Multi-Reference IM-SRG for Nuclei

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Outline

- The Similarity Renormalization Group
- In-Medium SRG
- Multi-Reference In-Medium SRG
- Ground States of Closed- and Open-Shell Nuclei
- Next Steps
- Conclusions

Prelude:

Similarity Renormalization Group

Review:

S. Bogner, R. Furnstahl, and A. Schwenk, Prog. Part. Nucl. Phys. **65** (2010), 94

E. Anderson, S. Bogner, R. Furnstahl, and R. Perry, Phys. Rev. **C82** (2011), 054001

E. Jurgenson, P. Navratil, and R. Furnstahl, Phys. Rev. **C83** (2011), 034301

R. Roth, S. Reinhardt, and H. H., Phys. Rev. **C77** (2008), 064003

H. H. and R. Roth, Phys. Rev. **C75** (2007), 051001

Similarity Renormalization Group

Basic Concept

continuous unitary transformation of the Hamiltonian to band-diagonal form w.r.t. a given "uncorrelated" many-body basis

- flow equation for Hamiltonian $H(s) = U(s)HU^\dagger(s)$:

$$\frac{d}{ds}H(s) = \left[\eta(s), H(s)\right], \quad \eta(s) = \frac{dU(s)}{ds}U^\dagger(s) = -\eta^\dagger(s)$$

- choose $\eta(s)$ to achieve desired behavior, e.g.,

$$\eta(s) = \left[H_d(s), H_{od}(s)\right]$$

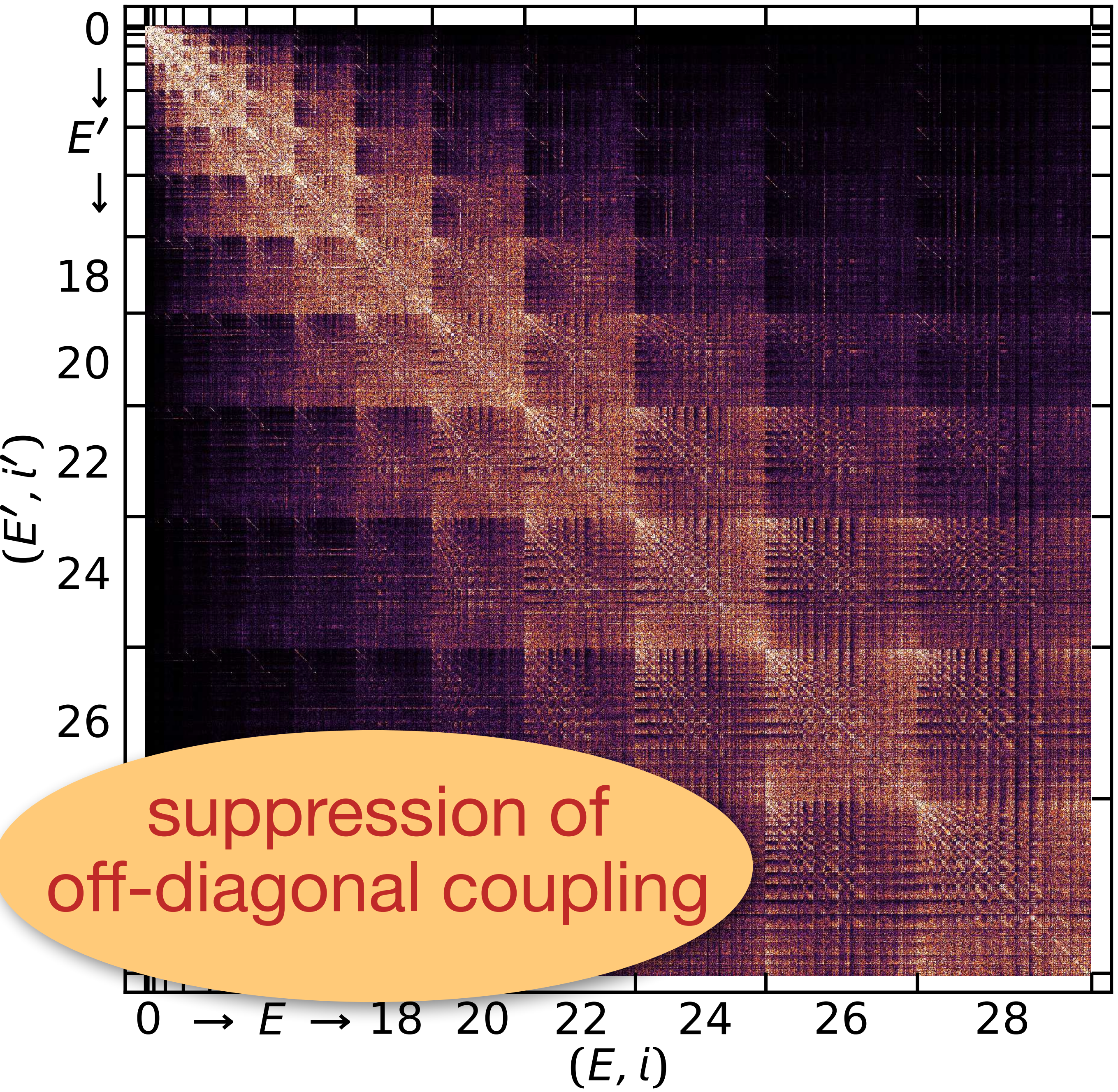
to suppress (suitably defined) off-diagonal Hamiltonian

- consistent evolution for all observables of interest

SRG in Three-Body Space

3B Jacobi-HO Matrix Elements

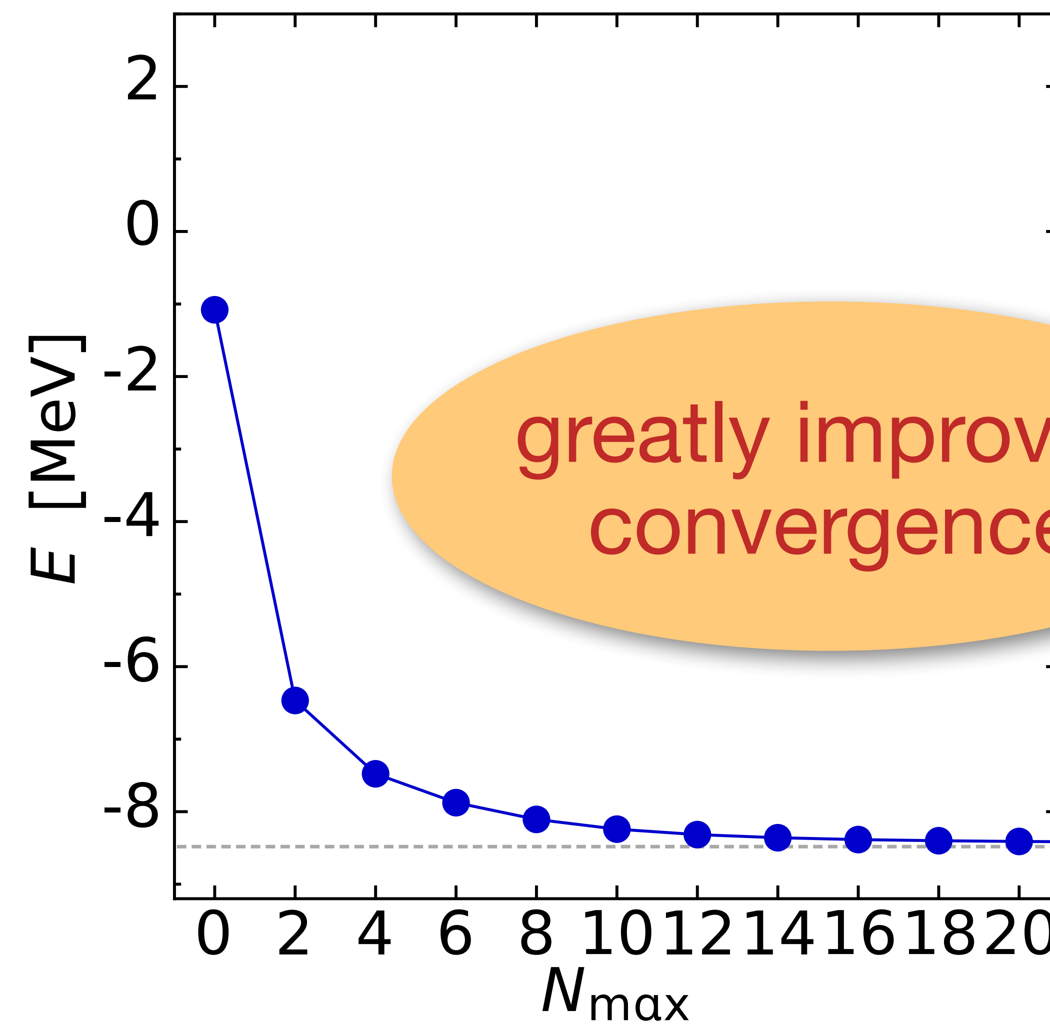
$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 28$ MeV



suppression of off-diagonal coupling

$\lambda = 1.33\ \mathrm{fm}^{-1}$

^{3}H ground-state (NCSM)



greatly improved convergence

[figures by R. Roth, A. Calci, J. Langhammer]

In-Medium SRG

S. K. Bogner, H. H., T. Morris, A. Schwenk, and K. Tuskiyama, to appear in Phys. Rept.

H. H., S. K. Bogner, S. Binder, A. Calci, J. Langhammer, R. Roth, and A. Schwenk, Phys. Rev. C **87**, 034307 (2013)

K. Tsukiyama, S. K. Bogner, and A. Schwenk, Phys. Rev. Lett. **106**, 222502 (2011)

Solving the Flow Equation

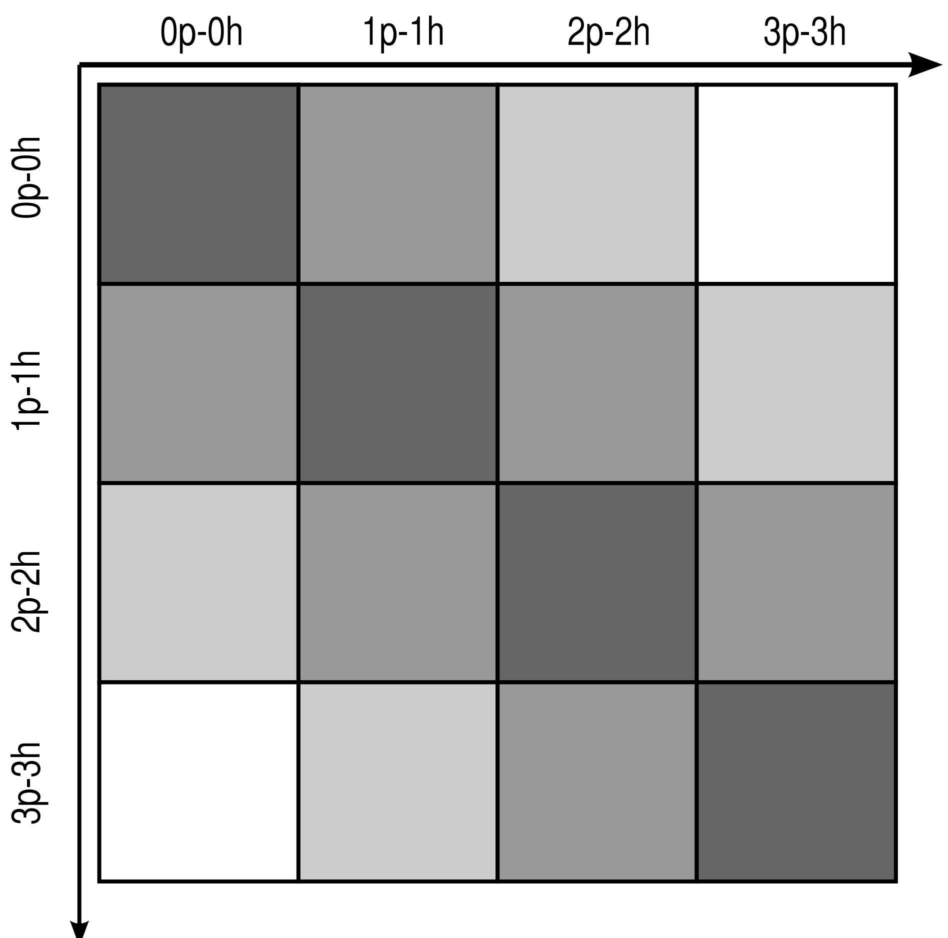
- operator flow equation:

$$\frac{d}{ds}H(s) = [\eta(s), H(s)]$$

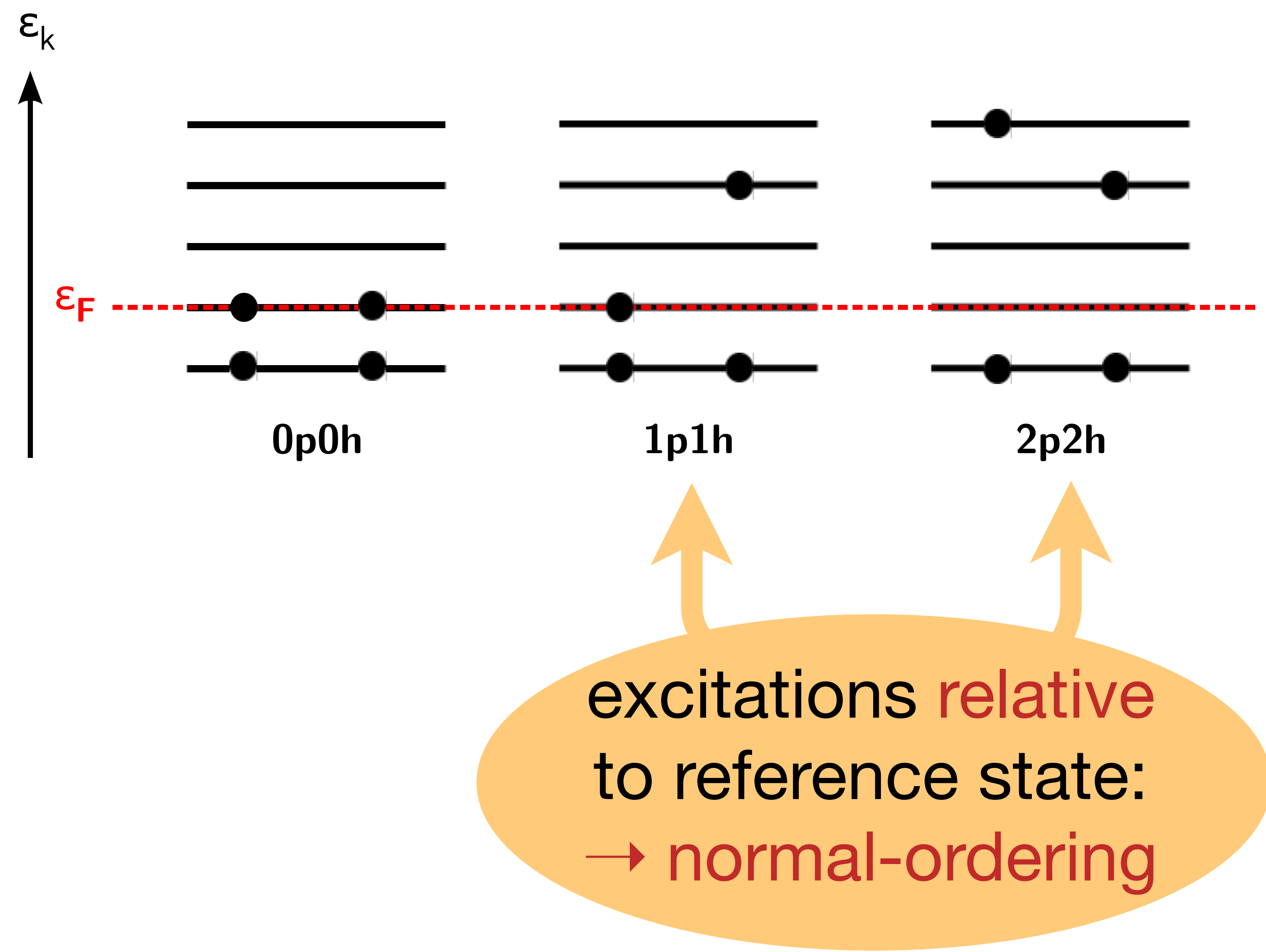
  - encodes information for all systems described by H
  - (“arbitrarily”) complex unitary transformation to solve everything at once

- truncation / organization scheme
  - introduces restrictions (particle number, …), uncertainties
- basis (operator algebra and/or Hilbert space)
- generator
  - introduce information from target system (e.g. reference state) to optimize transformation, reduce uncertainties

