

# Internally contracted multireference coupled-cluster theory

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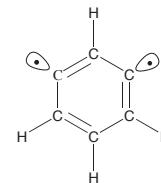
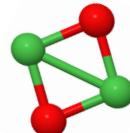
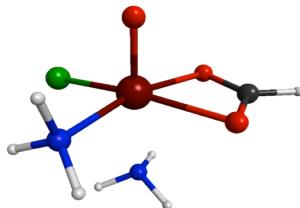
ESNT Workshop 2015 at CEA Saclay



# Target now: Molecules

Actually: Electrons in the field of (point-like) nuclei fixed in space

(Keywords: Born-Oppenheimer approximation, clamped nuclei approximation)



We solve for (mostly) stationary states of the electronic (non-relativistic) Hamiltonian

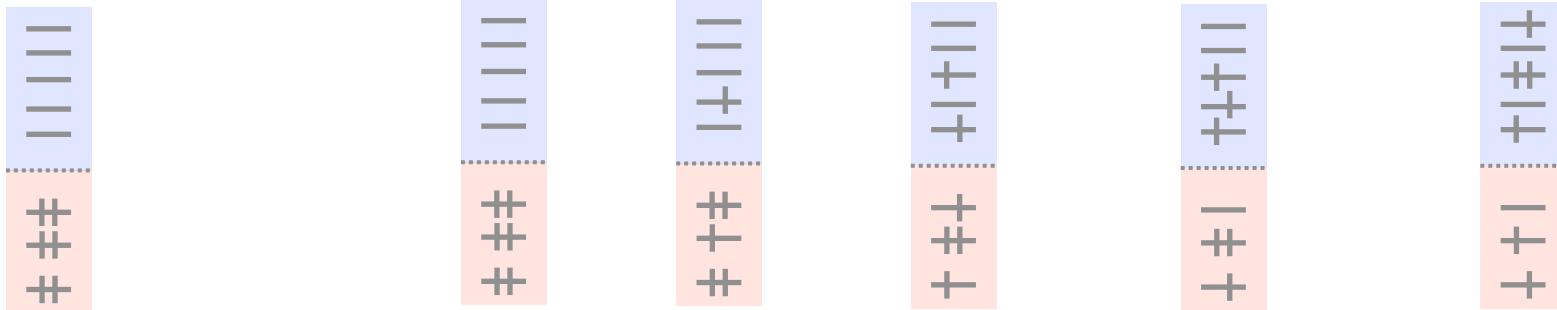
$$h_p^q = \langle p | \left( -\frac{1}{2} \Delta - \sum_I \frac{Z_I}{|\mathbf{r}_1 - \mathbf{R}_I|} \right) | q \rangle \quad g_{pr}^{qs} = \langle pr | \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} | qs \rangle$$

$$\hat{H} = \sum_{pq} h_p^q a_p^\dagger a_q + \frac{1}{4} \sum_{pqrs} g_{pr}^{qs} a_p^\dagger a_r^\dagger a_s a_q = h_p^q a_q^p + \frac{1}{4} g_{pr}^{qs} a_{qs}^{pr}$$

creation/annihilation of spin-orbitals

short-hand notation

# Many-electron states in molecules



$$|\Phi_0\rangle \implies |\Psi\rangle = c_0|\Phi_0\rangle + c_a^i|\Phi_i^a\rangle + c_{ab}^{ij}|\Phi_{ij}^{ab}\rangle + c_{abc}^{ijk}|\Phi_{ijk}^{abc}\rangle + c_{abcd}^{ijkl}|\Phi_{ijkl}^{abcd}\rangle + \dots$$

$$\propto |\Phi_0\rangle + |\Psi_{\text{korr}}\rangle$$

Coupled-Cluster (CC) ansatz:

$$|\Psi_{\text{CC}}\rangle = e^{\hat{T}}|\Phi_0\rangle = |\Phi_0\rangle + \hat{T}|\Phi_0\rangle + \frac{1}{2}\hat{T}^2|\Phi_0\rangle + \dots$$

 Coester, Kümmel (1958)  
 Čížek, Paldus (1966)  
 Bartlett (1978)

# Coupled-cluster expansion

**Ansatz:**

$$|\Psi_{CC}\rangle = e^{\hat{T}} |\Phi_0\rangle \quad \text{single determinant}$$

$$\hat{T} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 + \dots$$

**Features:**

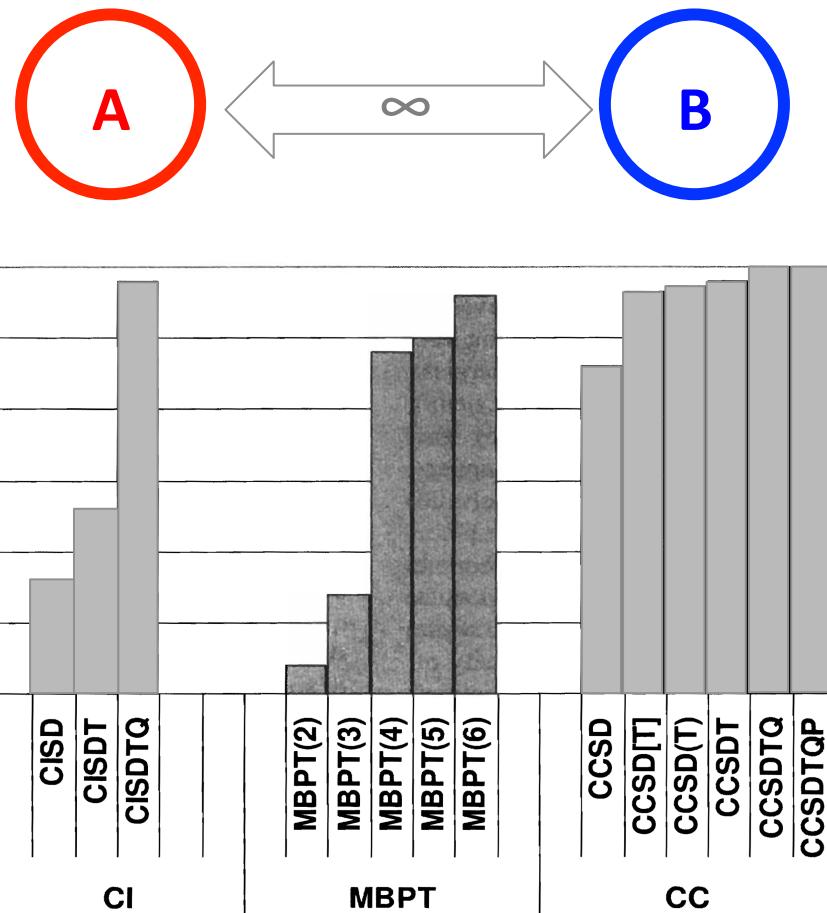
**separability**

$$\begin{aligned} |\Psi_{CC}^{AB}\rangle &= e^{\hat{T}_A + \hat{T}_B} |\Phi_0^{AB}\rangle \\ &= \hat{\mathcal{A}}\left(e^{\hat{T}_1} |\Phi_0^A\rangle\right)\left(e^{\hat{T}_2} |\Phi_0^B\rangle\right) \\ &= \hat{\mathcal{A}}|\Psi_{CC}^A\rangle|\Psi_{CC}^B\rangle \end{aligned}$$

