Symmetry-restored multi-reference mean-field theory for nuclei

Michael Bender

Université Bordeaux 1; CNRS/IN2P3; Centre d'Etudes Nucléaires de Bordeaux Gradignan
UMR5797, Chemin du Solarium, BP120, 33175 Gradignan, France

Workshop on
"Near-degenerate systems in nuclear structure and quantum chemistry from ab-initio many-body methods"
Espace de Structure Nucléaire Théorique, Saclay, 2 April 2015

What to expect from this talk

Bottom lines

- I use old-school phenomenological effective interactions (energy density functionals (EDFs)) – no *ab-initio* in this talk!
- I will not worry about getting *in-medium* correlations from a "bare" Hamiltonian.
- I will address nuclear structure phenomena that require multi-reference techniques. (It turns out that almost all do!)

Topics

- Why MR?
- From simple to increasingly complicated MR schemes.
- Technical issues.

Semantics

- single reference (SR) ≡ self-consistent mean field, "HF", "HFB"
- multi reference (MR) ≡ projection and/or Generator Coordinate Method

Nuclear physics jargon 101

- static correlations: what one gets by symmetry-breaking mean field
  - HFB-type pairing
  - deformation (quadrupole, octupole, ...)
  - high angular-momentum (nuclear physics jargon: high-spin) states
- dynamical correlations: correlations from "going beyond the mean field"
  - Correlations from symmetry restoration
  - shape mixing (because of deformation softness and/or shape coexistence)
  - excited states (vibrations, ...)
  - restoration of angular-momentum, parity and isospin selection rules for electromagnetic and weak transitions

Why worry?

- nuclei are self-bound systems ⇒ conserved quantum numbers
- most, if not all, nuclei exhibit shape coexistence Heyde & Wood, RMP83 (2011) 1467
- individual and various collective excitation modes on the same energy scale
- large amplitude collective motion

Symmetry restoration

particle-number projector

$$\hat{P}_{N_0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi_N \underbrace{e^{-i\phi_N N_0}}_{\text{weight}} \overbrace{e^{i\phi_N \hat{N}}}^{\text{rotation in gauge space}}$$

angular-momentum restoration operator

$$\hat{P}^J_{MK} = \frac{2J+1}{16\pi^2} \int_0^{4\pi} d\alpha \int_0^{\pi} d\beta \, \sin(\beta) \int_0^{2\pi} d\gamma \underbrace{\mathcal{D}^{*J}_{MK}(\alpha,\beta,\gamma)}_{\text{Wigner function}} \overbrace{\hat{R}(\alpha,\beta,\gamma)}^{\text{rotation in real space}}$$

K is the z component of angular momentum in the body-fixed frame.
Projected states are given by

$$|JMq\rangle = \sum_{K=-J}^{+J} f_J(K) \, \hat{P}^J_{MK} \, \hat{P}^Z \, \hat{P}^N \, |\mathrm{MF}(q)\rangle = \sum_{K=-J}^{+J} f_J(K) \, |JM(qK)\rangle$$

$f_J(K)$ is the weight of the component K and determined variationally

Axial symmetry (with the z axis as symmetry axis) allows to perform the α and γ integrations analytically, while the sum over K collapses, $f_J(K) \sim \delta_{K0}$

Configuration mixing by the symmetry-restored Generator Coordinate Method

Superposition of projected self-consistent mean-field states $|\mathrm{MF}(\mathbf{q})\rangle$ differing in a set of collective and single-particle coordinates $\mathbf{q}$

$$|NZJM\nu\rangle = \sum_{\mathbf{q}} \sum_{K=-J}^{+J} f_{J,\kappa}^{NZ}(\mathbf{q},K)\, \hat{P}_{MK}^{J}\, \hat{P}^{Z}\, \hat{P}^{N}\, |\mathrm{MF}(\mathbf{q})\rangle = \sum_{\mathbf{q}} \sum_{K=-J}^{+J} f_{J\nu}^{NZ}(\mathbf{q},K)\, |NZ\, JM(\mathbf{q}K)\rangle$$

with weights $f_{J\nu}^{NZ}(\mathbf{q},K)$.

$$\frac{\delta}{\delta f_{J\nu}^{*}(\mathbf{q},K)} \frac{\langle NZ\, JM\nu|\hat{H}|NZ\, JM\nu\rangle}{\langle NZ\, JM\nu|NZ\, JM\nu\rangle} = 0 \quad \Rightarrow \quad \text{Hill-Wheeler-Griffin equation}$$

$$\sum_{\mathbf{q}'} \sum_{K'=-J}^{+J} \left[\mathcal{H}_J^{NZ}(\mathbf{q}K,\mathbf{q}'K') - E_{J,\nu}^{NZ}\, \mathcal{I}_J^{NZ}(\mathbf{q}K,\mathbf{q}'K')\right] f_{J,\nu}^{NZ}(\mathbf{q}'K') = 0$$

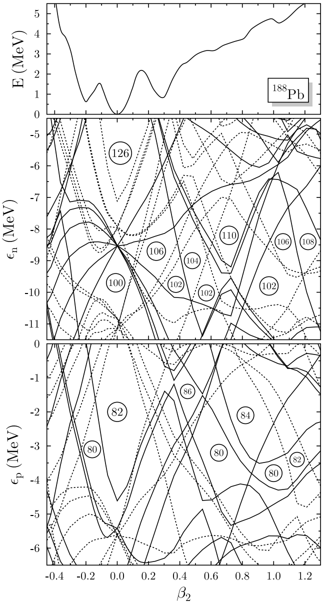
with

$$\mathcal{H}_J(\mathbf{q}K,\mathbf{q}'K') = \langle NZ\, JM\, \mathbf{q}K|\hat{H}|NZ\, JM\, \mathbf{q}'K'\rangle \quad \text{energy kernel}$$
$$\mathcal{I}_J(\mathbf{q}K,\mathbf{q}'K') = \langle NZ\, JM\, \mathbf{q}K|NZ\, JM\, \mathbf{q}'K'\rangle \quad \text{norm kernel}$$

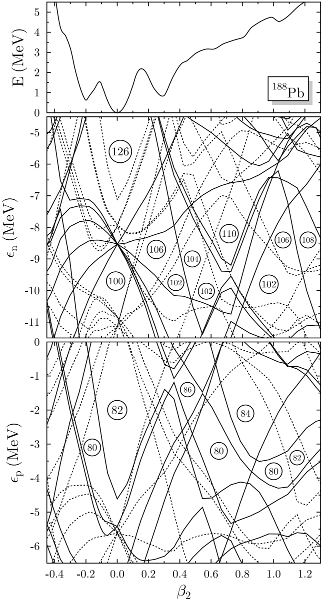
Angular-momentum projected GCM gives the

- correlated ground state for each value of J
- spectrum of excited states for each J

Horizontal vs. vertical expansion of correlations



Horizontal vs. vertical expansion of correlations



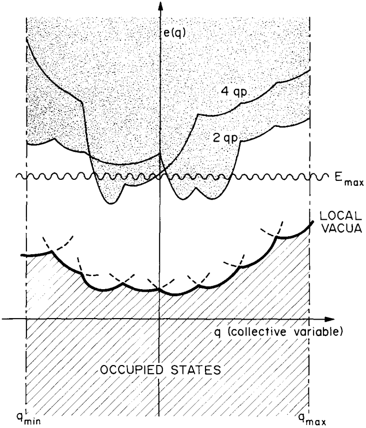


Fig. 1. Schematic plot of the energy versus the collective variable. The dark envelopes show the positions of the local vacua. The domain of the collective variable is defined by $q_{\min}$, $q_{\max}$ and the energy cut $E_{\max}$.

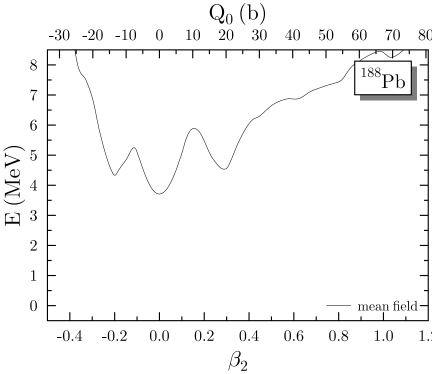
F. Dönau *et al*, NPA496 (1989) 333.

