Superfluidity-kill at overflow of trapped fermions. Quantal and semiclassical studies

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Outline

1. The physical context
2. General considerations about semiclassics for pairing
3. The slab model. Quantal and Thomas-Fermi approach
4. Cold atoms in double harmonic container
5. Wigner-Seitz cells in neutron stars with BCP functional
6. Finite nuclei at drip
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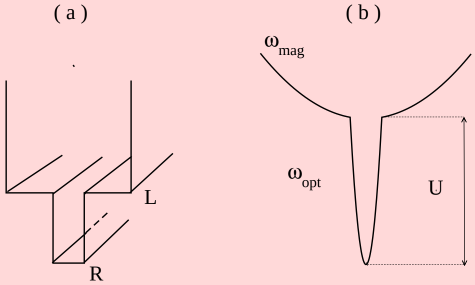
Physical context

Overflow situations of superfluid fermions in finite mean field potential →



nuclei (drip line), nuclei in Wigner Seitz cells in crust of neutron stars, Cold atoms

Semiclassics for pairing

In weak coupling, we have BCS:

$$\Delta_n = \sum_{n'} \langle n\bar{n}|v|n'\bar{n}'\rangle \frac{\Delta_{n'}}{2\sqrt{(\varepsilon_{n'}-\mu)^2+\Delta_{n'}}} \quad (1)$$

In LDA we have

$$\Delta(R,p) = \int \frac{d^3p'}{(2\pi\hbar)^3} V_{p,p'} \frac{\Delta(R,p')}{2\sqrt{(\varepsilon_{p'}-\mu(R))^2+\Delta^2(R,p')}} \quad (2)$$

where

$$\mu(R) = [\mu - V(R)]\ \Theta(\mu - V(R))$$

is the local Fermi energy. The condition for validity of LDA is that

coherence length is $<<$ oscillator length.

hbar corrections

$$\mathcal{C}_0{}^{\beta} = e^{-\mathcal{H}_W} + O(\hbar^2) \tag{3}$$

$$\mathcal{H}_W = \begin{pmatrix} h(\mathbf{R},\mathbf{p}) & \Delta(\mathbf{R},\mathbf{p}) \\ \Delta(\mathbf{R},\mathbf{p}) & -h(\mathbf{R},\mathbf{p}) \end{pmatrix} \tag{4}$$

O($\hbar^2$) gradient correction extremely complicated! Several pages!.
Similar to heat kernel expansion!

In novel Thomas-Fermi approximation, we take $\hbar \to 0$ of gap equation in configuration space

$$\langle r1r2|n\bar{n}\rangle = \langle r1|n\rangle\langle n|r2\rangle \tag{5}$$

Then for $\hbar \to 0$, we have

$$\{|n\rangle\langle n|\}_{Wigner} \to f_{E_n}(\mathbf{R},\mathbf{p}) = \frac{1}{g^{TF}(E)}\delta(E_n - H_{cl.}) \tag{6}$$

with $H_{cl.} = \frac{p^2}{2m^*} + V(R)$ and the level density

$$g^{TF}(E) = \int d^3R \int \frac{d^3p}{(2\pi\hbar)^3}\delta(E - H_{cl.}) \tag{7}$$

With this we can calculate pairing matrix element semiclassically and obtain for gap eq.:

$$\Delta(E) = \int dE' g^{TF}(E')V(E,E')\frac{\Delta(E')}{2\sqrt{(E'-\mu)^2 + \Delta^2(E')}} \tag{8}$$

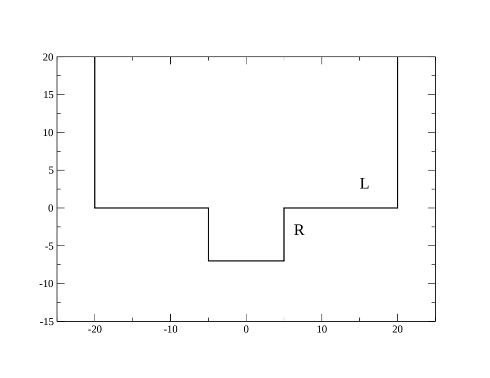
$$V(E,E') = \int d^3R \int\int \frac{d^3p}{(2\pi\hbar)^3}\frac{d^3p'}{(2\pi\hbar)^3} f_E(\mathbf{R},\mathbf{p}) v(\mathbf{p}-\mathbf{p}') f_{E'}(\mathbf{R},\mathbf{p}') \quad (9)$$

delta force $\rightarrow$

$$V(E,E') \sim -\frac{g}{g^{TF}(E)g^{TF}(E')} \int d^3R \sqrt{E-V(\mathbf{R})}\sqrt{E'-V(\mathbf{R})} \quad (10)$$