$$E^{AB} = E^A + E^B$$

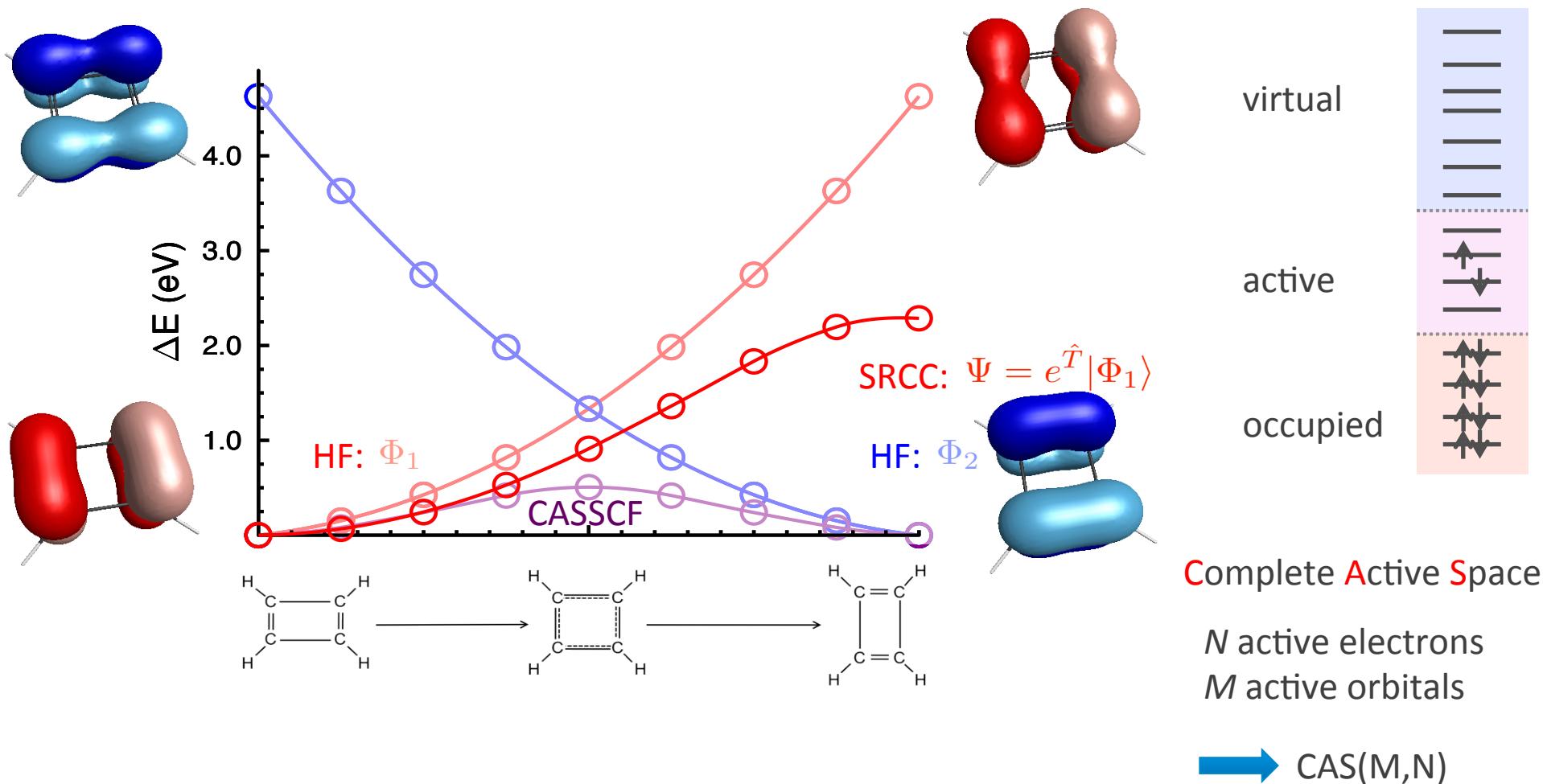
**fast convergence with cluster rank**

Percentage of  $E_{\text{corr}}$  recovered for set of small molecules  
Bartlett, Mol Phys 108, 2905 (2010)



# Multireference cases

Example: Automerization of cyclobutadiene



# Overview: Different approaches

$$|\Psi_{CC}\rangle = e^{\hat{T}} |\Phi_0\rangle \quad \xrightarrow{\hspace{1cm}} \quad |\Psi_{\text{ext-SRCC}}\rangle = e^{\hat{T}_{\text{ex}}} e^{\hat{T}_{\text{int}}} |\Phi_0\rangle$$

Adamowicz (1992), Piecuch, Kallay, Olsen, Paldus, Bartlett, ...

$$|\Psi_{ic}\rangle = e^{\hat{T}} \sum_{\mu} |\Phi_{\mu}\rangle c_{\mu}$$

[Cicek, 1967], Mukherjee (1975)

## Valence universal: Fock-Space MRCC

Haque, Mukherjee (1984)  
Stolarszyk, Monkhorst (1985)  
Kaldor (1986)

recent work by: Musiał, Bartlett

$$|\Psi_{JM}\rangle = \sum_{\mu} e^{\hat{T}(\mu)} |\Phi_{\mu}\rangle c_{\mu}$$

Jeziorski, Monkhorst (1981)

## State-universal

Piecuch, Jeziorski; Kucharski, Bartlett; Li, Paldus

## State-specific: ic-MRCC

Banerjee, Simons (1981)  
Nooijen et al. (2001, 2011)  
Olsen (2005)  
Evangelista, Gauss (2011)  
Hanauer, AK (2011)

compare: CT theory

Chan, Yanai (2006)

## State-specific

Mášik, Hubač (1998)  
Pittner et al. (2006)  
Mahapatra, Datta, Mukherjee (1998)  
Evangelista, Allen, Gauss et al. (2007)  
Das, Kallay, Mukherjee (2009)  
Hanrath (2005)

## Reviews about MRCC: For example ...

Lyakh, Musiał, Lotrich, Bartlett, Chem. Rev. 112, 182 (2012)  
Köhn, Hanauer, Mück, Jagau, Gauss, WIREs CMS 3, 176 (2013)

# Design of a multireference CC theory

Single reference:

$$|\Phi_0\rangle$$



$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle$$

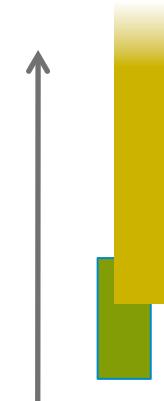
State-universal ansatz: (Jeziorski, Monkhorst)

$$\{|\Phi_\mu\rangle\}$$



$$|\Psi_{JM}\rangle = \sum_\mu e^{\hat{T}(\mu)} |\Phi_\mu\rangle c_\mu$$

Intruder state problems: State-specific versions



State-specific internally contracted ansatz

$$|\Psi_0\rangle = \sum_\mu |\Phi_\mu\rangle c_\mu$$



$$|\Psi_{ic}\rangle = e^{\hat{T}} \sum_\mu |\Phi_\mu\rangle c_\mu$$

+ Simpler, better control of intruders

- Questionable at state crossings (!)

# Internally contracted MRCC: Why?

$$|\Psi_{\text{ic}}\rangle = e^{\hat{T}} \sum_{\mu} |\Phi_{\mu}\rangle c_{\mu}$$

“internally contracted”: originally used in CI context by Siegbahn, Mayer, Werner (1980s)

## Promising features:

- Genuine multireference scheme (no bias within model space)
- Compact amplitude manifold (no scaling with model space size)
- Spin adaption or spin-free formulation is easy
- Orbital invariance



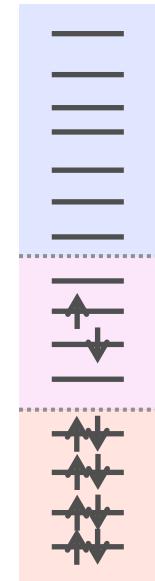
Evangelista, Gauss, JCP 134, 114102 (2011)  
Hanauer, Köhn, JCP 134, 204111 (2011)

## What is scary:

- Non-commuting operator manifold
- Linear dependencies

*Simplified approaches, see e.g.*

Banerjee, Simons, IJQC 19, 207 (1981)  
Laidig, Saxe, Bartlett, JCP 86, 887 (1987)  
Fink, Staemmler, TCA 87, 129 (1993)



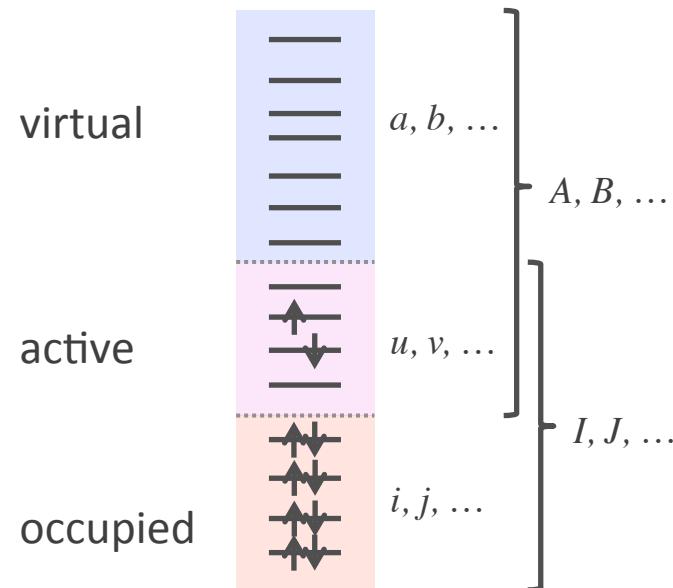
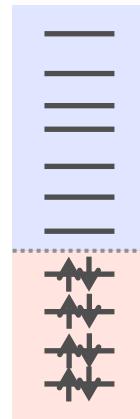
# ic-MRCC vis-à-vis single-reference CC

## Orbital space:

Hartree-Fock

virtual

occupied



CASSCF

## Reference wave function:

$$|\Phi_0\rangle$$

$$|\Psi_0\rangle = \sum_{\mu} |\Phi_{\mu}\rangle c_{\mu}$$

## Correlated wave function:

$$|\Psi\rangle = e^{\hat{T}} |\Phi_0\rangle$$

$$|\Psi_{\text{ic}}\rangle = e^{\hat{T}} |\Psi_0\rangle$$

## Cluster operator:

$$\hat{T} = \sum_{ia} t_a^i a_i^a + \frac{1}{4} \sum_{ijab} t_{ab}^{ij} a_{ij}^{ab} + \dots$$

$$\hat{T} = \sum_{IA} t_A^I a_I^A + \frac{1}{4} \sum_{IJAB} t_{AB}^{IJ} a_{IJ}^{AB} + \dots$$