Decoupling in A-Body Space



$\langle i | H | j \rangle$



Normal Ordering

- second quantization: $A^{k_1\ldots k_N}_{l_1\ldots l_N} = a^\dagger_{k_1}\ldots a^\dagger_{k_N} a_{l_N}\ldots a_{l_1}$
- particle- and hole density matrices:

$$\lambda^k_l = \left\langle \Phi \right| A^k_l \left| \Phi \right\rangle \longrightarrow n_k \delta^k_l\,, \quad n_k \in \{0,1\}$$
$$\xi^k_l = \lambda^k_l - \delta^k_l \qquad \longrightarrow -\bar{n}_k \delta^k_l \equiv -(1-n_k)\delta^k_l$$

- define normal-ordered operators recursively:

$$A^{k_1\ldots k_N}_{l_1\ldots l_N} = :A^{k_1\ldots k_N}_{l_1\ldots l_N}: + \lambda^{k_1}_{l_1} :A^{k_2\ldots k_N}_{l_2\ldots l_N}: + \text{singles}$$
$$+ \left(\lambda^{k_1}_{l_1}\lambda^{k_2}_{l_2} - \lambda^{k_1}_{l_2}\lambda^{k_2}_{l_1}\right) :A^{k_3\ldots k_N}_{l_3\ldots l_N}: + \text{doubles} + \ldots$$

- algebra is simplified significantly because

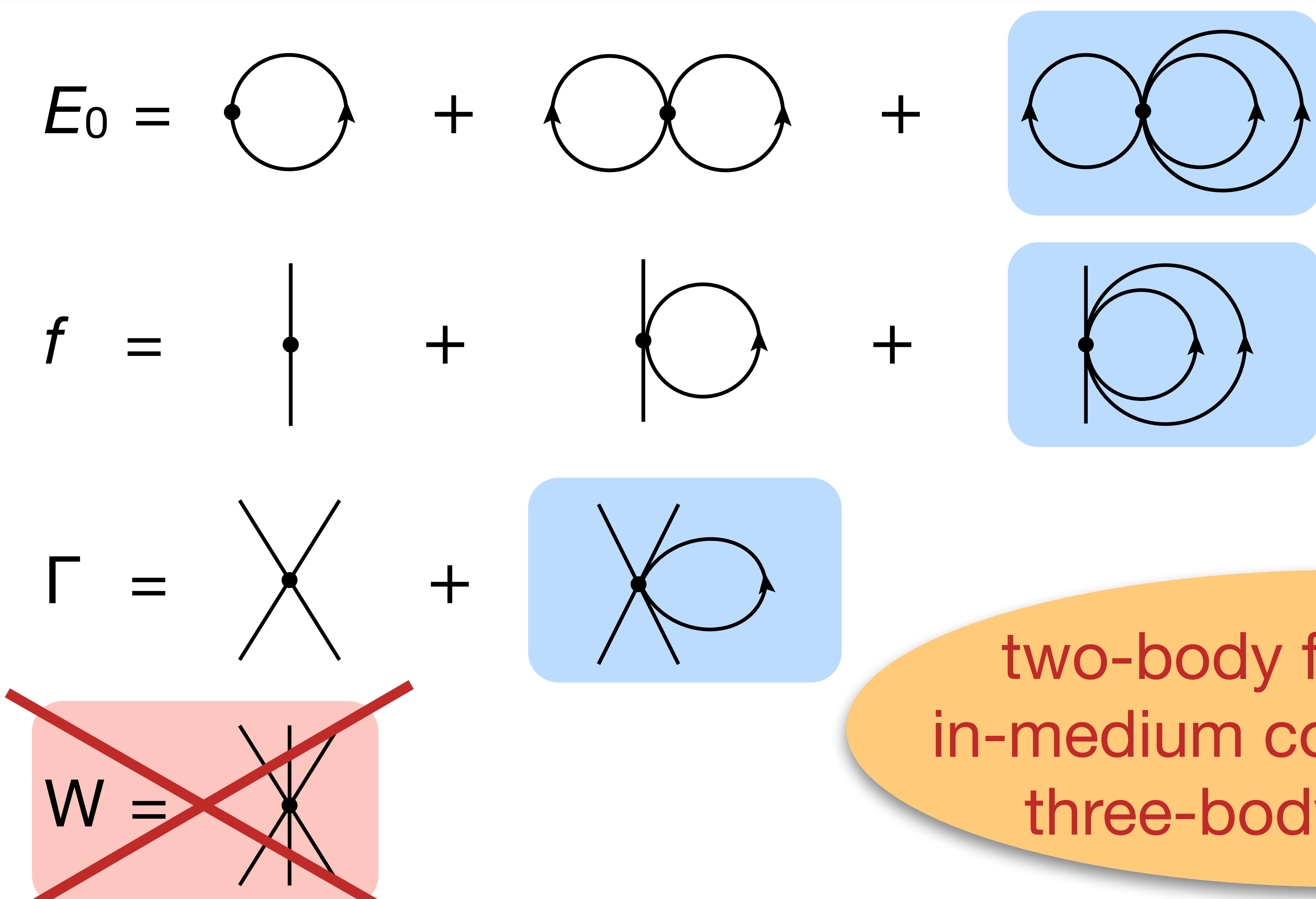
$$\left\langle \Phi \right| :A^{k_1\ldots k_N}_{l_1\ldots l_N}: \left| \Phi \right\rangle = 0$$

- Wick's theorem gives simplified expansions (fewer terms!) for products of normal-ordered operators

Normal-Ordered Hamiltonian

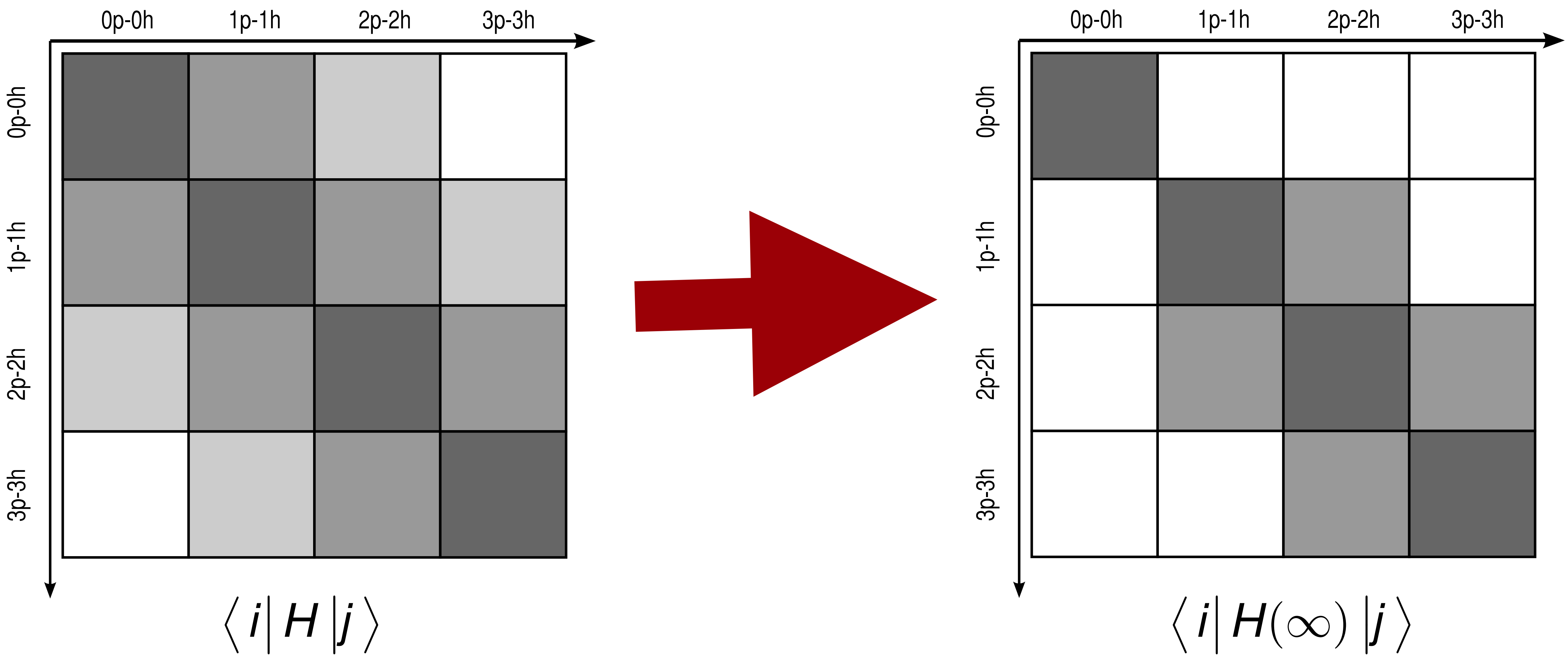
Normal-Ordered Hamiltonian

$$H = E_0 + \sum_{kl} f^k_l :A^k_l: + \frac{1}{4}\sum_{klmn}\Gamma^{kl}_{mn} :A^{kl}_{mn}: + \frac{1}{36}\sum_{ijklmn} W^{ijk}_{lmn} :A^{ijk}_{lmn}:$$



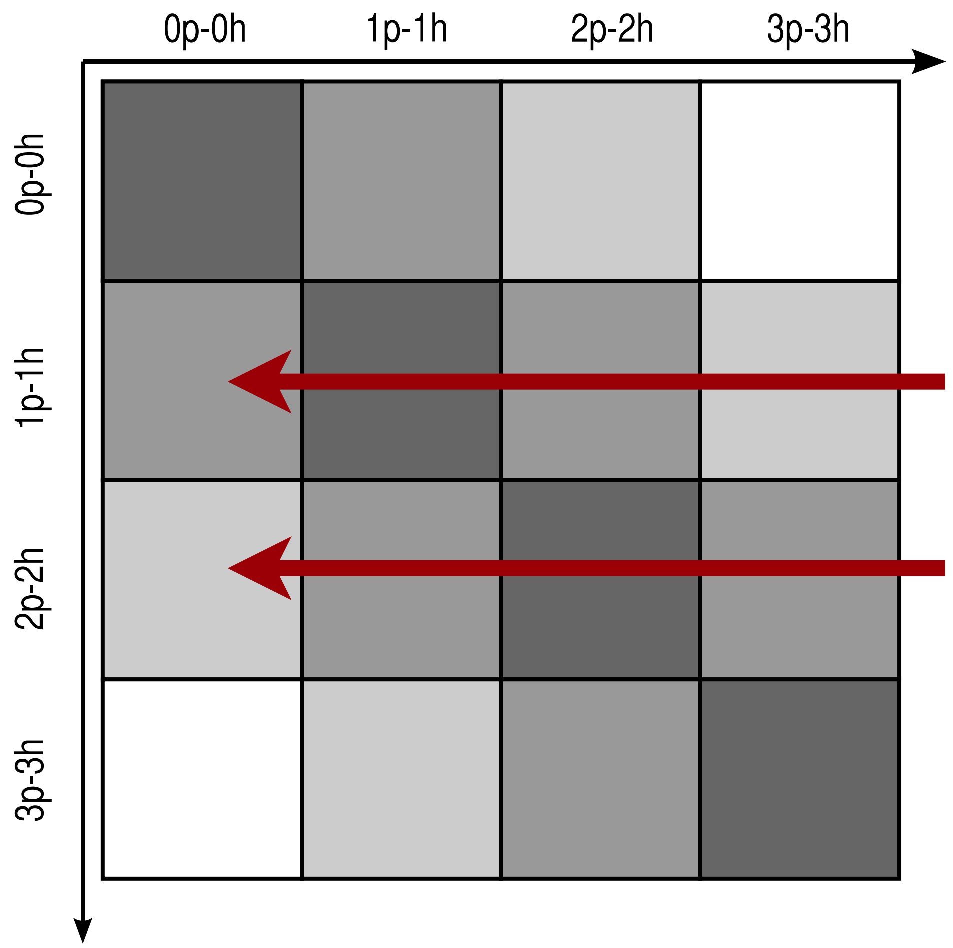
two-body formalism with in-medium contributions from three-body interactions

Decoupling in A-Body Space



aim: decouple reference state $|\Phi\rangle$ (0p-0h) from excitations

Decoupling in A-Body Space



$$\left\langle {}^{p}_{h} \middle| H \middle| \Psi \right\rangle = \sum_{kl} f^{k}_{l} \left\langle \Psi \middle| :A^{h}_{p} :: A^{k}_{l}: \middle| \Psi \right\rangle = -n_h \bar{n}_p f^{p}_{h}$$

$$\left\langle {}^{pp'}_{hh'} \middle| H \middle| \Psi \right\rangle = \sum_{klmn} \Gamma^{kl}_{mn} \left\langle \Psi \middle| :A^{hh'}_{pp'} :: A^{kl}_{mn}: \middle| \Psi \right\rangle \sim \Gamma^{pp'}_{hh'}$$

- define off-diagonal Hamiltonian (suppressed by IM-SRG flow):

$$H_{od} \equiv f_{od} + \Gamma_{od}, \quad f_{od} \equiv \sum_{ph} f^{p}_{h} :A^{p}_{h}: + \text{H.c.}, \quad \Gamma_{od} \equiv \frac{1}{4} \sum_{pp'hh'} \Gamma^{pp'}_{hh'} :A^{pp'}_{hh'}: + \text{H.c.}$$

➡ construct generator

Choice of Generator

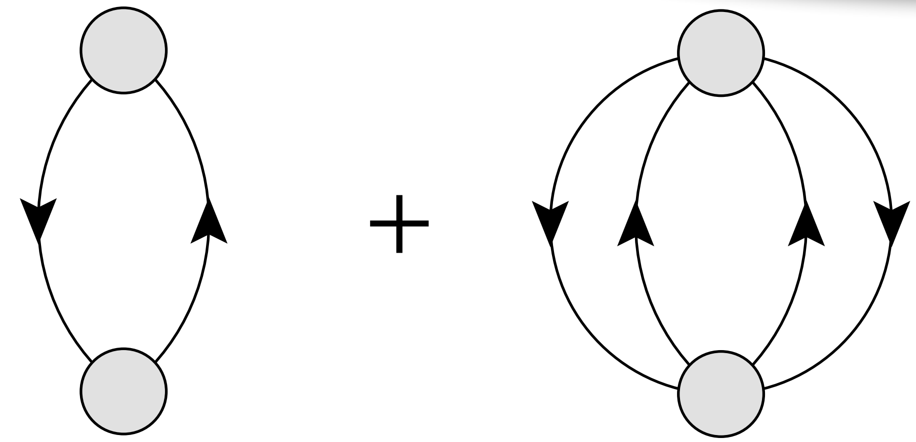
- Wegner: $$\eta^{I} = [H_d, H_{od}]$$
- White: (J. Chem. Phys. 117, 7472)
$$\eta^{II} = \sum_{ph} \frac{f^p_h}{\Delta^p_h} :A^p_h: + \frac{1}{4} \sum_{pp'hh'} \frac{\Gamma^{pp'}_{hh'}}{\Delta^{pp'}_{hh'}} :A^{pp'}_{hh'}: - \text{H.c.}$$
$\Delta^p_h, \Delta^{pp'}_{hh'}$: approx. 1p1h, 2p2h excitation energies
- "imaginary time": (Morris, Bogner)
$$\eta^{III} = \sum_{ph} \text{sgn}\left(\Delta^p_h\right) f^p_h :A^p_h: + \frac{1}{4} \sum_{pp'hh'} \text{sgn}\left(\Delta^{pp'}_{hh'}\right) \Gamma^{pp'}_{hh'} :A^{pp'}_{hh'}: - \text{H.c.}$$
- off-diagonal matrix elements are suppressed like $e^{-\Delta^2 s}$ (Wegner), e^{-s} (White), and $e^{-|\Delta| s}$ (imaginary time)
- g.s. energies ($s \to \infty$) differ by $\ll 1\%$

