



EFFECTIVE THEORIES AND FUNDAMENTAL THEORIES

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Outline

Case: Nuclear Forces
Effective Field Theories
Nuclear EFTs
Summary

There are few problems in theoretical physics which have attracted more attention than that of trying to determine the fundamental interaction between two nucleons. It is also true that scarcely ever has the world of physics owed so little to so many. (...) It is hard to believe that many of the authors are talking about the same problem or, in fact, that they know what the problem is.

> M. L. Goldberger Midwestern Conference on Theoretical Physics, Purdue University, 1960

The Nuclear Force Problem: Is the Never-Ending Story Coming to an End?

R. Machleidt

Department of Physics, University of Idaho, Moscow, Idaho, U.S.A.

Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

1935	Yukawa: Meson Theory	No
1950's	<i>The "Pion Theories"</i> One-Pion Exchange: o.k. Multi-Pion Exchange: disaster	renormalization-group invariance
	Many pions \equiv multi-pion resonances: $\sigma, \rho, \omega,$	split with particle physics
	The One-Boson-Exchange Model Refine meson theory:	Life with
1970's	Sophisticated 2π exchange models (Stony Brook, Paris, Bonn)	models: refined description
1980's	Nuclear physicists discover QCD Quark Cluster Models	of two-body scattering; three-body forces?
1990's and beyond	Nuclear physicists discover EFT Weinberg, van Kolck Back to Meson Theory! But, with Chiral Symmetry	

Quantum Monte Carlo Calculations of Light Nuclei

$$v_{ij}^{\pi} + v_{ij}^{R} = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^{p}$$

$$Steven C. Pieper and R. B. Wiringa$$
Physics Division, Argonne National Laboratory, Argonne, IL 60439;
email: spieper@anl.gov, wiringa@anl.gov
$$O_{ij}^{p=1,14} = [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L} \cdot \mathbf{S})^2] \otimes [1, \tau_i \cdot \tau_j]$$

$$O_{ij}^{p=15,18} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}] \otimes T_{ij}$$
, and $(\tau_{zi} + \tau_{zj})$

from T.B. Clegg

Barker et al., PRL 48 (1982) 918

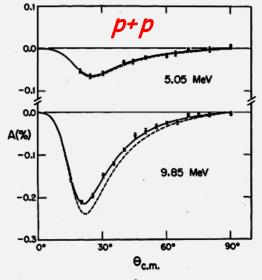
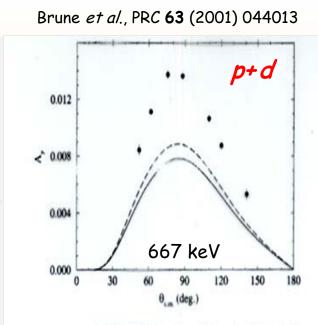
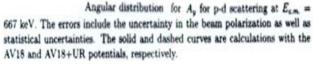


FIG. 2. Proton-proton analyzing power at 5.05 and 9.85 MeV plotted as a function of c.m. scattering angle. The solid curves through the data points are obtained from our phase-shift analyses. The dashed curves are the analyzing powers predicted from the Paris potential.







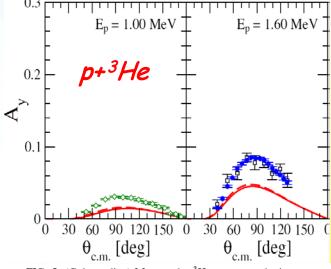
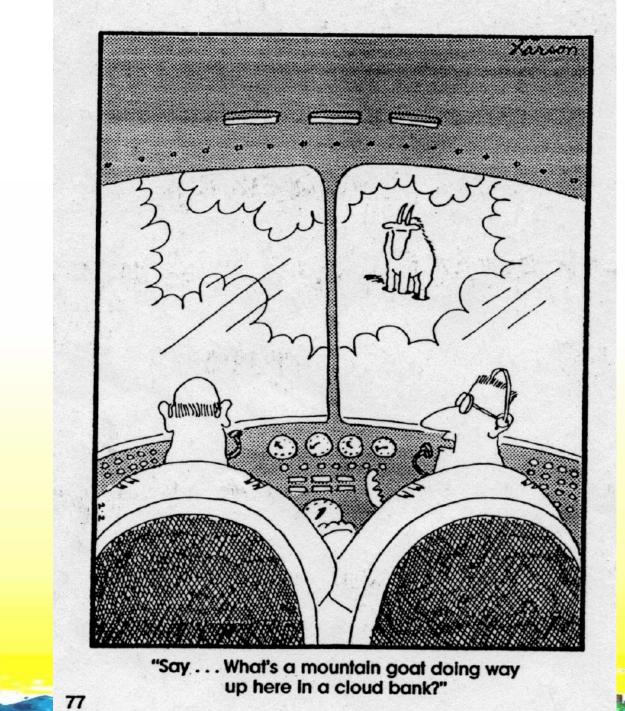


FIG. 5. (Color online) Measured p-³He proton analyzing power A_y (solid circles) at five different energies are compared with the data of Ref. [10] (open squares), Ref. [22] (open diamonds), and Ref. [67] (open circles). Curves show the results of theoretical calculations for the AV18 (dashed lines) and AV18/UIX (solid lines) potential models.



Time for a paradigm change, perhaps?

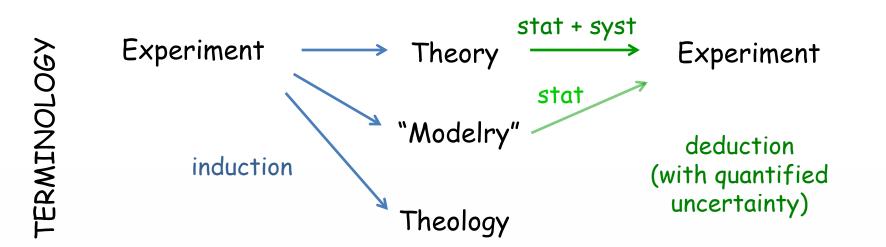
The Nuclear Force Problem: Is the Never-Ending Story Coming to an End?

