Semiclassical approach to pairing in the weak coupling regime: nuclei, cold atoms and neutron stars

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Int.J.Mod.Phys. **E20**, 399 (2011);
J.of Phys:Conf.Ser. **321**, 012024 (2011);
Phys.Rev.Lett. **107**, 205301 (2011) (arXiv 1110.1188)

INTRODUCTION

STABLE NUCLEI

177 even-even; 58 even-odd; 54 odd-even; 10 odd-odd

$$\Delta \sim \frac{1}{2}\big(E(A+1) - 2E(A) + E(A-1)\big) \qquad A \quad even$$

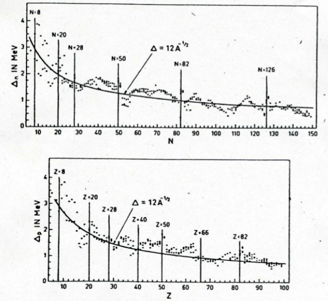


Fig. 15.5 Neutron and proton pairing energies as a function of *N* and *Z*, respectively (Zeldes *et al.* 1967).

THEORETICAL FRAMEWORK

BCS THEORY

$$\Delta_{n_c} = -\sum_{n'_c} V_{n_c n'_c} \frac{\Delta_{n'_c}}{2E_{n'_c}}, \quad N = \sum_{n_c} \left(1 - \frac{\epsilon_{n_c} - \mu}{E_{n_c}}\right)$$

where

$$V_{n_c n'_c} = \langle n_c \bar{n}_c | v | n'_c \bar{n}'_c \rangle \quad and \quad E_n = [(\epsilon_n - \mu)^2 + \Delta_n^2]^{1/2},$$

FURTHER IMPROVMENTS

Hartree-Fock-Bogolibov theory

Particle-vibration coupling

SEMICLASSICAL APPROXIMATIONS

LDA is the standard semiclassical limit to the pairing problem.

H.Kuchareck, P.Ring, P.Schuck, R.Bengtsson amd M.Girod,

Phys. Lett.**B216**, 240 (1989).

$$\Delta(\mathbf{R},\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} v(\mathbf{p}-\mathbf{p}')\kappa(\mathbf{R},\mathbf{p}')$$

$$\kappa(\mathbf{R},\mathbf{p}) = \frac{\Delta(\mathbf{R},\mathbf{p})}{2\sqrt{((p^2-p_F^2(\mathbf{R}))/2m^*)^2+(\Delta(\mathbf{R},\mathbf{p}))^2}}$$

The pairing matrix elements are evaluated using plane waves.

THE THOMAS-FERMI APPROXIMATION IN WEAK COUPLING

In the weak cooupling regime $\Delta/\mu << 1$. In this case the canonical basis can be replaced by the HF one:

$$H|n\rangle = \epsilon_n|n\rangle.$$

At equilibrium and for time reversal invariant systems canonical conjugation and time reversal operation are related by

$$\langle \mathbf{r}|\bar{n}\rangle = \langle n|\mathbf{r}\rangle \Rightarrow \langle \mathbf{r}_1\mathbf{r}_2|n\bar{n}\rangle = \langle \mathbf{r}_1|\hat{\rho}_n|\mathbf{r}_2\rangle,$$

$$V_{nn'} = \langle n\bar{n}|v|n'\bar{n}'\rangle = \int \langle \mathbf{r}_2|\hat{\rho}_n|\mathbf{r}_1\rangle\langle \mathbf{r_1}\mathbf{r_2}|v|\mathbf{r}'_\mathbf{1}\mathbf{r}'_\mathbf{2}\rangle\langle \mathbf{r}'_1|\hat{\rho}_n|\mathbf{r}'_2\rangle d\mathbf{r_1}d\mathbf{r_2}d\mathbf{r}'_\mathbf{1}d\mathbf{r}'_\mathbf{2}$$