Our implementation(s)

- Coordinate space representation on a 3d mesh using Lagrange-mesh techniques (in quantum chemistry jargon: a special case of a DVR)
- "HF+BCS" or "HFB" solved with two-basis method
- full space of occupied single-particle states
- Skyrme energy density functionals (old school) or Skyrme pseudo-potentials (new school)

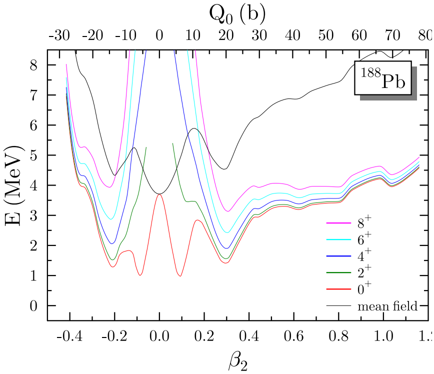
Typical applications

Configuration mixing via the projected Generator Coordinate Method



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

Spectroscopy from MR EDF



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

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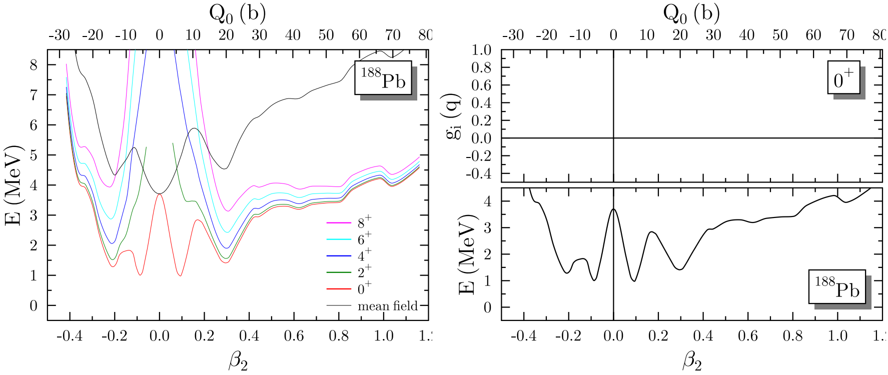

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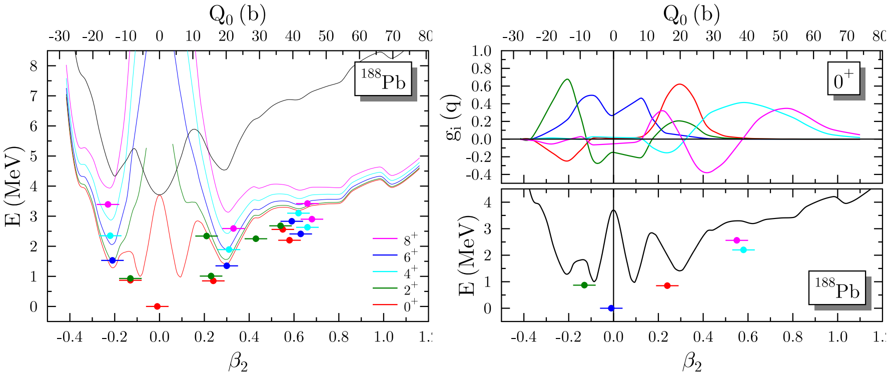
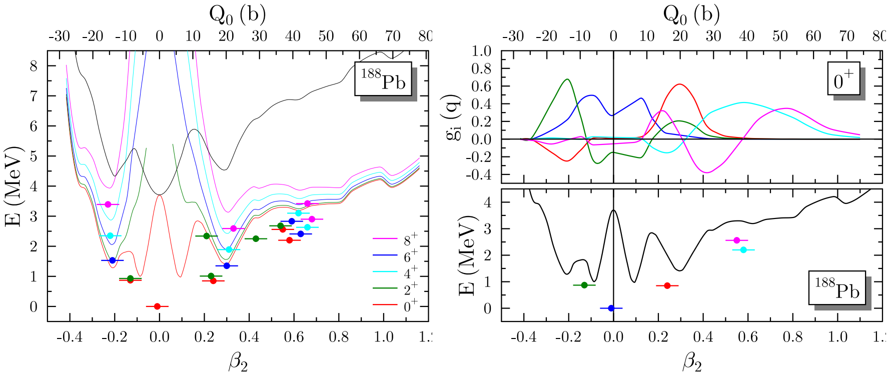


M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.

Attention: $g_i^2(q)$ is not the probability to find a mean-field state with intrinsic deformation q in the collective state



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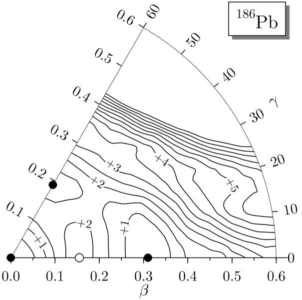
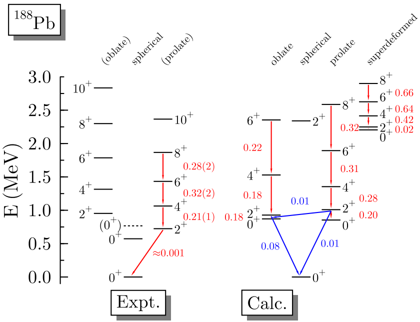
Possible role of non-axial degrees of freedom



M. B., P. Bonche, T. Duguet, P.-H. Heenen, Phys. Rev. C 69 (2004) 064303.
Experiment: T. Grahn *et al*, Phys. Rev. Lett. **97** (2006) 062501

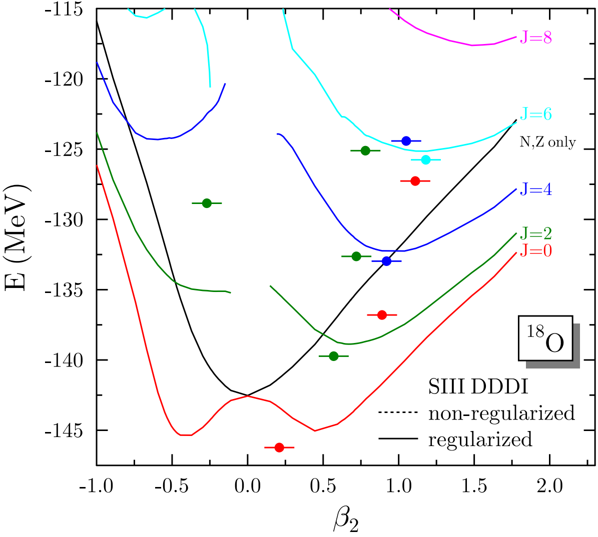


- in-band and out-of-band $E2$ transition moments directly in the laboratory frame with correct selection rules
- full model space of occupied particles
- only occupied single-particle states contribute to the kernels ("horizontal expansion")
- $\Rightarrow$ *no effective charges necessary*
- *no adjustable parameters*

$$B(E2; J'_{\nu'} \to J_\nu) = \frac{e^2}{2J'+1} \sum_{M=-J}^{+J} \sum_{M'=-J'}^{+J'} \sum_{\mu=-2}^{+2} |\langle J M \nu | \hat{Q}_{2\mu} | J' M' \nu' \rangle|^2$$

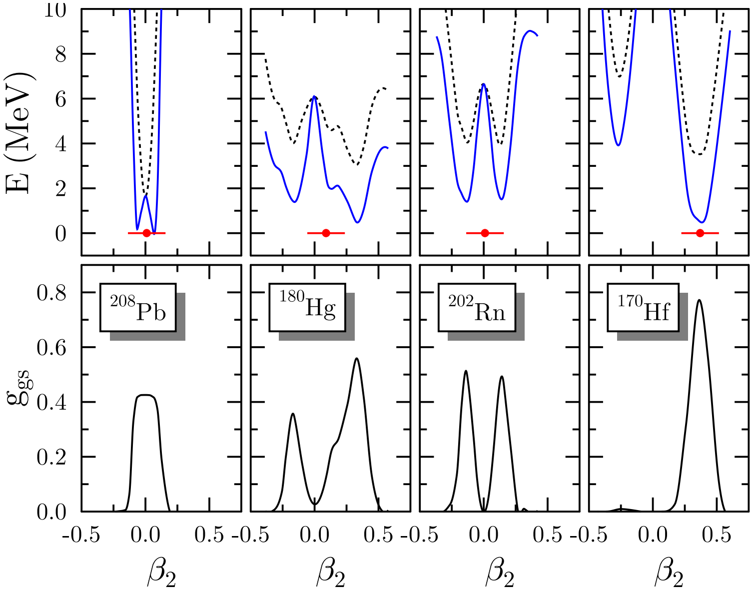
$$\beta_2^{(t)} = \frac{4\pi}{3R^2 A} \sqrt{\frac{B(E2; J \to J-2)}{(J\,0\,2\,0\,|(J-2)\,0)^2 e^2}} \quad \text{with} \quad R = 1.2\, A^{1/3}$$