# ic-MRCC vis-à-vis single-reference CC

**Correlated wave function:**

$$|\Psi\rangle = e^{\hat{T}} |\Phi_0\rangle$$

$$|\Psi_{\text{ic}}\rangle = e^{\hat{T}} |\Phi_0\rangle$$

**Cluster operator:**

$$\begin{aligned}\hat{T} &= \sum_{ia} t_a^i a_i^a \\ &+ \frac{1}{4} \sum_{ijab} t_{ab}^{ij} a_{ij}^{ab} \\ &+ \dots = \sum_{\rho} t_{\rho} \hat{\tau}_{\rho}\end{aligned}$$

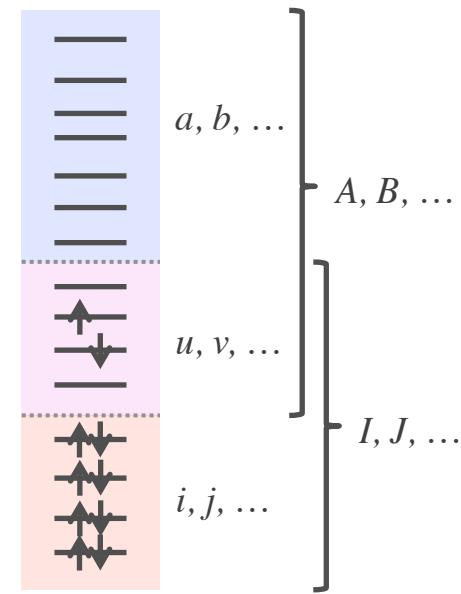
**Excitation types  
(singles&doubles):**

$$\hat{\tau}_{\rho} = \begin{cases} \hat{a}_i^a \\ \hat{a}_{ij}^{ab} \end{cases}$$

virtual

active

occupied



$$\begin{aligned}\hat{T} &= \sum_{IA} t_A^I a_I^A \\ &+ \frac{1}{4} \sum_{IJAB} t_{AB}^{IJ} a_{IJ}^{AB} \\ &+ \dots = \sum_{\rho} t_{\rho} \hat{\tau}_{\rho}\end{aligned}$$

$$\hat{\tau}_{\rho} = \begin{bmatrix} \hat{a}_i^a & \hat{a}_i^a \\ \hat{a}_{ij}^{ab} & \hat{a}_{ij}^{ab} \\ \hat{a}_{ij}^{ab} & \hat{a}_{ij}^{ab} \\ \hat{a}_{ij}^{ub} & \hat{a}_{ij}^{ub} \\ \hat{a}_{ij}^{ub} & \hat{a}_{ij}^{ub} \\ \hat{a}_{ij}^{uw} & \hat{a}_{ij}^{uw} \end{bmatrix} \begin{bmatrix} \hat{a}_i^u \\ \hat{a}_i^u \\ \hat{a}_{uj}^u \\ \hat{a}_{uj}^u \\ \hat{a}_{uj}^u \\ \hat{a}_{vj}^u \end{bmatrix}$$

avoid active-active excitations, instead:  
optimize

$$|\Psi_0\rangle = \sum_{\mu} |\Phi^{\mu}\rangle c_{\mu}$$

# ic-MRCC vis-à-vis single-reference CC

**Correlated wave function:**

$$|\Psi\rangle = e^{\hat{T}} |\Phi_0\rangle$$

$$|\Psi_{\text{ic}}\rangle = e^{\hat{T}} |\Psi_0\rangle$$

**Energy**

$$E = \langle \Phi_0 | e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Phi_0 \rangle$$

$$E = \langle \Psi_0 | e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Psi_0 \rangle_{\text{occupied}}$$

**Equations for reference function**

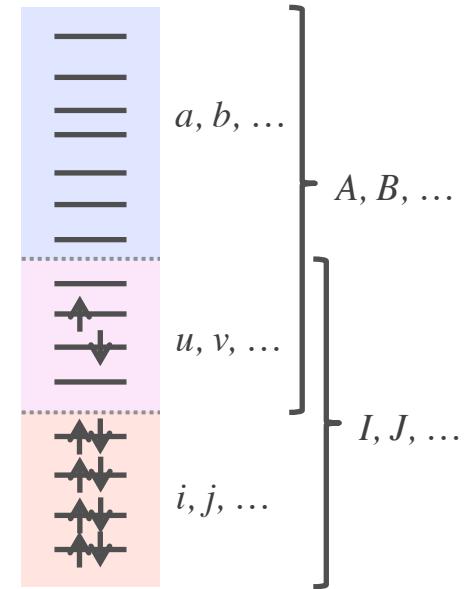
$$|\Psi_0\rangle = \sum_{\mu} |\Phi^{\mu}\rangle c_{\mu}$$

$$\sum_{\nu} \langle \Phi_{\mu} | e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Phi^{\nu} \rangle c_{\nu} = E c_{\mu} \quad \equiv \quad \hat{\mathbf{H}}^{\text{eff}} \mathbf{c} = E \mathbf{c}$$

**Amplitude equations**

$$0 = \langle \Phi_0 | (\hat{\tau}_{\rho})^{\dagger} e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Phi_0 \rangle$$

$$0 = \langle \Psi_0 | (\hat{\tau}_{\rho})^{\dagger} e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Psi_0 \rangle$$



Here it becomes  
more difficult!

# Linear dependencies

$$0 = \langle \Psi_0 | (\hat{\tau}_\rho)^\dagger e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Psi_0 \rangle$$

non-orthogonal

$$S_{\rho\sigma} = \langle \Psi_0 | (\hat{\tau}_\rho)^\dagger \hat{\tau}_\sigma | \Psi_0 \rangle \neq \delta_{\rho\sigma}$$

linear dependencies, e.g.

$$\begin{aligned}\hat{a}_{1\bar{1}}^{ab}(c_1|1\bar{1}\rangle + c_2|2\bar{2}\rangle) &= c_1|ab\rangle \\ \hat{a}_{2\bar{2}}^{ab}(c_1|1\bar{1}\rangle + c_2|2\bar{2}\rangle) &= c_2|ab\rangle\end{aligned}$$

$$\sum_u \hat{a}_{ui}^{ua} |\Psi_0\rangle = n_a \hat{a}_i^a |\Psi_0\rangle$$

Consequences:

covariant and contravariant quantities

$$\hat{A} = \sum_\rho A_\rho \hat{\tau}_\rho$$

$$\bar{A}_\rho = \langle \Psi_0 | (\hat{\tau}_\rho)^\dagger \hat{A} | \Psi_0 \rangle = \sum_\sigma S_{\rho\sigma} A_\sigma$$

projection  $\langle \Psi_0 | (\hat{\tau}_\rho)^\dagger$  has less degrees of freedom than operator basis  $\sum_\rho t_\rho \hat{\tau}_\rho$

particle ranks are not orthogonal, e.g.

$$\langle \Psi_0 | a_{vw}^{ui} a_i^w | \Psi_0 \rangle \neq 0$$

# Linear dependencies

project to orthogonalized, non-redundant basis:

technically identical to ic-MRCI, CASPT2 etc.  $S_{\rho\sigma} = \langle \Psi_0 | (\hat{\tau}_\rho)^\dagger \hat{\tau}_\sigma | \Psi_0 \rangle = S_{\rho_a \sigma_a}^{\text{act.}} \otimes 1_{\rho_i \sigma_i}^{\text{inact.}}$

$$\Rightarrow \hat{\tau}'_{\tilde{\rho}} = X_{\tilde{\rho}}^\sigma \tau_\sigma \quad \text{where} \quad \hat{\tau}' = \hat{\tau}^{\parallel} + \hat{\tau}^{\perp} \quad \text{with} \quad \hat{\tau}_{\tilde{\rho}}^{\perp} | \Psi_0 \rangle = 0$$

$$\Rightarrow \hat{T} = \hat{T}^{\parallel} + \hat{T}^{\perp}$$

we have the *freedom* to set  $\hat{T}^{\perp} = 0$  (will still converge to full CI)\*

\*Evangelista, Hanauer, Köhn, Gauss, JCP 136, 204108 (2012)

## Modified amplitude equations

$$0 = \langle \Psi_0 | (\hat{\tau}_{\tilde{\rho}}^{\parallel})^\dagger e^{-\hat{T}^{\parallel}} \hat{H} e^{\hat{T}^{\parallel}} | \Psi_0 \rangle$$

**! A nasty problem: near-linear dependencies !**

**Technical note:**  $X_{\tilde{\rho}}^\sigma = (S_\eta^{-\frac{1}{2}})_{\tilde{\rho}}^\sigma$  + Schmidt-orthog. betw. particle ranks

$$\Omega_{\tilde{\rho}}^{\parallel} = X_{\tilde{\rho}}^\sigma \langle \Psi_0 | (\tau_\sigma)^\dagger \bar{H} | \Psi_0 \rangle$$

$$\Delta t_\rho^{\parallel} = (X^\dagger)_{\tilde{\rho}}^{\tilde{\sigma}} \left( \Omega_{\tilde{\sigma}}^{\parallel} / \Delta \epsilon_{\tilde{\sigma}} \right)$$

$$\rightarrow \langle \Psi_0 | \hat{T}_1^\dagger \hat{T}_2 | \Psi_0 \rangle \stackrel{!}{=} 0$$

proper choice important for extensivity!