R. Machleidt

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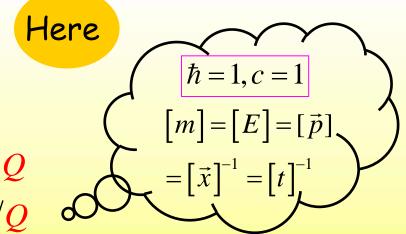
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	Sophisticated 2π exchange models	_ models:
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1980's	Nuclear physicists discover	of two-body scattering;
	QCD	
	Quark Cluster Models	three-body forces?
	Nuclear physicists discover (EFT)	
1990's	Weinberg, van Kolck	
and beyond	Back to Meson Theory!	
-	But, with Chiral Symmetry	



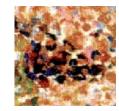
Post-quantum mechanics ("realistic"?) attitude: only observable quantities (S-matrix elements) matter

experiments only probe finite momenta Q*i.e.* only distances $\Delta r \ge 1/Q$



EFFECTIVE (FIELD) THEORIES

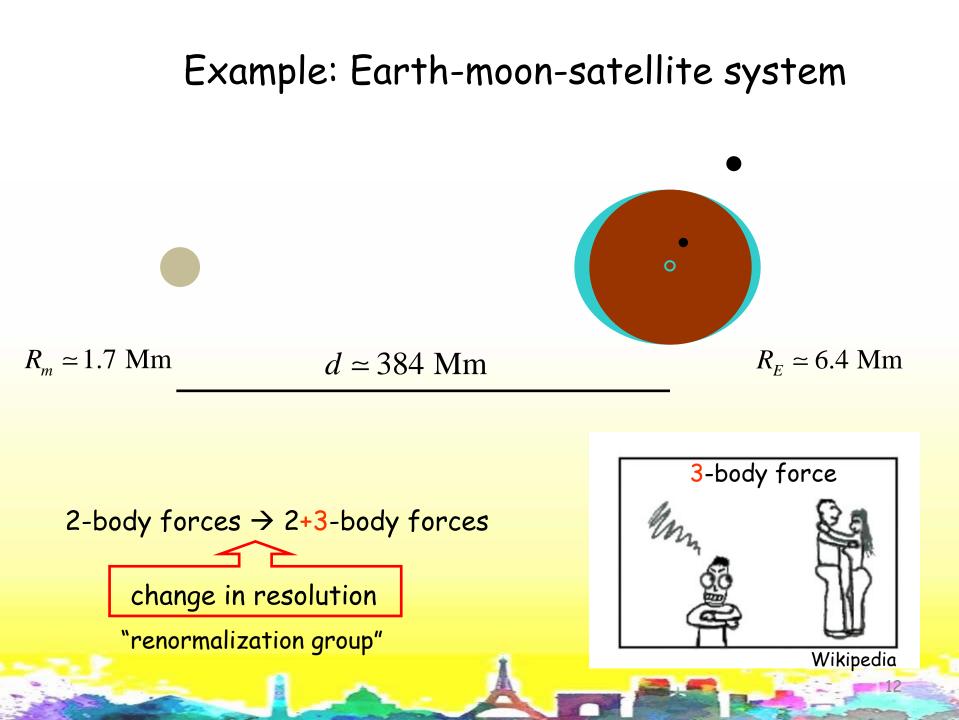
> Relevant degrees of freedom



> Relevant degrees of freedom

choose the coordinates that fit the problem

> All possible interactions



> Relevant degrees of freedom

choose the coordinates that fit the problem

> All possible interactions

what is not forbidden is compulsory

Symmetries

A farmer is having trouble with a cow whose milk has gone sour. He asks three scientists—a biologist, a chemist, and a physicist—to help him. The biologist figures the cow must be sick or have some kind of infection, but none of the antibiotics he gives the cow work. Then, the chemist supposes that there must be a chemical imbalance affecting the production of milk, but none of the solutions he proposes do any good either. Finally, the physicist comes in and says, "First, we assume a spherical cow..."