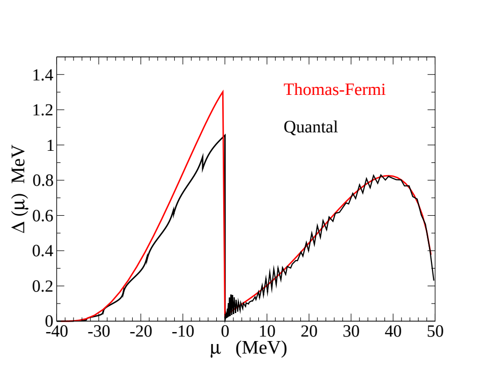
very simple!

For a check, we use a SLAB with following transverse profile
L=100fm, R=10fm

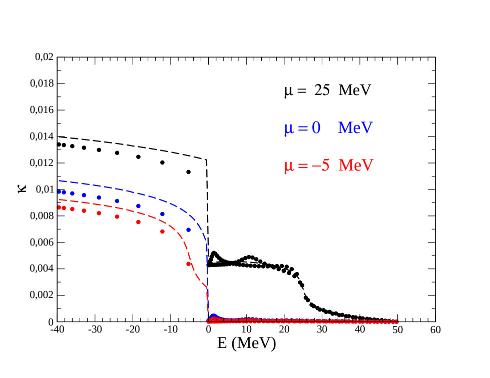


We will solve gap equation as a fct of filling, i.e. fct of μ

red: TF; black: quantal; slab with pocket, depth = - 40 MeV and cut off + 50 MeV

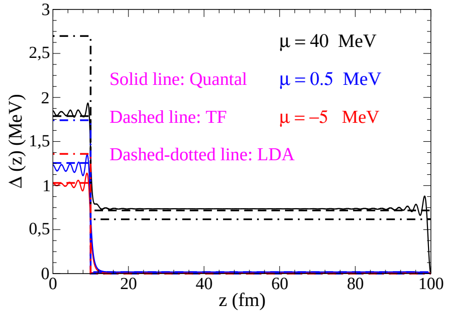


Energy dependence of pairing tensor $\kappa_n = \int d^2p_\perp \langle a^+_{n,p_\perp} a^+_{\bar{n},-p_\perp} \rangle$



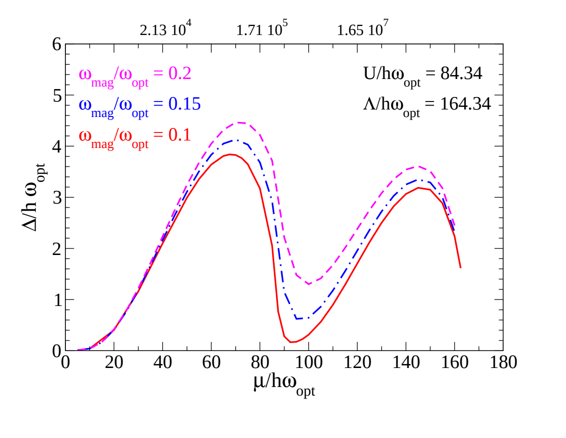
Very good agreement between quantal and TF!