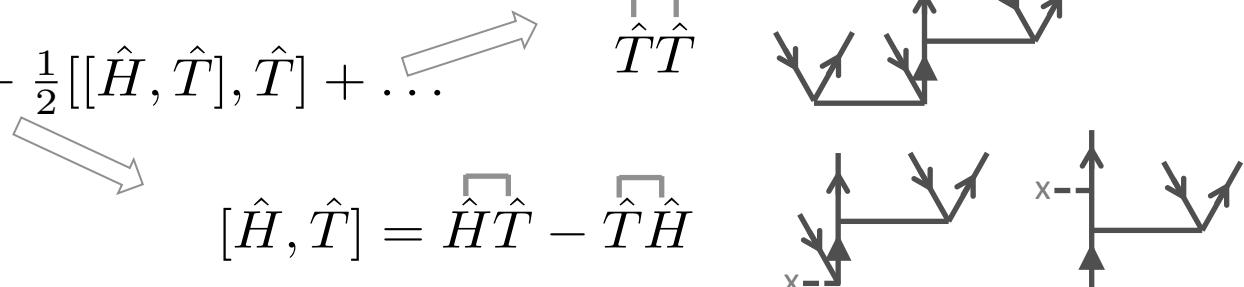
Hanauer, Köhn, JCP 134, 204111 (2011)  
 Hanauer, Köhn, JCP 137, 131103 (2012)

# Numerical evaluation

$$\sum_{\nu} \langle \Phi_{\mu} | e^{-\hat{T}} \hat{H} e^{\hat{T}} | \Phi^{\nu} \rangle c_{\nu} = E c_{\mu} \quad 0 = \langle \Psi_0 | (\hat{\tau}_{\tilde{\rho}}^{||})^{\dagger} e^{-\hat{T}^{||}} \hat{H} e^{\hat{T}^{||}} | \Psi_0 \rangle$$

$$e^{-\hat{T}} \hat{H} e^{\hat{T}} = \hat{H} + [\hat{H}, \hat{T}] + \frac{1}{2} [[\hat{H}, \hat{T}], \hat{T}] + \dots$$

$[\hat{\tau}_{\rho}, \hat{\tau}_{\sigma}] \neq 0$

$$[\hat{H}, \hat{T}] = \hat{H}\hat{T} - \hat{T}\hat{H}$$


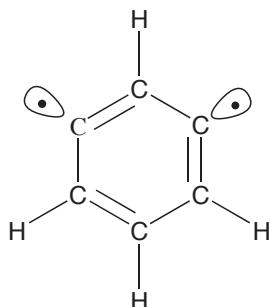
icMRCCSD Energy: up to 4-fold comm.

Equations: up to 8-fold comm.

$e^{-\hat{T}} \hat{H} e^{\hat{T}}$	CCSD	ic-MRCCSD / CAS(6,6)
$= \hat{H}$	2	27
$+ [\hat{H}, \hat{T}]$	13	690
$+ \frac{1}{2} [[\hat{H}, \hat{T}], \hat{T}]$	21	8 766
$+ \frac{1}{6} [[[\hat{H}, \hat{T}], \hat{T}], \hat{T}]$	8	70 997
$+ \frac{1}{24} [[[[\hat{H}, \hat{T}], \hat{T}], \hat{T}], \hat{T}]$	1	399 839
$+ \dots$	0	...

# Commutator approximation

**Singlet-Triplet separation of m-Benzyne:**  
 icMRCCSD, cc-pVDZ basis, CAS(2,2)  
 frozen core



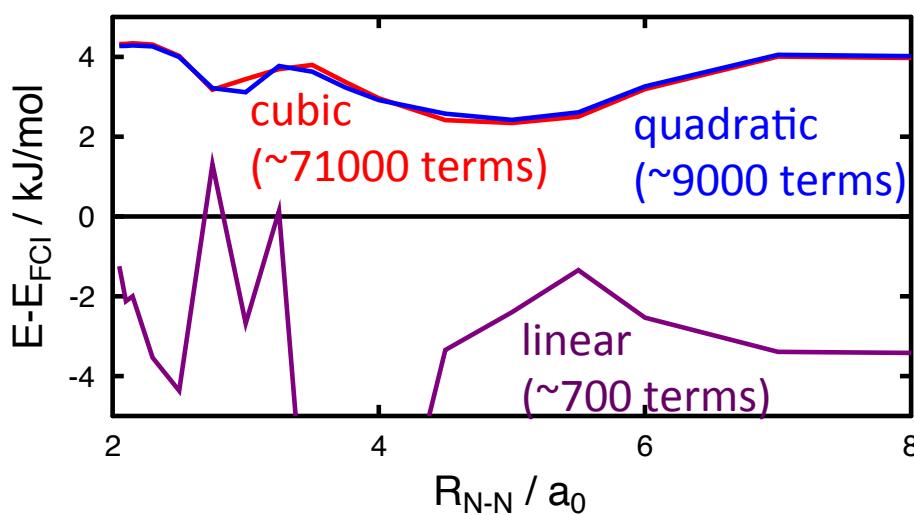
$$\langle \Psi_0 | (\tau'_\rho)^\dagger \hat{H} | \Psi_0 \rangle = 0$$

$$\sum_\nu \langle \Phi_\mu | \hat{H} | \Phi^\nu \rangle c_\nu = E c_\mu$$

$N_{\text{com}}(t)$	$N_{\text{com}}(E)$	$\Delta E_{\text{ST}} / \text{kcal mol}^{-1}$
4	4	17.289
3	3	17.289
2	2	17.415
2	4	17.415

**$N_2$  stretching:**

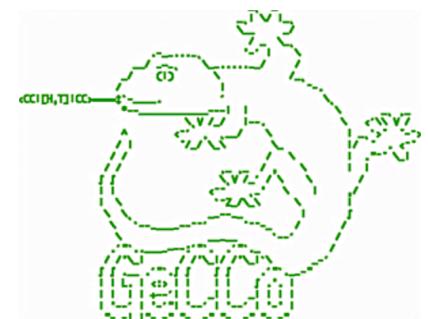
$N_2$ , cc-pVDZ, CAS(6,6)  
 ic-MRCCSD vs. FCI



# Numerical evaluation

## Dealing with complex equations:

- Derivation by **automated techniques**  
using GeCCo program  
(Integrals, CASSCF: DALTON, GAMESS, Molpro)
- **Approximations:** Truncate after double commutator  
tested: Evangelista, Gauss, JCP 134, 114102(2011)  
Hanauer, Köhn, J. Chem. Phys. 134, 204111 (2011)
- Present implementation (GeCCo):  
Certainly not perfect ...  
... BUT: **correct  $N^6$  scaling** (for ic-MRCCSD and fixed active space)





# GeCCo

Very new:

PYTHON powered  
input language for F90 back-end

define operators

symbolic algebra with operators

general order contraction kernel for numerical evaluation

```
#####
# operators associated with T
#####
new_target('T-Operators')

PRINT({STRING:'==== Cluster Operators ==='})

DEF_OP_FROM_OCC({LABEL: 'Tex', DESCRIPTOR: '[PV], H|P, V|P[PV], [HV] [HV] |VV, H[HV]'})
DEF_ME_LIST({LIST: 'TLST', OPERATOR: 'Tex', IRREP: 1, '2MS': 0, AB_SYM: +1})
PRINT_MEL_INFO({LIST: 'TLST'})

CLONE_OPERATOR({LABEL: 'Tp', TEMPLATE: 'Tex'})

#
# Formula for overlap matrix
#
EXPAND_OP_PRODUCT({LABEL: 'F_SMAT', NEW: True, OP_RES: 'SSCAL',
                    OPERATORS: ['C0^+', 'Tv^+', 'Tv', 'C0'],
                    IDX_SV: [1, 2, 3, 4] })
FACTOR_OUT({LABEL_RES: 'F_SMAT', LABEL_IN: 'F_SMAT', INTERM: 'F_GAM0' })
PRINT_FORMULA({LABEL: 'F_SMAT'})

OPTIMIZE({LABEL_OPT: 'FOPT_H_C0', LABELS_IN: 'F_H_C0'})

PRINT({STRING: 'Prepare diagonal ...'})
PRECONDITIONER({LIST_PRC: 'DOLST', LIST_INP: 'H0', MODE: 'dia-H'})

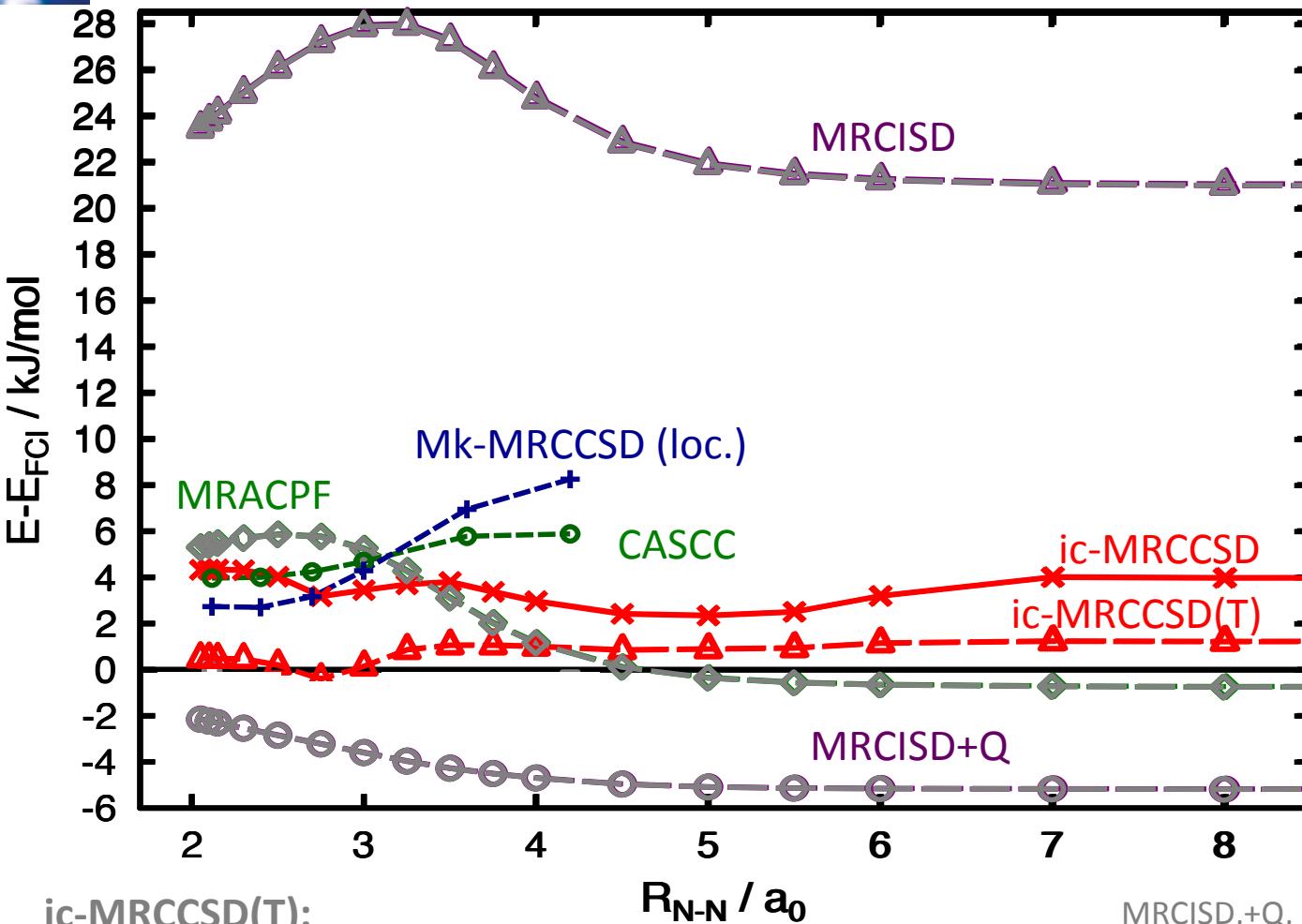
PRINT({STRING: 'Solving equations ...'})

SOLVE_EVP({LIST_OPT: 'C0LST', LIST_PRC: 'DOLST', OP_MVP: 'H_C0', OP_SVP: 'C0',
            FORM: 'FOPT_H_C0', MODE: 'DIA', N_ROOTS: 1})
```



# A few examples

# N<sub>2</sub> dissociation



ic-MRCCSD(T):

Hanauer, Köhn, JCP 136, 204107 (2012)

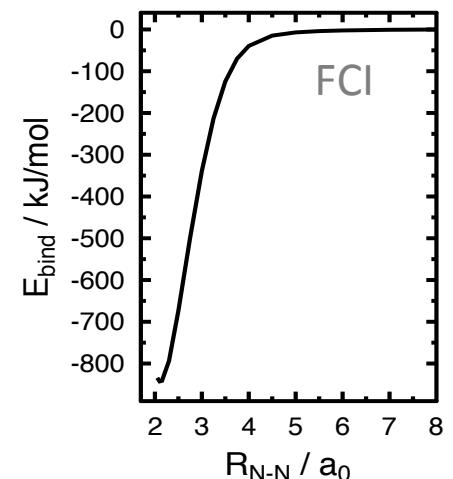
MRCISD,+Q, MRACPF: Werner, private comm.

MRAQCC: Yanai, Chan, JCP 127, 104107

CASCC: Lyakh et al., Mol. Phys. 105, 1335 (2007)

Mk-MRCCSD: Das, Kallay, Mukherjee, CP 392, 83 (2012)

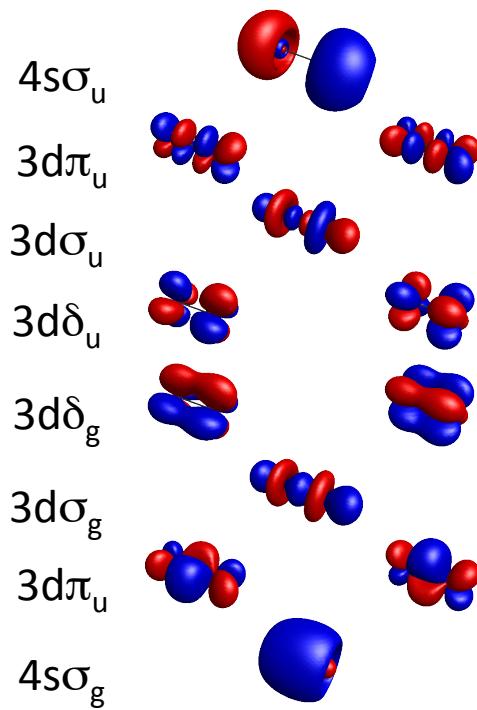
cc-pVDZ basis set



all MR methods use  
CAS(6,6)

