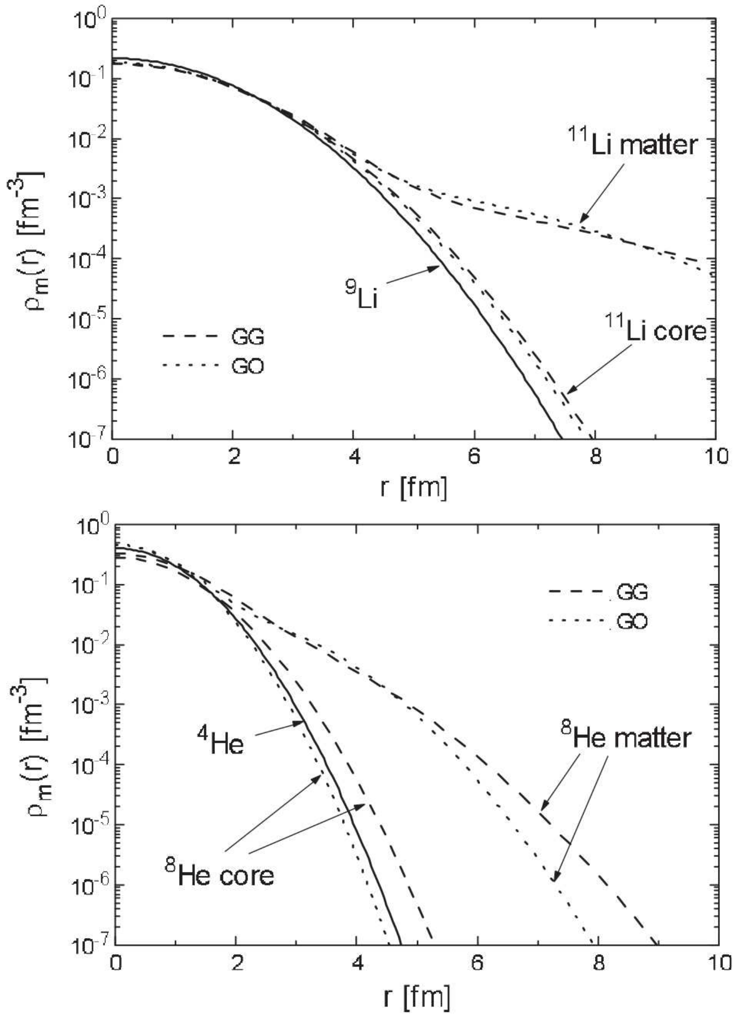
Example: isotope shifts for Li

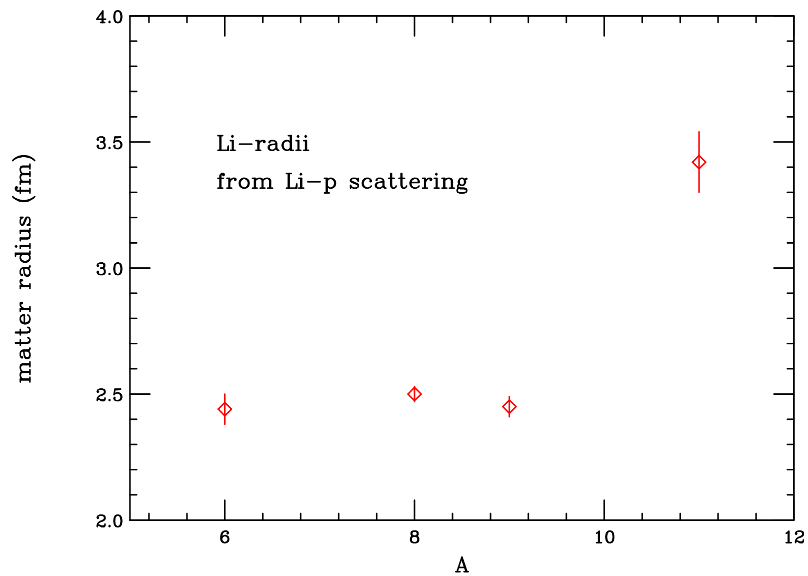
measured by laser spectroscopy with stored ions
Nörtershäuser *et al.*[9]
^{11}Li = Borromean nucleus (2n, ^{10}Li unbound)
2n-separation energy only 369KeV



Jump in R due to large-r tail from outermost proton

Extreme example of tail-importance: matter radii [10]





Densities from GeV/N p-nucleus scattering in inverse kinematics

How to proceed in presence of such a halo?

standard analysis with model for ρ not promising
density has too 'complicated' a shape
large-r tail affects R way too much

More promising approach

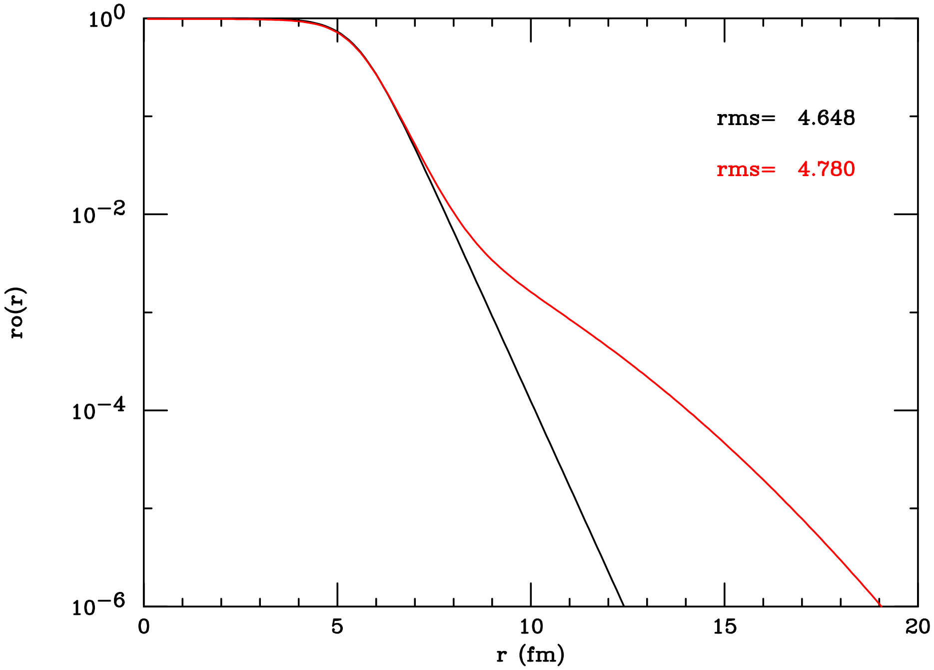
tail due to *one* shell with low separation energy
core in general more 'normal'