Spherical nuclei don't stay spherical "beyond the mean field"



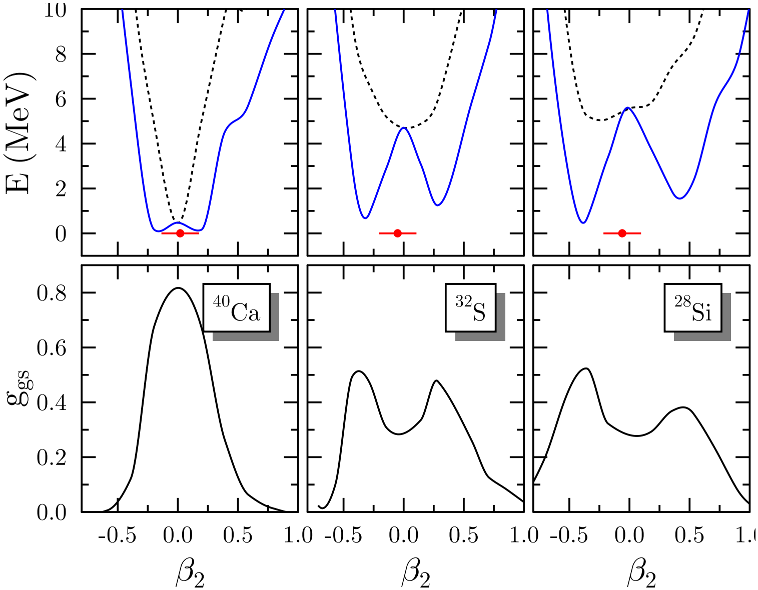
M. B., B. Avez, B. Bally, T. Duguet, P.-H. Heenen, D. Lacroix, unpublished

Typical cases



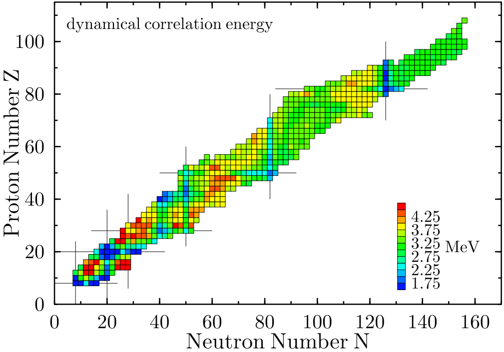
M. B., G. F. Bertsch, P.-H. Heenen, Phys. Rev. C 73 (2006) 034322

Typical cases



M. B., G. F. Bertsch, P.-H. Heenen, Phys. Rev. C 73 (2006) 034322

Static vs. dynamical quadrupole correlation energies



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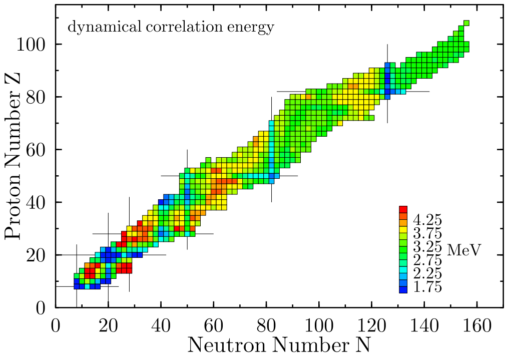

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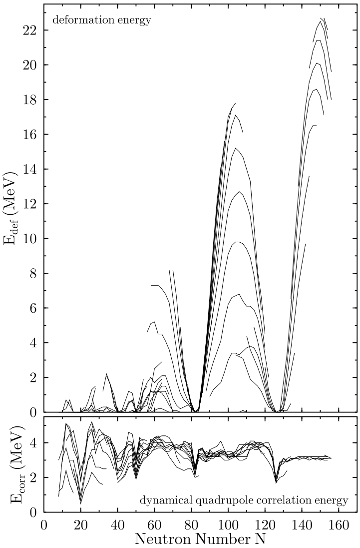
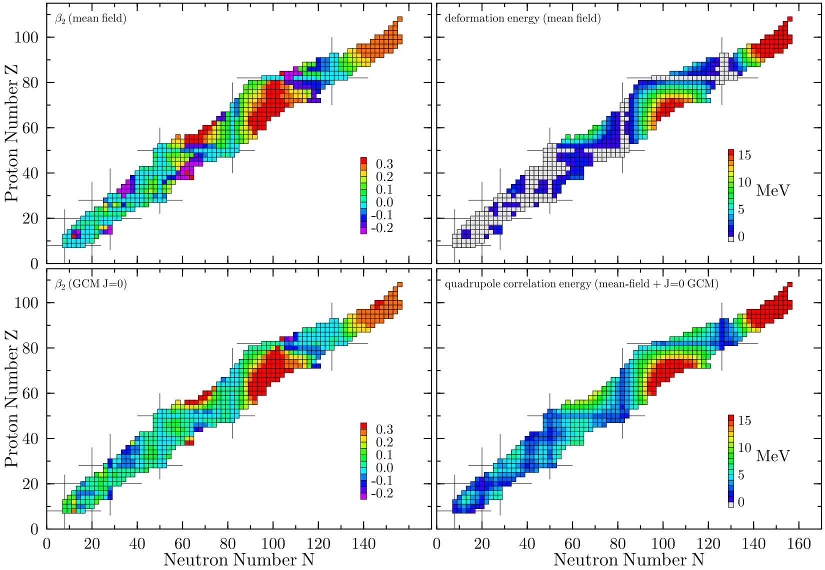
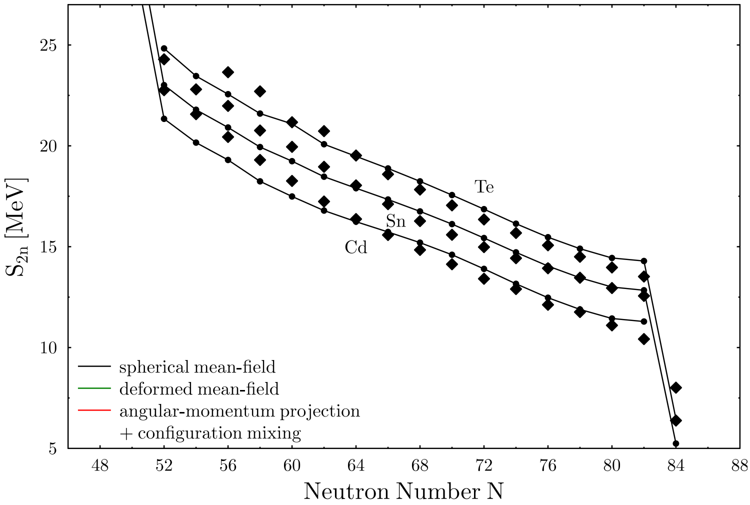


Intrinsic Deformation and Quadrupole Correlation Energy



M. B., G. F. Bertsch, P.-H. Heenen, Phys. Rev. C 73 (2006) 034322

Two-nucleon separation energies



data from M. B., G. F. Bertsch, P.-H. Heenen, Phys. Rev. C 73 (2006) 034322

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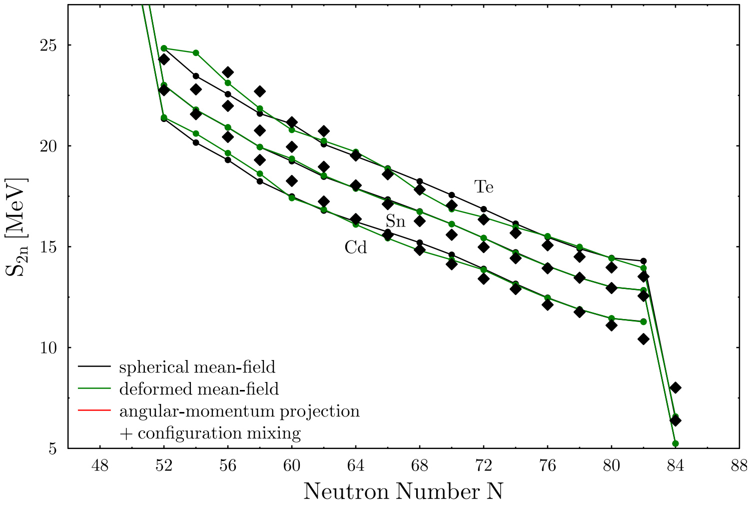