IM-SRG(2) Flow Equations

0-body Flow

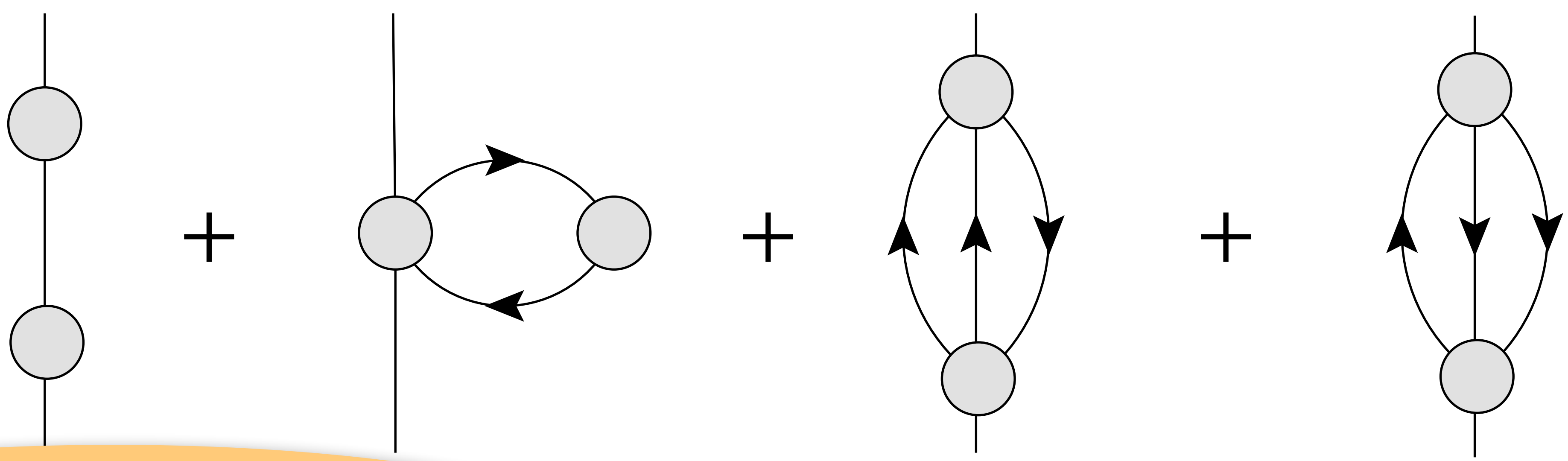
~ 2nd order MBPT for *H(s)*

$$\frac{dE}{ds} =$$



1-body Flow

$$\frac{df}{ds} =$$



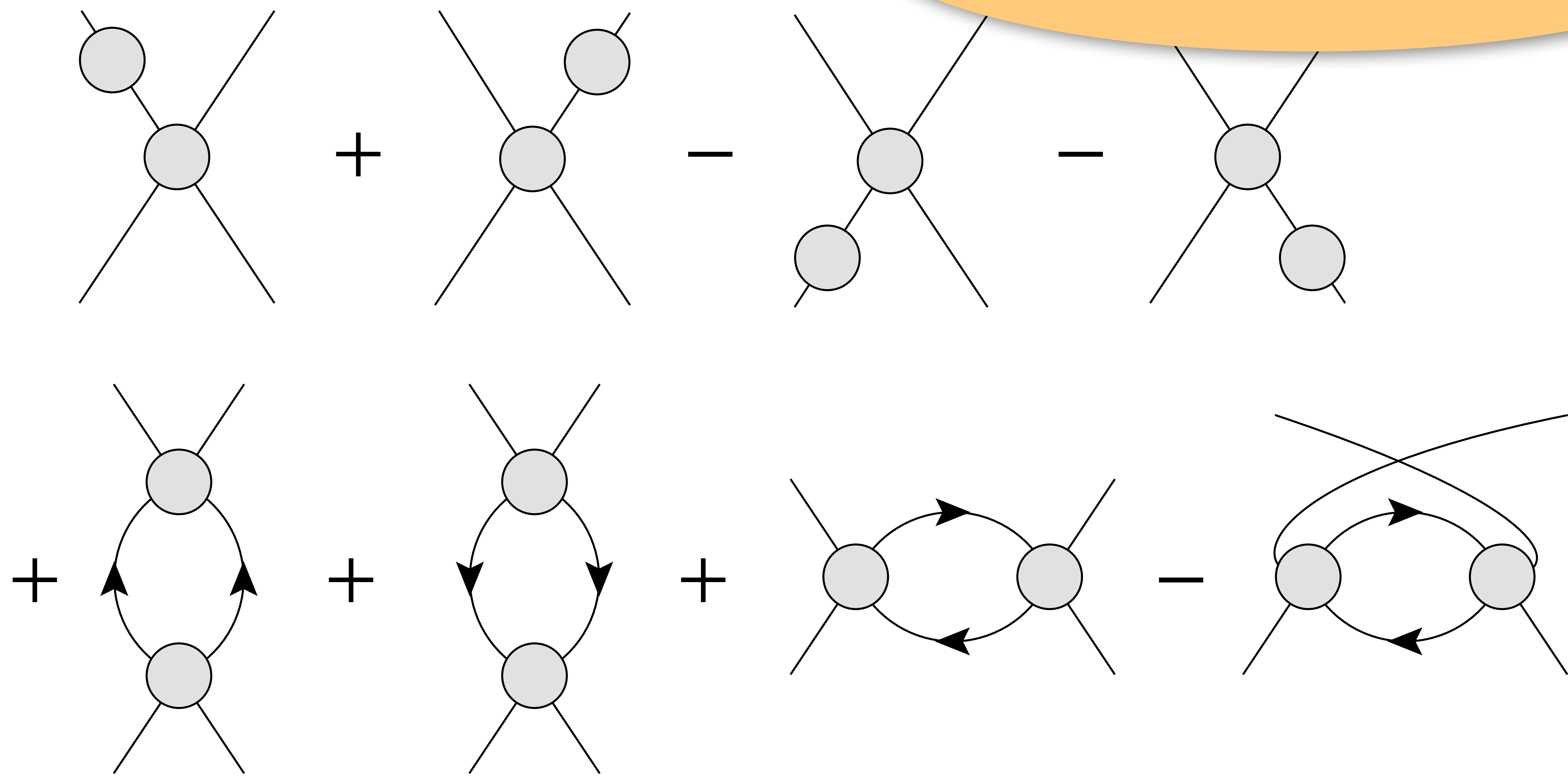
IM-SRG(2): truncate ops.
at two-body level

IM-SRG(2) Flow Equations

2-body Flow

$O(N^6)$ scaling
(before particle/hole distinction)

$$\frac{d\Gamma}{ds} =$$



s channel

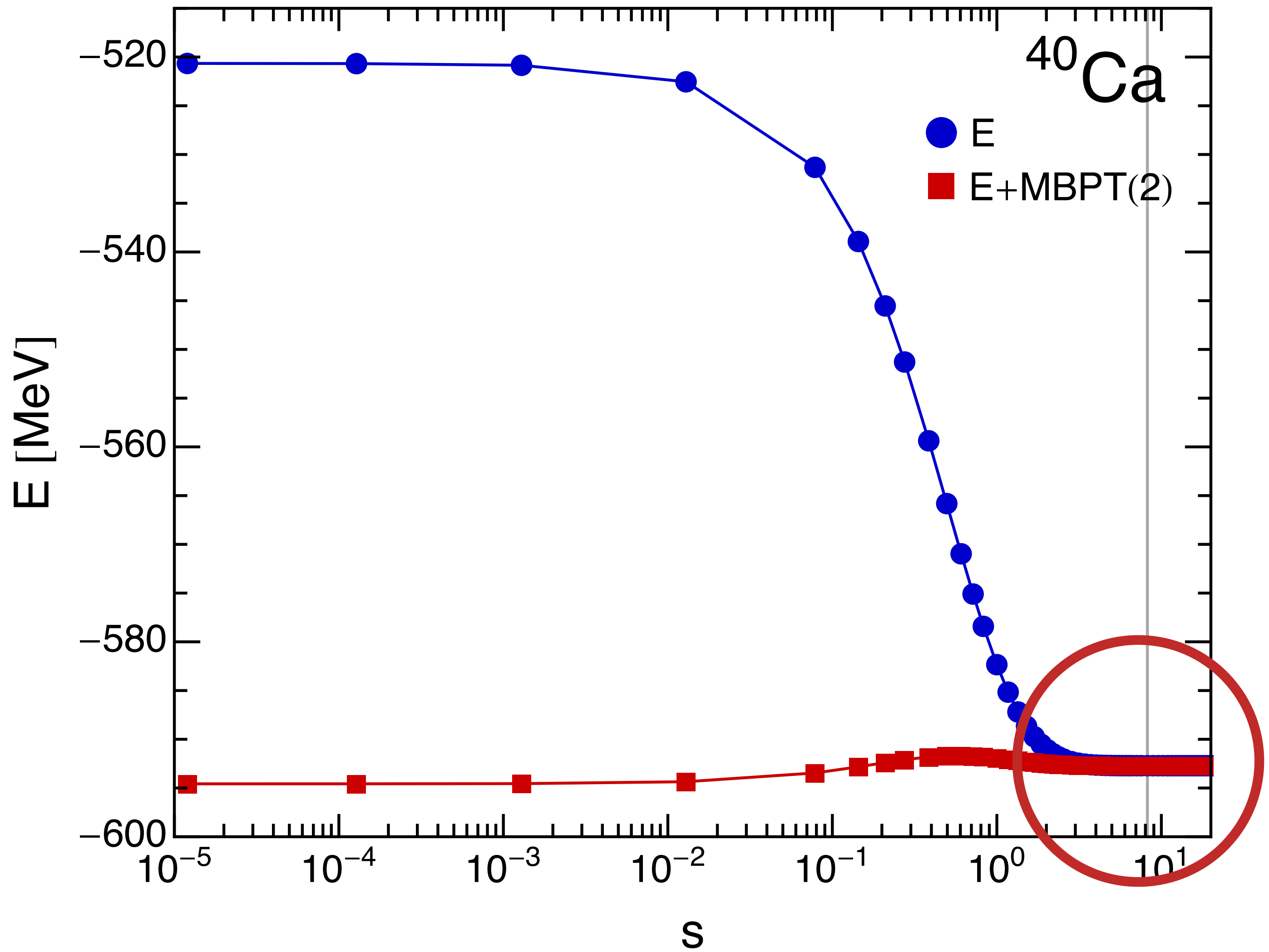
ladders

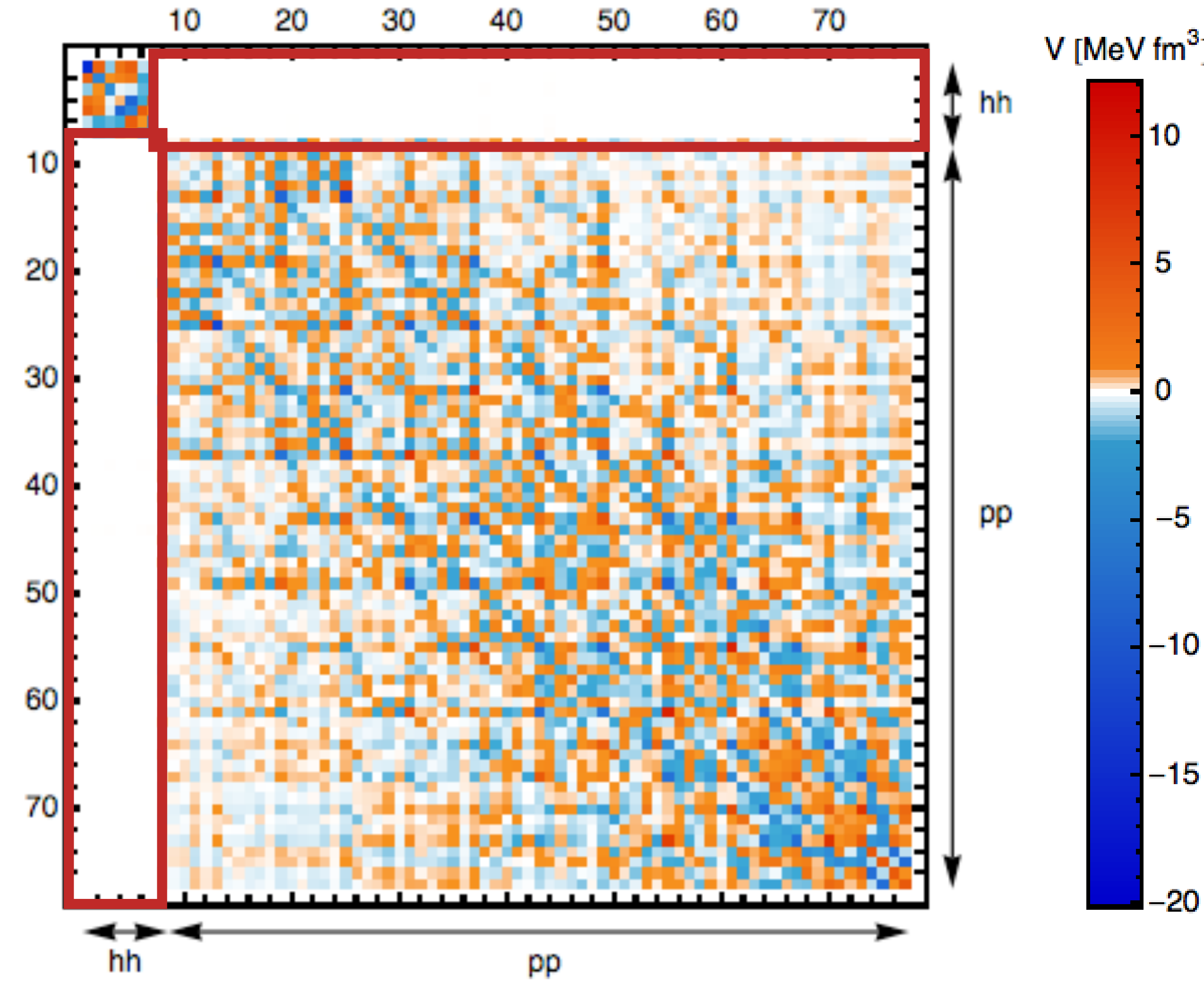
t channel

u channel

rings

Decoupling





N3LO, $\lambda = 2.0\ \mathrm{fm}^{-1}, e_{\mathrm{Max}} = 8$

non-perturbative resummation of MBPT series (correlations)

off-diagonal couplings are rapidly driven to zero

Multi-Reference IM-SRG

H. H., in preparation

H. H., S. Bogner, T. Morris, S. Binder, A. Calci, J. Langhammer, R. Roth, Phys. Rev. C **90**, 041302 (2014)

H. H., S. Binder, A. Calci, J. Langhammer, and R. Roth, Phys. Rev. Lett **110**, 242501 (2013)

Multi-Reference IM-SRG

- generalized Wick's theorem for arbitrary reference states (Kutzelnigg & Mukherjee)
- define irreducible n-body density matrices of reference state:

$$\rho^{kl}_{mn} = \lambda^{kl}_{mn} + \lambda^k_m \lambda^l_n - \lambda^k_n \lambda^l_m$$

$$\rho^{ijk}_{lmn} = \lambda^{ijk}_{lmn} + \lambda^i_l \lambda^{jk}_{mn} + \lambda^i_l \lambda^j_m \lambda^k_n + \text{permutations}$$

$$\ldots$$

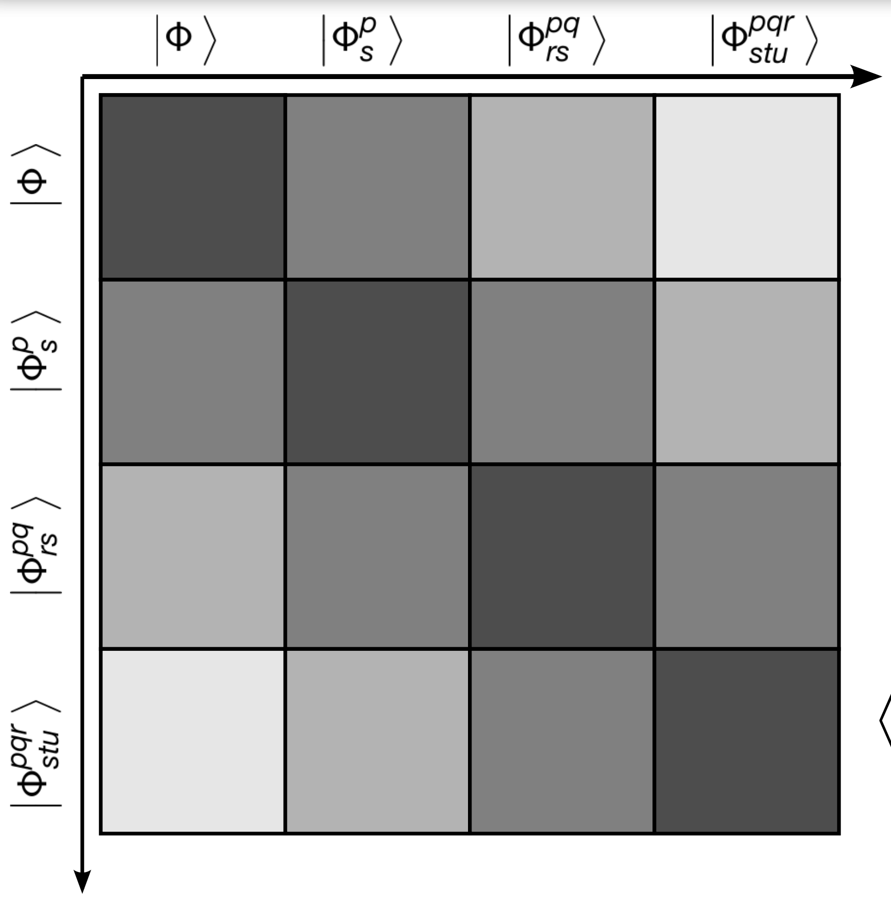
- irreducible densities give rise to additional contractions:

$$: A^{ab\ldots}_{cd\ldots} :: A^{kl\ldots}_{mn\ldots} : \quad \longrightarrow \quad \lambda^{ab}_{mn}$$

$$: A^{ab\ldots}_{cd\ldots} :: A^{kl\ldots}_{mn\ldots} : \quad \longrightarrow \quad \lambda^{ab}_{cm}$$

$$\ldots$$

Decoupling Revisited



$$\langle {}^{p}_{s} | H | \Phi \rangle \sim \bar{n}_p n_s f^{p}_{s}, \sum_{kl} f^{k}_{l} \lambda^{sk}_{pl}, \sum_{klmn} \Gamma^{kl}_{mn} \lambda^{skl}_{pmn}, \dots$$

$$\langle {}^{pq}_{st} | H | \Phi \rangle \sim \bar{n}_p \bar{n}_q n_s n_t \Gamma^{pq}_{st}, \sum_{kl} \Gamma^{pk}_{sl} \lambda^{tk}_{ql}, \sum_{kl} f^{k}_{l} \lambda^{stk}_{pql}, \sum_{klmn} \Gamma^{kl}_{mn} \lambda^{stkl}_{pqmn}, \dots$$

$$\langle {}^{pqr}_{stu} | H | \Phi \rangle \sim \dots$$

- truncation in irreducible density matrices based on, e.g.,
  - number of correlated vs. total pairs, triples, ... (caveat: highly collective reference states)
  - perturbative analysis (e.g. for shell-model like states)
- verify for chosen multi-reference state when possible