Parameterize ρ as model+tail

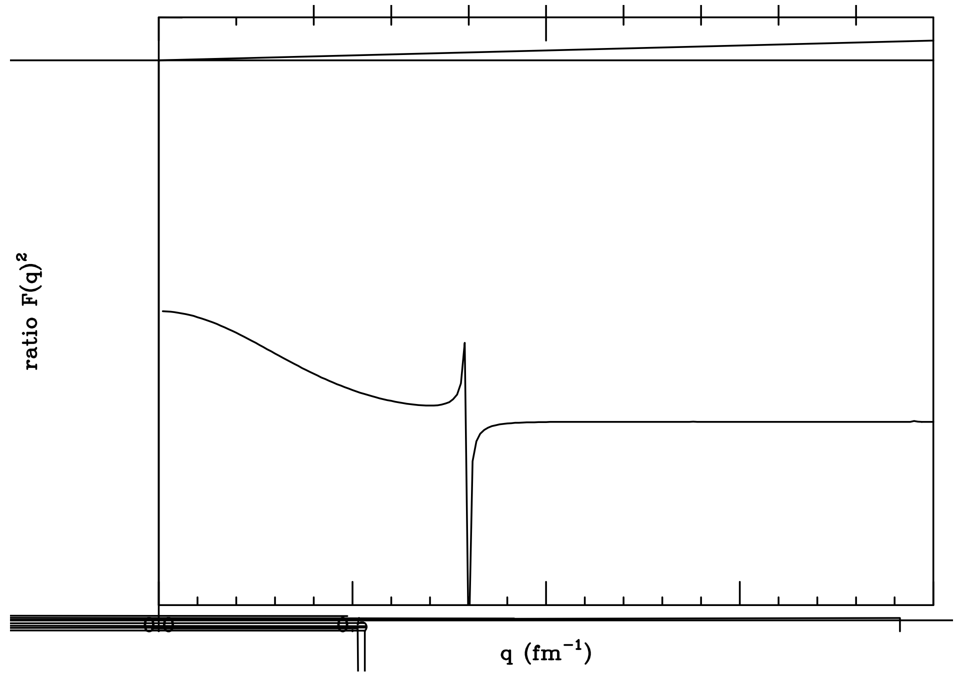
model = standard 2PF, 2PG, ...
tail = calculated in WS potential using experimental separation energy

Example

add tail to Pb density
  similar to findings from above example Li
compute change in cross section



10^{0}
10^{-2}
10^{-4}
10^{-6}
ro(r)
rms= 4.648
rms= 4.780
0
5
10
15
20
r (fm)



ratio $F(q)^2$
$q\ (fm^{-1})$

Determination of z

Lessons from the "early days"

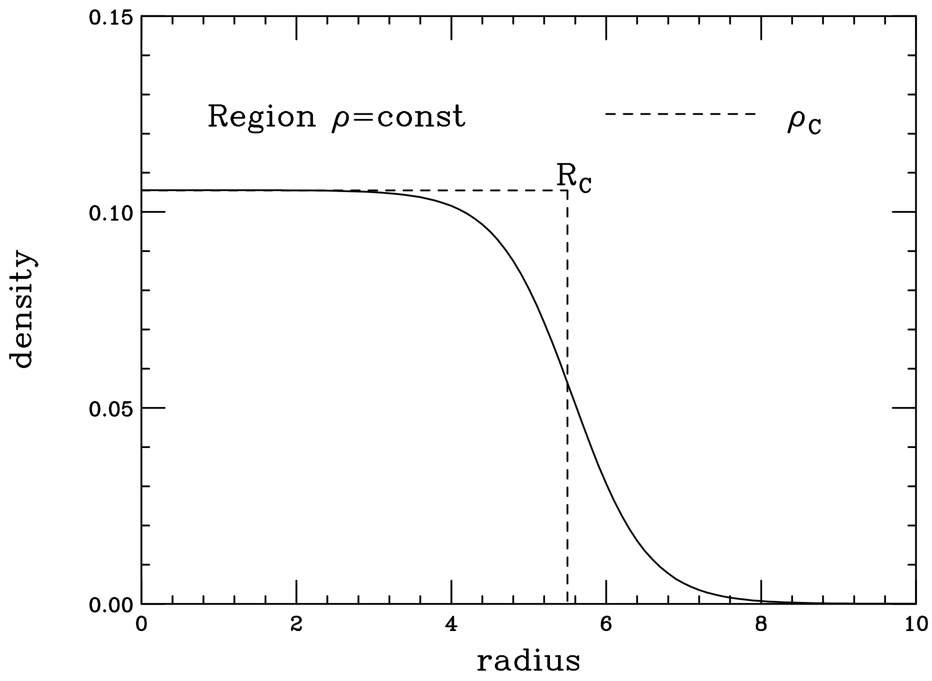
z obtained from $F(q)$ in 2. diffraction maximum