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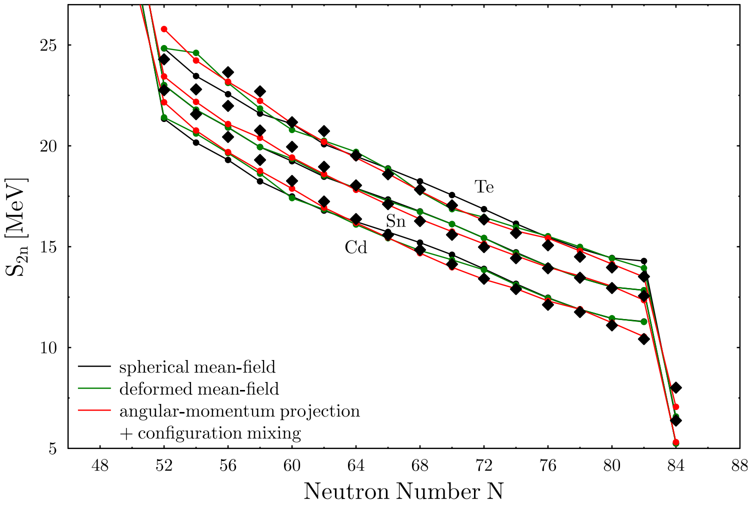
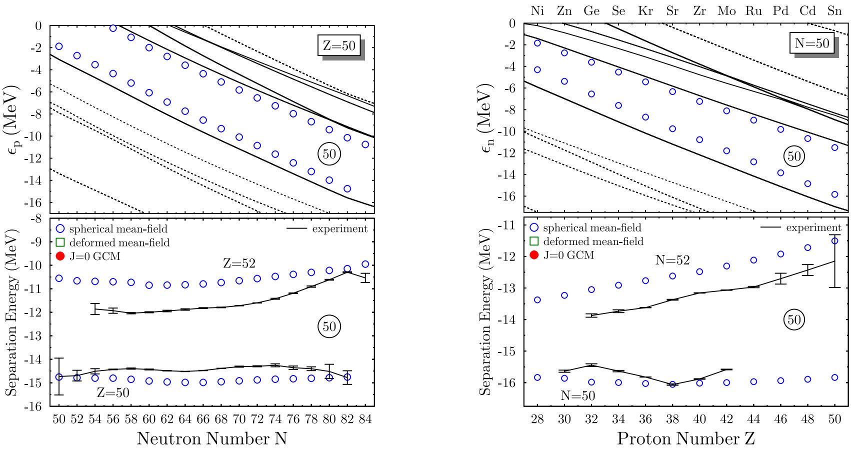


data from M. B., G. F. Bertsch, P.-H. Heenen, Phys. Rev. C 73 (2006) 034322

Eigenvalues of the single-particle Hamiltonian vs. S_{2q}



lower panel: $-S_{2p}(Z=50, N)/2$
The global linear trend is taken out subtracting
$\frac{N-82}{2} [S_{2p}(Z=50, N=50) - S_{2p}(Z=50, N=82)]$
using the spherical mean-field S_{2p}

M. B., G. F. Bertsch, P.-H. Heenen, PRC 78 (2008) 054312

lower panel: $-S_{2n}(Z, N=50)/2$
The global linear trend is taken out subtracting
$\frac{N-50}{2} [S_{2n}(Z=28, N=50) - S_{2n}(Z=50, N=50)]$
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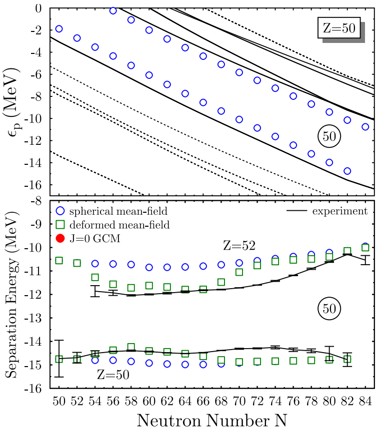
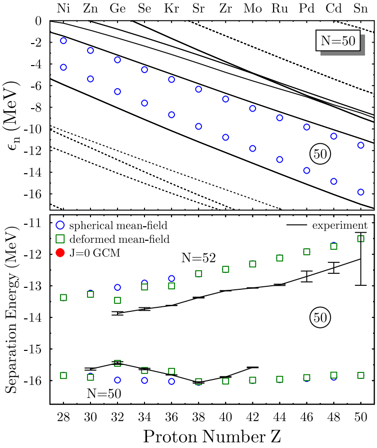
Eigenvalues of the single-particle Hamiltonian vs. S_{2q}





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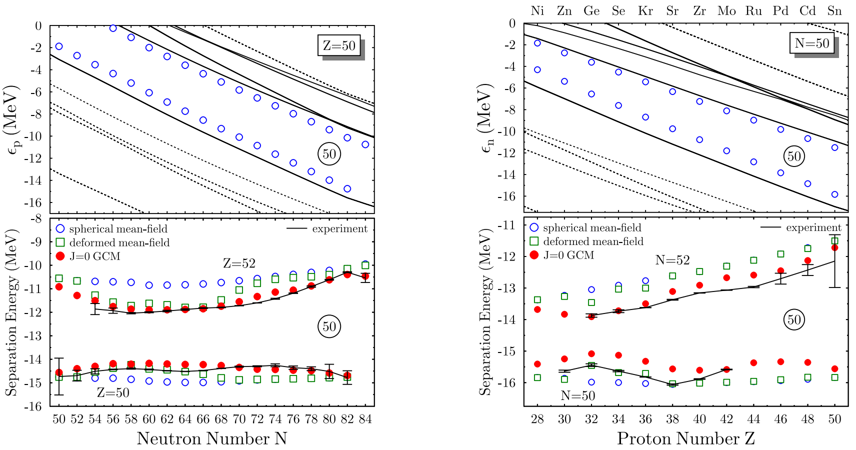

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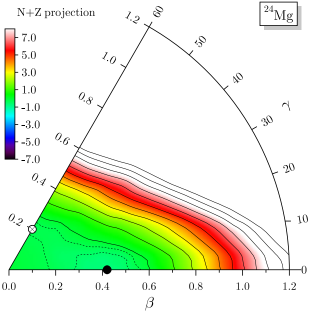
Angular momentum projection of triaxial states

mean-field deformation energy surface



Angular momentum projection of triaxial states

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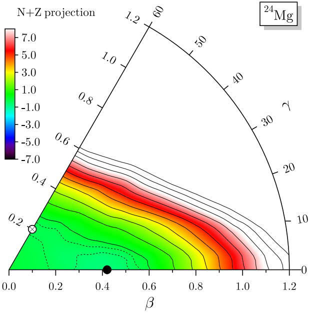
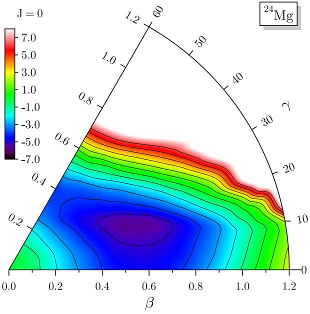


$J = 0$ projected deformation energy surface



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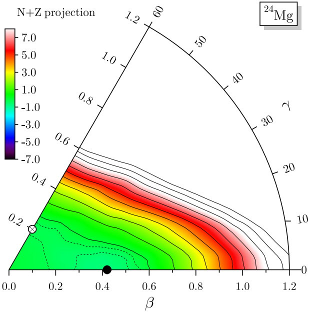