Generators

$$\eta^{I} = [H_d, \boxed{H_{od}}]$$

$$\eta^{II} = \sum_{pr} \frac{\boxed{\bar{n}_p n_r f^p_r + \ldots}}{\boxed{\Delta^p_r}} :A^p_r: + \frac{1}{4}\sum_{pqrs} \frac{\boxed{\bar{n}_p \bar{n}_q n_r n_s \Gamma^{pq}_{rs} + \ldots}}{\boxed{\Delta^{pq}_{rs}}} :A^{pq}_{rs}: - \text{H.c.}$$

$$\eta^{III} = \sum_{pr} \boxed{\mathrm{sgn}(\Delta^p_r)} (\boxed{\bar{n}_p n_r f^p_r + \ldots}) :A^p_r: + \frac{1}{4}\sum_{pqrs} \boxed{\mathrm{sgn}(\Delta^{pq}_{rs})} (\boxed{\bar{n}_p \bar{n}_q n_r n_s \Gamma^{pq}_{rs} + \ldots}) :A^{pq}_{rs}: - \text{H.c.}$$

- White generator: small energy denominators due to near-degeneracies
- imaginary time generator: sign choice depends on approximate energies
- definition of H^{od} subject to density truncations

Brillouin Generator

- consider unitary variations of the energy functional

$$E(s) = \left\langle \Phi \middle| H(s) \middle| \Phi \right\rangle$$

- define generator as the residual of the irreducible Brillouin condition (= gradient of E)

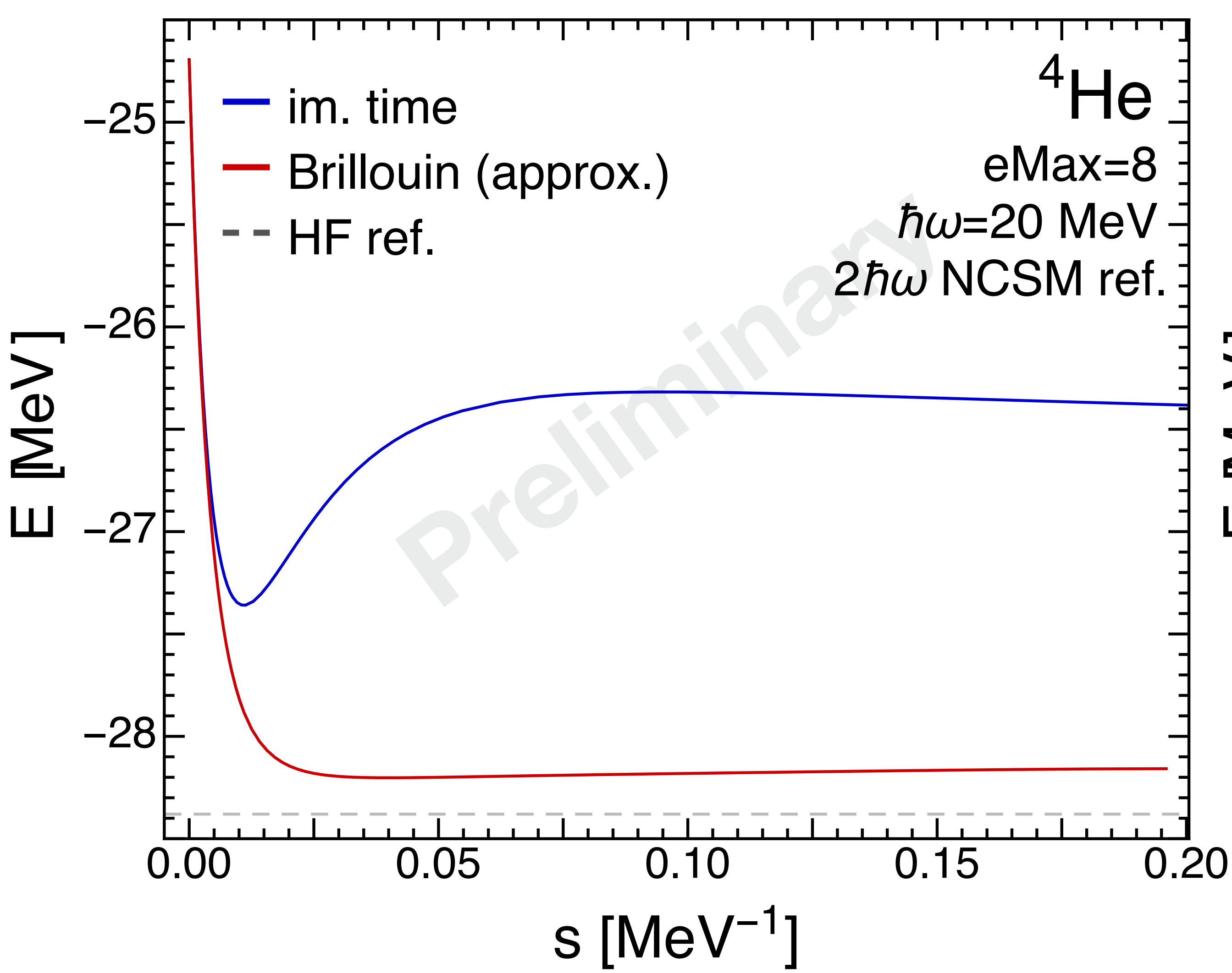
$$\begin{aligned}\eta^p_r &\equiv \left\langle \Phi \right| \left[:A^p_r:, H \right] \left| \Phi \right\rangle \\ \eta^{pq}_{rs} &\equiv \left\langle \Phi \right| \left[:A^{pq}_{rs}:, H \right] \left| \Phi \right\rangle\end{aligned}$$

- fixed point ($\eta = 0$) is reached when IBC is satisfied, energy stationary (cf. ACSE approach of Mazziotti et al.)

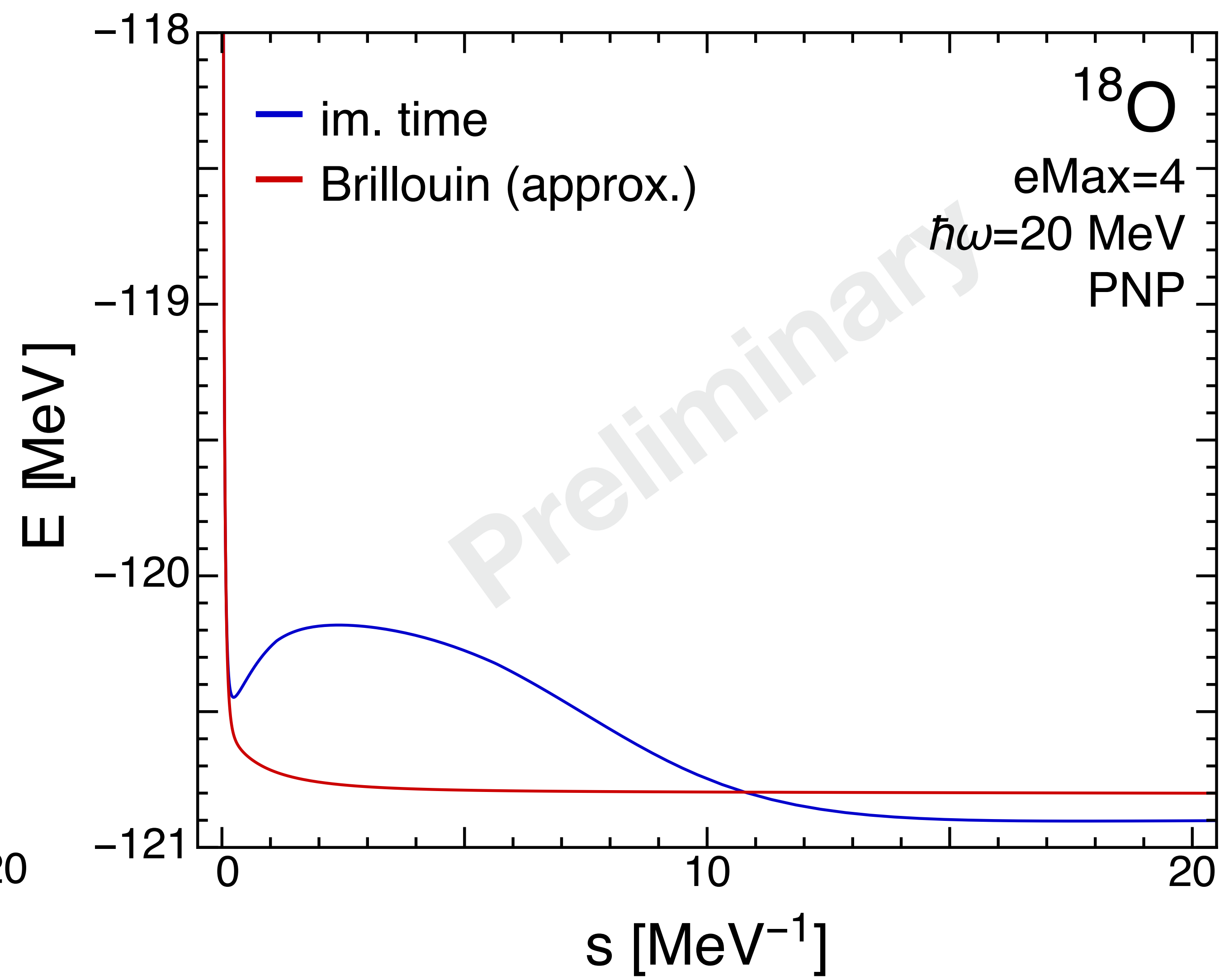
- Brillouin generator depends linearly on $\lambda^p_s, \lambda^{pq}_{st}, \lambda^{pqr}_{stu}$, higher irreducible density matrices are not required

Brillouin Generator

NN-only, λ=1.88 fm^{-1}



NN+3N-ind., λ=2.0 fm^{-1}



- norm of Brillouin generator decays monotonically
  (approximation: 2B "particle-hole"-like term switched off, 3B density not yet implemented)

➡ use in Magnus formulation of MR-IM-SRG

Multi-Reference Flow Equations

0-body flow:

$$\frac{dE}{ds} = \sum_{ab}(n_a - n_b)\left(\eta^a_b f^b_a - f^a_b \eta^b_a\right) + \frac{1}{4}\sum_{abcd}\left(\eta^{ab}_{cd}\Gamma^{cd}_{ab} - \Gamma^{ab}_{cd}\eta^{cd}_{ab}\right) n_a n_b \bar{n}_c \bar{n}_d$$

$$+ \frac{1}{4}\sum_{abcd}\left(\frac{d}{ds}\Gamma^{ab}_{cd}\right)\lambda^{ab}_{cd} + \frac{1}{4}\sum_{abcdklm}\left(\eta^{ab}_{cd}\Gamma^{kl}_{am} - \Gamma^{ab}_{cd}\eta^{kl}_{am}\right)\lambda^{bkl}_{cdm}$$

1-body flow:

$$\frac{d}{ds}f^1_2 = \sum_{a}\left(\eta^1_a f^a_2 - f^1_a \eta^a_2\right) + \sum_{ab}\left(\eta^a_b \Gamma^{b1}_{a2} - f^a_b \eta^{b1}_{a2}\right)(n_a - n_b)$$

$$+ \frac{1}{2}\sum_{abcdef}\left(\eta^{1a}_{bc}\Gamma^{bc}_{2a} - \Gamma^{1a}_{bc}\eta^{bc}_{2a}\right)(n_a \bar{n}_b \bar{n}_c + \bar{n}_a n_b n_c)$$

$$+ \frac{1}{4}\sum_{abcde}\left(\eta^{1a}_{bc}\Gamma^{de}_{2a} - \Gamma^{1a}_{bc}\eta^{de}_{2a}\right)\lambda^{de}_{bc} + \sum_{abcde}\left(\eta^{1a}_{bc}\Gamma^{be}_{2d} - \Gamma^{1a}_{bc}\eta^{be}_{2d}\right)\lambda^{ae}_{cd}$$

$$- \frac{1}{2}\sum_{abcde}\left(\eta^{1a}_{2b}\Gamma^{cd}_{ae} - \Gamma^{1a}_{2b}\eta^{cd}_{ae}\right)\lambda^{cd}_{be} + \frac{1}{2}\sum_{abcde}\left(\eta^{1a}_{2b}\Gamma^{bc}_{de} - \Gamma^{1a}_{2b}\eta^{bc}_{de}\right)\lambda^{ac}_{de}$$

Multi-Reference Flow Equations

2-body flow:

$$\frac{d}{ds}\Gamma^{12}_{34} = \sum_a \left(\eta^1_a\Gamma^{a2}_{34} + \eta^2_a\Gamma^{1a}_{34} - \eta^a_3\Gamma^{12}_{a4} - \eta^a_4\Gamma^{12}_{3a} - f^1_a\eta^{a2}_{34} - f^2_a\eta^{1a}_{34} + f^a_3\eta^{12}_{a4} + f^a_4\eta^{12}_{3a}\right)$$
$$+ \frac{1}{2}\sum_{ab}\left(\eta^{12}_{ab}\Gamma^{ab}_{34} - \Gamma^{12}_{ab}\eta^{ab}_{34}\right)(1 - n_a - n_b)$$
$$+ \sum_{ab}(n_a - n_b)\left(\left(\eta^{1a}_{3b}\Gamma^{2b}_{4a} - \Gamma^{1a}_{3b}\eta^{2b}_{4a}\right) - \left(\eta^{2a}_{3b}\Gamma^{1b}_{4a} - \Gamma^{2a}_{3b}\eta^{1b}_{4a}\right)\right)$$

two-body flow unchanged, $O(N^6)$ scaling preserved

Particle-Number Projected HFB State

- HFB ground state is a superposition of states with different particle number:

$$\left|\Psi\right\rangle = \sum_{A=N,N\pm2,\ldots} c_A \left|\Psi_A\right\rangle\,, \quad \left|\Psi_N\right\rangle \equiv P_N\left|\Psi\right\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i\phi(\hat{N}-N)}\left|\Psi\right\rangle$$

- calculate irreducible densities (project only once), e.g.,

$$\lambda^k_l = \frac{\left\langle\Psi\right| A^k_l P_N \left|\Psi\right\rangle}{\left\langle\Psi|\Psi\right\rangle}\,, \quad \lambda^{kl}_{mn} = \frac{\left\langle\Psi\right| A^{kl}_{mn} P_N \left|\Psi\right\rangle}{\left\langle\Psi|\Psi\right\rangle} - \lambda^k_m\lambda^l_m + \lambda^k_n\lambda^l_m$$

- work in natural orbitals (= HFB canonical basis):

$$\lambda^k_l = n_k\delta^k_l \left(= v_k^2\delta^k_l\right)\,, \quad 0 \leq n_k \leq 1$$

- in NO basis, $\lambda^{kl}_{mn}, \lambda^{klm}_{nop}$ require only $2N^2$, $3N^3$ storage