Understanding in PWBA:

consider $\rho(r)$ as result of folding $*$

fold uniform density ρ_c with gaussian G(r-r') of width z

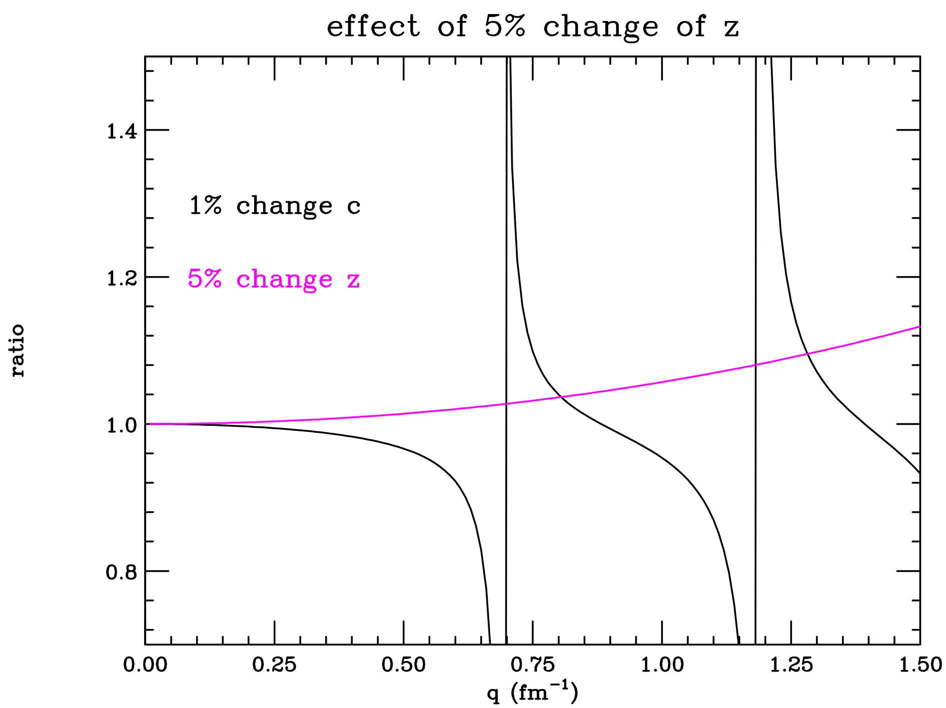
$\rho(r) = \rho_c(r) * G(r)$



Result for Fourier transform FT

$$F(q) = FT(\rho_c) \cdot FT(G)$$

Effect of $z \sim$ multiplication of $F_c(q)$ with FT of G *i.e.* $e^{-q^2 z^2}$



region of 2. diffraction maximum sensitive to z
shape of $F(q)$ for Δc and Δz different

but: effect not large
third diffraction maxi would be better

Particular interest in z for unstable nuclei

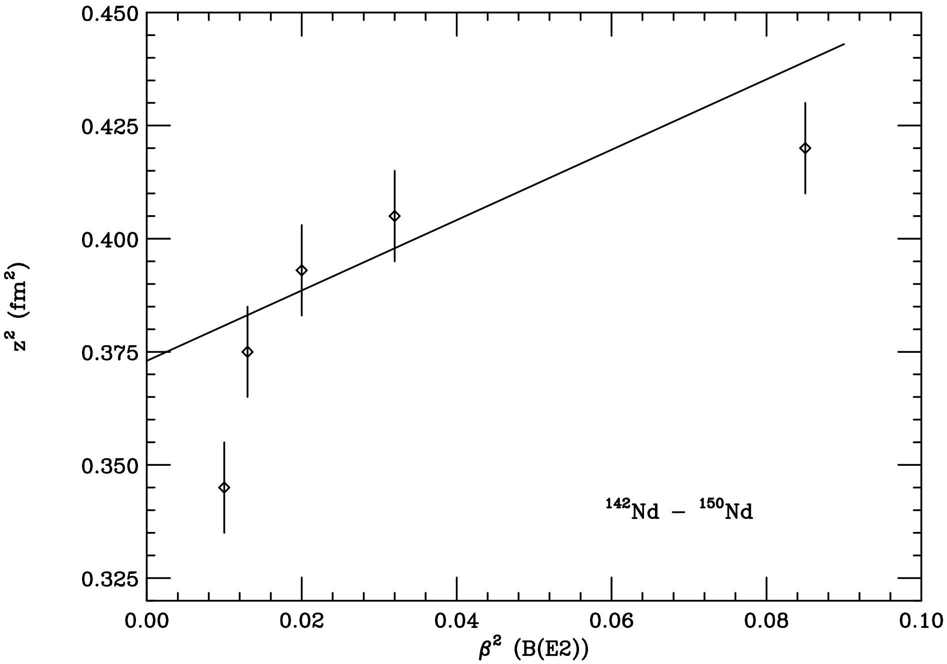
larger for halo-nuclei?