Position dependence of gap:

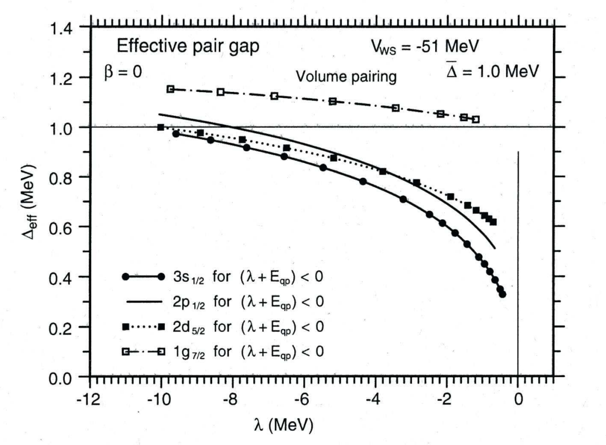


Agreement between quantal and TF slightly less good. May be $\hbar$ corrections necessary. Quite easy to incorporate!
In any case, if pocket becomes macroscopically large and also outer container, Fermigas approximation becomes exact. LDA still wrong because no coupling between outside and inside. In this example, TF is equivalent to Fermi gas approximation! In Fermi gas HFB=BCS!

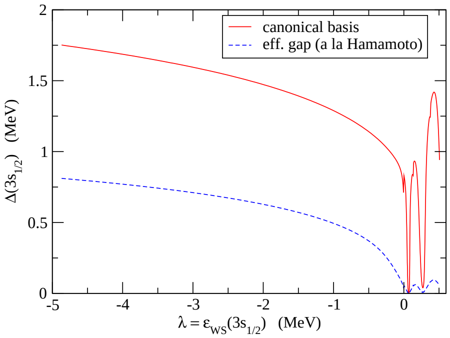
We now treat the example of cold atoms in the double harmonic well. We take parameter values from the literature corresponding approximately to the 6Li case.
When the narrow container is full (overflow), exactly 2×10^5 are in.
Similar scenario as for slab



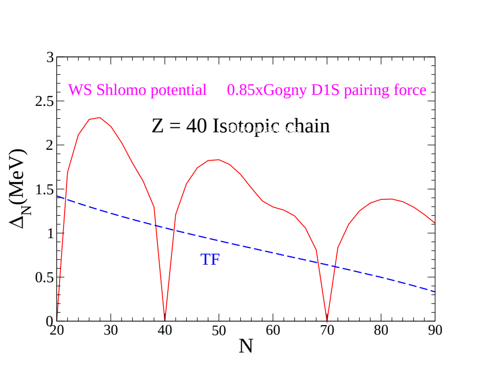
Finite nuclei at neutron drip [Hamamoto, PRC 71]:



More extended calculation by Hagino



In nuclei shell fluctuations are very strong but tendency can clearly be seen

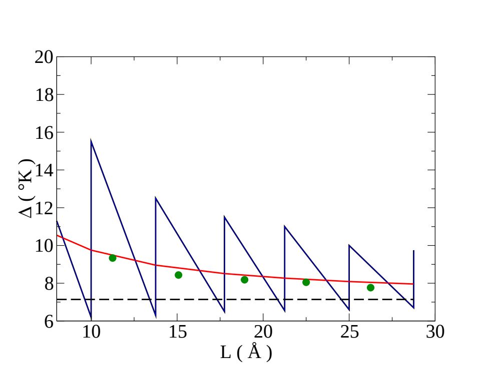


Size dependence of gap in finite Fermi systems:

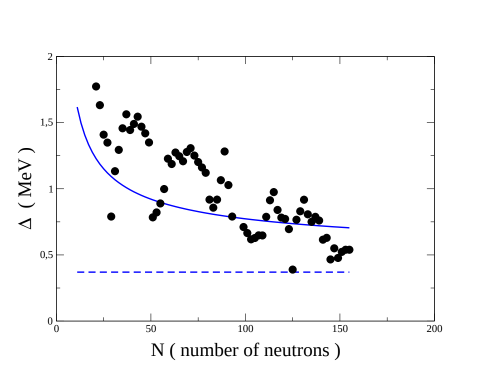
$$\Delta = \Delta_B e^{-C\frac{S}{V}} \sim \Delta_B[1 - C\frac{S}{V}] \tag{11}$$

$$C = \frac{1}{v_F^B g_F^B}\frac{8}{\pi}\frac{1}{k_F^B} \tag{12}$$

a) metallic films and grains



b) Size dependence of gaps in nuclei



1/R = $A^{-1/3}$ dependence!