$$\sum_{ij} \alpha_{ij} u_i v_j \rightarrow \vec{u} \cdot \vec{v}$$

no, say, u_1v_2

 $+\sum_{ij}\delta\alpha_{ij}u_iv_j$ $\left| \delta \alpha_{ij} \right| \ll 1$

amenable to perturbation theory

> Relevant degrees of freedom

choose the coordinates that fit the problem

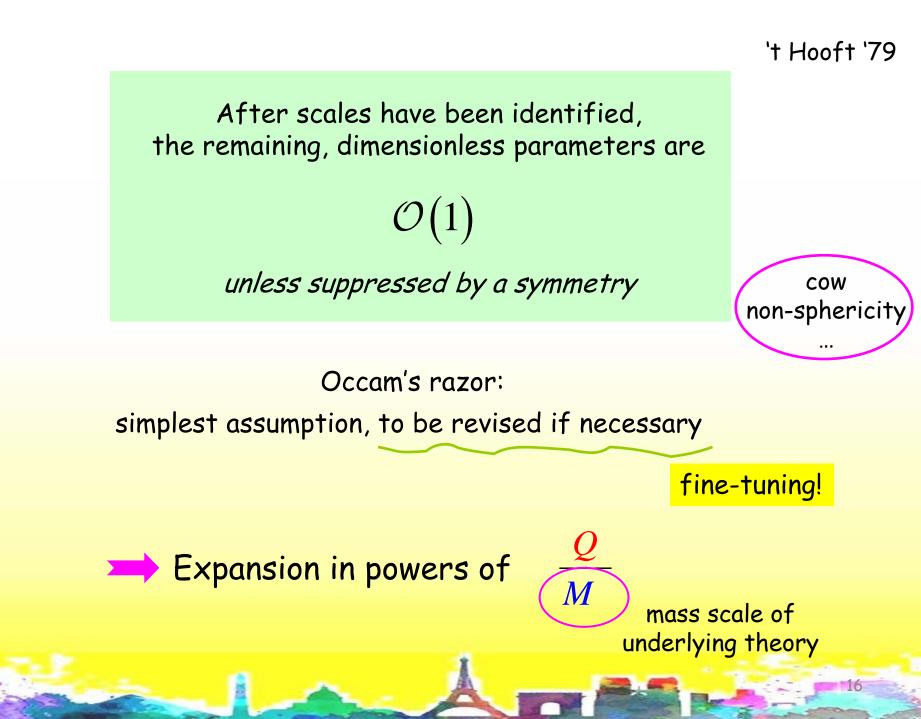
> All possible interactions

what is not forbidden is compulsory

Symmetries

not everything is allowed

> Naturalness



A classical example: the flat Earth
light object near surface of a large body

$$E \sim mgh \ll E_{und} \equiv mgR \qquad \begin{cases} \text{d.o.f.: mass } m \\ \text{sym: } V_{eff}(h, x, y) = V_{eff}(h) \end{cases}$$

$$V_{eff}(h) = m\sum_{i=0}^{\infty} g_i h^i = \text{const} + mg \{h + \eta h^2 + ...\} \qquad \text{(neglecting quantum corrections...)}}$$

$$naturalness: \quad \frac{mg_{i+1}h^{i+1}}{mg_ih^i} = \frac{E}{E_{und}} \times \mathcal{O}(1) = \frac{h}{R} \times \mathcal{O}(1) \Leftrightarrow g_{i+1} = \mathcal{O}\left(\frac{g}{R^i}\right)$$

$$V_{und}(h) = -GMm \frac{1}{R+h} = m\left(\frac{GM}{R^2}\right) \sum_{i=0}^{\infty} \left(\frac{-1}{R}\right)^{i-1} h^i \Rightarrow g_{i+1} = (-1)^i \frac{g}{R^i}$$

$$M \qquad \text{itself the first term in a low-energy EFT of general relativity...}$$

Going a bit deeper...

A short path to quantum mechanics

$$P = |\mathbf{A}_1 = |\mathbf{A}_2 + \mathbf{A}_2|^2 + \mathbf{A}_4|^2$$

sum over
all paths
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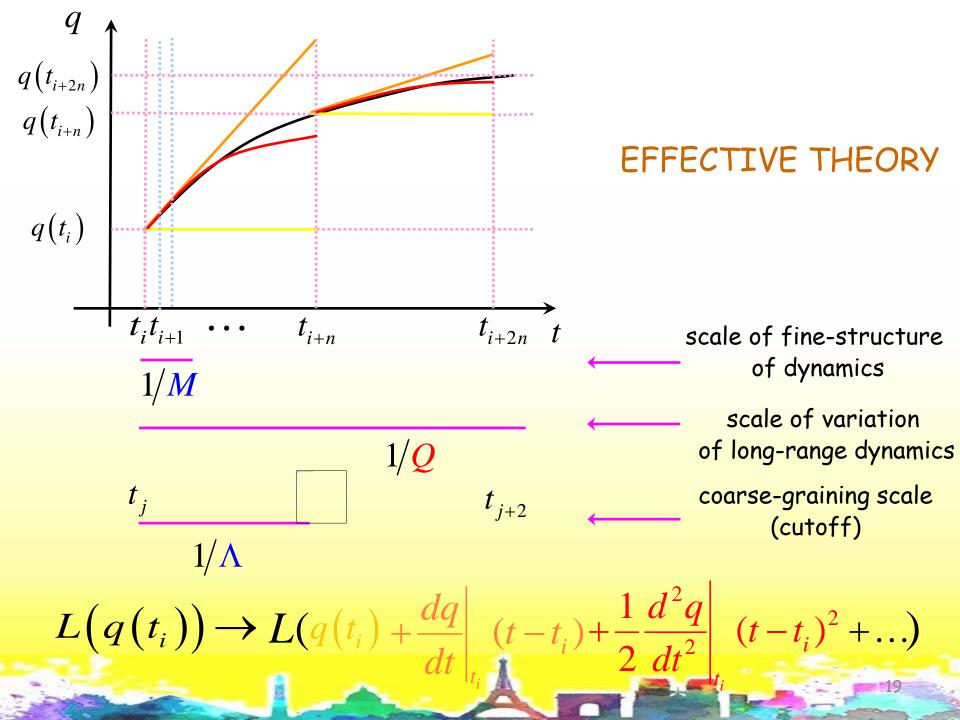
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$$P = |\mathbf{A}_1 = |\mathbf{A}_2 + \mathbf{A}_2|^2 + \mathbf{A}_4|^2$$

classical path $\delta\left(\int dt L(q(t))\right) = 0$

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More generally,

$$\begin{split} A &= \int Dq \, \exp\left(i\int dt \, L_{und}(q)\right) \times \int D\tilde{q} \,\,\delta\left(\tilde{q} - f_{\Lambda}(q)\right) \\ &= \int D\tilde{q} \,\,\exp\left(i\int dt \, L_{EFT}(\tilde{q})\right) \qquad \prod_{i} \int d\tilde{q}(t_{i}) \,\delta\left(\tilde{q}(t_{i}) - f\left(q(t_{i})\right)\right) \end{split}$$

$$L_{EFT}\left(\tilde{q}\right) = \sum_{d,n=0}^{\infty} c_{d,n}(\boldsymbol{M},\boldsymbol{\Lambda}) O_{d,n}\left(\tilde{q},\left(\frac{d^{d}\tilde{q}}{dt^{d}}\right)^{n}\right)$$