large for deformed nuclei

$$z_{eff}^2 = z_0^2 + \frac{3}{4\pi^3}\beta^2 c^2 + O(\beta^3)$$

Particularly useful for series of isotopes

Example: Nd-isotopes [1]



could be best way to get at deformation, is only way for I=0 nuclei

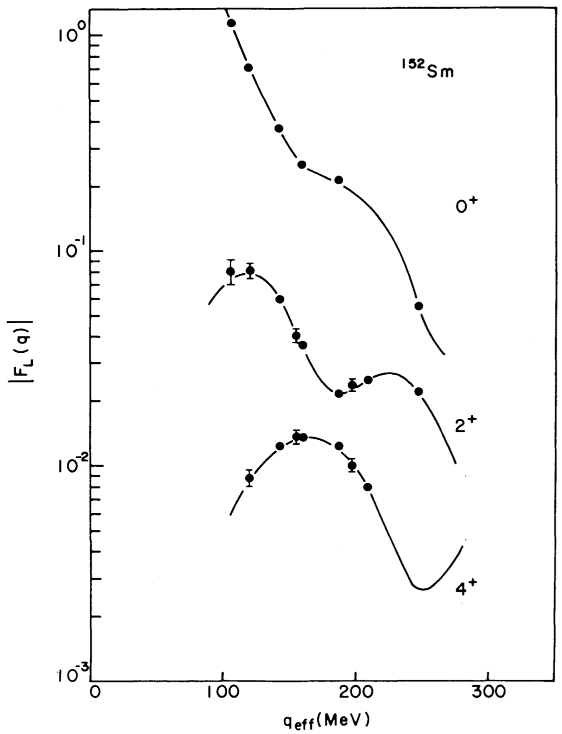
note: need rather *precise* z's

Challenge beyond L: energy-resolution

near 1. minimum, inelastic F's peak
  have typically several 10% of probability
must separate elastic/inelastic scattering
exotic nuclei typically far from magic
  low excitation energies
also needed for data on transition form factors

Problem: long targets

required to get high L
makes small $\Delta E/E$ difficult



Advantage of fixed-target experiments

resolution needed $\sim E^*$
$\Delta E/E \quad \sim$ 10 times easier
  than for collider experiments

Most realistic solution

magnetic spectrometer
deflection $\perp$ to scattering plane
decouples coordinate and E

Additional considerations $\rightarrow$ precise z

a priori correlation $c \leftrightarrow z$ not too strong
nevertheless: more precise c (or R) allows for more precise z
independent knowledge on R-type observable $\rightarrow$ smaller δz

For stable (reference)-isotope

add muonic X-ray information
available for *many* nuclei
yields *very* precise moments of ρ
for larger Z also yield higher moments of type $\langle r^4 \rangle$, $\langle r^6 \rangle$
(Barrett moments, to be precise)

For precise isotope shifts of R-type observables

include also electronic isotope shifts
occasionally easier to measure than (e,e) cross sections
particularly for exotic nuclei
need to know e-wave function at origin
$\Psi(r=0)$ calibrated using stable isotopes

General rule: add as much independent R-type info as possible

also helps to mitigate problems with absolute normalization of σ

Central depression w

large for heavy nuclei away from valley? Bubble nuclei??

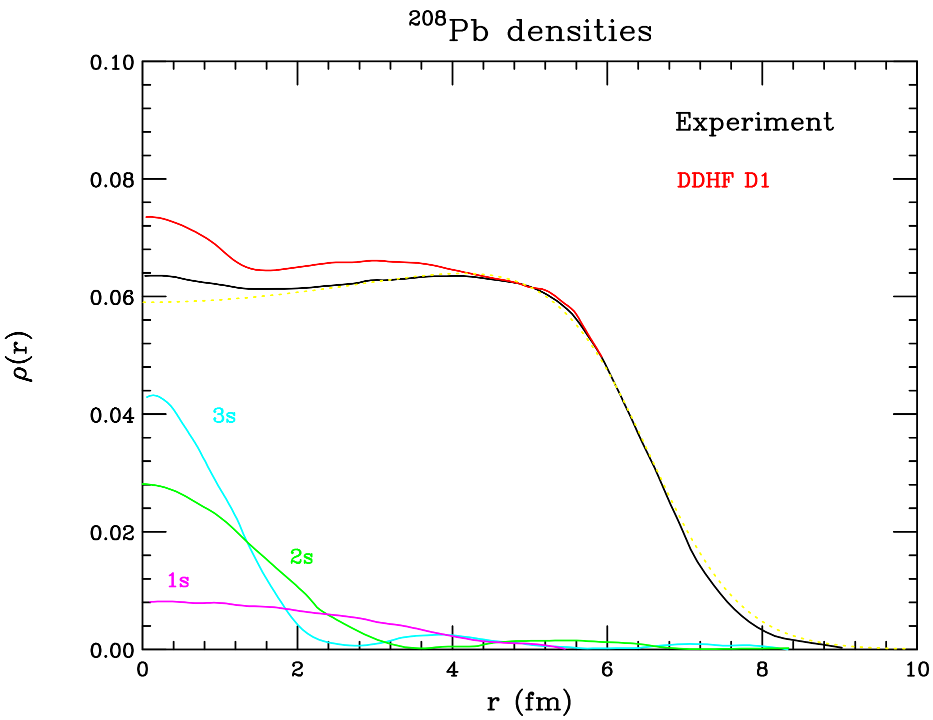
predicted for super-heavy, proton-rich nuclei

