Energy density functional studies of elastic and inelastic scattering off nuclei

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work carried out while being at the

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Workshop on

Electron-radioactive ion collisions: theoretical and experimental challenges Espace de Structure Nucléaire et réactions Théorique, Saclay, 26 April 2016



Bottom lines

- Method: Symmetry-restored Generator Coordinate Method based on deformed HFB states using an effective Skyrme interaction.
- Goal: study the impact of deformation, shape fluctuations and shape mixing (which all are "long-range" correlations) on observables.
- no in-medium ("short-range") correlations that are explicitly dealt with.

Outline

- What is a symmetry-restored Generator Coordinate Method based on deformed HFB states and why using it.
- Calculation of densities and transition densities in the laboratory frame – proof of principle.
- From simple to increasingly complicated MR schemes
 - possibilities for the future.

I assume you have heard about Hartree-Fock and Hartree-Fock Bogoliubov.

 \Rightarrow Single-Reference (SR) Energy Density Functional (EDF) Methods

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I assume you have heard about Hartree-Fock and Hartree-Fock Bogoliubov.

 \Rightarrow Single-Reference (SR) Energy Density Functional (EDF) Methods

I will talk about mixing HF & HFB states

 \Rightarrow Multi-Reference (MR) Energy Density Functional (EDF) Methods

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Horizontal vs. vertical expansion of correlations



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Horizontal vs. vertical expansion of correlations







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F. Dönau et al, NPA496 (1989) 333.

particle-number projector



angular-momentum restoration operator

rotation in real space

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$$\hat{P}_{MK}^{J} = \frac{2J+1}{16\pi^2} \int_0^{4\pi} d\alpha \int_0^{\pi} d\beta \, \sin(\beta) \int_0^{2\pi} d\gamma \, \underbrace{\mathcal{D}_{MK}^{*J}(\alpha,\beta,\gamma)}_{Wigner \, function} \quad \widehat{\hat{R}(\alpha,\beta,\gamma)}$$

 ${\cal K}$ is the z component of angular momentum in the body-fixed frame. Projected states are given by

$$|JMq
angle = \sum_{K=-J}^{+J} f_J(K) \ \hat{P}^J_{MK} \ \hat{P}^Z \ \hat{P}^N |\mathsf{MF}(q)
angle = \sum_{K=-J}^{+J} f_J(K) \ |JM(qK)
angle$$

 $f_J(K)$ is the weight of the component K and determined variationally

Axial symmetry (with the z axis as symmetry axis) allows to perform the α and γ integrations analytically, while the sum over K collapses, $f_J(K) \sim \delta_{K0}$

Configuration mixing by the symmetry-restored Generator Coordinate $\ensuremath{\mathsf{Method}}$

Superposition of projected self-consistent mean-field states $|\mathsf{MF}(q)\rangle$ differing in a set of collective and single-particle coordinates q

$$|NZJM\nu\rangle = \sum_{\mathbf{q}} \sum_{K=-J}^{+J} f_{J,\kappa}^{NZ}(\mathbf{q},K) \hat{P}_{MK}^{J} \hat{P}^{Z} \hat{P}^{N} |\mathsf{MF}(\mathbf{q})\rangle = \sum_{\mathbf{q}} \sum_{K=-J}^{+J} f_{J\nu}^{NZ}(\mathbf{q},K) |NZJM(\mathbf{q}K)\rangle$$
with weights $f_{J\nu}^{NZ}(\mathbf{q},K)$.