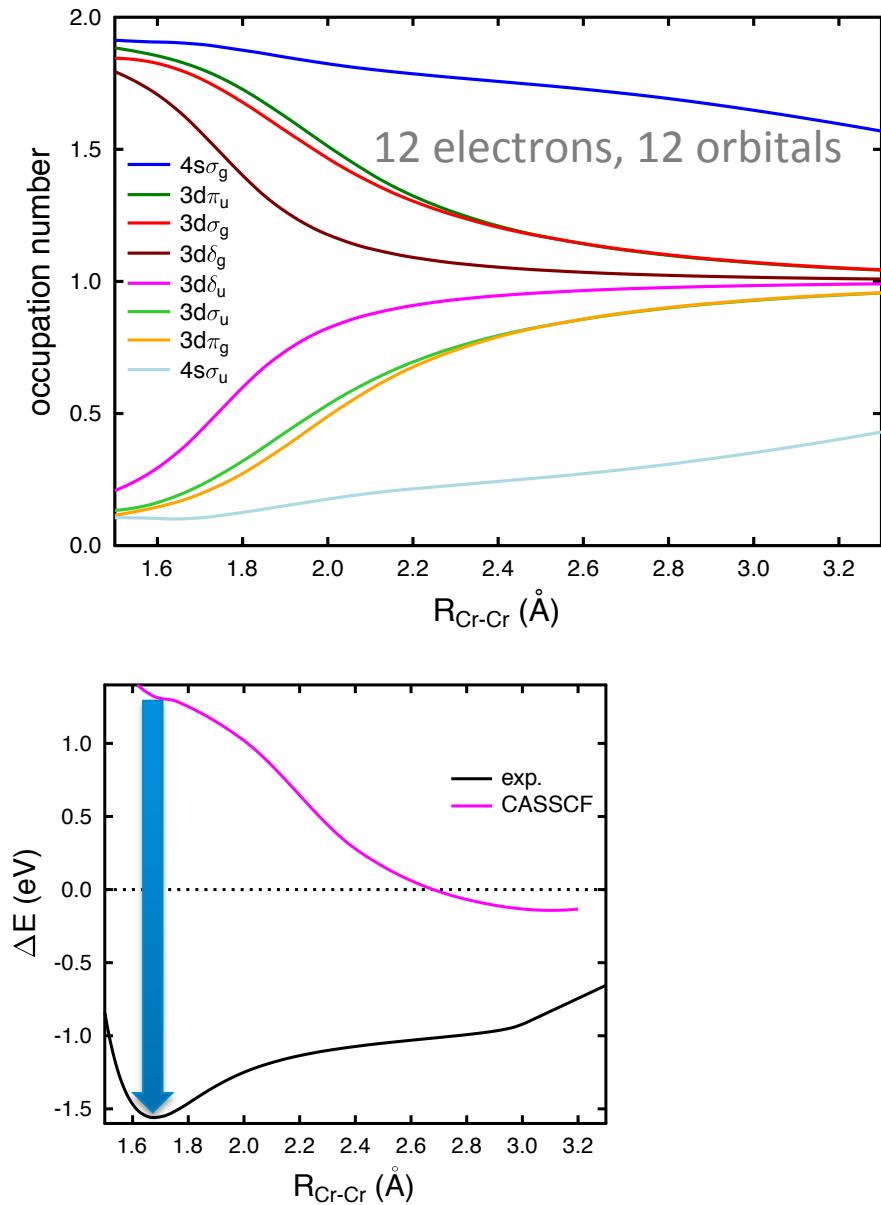
# Chromium dimer

Strong multireference character

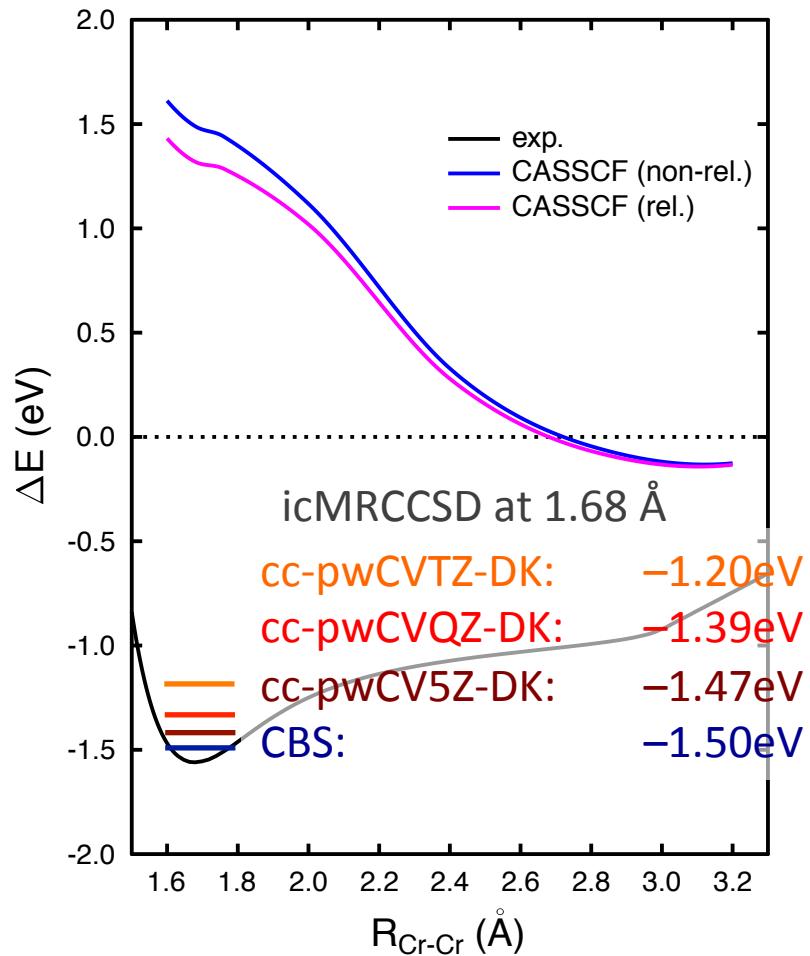


Huge contribution from dynamic correlation

3s3p-correlation important!!

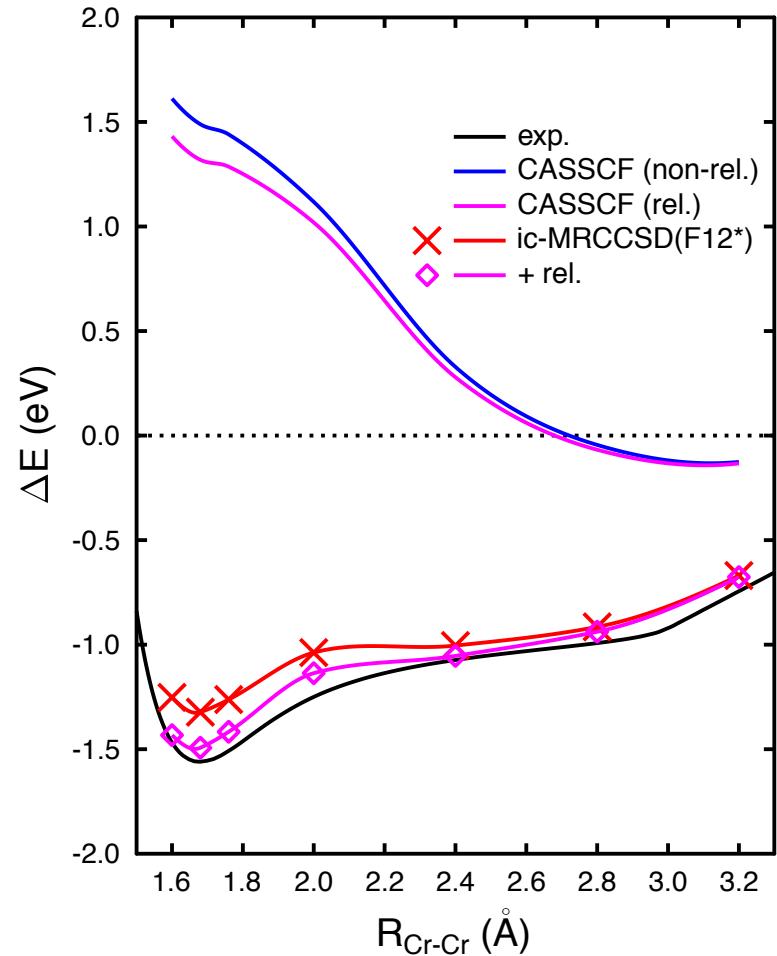


# Some preliminary results



2<sup>nd</sup>-order DKH transf. for scalar rel. effects

cc-pwCVTZ with geminal terms



# F12 methods

Molecular CC calculations need large basis sets!

Example:

CCSD(T) atomization energies  
(20 small molecules)

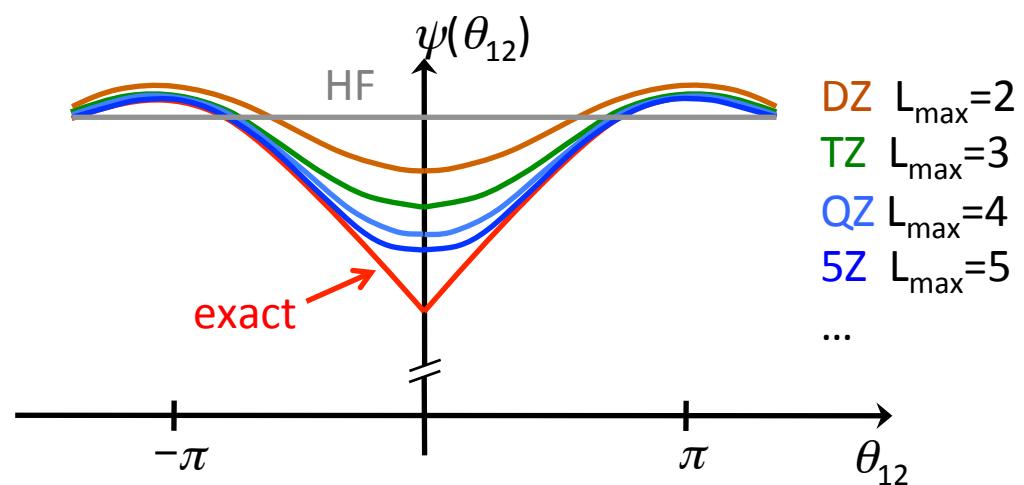
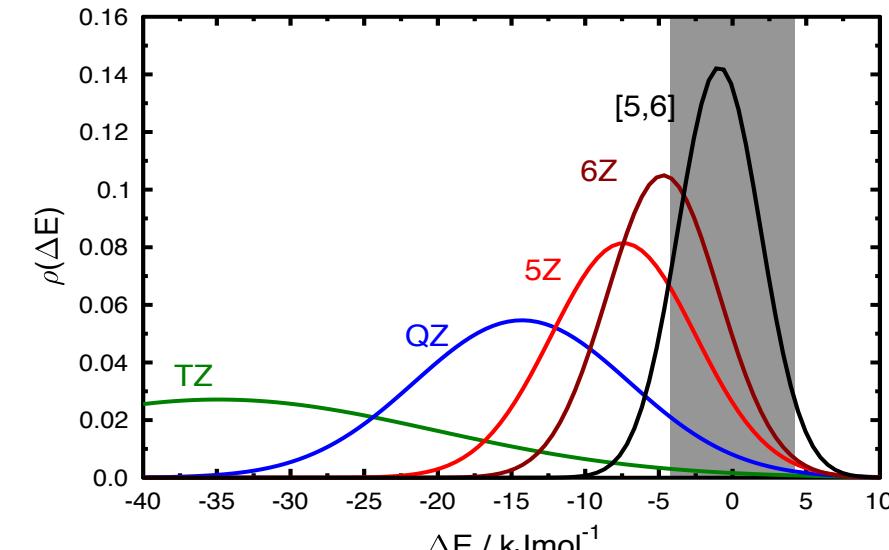
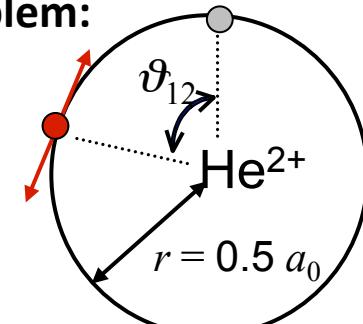
Error in electronic contribution  
compared to reference values  
(derived from experiment)