Ground States of Closed and Open-Shell Nuclei

H. H., in preparation

H. H., S. Bogner, T. Morris, S. Binder, A. Calci, J. Langhammer, R. Roth, Phys. Rev. C **90**, 041302 (2014)

H. H., S. Binder, A. Calci, J. Langhammer, and R. Roth, Phys. Rev. Lett **110**, 242501 (2013)

H. H., S. K. Bogner, S. Binder, A. Calci, J. Langhammer, R. Roth, and A. Schwenk, Phys. Rev. C **87**, 034307 (2013)

Hamiltonians

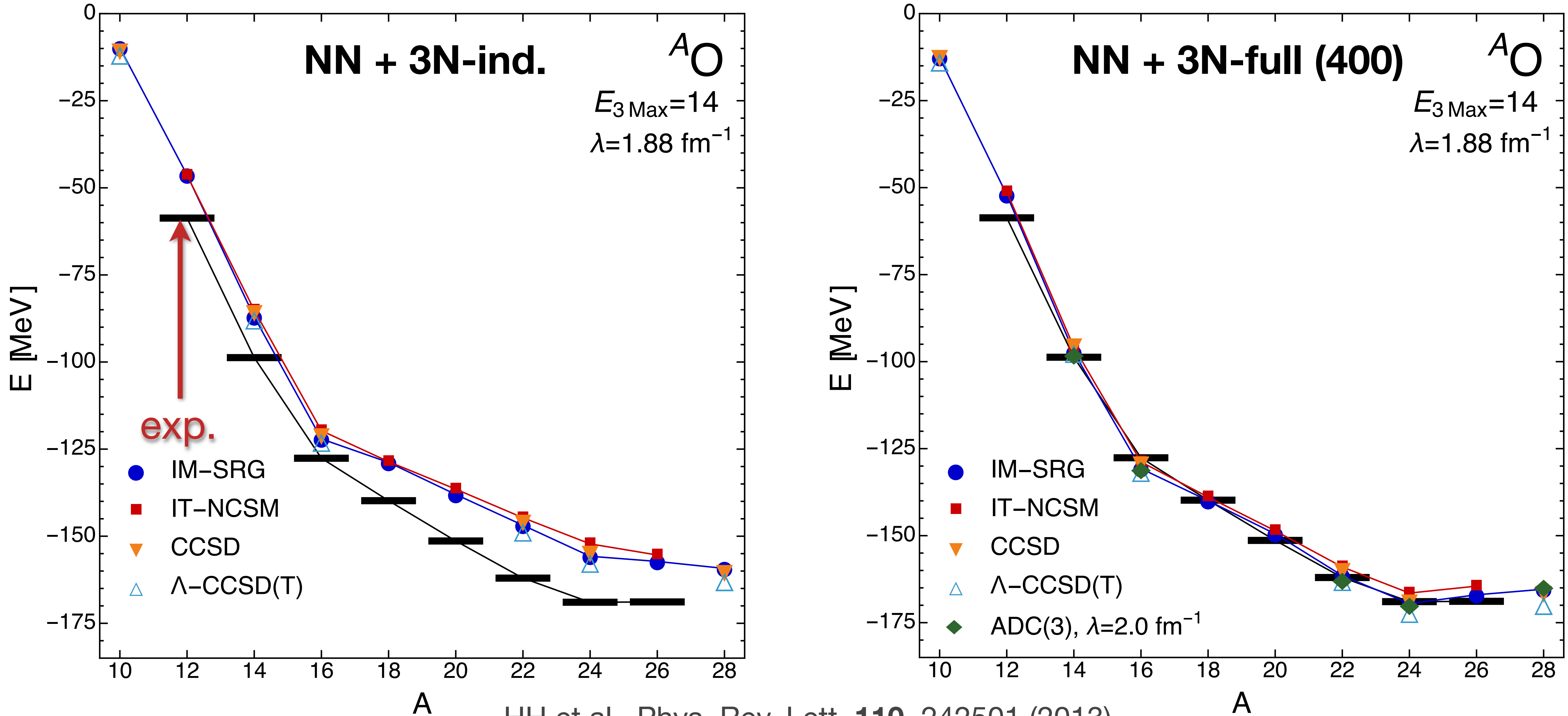
Initial Hamiltonian

- NN: chiral interaction at N^3LO (Entem & Machleidt)
- 3N: chiral interaction at N^2LO (c_D, c_E fit to ^3H, ^4He energies, β decay)

SRG-Evolved Hamiltonians

- **NN + 3N-induced:** start with initial NN Hamiltonian, keep two- and three-body terms
- **NN + 3N-full:** start with initial NN + 3N Hamiltonian, keep two- and three-body terms

Results: Oxygen Chain



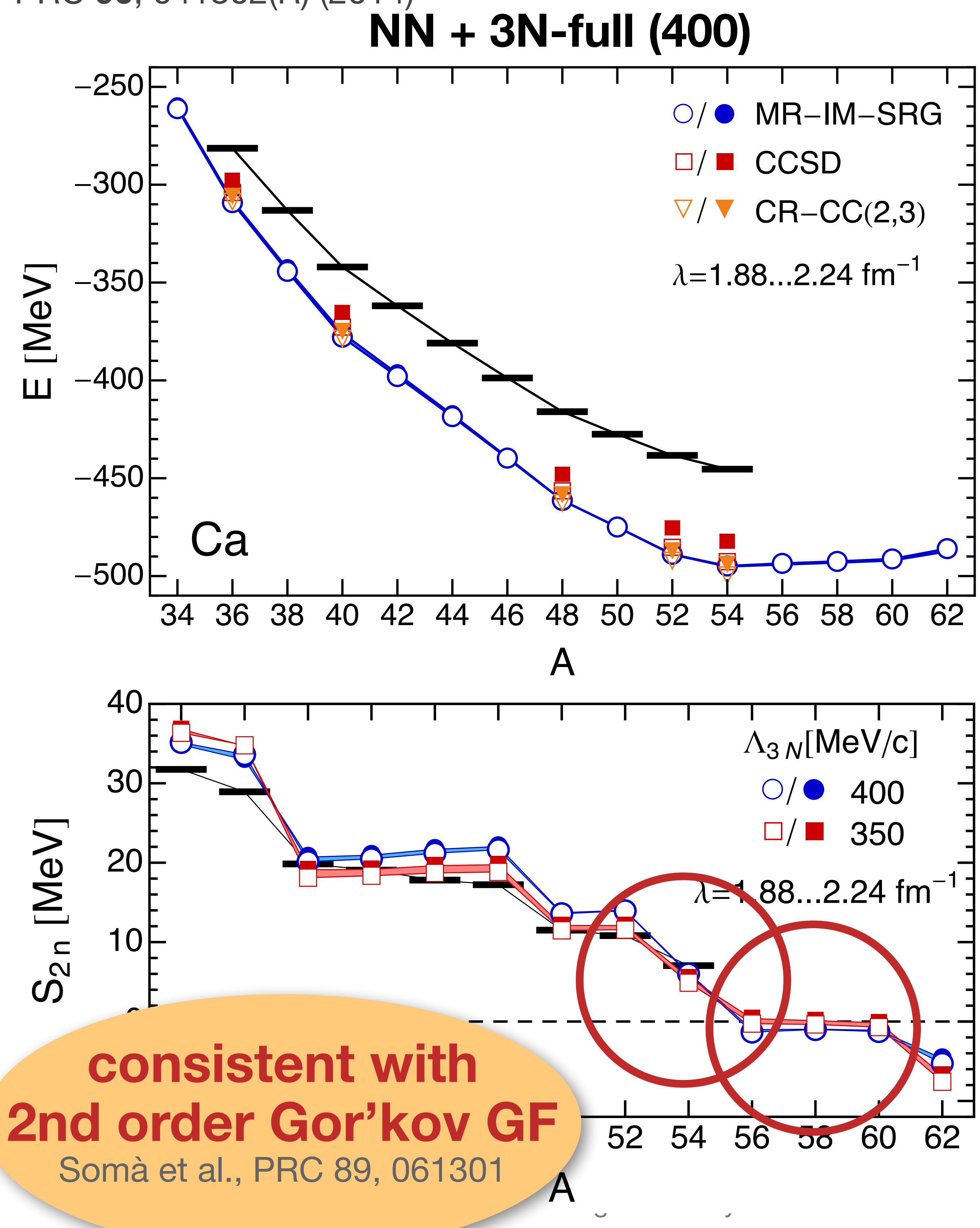
HH et al., Phys. Rev. Lett. **110**, 242501 (2013)

ADC(3): A. Cipollone et al., Phys. Rev. Lett. **111**, 242501 (2013)

- Multi-Reference IM-SRG with number-projected Hartree-Fock-Bogoliubov as reference state (pairing correlations)
- consistent results from different many-body methods

Two-Neutron Separation Energies

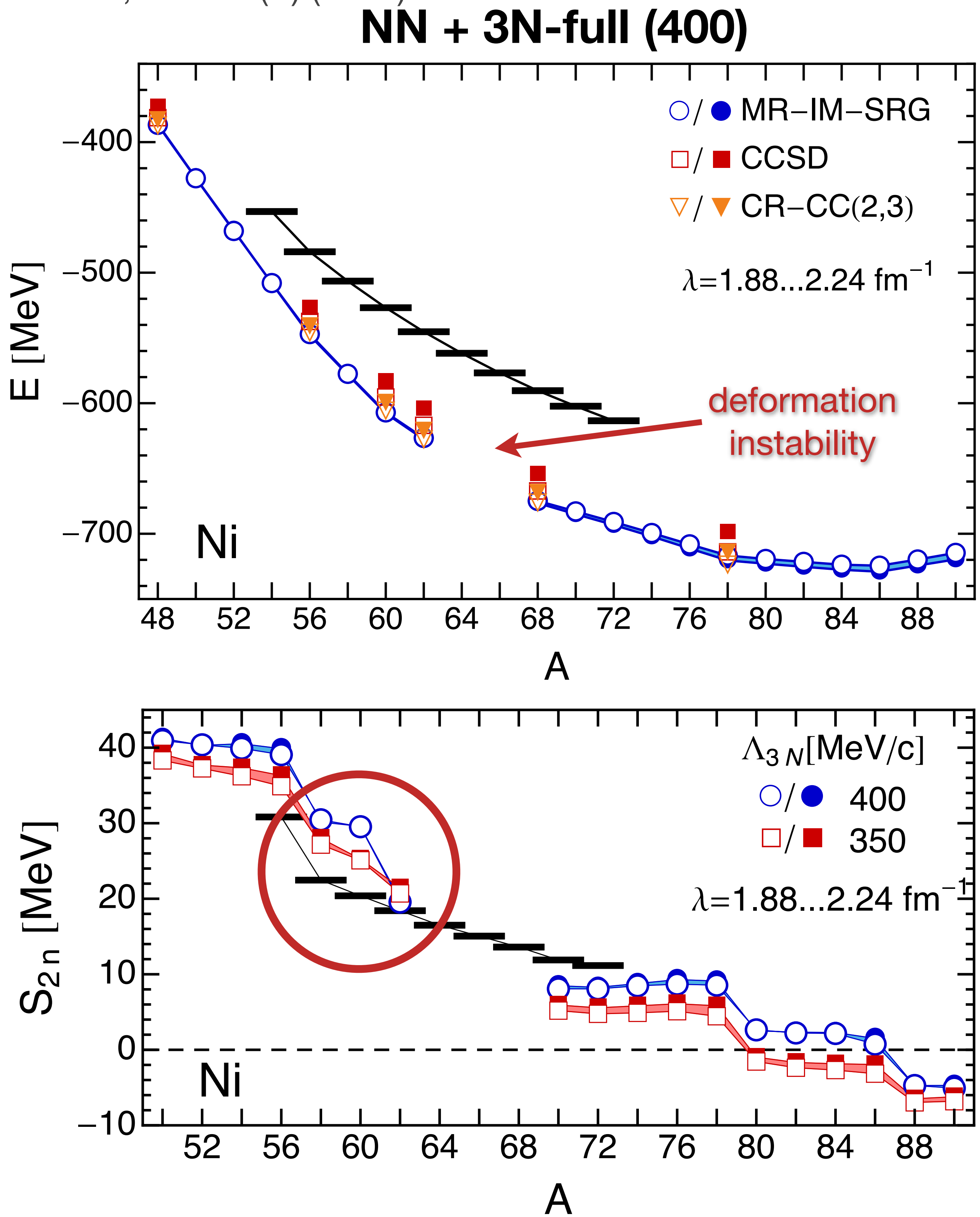
PRC **90**, 041302(R) (2014)



- differential observables (S_{2n}, spectra,...) filter out interaction components that cause overbinding
- predict flat trends for g.s. energies/S_{2n} beyond ^{54}Ca - await experimental data
- ^{52}Ca, ^{54}Ca robustly magic due to 3N interaction
- no continuum coupling yet, other S_{2n} uncertainties < 1MeV

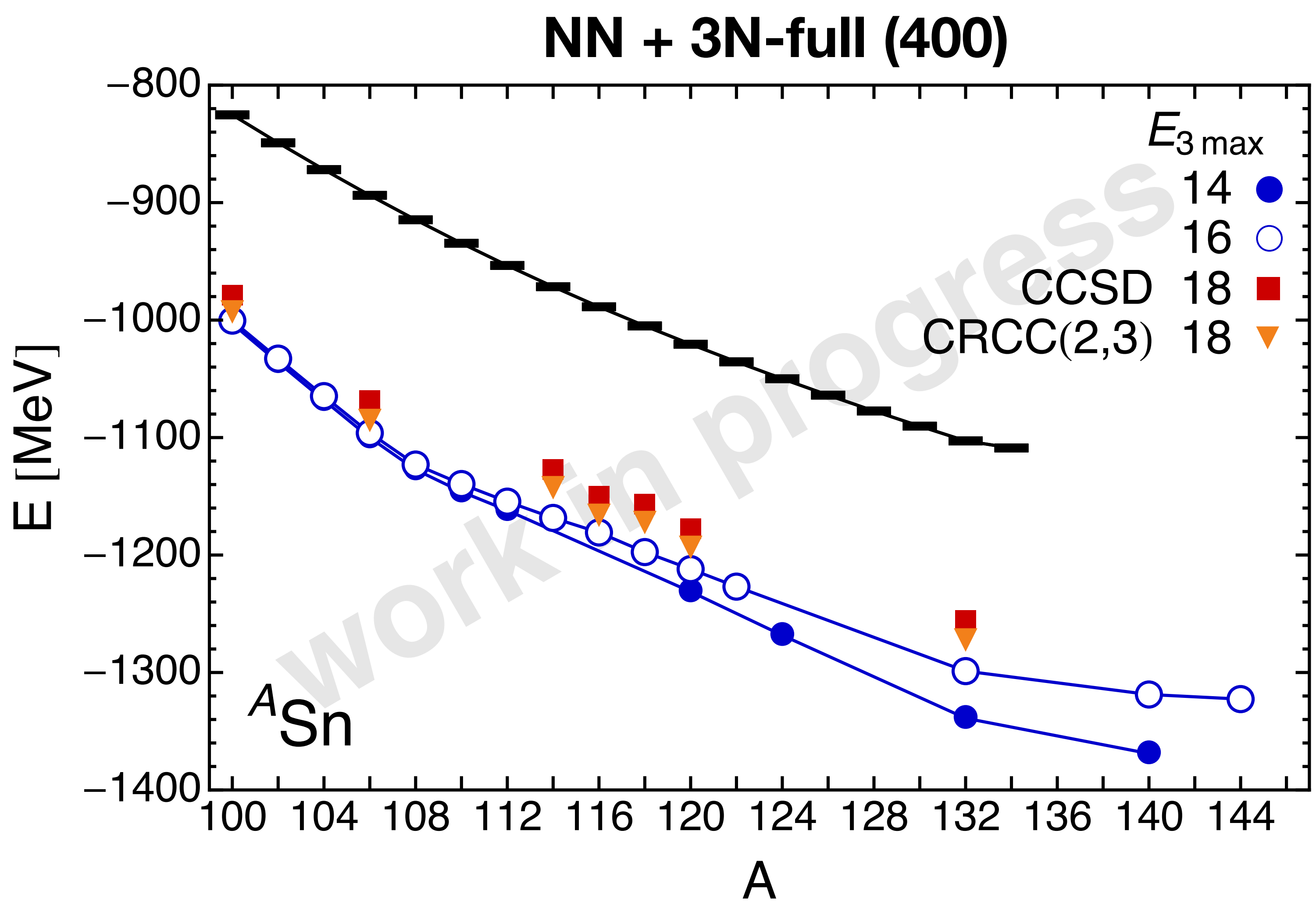
Two-Neutron Separation Energies

PRC **90**, 041302(R) (2014)