Lesson from 'early days'

not clear if w describes central depression

biggest effect of $(1 + wr^2/c^2)$ occurs in tail of $\rho(r)$ at *large* r

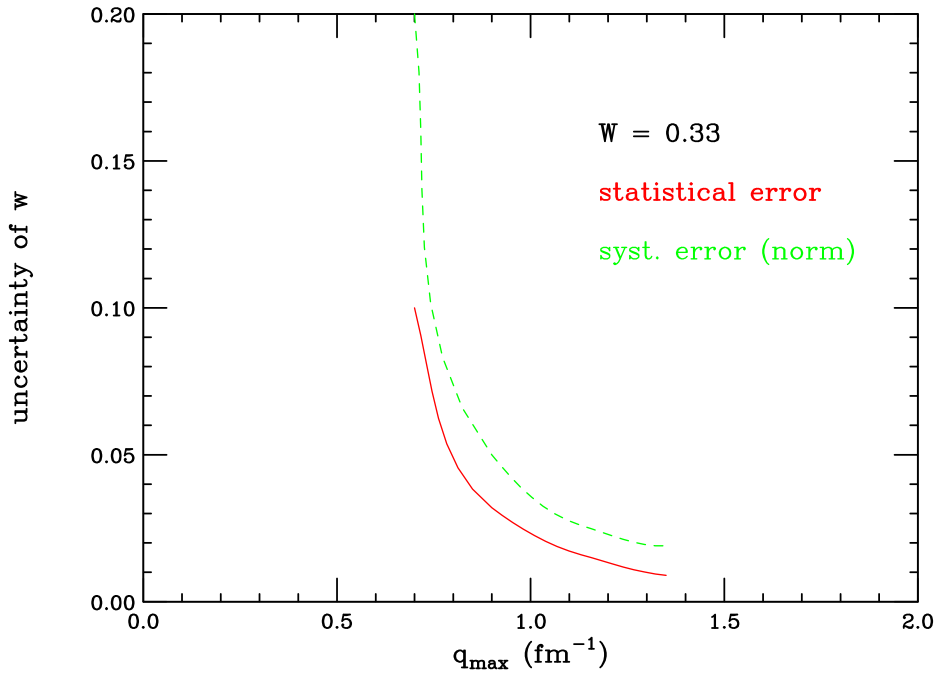
Example: ^{208}Pb: $w = 0.33 \rightarrow$ depression?? Not there in modelindependent ρ [11]



w essentially compensates too fast fall-off of Gaussian distribution

Sensitivity to w with 3PG (phase-shift calculation)

use data with 2% stat. errors, up to q_{max}
fit with 3PG
derive uncertainly of w, syst. error due to norm change



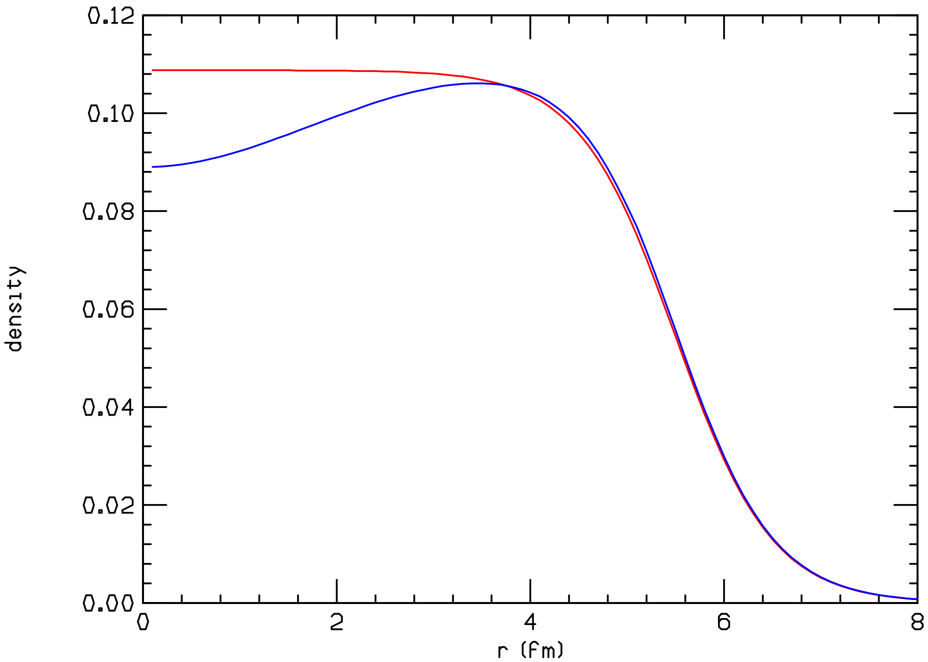
w with precise data to $q = 1.5fm^{-1}$ measurable
but: represents depression, or shape $\rho(r)$ at large r?

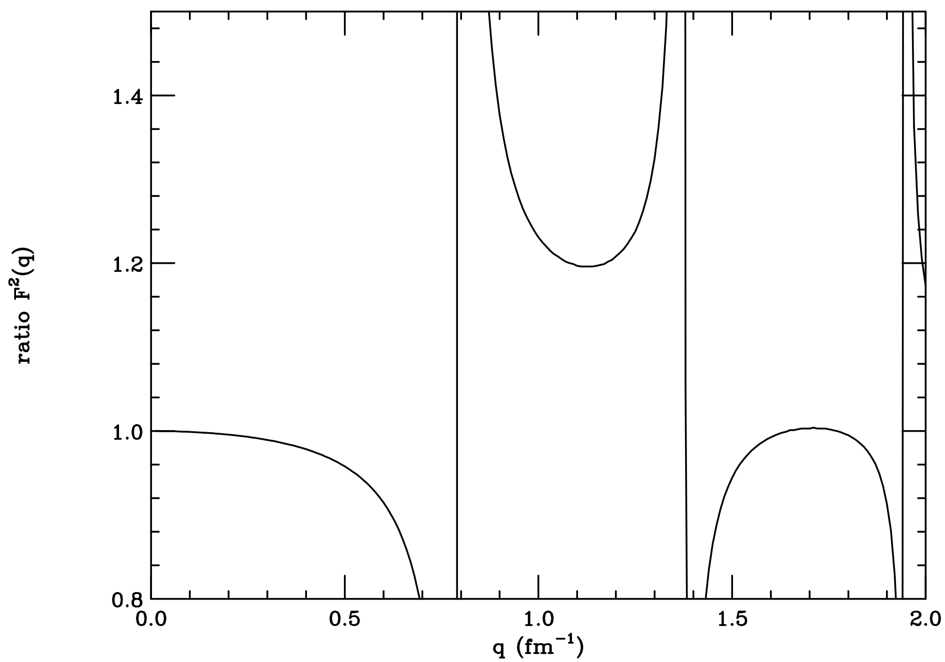
My conclusion: w mainly used to affect large-r tail, *not* central depression
$\rightarrow$ keep away from $w \sim$ central depression
w affects *both* small and large r