Wilson or low-energy coefficients (LECs) operators or interactions

$$c_{d,n} \sim c_{0,0} / M^{dn}$$
 natural

$$\left\{ \begin{aligned} q: \Delta t < 1/M \\ \tilde{q}: \Delta t \sim 1/E > 1/M \end{aligned} \right.$$

e.g.
$$L_{EFT}\left(\tilde{q}\right) = \frac{m}{2} \left(\frac{d\tilde{q}}{dt}\right)^2 + \frac{m\omega^2}{2}\tilde{q}^2 + c_{0,0}\tilde{q}^4 + c_{1,2}\tilde{q}^2 \left(\frac{d\tilde{q}}{dt}\right)^2 + \dots$$

"second quantization" + Lorentz invariance

$$q(t) \rightarrow \psi(\vec{r}, t), \psi^{*}(\vec{r}, t) \rightarrow \psi(x), \psi^{*}(x) \quad \stackrel{\text{representation}}{\text{of } SO(3,1)}$$

$$dt \rightarrow dt \, d^{3}r \qquad \equiv d^{4}x \qquad \text{scalar}$$

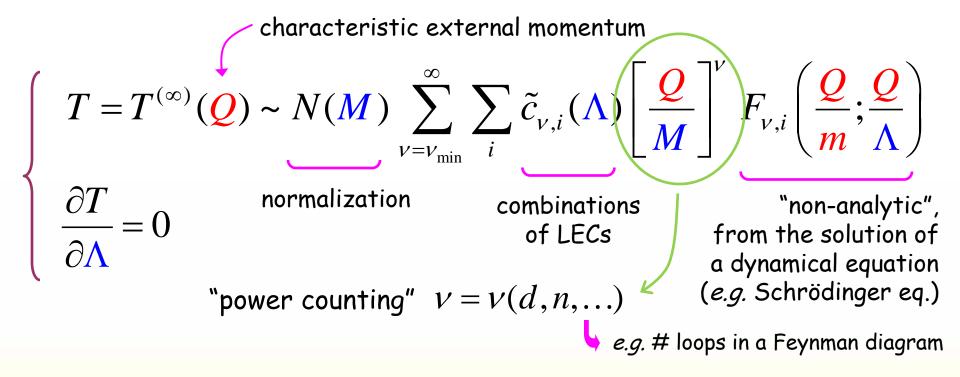
$$\frac{d}{dt} \rightarrow \frac{\partial}{\partial t}, \frac{\partial}{\partial \vec{r}} \qquad \rightarrow \frac{\partial}{\partial x^{\mu}} \qquad \text{vector}$$

$$dt \, L(q(t)) \rightarrow \int dt \int d^{3}r \, \mathcal{L}(\psi(\vec{r}, t)) = \int d^{4}x \, \mathcal{L}(\psi(x)) \qquad \text{scalar}$$

EFFECTIVE FIELD THEORIES

Euler + Heisenberg '36 Weinberg '67 ... '79 Wilson, early 70s

....



For $Q \ll M$, truncate consistently with RG invariance so as to allow systematic improvement (perturbation theory):

$$T = T^{(\overline{\nu})} \left[1 + \mathcal{O}\left(\frac{\underline{Q}}{\underline{M}}, \frac{\underline{Q}}{\Lambda}\right) \right] \qquad \frac{\Lambda}{T^{(\overline{\nu})}} \frac{\partial T^{(\overline{\nu})}}{\partial \Lambda} = \mathcal{O}\left(\frac{\underline{Q}}{\Lambda}\right)$$

N.B. Want large "model space" to reduce cutoff errors $\Lambda \ge M$ but no need for (possibly ill-defined) $\Lambda \to \infty$

Two possibilities:

> know <u>and</u> can solve underlying theory --

get c_i 's in terms of parameters in \mathcal{L}_{und} by matching

know <u>but</u> cannot solve, or do <u>not</u> know, underlying theory -invoke Weinberg's "folk theorem":

The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general *S* matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content.

S. Weinberg '79

Why is this useful?

Because in general the appropriate degrees of freedom below M φ are not the same as above $\phi = (\phi_H, \phi_L)$

Examples:

- M is mass of physical particle -virtual exchange in coefficients C_i (Appelquist-Carazzone decoupling theorem)
- M is scale associated with breaking of continuous symmetry -appearance of massless Goldstone bosons or gauge-boson mass (Goldstone's theorem, Higgs mechanism)
- M is scale of confinement -- rearrangement of whole spectrum
- M is radius of Fermi surface -- BCS behavior

Bira's Recipe for an EFT

- 1. identify degrees of freedom and symmetries
- 2. construct most general Lagrangian
- 3. assume certain scales, do power counting
- 4. calculate observables in successive orders with all momenta $Q < \Lambda$
- 5. relate $c_i(\Lambda), \Lambda$ to observables and check they are independent of Λ
- 6. check convergence:

not a model form factor

if good, declare victory

if not, repeat from 3; if problem remains, repeat from 1

"Modern S-matrix theory" - S. Weinberg

- ✓ No dependence on specific fields
- ✓ Quantum field theory a tool to generate most general S matrix

"New conceptualization" of renormalization

- ✓ Reg + renorm is the process of connecting Lag to observables
- \checkmark No infinities, nothing under the rug
- ✓ Choice of reg is psychology, RG invariance is physics
- \checkmark Importance of lowest-dimension operators explained