Digression: new BCP functional (together with L Robledo)
What is it?
Baldo: nuclear matter, neutron matter:

$$E_{pot}^{\infty} = a_1\rho + a_2\rho^2 + ... \tag{13}$$

finite nuclei $\rightarrow$

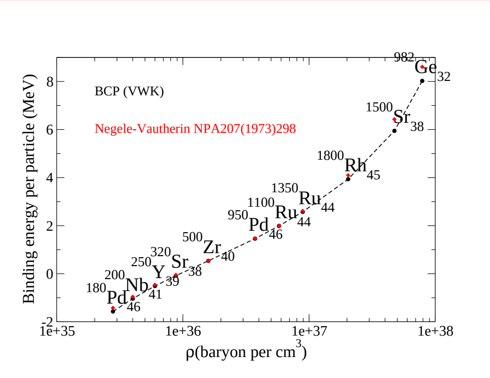
$$a_2\rho^2 \rightarrow \quad \int d^3r \int d^3r' \rho(\mathbf{r}) v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}') \tag{14}$$

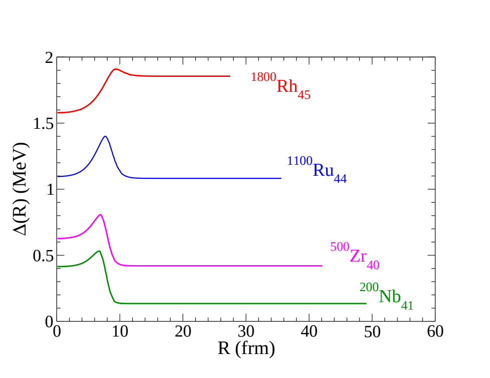
$$v(r) = v_0 e^{(r/r_0)^2} \tag{15}$$

r_0 the only finite range parameter! Surface energy!

Wigner-Seitz cells fully selfconsistently with BCP functional with ETF + TF pairing

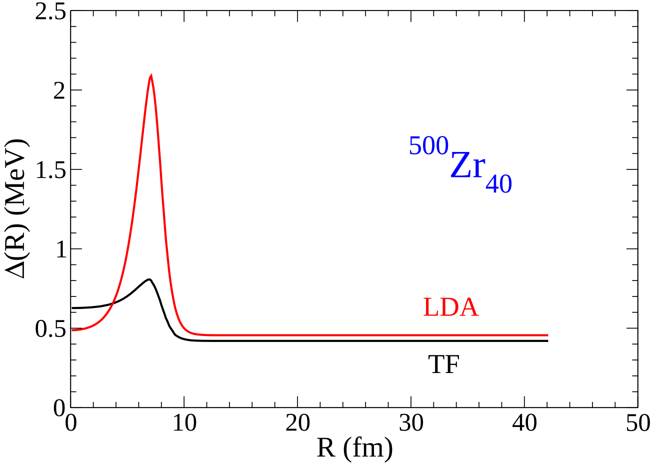
comparison with Negele Vautherin:



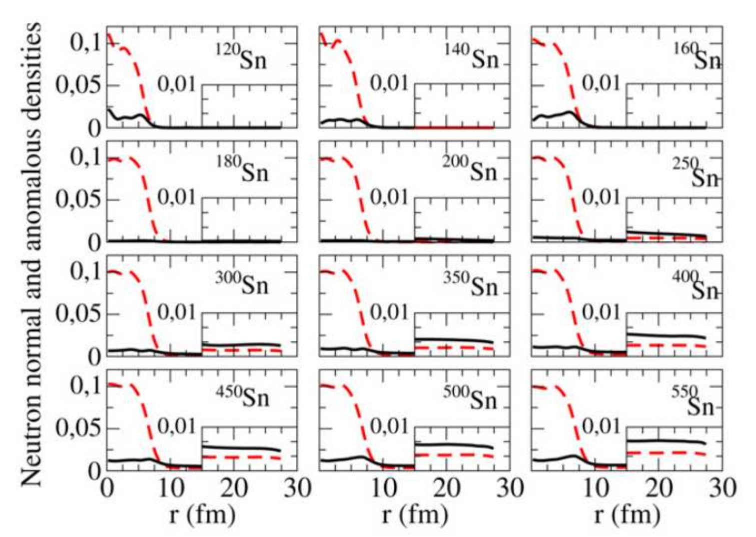


Gap becomes very small also inside cluster at drip! [Grasso, Khan, Margueron, van Giai]