Better parameterization of eventual central dip:

$$\begin{aligned}\rho(r) &= 2PF - ae^{-r^2/b^2} \quad \text{for } r \leq c \\ &= 2PF \qquad\qquad\quad\; \text{for } r \geq c\end{aligned}$$

Result for $\rho(r)$ and F^2-ratio in PWBA





Fairly characteristic change in F^2

increase of F^2 in 2. maximum, little change in third maxi
comparatively little change below first mini, above second mini

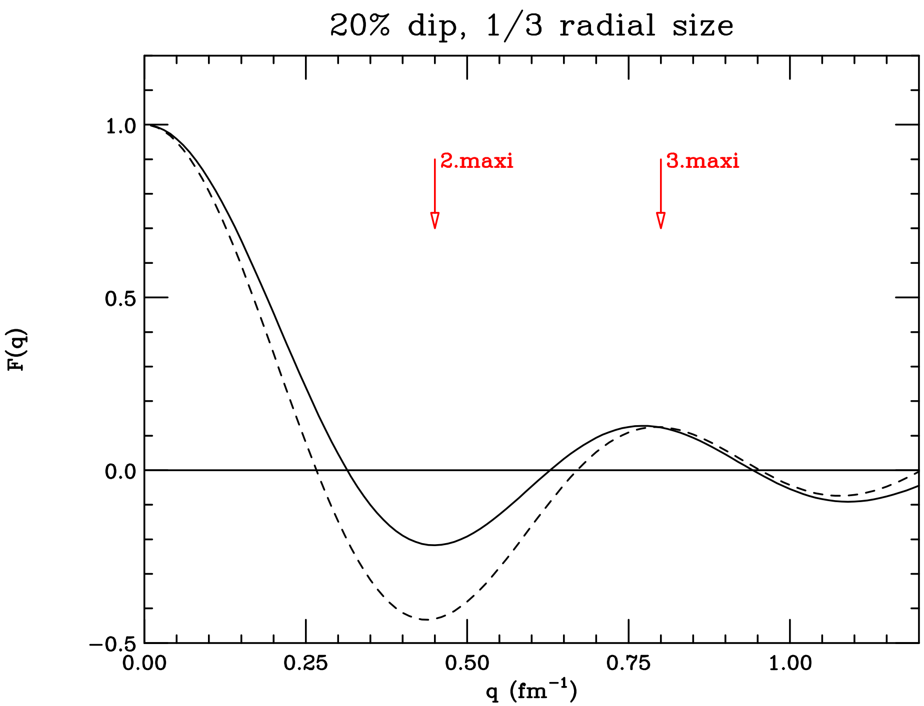
Measurable if can reach third maxi of F^2

Qualitative understanding from q-dependent term in integrand for F(q)

compare for $r_1 = 0.3 r_0$

$sin(qr_0)/qr_0$

$0.8\ sin(qr_0)/qr_0 + 0.2\ sin(qr_1)/qr_1$



Confirms: main signal of central dip = enhancement in second maxi of F(q)

should be within reach of experiment

Light vs. heavy nuclei

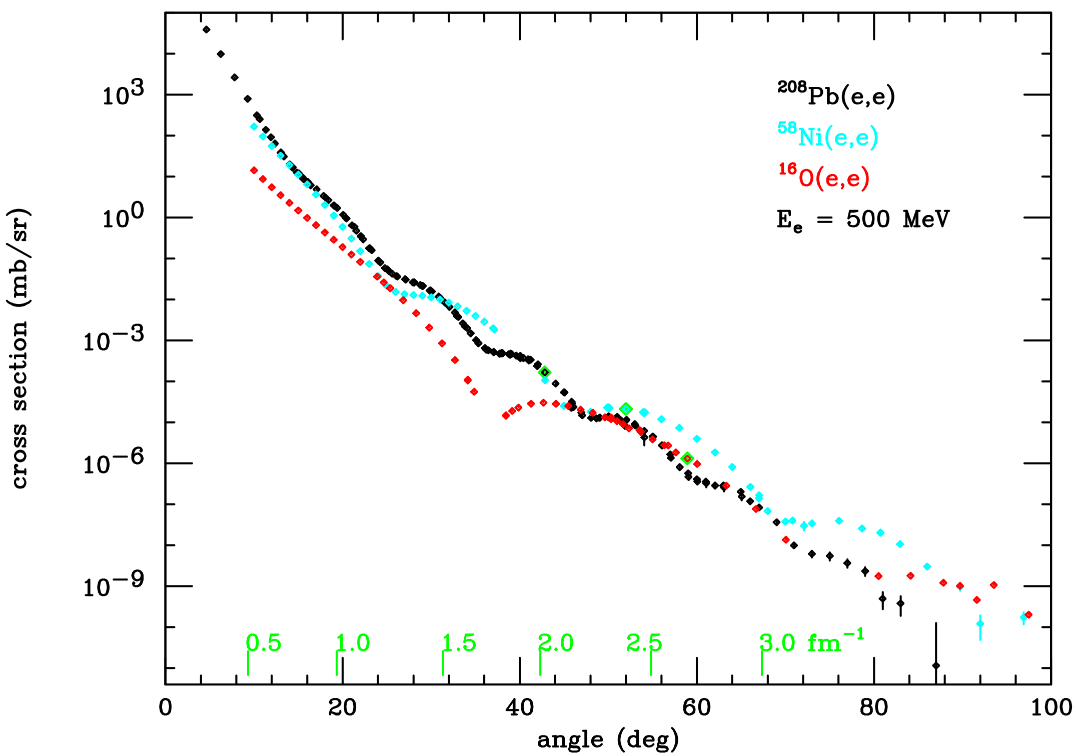
above studies mainly performed for A=208

how about lighter nuclei?