$H|n\rangle = \epsilon_n|n\rangle$ can be written in terms of $\hat{\rho}_n$ as:

$$(H - \epsilon_n)\hat{\rho}_n = 0.$$

Taking the Wigner transform of this latter equation, we obtain in the $\hbar \to 0$ limit: $(H_{cl.} - \epsilon)f_\epsilon(\mathbf{R}, \mathbf{p}) = 0$, which solution is

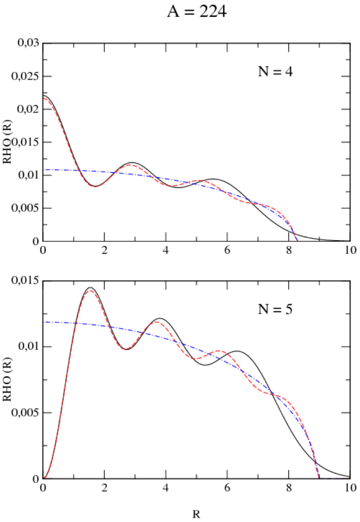
$$f_E(\mathbf{R}, \mathbf{p}) = \frac{1}{g^{TF}(E)}\delta(E - H_{cl.}) + O(\hbar^2).$$

with

$$H_{cl.} = \frac{p^2}{2m^*(\mathbf{R})} + V(\mathbf{R}) \quad and \quad g^{TF}(E) = \frac{1}{(2\pi\hbar)^3}\int d\mathbf{R}d\mathbf{p}\delta(E - H_{cl.}).$$

X. Viñas, P. Schuck, M. Farine and M. Centelles, Phys. Rev. **C67**, 054307 (2003).

HARMONIC OSCILLATOR POTENTIAL



The gap equation in semiclassical TF approximation reads:

$$\Delta(E) = -\int_0^{\infty} dE' g^{TF}(E') V(E,E') \kappa(E'),$$

with

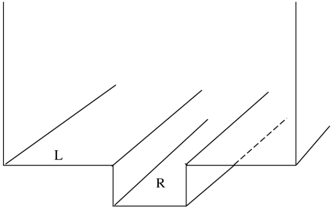
$$\kappa(E) = \frac{\Delta(E)}{2\sqrt{(E-\mu)^2 + \Delta^2(E)}}.$$

and

$$V(E,E') = \int \frac{d\mathbf{R} d\mathbf{p}}{(2\pi\hbar)^3} \int \frac{d\mathbf{R}' d\mathbf{p}'}{(2\pi\hbar)^3} f_E(\mathbf{R},\mathbf{p}) f_{E'}(\mathbf{R}',\mathbf{p}') v(\mathbf{R},\mathbf{p};\mathbf{R}',\mathbf{p}'),$$

with $v(\mathbf{R},\mathbf{p};\mathbf{R}',\mathbf{p}')$ the double Wigner transform of $< \mathbf{r_1 r_2}|v|\mathbf{r_1' r_2'} >$ which for a local translationally invariant force yields $v(\mathbf{R},\mathbf{p};\mathbf{R}',\mathbf{p}') = \delta(\mathbf{R}-\mathbf{R}')v(\mathbf{p}-\mathbf{p}')$

A SIMPLE EXEMPLE: SLAB GEOMETRY



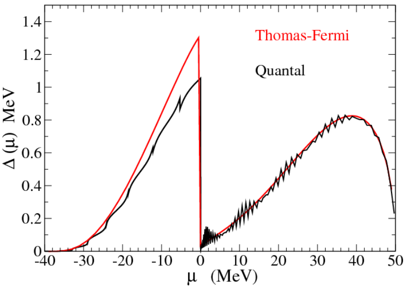
Mean field potential

$$V(x) = 0 \quad -L \leq x \leq -R \quad or \quad R \leq x \leq L; \quad V(x) = V_0 \quad -R \leq x \leq R$$

$$V_0 = -40 MeV \quad L = 100 fm \quad R = 10 fm$$

Pairing force

$$V_p = -g\delta(\mathbf{r} - \mathbf{r}') \quad g = 150 MeVfm^3 \quad \Lambda = 50 MeV$$