$$\frac{\delta}{\delta f_{J\nu}^{*}(\mathbf{q},K)} \frac{\langle NZ JM\nu | \hat{H} | NZ JM\nu \rangle}{\langle NZ JM\nu | NZ JM\nu \rangle} = 0 \quad \Rightarrow \quad \text{Hill-Wheeler-Griffin equation}$$

$$\sum_{\mathbf{q}'}\sum_{\mathbf{K}'=-J}^{+J} \left[\mathcal{H}_{J}^{NZ}(\mathbf{q}\mathbf{K},\mathbf{q}'\mathbf{K}') - E_{J,\nu}^{NZ} \mathcal{I}_{J}^{NZ}(\mathbf{q}\mathbf{K},\mathbf{q}'\mathbf{K}') \right] f_{J,\nu}^{NZ}(\mathbf{q}'\mathbf{K}') = 0$$

with

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$$\begin{aligned} \mathcal{H}_{J}(\mathbf{q}K,\mathbf{q}'K') &= \langle NZ \; JM \; \mathbf{q}K | \hat{H} | NZ \; JM \; \mathbf{q}'K' \rangle & \text{energy kerne} \\ \mathcal{I}_{J}(\mathbf{q}K,\mathbf{q}'K') &= \langle NZ \; JM \; \mathbf{q}K | NZ \; JM \; \mathbf{q}'K' \rangle & \text{norm kernel} \end{aligned}$$

Angular-momentum projected GCM gives the

- \blacktriangleright correlated ground state for each value of J
- spectrum of excited states for each J

- full space of occupied single-particle states
- effective Skyrme interactions / energy density functionals
- "HF+BCS" or "HFB" solved with two-basis method
- Coordinate space representation on a 3d mesh using Lagrange-mesh technique



FIG. 9. (Color online) Radial density profile of 34 Ne in different box sizes with dx = 0.8 fm.



FIG. 10. (Color online) Absolute difference between the total rms radii calculated on a 3D mesh with respect to those of LENTEUR as a function of the step size dx for the spherical nuclei ⁴⁰Ca (a). ¹¹²Sn (b), and ²⁰⁰Pb (c) and those calculated with the Skyrme parametrizations as indicated.

W. Ryssens, P.-H. Heenen, M. B., PRC 92 (2015) 064318



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

Spectroscopy from MR EDF



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

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Attention: $g_i^2(q)$ is not the probability to find a mean-field state with intrinsic deformation q in the collective state

M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303. Experiment: T. Grahn *et al*, Phys. Rev. Lett. **97** (2006) 062501



- in-band and out-of-band E2 transition moments directly in the laboratory frame with correct selection rules
- full model space of occupied particles
- only occupied single-particle states contribute to the kernels ("horizontal expansion")
- \blacktriangleright \Rightarrow no effective charges necessary
- no adjustable parameters

$$B(E2; J'_{\nu'} \to J_{\nu}) = \frac{e^2}{2J'+1} \sum_{M=-J}^{+J} \sum_{M'=-J'}^{+J'} \sum_{\mu=-2}^{+2} |\langle JM\nu | \hat{Q}_{2\mu} | J'M'\nu' \rangle|^2$$
$$\beta_2^{(t)} = \frac{4\pi}{3R^2A} \sqrt{\frac{B(E2; J \to J-2)}{(J020|(J-2)0)^2 e^2}} \quad \text{with} \quad R = 1.2 A^{1/3}$$

Spherical nuclei don't stay spherical "beyond the mean field"



M. B., B. Avez, B. Bally, T. Duguet, P.-H. Heenen, D. Lacroix, unpublished

charge radii: Experimental signatures of shape mixing



M. B., G. F. Bertsch, P.-H. Heenen, PRC 78 (2008) 054312

T. Cocolios, ..., M. B., P. H. Heenen PRL 106 (2011) 052503.

Transition matrix element for tensor operator $\hat{T}^\lambda_\mu = \hat{a}^\dagger_{\bf r} \hat{a}_{\bf r} \; T^\lambda_\mu ({\bf r})$

$$\langle J_f M_f \nu_f | \hat{T}^{\lambda}_{\mu} | J_i M_i \nu_i \rangle = \int d^3 r \, \rho^{J_f M_f \nu_f}_{J_i M_i \nu_i}(\mathbf{r}) \, T^{\lambda}_{\mu}(\mathbf{r}) \, .$$