- flat trends for g. s. energies and S_{2n} (similar to Ca)
- deformation instability in ^{64,66}Ni calculations - issue with “shell” structure
- further evidence from 3N cutoff variation
- no continuum coupling yet, other S_{2n} uncertain-ties < 1MeV

The *Ab Initio* Mass Frontier: Tin



E_{3max}	memory (float) [GB]
14	5
16	~20
18	100+

- systematics of overbinding similar to Ca/Ni
- not converged with respect to 3N matrix element truncation:

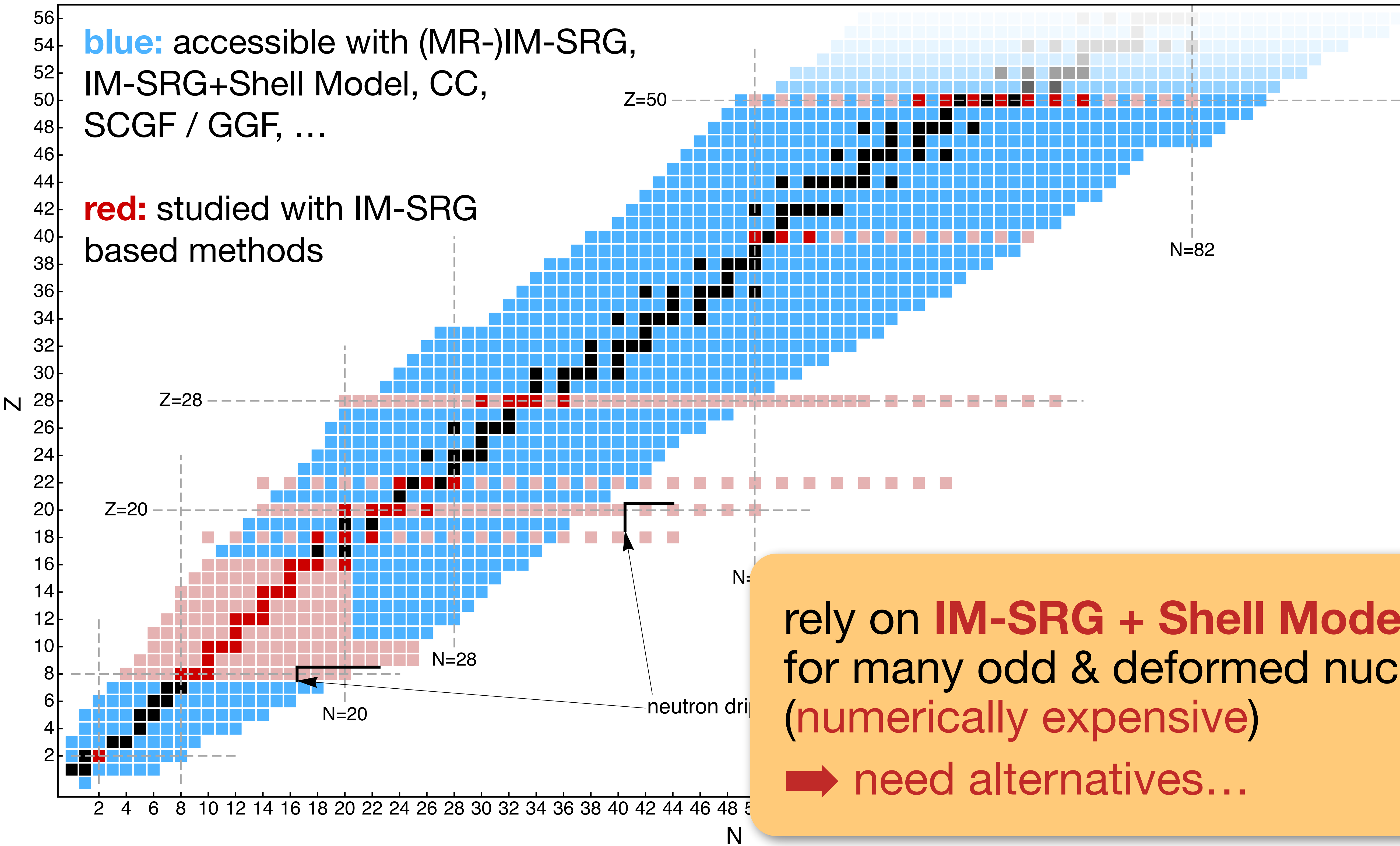
$$e_1 + e_2 + e_3 \leq E_{3max}$$

($e_{1,2,3}$: SHO energy quantum numbers)

- need technical improvements to go further

Next Steps

Reach of Ab Initio Methods



rely on **IM-SRG + Shell Model** for many odd & deformed nuclei (numerically expensive)

➡ need alternatives…

Equations-of-Motion for Excitations

- describe “excited states” based on reference state:

$$\left|\Psi_k\right\rangle \equiv R_k \left|\Psi_0\right\rangle$$

- **(MR-)IM-SRG effective Hamiltonian** in EOM approach:

$$\left[H(\infty), R_k\right] = \omega_k R_k\,, \quad \omega_k = E_k - E_0$$

- computational effort scales **polynomially**, vs. factorial scaling of Shell Model

- can exploit Multi-Reference capabilities (commutator formulation identical to flow equations)

➡ **complementary** to Shell Model

EOM Applications

- particle-hole excitations (TDA, RPA, Second RPA, ...)

$$R_k = \sum_{ph} R^{(k)}_{ph} :a^\dagger_p a_h: + \sum_{pp'hh'} R^{(k)}_{pp'hh'} :a^\dagger_p a^\dagger_{p'} a_{h'} a_h: + \ldots$$

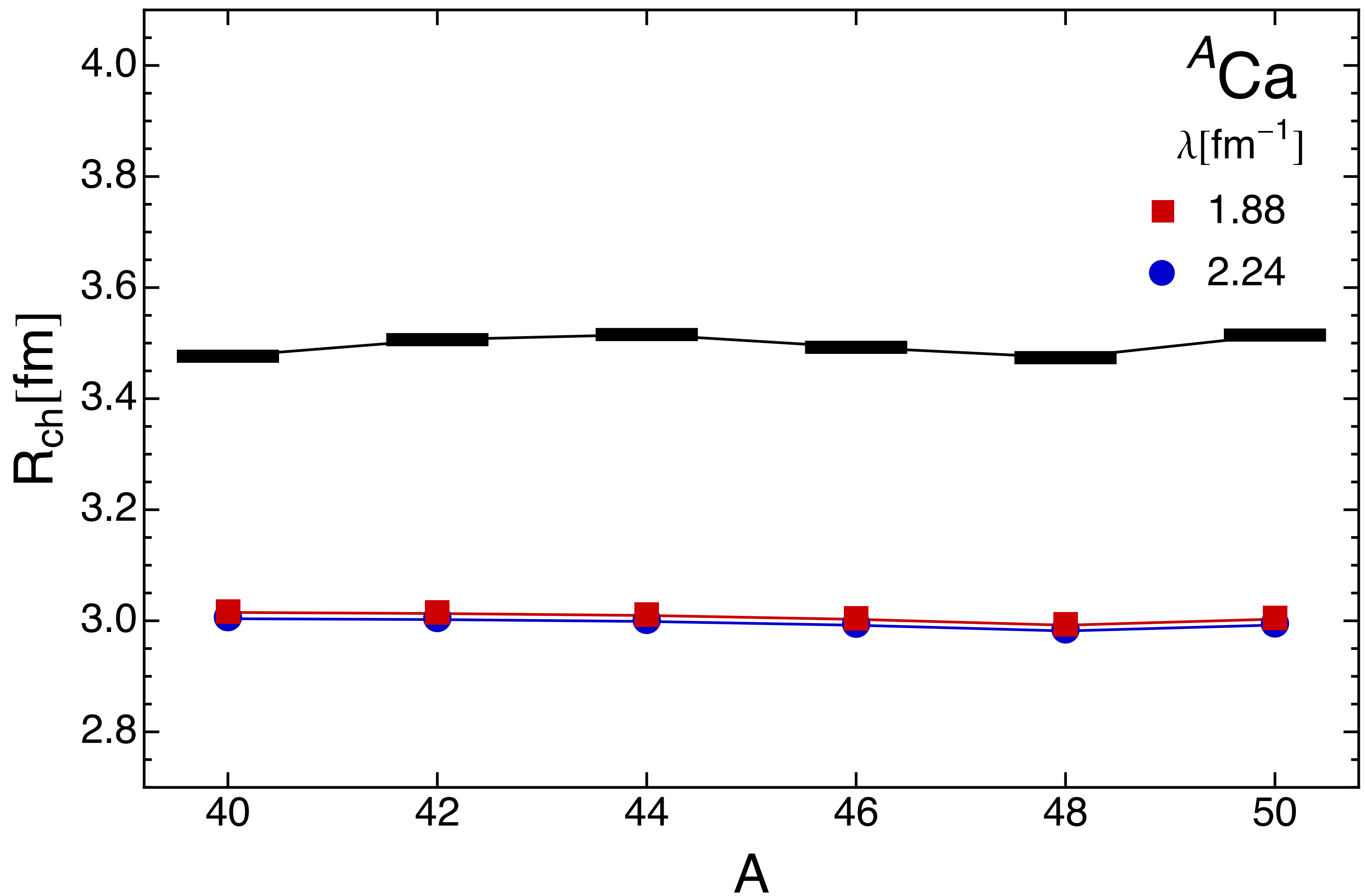
➡ **giant resonances**

- particle attachment (analogous for removal):

$$R_k = \sum_{ph} R^{(k)}_{p} :a^\dagger_p: + \sum_{pp'h} R^{(k)}_{pp'h} :a^\dagger_p a^\dagger_{p'} a_h: + \ldots$$

➡ **ground and excited states in odd nuclei**

Effective Operators



- small radii: interaction issue (power counting, regulators, LECs, ...), also consider currents?
- implementation of electromagnetic & weak transition operators in progress; aim for consistent treatment: chiral EFT, SRG, IM-SRG (& Shell Model code !)

Magnus Series Formulation

- construct unitary transformation explicitly:

$$U(s) = \mathcal{S} \exp \int_0^s ds' \eta(s') \equiv \exp \Omega(s)$$

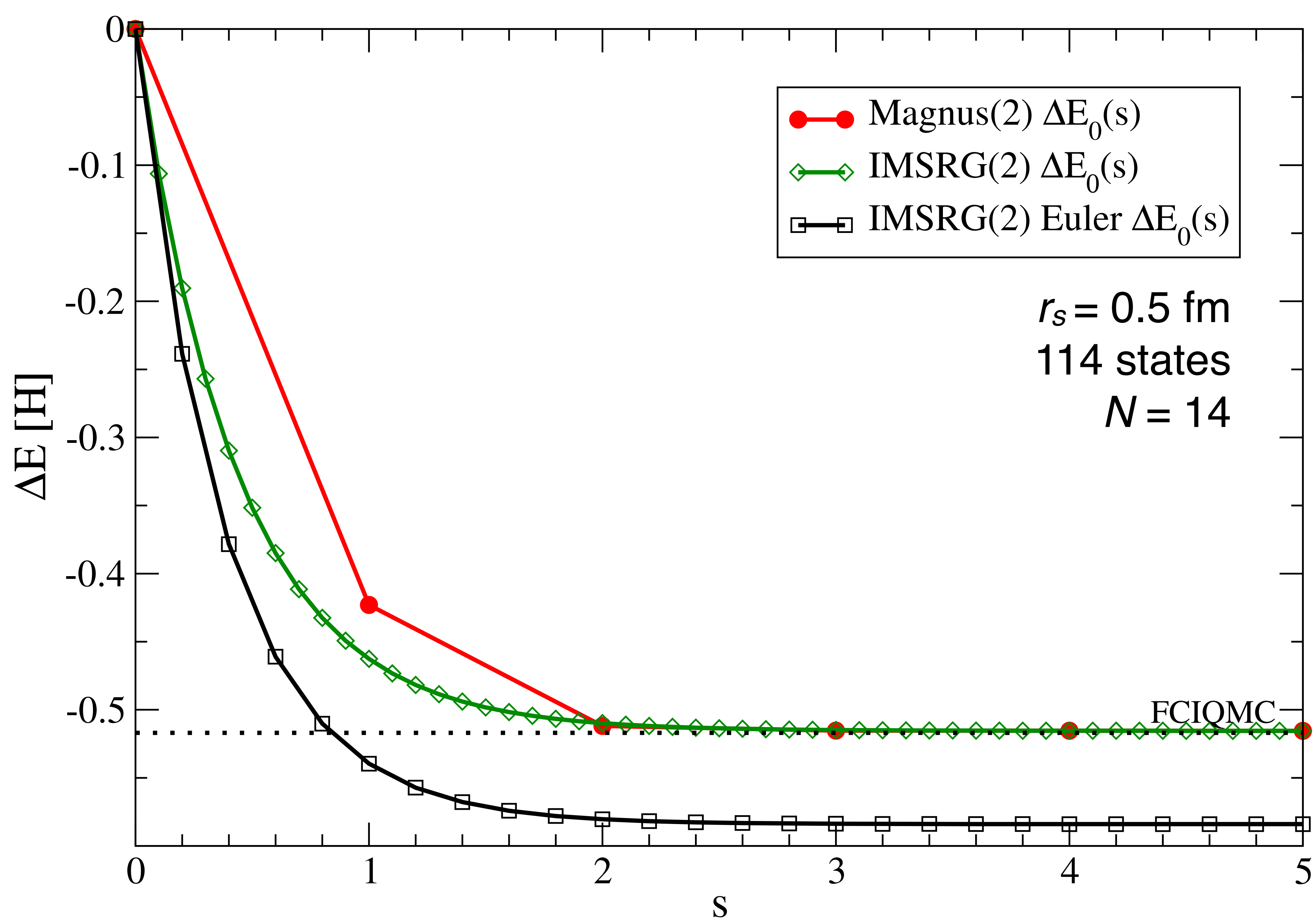
- flow equation for Magnus operator :

$$\frac{d}{ds}\Omega = \sum_{k=0}^{\infty} \frac{B_k}{k!} \mathrm{ad}_{\Omega}^{k}(\eta) \,, \quad \mathrm{ad}_{\Omega}(O) = [\Omega, O]$$

(B_k: Bernoulli numbers)

- construct $O(s) = U(s) O_0 U^\dagger(s)$ using Baker-Campbell-Hausdorff expansion (Hamiltonian + effective operators)
- generate systematic approximations to (MR-)IM-SRG(3)
- simple integrator sufficient (Euler!) - **unitarity built in**

Example: Homogenous Electron Gas



T. D. Morris, N. Parzuchowski, S. K. Bogner, in preparation

Conclusions

Conclusions

- IM-SRG is a powerful *ab initio* framework for closed- and open-shell, medium-mass & (heavy) nuclei
- derivation of Shell-Model interactions (see talk by J. Holt)
  ➡ immediate access to spectra, odd nuclei, intrinsic deformation (at Shell Model numerical cost)
- in progress:
  - EOM for odd nuclei and excited states
  - effective transition operators
  - approximate IM-SRG(3)
- new perspectives for old (?) problems: evolution of long-range correlations, construction of density functionals...