Gap Equation

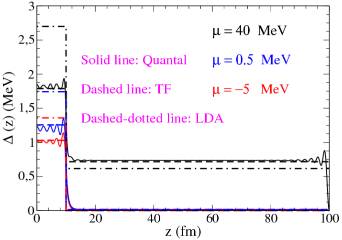
$$\Delta_n = -\sum_{n'} \Theta(\Lambda - \varepsilon_{n'}) V_{nn'} K_{n'}; \quad K_n = \frac{m}{4\pi\hbar^2} \Delta_n \ln \frac{\Lambda - \mu + \sqrt{(\Lambda-\mu)^2 + \Delta_n^2}}{\varepsilon_n - \mu + \sqrt{(\varepsilon_n - \mu)^2 + \Delta_n^2}}$$

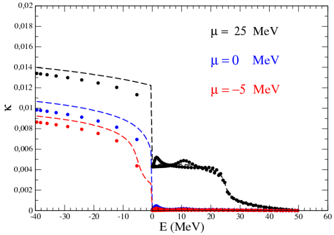
$$\Delta(z) = -gK(z)$$

$$K(z) = \sum K_n |\varphi_n(z)|^2 \quad K(z) = \int_{V_0}^{\Lambda} dE g^{TF}(E) K(E) \rho_E^{TF}(z)$$

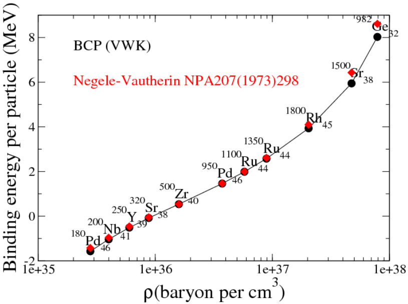
.

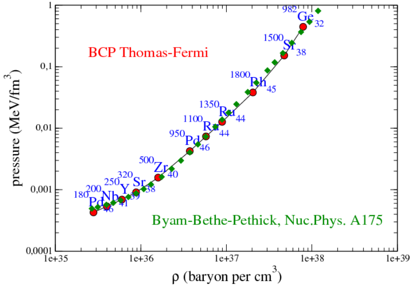
Gap and Pairing Tensor



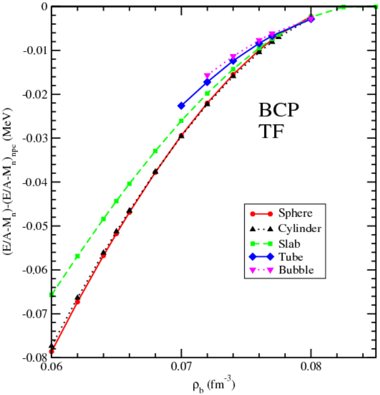


TF approach to the inner crust of neutron stars
BCP Energy Density Functional
Allows to compute not in spherical symmetry but also with planar (slabs) and cylindrical (rods) geometries