Transition density in the laboratory between GCM states $|J_iM_i\nu_i\rangle$ and $|J_fM_f\nu_f\rangle$ assuming axial HFB states

$$\rho_{J_{i}M_{i}\nu_{i}}^{J_{f}M_{i}\nu_{f}}(\mathbf{r}) = \sum_{q_{f},q_{i}} f_{\nu_{f},q_{i}}^{J_{f}*} \langle q' | \hat{P}_{0M_{f}}^{J_{f}} \hat{\rho}(\mathbf{r}) \hat{P}_{0M_{i}}^{J_{i}\dagger} \hat{P}^{N} \hat{P}^{Z} | q \rangle f_{\nu_{i},q}^{J_{i}0}$$

with

$$\begin{aligned} \langle q' | \hat{P}_{0M_{f}}^{J_{f}} \hat{\rho}(\mathbf{r}) \hat{P}_{0M_{i}}^{J_{i}\dagger} \hat{P}^{N} \hat{P}^{Z} | q \rangle \\ &= \frac{\hat{J}_{i}^{2} \hat{J}_{f}^{2}}{(8\pi^{2})^{2}} \int d\Omega' \ D_{0M_{f}}^{J_{f}*}(\Omega') \sum_{K} D_{K0}^{J_{i}}(\Omega') \int d\Omega'' \ D_{0K}^{J_{i}}(\Omega'') \langle q' | \hat{\rho}(\tilde{\mathbf{r}}_{\Omega'}) \hat{P}^{N} \hat{P}^{Z} \hat{R}^{\dagger}(\Omega'') | q \rangle \\ &\equiv \frac{\hat{J}_{f}^{2}}{8\pi^{2}} \int d\Omega' \ D_{0M_{f}}^{J_{f}*}(\Omega') \sum_{K} D_{KM_{i}}^{J_{i}}(\Omega') \hat{R}^{\dagger}(\Omega') \rho_{q'q}^{J_{f}J_{i}K0}(\mathbf{r}) \end{aligned}$$

For the density of the GCM state |JM
u
angle one obtains

$$\rho_{JM\nu}^{JM\nu}(\mathbf{r}) = \sum_{q_f,q_i} f_{\nu,q'}^{J*} f_{\nu,q}^{J0} \sum_{\lambda} Y_{\lambda 0}(\hat{\mathbf{r}}) \langle JM\lambda 0 | JM \rangle \sum_{K} \langle J0\lambda K | JK \rangle \int d\hat{\mathbf{r}}' \rho_{q'q}^{JJK0}(\mathbf{r},\hat{\mathbf{r}}') Y_{\lambda K}^*(\hat{\mathbf{r}}')$$

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Pioneering mean-field work for ground states of spherical nuclei

PRASICAL REVIEW C.

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VOLUME 1. NUMBER 4

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VOLUME 5, NUMBER 3

Hartree-Fock Calculations with Skyrme's Interaction. I. Spherical Nuclei*

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and D, M, Brick Department of Theoretical Physics, University of Oxford, Oxford, United Ringdom disceived 15 Sceenber 1971)



FIG. 6. Charge distribution of ²⁸⁸Pb calculated with interactions I and II. The experimental charge distribution is that of Ref. 26



VOLUME 21. NUMBER 4.

FIG. 13. The proton distribution p, as well as the charge density distribution per are plotted together with the density extracted from electron scattering experiment. In the lower part of the figure the contribution to the central density originating from the three s1/2 HF states is also displayed.

PHYSICAL REVIEW C

APRIL 1980

JANUARY 1972

VOLUME 5, NUMBER 1 Relativistic Self-Consistent Meson Field Theory of Soberical Nuclei





FIG. 1. Comparison of theoretical nuclear charge distributions with experimental (analytic) curves,