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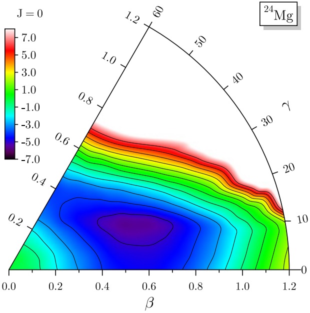
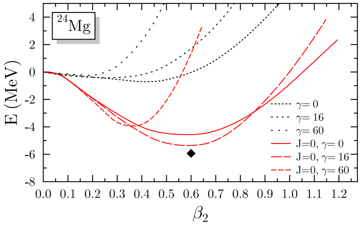
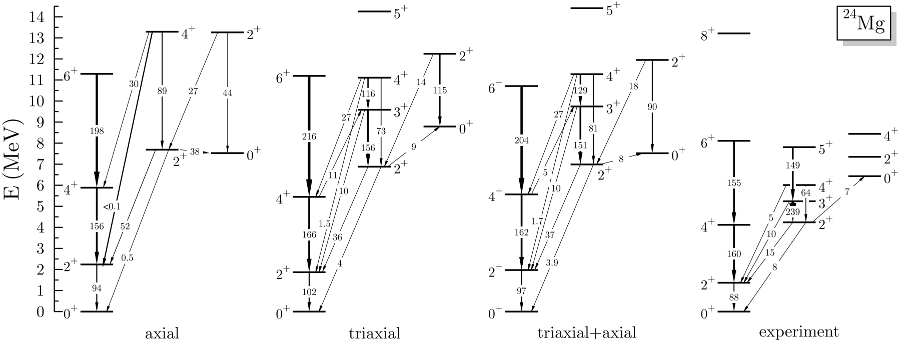




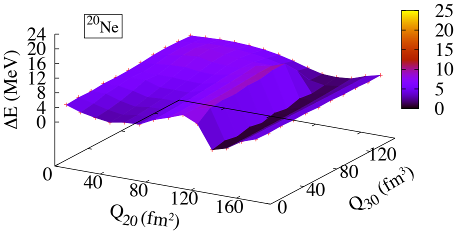
M. B. and P.-H. Heenen, Phys. Rev. C **78** (2008) 024309

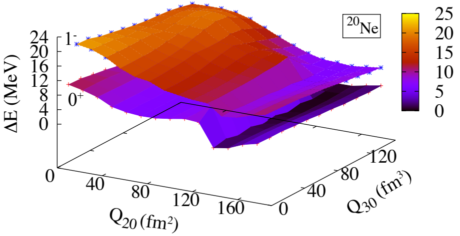
Mixing of angular-momentum projected triaxial states of different intrinsic deformation



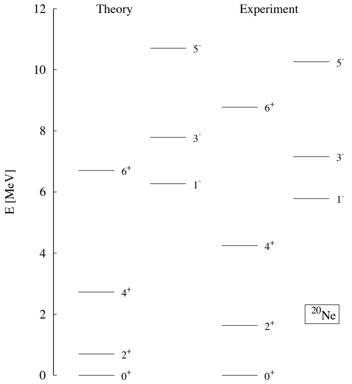
M. B. and P.-H. Heenen, Phys. Rev. C **78** (2008) 024309

Other collective degrees of freedom: Octupole deformation



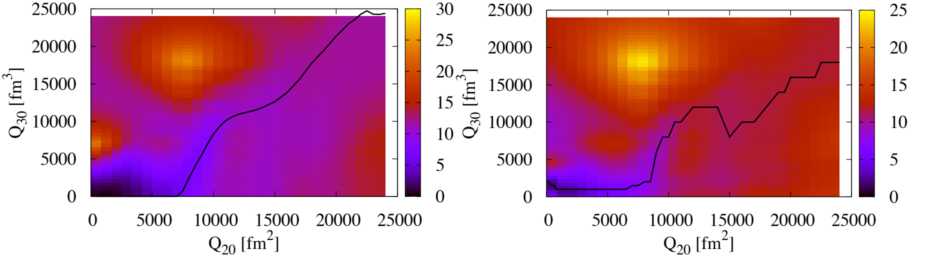


V. Hellemans, M. B., P.-H. Heenen, to be published.



- Skyrme SLy5
- axial Slater determinants (no pairing).
- *J* and parity projection

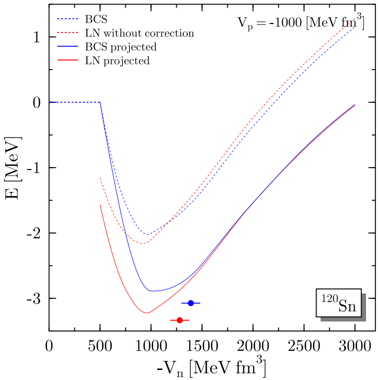
Towards the MR description of fission: the case of ^{180}Hg



V. Hellemans, M. B., P.-H. Heenen, to be published.

- Skyrme SLy5s1 (recent parametrization with optimized surface tension) R. Jodon, K. Bennaceur, M. B., J. Meyer, in preparation.
- axial states
- N, Z, J & parity projection

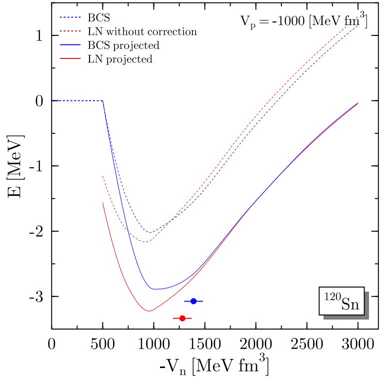
Other collective degrees of freedom: dynamical pairing



Mixing of particle number projected spherical states in ^{120}Sn with different amount of neutron pairing correlations

M. B., T. Duguet, IJMPE16 (2007) 222

Other collective degrees of freedom: dynamical pairing



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VAQUERO, EGIDO, AND RODRÍGUEZ

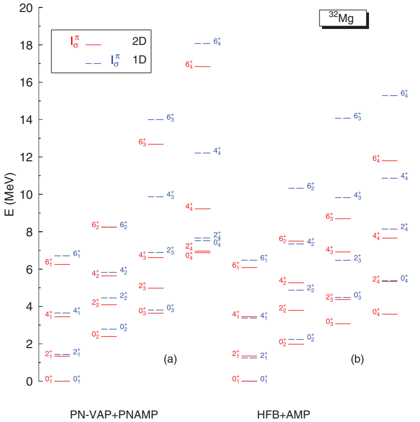


FIG. 12. (Color online) Spectra of ^{32}Mg in the PN-VAP + PNAMP approach (left) and the HFB + AMP approach (right).

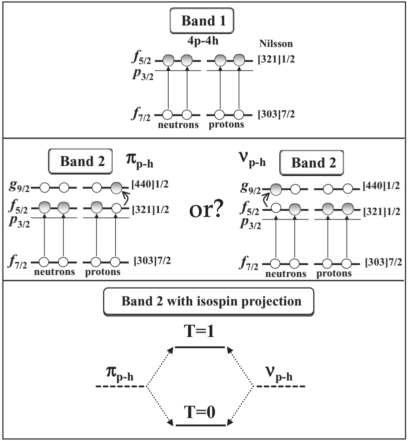
dynamical pairing combined with quadrupole GCM and projection on N, Z, and J

N. López Vaquero, J. L. Egido, T. R. Rodríguez, PRC 88 (2013) 064311.

isospin is a particular case:

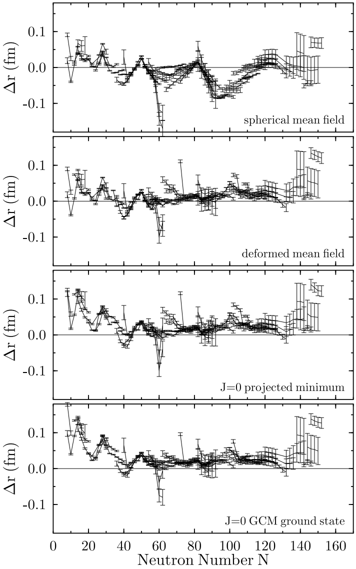
- physically broken by the Coulomb interaction, the difference of proton and neutron masses, and small contributions of the strong interaction
- spuriously broken by mean-field methods, with some severe consequences for calculated properties of nuclei close to the $N = Z$ line.
- spurious breaking can be removed by diagonalizing the symmetry-breaking Hamiltonian in a basis of isospin-projected mean-field states.
- isospin projection mixes proton and neutron single-particle states.
- controlling the physical isospin breaking has relevance for tests of the standard model through the analysis of superallowed $0^+ \to 0^+$ Fermi β decay