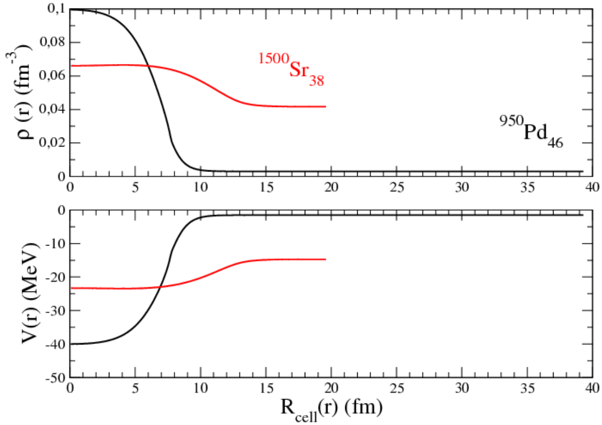




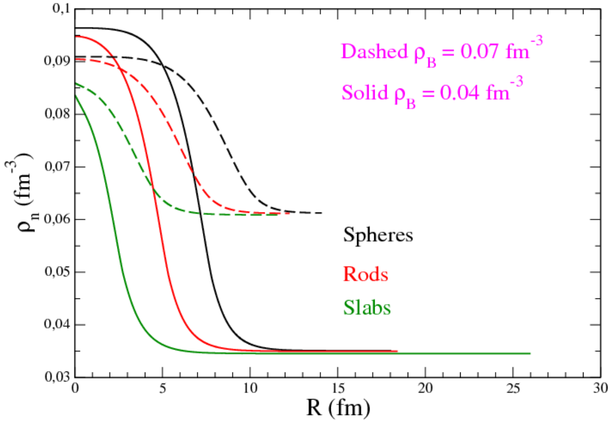
TF approach to the inner crust of neutron stars
BCP Energy Density Functional
Close to transition density some pasta phases appears within the BCPM model



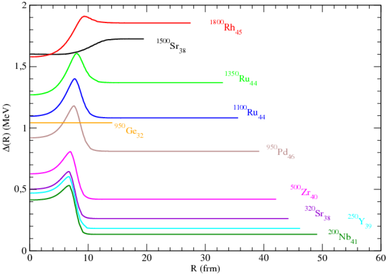
TF approach to the inner crust of neutron stars
BCP Energy Density Functional



TF approach to the inner crust of neutron stars
BCP Energy Density Functional

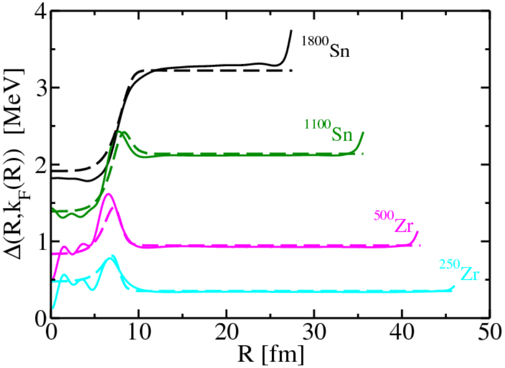


Semiclassical pairing in Wigner-Seitz cells

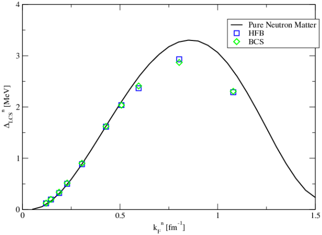


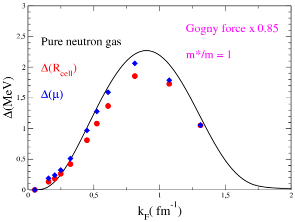
$$\Delta(\mathbf{R}) = \frac{1}{\kappa(\mathbf{R})}\int \frac{d\mathbf{p}}{(2\pi\hbar)^3}\Delta(\mathbf{R},\mathbf{p})\kappa(\mathbf{R},\mathbf{p}); \quad \Delta(\mathbf{R},\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} v(\mathbf{p}-\mathbf{p}')\kappa(\mathbf{R},\mathbf{p}')$$

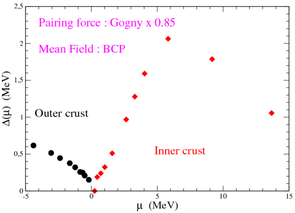
TF approach to the inner crust of neutron stars
BCP Energy Density Functional
Comparison with HFB calculations



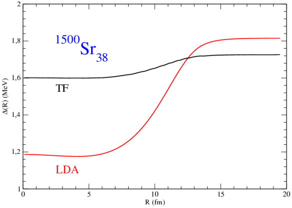
Semiclassical pairing in Wigner-Seitz cells

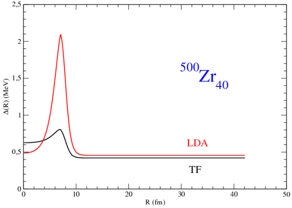


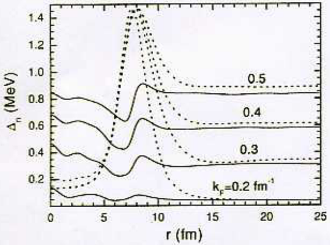




Semiclassical pairing in Wigner-Seitz cells
Comparison between TF and LDA







ctions for various k_F values.