Acknowledgments

S. Bogner, T. Morris,
N. Parzuchowski, F. Yuan
NSCL, Michigan State University

E. Gebrerufael, K. Hebeler,
R. Roth, A. Schwenk, J. Simonis,
C. Stumpf, K. Vobig
TU Darmstadt, Germany

A. Calci, J. D. Holt, R. Stroberg
TRIUMF, Canada

S. Binder, K. Wendt
UT Knoxville & Oak Ridge National Laboratory

G. Papadimitriou
Iowa State University

R. Furnstahl, S. König, S. More,
R. Perry
The Ohio State University

P. Papakonstantinou
IBS / Rare Isotope Science Project, South Korea

T. Duguet, V. Somà
CEA Saclay, France

NUCLEI
Nuclear Computational Low-Energy Initiative

Ohio Supercomputer Center

ICER

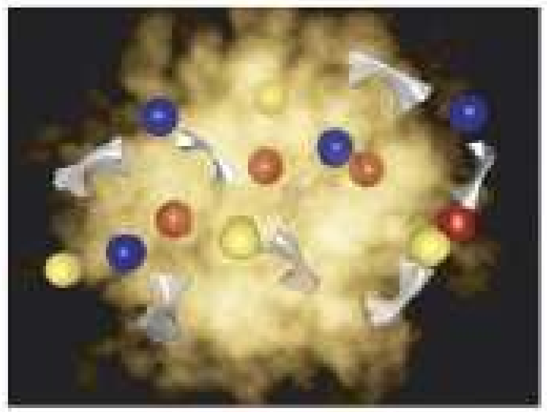
Supplements

Scales of the Strong Interaction

momentum transfer (resolution)

QCD

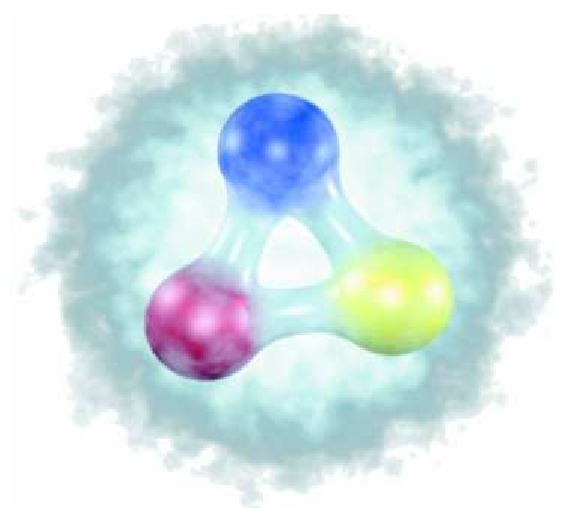
Chiral EFT

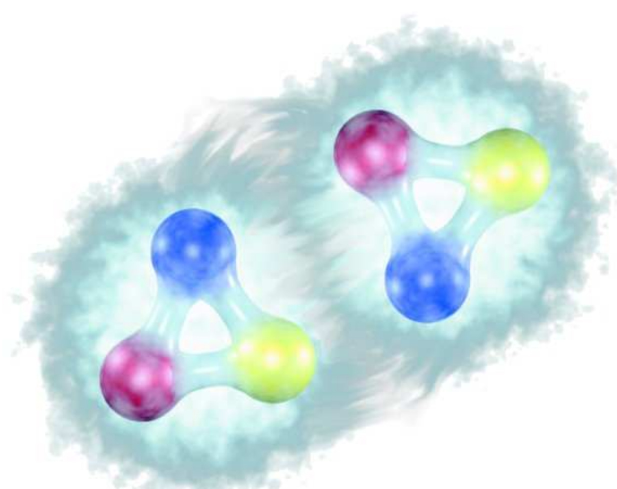


quarks, gluons

chiral phase transition

(de)confinement phase transition





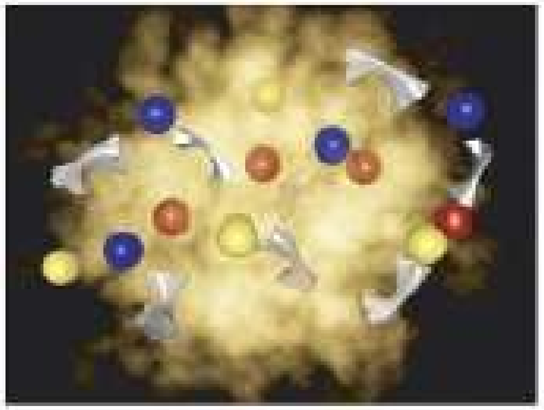
Weinberg's 3rd Law of Progress in Theoretical Physics:
"You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

Scales of the Strong Interaction

momentum transfer (resolution)

QCD

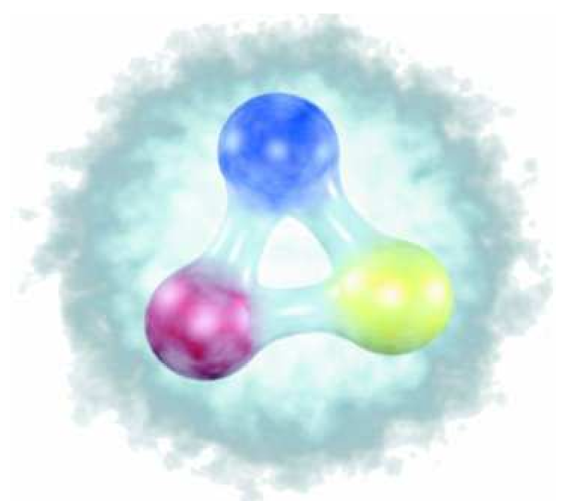
Chiral EFT



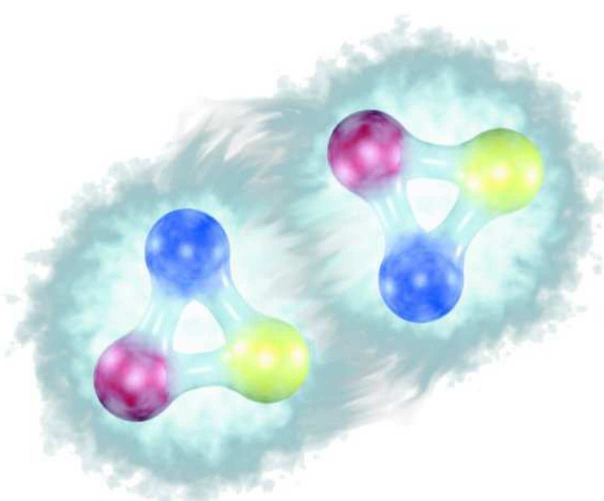
quarks, gluons

chiral phase transition

(de)confinement phase transition



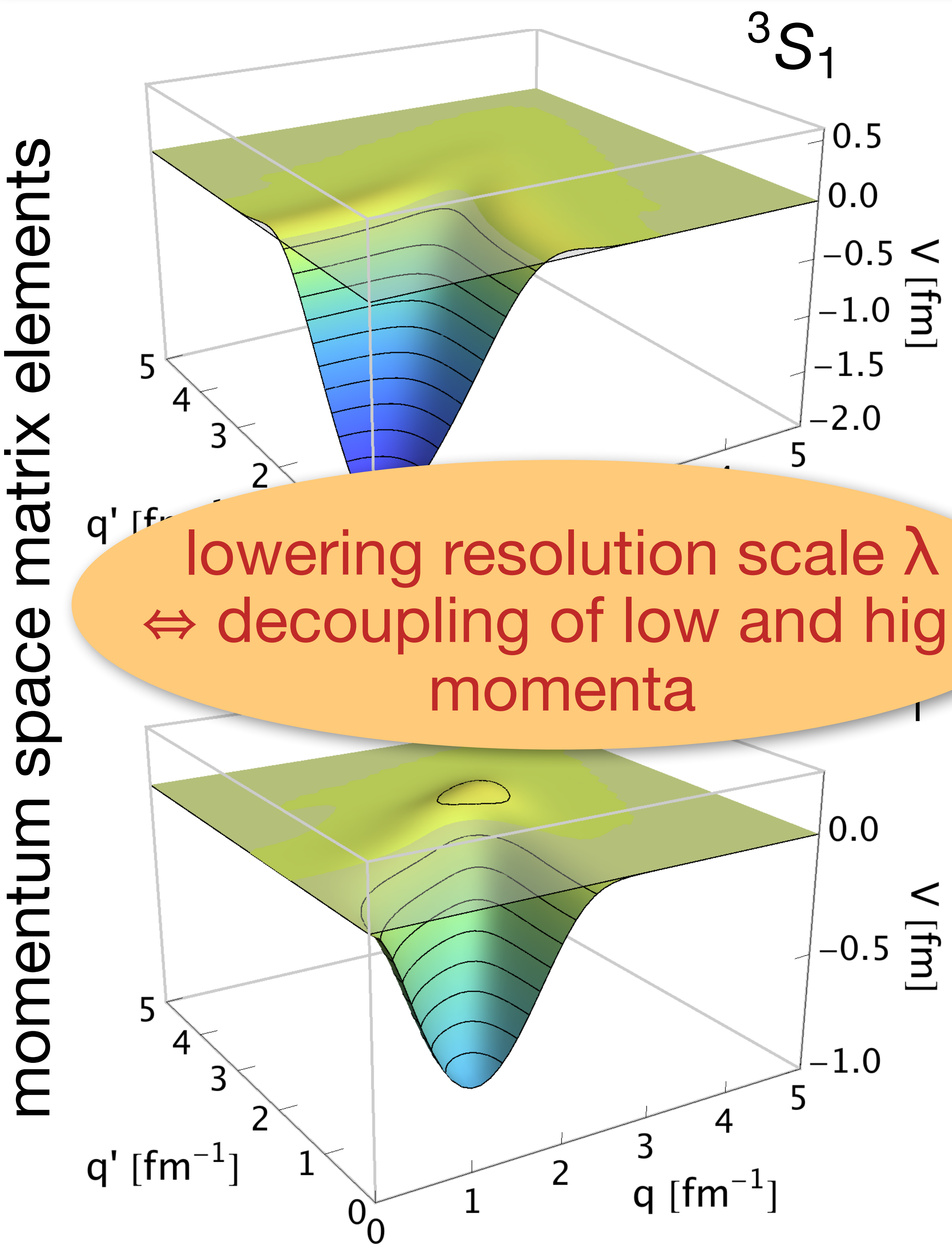
pions (π), nucleons (N), ...



chiral symmetry

spontaneously broken,
π is Goldstone boson

SRG in Two-Body Space



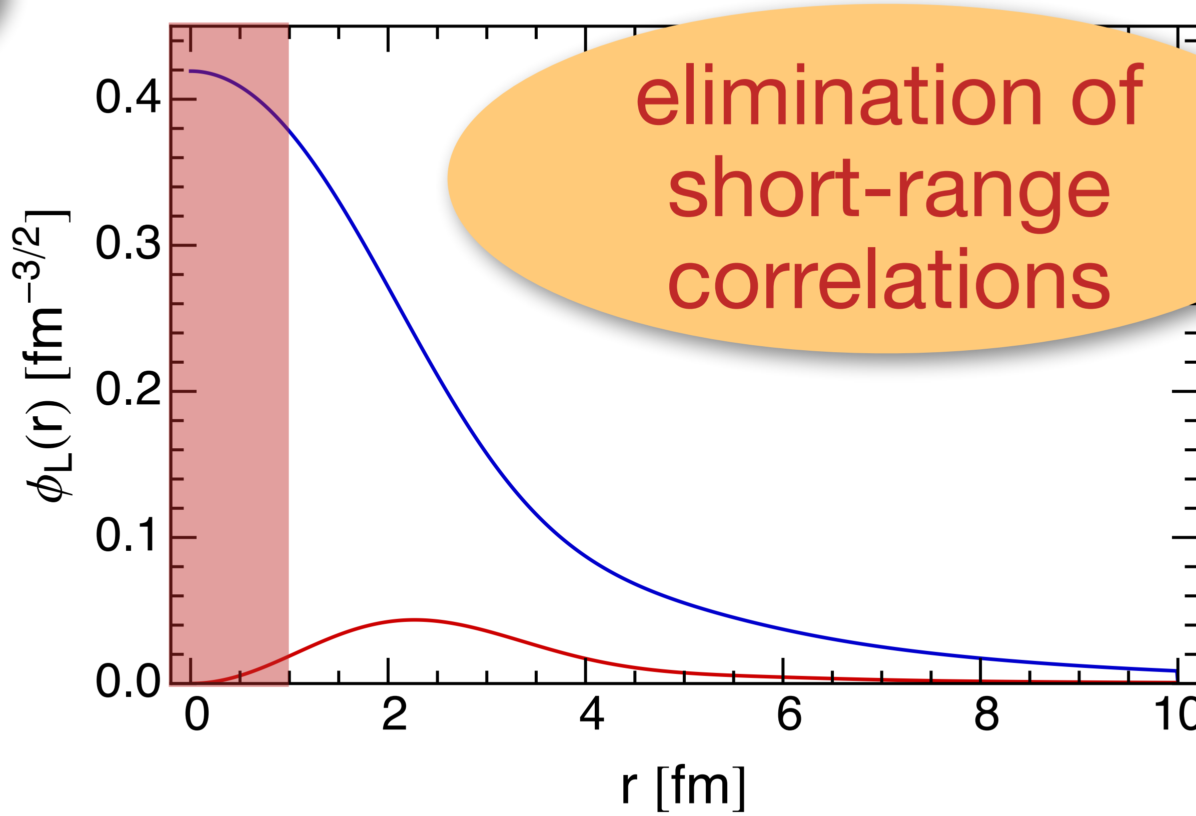
lowering resolution scale λ
⇔ decoupling of low and high momenta

λ = 1.8 fm^{-1}

$$\eta(\lambda) = 2\mu\left[T_{\rm rel}, H(\lambda)\right]$$

$$\lambda = s^{-1/4}$$

deuteron wave function



elimination of short-range correlations

Induced Interactions

- SRG is a unitary transformation in A-body space
- up to A-body interactions are induced during the flow:

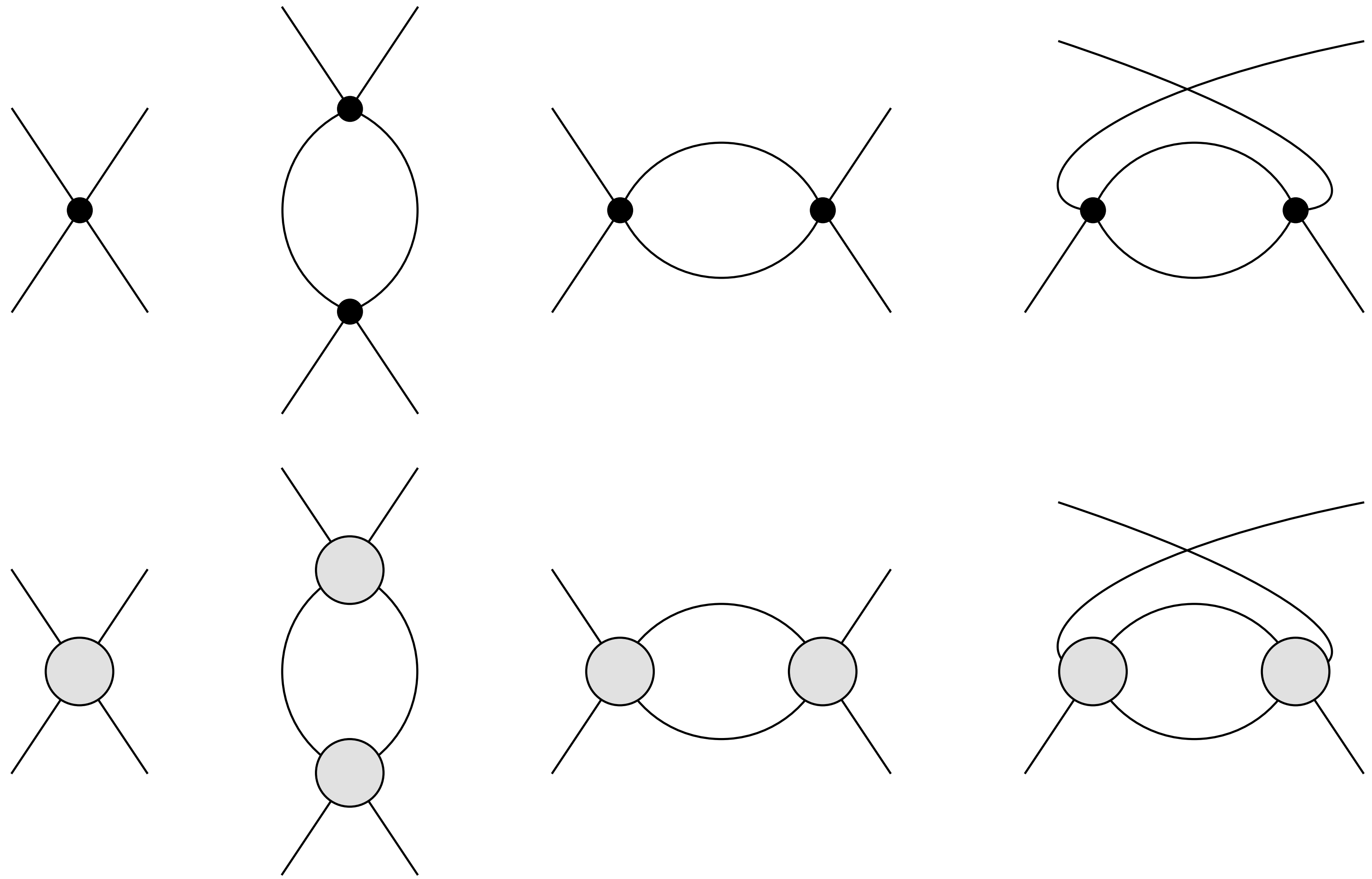
$$\frac{dH}{d\lambda} = [[\sum a^\dagger a, \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}], \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}] = \ldots + \sum \underbrace{a^\dagger a^\dagger a^\dagger aaa}_{\text{3-body}} + \ldots$$