Example: rotational bands of ^{56}Ni

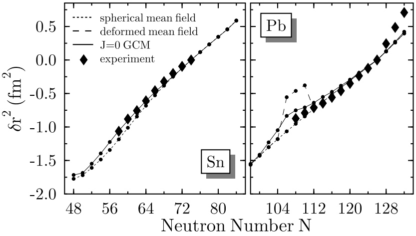


W. Satuła, J. Dobaczewski, W. Nazarewicz, M. Rafalski, PRC 81 (2010) 054310

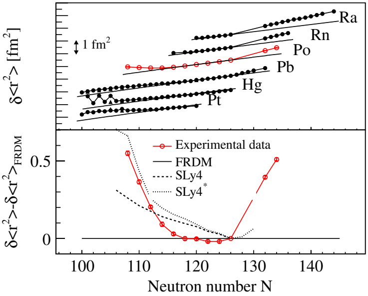
charge radii: Experimental signatures of shape mixing



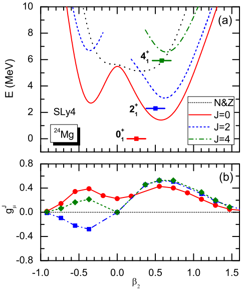
M. B., G. F. Bertsch, P.-H. Heenen, PRC 78 (2008) 054312



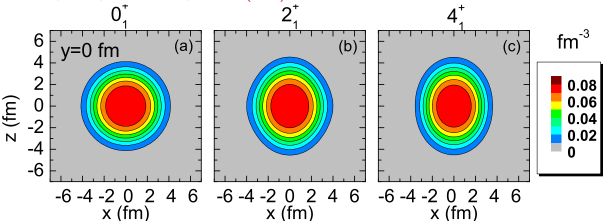
M. B., G. F. Bertsch, P.-H. Heenen, PRC 78 (2008) 054312



T. Cocolios, . . . , M. B., P.-H. Heenen, PRL 106 (2011) 052503.



J. M. Yao, M. B., P.-H. Heenen, PRC 91 (2015) 024301



Contour plots of the 3D proton densities (in fm^{-3}) in the $y = 0$ plane for the 0_1^+ (a), 2_1^+ (b), 4_1^+ (c) states (with $M = 0$) in ^{24}Mg.

Transition density in the laboratory between GCM states $|J_i M_i \mu_i\rangle$ and $|J_f M_f \mu_f\rangle$ assuming axial HFB states

$$\rho^{J_f M_f \mu_f}_{J_i M_i \mu_i}(\mathbf{r}) = \sum_{q_f, q_i} f^{J_f *}_{\mu_f, q'} \langle q' | \hat{P}^{J_f}_{0M_f} \hat{\rho}(\mathbf{r}) \hat{P}^{J_i \dagger}_{0M_i} \hat{P}^N \hat{P}^Z | q \rangle f^{J_i 0}_{\mu_i, q}$$

with

$$\begin{aligned} &\langle q' | \hat{P}^{J_f}_{0M_f} \hat{\rho}(\mathbf{r}) \hat{P}^{J_i \dagger}_{0M_i} \hat{P}^N \hat{P}^Z | q \rangle \\ &= \frac{\hat{J}_i^2 \hat{J}_f^2}{(8\pi^2)^2} \int d\Omega' \, D^{J_f *}_{0M_f}(\Omega') \sum_K D^{J_i}_{K0}(\Omega') \int d\Omega'' \, D^{J_i}_{0K}(\Omega'') \langle q' | \hat{\rho}(\tilde{\mathbf{r}}_{\Omega'}) \hat{P}^N \hat{P}^Z \hat{R}^\dagger(\Omega'') | q \rangle \\ &\equiv \frac{\hat{J}_f^2}{8\pi^2} \int d\Omega' \, D^{J_f *}_{0M_f}(\Omega') \sum_K D^{J_i}_{KM_i}(\Omega') \hat{R}^\dagger(\Omega') \rho^{J_f J_i K 0}_{q' q}(\mathbf{r}) \end{aligned}$$

For the density of the GCM state $|JM\mu\rangle$ one obtains

$$\rho^{JM\mu}_{JM\mu}(\mathbf{r}) = \sum_{q_f, q_i} f^{J*}_{\mu, q'} f^{J0}_{\mu, q} \sum_\lambda Y_{\lambda 0}(\hat{\mathbf{r}}) \langle JM\lambda 0 | JM \rangle \sum_K \langle J0\lambda K | JK \rangle \int d\hat{\mathbf{r}}' \, \rho^{JJK0}_{q' q}(r, \hat{\mathbf{r}}') Y^*_{\lambda K}(\hat{\mathbf{r}}')$$

Recent developments

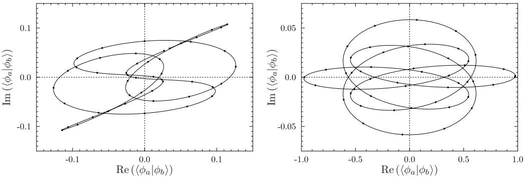
Sign problem (of the overlap)

Onishi's formula: $\langle q|q'\rangle^2 = \det(D)$ ($\Rightarrow$ undetermined sign)

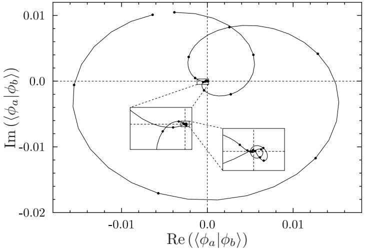
Starting point: L. Robledo, PRC 79 (2009) 021302; PRC 84 (2011) 014307: calculate overlap in some special cases from Pfaffian

Examples: 3d J projection: α, β held fixed at some values, γ varied

one-quasiparticle states in ^{25}Mg



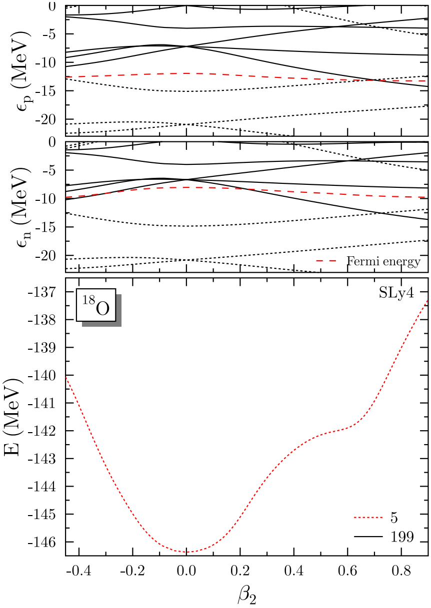
^{24}Mg cranked to $I = 8\hbar$



B. Avez & M. B., PRC 85 (2012) 034255

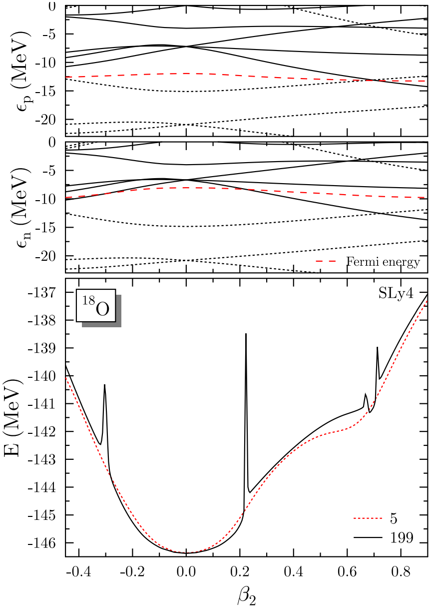
Similar expressions derived by Bertsch & Robledo, PRL108 (2012) 042505.