N. Chamel et al, Phys. Rev. **C81**, 045804 (2010)

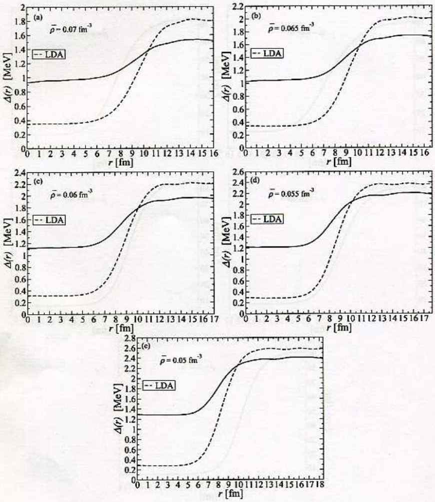
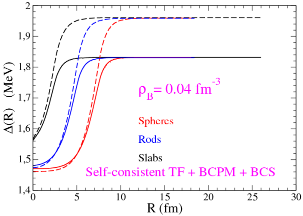
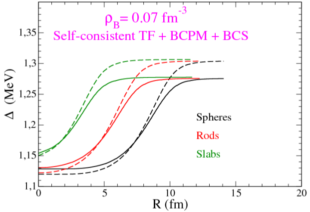


FIG. 4. Angle-averaged neutron pairing field $\Delta(r)$ (solid line) and LDA pairing field (dashed line) for the different crustal layers at $T = 0$.

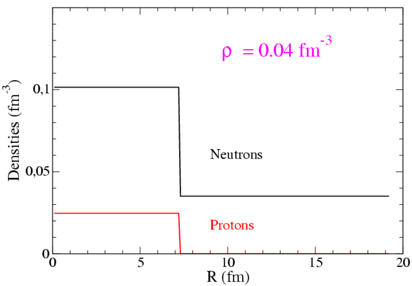
Semiclassical pairing in Wigner-Seitz cells

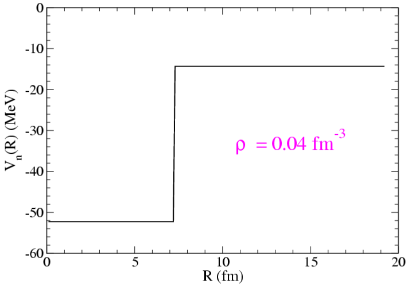
Gaps in Pasta Phases





Compressible Liquid Drop Model (CLDM)





$$
\begin{aligned}
\mathcal{E} &= \frac{E}{V_c}\chi\mathcal{E}\left(\rho_n,\rho_p\right) + (1-\chi)\,\mathcal{E}\left(\rho_d,0\right) + \frac{3\chi\sigma_s}{R} + n_s\mu_s \\
&+ \frac{4\pi}{5}e^2R^2\rho_p^2\chi\left(1-\frac{3}{2}\chi^{1/3}+\frac{1}{2}\chi\right) + \frac{3}{4}\left(3\pi^2\right)^{1/3}\chi^{4/3}\rho_p^{4/3}
\end{aligned}
$$

Pairing in the Compressible Liquid Drop Model (CLDM)

Gap equations

$$-V_1 \leq E \leq -V_2 \qquad \Delta_1(E) = \Delta_N(\mu)$$

$$-V_2 \leq E \leq \Lambda \qquad \Delta_2(E) = \frac{\chi \tilde{g_N}(E)\Delta_N(\mu) + (1-\chi)\tilde{g_g}(E)\Delta_g(\mu)}{\chi \tilde{g_N}(E) + (1-\chi)\tilde{g_g}(E)}$$

where

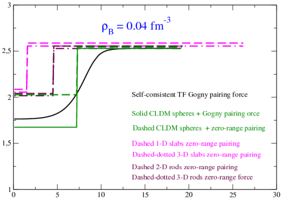
$$3-D \quad \tilde{g_N}(E) = \sqrt{E+V_1} \quad \tilde{g_g}(E) = \sqrt{E+V_1} \quad \chi = \frac{R^3}{R_c^3}$$