- state-of-the-art: evolve in three-body space, truncate induced four- and higher many-body forces
  (Jurgenson, Furnstahl, Navratil, PRL 103, 082501; Hebeler, PRC 85, 021002; Wendt, PRC 87, 061001)
- λ-dependence of eigenvalues is a diagnostic for size of omitted induced interactions

In-Medium SRG Flow: Diagrams

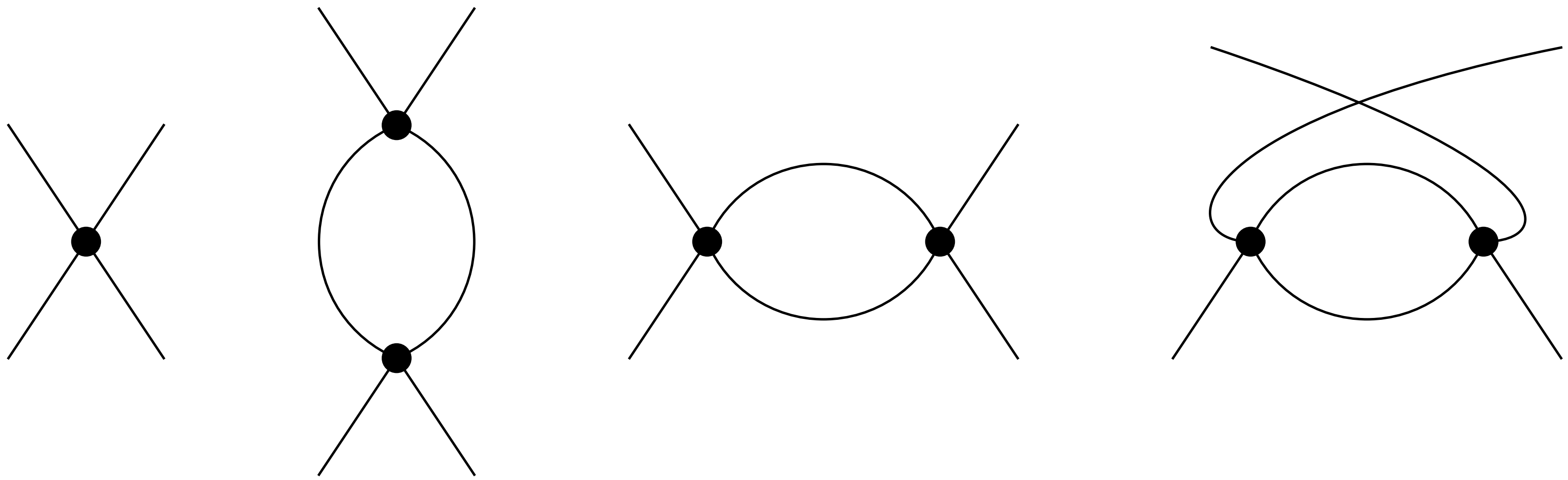
$\Gamma(\delta s) \sim$

$\Gamma(2\delta s) \sim$

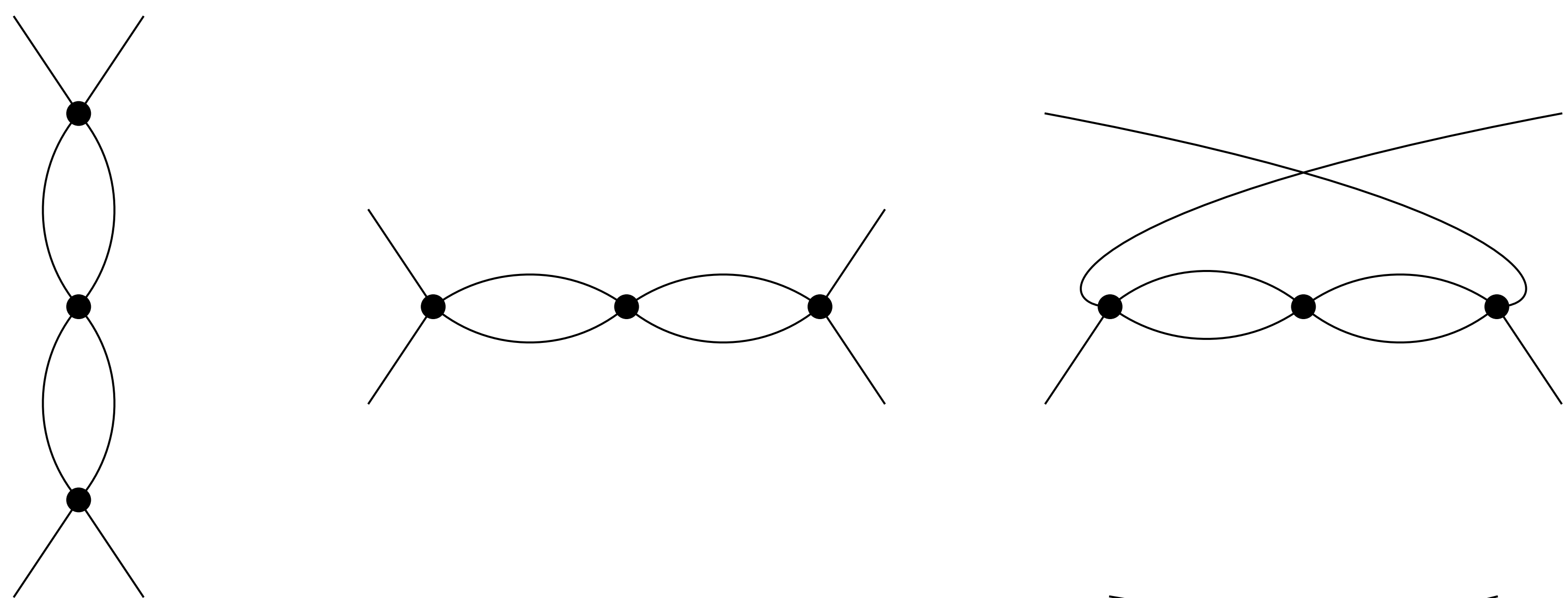


In-Medium SRG Flow: Diagrams

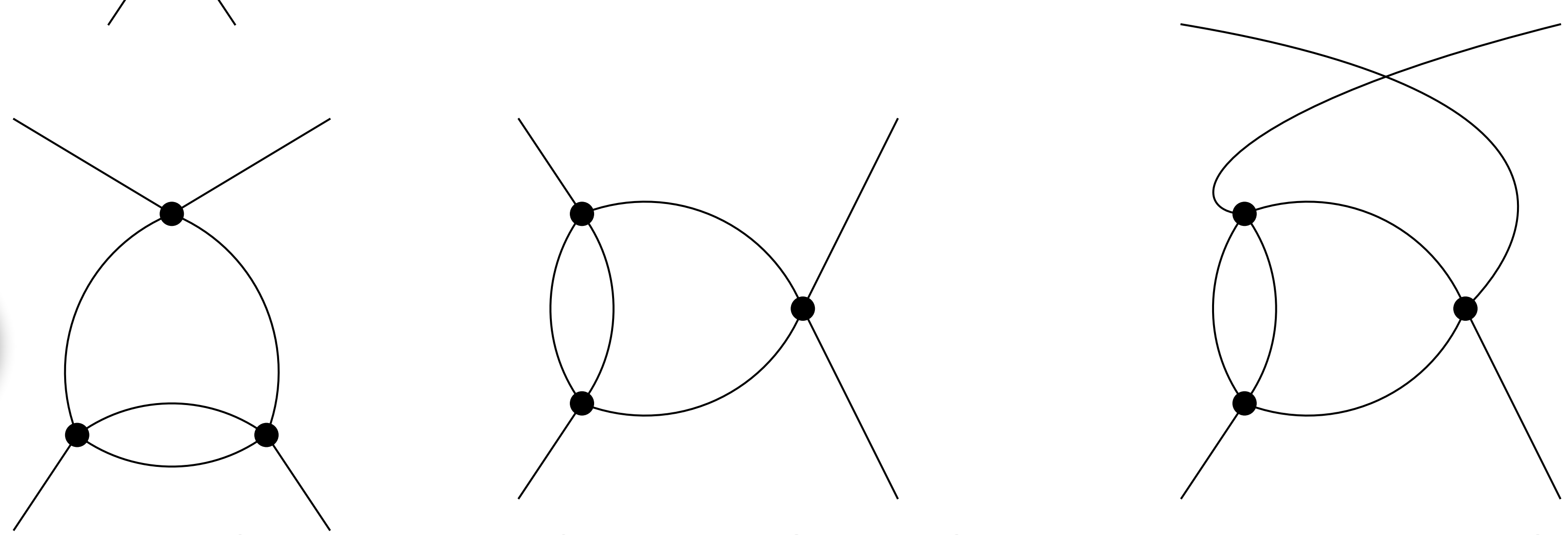
$\Gamma(\delta s) \sim$



$\Gamma(2\delta s) \sim$

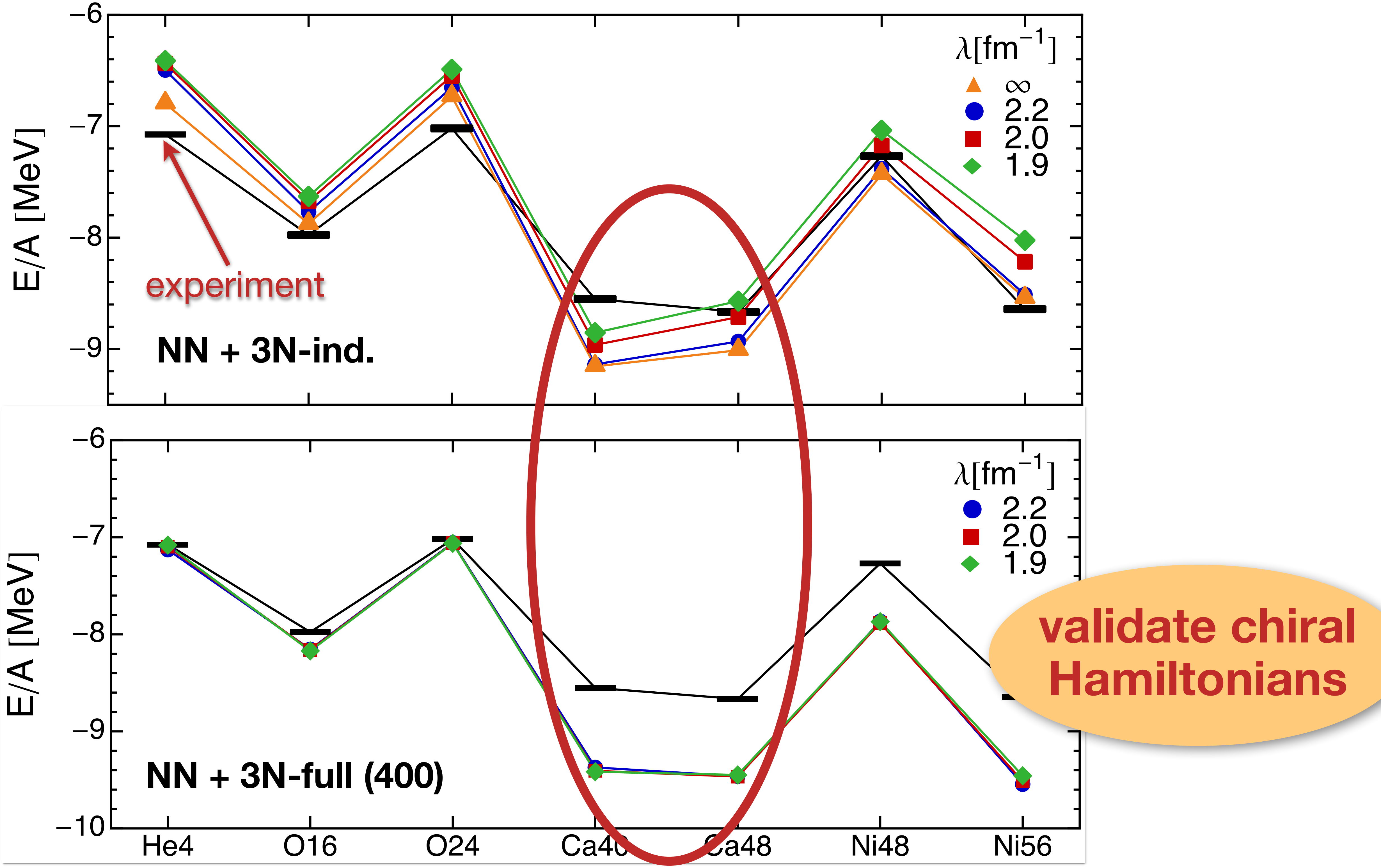


non-perturbative resummation



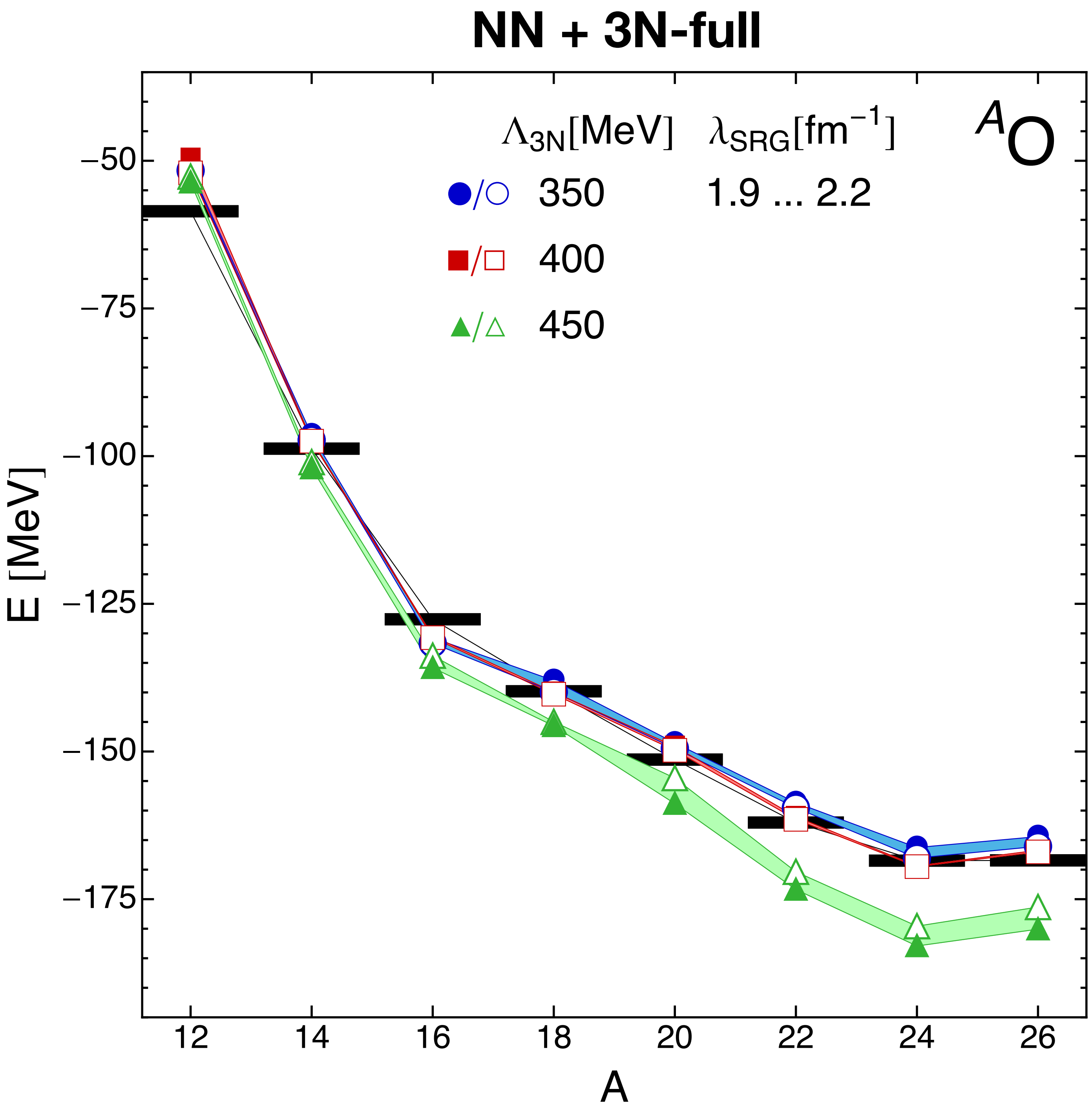
& many more...

Results: Closed-Shell Nuclei



Phys. Rev. C **87**, 034307 (2013), arXiv: 1212.1190 [nucl-th]

Variation of Scales



Phys. Rev. Lett. **110**, 242501 (2013)

- variation of initial 3N cutoff only
- diagnostics for chiral interactions
- dripline at A=24 is robust under variations
- (leading) continuum effects too small to bind ^{26}O