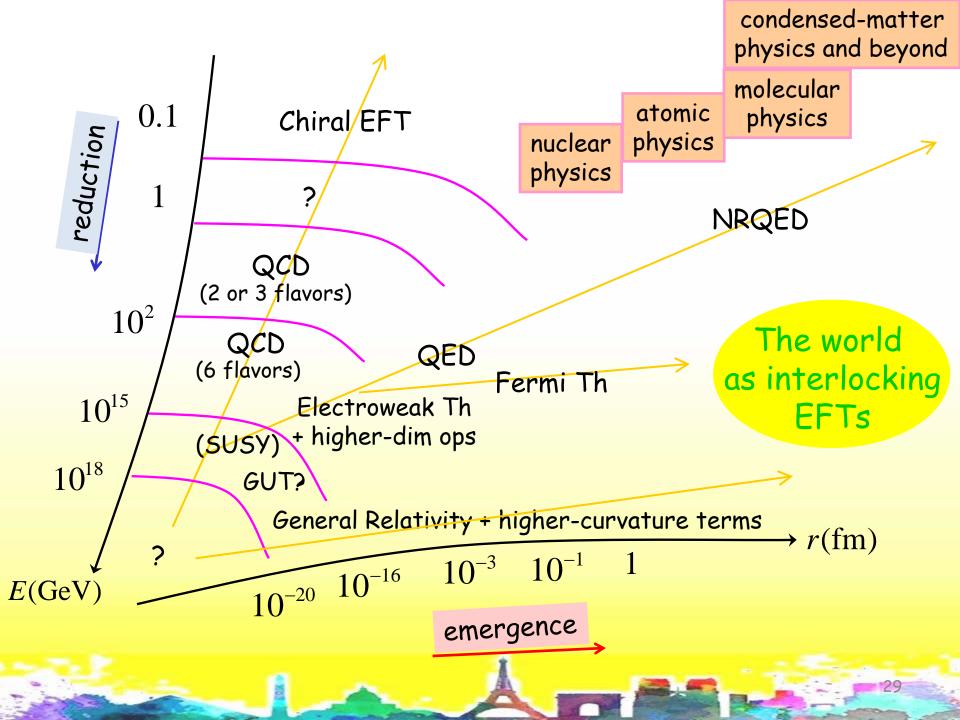
The mother of all models

- models have fewer, but *ad hoc*, interactions and do not necessarily match the underlying theory
- models with the correct symmetry pattern can be reproduced by EFT with an infinite number of constraints in the LECs
- models useful in the identification of relevant degrees of freedom and symmetries, but plagued with uncontrolled errors

A significant change in physicists' attitude towards what should be taken as a guiding principle in theory construction is taking place in recent years in the context of the development of EFT. For many years (...) renormalizability has been taken as a necessary requirement. Now, considering the fact that experiments can probe only a limited range of energies, it seems natural to take EFT as a general framework for analyzing experimental results.