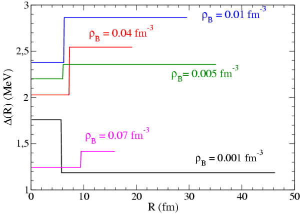
$$2-D \quad \tilde{g_N}(E) = 1 \quad \tilde{g_g}(E) = 1 \quad \chi = \frac{R^2}{R_c^2}$$

$$1-D \quad \tilde{g_N}(E) = \frac{1}{\sqrt{E+V_1}} \quad \tilde{g_g}(E) = \frac{1}{\sqrt{E+V_1}} \quad \chi = \frac{R}{R_c}$$

Semiclassical pairing in Wigner-Seitz cells

Gaps in Pasta Phases





ρ_B= 0.07 fm^{-3} 7.696684 -0.134515 7.562169 MeV SLABS

ρ_B= 0.07 fm^{-3} 7.693298 -0.134406 7.558892 MeV SPHERES

ρ_B= 0.07 fm^{-3} 7.693138 -0.134319 7.558819 MeV RODS

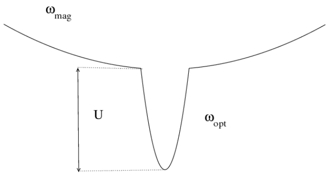
ρ_B= 0.04 fm^{-3} 5.851508 -0.365913 5.485595 MeV SLABS

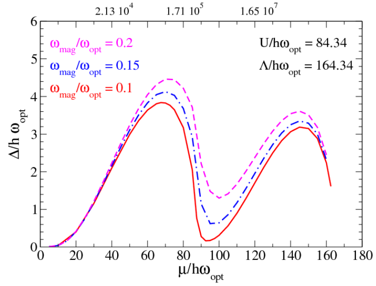
ρ_B= 0.04 fm^{-3} 5.807168 -0.365727 5.441441 MeV RODS

ρ_B= 0.04 fm^{-3} 5.800047 -0.366284 5.433763 MeV SPHERES

COLD ATOMS

Ketterle's trap, Phys. Rev. Lett. **81**, 2194 (1998)





$$\omega_{opt} = 2\pi \times 1000Hz; \quad g = -\hbar\omega_{opt} \quad \Lambda = 164.34\hbar\omega_{opt}$$

CONCLUSIONS

- We have presented here a Thomas-Fermi theory for pairing in finite Fermi systems for weak coupling situations with $\Delta/\varepsilon_F << 1$.
- This Thomas-Fermi theory differs from the usual Local Density Approximation. This essentially stems from the fact that we approximate the gap equation in configuration space and, thus, keep the size dependence of the matrix elements of the pairing force. This is not the case in LDA where the matrix elements of the force are always evaluated in plane wave basis.
- This semiclassical approach to pairing is only based on the usual validity criterion of Thomas-Fermi theory, namely that the Fermi wave length is smaller than the oscillator length. At no point the Local Density Approxiamtion condition that the extension of the Cooper pairs (coherence length) must be smaller than the oscillator length enters the theory. Thus, the present Thomas-Fermi approach yields for all pairing quantities the same quality as Thomas-Fermi theory does for quantities in the normal fluid state.

- The gap values obtained represent very well the mass number dependence in N and Z of the average gap for the D1S Gogny force employed in this work. Essentially the obtained gap values correspond to nuclei where the discrete quantal single particle level density has been replaced by a Thomas-Fermi smoothed continuous level density.
- We presented the full chart of the N, Z dependence of the average gap at the Fermi surface using, as mentioned, the D1S force for the pairing field and also for the mean field and effective mass, as obtained from the D1S force using extended TF theory.
- An interesting feature of our study is that the average gap breaks down going to the drip line. This surprising result is confirmed by quantal calculations, though strongly masked by shell fluctuations.
- A similar fact can also appears for cold atoms in a double trap.