Values from: Helgaker, Jørgensen, Olsen,  
“Electronic structure theory”, Wiley (2000)

**Root of the problem:**

Behavior of  $\psi$   
for  $\mathbf{r}_1 \rightarrow \mathbf{r}_2$  due to

$$\hat{V}_{ee} \propto \frac{1}{r_{12}}$$



# F12 theory: concepts

**Ansatz for pair-function:**

$$|u_{ij}\rangle = \sum_{a>b} t_{ab}^{ij} |ab\rangle + \sum_{k>l} c_{kl}^{ij} \hat{Q}_{12} f(r_{12}) |kl\rangle$$

**Correlation factor:**

F12-theory (Ten-no):

$$f(r_{12}) = \frac{1}{\gamma} (1 - e^{-\gamma r_{12}})$$

$\gamma \approx 1 \text{ a}_0^{-1}$  is a fixed basis set parameter

**Combination with CC theory** (Kutzelnigg, Klopper, Noga)

$$T_{2'} = \frac{1}{8} c_{kl}^{ij} \langle \alpha\beta | \hat{Q}_{12} f(r_{12}) | kl \rangle a_{ij}^{\alpha\beta}$$

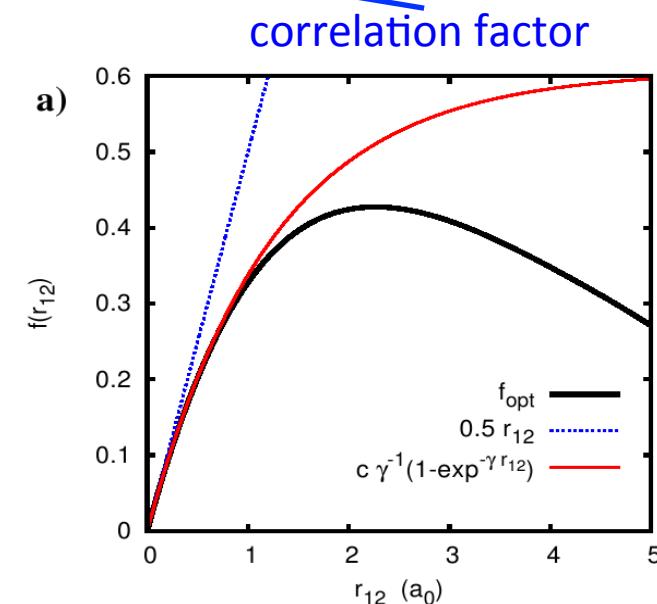
**Efficient combination with CC theory possible:**

CCSD-F12a/b (Werner, Adler, Knizia),  
CCSD(F12\*) (Hättig, Köhn, Tew)

**Analogous development: icMRCCSD(F12\*)**

Liu, Hanauer, Köhn, Chem. Phys. Lett. 565, 122 (2013)

$$T_{2'} = \frac{1}{8} c_{KL}^{IJ} \langle \alpha\beta | \hat{Q}_{12} f(r_{12}) | KL \rangle a_{IJ}^{\alpha\beta}$$



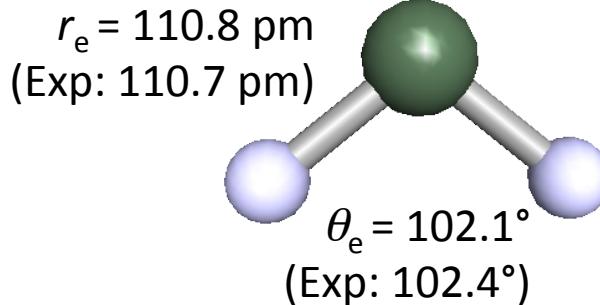
From: Tew, Hättig, Klopper, Bachorz. In: "Recent Progress in Coupled Cluster Methods ...", Csarsky, Pittner, Paldus (Eds.), Springer

Kutzelnigg (1985), Kutzelnigg, Klopper (1987, 1991),  
Ten-no (2004), May, Manby (2004), Tew, Klopper  
(2005)

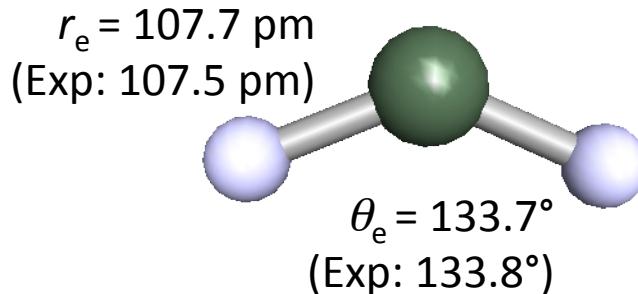
# icMRCCSD(F12\*) theory

$\text{CH}_2$

Singlet

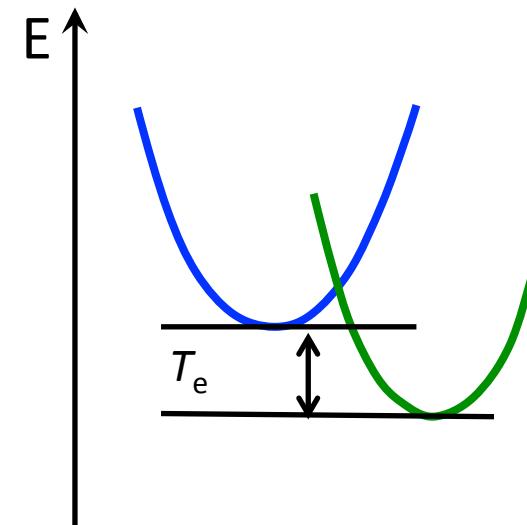


Triplet



CAS(2,2)

ic-MRCCSD(F12\*)+(T)/cc-pVQZ-F12  
(frozen core, numerical gradients)



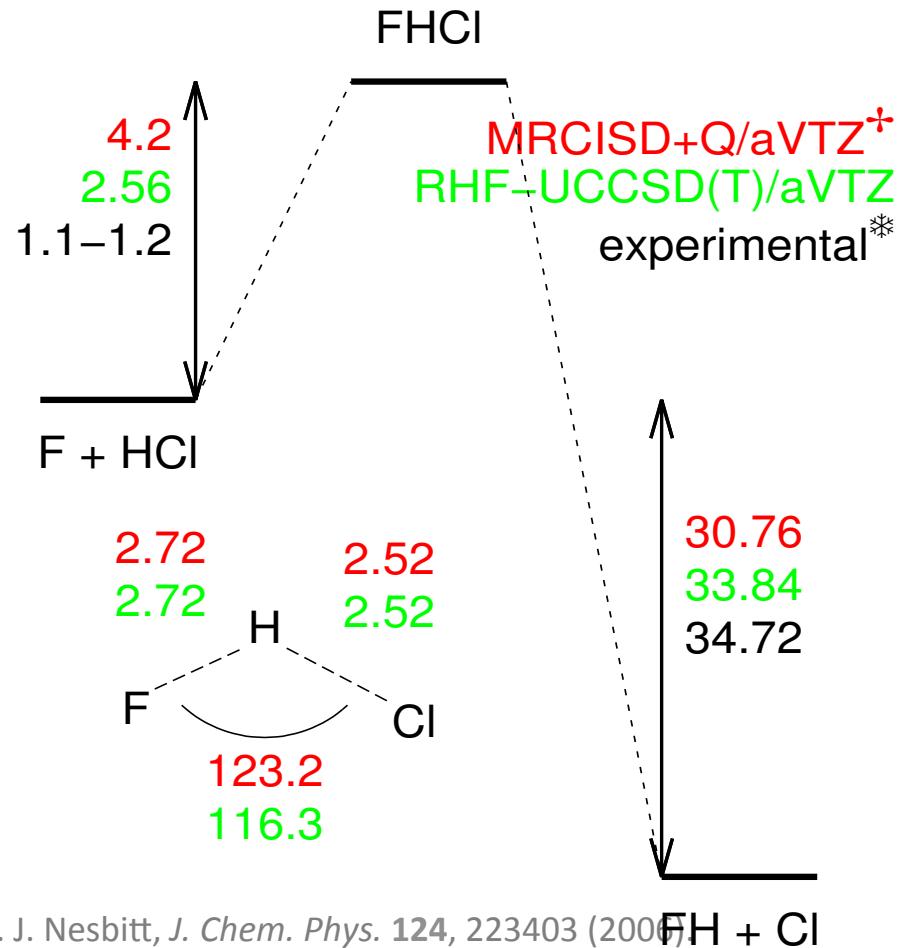
$T_e = 37.2 \text{ kJ/mol}$   
(Exp: 37.7 kJ/mol)

Exp. values: Petek et al, JCP 91, 6566 (1989)  
Bunker, Sears, JCP 83, 4866 (1985)  
Bunker, Jensen, JCP 79, 1224 (1983)

# The $\text{F} + \text{HCl} \rightarrow \text{Cl} + \text{HF}$ reaction

Traditional theoretical calculations overestimate experimental barrier height!  $\dagger\star$