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Image: A matrix

- ... using approximate angular-momentum projection ("rotational model"):
 - > Z. Zaringhalam and J. W. Negele, Nucl. Phys. A 288, 417 (1977).
 - E. Moya de Guerra and A. E. L. Dieperink, Phys. Rev. C 18, 1596 (1978).
 - E. Moya de Guerra and S. Kowalski, Phys. Rev. C 20, 357 (1979)
 - E. Moya de Guerra and S. Kowalski, Phys. Rev. C 22, 1308 (1980).
 - E. Moya de Guerra, Ann. Phys. (NY) 128, 286 (1980).
 - M. Nishimura, D. W. L. Sprung, and E. Moya De Guerra, Phys. Lett. B 161, 235 (1985).
 - E. Moya de Guerra, Phys. Rep. 138, 293 (1986).
 - E. Graca, P. Sarriguren, D. Berdichevsky, D. W. L. Sprung, E. Moya De Guerra, M. Nishimura, Nucl. Phys. A 483, 77 (1988).
 - D. Berdichevsky, P. Sarriguren, E. Moya de Guerra, M. Nishimura, and D. W. L. Sprung, Phys. Rev. C 38, 338 (1988).
- ... or using exact projection of HF states in a schematic single-particle basis:
 - Y. Abgrall, P. Gabinski, and J. Labarsouque, Nucl. Phys. A 232, 235 (1974).

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PHYSICAL REVIEW C 86, 014310 (2012)

Beyond-mean-field study of the possible "bubble" structure of ³⁴Si

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PHYSICAL REVIEW C 91, 024301 (2015)

Beyond-mean-field study of elastic and inelastic electron scattering off nuclei

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Laboratory densities of ³⁴Si



FIG. 1. (Color online) Energy curves for the particle-numberprojected HFB states (N&Z) and particle-number and angularmomentum projected states (J = 0, 2, 4, 6 curves) for ³⁴Si as a function of the intrinsic quadrupole deformation of the mean-field states they are projected from. The solid square dots correspond to the lowest GCM solutions, which are plotted at their average deformation $\sum_{\alpha} q |g'_{\alpha}(q)|^2$ (see text).



FIG. 5. (Color online) Neutron, proton, and total radial densities at x = y = 0 for ³⁴Si for the spherical HF state (a), the spherical HFB +LN state (b), and its projection on good particle numbers (c), as well as for the GCM 0⁺₁ ground state (d). Neutron and proton densities have been rescaled with the factors A/N and A/Z, respectively.

J. M. Yao, S. Baroni, M. B., P.-H. Heenen, PRC 86 (2012) 014310

Laboratory densities of ³⁴Si







FIG. 7. (Color online) Comparison of point-proton densities (a) with the folded charge densities (b) for 34 Si for the same states as in Fig. 5.

J. M. Yao, S. Baroni, M. B., P.-H. Heenen, PRC 86 (2012) 014310

Follow-up work using the relativistic mean-field model

Physics Letters B 723 (2013) 459-463

Does a proton "bubble" structure exist in the low-lying states of ³⁴Si?

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Fig. 4. (Color online.) Comparison of proton density distributions of ground state from both mean-field and beyond mean-field calculations for ³⁴5i. The nonrelativistic results, taken from Ref. [9], are given for comparison.



Fig. 5. (Color online.) Charge density distributions in ^{34}Si and ^{36}S from the relativistic calculations using the PC-PK1 force. The experimental data for ^{36}S are taken from Ref. [24].

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Elastic and inelastic scattering off ²⁴Mg – densities



FIG. 1. (Color online) (a) Total energy (normalized to the 0 $\hat{\eta}^*$ state) for the particle-number-projected HFB states (N&Z) and for the particle-number and angular-momentum-projected states (turves for J = 0, 2, and 4) for ^{3M}Mg as a function of the intrinsic mass quadrupole deformation of the mean-field states. The solid squares indicate the lowest GCM solutions, which are plotted at their average deformation $\hat{\beta}_{J\mu}$. (b) Collective wave functions $g'_{\mu,q}$ [cf. Eq. (5)] of the 0 $\hat{\eta}^*$, 2 $\hat{\eta}^*$, and 4 $\hat{\eta}^*$ states.

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FIG. 3. (Color online) Contour plots of calculated TPD $\rho_{01}^{\alpha'}(\mathbf{r})$, Eq. (21), in fm⁻³ in the y = 0 plane for the inelastic scattering from the ground state to the 2_1^+ (a) and the 4_1^+ (b) states with M = 0 in ²⁴Mg.