Fall-off of $d\sigma/d\Omega$

quite similar for different A

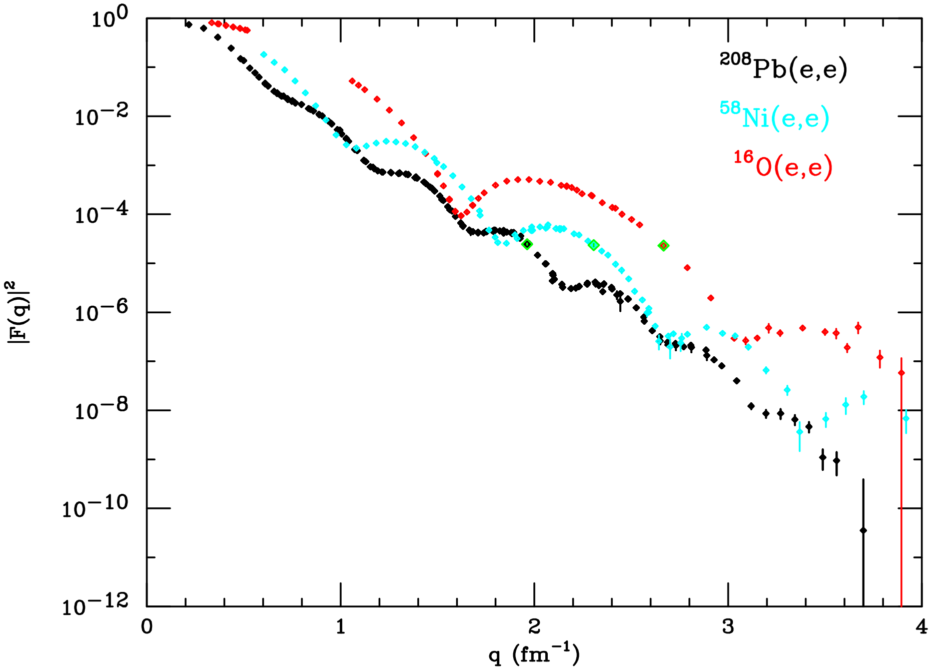
somewhat slower for small A



Relevant for accuracy of $\rho(r)$:

smallest $F(q)$ measured, not q or number of diffraction maxima

$$\rho(r, q_{max}) = \int_0^{q_{max}} F(q) \, \frac{sin(qr)}{qr} \, q^2 dq$$



For small A

need larger q to get same, small $F(q)$
need larger q for same completeness error $\rightarrow$ somewhat lower σ
determination of c, z, 'w' somewhat more difficult

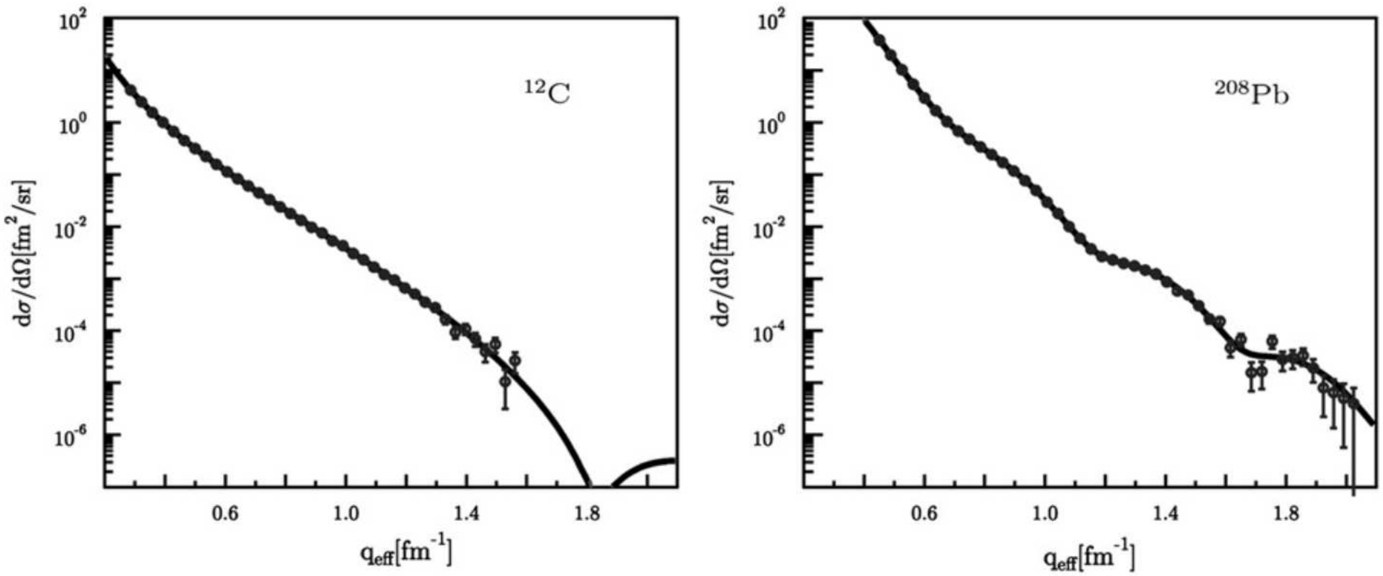
Need/can determine further integral quantities?