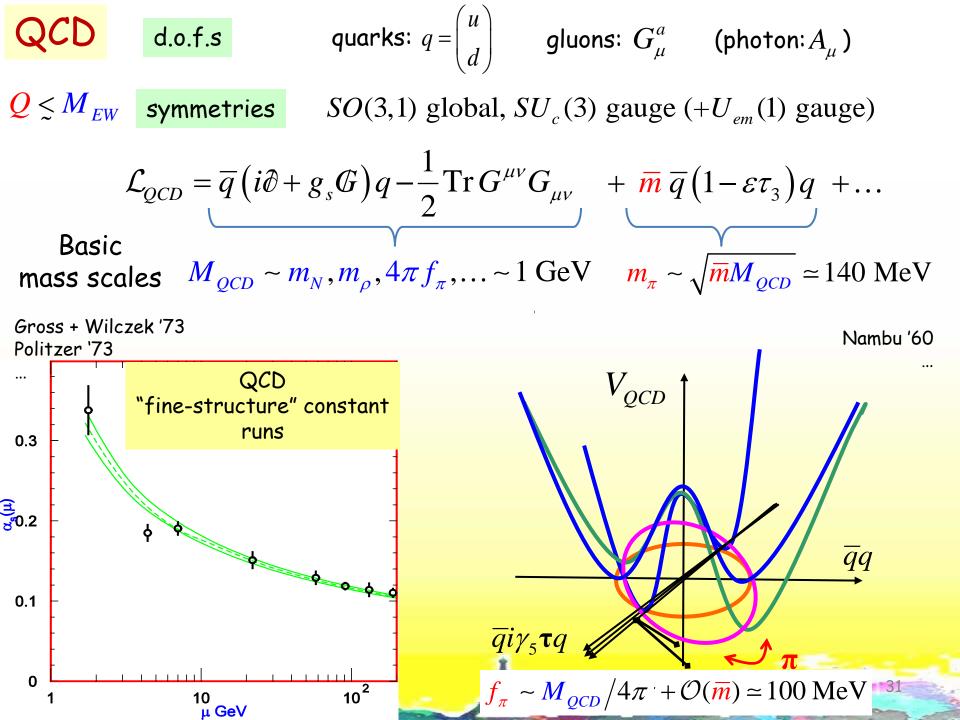
T.Y. Cao

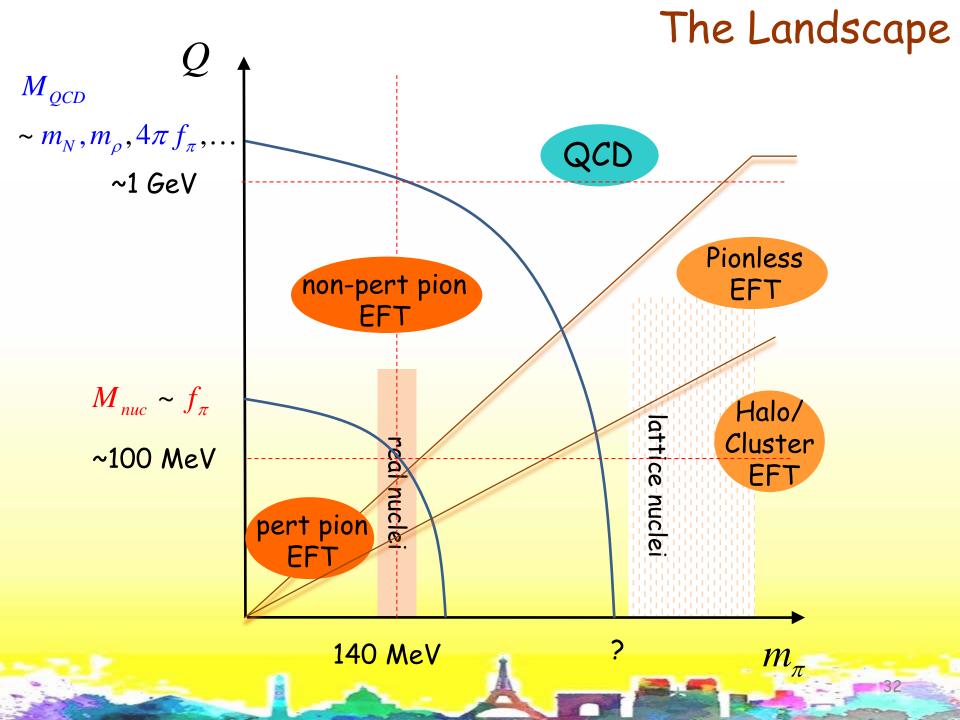
Renormalization, From Lorentz to Landau (and Beyond), L.M. Brown (ed), 1993



(fundamental - effective) theory \approx theology

but back to nuclear forces:





d.o.f.snucleons:
$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$
(+ Delta isobar, Roper)Chiral EFT
 $Q \sim m_{\pi} \ll M_{QCD}$ pions: $\pi = \begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ -i(\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix}$ (photon: A_{μ})symmetries

SO(3,1) global, $SU(2)_L \times SU(2)_R$ global (+ $U_{em}(1)$ gauge)

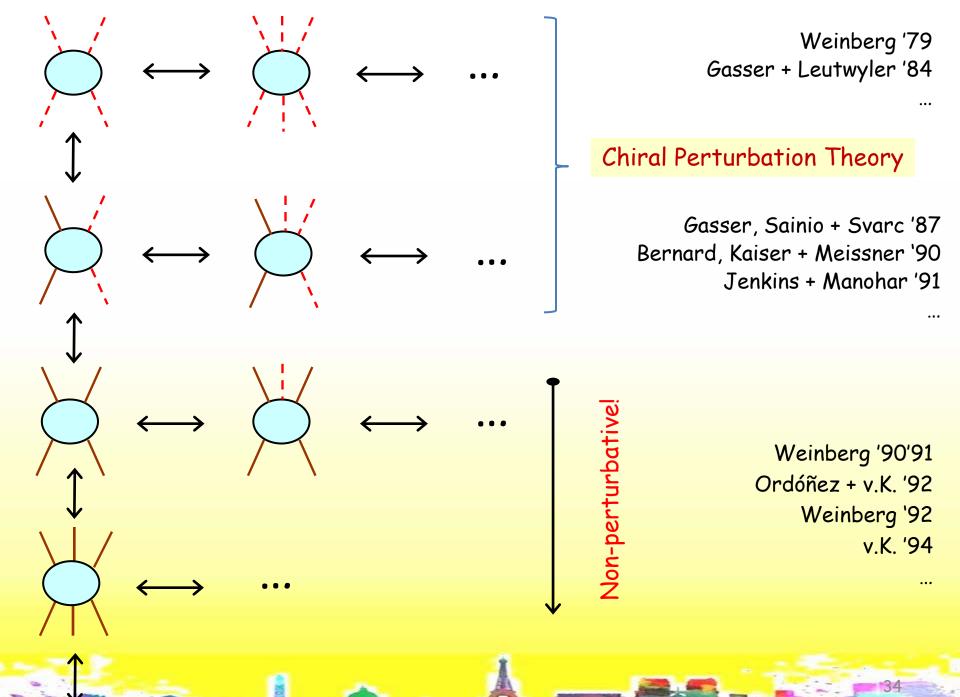
$$\begin{aligned} \mathcal{L}_{\chi EFT} &= \frac{1}{2} D_{\mu} \pi \cdot D^{\mu} \pi - \frac{m_{\pi}^{2}}{2} \frac{\pi^{2}}{1 + \pi^{2}/4f_{\pi}^{2}} + N^{+} \left(i \mathcal{D}_{0} + \frac{\vec{\mathcal{D}}^{2}}{2m_{N}} \right) N + \frac{g_{A}}{2f_{\pi}} N^{+} \vec{S} \tau N \cdot \cdot \vec{D} \pi \\ &+ C_{0} N^{+} N N^{+} N + C_{2}' N^{+} N \left(\vec{\mathcal{D}} N^{+} \right) \cdot \vec{\mathcal{D}} N + \dots \end{aligned}$$

$$\boldsymbol{D}_{\mu} = \left(1 + \boldsymbol{\pi}^{2} / 4 f_{\pi}^{2}\right)^{-1} \partial_{\mu} \qquad \boldsymbol{\mathcal{D}}_{\mu} = \partial_{\mu} + \frac{i}{2 f_{\pi}^{2}} \left(\boldsymbol{\pi} \times \boldsymbol{D}_{\mu} \boldsymbol{\pi}\right) \cdot \mathbf{t}^{(I)}$$