Experimental ro-vibrational distribution not reproduced by quantum dynamics.  $\star$



$\dagger$ M. P. Deskevich, M. Y. Hayes, K. Takahashi, R. T. Skodje and D. J. Nesbitt, *J. Chem. Phys.* **124**, 223403 (2006)

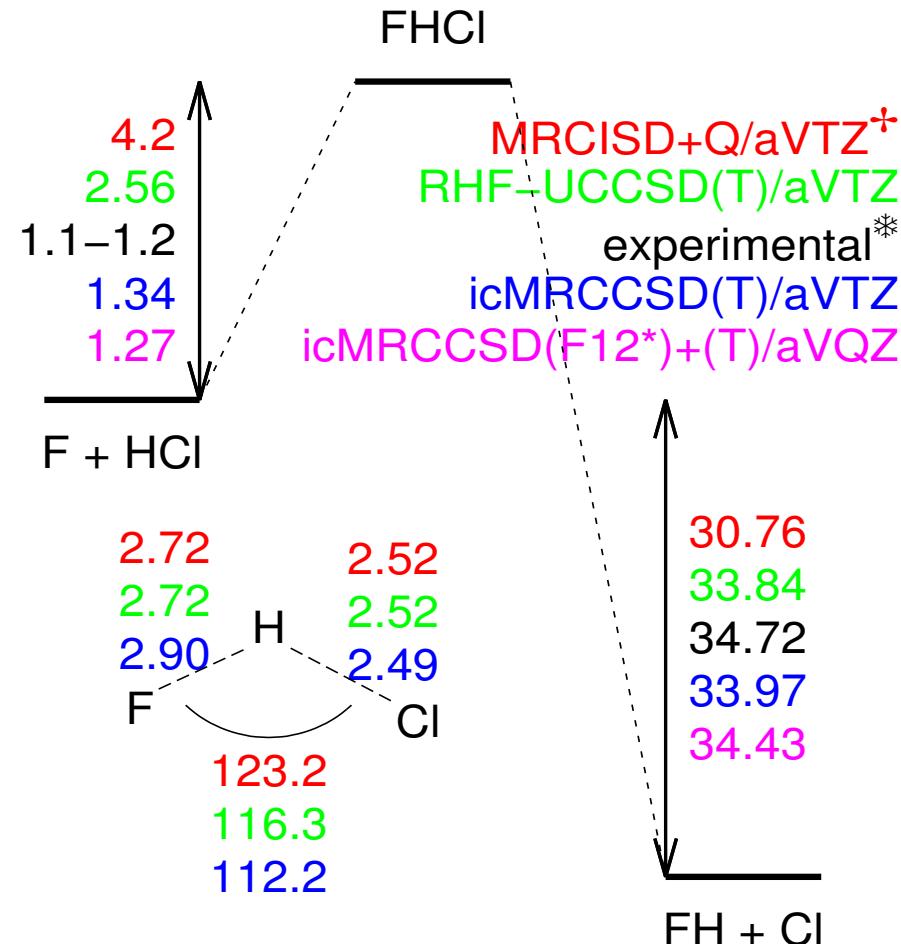
$\star$ A. Li, H. Guo, Z. Sun, J. Klos and M. H. Alexander, *Phys. Chem. Chem. Phys.* **15**, 15347 (2013).

\*N. Jonathan *et al.*, *Chem. Phys. Lett.* **7**, 257 (1970); A. M. G. Ding *et al.*, *Faraday Discuss. Chem. Soc.* **55**, 252 (1973).

# The $\text{F} + \text{HCl} \rightarrow \text{Cl} + \text{HF}$ reaction

Barrier height and exothermicity obtained with icMRCCSD(T) theory are very close to the experimental values!

CAS(3,3) sufficient!  
[ MRCI uses CAS(15,9) ]



<sup>†</sup>M. P. Deskevich, M. Y. Hayes, K. Takahashi, R. T. Skodje and D. J. Nesbitt, *J. Chem. Phys.* **124**, 223403 (2006).

<sup>\*</sup>N. Jonathan *et al.*, *Chem. Phys. Lett.* **7**, 257 (1970); A. M. G. Ding *et al.*, *Faraday Discuss. Chem. Soc.* **55**, 252 (1973).

# Active-space size

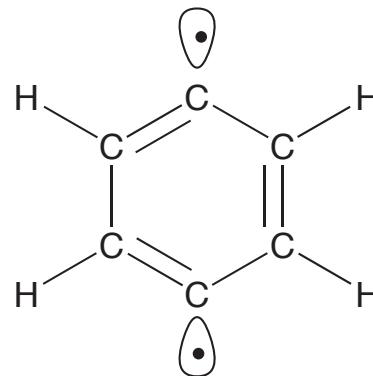
Singlet-Triplet splitting of p-benzyne

$\Delta E_{S-T}$ /kcal mol <sup>-1</sup>	icMRCCSD	icMRCCSD(T)
CAS(2,2)	3.7	4.8
CAS(8,8)	4.7	4.9

cc-pVDZ basis

Structures: RHF/UHF-CCSD(T)/cc-pVDZ

from: Evangelista et al., JCP 127, 024102 (2007)



keep the active space as  
small as physically  
necessary

Hanauer, Köhn, JCP 136, 204107 (2012)

see also: Evangelista, Hanauer, Köhn, Gauss, JCP 136, 204108 (2012)



# Additional aspects (now and future)

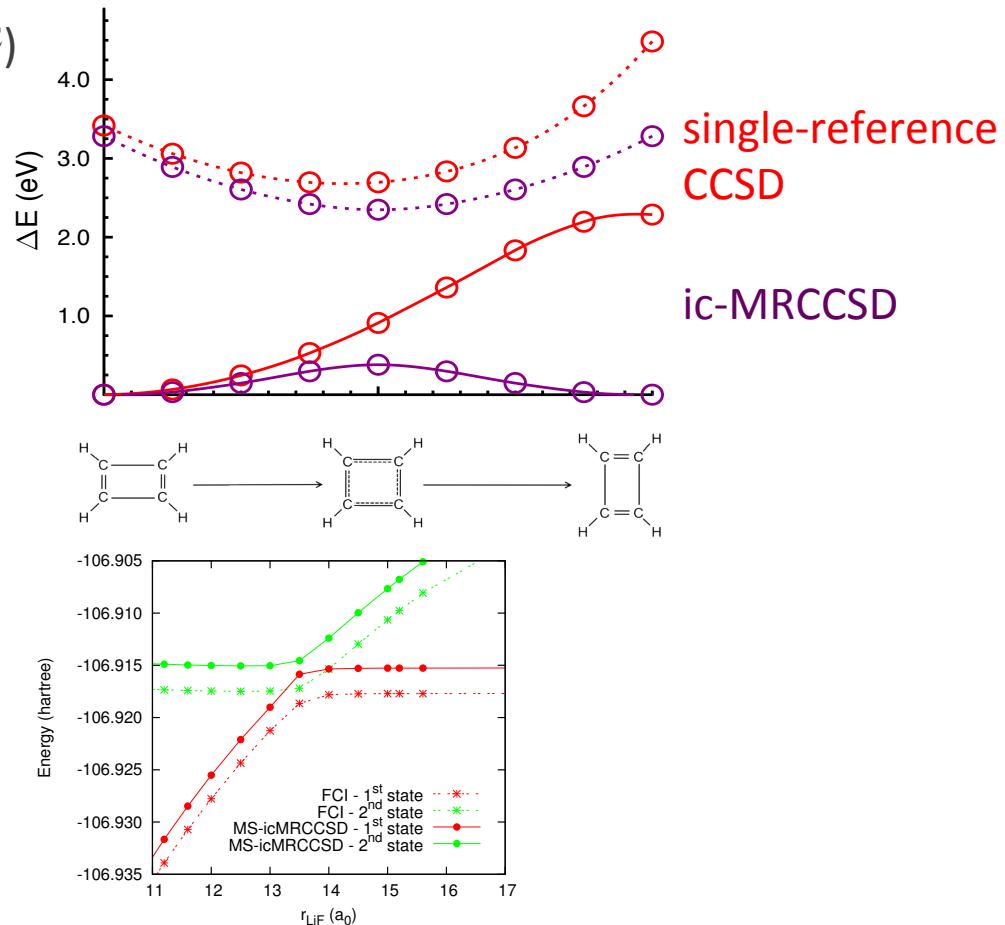
Full spin adaptation (eigenfunctions of  $S^2$ )

Excited states via linear-response  
and/or EOM frameworks

Multi-state extension

Response properties

Lower-order approximations (perturbation theory, CEPA-like theories)



## Success and criticism

Full analogy to single-reference CC

$$|\Psi_{\text{ic}}\rangle = e^{\hat{T}} \sum_{\mu} |\Phi_{\mu}\rangle c_{\mu}$$

Proper formal scaling, compact excitation manifold

Fully extensive energy, orbital invariance

Very accurate results

### Problems, criticism:

Threshold for near-linear dependencies

Commutator approximation

Some consistency problems remain (for dissociating singlets)

Efficiency