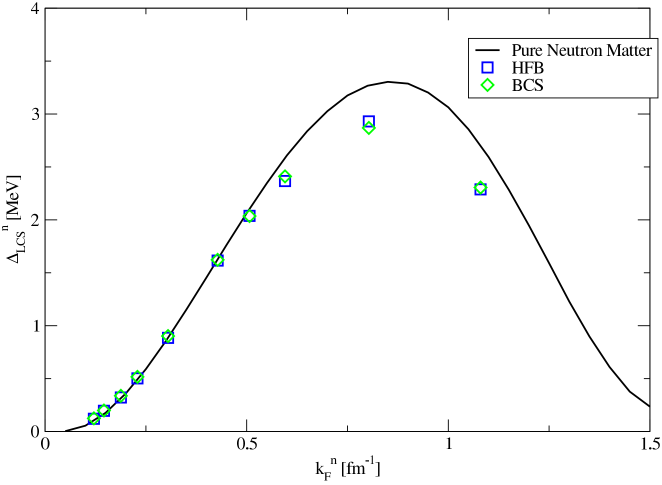
Comparison of LDA and TF for 500Zr



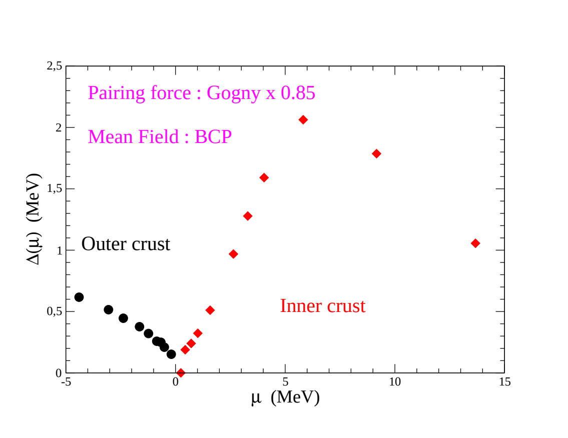
WS cell results by Grasso, Khan, Margueron, van Giai



HFB versus BCS in WS cells: (Alessandro Pastore)

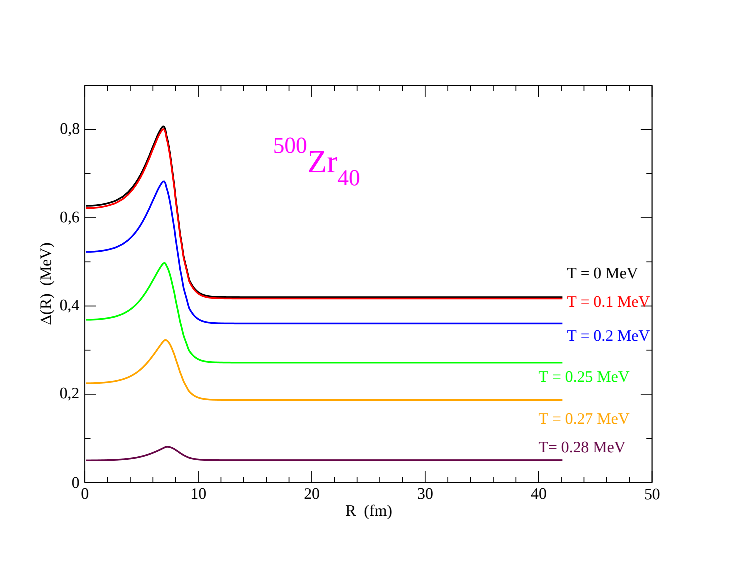


Gaps as a function of μ in WS cells



The dip again corresponds to the overflow (drip) point.

Same investigation can also be performed at finite temperature



gap goes to zero at Tc inside as outside!

Conclusions

Novel TF approach for pairing performs very well.

Drip and overflow situations reduce or kill pairing.

semiclassical treatment of WS cells with BCPM

Futur: TF for HFB

THANK YOU