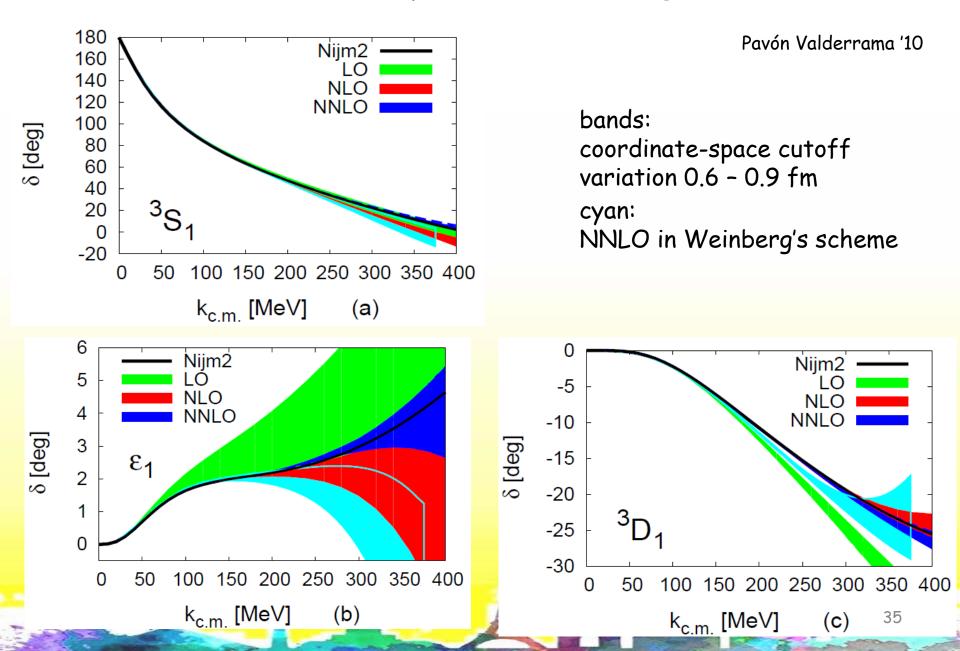
chiral covariant derivatives

other spin/isospin , more derivatives, powers of pion mass, Deltas and Ropers, few-body forces, *etc.*

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Example: NN scatterng



Re-inserted nuclear physics in the context of particle physics

Took about 10 years to be accepted by nuclear community

... and then only for the wrong reasons: results comparable to those of phenomenological potentials when treated as a phenomenological potential

Ab Initio Path to Heavy Nuclei

Example: ground energies of many nuclei

Sven Binder,^{1, *} Joachim Langhammer,¹ Angelo Calci,¹ and Robert Roth¹ ¹Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany (Dated: December 20, 2013)

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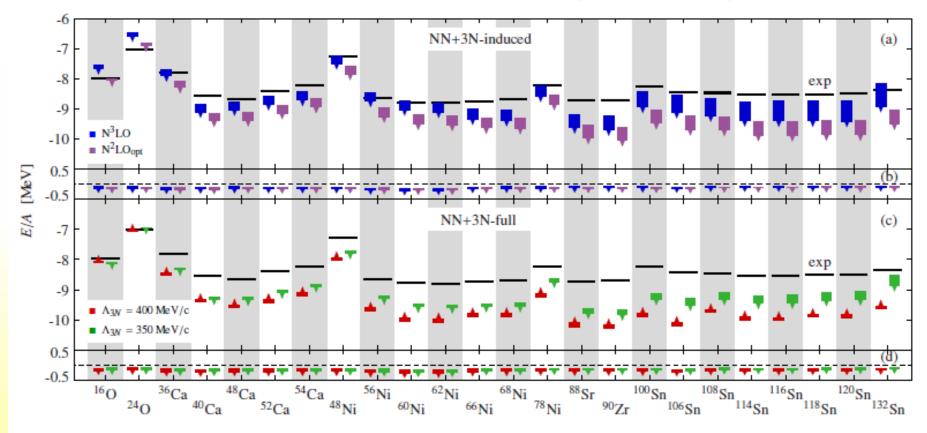


FIG. 5: (Color online) Ground-state energies from CR-CC(2,3) for (a) the NN+3N-induced Hamiltonian starting from the N³LO and N²LOoptimized NN interaction and (c) the NN+3N-full Hamiltonian with $\Lambda_{3N} = 400 \text{ MeV/c}$ and $\Lambda_{3N} = 350 \text{ MeV/c}$. The boxes represent the spread of the results from $\alpha = 0.04 \text{ fm}^4$ to $\alpha = 0.08 \text{ fm}^4$, and the tip points into the direction of smaller values of α . Also shown are the contributions of the CR-CC(2,3) triples correction to the (b) NN+3N-induced and (d) NN+3N-full results. All results employ $\hbar\Omega = 24 \text{ MeV}$ and 3N interactions with $E_{3max} = 18$ in NO2B approximation and full inclusion of the 3N interaction in CCSD up to $E_{3max} = 12$. Experimental binding energies [32] are shown as black bars.