can determine *more* than c, z, dip?

depends strongly on achievable luminosity L
depends on quality of e-spectrometer
evaluation beyond scope of present study

Use result of study for ELISe (Antonov *et al.* [12])

assume L $= 10^{28}$, 100msr, 4 weeks

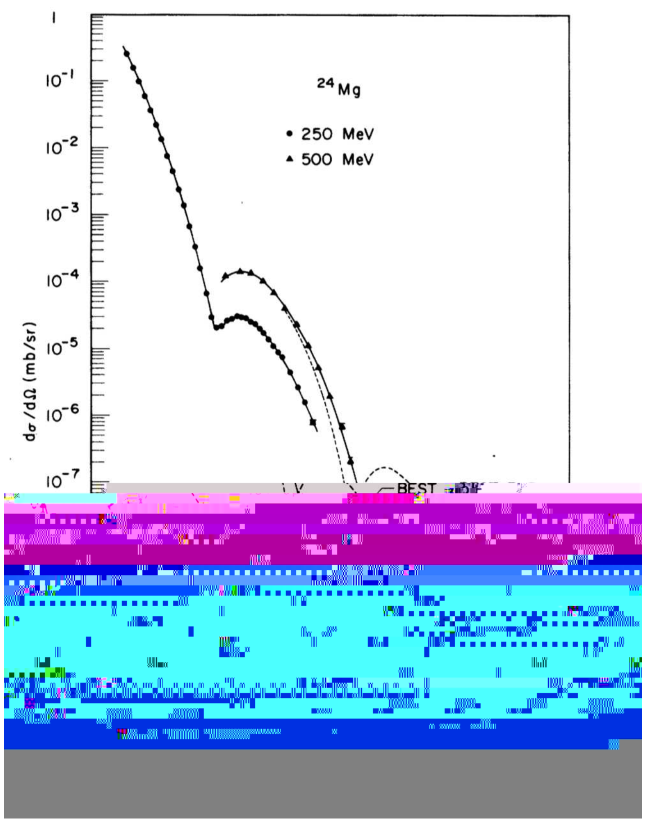


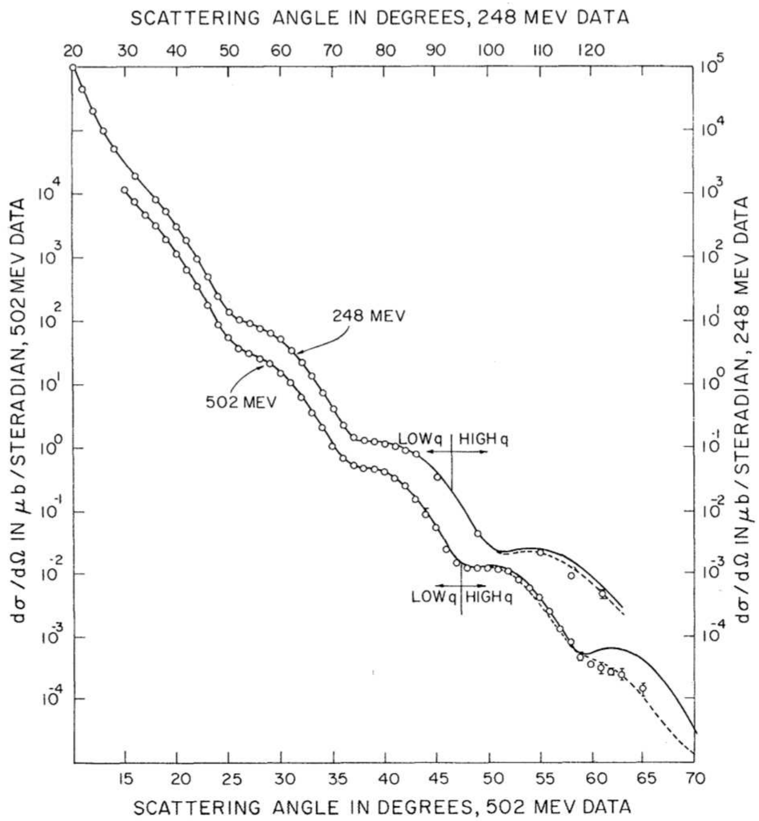
$q_{max} \sim 1.5(2.0) fm^{-1}$ seems to be limit for A=12(208)

Implications for quantities that can be determined

experience from past fits to stable isotope data:
3PF, 3PG typically flexible enough for $q_{max} \sim 2fm^{-1}$

Examples: Mg, Pb [13, 14]





Data from ETIC-type collider can be described using 3pF, 3pG, ...

For exotic nuclei can determine R, z, central dip, large-r tail

(e,e) and trapped ions: conclusions

great interest + potential

Limitations due to achievable luminosity

interpretation of data with models for $\rho(r)$
extraction of integral properties: R, z, dip, halo
look feasible given present estimates for L

Precise results for rms-radius R

need precise (absolute) cross sections
only achievable via *ratio* measurements to stable isotopes
requires precise monitors for (relative) luminosity

For precise extraction of R

use realistic large-r tail of ρ (not properly done in past)
reanalyze data for (stable) reference nucleus

For nuclei far from valley

must expect halo's, *i.e.* long large$r$ tail
parameterize as halo (R(r) from WS potential) + model-density
emphasizes interest in measuring SE via precise masses, (e,e'p)

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