"Pole problem"



- pure particle-number projection

"Pole problem"



- pure particle-number projection
- first hints from Hamiltonian-based approaches in the form of failure of approximations: Dönau, PRC 58 (1998) 872; Almehed, Frauendorf, Dönau, PRC 63 (2001) 044311.
- analysis in density-dependent-Hamiltonian-based approach: Anguiano, Egido, Robledo NPA696 (2001) 467
- First analysis in a strict energy density functional (EDF) framework and of EDF-specific consequences by Dobaczewski, Stoitsov, Nazarewicz, Reinhard, PRC 76 (2007) 054315
- Further analysis of the EDF case by Lacroix, Duguet, Bender, PRC 79 (2009) 044318; Bender, Duguet, Lacroix, PRC 79 (2009) 044319; Duguet, Bender, Bennaceur, Lacroix, Lesinski, PRC 79 (2009) 044320; Bender, Avez, Bally, Duguet, Heenen, Lacroix, *still in preparation*

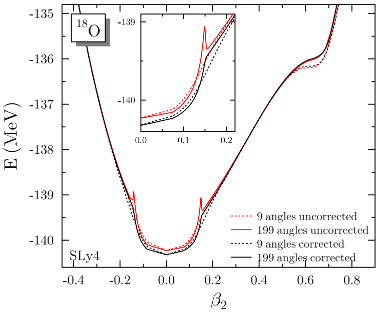
The origin of the problem in a nutshell

- All standard energy density functionals (EDF) used for mean-field models and beyond do not correspond to the expectation value of a Hamiltonian for at least one of the following reasons:
  - density dependences
  - the use of different effective interactions in the particle-hole and pairing parts of the energy functional
  - the omission, approximation or modification of specific exchange terms

  that are all introduced for phenomenological reasons and/or the sake of numerical efficiency.
- consequence: breaking of the exchange symmetry ("Pauli principle") under particle exchange when calculating the energy, leading to non-physical interactions of a given nucleon or pair of nucleons with itself, or of three nucleons among themselves etc.
- the resulting self-interactions and self-pairing-interactions remain (usually) hidden in the mean field
- in the extension to symmetry-restored GCM, these terms cause
  - discontinuities and divergences in symmetry-restored energy surfaces
  - breaking of sum rules in symmetry restoration
  - potentially multi-valued EDF in case of standard density-dependences

Second problem: non-analytical density dependences

Non-viability of non-integer density dependences



Duguet, Lacroix, M. B., Bennaceur, Lesinski, PRC 79 (2009) 044320

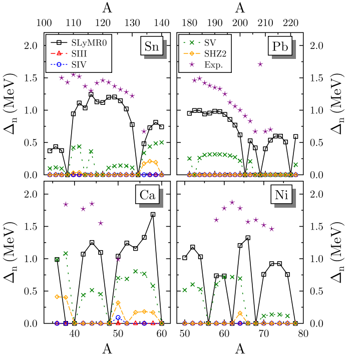
- in symmetry restored GCM, the local densities $\rho^{qq'}(\mathbf{r})$ are in general complex
- $\left[\rho^{qq'}(\mathbf{r})\right]^{\alpha}$ is a multi-valued non-analytical function
- spurious contribution from branch cuts (see Dobaczewski*et al.* PRC 76 (2007) 054315, and Duguet *et al.* PRC 79 (2009) 044320 for complex plane analysis)
- (partial) workaround when conserving specific symmetries: use particle-number projected densities for density dependence instead (stategy currently used by L. Egido and collaborators).
  - difficult to justify formally.
  - leads to unphysical excitation spectra when using time-reversal-invariance breaking reference states.

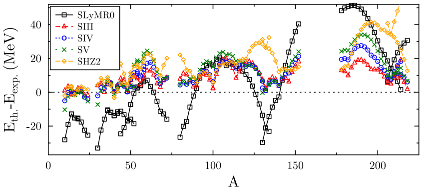
What to do?

1. construct the EDF from a density-dependent Hamiltonians with special *ad-hoc* treatment of the density entering the density-dependent terms for which numerically efficient high-quality parameterizations can be easily constructed. Problem: cannot be defined for all possible confifuration mixings of interest [Robledo, J. Phys. G 37 (2010) 064020] and/or fails when breaking time-reversal invariance [Bender, Avez, Bally, Heenen, in preparation].
2. introducing a regularization scheme of the EDF that allows for the use of (almost) standard functionals [Lacroix, Duguet, & Bender, PRC 79 (2009) 044318] for which numerically efficient high-quality parameterizations can be easily constructed [Washiyama, Bennaceur, Avez, Bender, Heenen, & Hellemans, PRC 86 (2012) 054309]. Problems: complicated formalism. Almost works sometimes. Sometimes fails spectactularly.
3. constructing the EDF as expectation value of a strict Hamiltonian. New problems: numerically much more costly due to Coulomb exchange & pairing; no available parameterizations of high quality (the difficulties to construct such parametrizations was the main motivation to use EDFs with density-dependent coupling constants in the 1970s).

Toy Skyrme Hamiltonian: SLyMR0

$$
\begin{aligned}
\hat{v} = {} & t_0 \left(1 + x_0 \hat{P}_\sigma\right) \hat{\delta}_{r_1 r_2} \\
& + \frac{t_1}{2} \left(1 + x_1 \hat{P}_\sigma\right) \left(\hat{\mathbf{k}}_{12}^{\prime\, 2} \hat{\delta}_{r_1 r_2} + \hat{\delta}_{r_1 r_2} \hat{\mathbf{k}}_{12}^{2}\right) \\
& + t_2 \left(1 + x_2 \hat{P}_\sigma\right) \hat{\mathbf{k}}_{12}^{\prime} \cdot \hat{\delta}_{r_1 r_2} \, \hat{\mathbf{k}}_{12} \\
& + \mathrm{i}\, W_0 \left(\hat{\boldsymbol{\sigma}}_1 + \hat{\boldsymbol{\sigma}}_2\right) \cdot \hat{\mathbf{k}}_{12}^{\prime} \times \hat{\delta}_{r_1 r_2} \hat{\mathbf{k}}_{12} \\
& + u_0 \left(\hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} + \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_1 r_2} + \hat{\delta}_{r_2 r_1} \hat{\delta}_{r_3 r_1}\right) \\
& + v_0 \left(\hat{\delta}_{r_1 r_3} \hat{\delta}_{r_2 r_3} \hat{\delta}_{r_3 r_4} + \hat{\delta}_{r_1 r_2} \hat{\delta}_{r_3 r_2} \hat{\delta}_{r_2 r_4} + \cdots\right) \\
& + \text{Coulomb}
\end{aligned}
$$





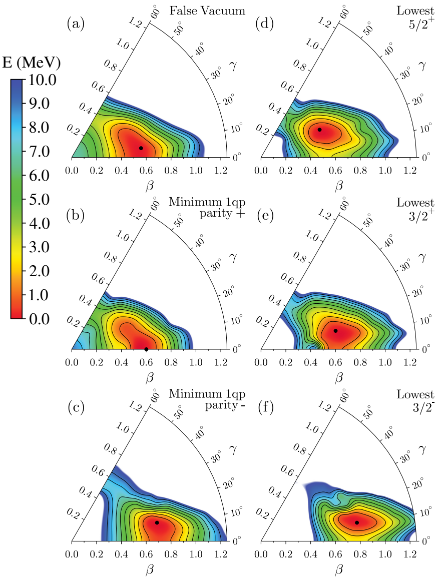
- impossible to fullfil usual nuclear matter constraints, to have stable interactions and attractive pairing
- no "best fit" possible
- very bad performance compared to standard general functionals

J. Sadoudi, M. Bender, K. Bennaceur, D. Davesne, R. Jodon, and T. Duguet, Physica Scripta T154 (2013) 014013

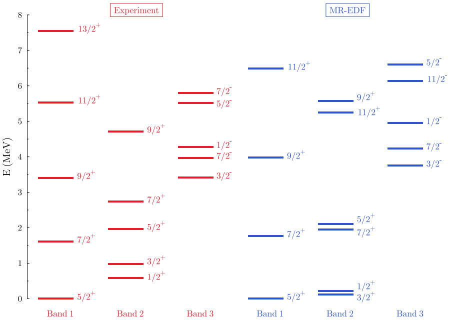
Nuclear physics jargon: breaking time-reversal invariance of the reference states