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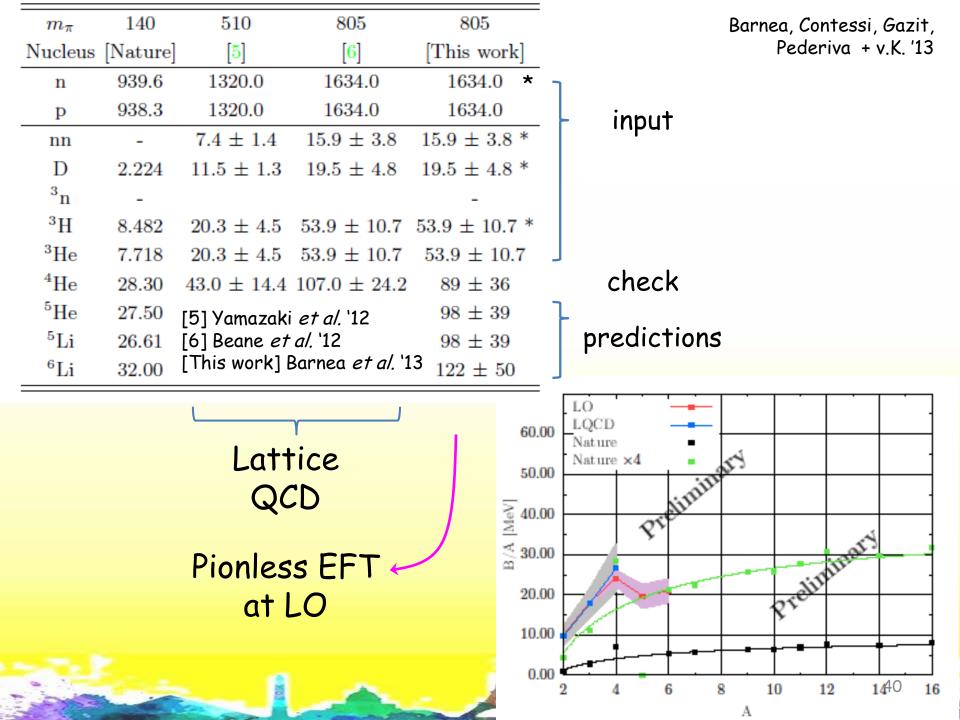
But that's another talk...

Here only two points:
connection with underlying theory
interplay with experimental data

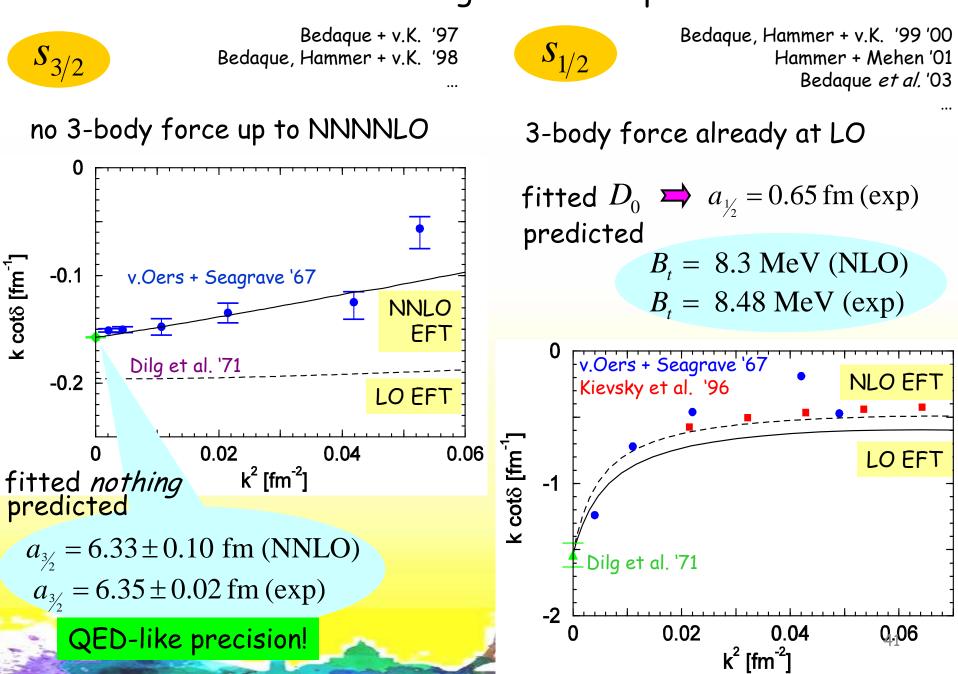
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symmetries
 $SO(3,1)$ global $(+U_{em}(1)$ gauge)

$$\mathcal{L}_{\chi EFT} = \frac{1}{2} D_{\mu} \boldsymbol{\pi} \cdot D^{\mu} \mathcal{D}_{\pi} \frac{m_{\pi}^{2}}{z_{FT}} \frac{N^{+}}{2} \left(\frac{\boldsymbol{\pi}^{2}}{12} + \frac{\nabla^{2}}{z_{T}} \right)^{+} \mathcal{N} \left(i\mathcal{D}_{C} + \frac{\mathcal{P}^{2}}{2m_{N}} \right)^{+} \mathcal{N} + \frac{g_{A}}{2f_{\pi}} N^{+} \vec{S} \boldsymbol{\tau} N \cdot \vec{D} \boldsymbol{\pi}$$
$$+ C_{0} N^{+} N N^{+} N + C_{2}^{t} \mathcal{D}_{0}^{+} \mathcal{M} \left(\mathcal{D} \mathcal{M}^{+} \right) \mathcal{N} \mathcal{D} \mathcal{N} + \dots$$
other spin/isospin, isospin, other spin/isospin, isospin, more derivatives, etc.

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nd scattering with NN input



Many other successes, but still ignored by nuclear community (perhaps because it cannot be treated as a phenomenological potential)

Summary

EFT is a general framework for theory construction

- same method across scales
- ✓ model independent
- controlled expansion

EFT is (very slowly) becoming the paradigm in nuclear physics

- v encodes QCD (and, more generally, B/SM)
- ✓ incorporates hadronic physics
- ✓ generates nuclear structure