FIG. 4. (Color online) Elastic C0 form factor $|F_0(q)|^2$ for the 0_1^+ ground state of ²⁴Mg, in comparison with several calculations: he C0 form factor obtained by particle-number and J = 0 projection of a single HFB state with either $B_2 = 0$ (spherical shape; light blue dash-dotted curve) or $\beta_2 = 0.55$ (minimum of J = 0 projected energy curve; dark blue dashed curve) and from the full projected GCM calculation (red solid curve). The inset shows the corresponding charge density. Data (solid triangles and circles) are taken from Ref. [89].

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FIG. 5. (Color online) Longitudinal C2 form factor $|F_2(q)|^2$ for the transition from the ground state to the 2_1^+ state for ${}^{34}Mg$, in comparison with available data. The form factor calculated with only one single configuration of $\beta_2 = 0.55$ and the form factor of the transition proton density from full GCM calculations are given for comparison. The inset shows the corresponding transition densities. Data are taken from Ref. [90] (squares) and Ref. [89] (circles and triangles).

Elastic and inelastic scattering off ⁵⁸Ni



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FIG. 10. (Color online) Data for the elastic form factor $|F_0(q)|^2$ for the ground state of ⁵⁸Ni taken from Ref. [99] in comparison with the form factor obtained from four different calculations: projection of a single HFB configuration with either $\beta_2 = 0$ (spherical shape) or $\beta_2 = \mp 0.21$ (oblate and prolate minima of the J = 0 energy curve) and full GCM of projected states.



FIG. 11. (Color online) Comparison between the charge distribution of the ground state of ⁵⁸Ni obtained using single projected mean-field configurations or the full GCM basis and the experimental data [100]. A previous calculation using a one-dimensional Bohr Hamiltonian based on an HFB calculation with the Gogny DI force (1DBH) [101] is also shown. The insets magnify the profile of the charge density at very small radii and in the nuclear surface.

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FIG. 12. (Color online) (a) Elastic form factor $|F_0(q)|^2$ for the J = 0 state of ⁵⁸Ni projected from a single HFB configuration with prolat deformation of β_2 increasing from 0.0 to 0.7, respectively. (b) Charge distributions corresponding to the form factors displayed in panel (a).

Elastic and inelastic scattering off Ni isotopes





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FIG. 18. (Color online) Calculated inelastic Coulomb form factors $|F_L(q)|^{-1}$ for the transition from the ground state to the J_1^+ (L = J = 2.4) state in ^{34–6}Ni, in comparison with available data, taken from Ref. [105] (up triangles), Ref. [106] (squares and diamonds), Ref. [107] (vices), and Ref. [108] (left) and right triangles).

- X. Y. Wu, J. M. Yao, and Z. P. Li, Phys. Rev. C 89, 017304 (2014).
- H. Mei, K. Hagino, J. M. Yao, and T. Motoba, Phys. Rev. C 90, 064302 (2014).
- X.-Y. Wu and X.-R. Zhou, Phys. Rev. C 92, 054321 (2015).
- Y. Fukuoka, S. Shinohara, Y. Funaki, T. Nakatsukasa, and K. Yabana, Phys. Rev. C 88, 014321 (2013).

What could be done in the future: triaxiality, odd-A nuclei,



Angular-momentum and particle-number projected GCM of blocked triaxial one-quasiparticle states



- B. Bally, doctoral thesis, Université de Bordeaux (2014)
- B. Bally, B. Avez, M. B., and P.-H. Heenen, PRL 113 (2014) 162501

Take-away messages

- MR techniques provide useful tools to describe correlations related to the finiteness and self-boundedness of atomic nuclei
- symmetry restoration of symmetry-breaking reference states
- GCM-type mixing of (symmetry-restored) states
- Construction of laboratory densities, transition densities, and their form factors is feasible.

Questions to the audience

- What would be an interesting extension?
 - non-axiality?
 - single-particle excitations?
 - odd nuclei?
 - currents related to magnetic excitations?

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