- one non-paired nucleon: odd-A nuclei
- one or several broken pairs: states with seniority $\neq 0$ (and usually $J \neq 0$) in even-even nuclei
- disturbed pairs: angular-momentum optimized collectively rotating states (in nuclear physics jargon: self-consistent cranking)

$$\hat{H} \to \hat{H} - \boldsymbol{\omega} \cdot \hat{\mathbf{J}}$$



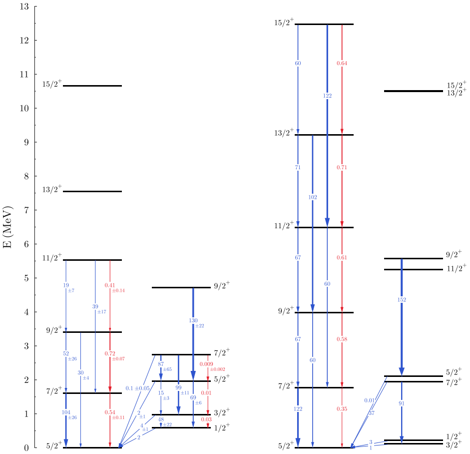
Angular-momentum and particle-number projected GCM of blocked triaxial one-quasiparticle states



B. Bally, doctoral thesis, Université de Bordeaux (2014)

B. Bally, B. Avez, M. B., and P.-H. Heenen, PRL 113 (2014) 162501

First "beyond-mean-field" results for odd-A nuclei with SLyMR0

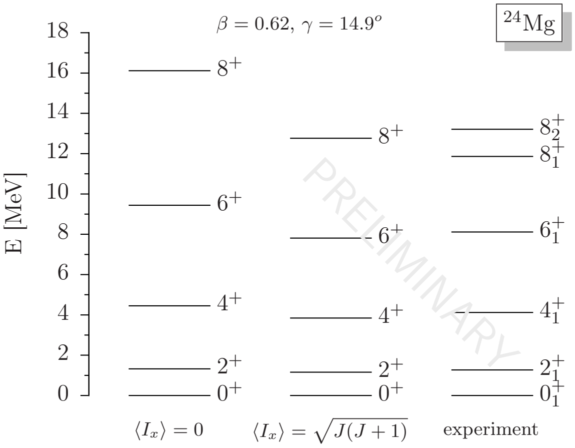


B. Bally, B. Avez, M. B., P.-H. Heenen (to be published)

Data from Nuclear Data Sheets 110 (2009) 1691

- spectroscopic quadrupole moment Q_s of the $5/2^+$ ground state:
  Exp: 20.1 ± 0.3 e fm^2
  Calc: 23.25 e fm^2
- magnetic moment μ of the $5/2^+$ ground state in nuclear magnetons:
  Exp: -0.855
  Calc: -1.054

Projection of cranked states

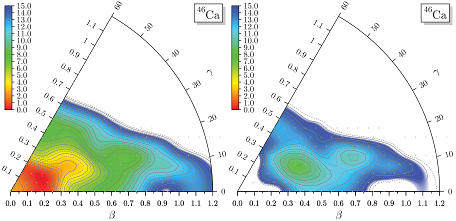


B. Bally, B. Avez, M. B., P.-H. Heenen (unpublished)

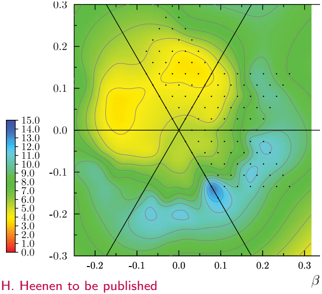
- compression of excitation spectrum
- projecting the states cranked to I does not lead to the lowest energy for $J = I$ states.

seniority 0

N, Z, $J = 0$ projected N, Z, $J = 6$ projected



seniority 2, lowest N, Z, $J = 6$ projected

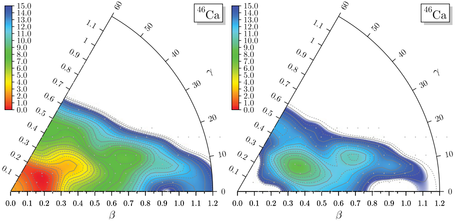


M. B.& P.-H. Heenen to be published

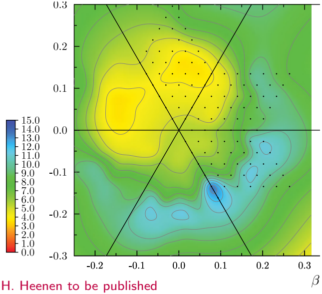
MR EDF with diabatic states: ^{46}Ca

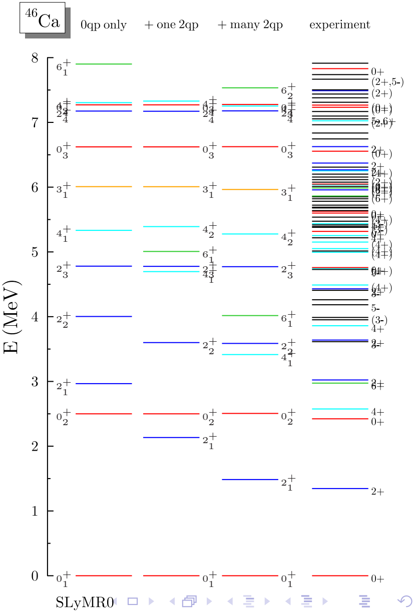
seniority 0

N, Z, $J = 0$ projected $\quad$ N, Z, $J = 6$ projected



seniority 2, lowest N, Z, $J = 6$ projected





M. B.& P.-H. Heenen to be published

Ongoing explorative work on the effective interaction

- construction of most general central 3-body contact interaction of 2nd order in gradients J. Sadoudi, T. Duguet, J. Meyer, M. B., PRC 88 (2013) 064326
- parameter adjustment of a standard 2-body + central 3-body contact interaction up to 2nd order in gradients: SLyMR1 R. Jodon, Thesis, Lyon (2014). Better than SLyMR0, but still serious deficiencies.
- construction of most general spin-orbit + tensor 3-body force of 2nd order in gradients has been started by J. Sadoudi.

What to add?

- two-body contact terms of 4th (and even 6nd) order in gradients

  F. Raimondi, B. G. Carlsson, J. Dobaczewski, J. Toivanen PRC84 (2011) 064303

  D. Davesne, A. Pastore, J. Navarro, JPG 40 (2013) 095104

  P. Becker, D. Davesne, J. Meyer, A. Pastore, J. Navarro, JPG 42 (2015) 034001
- non-local finite range two-body terms

  F. Raimondi, K. Bennaceur, J. Dobaczewski, JPG41 (2014) 055112
- semi-contact three-body force

  D. Lacroix, K. Bennaceur, PRC91 (2015) 011302R

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  D. Lacroix, K. Bennaceur, PRC91 (2015) 011302R
- or try a different strategy altogether: explicit in-medium correlations from MBPT

  T. Duguet, M. Bender, J.-P. Ebran, T. Lesinski, V. Somà, arXiv:1502.03672

Take-away messages

- MR techniques provide useful tools to describe correlations related to the finiteness and self-boundedness of atomic nuclei
- symmetry restoration of symmetry-breaking reference states
- GCM-type mixing of (symmetry-restored) states
- The last 10 years brought a huge step forward in the advancement of implementation of multi-dimensional MR techniques
- there are interesting formal problems that we just start to identify and understand, and that remain to be solved

Acknowledgements

The work presented here would have been impossible without my collaborators

Founding fathers

Paul Bonche (deceased)	SPhT, CEA Saclay
Hubert Flocard (retired)	CSNSM Orsay
Paul-Henri Heenen	Université Libre de Bruxelles

formal aspects

Thomas Duguet	Irfu/CEA Saclay & KU Leuven & NSCL/MSU
Denis Lacroix	IPN Orsay

code development and benchmarking

Benoît Avez	CEN Bordeaux Gradignan
Benjamin Bally	CEN Bordeaux Gradignan, now SPhN, CEA Saclay
Veerle Hellemans	Université Libre de Bruxelles
Jiangming Yao	Université Libre de Bruxelles

development and benchmarking of new functionals

Karim Bennaceur	IPN Lyon & Jyväskylä
Dany Davesne	IPN Lyon
Robin Jodon	IPN Lyon
Jacques Meyer	IPN Lyon
Alessandro Pastore	IPN Lyon, now CEA Bruyères-le-châtel
Jeremy Sadoudi	CEN Bordeaux Gradignan
Kouhei Washiyama	Université Libre de Bruxelles